



# Time Dependent Flexural Analysis of Reinforced Concrete Members

Prepared by

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# ABSTRACT

Concrete is one of the most widely used building materials in the construction industry in the world. Time dependent behaviour of concrete is the major concern for the structural engineers due to its significant effect in the long term serviceability and durability. Reinforced concrete (RC) members are prone to the effect of time dependent deformations that are known as shrinkage and creep, can produce substantial deformations and deflections to the structure.

The mechanics of quantifying the serviceability deflection of RC beams is complex due to flexural cracking and the associated partial interaction (PI) behaviour of slip between the reinforcement and adjacent concrete. Add the additional complexity of time dependent concrete shrinkage to this partial-interaction (PI) behaviour and the problem becomes very complex.

Current design and analysis techniques to quantify serviceability deflection of reinforced concrete (RC) members are generally built on two major principles which are full interaction (FI) through the use of moment curvature approaches; and a uniform longitudinal shrinkage strain  $\epsilon_{sh}$  within the member to simplify the analysis technique. Both of the premises are gross approximations and with regard to the first premise, RC beams are subject to flexural cracking and the associated partial interaction (PI) behaviour of slip between the reinforcement and adjacent concrete. Furthermore with regard to the second premise, numerous tests have shown that  $\epsilon_{sh}$  varies along both the depth and width of the beam and which is far from uniform. Hence there are two major sources of error in the quantification of serviceability deflections of RC beams for design and which are due to the PI mechanisms that occur in practice; and that due to the time dependent material properties of creep and shrinkage.

This thesis deals with the development of PI numerical mechanics models with non-linear shrinkage strain variations achieved from a moisture diffusion model developed in this study and that is required to simulate the PI behaviour of RC beams in order to considerably reduce the source of error occurred due to the application of numerical mechanics model. Hence this new mechanics model will allow: the development of better design mechanics rules for serviceability deflection; and also assist in the better quantification of non-linear shrinkage and creep by removing or considerably reducing

the existing mechanics source of error. Importantly, this research provides mechanics solutions for all the facets that control the serviceability time dependent behaviour of RC beams and it is envisaged that these numerical mechanics solutions can provide researchers with the tools to develop simple design procedures as they simulate the major mechanisms influencing cracking and tension stiffening in reinforced concrete beams. Current shrinkage test methodology is having some limitations that are all surfaces are exposed to the environment and they are small scaled which leads to a uniformity of shrinkage strain and which are not present in real size RC beams. Therefore in this thesis, a new form of experimental setup for shrinkage have been proposed to better quantify the shrinkage variations along both the width and depth of RC members with varying the sizes and surface boundary conditions.

# STATEMENT OF ORIGINALITY

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## **List of Publication**

Based on the research work one journal paper has been submitted for publication in Proceedings of the Institution of Civil Engineers – Structures and Buildings.

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# Table of Contents

ABSTRACT.....	I
STATEMENT OF ORIGINALITY.....	III
ACKNOWLEDGEMENT.....	IV
LIST OF PUBLICATIONS.....	V
LIST OF FIGURES.....	IX
LIST OF TABLES.....	XIV

## **Chapter 1 Introduction..... 1**

1.1 Introduction.....	1
1.2 Scope of the Research.....	6
1.3 Aims and Objectives of the Research.....	6
1.4 Structure of the Thesis.....	7

## **Chapter 2 Literature Review ..... 9**

Introduction.....	9
2.1 Shrinkage and its types.....	9
2.2 Effect of Shrinkage and Creep on Structures.....	12
2.3 Prediction of Shrinkage Strains.....	14
2.4 Using Moisture Diffusion to quantify Shrinkage.....	17
2.4.1 Moisture and Humidity Diffusion in Concrete.....	18
2.4.2 Shrinkage strain in Concrete.....	22
2.4.3 Correlations in between with Moisture loss, Humidity and Shrinkage strain of Concrete.....	25
2.5 Member Behaviour.....	37
2.5.1 Models to predict long term Deflections.....	37
2.6 Summary.....	41

<b>Chapter 3 Journal Paper on Non-linear Shrinkage .....</b>	<b>43</b>
<b>Chapter 4 Simulating Shrinkage Strain using Moisture Diffusion.....</b>	<b>84</b>
4.1 Introduction.....	84
4.2 Moisture diffusion equation.....	84
4.3 Moisture diffusion coefficient .....	86
4.4 Finite difference method on moisture diffusion analysis .....	87
4.5 Relationship between pore relative humidity and free shrinkage strain of concrete .....	88
4.6 Quantification of moisture diffusion coefficient .....	90
4.7 Four way flow in rectangular beam.....	92
4.8 Three way flow in a beam .....	94
4.9 Two way flow in a beam .....	95
4.10 One way flow in a beam .....	96
4.11 First simulation with Asad, Baluch et al. (1997).....	98
4.12 Second simulation with Jafarifar (2012).....	100
4.13 Third simulation with Kim and Lee (1999).....	105
<b>Chapter 5 Proposed Experimental Work.....</b>	<b>114</b>
5.1 Introduction.....	114
5.2 Purpose of tests .....	114
5.3 Sizes of specimen.....	115
5.4 Testing for material properties.....	116
5.5 Standard shrinkage test .....	116
5.6 Instrumentation in details.....	116
5.7 Concluding remarks .....	130



