# PUBLISHED VERSION

http://hdl.handle.net/2440/100080

PHYSICS LETTERS B

CrossMark

# Electromagnetic contribution to charge symmetry violation in parton distributions

X.G. Wang *, A.W. Thomas, R.D. Young

*ARC Centre of Excellence for Particle Physics at the Terascale and CSSM, Department of Physics, University of Adelaide, Adelaide SA 5005, Australia*

A B S T R A C T

We report a calculation of the combined effect of photon radiation and quark mass differences on charge symmetry violation (CSV) in the parton distribution functions of the nucleon. Following a recent suggestion of Martin and Ryskin, the initial photon distribution is calculated in terms of coherent radiation from the proton as a whole, while the effect of the quark mass difference is based on a recent lattice QCD simulation. The distributions are then evolved to a scale at which they can be compared with experiment by including both QCD and QED radiation. Overall, at a scale of 5 GeV$^2$, the total CSV effect on the phenomenologically important difference between the $d$ and $u$-quark distributions is some 20% larger than the value based on quark mass differences alone. In total these sources of CSV account for approximately 40% of the NuTeV anomaly.

## 1. Introduction

Charge symmetry (CS) refers to the invariance of the QCD Hamiltonian under the operator $e^{i\pi I_2}$, a rotation by 180 deg about the 2-axis in iso-space. Under this operation $u$-quarks rotate into $d$-quarks and vice-versa, while protons and neutrons are also interchanged. As a result, if QCD were to respect this symmetry, the up-quark distribution in the proton, $u^p$, and the down quark distribution in the neutron, $d^n$, would be identical. Similarly one would have $d^p = u^n$. Precisely these relations have been almost universally *assumed* for the past 40 years, as without such an assumption it would have been impossible to separate the flavor dependence of the parton distribution functions (PDFs).

Studies of strongly interacting systems have established that CS is typically respected at the level of a fraction of a percent [1], much better than isospin symmetry, which requires the invariance of the Hamiltonian under all rotations in iso-space. Nevertheless, as one uses tests of symmetries to probe for physics beyond the Standard Model, or aims for higher precision at the LHC, it is vital to know just how well the PDFs satisfy CS. Furthermore, subtle tests of such a symmetry can also yield information on how QCD itself works. Thus the study of charge symmetry violation (CSV) in PDFs may also lead to a deeper understanding of the structure of the nucleon itself. For reviews of CSV in PDFs we refer to Londergan et al. [2,3].

There are two dominant sources of CSV in the nucleon, the electromagnetic interaction and the mass differences of the $u$ and $d$ quarks, $\delta m = m_d - m_u$. The first investigations of CSV in the PDFs were based on the effect of $\delta m$ within the MIT bag model [4–6]. These calculations showed CSV violating effects as large as 5% at large-$x$, while they were at the percent level in the momentum fractions:

$$\delta U = \int_0^1 dx\, x \delta u(x) \; ; \; \delta D = \int_0^1 dx\, x \delta d(x) \; , \qquad (1)$$

where the CSV PDFs are $\delta u = u^p - d^n$ and $\delta d = d^p - u^n$. The major effect, which could be understood in terms of the dominant role played by di-quark correlations [7], arose from the mass difference between the $dd$ and $uu$ spectator pairs to the struck $u$-quark in a neutron and $d$-quark in a proton. It was found that $\delta u$ and $\delta d$ had a similar magnitude and opposite sign.

In the context of the NuTeV experiment [8], where these published effects were sufficiently large to reduce the anomaly to $2\sigma$ or less [9,10] considerable work was carried out to establish the extent to which these results were model independent. Recently, lattice QCD studies of these moments [11,12] (although necessarily the charge conjugation positive combination, rather than the valence combination calculated in the bag model) confirmed the sign and magnitude of the pioneering calculations.

The importance of QED radiation on DIS processes was recognised more than 40 years ago in the context of photon radiation from quarks in charged current neutrino interactions [13]. In the

* Corresponding author.
*E-mail address:* xuan-gong.wang@adelaide.edu.au (X.G. Wang).

context of DGLAP evolution of PDFs, Spiesberger [14] summarised the potentially large effects associated with mass singularities involving $\ln(Q_0^2/m_q^2)$, where $m_q$ is a light quark current mass. To avoid such problems he proposed to redefine the PDFs at the starting scale, $Q_0^2$, to include the effects of these singularities. The residual effects of photon radiation are then relatively small and at any given scale, $Q^2$, could be shown to be equivalent to a shift of scale of the PDFs by a charge-dependent factor (analogous to "dynamical rescaling").

