Effect of 3D Stress States at Crack Front on Deformation, Fracture and Fatigue Phenomena

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Abstract

Theoretical, numerical and experimental studies involving elastic plate components, weakened by through-the-thickness cracks and subjected to loading parallel to the plane of the plate, are often based on plane stress or plane strain simplifications. These simplifications essentially reduce the dimensionality of the physical three-dimensional problem and enable the achievement of effective analytical and numerical solutions for many important practical problems. The influence of various three-dimensional effects, such as the variation of stresses across the plate thickness, effects of the three-dimensional corner (vertex) singularities and coupling of fracture modes II and III, on the deformation and stresses near the crack front are at present largely ignored or viewed as negligible for all practical purposes. As a result of this view, the outcomes of experimental studies and fracture tests are also commonly analysed within the framework of the plane theories of elasticity. Nevertheless, a number of theoretical and experimental studies over the past two decades have demonstrated that the predictions made within these theories can be unsatisfactory and the effect of three-dimensional stress states at the crack front on deformation, fatigue and fracture of plate components can be significant.

This thesis aims to elucidate the role of three-dimensional stress states in the deformation, fracture and fatigue phenomena further. The main outcomes of this thesis are: (1) the development and validation of a simplified method for the evaluation of the fatigue crack front shapes and their effect on the steady-state fatigue crack growth rates in plate components; (2) investigation of the effect of three-dimensional corner (vertex) singularities on the stress intensities and displacement field near the crack front; and (3) development and validation of a new experimental approach for the evaluation of mode I and mode II stress intensity factors from the measurement of the out-of-plane displacements in the near crack tip region, which are affected by three-dimensional effects, and, in particular, by the 3D corner (vertex) singularity.

This new research is important in many engineering contexts. For example, the new theoretical model, which takes into account the actual shape of the crack front, can be utilised in advanced fatigue life calculations, as well as in failure investigations. The latter is possible as the shape of the fatigue crack front can now be related to the parameters of fatigue loading. The new experimental approach developed in this thesis can be useful in fracture characterisation of thick plate components with through-cracks. This approach specifically addresses the situation when the K-dominance zone, or William's solution convergence domain, are relatively small. In this case, the data extraction region can be affected by the three-dimensional stress states leading to significant errors in the evaluation of the stress intensity factors when using traditional approaches.

This thesis is presented in the form of a compendium of published papers that are the summation of the research undertaken by the author. The five articles which form the main body of the thesis are united by a common theme, which is the investigation of three-dimensional effects near the crack front on stresses and displacements, fracture and fatigue phenomena. Two appendices are also included; they represent a compilation of the candidate's publications related to the main topic of the thesis.

Declaration

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List of Publications

Journal publications

1. **He, Z,** Kotousov, A & Branco, R 2014, ‘A simplified method for the evaluation of fatigue crack front shapes under mode I loading’, *International Journal of Fracture*, vol. 188, no. 2, pp. 203-211.

2. **He, Z,** Kotousov, A & Berto, F 2015, ‘Effect of vertex singularities on stress intensities near plate free surfaces’, *Fatigue and Fracture of Engineering Materials and Structures*, vol. 38, no. 7, pp. 860-869.

3. **He, Z,** Kotousov, A, Fanciulli, A, Berto, F & Nguyen, G 2015, ‘On the evaluation of stress intensity factor from displacement field affected by 3D corner singularity’, *International Journal of Solids and Structures*, vol. 78-79, pp. 131-137.

4. **He, Z,** Kotousov, A, Fanciulli, A & Berto, F 2015, ‘An experimental method for evaluating mode II stress intensity factor from near crack tip field’, *International Journal of Fracture*, published online.

5. **He, Z** & Kotousov, A 2015, ‘On evaluation of stress intensity factor from far and near vertex fields’, *Experimental Mechanics*, under review.

6. **He, Z,** Kotousov, A, Berto, F & Branco, R 2015, ‘A brief review of recent three-dimensional studies of brittle fracture’, *Physical Mesomechanics*, accepted: 18 December 2015.

7. Gardeazabal, D, **He, Z** & Kotousov, A 2014, ‘On influence of non-singular stress states on brittle fracture’, *International Journal of Fracture*, vol. 185, no. 1, pp. 201-208.

8. Kotousov, A, **He, Z** & Fanciulli, A 2015, ‘Application of digital image correlation technique for investigation of the displacement and strain fields within a sharp notch’, *Theoretical and Applied Fracture Mechanics*, vol. 79, pp. 51-57.

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1. Kotousov, A, **He, Z** & Gardeazabal, D 2013, ‘On scaling of brittle fracture’, in CH Wang (ed), *Proceedings of the 8th International Conference on Structural Integrity and Fracture*, CD-ROM, Melbourne, Australia, pp. 93-96.

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3. **He, Z** & Kotousov, A 2015, ‘Estimation of stress intensity factors from near crack tip field’, *International Scholarly and Scientific Research and Innovation*, *Proceedings of the 17th International Conference on Applied Mechanics and Mechanical Engineering*, Stockholm, Sweden, pp. 526-530.

4. **He, Z,** Kotousov, A & Branco, R 2015, ‘Evaluation of fatigue crack front shape for a specimen with finite thickness’, *Proceedings of the 4th International Conference on Advances in Mechanics Engineering*, Madrid, Spain.

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Chapter 1

Introduction

1 Introduction

1.1 Fracture Mechanics

In designing a structural component or machine part, it is very important to evaluate the most probable failure mechanisms and critical loading conditions leading to failure, as well as the overall lifetime of the element being designed. In general, such evaluations require a basic understanding of the essential stages of fracture processes, such as defect initiation, propagation, possible interactions and final fracture, which take place across several spatial scales. This multi-scale nature of fracture, in particular, represents a significant challenge for the development of adequate predictive models. The understanding and development of models for various fracture and fatigue stages constitutes the scope of fracture mechanics as an engineering discipline. It can be stated that fracture mechanics is one of the most decisive areas of engineering as “the lives of many persons, and the property of many more, will be saved if the truth of the matter be discovered – lost if it be not” (Biggs 1960).

From a macroscopic point of view, fracture of a solid can be considered as the propagation of a macro-crack leading to the separation of the solid into two or several parts. However, the macro-crack propagation is a result of many physical phenomena at the atomic and microscopic scales, such as generation and emission of dislocations, damage formation and accumulation, nucleation, growth and coalescence of micro-cracks under monotonic or cyclic loading, sometimes in the presence of an aggressive environment. Therefore, fracture mechanics is concerned with a very wide and diverse range of phenomena, which can be roughly categorised into atomic, micro-, meso- and macro-phenomena. Because of the complexity of these interacting phenomena, at the current stage, and probably in the foreseeable future, there will be no commonly accepted theory that can satisfactorily describe fractures of solids and structures (Erdogan 2000).

From a practical point of view, the macroscopic theories are of particular interest, because these theories are capable of providing quantitative evaluations of the integrity and reliability of structural components and machines. The importance of such evaluations would appear to be self-evident. The macroscopic theories are based on the fundamental results of continuum solid mechanics and, in particular, on the linear theory of elasticity. Within this theory, it is assumed that a continuous solid comprises flaws (sufficiently large in comparison with the characteristic size of the micro-structure), which can propagate and lead to fracture. From an engineering perspective, these macroscopic theories provide a link between the applied loading, material properties, environmental conditions, and the geometry of the solids and flaws from one side and fracture initiation and propagation from the other side. This PhD project adopts the macroscopic approach and is largely based on phenomenological theory: Linear Elastic Fracture Mechanics (LEFM), or Brittle Fracture Mechanics. The Literature Review (Chapter 2) will provide key aspects of this theory.

1.2 Brittle Fractures

The current thesis is mainly concerned with brittle fractures, which are the primary concern of engineers and designers. The following two typical examples describe a devastating history of experiences with such types of fractures. These examples support the well-known engineering rule, introduced into the design of large scale-structures after World War II, that any possibility of fracture of a load-bearing structure in a brittle manner must be avoided.

The classic example, which is mentioned in almost all texts, is a fracture of a main chain of the Montrose suspension bridge on the 19th of March, 1830, in Great Britain (Erdogan 2000). On an afternoon that a large crowd of people gathered to watch a boat race, the main chain was suddenly ruptured and three people were instantly killed and many others were injured. There have been many more similar accidents resulting from brittle fractures over the past two hundred years.

A more recent incident, which is also cited frequently in fracture mechanics text books and review papers, relates to the first commercial passenger jet airplane (Comet I), which had a significant impact on the future development of the aerospace industry. On the 10th of January, 1954, a Comet I airplane aircraft disintegrated in the air and subsequently crashed into the Mediterranean Sea. Soon after, on 8 April, 1954, another aircraft of the same type disintegrated in the air and fell into the Mediterranean sea near Naples. In order to determine the cause of the accidents, a water tank was built in which a Comet fuselage was subjected to cyclic pressurisation. The fuselage had been subjected to 1230 pressure cycles in flight. After 1830 cycles of pressurisation-depressurisation, the fuselage burst open in a catastrophic (brittle) manner (Liebowitz 1969).

The above classic examples reveal and highlight the main features of brittle fractures: such fractures normally occur unexpectedly and propagate very fast, so there is no time to react to and confine this kind of event. Brittle fractures are usually associated with large-scale damage and, as a rule, they result in catastrophic consequences, loss of lives and significant damage to property and/or the environment. The ultimate aim of the studies undertaken within this PhD project is to reduce the possibility of such fractures further by developing more accurate predictive models and incorporating important factors, which are normally disregarded in the classic approaches to failure evaluation.

1.3 Historical Development of the Field

The role of fractures in human history cannot be underestimated. It is naturally ‘embedded’ into human life and various human activities, such as building, destroying or surviving. However, the first known theoretical work devoted to fracture was that of Griffith (1921), who analysed the crack propagation in glass fibres. Griffith stated that an existing crack will propagate if the potential energy of the stressed structure or component is decreased. He developed a fracture criterion, which simply indicates that at fracture initiation there must be a balance between (1) the decrease of elastic strain energy of the system due to the crack extension and (2) the energy required to form new crack surfaces. This criterion still forms the foundation of modern theories of brittle fracture.

Zener and Hollomon (1944) were the first who applied the Griffith concept to the brittle fracture of metallic materials. Not long after that, Irwin modified and extended the Griffith-type energy balance criterion, not only to brittle materials but also to materials that exhibit plastic behaviour (Anderson 1995). He recognised that even for materials exhibiting a 'purely brittle' behaviour, there still exists evidence of the accumulation of plastic deformation around the crack tip. Irwin pointed out that the decrease of the stored elastic strain energy must be counteracted by the surface energy plus the dissipative energy (which includes the work of plastic deformations). A similar justification to modify the Griffith's criterion was proposed by Orowan (1949). Furthermore, Irwin and Orowan have found that for ductile materials the energy needed to create new crack surfaces is quite small in comparison with the dissipative energy.

Another contribution by Irwin to the early development of fracture mechanics took place in the middle 1950s. He showed the equivalence between the energy approach and the stress intensity concept. The latter concept states that fracture initiation occurs when the stress intensity factor reaches the critical stress intensity factor (K_C), which is called fracture toughness (Irwin 1958). The theory of LEFM, which is a primary focus of this thesis, was formulated in terms of the stress intensity factor (Ewalds & Wanhill 1984). This is because the basic characteristics of the stress states in the vicinity of a crack tip in a relatively brittle material are always the same. Thus, the determination of fracture toughness (or the critical characteristic) of the material and stress intensity factor for a particular situation makes it possible to evaluate the risk of brittle fracture and loading conditions leading to such fractures.