## **Chapter 6 Long term Beam Deflection using Segmental Approach... 131**

6.1	Introduction.....	131
6.2	Partial interaction segmental analysis.....	132
6.3	Prior to cracking segmental analysis .....	133
6.4	Accommodation of cracking in the segmental approach.....	135
6.5	Partial interaction tension stiffening model.....	137
6.6	Partial-interaction segmental model .....	143
6.7	Constant longitudinal shrinkage along depth and width.....	144
6.8	Variation in longitudinal shrinkage strain along depth.....	147
6.9	Variation in longitudinal shrinkage strain along depth and width.....	149
6.10	Parametric study .....	151
6.11	Application to test specimens .....	154

## **Chapter 7 Conclusions and Recommendations..... 158**

<b>References.....</b>	<b>161</b>
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## List of Figures

Figure 1.1: Concrete strain component under sustained compressive stress (Gilbert and Ranzi 2011) .....	1
Figure 1.2: Thesis layout of this research.....	7
Figure 2.1: Shrinkage stages and types of concrete .....	10
Figure 2.2: Shrinkage strain components in normal strength concrete (Sakata et al., 2004 cited in Gribniak et al., 2008).....	11
Figure 2.3: Experimental and predicted shrinkage strain in a plain rectangular concrete prism (80 × 150 × 500 mm) using different codes and shrinkage prediction models .....	16
Figure 2.4: Experimental and predicted shrinkage strain in a standard concrete prism (50 × 50 × 300 mm) using different codes and shrinkage prediction models .....	17
Figure 2.5: Numerical moisture profiles compared with the experimental test results: (a) Plain CC mix; (b) Plain RCC mix; (c) SFR-CC mix (2.5%); (d) SFR-RCC mix (2.5%) (Jafarifar 2012).....	19
Figure 2.6: Shrinkage strain variation along the thickness of concrete specimen for two different mixes and comparison with the analytical results (Kim and Lee 1998).....	23
Figure 2.7: Calculated relative humidity compared to modified experimental results due to moisture diffusion only a) curing period = 3 days b) curing period = 28 days (Kim and Lee 1999).....	26
Figure 2.8: Relationship between relative humidity and moisture diffusion on different moist curing period (Kim and Lee 1999) .....	27

Figure 2.9: Loss of moisture in concrete due to drying (Kim and Lee 1999) .....	28
Figure 2.10: Comparisons between experimental and numerical results (ambient temperature 200 C): a) Exposed at 3 days (w/c = 0.28) b) Exposed at 28 days (w/c = 0.28) c) Exposed at 3 days (w/c = 0.40) d) Exposed at 28 days (w/c = 0.40) e) Exposed at 3 days (w/c = 0.68) (Kang et al 2012) .....	29
Figure 2.11: Relationship between ultimate shrinkage and relative humidity for cement paste and mortar specimens (Bissonnette, Pierre et al. 1999) .....	30
Figure 2.12: Relation between shrinkage and weight loss for various types and sizes of paste, mortar and concrete specimens (Bissonnette, Pierre et al. 1999) .....	30
Figure 2.13: Relationship between free shrinkage strain and interior relative humidity (RH) of a) C30 concrete and b) C80 concrete (Zhang, J, Dongwei and Wei 2010) .....	32
Figure 2.14: Calculated free shrinkage strains and measured relative humidity at different depths of the slab from exposed surface against drying period (Zhang, J, Dongwei and Wei 2010) .....	33
Figure 2.15: Relationship between free shrinkage strain and moisture loss (ACI209R-92 1997; Asad, M 1995) .....	34
Figure 2.16: Experimental results of different concrete specimens drying shrinkage as a function of weight loss (Granger, Torrenti and Acker 1997) .....	36
Figure 4.1: Flow chart diagram to perform moisture diffusion modelling process .....	89
Figure 4.2: a) Gilbert and Nejadi (2004) Beam B1a b) Sectional elevation of beam B1a at A-A c) Four way flow in beam B1a d) Three way flow in beam B1a e) Two way flow in beam B1a f) One way flow in beam B1a .....	90