In the modern era, Martin et al. [MRST] [15] and Glück et al. [16] sought to improve on the work of Spiesberger by including a photon distribution at the starting scale. In both cases this meant that CS was violated at that scale and both the initial photon distribution and the CSV PDFs were estimated in terms of the large logarithms associated with the mass singularities. Again in the context of the NuTeV experiment, it is important that the sign of the CSV associated with QED radiation was the *same* as that arising from $\delta m$ [16], even though they enter with opposite signs in the neutron–proton mass difference. It is therefore vital [17] to have a consistent treatment of both effects and this is the aim of the present work.

The appearance of current masses in the QED logarithm is at odds with the modern understanding of non-perturbative QCD. At low scales the phenomenon of spontaneous chiral symmetry breaking [18] means that what naturally appears is a constituent quark mass, rather than the current quark mass. For example, a naive evaluation of the electromagnetic self-energy of a quark including the non-perturbative quark propagator, naturally yields a result proportional to $e_q^2 \alpha M(0)$, where $M(0) \sim 0.4$ GeV [19] and $e_q$ is the charge of the quark in units of the positron charge. Since the quark–photon splitting function is derived by cutting this self-energy diagram, one is rather led to a correction to the PDFs at the starting scale of order $\ln(Q_0^2/M(0)^2)$, which is necessarily much smaller than proposed in Refs. [15,16].

Very recently Martin and Ryskin [20] re-examined the issue of the initial photon distribution. They observed that at the low scale $Q_0^2$ the major part of the input photon distribution "comes from the *coherent* emission of the photon from the 'elastic' proton". In the present work we use this insight to make a new and more consistent calculation of the electromagnetic contributions to CSV. Since the scale at which typical, valence dominated quark models like NJL are matched to QCD is somewhat lower that used in Refs. [15,16], we modify the initial quark distribution and generate the initial photon distribution for the coherent radiation from the elastic proton. The evolution from that scale to a typical scale at which one might compare with experiment, say 5 GeV$^2$, then includes incoherent radiation of both gluons and photons from the quarks. In comparison with simply adding the effect of the quark mass difference to the original estimates of QED radiation by MRST and Glück et al., the extent of CSV on the $u$-quarks increases a little, while that for the $d$-quarks decreases in magnitude. Overall, at a scale of 5 GeV$^2$, the total CSV effect on the difference between the $d$ and $u$-quark distributions, which is the combination relevant to the NuTeV experiment, is some 20% larger than the value based on quark mass differences alone.

## 2. Quark distribution functions

The dynamical parton distributions, generated radiatively from valence-like inputs at low scales, are determined from global fit by Glück, Reya and Vogt (GRV) [21], taking into account small-$x$ data on deep inelastic and other hard scattering processes. The leading order (LO) input distributions of proton at $Q_0^2 = \mu_{LO}^2 = 0.26$ GeV$^2$ are then given by

$$xu_v(x, \mu_{LO}^2) = 1.239 x^{0.48}(1-x)^{2.72}$$

$$\times (1 - 1.8\sqrt{x} + 9.5x)$$

$$xd_v(x, \mu_{LO}^2) = 0.614(1-x)^{0.9} xu_v(x, \mu_{LO}^2)$$

$$x\Delta(x, \mu_{LO}^2) = 0.23 x^{0.48}(1-x)^{11.3}$$

$$\times (1 - 12.0\sqrt{x} + 50.9x)$$

$$x(\bar{u}+\bar{d})(x, \mu_{LO}^2) = 1.52 x^{0.15}(1-x)^{9.1}$$

$$\times (1 - 3.6\sqrt{x} + 7.8x)$$

$$xg(x, \mu_{LO}^2) = 17.47 x^{1.6}(1-x)^{3.8}$$

$$xs(x, \mu_{LO}^2) = x\bar{s}(x, \mu_{LO}^2) = 0 , \tag{2}$$

where $\Delta \equiv \bar{d} - \bar{u}$. The corresponding next-to-leading order (NLO) input at $Q_0^2 = \mu_{NLO}^2 = 0.40$ GeV$^2$ is