One important application of LEFM is in fatigue crack growth modelling. In the 1960s, Paris (Paris et al. 1961; Paris & Erdogan 1963) hypothesised that the crack growth rate under fatigue loading can be characterised by the range of stress intensity factors, or the difference between the maximum and minimum stress intensity factors. Forman et al. (1967) and Walker (1970) improved Paris' law by incorporating the effects of different stress ratios. In the early 1970s, Elber (1970, 1971) discovered crack closure and proposed that the effective stress intensity factor range should control crack growth. This concept was further extended by many other researchers (e.g. Kotousov & Codrington 2010).

Other applications of LEFM include stress-corrosion cracking (Newman & Procter 1990), dynamic fracture mechanics (Freund 1998), creep (Nicholson & Formby 1975) and visco-elastic fracture (Anderson 1995). It is important to note that these theoretical developments have had a large impact on industry, which utilised these theoretical developments in design, manufacturing and maintenance of machine and structural components (Cotterell 2002).

Conventional approaches, used prior to 1960 in structure and component design, were mainly based on ultimate tensile strength, the von Mises yield criterion or Tresca yield criterion. Until the 1960s, the 'safe life' criterion was developed and widely accepted in structural design. The purpose of this development was to ensure that the time for initiation of a cracked structure or component should be greater than the designed operational life (Erdogan 2000). The application of this criterion has, to some extent, helped to reduce the occurrence of structural failures of critical components and machine elements. In the 1970s, the so-called 'damage tolerance' method was proposed, which requires that "at any given time, the remaining crack propagation life must be longer than the designed operational life" (FAA Technical Centre 1993). The application of this concept is entirely based on the fracture mechanics approach. Nowadays, fracture mechanics has become an essential tool for engineers and researchers in a number of fields including (1) the design of machine structural or critical components, (2) material selection and alloy development and (3) evaluation of structural defects and mechanical damage (Jackson 1978).

1.4 Objectives of the Research

Classic LEFM is now widely applied to evaluate the failure of materials or structures as long as the material exhibits elastic (or brittle) behaviour under the specified range of loading (Cotterell 2002). Normally, high-strength steels, some plastics, glass, concrete and ice are considered as brittle materials. Ductile materials normally demonstrate this type of fracture at sufficiently low temperatures or at low strain loading rates, resulting in a small plastic or process zone. Within the framework of LEFM, it is commonly assumed that a single value of the stress intensity factor can be used as a sole parameter to characterise brittle fracture, as well as fatigue crack growth (Anderson 1995). LEFM is also widely applied to the evaluation of the fatigue life of structural components, as the large portion of the life time corresponds to relatively small defects. The plastic zone size for such defects remains relatively small, even for highly ductile materials. The latter justifies the application of LEFM to fatigue calculations. The LEFM is currently imbedded into many fatigue evaluation procedures, codes and standards.

Over the past sixty years, a great number of experimental, numerical and analytical techniques have been developed for solving crack problems, specifically for the calculation of the stress intensity factor for various geometries and loading conditions. These include experimental methods of caustics, speckle interferometry, photoelasticity and digital image correlation. Numerical investigations of crack problems have predominantly utilised the Boundary Element or Finite Element method. In addition, numerous analytical methods have also been developed for relatively simple crack geometries. Among these analytical results and methods are the well-known Williams' asymptotic crack tip solutions (Williams 1952, 1957) and Westergaard's exact analytical solutions for an embedded crack in an infinite elastic plate (Westergaard 1939).

Despite the significant progress in fracture mechanics, there are still many questions and general problems that need to be addressed. For example, as was stated in the literature review (see Chapter 2), most of the theoretical results, numerical and experimental studies are currently based on the two-dimensional (2D) framework of the linear theory of elasticity. The current standards and codes for fracture and fatigue evaluation also implement the solutions obtained within plane stress or plane strain assumptions. However, recent extensive studies have demonstrated the importance of three-dimensional (3D) effects in several critical areas (Branco & Antunes 2008; Kotousov et al. 2013; Pook 2013). These areas include fatigue crack growth, which can be affected by a curved crack front shape and fracture initiation, which can be influenced by 3D stress states, in particular, by the coupled modes. Consequently, the overall objective of this PhD project is to elucidate the role of 3D effects in various fatigue and fracture phenomena and, through this new understanding, suggest improvements to the current predictive and experimental techniques. These more accurate techniques are needed to support modern designs and developments in the civil, mechanical, aerospace and chemical industries.

1.5 Details of Publications Included in the Thesis

Five chapters of this thesis represent research articles published or submitted for publication by the candidate in collaboration with his colleagues. Two papers relevant to the topic of the thesis are provided in the appendices. In these papers the candidate had a significant but not a primary role. The thesis is broadly divided into three main parts. The first part (Chapter 3) is devoted to the development and validation of a simplified model for the evaluation of the fatigue crack front shapes and its effect on the steady-state fatigue crack growth rates. The investigation of the effect of 3D corner singularity on stress intensities and the displacement field near the crack front is the focus of the second part (Chapters 4 and 5). The third part (Chapter 5, 6 and 7) is dedicated to the development and verification of a new experimental approach for the evaluation of mode I/II stress intensity factors from the measurement of the out-of-plane displacement field near the crack tip. This is a fully new development, which has never been attempted previously worldwide, and represents one of the main outcomes of this thesis.

These three parts of the thesis are coherent and unified on the investigation of three-dimensional effects near the crack front on stress and displacement fields, fracture and fatigue phenomena. Brief summaries of each individual article are provided below:

Chapter 3: A simplified method for the evaluation of fatigue crack front shapes under mode I loading

In this chapter, a simplified mathematical model for the evaluation of the front shapes of through-the-thickness fatigue cracks is developed. The present model is based on the plasticity-induced crack closure concept, 3D finite element simulations and analytical results obtained by Codrington and Kotousov (2009). These analytical results for the steady state regime of fatigue crack growth in plates of finite thickness, allow for a significant reduction in the complexity of the evaluation procedure. All previous results were obtained using direct numerical simulations (Branco & Antunes 2008; Camas et al. 2012; Ševčík et al. 2012), which are time-consuming, difficult to verify or reproduce and not very suitable for parametric studies. The developed method is validated against the outcomes of experimental studies. A significant part of Chapter 3 is also dedicated to the investigation of the influence of various parameters (e.g. plate thickness and maximum applied load) on the crack front shapes at steady-state propagation, as well as the analysis of the differences in the results of fatigue crack growth evaluation obtained with two- and three-dimensional approaches. One important application of the developed method and the parametric study is in forensic engineering since it is able to evaluate the loading conditions in various failure investigations, based on the good prediction of the evolution of fatigue crack front shapes.

Chapter 4: Effect of vertex singularities on stress intensities near plate free surfaces

An extensive investigation using finite element is conducted for the first time in Chapter 4 to investigate the influence of 3D corner (vertex) singularities on the stress intensity for a straight through-the-thickness crack in sufficiently large elastic plates subjected to mode I/II loading. It is demonstrated that the asymptotic behaviour of stress intensity factor within a very thin layer near the free surface (which occupies 5% of the total plate thickness) is governed by the difference in the strength of the corner and edge singularities. Furthermore, the numerical approach and calculations are verified using the invariant properties of J-integral. The theoretical analysis agrees well with experimental results. This indicates that the brittle fracture is normally initiated in the middle section of the straight crack front if the crack is subjected to mode I loading, and near the free surfaces if it is subjected to mode II loading.

Chapter 5: On the evaluation of stress intensity factor from displacement field affected by 3D corner singularity

Chapter 5 presents a new experimental approach for evaluating mode I stress intensity factor from out-of-plane displacement in the area very close to the crack tip. Extensive numerical studies are conducted to establish a link between the applied stress intensity factor and the displacement field in the area of interest. This displacement field does not follow either plane stress or plane strain assumptions. A simple fitting equation is proposed, which links the value of the out-of-plane displacement with the applied stress intensity factor, Young's modulus, Poisson's ratio, plate thickness and the strength of corner singularity, which affects the displacement field close to the crack front. Further experimental study confirms that the out-of-plane displacement field follows the proposed theoretical solution, in turn forming a foundation for the development of a new experimental procedure.

Chapter 6: An experimental method for evaluating mode II stress intensity factor from near crack tip field

In this chapter, the new experimental approach as presented in Chapter 5 is further extended for evaluating the mode II stress intensity factor. It is found from the careful experimental and numerical studies that the out-of-plane displacement field near the crack tip (for both mode I and II conditions) is not significantly affected by the higher order non-singular terms of crack tip asymptotic expansion. This implies that the accuracy of the determination of stress intensity factor from the near crack tip region could essentially be much better in comparison with the existing experimental techniques, which rely on curve fitting of the leading (singular) and higher order non-singular terms in the asymptotic expansion. One essential advantage of this new experimental approach is that it can be applied to various specimen geometries and loading conditions without changing the theoretical framework.

Chapter 7: On evaluation of stress intensity factor from far and near vertex fields

Chapter 7 compares two different experimental approaches (the newly developed approach as presented in Chapter 5 and the traditional approach which is based on the classic plane stress power series expansion, e.g. Williams' solution) for the evaluation of stress intensity factor for a single edge cracked rectangular plate subjected to symmetric three-point bend (mode I) loading. The effect of the distance of the data points from the crack tip on the accuracy of the evaluation of stress intensity factor from Williams' solution is further investigated. The outcomes of this study indicate that, in order to achieve an accurate evaluation of stress intensity factor based on the classic 2D asymptotic power series expansion, the experimental measurements have to be extracted from outside the 3D region. In contrast, the measurement for the new developed approach can be conducted anywhere in the near crack tip region, as the magnitude of the out-of-plane displacement in this region is essentially constant, which makes this new approach particularly attractive for experimental techniques.

Chapter 8: Conclusions and future work

In this closing chapter, the main research outcomes of the thesis are summarised and the recommendations regarding future work are provided. These recommendations focus on the possible application and immediate extension of the outcomes achieved in the current thesis.

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Chapter 2

Literature Review

2 Literature Review

Nomenclature

$A_1(z)$	Mode I edge stress intensity function
a	Crack length
$B(t)$	Density of the dislocations at point t
E	Young’s modulus
G_w	The displacement kernel
h	Half plate thickness
$K_I(z)$	Local mode I stress intensity factor along the crack front direction
K_I^∞	Remote (applied) mode I stress intensity factor
$K_{II}(z)$	Local mode II stress intensity factor along the crack front direction
K_{II}^∞	Remote (applied) mode II stress intensity factor
$K_n(\cdot)$	Modified Bessel functions of n^{th} order
$K_O(z)$	Local mode O stress intensity factor of the out-of-plane coupled mode
K_λ	Stress intensity factor at the corner point
r, θ, z	Cylindrical polar coordinates with origin at the middle of the crack front
r, θ, ϕ	Spherical coordinates with origin at the corner point
u_z	Out-of-plane displacement
x, y, z	Cartesian coordinates with origin at the middle of the crack front
β	Intersection angle between the crack front and a free surface
λ	Strength of the corner singularity
ν	Poisson’s ratio
σ_{yy}^∞	Remote (applied) tensile stress, which acts perpendicular to the crack front
$\sigma_{\alpha\beta}$	In-plane stress components in polar coordinate system $(\alpha, \beta = r, \theta)$

2.1 Introduction

Given that fracture mechanics covers a very large area and incorporates many phenomena, methods and techniques, this literature review will focus on some specific aspects, in particular classical Linear Elastic Fracture Mechanics and three-dimensional aspects of crack problems, which are important for the current work. The research gaps will be identified from a careful review of the literature. The latter will be addressed in the following chapters of the current thesis.

2.2 Classical Linear Elastic Fracture Mechanics

Linear Elastic Fracture Mechanics (LEFM) is one of the most successful concepts of continuum mechanics and nowadays is widely used for solving different types of fracture problems, such as "brittle fracture, fatigue crack growth, stress-corrosion cracking, dynamic fracture and, creep- and visco-elastic fracture" (Cotterell 2002). Thus a fundamental understanding of some basic concepts and key solutions in LEFM is essential for research in this area. This section introduces Williams' crack tip solution, obtained within the framework of the plane theory of elasticity. Furthermore, it presents the classic concepts of LEFM, including the stress intensity concept and the concept of crack initiation. These concepts of fracture theory are commonly utilised in many standards, fatigue and fracture evaluation codes (e.g. Standards Association of Australia 1972; Forman et al. 1988). These concepts will be examined critically in this thesis.

