



DOCTORAL THESIS

Robust Rotation Search in Computer Vision

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*A thesis submitted in fulfilment of the requirements
for the degree of Doctor of Philosophy*

in the

Faculty of Engineering, Computer and Mathematical Sciences
School of Computer Science

August 2016

Declaration

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Abstract

School of Computer Science

Doctor of Philosophy

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by Álvaro Joaquín PARRA BUSTOS

Rotation search is a fundamental problem which has significant importance in geometric computer vision. In many practical settings, data or measurements for rotation search are usually contaminated with large errors, leading to the existence of *outliers* in the data. As a consequence, traditional least-squares rotation estimation methods are not suitable in many practical applications. A more appropriate approach is to search for the rotation based on a robust criterion. However, optimisation problems involving robust criteria are hard to solve since the objective functions are usually non-differentiable and non-convex.

This thesis makes several fundamental contributions in robust rotation search. In contrast to approximations or local methods that are typically used by current practitioners, the presented methods in this thesis guarantee global optimality. The main challenge for robust rotation search algorithms is to find an optimal result in reasonable time (to be practical in out-of-lab applications). The work in this thesis is a contribution in this direction.

To efficiently solve robust rotation search, several strategies are presented based on new insights into the geometry of rotations, from the perspective of global optimisation. Firstly, for point set registration on horizontally levelled data, the presented algorithms make it possible to globally find the best rotation in *real-time*. Secondly, for the fully unconstrained 3D rotation search problem, the presented algorithms outperform previous methods by an order of magnitude. The final contribution of this thesis is an algorithm to safely remove true outliers when rotation is computed on outlier contaminated point correspondences. Substantial speed-up can be obtained when the proposed outlier removal is used as a preprocessor to globally optimal algorithms. Since no inliers are discarded, global optimality is guaranteed.

The contributions in this thesis can impact on computer vision problems where rotation search is invoked as a subroutine. This thesis presents examples from 3D point cloud registration and image stitching.

Acknowledgements

I am pleased to thank to my supervisor, Dr. Tat-Jun Chin, for all his advice, comments, and critical revisions throughout this thesis and the articles we published during my PhD. I really appreciate his interest in this research. Working with him has certainly been an enriching life experience. I would also like to thank to my co-supervisor Prof. David Suter for his advice, revisions and comments during research meetings. I extend this thanks to Anders Eriksson for his valuable comments on ideas and works that are part of this thesis.

I would also like to thank my Master's supervisor Dr. Julián Ortiz for his constant advice.

I would like to express my sincere appreciation to my parents for all they support and encouragement. I extend this thanks to my tata and tía Maruja for all the unconditional support. I want to thanks to Carola Cardoso for her help, company and encouragement.

I am also grateful to family and friends that visited me and shared amazing moments. A big thank to Pamela Cordero and Nicolás Seitz.

During these years I have shared with incredible people. I am very grateful to Exequiel Sepúlveda and his family for their constant support. A big thank also to Andrés Figueroa and Paula Núñez for the shared moments, trips and help.

I also would like to extend thanks to Quoc-Huy Tran, Trung T. Pham, and Roberto Shinmoto for the general advice and encouragement, and to Russell Disher for his hospitality. Many thanks to Sergio Palacio, Carl Vail, Greg Rowlands and Kingsley Denton for their help.

Last I would like to thank CONICYT Becas-Chile for founding my PhD.

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Abbreviations

BnB	B ran ch and B ound
CSM	C onsensus S et M aximisation
DoF	D egrees of F reedom
GM	G eometric M atching
LS	L east S quares
SLAM	S imultaneous L ocalisation A nd M apping

Publications

This thesis is in part result of the work presented in the following papers:

- Álvaro Parra Bustos, Tat-Jun Chin, Anders Eriksson, Hongdong Li and David Suter: Fast rotation search with stereographic projections for 3D registration. IEEE Transactions on Pattern Analysis and Machine Intelligence. Accepted on 23 Dec 2015.
(DOI: [10.1109/TPAMI.2016.2517636](https://doi.org/10.1109/TPAMI.2016.2517636))
- Álvaro Parra Bustos and Tat-Jun Chin: Guaranteed Outlier Removal for Rotation Search. In International Conference on Computer Vision (ICCV) 2015: 2165-2173
- Álvaro Parra Bustos, Tat-Jun Chin and David Suter: Fast rotation search with stereographic projections for 3D registration. In Computer Vision and Pattern Recognition (CVPR) 2014: 3930-3937
(DOI: [10.1109/CVPR.2014.502](https://doi.org/10.1109/CVPR.2014.502))
- Tat-Jun Chin, Álvaro Parra Bustos, Michael Brown and David Suter: Fast rotation search for real-time interactive point cloud registration. In ACM SIGGRAPH Symposium on Interactive 3D Graphics and Games (I3D) 2014: 55-62
(DOI: [10.1145/2556700.2556712](https://doi.org/10.1145/2556700.2556712))

For my parents.