$$xu_v(x, \mu_{NLO}^2) = 0.632 x^{0.43}(1-x)^{3.09}(1 + 18.2x)$$

$$xd_v(x, \mu_{NLO}^2) = 0.624(1-x)^{1.0} xu_v(x, \mu_{NLO}^2)$$

$$x\Delta(x, \mu_{NLO}^2) = 0.20 x^{0.43}(1-x)^{12.4}$$

$$\times (1 - 13.3\sqrt{x} + 60.0x)$$

$$x(\bar{u}+\bar{d})(x, \mu_{NLO}^2) = 1.24 x^{0.20}(1-x)^{8.5}$$

$$\times (1 - 2.3\sqrt{x} + 5.7x)$$

$$xg(x, \mu_{NLO}^2) = 20.80 x^{1.6}(1-x)^{4.1}$$

$$xs(x, \mu_{NLO}^2) = x\bar{s}(x, \mu_{NLO}^2) = 0. \tag{3}$$

## 3. Photon distribution functions

The additional contribution to the valence quark charge asymmetries arises from radiative QED effects. The so-called DGLAP evolution equations are modified by introducing the photon PDF, $\gamma^N(x, Q^2)$. Following Martin and Ryskin, as shown in Ref. [20], the major part of the input photon PDF, $\gamma^p(x, Q_0^2)$, comes from the *coherent* emission of the photon from the elastic proton. Below the model scale, $Q_0^2$, we assume that the contribution from *incoherent* emission of photons from quarks within the nucleon is negligible. Above the model scale we utilise the APFEL program [22] to perform combined LO/NLO QCD and LO QED evolution in the variable–flavor–number scheme (VFNS). That is, in that region both QCD and QED radiation is treated as incoherent radiation from the quarks.

The coherent emission from the proton is given by [20]

$$\gamma_{coh}^p(x, Q_0^2) = \frac{\alpha}{2\pi} \frac{1+(1-x)^2}{x} \int_0^{|t|<Q_0^2} dq_t^2 \frac{q_t^2}{(q_t^2 + x^2 m_p^2)^2} F_1^2(t) , \tag{4}$$

where $q_t$ is the transverse momentum of the emitted photon and

$$t = -\frac{q_t^2 + x^2 m_p^2}{1-x} . \tag{5}$$

$F_1$ is the Dirac form factor of proton. Letting $Q^2 = -t$, $F_1(Q^2)$ is given by

$$F_1(Q^2) = \frac{4M_p^2 G_E^p(Q^2) + Q^2 G_M^p(Q^2)}{Q^2 + 4M_p^2} , \tag{6}$$

with a dipole parametrisation for the electric and magnetic form factors,

$$G_E^p(Q^2) = \frac{G_M^p(Q^2)}{\mu_p} = \frac{1}{(1 + \frac{Q^2}{\Lambda^2})^2} , \tag{7}$$

where $\mu_p = 2.793$ and $\Lambda^2 = 0.71$ GeV$^2$.

**Table 1**
The momentum fraction carried by the photon and the corresponding $\beta$ parameters.

| $Q_0^2$ (GeV$^2$) | $p_\gamma(Q_0^2)$ | $\beta^u$ | $\beta^d$ |
|---|---|---|---|
| 0.26 | 0.00105 | 0.0584 | 0.0292 |
| 0.40 | 0.00113 | 0.0614 | 0.0307 |

For the neutron, we neglect the small $F_1$ form factor of the neutron and set

$$\gamma_{coh}^n(x, Q_0^2) = 0 \,. \tag{8}$$

The proton momentum fraction carried by the photon is

$$p_\gamma(Q_0^2) = \int_0^1 x\gamma_{coh}^p(x, Q_0^2)dx \,. \tag{9}$$

Correspondingly, the initial distribution functions of the valence quarks in proton should be modified as

$$u_v^p(x, Q_0^2) = \left[u_v^p(x, Q_0^2)\right]_{GRV} - \beta^u f(x, Q_0^2) \,,$$
$$d_v^p(x, Q_0^2) = \left[d_v^p(x, Q_0^2)\right]_{GRV} - \beta^d f(x, Q_0^2) \,, \tag{10}$$