2.2.1 Williams' Crack Tip Solution

In 1952, Williams first demonstrated that the in-plane elastic stress components at the apex of an isotropic sharp corner can be singular. He considered the problem of a sharp corner with opening angles ψ (see Fig. 1a) subjected to symmetric and antisymmetric loading (or fracture mode I and mode II). In the specific case of a sharp crack (in which the opening angle equals to 2π, see Fig. 1b), by enforcing boundary conditions at radial edges, Williams represented the in-plane stress components in the radial and tangential directions in terms of a power series (Williams 1952, 1957):

$$\sigma_{\alpha\beta} = \sum_{n=1}^{\infty}\left(c_n r^{\frac{n}{2}-1} f_{\alpha\beta}(n,\theta)\right) \tag{1}$$

where, r and θ are polar coordinates as shown in Fig. 1b, the crack tip is located at the origin of the coordinate system, and the crack edges coincide with the line $\theta = \pi$, $f_{\alpha\beta}$ are dimensionless functions and $(\alpha,\beta) = (r,\theta)$.

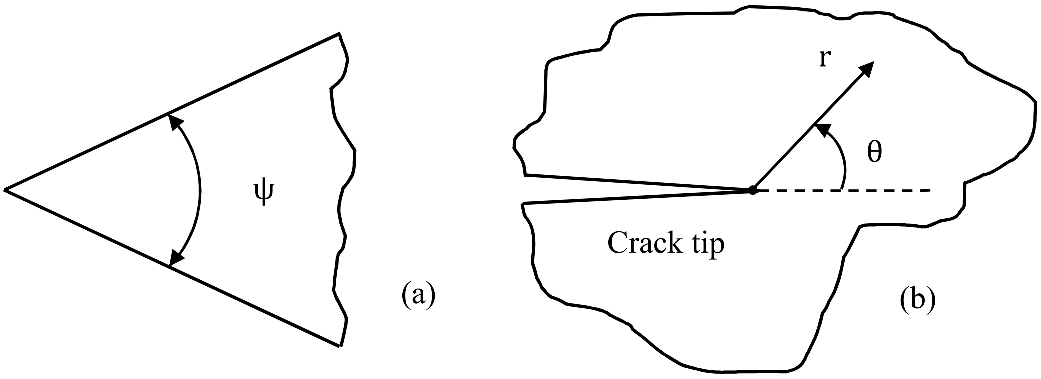

Figure 1: (a) Plate corner geometry characterised by a notch opening angle ψ; (b) Special case of the geometry at $\psi = 2\pi$ (crack).

With r approaching zero, the first term ($n = 1$) in Eq. (1) tends to infinity, while the higher order terms ($n \geq 2$) remain finite. Thus, the stress states in the close vicinity to the crack tip are dominated by the first term in the Williams' crack tip solution, which is proportional to $1/\sqrt{r}$ or the classic inverse square root singularity.

2.2.2 Stress Intensity Factor

In 1957, based on Westergaard's solution for plane problems (Westergaard 1939), Irwin (1957, 1958) first proposed the stress intensity concept in fracture mechanics. This new concept is based on the similarity of the distribution of stress in close vicinity to a crack tip for all crack problems. The stress field can be represented as:

$$\sigma_{\alpha\beta} = \frac{K}{\sqrt{2\pi r}} g_{\alpha\beta}(\theta) \tag{2}$$

where, $g_{\alpha\beta}$ are dimensionless functions of the angular position θ only and K is the stress intensity factor (SIF), which depends on the particular geometry of the problem as well as loading conditions (subscripts I and II are often used to denote the in-plane loading or fracture modes). Dimensional analysis shows that for any symmetrical geometry, K must be in direct proportion to the stress and the square root of a characteristic length, or mathematically:

$$K = \sigma\sqrt{\pi a}\, f\left(\frac{a}{W}\right) \tag{3}$$

where, a is the crack length, and f is a dimensionless parameter that depends on the geometry of the solid body and crack. Several very powerful analytical and numerical methods for the calculation of the stress intensity factor have been developed in the past. Many texts and handbooks (Broek 1982; Anderson 1995; Perez 2004) provide the explicit relationships for K values for a wide range of geometries and loading conditions.

2.2.3 Concept of Crack Initiation in LEFM

In accordance with LEFM, crack propagation (extension) occurs when the stress intensity factor reaches a critical value $\mathrm{K_c}$, which is called the fracture toughness (Irwin 1958). The value of $\mathrm{K_c}$ can be determined experimentally by measuring, for example, the fracture stress for a large plate with a through-the-thickness crack of known length.

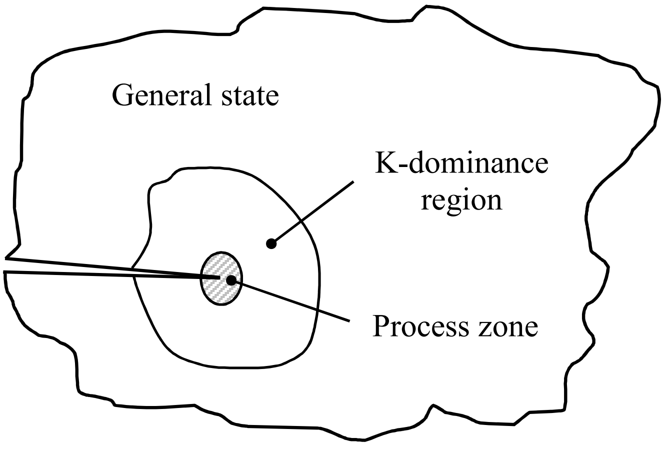

Figure 2: Characteristic regions around the tip of a crack.

Applications of the theory of classic LEFM to the failure assessment of cracked components are based on the following loose argument (Kotousov 2010). Consider a cracked plate made from reasonably brittle materials, three characteristic regions can normally be identified (see Fig. 2): the region immediately surrounding the crack tip forms the so-called process zone, in which all of the large nonlinear and non-elastic deformations are assumed to occur; a K-dominance region, where a linear elastic asymptotic stress field of the form $\mathrm{Kr}^{-1/2}$ is expected to be accurate; and the region of general stress state, where the stress field depends on the geometry of the solid and boundary conditions. Material failure normally starts within the process zone. When the process zone is fully encapsulated by the K-dominance region, all nonlinear phenomena in the process zone are likely to be controlled by the fields in the region of K-dominance, subsequently, the conditions for the failure initiation are function of the stress intensity factor only and nothing else (Lazzarin & Livieri 2001; Kotousov 2007a).

A direct consequence of the selection of K as a single fracture controlling parameter in LEFM is the scaling law. In general, the question of scaling occupies a central position in science, physics and engineering. Without understanding how strength changes with the size of a specimen or a structure, it is virtually impossible to apply any test results obtained at the laboratory specimen scale to predict and avoid fractures at smaller or larger scales: the scales of actual engineering applications. LEFM predicts that the strength increases or reduces as the inverse square-root of the scale factor. However, the LEFM prediction was found to be unsatisfactory through several hundreds of test results for appropriately brittle and quasi-brittle materials reported over the past fifty years (Sinclair & Chambers 1987). In particular, significant variations in the apparent fracture toughness for the same material are often observed in experimental studies conducted with different specimen configurations, crack sizes and loading conditions (Li & Zhang 2006; Smith et al. 2006; Sun & Qian 2009). To address this issue various scaling laws (Carpinteri 1994; Bazant & Chen 1997) and a number of two-parameter criteria (which utilise non-singular terms of Williams' crack tip solution) have been proposed in the past (Williams 1957; Nakamura & Parks 1992; Hutař et al. 2004; Kim & Cho 2009).

One of the objectives of the current project is to investigate the effectiveness of the two-parameter criteria, which could account for various effects induced by the specimen geometry and loading on initiation of brittle fracture. A joint paper addressing this objective was published in the International Journal of Fracture (Gardeazabal et al. 2014). Note that this paper is included in the Appendix. It was found that brittle fracture conditions can be successfully predicted with various two-parameter criteria, and there are no significant advantages in the use of T-stress (the uniform in-plane stress acting parallel to the crack direction) as the additional parameter in fracture criteria or the first non-singular term c_3 of Williams' crack tip solution, see Eq. (1).

2.3 Three-Dimensional Effects in Fracture Mechanics

It was not surprising that in problems with through-cracks the two-dimensional theories (or classic LEFM) often lead to peculiar results due, in part, to the fact that these are approximate theories even when the governing equations of these theories are solved exactly (Yang & Freund 1985). The stress states near an actual crack tip are always three-dimensional (3D) and an account for 3D effects may shed more light on many fracture and fatigue phenomena. This section provides a review of the outcomes of past studies devoted to the investigation of 3D crack problems. These include some important results related to the displacement field near the front of a through-the-thickness crack, variations of stress intensity factors across the plate thickness, coupled modes, 3D corner (vertex) singularity as well as fatigue crack growth and the crack path. These results highlight the essential differences between the classic 2D view on brittle fracture (provided in the previous section) and a more realistic approach, which considers the plane problems of elasticity within a 3D framework of linear elasticity.

2.3.1 3D Displacement Field near the Crack Front

Many analytical, numerical and experimental studies in the past have focused on the characterisation of 3D displacement, stress and strain fields near a crack front. Fig. 3 presents several results of evaluating the out-of-plane surface displacements around the crack tip sourced from an experimental study (Humbert et al. 2000), an analytical solution based on the first order plate theory (Kotousov

& Tan 2004) and numerical simulation (Pfaff et al. 1995). The cracks in these studies were subjected to pure mode I loading and the material in all these studies is considered to be linear-elastic or sufficiently brittle.

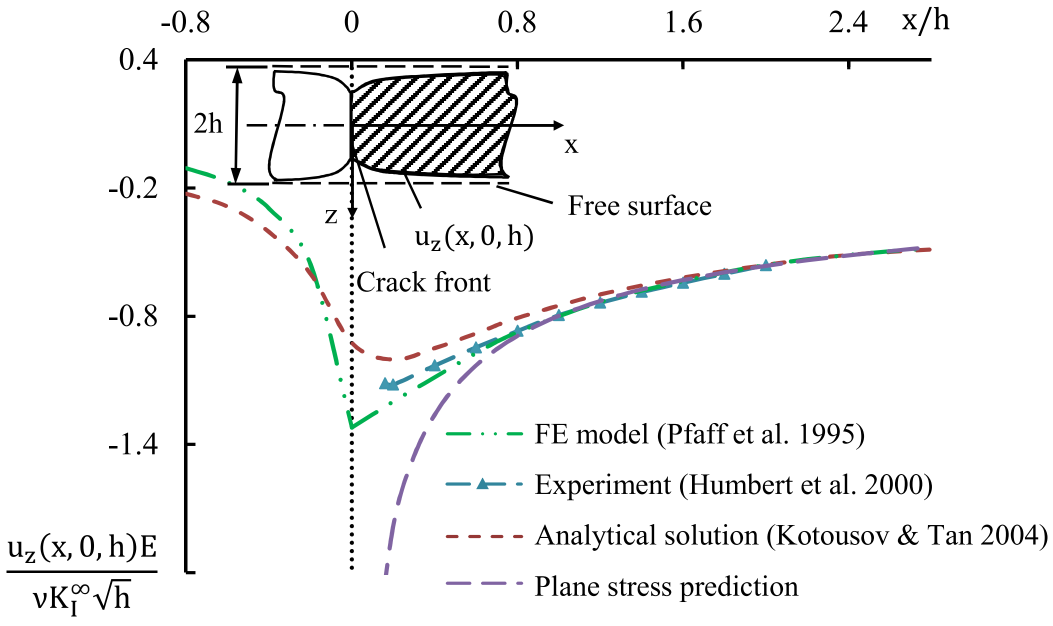

Figure 3: Normalised out-of-plane surface displacements along the crack direction as a function of x/h.