Figure 4.3: Moisture diffusivity vs moisture content or pore relative humidity, $h$ for the beams tested by Gilbert and Nejadi (2004) .....	92
Figure 4.4: Shrinkage strain profile in a four way flow Gilbert and Nejadi (2004) Beam B1a after 100 days, 250 days and 400 days of drying.....	94
Figure 4.5: Shrinkage strain profile in a three way flow Gilbert and Nejadi (2004) Beam B1a after 100 days, 250 days and 400 days of drying.....	95
Figure 4.6: Shrinkage strain profile in a two way flow Gilbert and Nejadi (2004) Beam B1a after 100 days, 250 days and 400 days of drying.....	96
Figure 4.7: Shrinkage strain profile in a one way flow Gilbert and Nejadi (2004) Beam B1a after 100 days, 250 days and 400 days of drying.....	97
Figure 4.8: Relationship between moisture content or pore relative humidity, $h$ and moisture diffusivity $D$ .....	98
Figure 4.9: Numerical simulation of experimental and predicted values of moisture loss at 1 cm from the drying surface using finite difference method .....	100
Figure 4.10: Moisture diffusivity versus pore relative humidity or moisture content for various types of concrete mixes (Jafarifar 2012) .....	101
Figure 4.11: Numerical moisture profiles compared with the experimental results simulated using finite difference method: (a) Plain CC mix; (b) Plain RCC mix; (c) SFR-CC mix; (d) SFR-RCC mix .....	104
Figure 4.12: Relationship between moisture diffusivity and moisture content for three different types of concrete a) H ( $w/c = 0.28$ ) b) M ( $w/c = 0.40$ ) c) L ( $w/c = 0.68$ ) with moist cured for 3 days and d) H ( $w/c = 0.28$ ) e) M ( $w/c = 0.40$ ) f) L ( $w/c = 0.68$ ) with moist cured for 28 days .....	109

Figure 4.13: Numerical simulation of experimental results for three different types of concrete after moist cured for 3 days .....	111
Figure 4.14: Numerical simulation of experimental results for three different types of concrete after moist cured for 28 days .....	113
Figure 5.1: Arrangements of demec gauge points along the depth at front (shown only) and rear surfaces and at both ends through the depth as well as width of the prism for measurements of shrinkage strains and total deformations of the prisms in one direction moisture diffusion process.....	119
Figure 5.2: Arrangements of demec gauge points along the depth at front (shown only) and rear surfaces of the prisms and at both ends through the depth as well as width of the prisms for measurements of shrinkage strains and total deformations of the prisms in two direction moisture diffusion processes .....	121
Figure 5.3: Arrangements of demec gauge points along the depth at front (shown only) and rear surfaces of the prisms and at both ends through the depth as well as width of the prisms for measurements of shrinkage strains along its depth and total deformations of the prisms in three direction moisture diffusion processes .....	123
Figure 5.4: Arrangements of demec gauge points along the width at top (shown only) and bottom surfaces of the prisms and at both ends through the depth as well as width of the prisms for measurements of shrinkage strains along its width and total deformations of the prisms in three direction moisture diffusion processes .....	125
Figure 5.5: Arrangements of demec gauge points along the depth at front (shown only) and rear surfaces of the prisms and at both ends through the depth as well as width of the prisms for measurements of shrinkage strains along its depth and total deformations of the prisms in four direction moisture diffusion processes.....	127

Figure 5.6: Arrangements of demec gauge points along the width at top (shown only) and bottom surfaces of the prisms and at both ends through the depth as well as width of the prisms for measurements of shrinkage strains along its width and total deformations of the prisms in four direction moisture diffusion processes.....	129
Figure 6.1: A standard multi-crack segmental analysis .....	133
Figure 6.2: Separating elements of RC beam (Concrete element) .....	135
Figure 6.3: Separating elements of RC beam (Reinforcement element).....	135
Figure 6.4: Flexural properties ( $M/\theta$ , $M/\chi$ and $M/EI$ ) .....	136
Figure 6.5: Cracked segmental analysis .....	137
Figure 6.6: Tension stiffening prism .....	138
Figure 6.7: Local deformation of $n^{th}$ segment in prism.....	140
Figure 6.8: Tension stiffening analysis .....	141
Figure 6.9: Linear shrinkage strain over width and depth (Concrete element).....	146
Figure 6.10: Linear shrinkage strain over width and depth (Reinforcement element)..	146
Figure 6.11: Non-linear shrinkage strain over depth (Concrete element).....	148
Figure 6.12: Non-linear shrinkage strain over depth (Reinforcement element).....	149
Figure 6.13: Non-linear shrinkage strain over width and depth (Concrete element 2b1) .....	150

Figure 6.14: Non-linear shrinkage strain over width and depth (Concrete element 2b2)	151
Figure 6.15: Non-linear shrinkage strain over width and depth (Reinforcement element)	151
Figure 6.16: Influence of slice number along the width of beam on member deflection	154
Figure 6.17: Predicted deflection of six beams tested by Gilbert and Nejadi (2004) ...	156
Figure 6.18: Influence of exposed surfaces on member deflection.....	157

## **List of Tables**

Table 5.1: Specimen size details with V/S ratios for one up to four direction diffusion processes.....	115
Table 5.2: Test details for material properties.....	116