where $f(x, Q^2)$ at $Q^2 = 1$ GeV$^2$ is taken from Ref. [23],

$$f(x, Q^2 = 1\,\text{GeV}^2) = x^{-0.5}(x - 0.0909)(1 - x)^4 \,. \tag{11}$$

The quantity $f(x)$ is chosen since its $x$ dependence has roughly the same form as the MRST initial valence quark parton distribution functions in both the limit $x \to 0$ and $x \to 1$ at $Q^2 = 1$ GeV$^2$. The first moment of $f(x)$ is fixed to be zero, in agreement with the valence quark normalisation. $f(x, Q_0^2)$ at $Q_0^2 = 0.26$ GeV$^2$ and $Q_0^2 = 0.40$ GeV$^2$ are obtained by LO and NLO QCD evolution from $Q^2 = 1$ GeV$^2$, respectively.

The coefficients $\beta^u$ and $\beta^d$ are determined by assuming that the momentum loss of the valence $u$ and $d$ quarks in the proton are $\frac{2}{3}p_\gamma$ and $\frac{1}{3}p_\gamma$, respectively,

$$\beta^u \int_0^1 dx\, xf(x, Q_0^2) = 2\beta^d \int_0^1 dx\, xf(x, Q_0^2) = \frac{2}{3}p_\gamma(Q_0^2) \,. \tag{12}$$

The momentum fraction carried by the photon and the corresponding $\beta$ parameters are shown in Table 1.

## 4. Results

In each case, we evolve to the final scales $Q^2 = 4$ GeV$^2$, 10 GeV$^2$ and 20 GeV$^2$. The pure QED contributions to the isospin-violating majority and minority valence distributions, $x\delta u_v$ and $x\delta d_v$, respectively, are shown in Fig. 1. The corresponding contributions to the second moments of $\delta u_v$ and $\delta d_v$ are given in Table 2.

At the initial scale, both $\delta U_v$ and $\delta D_v$ are negative in the valence region. At higher scales, $\delta U_v$ will decrease and therefore always remain negative. The sign of $\delta D_v$ depends on the final scale $Q^2$. There exists a critical scale, $Q_c^2$, above which $\delta D_v$ will be positive. At $Q^2 = 10$ GeV$^2$, which is appropriate for the NuTeV experiment, the QED contributions to the second moments for both the $u$ and $d$ quarks are significantly smaller than the predictions of Refs. [15] and [16].

At $Q^2 = 4$ GeV$^2$, the QCD contributions to the second moments are derived in Ref. [24] by extrapolating the first lattice simulations [11] to the physical point,

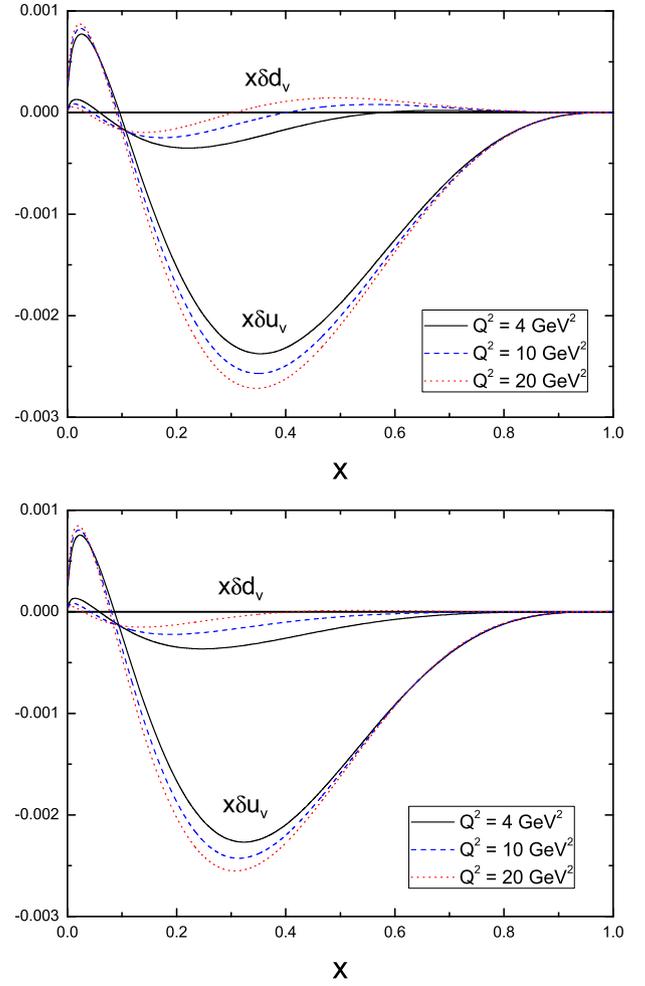$$\delta U_v = -0.0023(7) \,, \quad \delta D_v = 0.0017(4) \,, \tag{13}$$