As can be seen in Fig. 3, relatively far from the crack tip ($r > h$), all the studies are in good agreement and predict that the out-of-plane displacement will follow the classic plane stress solution of the theory of elasticity. However, in the close vicinity of the crack tip, the displacement field is very different from the predictions of the classic plane stress ($u_z \sim r^{-1/2}$) or plane strain ($u_z \equiv 0$) theory. The differences between the numerical and analytical results obtained within the first order plate theory in the near crack tip area can be explained by the modelling assumptions and simplification of this theory as well as by the influence of the 3D corner (vertex) singularity on the surface displacement near the crack tip. The analytical solution obtained within the first order plate theory is not able to capture and describe the 3D corner singularity effect. This 3D effect will be discussed later in the thesis. Moreover, the relationship between the applied stress intensity factor and the out-of-plane displacement near the vertex will be utilised in the development of a new experimental approach for the evaluation of mode I and mode II stress intensity factors.

2.3.2 Variation of SIF across Plate Thickness

By using finite element (FE) techniques, the variation of local (3D) stress intensity factors along the plate thickness direction for cracks under different loading modes (mode I, II, III or mixed modes) have been thoroughly investigated and reported in a number of papers (Chao and Liu 1997; Pook 2001; Hutař et al. 2010a; Kotousov et al. 2010; Garcia-Manrique et al. 2013). Some representative examples are given in the following section.

In 2007, She and Guo (2007) conducted a series of numerical analyses on straight cracked plates stressed in mode I and mode II, respectively. The local stress intensity factors were then solved by the quarter point displacement extrapolation method (Barsoum 1976). Fig. 4 schematically shows the variation of the normalised local stress intensity factor $\mathrm{K_I(z)/K_I^\infty}$ along the crack front for models stressed in mode I for different Poisson's ratio of the material ($\nu = 0.15, 0.3, 0.5$), where $\mathrm{K_I^\infty}$ is the remote applied stress intensity factor, and $\mathrm{K_I(z)}$ is the local stress intensity factor along the crack front direction. The numerical results presented in Fig. 4 show an increase above 2D solution at the centre line ($\mathrm{z/h} = 0$), while the values fall towards the expected value of zero as the free surface is approached ($\mathrm{z/h} = 1$). Similarly, Fig. 6 shows the normalised local stress intensity factor $\mathrm{K_{II}(z)/K_{II}^\infty}$ along the thickness direction for a model stressed in mode II. The dependence of $\mathrm{K_{II}(z)}$ has an opposite tendency from mode I loading near the intersection of the crack front with the free surfaces. Note that the influence of Poisson's ratio on the stress intensity factor for mode II is very weak and the results are not shown for the sake of the clarity of the Figure.

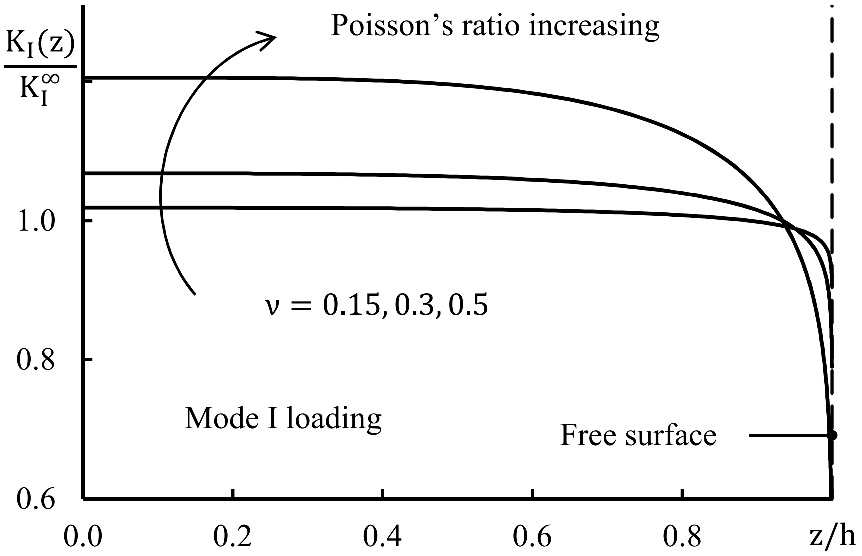

Figure 4: Normalised local stress intensity factors along the crack front for models stressed in mode I with different Poisson's ratio.

The numerical results presented above (Fig. 4 and 6), as well as in many other papers, have many implications in fatigue and fracture. For example, they can explain why the initiation of fracture for mode I loading is likely to occur in the central part of the specimen, while that for mode II loading has to be closer to the free surface, as has been reported in many experimental studies (Doquet et al. 2010; Seitl et al. 2013).

In this thesis, we will apply 3D J-integral to validate this stress intensity factor behaviour. The relationship between the local stress intensity factor near the vertex and the 3D corner singularity will also be established.

2.3.3 Coupled Modes

One of the characteristic features of 3D solutions of elastic problems with cracks is the existence of coupled modes (Nakamura & Parks 1989). These coupled modes cannot be identified within the framework of plane (2D) theory of elasticity and are largely ignored or considered to be negligible in previous theoretical and experimental studies (Kotousov et al. 2013).

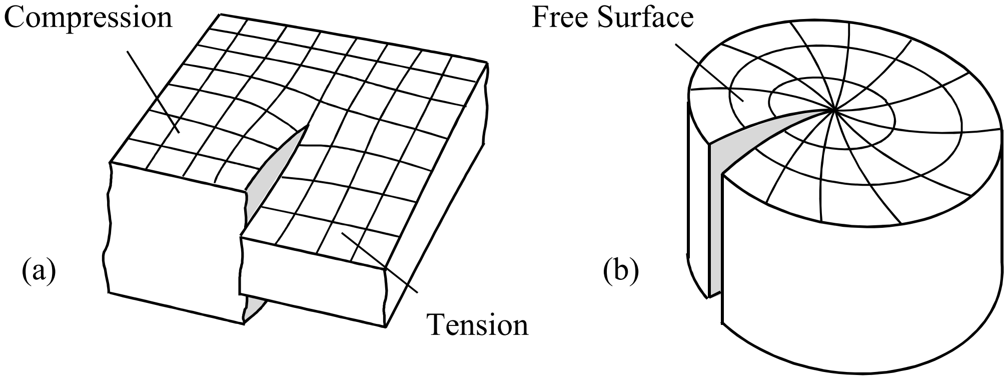

Figure 5: Deformation of the through the thickness crack subjected to (a) shear and (b) anti-plane loading (Kotousov et al. 2013).

The coupled mode in shear loading was called the out-of-plane mode, or mode O (first identified by Kotousov 2005), to distinguish it from the tearing mode (mode III). The out-of-plane mode is generated due to the 3D effects linked to Poisson's ratio of the material (see Fig. 5a), and it has the same characteristic equation for the degree of stress singularities as the conventional mode III (Kotousov et al. 2012, 2013). However, there are essential differences between these two modes. The out-of-plane singular mode is a localised fracture mode, which rapidly decays with the distance from the crack front and it is significantly affected by Poisson's ratio in contrast with fracture mode III. A similar coupled mode is generated in the case of the existence of mode III loading. In this case, the characteristic equation describing the degree of singularity of the coupled mode is the same as for mode II. However, the intensity of this coupled mode is not significantly affected by Poisson's ratio, as it is generated due to a redistribution of the out-of-plane shear stresses near to the free plate surfaces, see Fig. 5b.

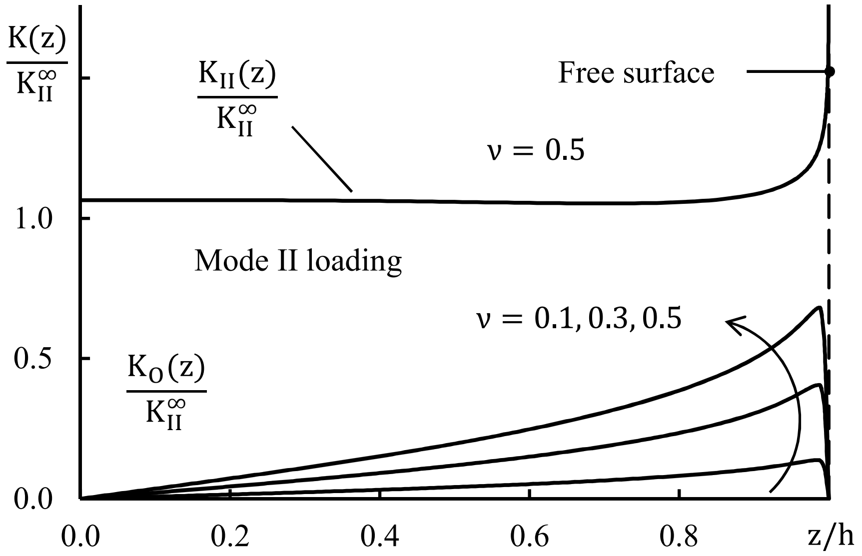

Figure 6: Normalised local stress intensity factors along the crack front for models stressed in mode II with different Poisson's ratios (Berto et al. 2011b).

A large computational effort was recently directed to the characterisation of these coupled modes for various structural components (Harding et al. 2010; Berto et al. 2013). Fig. 6 shows the numerical results of the normalised intensities of the out-of-plane mode $K_O(z)/K_{II}^{\infty}$ at different Poisson's ratios ($\nu = 0.1, 0.3, 0.5$). These results have been obtained by Berto et al. (2011b) and reproduced for validation purposes by He et al. (2015a) utilising a standard numerical procedure. It can be seen in the Figure that the intensities of the coupled mode are comparable with the intensities of the primary mode (mode II). The maximum values of the coupled mode are located in the vicinity of the plate free surfaces. The intensity K_O is limited to zero when $z/h \rightarrow 1$ due to the free from stress boundary conditions at the free surfaces, which negate the out-of-plane shear stress components.

Recent numerical studies by Berto et al. (2011c) demonstrated that the singular coupled modes can even be generated by non-singular (in a 2D sense) shear or anti-plane loading. In other words, the intensity of the coupled modes can be non-zero, even when the in-plane loading is characterised by zero values of applied mode I/II stress intensity factors ($K_I = K_{II} = 0$). The existence of these additional modes can help to explain why the apparent fracture toughness of the plates in mode II is much less than that in mode I, as often observed in experimental studies (Sanford 2005). However, this area lacks experimental studies to verify all theoretical findings experimentally, and, subsequently, one of the objectives of this project is to fill this gap. A joint paper was published in the Journal of Theoretical and Applied Fracture Mechanics (Kotousov et al. 2015). Note that this paper is included in the Appendix. Furthermore, the proposed out-of-plane displacement equation, which will be utilised for evaluating mode II stress intensity factors, also incorporates the contribution of the out-of-plane singular mode K_O.

2.3.4 3D Corner (Vertex) Singularity

Another characteristic feature of 3D solutions of elastic problems with cracks is the existence of the three-dimensional (3D) corner singularity, sometimes called vertex singularity. The 3D corner singularity was first discovered in the late 1970s and early 1980s by Benthem and several other researchers, who employed a finite difference scheme and the eigenfunction expansion method to demonstrate that at the intersection/vertex of the crack front and a free surface, the square root singularity disappears and, at such a point, one has to deal with a 3D corner singularity (Bazant & Estenssoro 1979; Benthem 1977, 1980). As opposed to the in-plane singularities, whose strength is described by universal square root behaviour, the strength of the corner singularity depends on Poisson's ratio as well as the intersection angle between the crack front and the free surface. For 3D corner singularity, the polar coordinates in Fig. 1 are replaced by the spherical coordinates (r, θ, ϕ) with the origin at the corner point. The angle ϕ is measured from the crack front. Stresses are proportional to K_λ/r^λ and displacements to $K_\lambda/r^{\lambda-1}$, where λ is a parameter defining the 3D corner singularity, and r is measured from the corner point. The problem of corner singularity has been well documented in a number of articles over the last thirty years (Huang 2004; Vu et al. 2015).