**Fig. 1.** (Colour online.) The pure QED contributions to isospin-violating majority $x\delta u_v$ and minority $x\delta d_v$ valence parton distributions at $Q^2 = 4$ GeV$^2$, 10 GeV$^2$ and 20 GeV$^2$. (Upper) LO QCD evolution from LO initial distributions at $Q_0^2 = 0.26$ GeV$^2$; (Lower) NLO QCD evolution from NLO initial distributions at $Q_0^2 = 0.40$ GeV$^2$. In both cases, the QED evolution is of LO in $\alpha$.

where the number in brackets indicates the error in the last significant figure. These values are in good agreement with previous phenomenological estimates of CSV, both those calculated within the MIT bag model [5,10] and those found in the MRST analysis [23]. Using the simplest phenomenological parametrisation

$$\delta q_v(x, Q^2) = \kappa_q f(x, Q^2) \,, \tag{14}$$

where $f(x, Q^2)$ is obtained by NLO QCD evolution from $Q_0^2 = 1$ GeV$^2$, and the normalisation factors are determined by taking the constraint, Eq. (13),

$$\kappa_u = -0.26(8) \,, \quad \kappa_d = 0.19(4) \,. \tag{15}$$

Combining with the pure QED contributions, given by the lower plot of Fig. 1, we show the total isospin violating distributions at $Q^2 = 4$ GeV$^2$ and 10 GeV$^2$ in Fig. 2. Here we see that the influence of QED has only a small effect on the down-quark (or minority quark) CSV. For the up (or majority) quark, we see that the QED effects enhance the overall magnitude of the quark-mass induced CSV.

Incorporating this new determination of partonic CSV, including the effect of QED, we revisit the total effect of CSV in the extraction of $\sin^2 \theta_W$ by the NuTeV Collaboration. The total correction, $\Delta s_W^2$, to $s_W^2$ arising from charge symmetry violation may be calculated

**Table 2**

The pure QED contributions to the second moments of $\delta u_v$ and $\delta d_v$ at $Q^2 = 4$ GeV$^2$, 10 GeV$^2$ and 20 GeV$^2$. The photon and valence quark distribution functions at the initial scale are given by Eqs. (4), (8) and (10), respectively.

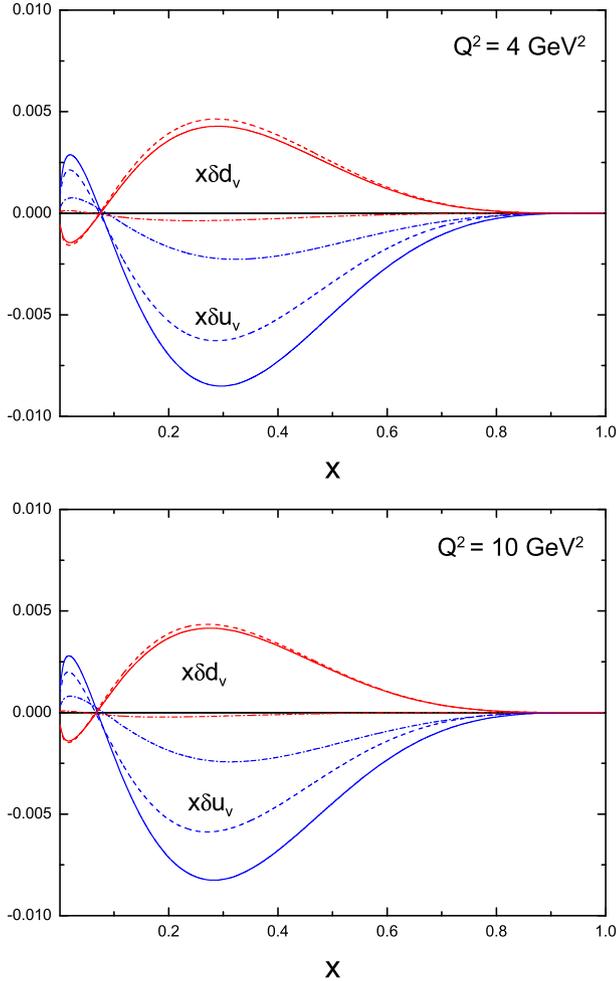| $Q_0^2$ (GeV$^2$) | $Q^2 = 4$ GeV$^2$ | | $Q^2 = 10$ GeV$^2$ | | $Q^2 = 20$ GeV$^2$ | |
|---|---|---|---|---|---|---|
| | $\delta U_v$ | $\delta D_v$ | $\delta U_v$ | $\delta D_v$ | $\delta U_v$ | $\delta D_v$ |
| 0.26 | −0.00099 | −0.00009 | −0.00107 | −0.00003 | −0.00113 | 0.00001 |
| 0.40 | −0.00089 | −0.00013 | −0.00095 | −0.00007 | −0.00099 | −0.00003 |