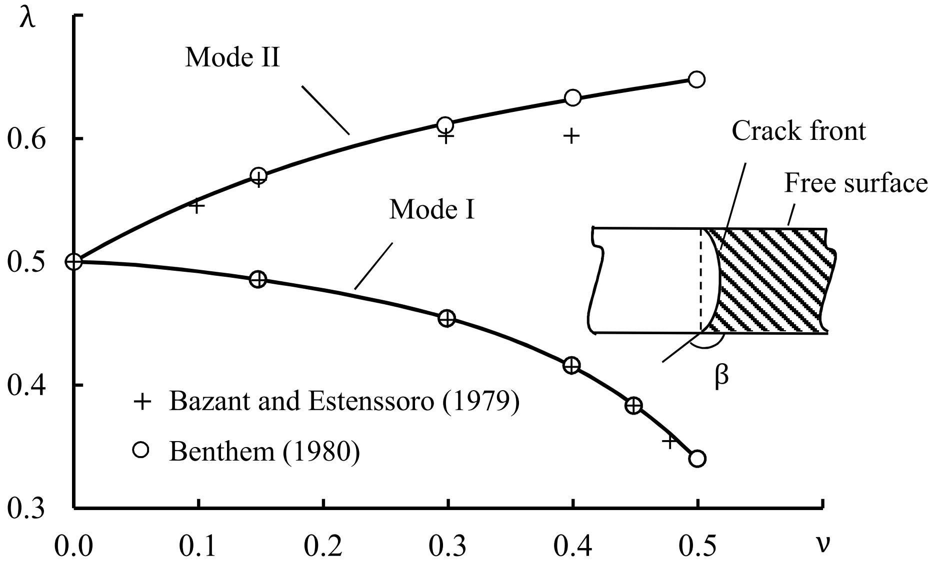

Figure 7: Effect of Poisson's ratio on λ, when intersection angle $\beta = 90°$.

Because of serious difficulties in analytical modelling, the results for the behaviour of the corner singularity have mainly been obtained using different semi-analytical methods or direct numerical techniques, in particular, finite element analysis (Shivakumar & Raju 1990; Pook 2000). The dependences of λ as a function of Poisson's ratio ν (when intersection angle $\beta = 90°$) are given in Fig. 7. Similar to the in-plane singularities, the 3D corner singularity can be uncoupled in the case of mode I and mode II loading. It is clearly seen from Fig. 7 that Poisson's ratio affects the strength of the 3D singularity for mode I/II loading in an opposite way. For mode I loading, a higher Poisson's ratio leads to a lower strength of singular behaviour, while the tendency is opposite for mode II loading.

The problem of 3D corner singularity was re-examined by de Matos and Nowell (2008). A detailed numerical study of the 3D corner singularity was also conducted recently by Pook (2013). In particular, all these studies verified the earlier findings of Nakamura and Parks (1988), i.e. the size of the 3D corner singularity field in a thin plate appears to be about 3~5 percent of the thickness in spherical radius and depends very little on the Poisson's ratio. Despite the limited volume of the cracked plate affected by the 3D corner singularity, this 3D effect can have a significant influence on fracture and fatigue phenomena (Pook 1994; Hutař et al. 2009, 2010b; Branco et al. 2012; Ševčík et al. 2012). For example, Heyder et al. (2005) suggested that the presence of the corner singularity might lead to a curved fatigue crack front in the vicinity of the free surface. One specific objective of this project is to undertake a study using the finite element method of the effects of corner singularity on stress intensities and displacement fields around the tip of a through-the-thickness crack in a linear-elastic plate of finite thickness. This study is important not just from a theoretical point of view, but also for its many practical implications, such as the development of a new experimental technique for the evaluation of in-plane stress intensity factors from the measurements of the near crack tip displacement field, which is affected by 3D stress states, in particular, the 3D corner singularity. The outcomes of this study have been published or submitted for publications in high quality journals, including the Fatigue and Fracture of Engineering Materials and Structures (He et al. 2015a), International Journal of Solids and Structures (He et al. 2015b), International Journal of Fracture (He et al. 2015c) and Experimental Mechanics (He et al. 2015d).

2.3.5 Fatigue Crack Growth and Crack Path

Fatigue is undoubtedly one of the most common failure mechanisms of engineering structures (Cui 2002). Around 50% ~ 90% of mechanical failures are reportedly due to fatigue each year (Stephens et al. 2001). The process of fatigue consists of three essential stages: initiation, propagation and final fracture. The propagation phase normally accounts for a significant portion of the overall lifetime, especially for specimens or structures with pre-existing cracks or sharp notches.

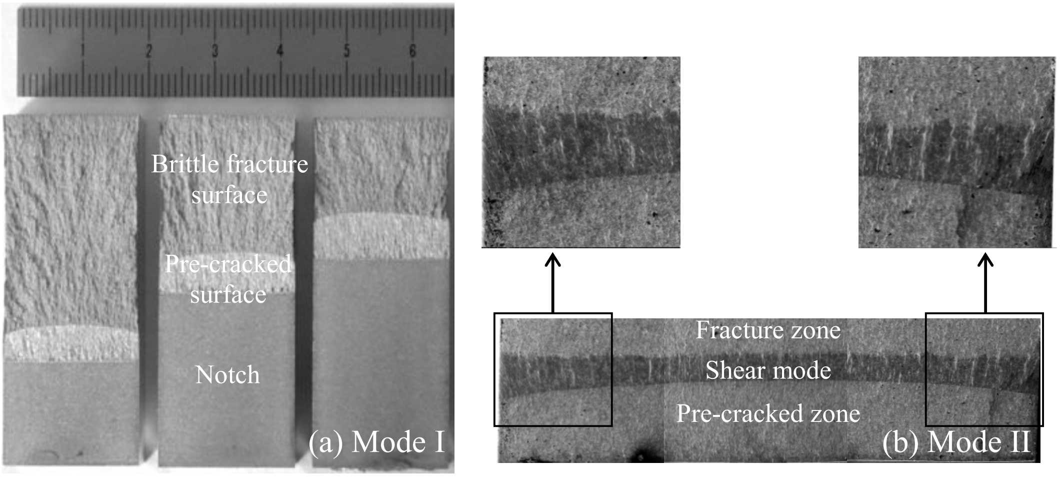

Figure 8: Fatigue crack growth of (a) Aluminium alloy 2024 loaded in mode I (Seitl et al. 2013), (b) maraging steel loaded in mode II (Doquet et al. 2010).

Propagation of fatigue cracks in plate and shell components is often treated as a two-dimensional process. In other words, the crack front is simply assumed to be straight and the variation of the stresses along the plate thickness direction to be negligible. These common assumptions essentially reduce the dimensionality of the problem and in many cases provide satisfactory predictions for many practical problems (Kotousov 2007b). However, as demonstrated in many theoretical and experimental papers, the actual stress field near the crack tip is always three-dimensional (3D) and, as a result, the fatigue crack front is not straight. It is normally curved near the free surface (Hou 2011). For example, Seitl et al. (2013) conducted an experimental study on the fatigue crack growth behaviour of Aluminium alloy 2024 under cyclic mode I loading. Fig. 8a shows crack surfaces of specimens for the various crack lengths to the specimen width ratio after failure.

The authors concluded that the crack grows non-uniformly from its initial location through the thickness of the specimen. Similarly, in 2010, Doquet et al. (2010) conducted a fatigue crack growth test on maraging steel. Fig. 8b illustrates the crack growth of maraging steel under cyclic mode II loading. The authors pointed out that at low cyclic loading amplitudes, shear mode cracks only initiated at a few discrete points close to the free surfaces; while at higher cyclic loading amplitudes, shear mode cracks can occur throughout the whole crack front.

In this thesis, we will develop a simplified mathematical mode for evaluating fatigue crack front shapes under mode I loading. Its influence on fatigue life predictions will be fully investigated.

2.4 Methods for 3D Analysis of Through-Cracks in Plates

Numerous analytical and numerical techniques have been developed in the past for the accurate evaluation of 3D crack problems. This section introduces an analytical solution based on refined plate theory, a 3D exact analytical solution from Yosibash, as well as a numerical approach. This numerical approach has been widely used in many works devoted to the 3D aspects of fracture mechanics and is adopted in the current project as well.

2.4.1 Analytical Solution Based on Refined Plate Theory (Kotousov)

Taking advantage of the distributed dislocation technique (Codrington et al. 2008) and the 3D elastic solution for an edge dislocation in an elastic plate of arbitrary thickness (Kotousov & Wang 2002), Kotousov and Tan (2004) developed a semi-analytical approach for the analysis of 3D crack problems. In accordance with this solution for an embedded crack (a) in an infinite plate of finite thickness

(2h), loaded with a tensile stress σ_{yy}^{∞}, which acts perpendicular to the crack front, the out-of-plane displacement u_z at any point can be calculated as:

$$u_z(x, y, z) = \frac{z}{h}\int_{-\frac{a}{2}}^{\frac{a}{2}} B(t) G_w(x - t, y) dt - \frac{\nu h \sigma_{yy}^{\infty}}{E} \tag{4}$$

where, $B(t)$ is the density of the dislocations at point t $(-a/2 \leq t \leq a/2)$, ν is Poisson's ratio and E is the elastic modulus, the displacement kernel is given below:

$$G_w(x, y) = \nu \kappa h \frac{1}{2\pi} \cos\phi\left((\kappa r)^{-1} - K_1(\kappa r)\right) \tag{5}$$

where, $\kappa = \frac{1}{h}\sqrt{\frac{6}{1-\nu}}$, $r = \sqrt{x^2 + y^2}$, $K_n(\cdot)$ is the modified Bessel functions of n^th^ order. The detailed description of the derivation procedure can be found from Kotousov and Tan (2004). This analytical solution assumes that the out-of-plane strain is uniform along the plate thickness.

2.4.2 3D Exact Analytical Solution (Yosibash)

A 3D exact analytical solution was obtained by Omer and Yosibash (2005), who utilised the general mathematical framework provided in Costable et al. (2004) to derive the 3D asymptotic expansion near the crack front. In accordance with this solution for a crack under pure mode I loading, the asymptotic three-dimensional displacement field ũ can be expressed as:

$$\tilde{u} = A_1(z) r^{\frac{1}{2}} \begin{pmatrix} u_{r1}(\theta) \\ u_{\vartheta 1}(\theta) \\ 0 \end{pmatrix} + \partial_3^1 A_1(z) r^{\frac{3}{2}} \begin{pmatrix} u_{r2}(\theta) \\ u_{\vartheta 2}(\theta) \\ u_{z2}(\theta) \end{pmatrix} + \partial_3^2 A_1(z) r^{\frac{5}{2}} \begin{pmatrix} u_{r3}(\theta) \\ u_{\vartheta 3}(\theta) \\ u_{z3}(\theta) \end{pmatrix} + \cdots \tag{6}$$

where, $\partial_3^j \equiv \frac{\partial^j}{\partial z}$, $A_1(z)$ represents the mode I edge stress intensity function and is associated with mode I loading, r, θ and z are cylindrical polar coordinates with the original point at the middle of the crack front, and $u_{\alpha\beta}(\theta)$ are functions of angular position θ. The detailed description of the derivation procedure can be found from Omer and Yosibash (2005). This analytical solution explicitly assumes a plane strain condition or zero out-of-plane displacement along the crack front, which is contradictory to the results obtained from many previous analytical, numerical and experimental studies (see Fig. 3).

This 3D exact analytical solution has been adopted in many later works devoted to the 3D aspects of fracture mechanics. For example, using the explicit solution, Omer and Yosibash (2005) showed that for more general 3D states of stress under mode I loading, the pointwise path-area J_{X1}-integral is still path-independent.

2.4.3 FE Method for 3D Analysis and Fatigue Crack Growth Modelling

Numerical investigations of 3D crack front fields have predominantly utilised the finite element (FE) method. One of the first comprehensive numerical studies of the 3D stress state near the crack front was carried out by Nakamura and Parks (1988, 1989). In the FE model, a circular disk surrounding the crack front was considered and simulated using the boundary layer approach, see Fig. 9a. The maximum radius of the disk is five times the plate thickness, which is large enough to guarantee that (1) the boundary conditions applied at the outer surface of the disk correspond to those of a far-field 2D plane stress solution (e.g. Williams' crack tip solution), and (2) the in-plane geometry of the FE model does not affect the 3D stress state near the crack front. The straight crack front is located at the centre of the disk, as shown in Fig. 9b. This numerical approach has been verified and extended in many other works devoted to the 3D aspects of fracture mechanics (She

& Guo 2007; Berto et al. 2011a, 2012). In particular, all these studies agree that the region of 3D effects is confined to approximately one-half of the plate thickness around the crack front, before converging into a 2D plane stress field at a radial distance roughly equal to the plate thickness.

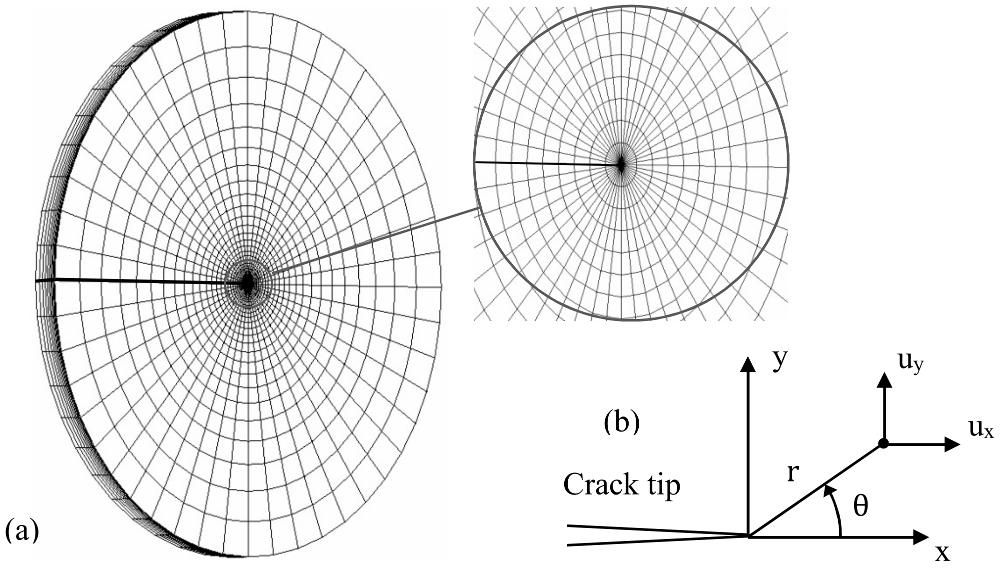

Figure 9: (a) Geometry and mesh division of the 3D finite element model; (b) Coordinate system with the origin at the centre of the disk.

Numerical techniques have also been successfully employed to predict crack shape evolution and fatigue life (Camas et al. 2012; Hutař et al. 2012; Zheng et al. 2014; Dundar & Ayhan 2015; Pathak et al. 2015). For example, Citarella and Buchholz (2008) investigated the 3D fatigue crack growth behaviour in three-point bending specimens, using the dual boundary element method. Ševčík et al. (2012) numerically estimated the fatigue crack front shape in a semi-infinite plane media by assuming a constant stress intensity factor throughout the curved crack front. Branco and Antunes (2008) developed an iterative numerical procedure to study the shape evolution of fatigue cracks in plates based on the remeshing technique developed by Smith and Cooper (1989). This procedure was modified and advanced in a number of subsequent papers (Antunes et al. 2010; Branco et al. 2013, 2015). The crack shape evolution in this procedure is governed by the effective stress intensity factor, which takes into account the plasticity induced crack closure effect. The procedure comprises four main steps: (1) development of a representative 3D finite element model; (2) calculation of the effective stress intensity factors along the crack front; (3) calculation of the crack front advance, normally by using Paris-type law; and (4) determination of a new crack front. The process is repeated until a specified crack front shape or final fracture is achieved. The evolutions of fatigue crack front shapes obtained from these iterative numerical simulations were validated successfully with experimental results. However, this numerical procedure relies on a number of parameters, which are not directly related to the relevant physical problems but rather to the computational procedure itself. In addition, to the best of our knowledge, there are no simple and transparent ways to reproduce independently or verify the computational procedure, as well as the reported numerical results and conclusions. Thus, one of the objectives of this project is to develop a simplified method to elucidate the role of the 3D effects on the formation of fatigue crack front under steady-state growth. This objective has been addressed in a recently published paper by the author (He et al. 2014).

2.5 Research Gaps and Specific Objectives

As can be seen from the present literature review, most of the theoretical, numerical and experimental studies of elastic plate components, weakened by through-the-thickness cracks and subjected to loading parallel to the plane of the plate, have utilised the two-dimensional framework of fracture mechanics and especially, LEFM. As a rule, all fundamental results are based on the plane theory of elasticity. The influence of various three-dimensional effects, such as the fatigue crack front curvature, variation of stresses across the plate thickness, effects of the three-dimensional corner singularity and coupling of fracture modes II and III, on the deformation, strains and stresses near the crack front are either not identified or are simply neglected within the plane theory of elasticity for practical purposes. However, many previous studies have demonstrated that the (2D) solutions made within these theories can be unsatisfactory (e.g. Sinclair & Chambers 1987; Kumar et al. 2011) and the effect of 3D stress states on deformation and the fracture of plates can be quite significant (e.g. Branco & Antunes 2008; Kotousov et al. 2013). Therefore, the current project aims to investigate and understand the role of three-dimensional stress states in deformation, fatigue and fracture phenomena. The following specific objectives, united under the umbrella of 3D fracture mechanics, have been investigated during candidature.

Objective 1: Develop a simplified mathematical model for the evaluation of the front shapes of through-the-thickness fatigue cracks

Propagation of through-the-thickness cracks in plates is often treated as a 2D process. In other words, the crack front is assumed to be straight and the variation of the stresses along the thickness direction to be negligible. Despite a large number of 3D analytical, numerical and experimental studies conducted over the last decade, it is still unclear how adequate these common assumptions and simplifications are, and what influence the account for the crack front shapes has on fatigue life predictions. Thus, one specific objective of the project is concerned with the 3D analysis of fatigue crack front shapes. The outcomes of this study may have a direct impact on fatigue assessment procedures, as well as post-mortem investigations of failed-in-operation structural components and machines.

Objective 2: Investigate the effect of 3D corner singularity on stress intensities and out-of-plane displacements field near straight crack front

Many experimental and analytical solutions have demonstrated that the state of stress changes from plane stress to 3D near the crack tip. This 3D region of the 3D stress state is confined to approximately one-half of the plate thickness around the crack tip. One specific objective of the project is to investigate the influence of corner singularity on the stress intensities and out-of-plane displacement field near crack front. In particular, there are different views in the reviewed literature on the state of strain at the tip of the crack. This study is important not only from a theoretical point of view but also has many practical implications.

Objective 3: Develop a new experimental method for evaluating plane stress intensity factors from near crack tip fields

The classic plane solutions of the theory of elasticity fail to describe the displacement, stress and strain fields in the close vicinity of the crack front. Subsequently, the existing techniques for determining stress intensity factors fully rely on the measurement taken outside the region affected by 3D effects. One specific objective of the project is to develop and validate an experimental procedure for accurate evaluation of mode I/II stress intensity factors from the near crack tip field. The proposed procedure will have several advantages over the existing experimental techniques.

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Chapter 3

A Simplified Method for the Evaluation of Fatigue Crack Front Shapes under Mode I Loading

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Contribution to the Paper	Created FE models, performed all analyses, interpreted data and wrote manuscript		
Overall percentage (%)	60		
Certification:	This paper reports on original research I conducted during the period of my Higher Degree by Research candidature and is not subject to any obligations or contractual agreements with a third party that would constrain its inclusion in this thesis. I am the primary author of this paper.		
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1. the candidate's stated contribution to the publication is accurate (as detailed above);
2. permission is granted for the candidate in include the publication in the thesis; and
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Name of Co-Author	Andrei Kotousov		
Contribution to the Paper	Supervised work development, participated in work discussions and co-wrote manuscript		
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Chapter 4

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Chapter 5

On the Evaluation of Stress Intensity Factor from Displacement Field Affected by 3D Corner Singularity

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On the evaluation of stress intensity factor from displacement field affected by 3D corner singularity

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ABSTRACT

Three-dimensional effects near crack front were subject of many analytical, numerical and experimental studies over the past 50 years. These studies were facilitated by obvious discrepancies between experimental evidences and predictions of two-dimensional theories of elasticity, which singular solutions currently form through a framework of contemporary Fracture Mechanics. In particular, the classical plane solutions of the theory of elasticity fail to describe the displacement, stress and strain fields in the close vicinity of the crack tip. Subsequently, the existing experimental techniques for determining stress intensity factor fully rely on the measurements taken outside the region affected by the three-dimensional effects, and, in particular, by 3D corner (vertex) singularity. In the present paper, we attempt for the first time to develop and validate an experimental procedure for accurate evaluation of mode I stress intensity factor from surface measurements of the transverse (out-of-plane) displacements over the area encapsulating the crack tip with a radius less than five per cent of the plate thickness. The proposed procedure can have several advantages in comparison with the existing experimental techniques. These are discussed in the paper.

1. Introduction

Theoretical, numerical and experimental studies of elastic plate components weakened by through-the-thickness cracks and subjected to loading parallel to the plane of the plate are often based on plane stress or plane strain simplifications. These simplifications essentially reduce the dimensionality of the actual three-dimensional (3D) problem and allow obtaining exact and effective analytical and numerical solutions for practically important problems (Kotousov, 2007). The influence of various three-dimensional effects, such as the effects of the plate thickness, 3D corner (vertex) singularity or coupling of fracture modes II and III, on the displacements, strains and stresses near the crack front are presently largely ignored or considered to be negligible for all practical purposes (Kotousov, 2010; Kotousov et al., 2010). Nevertheless, experimental evidences collected over the past five decades demonstrate that the three-dimensional stress states can play an important role in fatigue and fracture phenomena (Aliha and Saghafi, 2013; Pook, 2013a, 2013b; He et al., 2015; Kotousov et al., 2015). Subsequently, many researchers in the past attempted to explain various experimental observations, tendencies or test results attracting the three-dimensional considerations. One such characteristic example is the evolution of the fatigue crack front shape in fatigue test specimens. This phenomenon, which was a subject of recent extensive studies (Branco et al., 2012; Camas et al., 2012; Sevcik et al., 2012; He et al., 2014), was linked to the influence of 3D corner singularity on the energy release rate and stress state near the free surface.