**Fig. 2.** (Colour online.) The isospin-violating majority $x\delta u_v$ and minority $x\delta d_v$ valence parton distributions at $Q^2 = 4$ GeV$^2$ and 10 GeV$^2$. Dash-dotted, dashed and solid curves represent pure QED, pure QCD and the total contributions, respectively.

**Table 3**

The QED and QCD corrections to $\Delta s_W^2$ arising from valence quark charge symmetry violation.

| $\Delta s_W^2$ | $\delta u_v$ | $\delta d_v$ | Total |
|---|---|---|---|
| QED | −0.00043(6) | 0.00004(2) | −0.00039(6) |
| QCD | −0.00102(31) | −0.00074(17) | −0.00176(35) |

significantly improved estimate of the uncertainty associated with the QED contribution.

## 5. Conclusion

In summary, we have revisited the electromagnetic contribution to charge symmetry violation (CSV) in the parton distribution functions of the nucleon, which contributes the largest uncertainty associated with the CSV correction to the NuTeV anomaly. At very low $Q^2$ we treat the radiation of photons from the nucleon coherently, following the suggestion of Martin and Ryskin [20], while above the scale typically associated with valence dominated quark models the photon emission is associated with the individual quarks, through QED evolution [22]. The resulting electromagnetic contribution to the combination of second moments relevant to the NuTeV anomaly, namely $\delta D_v - \delta U_v$ is of order 0.0010 (at 10 GeV$^2$). When used with the NuTeV functional this yields a correction of less than 10% of the NuTeV anomaly. Adding the latest lattice QCD estimate of this moment [24], which is consistent with the older model dependent calculations [4–6], results in a total CSV correction to $\Delta s_W^2$ of $-0.0022 \pm 0.0004$, which constitutes a reduction in the anomaly of more than 40%. If one were to add the isovector EMC from Ref. [26], the total correction would be $-0.0041 \pm 0.0007$ and comparing with the quoted anomaly, $-0.0050 \pm 0.0016$, the discrepancy with the Standard Model appears to be resolved. The major remaining issue is the potential asymmetry between the $s$ and $\bar{s}$ distributions [27–30] and resolving that issue is now of even greater importance.

using the very convenient functional, $F[s_W^2, \delta q; x]$, provided by the collaboration [25]

$$\Delta s_W^2 = \int_0^1 F[s_W^2, \delta q; x] x \delta q(x, Q^2) dx \qquad (16)$$

at the central value $Q^2 = 10$ GeV$^2$. The individual contributions to $\Delta s_W^2$ are listed in Table 3. Therefore, the total correction arising from valence quark charge symmetry violation becomes

$$\Delta s_W^2 |_{total} = \Delta s_W^2 |_{QED} + \Delta s_W^2 |_{QCD} = -0.0022 \pm 0.0004, \qquad (17)$$

where the error is calculated by combining the errors on the individual contributions in quadrature. For the electromagnetic contribution the errors are taken as the differences between matching at $\mu_{LO}^2$ and $\mu_{NLO}^2$, while for the quark mass contribution the errors arise from Eq. (15). This value is consistent with that reported by Bentz et al. [17], namely $\Delta s_W^2 = -0.0026 \pm 0.0011$, but now with a