A comprehensive retrospective review of theoretical and experimental investigations of three-dimensional effects at cracks and sharp notches was recently presented by Pook (2013a, 2013b). Subsequently, in this section we will provide only a brief summary of the previous works, which are important for the purpose of the current paper. It seems Hartranft and Sih (1969) were the first who applied the eigen-function expansion approach to analyse crack problems utilising the three-dimensional framework of the theory of elasticity. This approach was further extended by Yosibash and his colleagues (Omer and Yosibash, 2005; Yosibash and Shannon, 2014) who derived an asymptotic solution in the vicinity of a crack front in a three-dimensional elastic domain. However, this solution ignores the singular stress state associated with 3D corner (vertex) singularity, which significantly affects the displacement field at the crack front close to the free surface. Based on the first-order (or Kane and Mindlin) theory Yang and Freund (1985) obtained an analytical solution describing the three-dimensional displacement and stress fields near the tip of a semi-infinite crack. This theory was also utilised by

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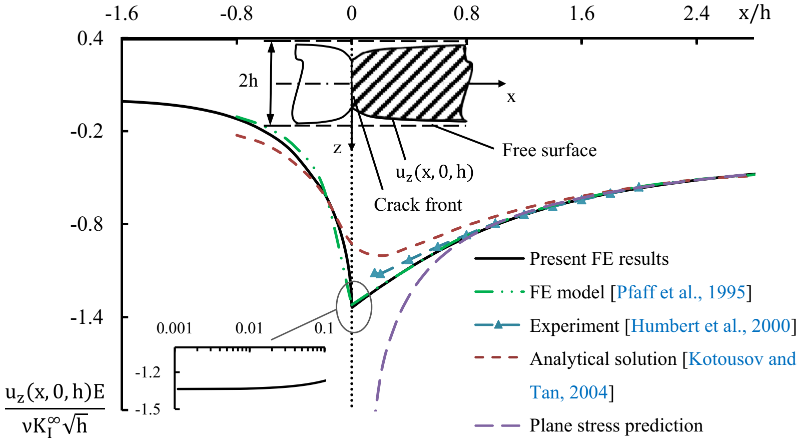


Fig. 1. Normalised transverse surface displacements along the crack direction as a function of x/h. In this figure E is Young's modulus, ν is Poisson's ratio, K_I^∞ is the remotely applied stress intensity factor in mode I and h is the half plate thickness.

Kotousov and his colleagues who first developed a three-dimensional solution for an edge dislocation in an elastic plate of arbitrary thickness (Kotousov and Wang, 2002a), and then implemented this analytical solution for the analysis of three-dimensional crack problems with the distributed dislocation technique (Kotousov and Tan, 2004; Codrington et al., 2008; Kotousov, 2007). In particular, the obtained analytical three-dimensional solutions broadly support a hypothesis proposed by Gregory and Wan (1988), which suggests that any three-dimensional solution of a plane problem can be divided into an interior state, which can be described by a corresponding solution of plane stress problem, and a three-dimensional decaying state. The decaying state can be expressed as:

$$\sigma \sim Me^{-\alpha d/h} \tag{1}$$

where, d is the minimum distance of the observation point from the edge of the plate, M is the maximum modulus of the prescribed edge tractions and α is a positive constant, which is independent of the half plate thickness h. For through-the-thickness cracks $\alpha \approx 1$, which is consistent with analytical solutions obtained in the frame of the first order plate theory (Kotousov and Wang, 2002a, 2002b; Kotousov, 2007) as well as experimental and numerical studies (Kotousov, 2010). These studies will be very briefly discussed next.

A number of experimental studies investigated the three-dimensional strain and displacement fields in the close vicinity of the crack tip with advanced measurement techniques such as method of caustics (Rosakis and Ravi-Chandar, 1986) and laser interferometry (Humbert et al., 2000; Pfaff et al., 1995) to name a few. However, it seems there were no experimental studies focusing on the area affected by 3D corner singularity. This area encapsulates the crack tip with a radius less than five per cent of the plate thickness. Numerical investigations were mainly based on the finite element (FE) method. One of the first comprehensive numerical studies of the three-dimensional stress state near the front of a through-the-thickness crack was conducted by Nakamura and Parks (1988). The numerical approach and results of this study have been verified and extended in many other works, for example, She and Guo (2007) and Berto et al. (2011a). Among the latest developments we point out a disk-shaped domain integral method (Nejati et al., 2015), which can provide a very accurate and efficient numerical solutions to three-dimensional crack problems.

A new experimental procedure to be presented in this paper utilises the measurements of the transverse surface displacements in the close vicinity of the crack tip for the evaluation of the stress intensity factor. Subsequently, Fig. 1 presents several results on evaluating the transverse surface displacements around the crack tip sourced from: an experimental study (Humbert et al., 2000), analytical solutions based on the first order plate theory (Kotousov and Tan, 2004) and numerical simulations (Pfaff et al., 1995). The cracks in these studies were subjected to pure mode I loading and the material in all these studies is considered to be linear-elastic or sufficiently brittle.

As it can be seen in Fig. 1, relatively far from the crack tip ($r > h$) all studies are in a good agreement and predict that the transverse displacements follow the classical plane stress solution of the theory of elasticity. However, in the close vicinity of the crack tip the displacement field is very different to the predictions of the classical plane stress ($u_z \sim r^{-1/2}$) or plane strain ($u_z \equiv 0$) theory. The differences between the numerical and analytical results obtained within the first order plate theory in the near crack tip area (Fig. 1) can be explained by the modelling assumptions and simplifications of this theory as well as by the influence of, so-called, 3D corner singularity on the surface displacement field near the crack tip. The analytical solutions obtained within the first order plate theory are not capable to capture and describe this three-dimensional effect. The 3D corner singularity was first discovered by Benthem who utilised a semi-analytical approach to represent the stress field at a corner point (Benthem, 1977). He demonstrated that at the intersection of the crack front and a free surface, the square root singularity disappears, and at such a point, one has to deal with a three-dimensional corner singularity. As opposite to the edge singularities, which have a universal square root power, the strength of the corner singularity depends on Poisson's ratio as well as on the angle at which the crack front intersects the free surface. Later, various numerical approaches were developed to investigate the corner singularity behaviour for different geometries and loading conditions. These include the finite different schemes (Benthem, 1980), eigenfunction expansion method (Bazant and Estenssoro, 1979), logarithmic regression of stress or displacement fields in the vicinity of the crack tip (Hutar et al., 2009, 2010). In particular, these advanced approaches verified the earlier findings of Nakamura and Parks (1988) that the size of the corner singularity field in a thin plate appears to be about 3–5 per cent of the thickness in spherical radius and depends very little on the Poisson's ratio. Despite on the limited volume of the cracked plate affected by the 3D corner singularity this three-dimensional effect can have a significant influence on fracture phenomena, for example, on the evaluation of the shape of fatigue crack fronts and fatigue crack growth rates as it was mentioned earlier (Branco et al., 2012; Camas et al., 2012; Sevcik et al., 2012; He et al., 2014).

In this paper, we attempt to develop a new experimental procedure for the evaluation of mode I stress intensity factor by extracting and analysing the out-of-plane (transverse) displacements very near the crack tip, specifically from the surface area encapsulating the crack tip with a radius less than five per cent of the plate thickness. This area is controlled by three-dimensional effects, including 3D corner singularity. The elastic displacement field does not follow

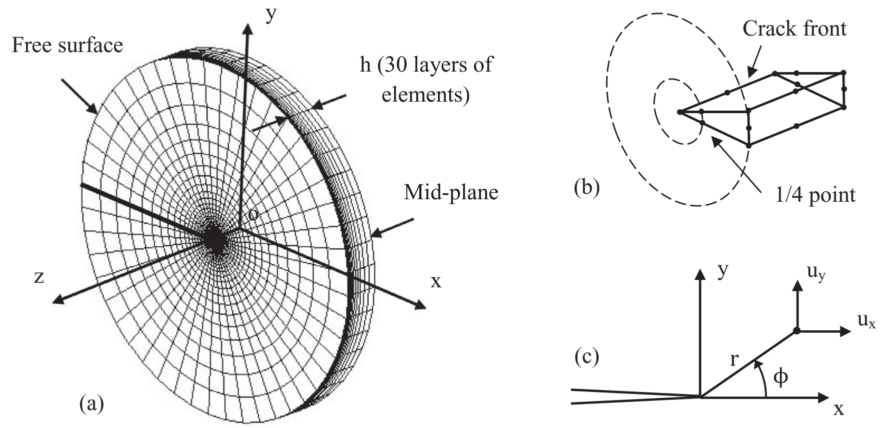


Fig. 2. 3D finite element model. (a) Geometry and mesh division of the model. (b) 3D 1/4 point singular element around the crack front. (c) Coordinate system with the origin at the mid-plane.

either plane stress or plane strain assumptions (see Fig. 1); and, it is normally excluded from the data extraction region in current experimental techniques. Of course, the new procedure can only be applied to relatively brittle materials or thick plate components or at low levels of the applied loading at which the process zone or zone of plastic deformations is much smaller than $0.1h$ (h is the half thickness of the specimen).

In the existing experimental techniques, which are based on fitting the experimental data to the classical plane stress asymptotic expansion of displacement or strain fields near the crack tip, the evaluation of the stress intensity factor can often be affected by the higher order terms of this asymptotic expansion, specifically when experimental measurements to be taken relatively far from the crack tip (Desai et al., 2012). Therefore, one of the possible advantages of the proposed experimental procedure for the evaluation of stress intensity factor is that the influence of the non-singular asymptotic terms on the accuracy of the evaluation can be effectively controlled and minimised by selecting a smaller size of the data extraction region where the effect of the higher order terms on the transverse displacement field is much lesser.

The paper is structured as follows: in the next Section we briefly present Finite Element (FE) methodology for investigating 3D displacement field in the near crack tip field. We validate this FE methodology against results obtained from the previous studies. Section 3 provides outcomes of the numerical simulations for a through-the-thickness crack subjected to mode I loading. In particular, it is demonstrated that the transverse surface displacement in the area controlled by 3D corner singularity is constant along the radial direction; and the magnitude of this displacement is a function of the applied stress intensity factor, Young's modulus, Poisson's ratio and plate thickness. The obtained numerical results form a conceptual basis for a new experimental procedure for the evaluation of the stress intensity factor by measuring the transverse displacement very near the crack tip, specifically from the surface area encapsulating the crack tip with radius less than five per cent of the plate thickness. Finally in Section 4, we compare the experimental and theoretical results. A very good agreement between these results confirms the possibility of the accurate evaluation of stress intensity factors from the surface measurements in the area controlled by three-dimensional effects. A discussion of main outcomes achieved in this paper is provided in the conclusion section.

2. Details of finite element model and verification

In this section we will briefly outline the main features of the Finite Element modelling approach adopted in the current work. Further, we will consider several verification examples. These examples will provide some confidence in the numerical simulations of the transverse displacement field to be presented in the next Section.

2.1. Geometry

The problem considered here is an elastic plate of finite thickness $2h$ containing a through-the-thickness crack subjected to remote loading. A circular disk area surrounding the crack-tip is selected to evaluate the stress and displacement fields near the crack tip. The radius of the disk is five times larger than the plate thickness, which, in accordance with the previous studies, is sufficient to guarantee that the truncated geometry does not affect the three-dimensional stress and displacement fields near the crack front (Harding et al., 2010; Berto et al., 2011a, 2011b). The straight crack front is located at the centre of the disk along the z-axis ($x = y = 0$). The finite element model also utilises the symmetry of the considered problem with respect to the z-plane, see Fig. 2a. So, the symmetry boundary conditions are enforced at $z = 0$.

2.2. Finite element model

The FE analysis was carried out using the ANSYS 14.5 package. An initial arrangement of 15-node trapezoidal elements with four midside nodes at the quarter points (see Fig. 2b) is developed around the crack front, which is surrounded by a radial array of 20-node brick elements. Each finite element spanned an angular sweep of 7.5°. In order to accurately capture the 3D displacement distribution, a higher mesh density was applied close to the corner point and few mesh convergence tests were successfully passed. The model consists of 30 layers of elements, which thicknesses are gradually decreased from the mid-plane to the free surface to more accurately represent the strong variations of the stress components and displacements near the free surface. The mesh has a total of 113,000 elements and 460,000 nodes. In all numerical simulations, the material is assumed to be isotropic and linear-elastic with a dummy value of Young's modulus, E, and Poisson's ratios, ν, ranging from 0.1 to 0.5.