## References

[1] G.A. Miller, B.M.K. Nefkens, I. Slaus, Phys. Rep. 194 (1990) 1.
[2] J.T. Londergan, J.C. Peng, A.W. Thomas, Rev. Mod. Phys. 82 (2010) 2009.
[3] J.T. Londergan, A.W. Thomas, Prog. Part. Nucl. Phys. 41 (1998) 49.
[4] E. Sather, Phys. Lett. B 274 (1992) 433.
[5] E.N. Rodionov, A.W. Thomas, J.T. Londergan, Mod. Phys. Lett. A 9 (1994) 1799.
[6] J.T. Londergan, G.T. Carvey, G.Q. Liu, E.N. Rodionov, A.W. Thomas, Phys. Lett. B 340 (1994) 115.
[7] F.E. Close, A.W. Thomas, Phys. Lett. B 212 (1988) 227.
[8] G.P. Zeller, et al., NuTeV Collaboration, Phys. Rev. Lett. 88 (2002) 091802;
G.P. Zeller, et al., Phys. Rev. Lett. 90 (2003) 239902 (Erratum).

[9] J.T. Londergan, A.W. Thomas, Phys. Rev. D 67 (2003) 111901.
[10] J.T. Londergan, A.W. Thomas, Phys. Lett. B 558 (2003) 132.
[11] R. Horsley, Y. Nakamura, D. Pleiter, P.E.L. Rakow, G. Schierholz, H. Stuben, A.W. Thomas, F. Winter, et al., Phys. Rev. D 83 (2011) 051501.
[12] I.C. Cloët, R. Horsley, J.T. Londergan, Y. Nakamura, D. Pleiter, P.E.L. Rakow, G. Schierholz, H. Stuben, et al., Phys. Lett. B 714 (2012) 97.
[13] J.E. Kiskis, Phys. Rev. D 8 (1973) 2129.
[14] H. Spiesberger, Phys. Rev. D 52 (1995) 4936.
[15] A.D. Martin, R.G. Roberts, W.J. Stirling, R.S. Thorne, Eur. Phys. J. C 39 (2005) 155.
[16] M. Glück, P. Jimenez-Delgado, E. Reya, Phys. Rev. Lett. 95 (2005) 022002.
[17] W. Bentz, I.C. Cloët, J.T. Londergan, A.W. Thomas, Phys. Lett. B 693 (2010) 462.
[18] A.W. Thomas, W. Weise, The Structure of the Nucleon, Wiley–VCH, Berlin, Germany, 2001, 389 pp.
[19] A. Bashir, L. Chang, I.C. Cloët, B. El-Bennich, Y.X. Liu, C.D. Roberts, P.C. Tandy, Commun. Theor. Phys. 58 (2012) 79.
[20] A.D. Martin, M.G. Ryskin, Eur. Phys. J. C 74 (2014) 3040.
[21] M. Glück, E. Reya, A. Vogt, Eur. Phys. J. C 5 (1998) 461.
[22] V. Bertone, S. Carrazza, J. Rojo, Comput. Phys. Commun. 185 (2014) 1647.
[23] A.D. Martin, R.G. Roberts, W.J. Stirling, R.S. Thorne, Eur. Phys. J. C 35 (2003) 325.
[24] P.E. Shanahan, A.W. Thomas, R.D. Young, Phys. Rev. D 87 (2013) 094515.
[25] G.P. Zeller, et al., Phys. Rev. D 65 (2002) 111103(R);
G.P. Zeller, et al., Phys. Rev. D 67 (2003) 119902 (Erratum).
[26] I.C. Cloet, W. Bentz, A.W. Thomas, Phys. Rev. Lett. 102 (2009) 252301.
[27] A.I. Signal, A.W. Thomas, Phys. Lett. B 191 (1987) 205.
[28] W. Melnitchouk, M. Malheiro, Phys. Lett. B 451 (1999) 224.
[29] M. Traini, Phys. Lett. B 707 (2012) 523.
[30] D. Mason, et al., NuTeV Collaboration, Phys. Rev. Lett. 99 (2007) 192001.