2.3. Boundary conditions

As it was mentioned in Section 1, the three-dimensional stress region is confined to approximately one-half of the plate thickness around the crack front (Harding et al., 2010; Kotousov, 2010). Therefore, the corresponding boundary displacements on the outer surface of the disk (outside the region of three-dimensional effects) can be prescribed in accordance with Williams's (1957) plane stress asymptotic solutions as follows:

$$u_x(r,\phi) = \frac{r^{1/2}}{\mu\sqrt{8\pi}} K_{\mathrm{I}}^{\infty}\left[\left(\kappa - \frac{1}{2}\right)\cos\left(\frac{1}{2}\phi\right) - \frac{1}{2}\cos\left(-\frac{3}{2}\phi\right)\right] + \text{higher order terms} \tag{2a}$$

$$u_y(r,\phi) = \frac{r^{1/2}}{\mu\sqrt{8\pi}} K_{\mathrm{I}}^{\infty}\left[\left(\kappa + \frac{1}{2}\right)\sin\left(\frac{1}{2}\phi\right) + \frac{1}{2}\sin\left(-\frac{3}{2}\phi\right)\right] + \text{higher order terms} \tag{2b}$$

where, r is the distance from the crack tip (see Fig. 2c), ϕ is the angle measured from the symmetry line, $\mu = E/2(1+\nu)$ is the shear modulus and $\kappa = (3-\nu)/(1+\nu)$ is Kolosov's constant for plane stress condition, which prevails far from the crack tip ($r > h$), and K_{I}^{∞} is the remotely applied mode I stress intensity factor. Particular values of K_{I}^{∞} and E are not important for the further analysis as all results will be presented in a dimensionless form. The higher order (or non-singular) terms in the Williams' asymptotic crack tip expansion produce zero transverse displacements at the crack tip (it was also verified with the current numerical simulations). Therefore, similar to the previous studies (Nakamura and Parks, 1988; Berto et al., 2011a, 2011b) in this work we will focus on the effect of the leading (singular) term of the asymptotic expansion (2) on the transverse displacement field.

2.4. Verification of the numerical approach

The developed FE approach was verified against the previously published results. The comparison of the normalised transverse surface displacement along the crack direction as a function of x/h is incorporated in Fig. 1. As it can be seen from this figure, the present results agree well with the outcomes of the numerical studies reported in the literature (Pfaff et al., 1995). Further validation of the developed FE model and numerical approach was conducted for the variation of the stress intensity factors across the crack front, which was well documented in many previous studies (e.g., Nakamura and Parks, 1988; Kotousov, 2010), and by comparing the strength (or power) of the 3D corner and edge singularities with published values. The latter will be discussed in the next section.

3. Transverse displacement field under mode I loading

This section presents the outcomes of numerical investigations of the transverse strain (ε_z) and displacement (u_z) fields in the close vicinity of a crack front, when the crack is subjected to remote mode I loading. Due to linearity of the formulated elastic problem, and based on dimensionless considerations, the numerical results can be generalised by introducing a dimensionless displacement function, which is defined as:

$$\hat{U}_z = \frac{u_z(r,\phi,z)E}{K_{\mathrm{I}}^{\infty}\sqrt{h}} \tag{3}$$

as K_{I}^{∞}, E and h are the only parameters of the problem having a physical dimension.

One interesting finding from the current numerical analysis is that the transverse displacements very close to the crack tip are independent of the angular position ϕ, and are almost constant along the radial direction confined by one tenth of the half plate thickness ($r/h < 0.1$). The difference in the values of transverse displacement within this region is less than five per cent. Therefore, it can be stated that the transverse displacement field is a function of the transverse coordinate only (i.e. when $r/h < 0.1$), or in other words, in the region (very near the crack tip) $u_z(r,\phi,z) \approx u_z(z)$. Typical results of numerical simulations are shown in Fig. 3 for the dimensionless transverse displacement field, $\hat{U}_z$, at Poisson's ratio $\nu = 0.3$. Another interesting feature, which has been confirmed with the present FE simulations, is that the effect of the higher order terms of plane stress asymptotic expansion (2) on the transverse displacements decreases as approaching the crack tip, as discussed earlier.

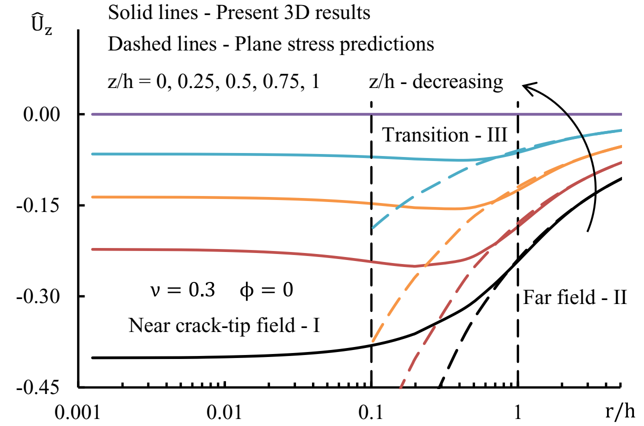
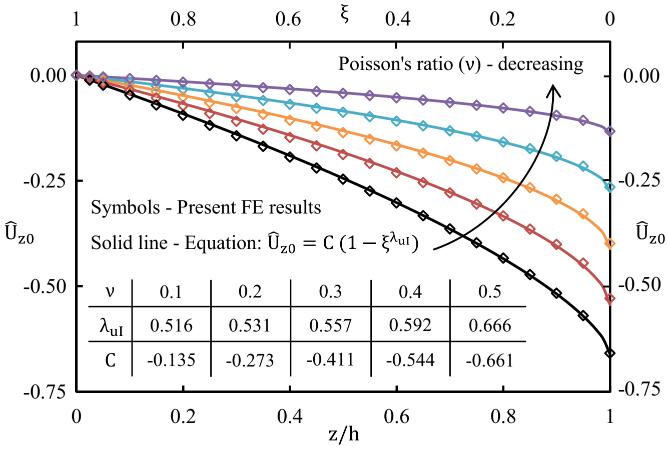


Fig. 3. Distribution of the dimensionless transverse displacement $\hat{U}_z$ along the bisector line $\phi = 0$, at different positions z along the crack front.



ν	0.1	0.2	0.3	0.4	0.5
$\lambda_{u\mathrm{I}}$	0.516	0.531	0.557	0.592	0.666
C	-0.135	-0.273	-0.411	-0.544	-0.661

Fig. 4. Dimensionless transverse displacement $\hat{U}_{z0}$ along the crack front for various Poisson's ratio ν.

From Fig. 3, we also can see three characteristic regions: near the crack tip region – I, which is encapsulated by a radial distance of approximately 0.1h from the crack tip; far field region – II, where the displacement and stress fields follow the two-dimensional plane stress solution, and, in particular, comply with the classical crack tip asymptotic expansion. This region is located outside the cylindrical domain ($r/h > 1$) encapsulating the crack tip; the third characteristic region – III, is the transition zone, in which the displacement converges from plane stress to the near-tip transverse displacement field.

Next, we provide further evidence demonstrating that this near crack-tip transverse displacement field can be linked to the corner singularity, which, in particular, affects the surface displacements near the corner point. Fig. 4 shows the dimensionless transverse displacement $\hat{U}_{z0}$ as a function of the position along the crack front z/h (or $\xi = 1 - z/h$), for different values of Poisson's ratio ν (0.1, 0.2, 0.3, 0.4 and 0.5). Due to the symmetry, the results are shown only for a half of the plate thickness, or for $0 \le z/h \le 1$. It is found that, within the near crack-tip field, the transverse displacement also follows the corner singular solutions, or $u_z \propto \xi^{\lambda_{u\mathrm{I}}}$. Here, $\lambda_{u\mathrm{I}}$ is the strength of the corner singularity (or power of the displacement function at the free surface), which is associated with mode I loading. The values of the power of the displacement function, which were calculated from the current numerical simulations, are listed in Fig. 4 for a range of Poisson's ratios. These values are also in a very good agreement with

Table 1
Reference values for (mode I) power of the stress, $\lambda_{\sigma 1}$, and displacement, λ_{ul}, at the free surface for various Poisson's ratios, ν.

Poisson's ratio (ν)	de Matos and Nowell (2008)[a] $\lambda_{\sigma 1}+1$ ($\phi=\pi/4$)	λ_{ul} ($\phi=3\pi/4$)	λ_{ul} ($\phi=\pi$)	Benthem (1977, 1980) $\lambda_{\sigma 1}+1$	Shivakumar and Raju (1990) λ_{ul} ($\phi=\pi$)	Present results λ_{ul}
0.1	0.515	0.512	0.511	–	–	0.516
0.2	0.533	0.527	0.526	–	–	0.531
0.3	0.566	0.551	0.549	0.548	0.548	0.557
0.4	0.599	0.587	0.588	0.586	0.583	0.592
0.5	–	0.659	0.667	0.668	–	0.666

[a] Mode I power of 3D corner singularity was extracted along various radial directions, ϕ.

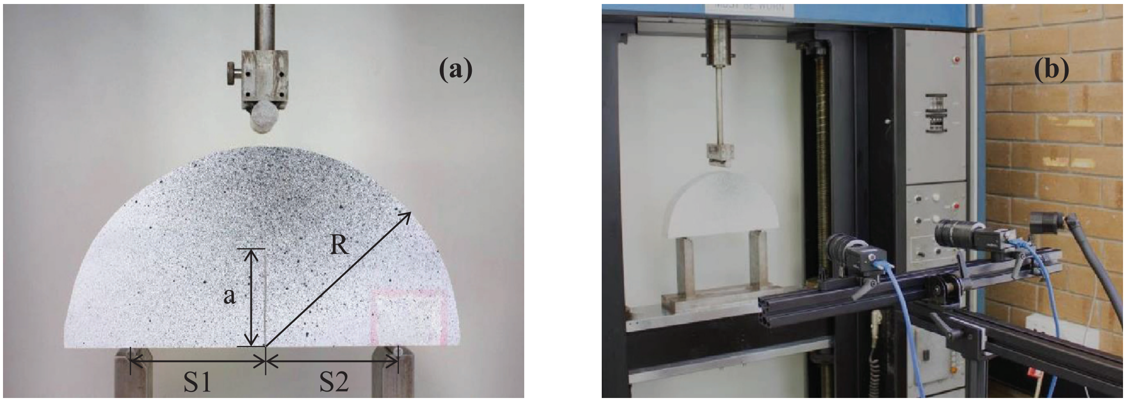


Fig. 5. (a) Edge cracked semi-circular specimen under (symmetric) three-point bend loading. (b) Experimental set up for the measurement of transverse surface displacement.

the previously published results, see Table 1. The differences between the present results and values sourced from references (Benthem, 1977, 1980; de Matos and Nowell, 2008; Shivakumar and Raju, 1990) are less than 1 per cent.

Again, based on linearity of the elastic problem under consideration and attracting dimensionless considerations, for the whole range of Poisson's ratios, the transverse displacement very near the crack front can be very accurately represented by the following equation:

$$u_z(r,\phi,z) \approx u_z(z) \approx -\frac{1.34\cdot\nu}{E} K_I^{\infty} \sqrt{h}\cdot(1-\xi^{\lambda_{ul}}) \tag{4a}$$

At the surface, $z=\pm h$, the above equation becomes:

$$u_z(r,\phi,\pm h) \approx u_z(\pm h) \approx -\frac{1.34\cdot\nu}{E} K_I^{\infty} \sqrt{h} \tag{4b}$$

The transverse strain associated with the displacement field (4) near the crack front can be approximated as:

$$\varepsilon_z(z) \approx -\frac{1.34\cdot\nu\cdot\lambda_{ul}}{E\sqrt{h}} K_I^{\infty}\cdot\xi^{\lambda_{ul}-1} \tag{5}$$

It is clear that the plane strain solution predicting $\varepsilon_z(z)=0$ can only be partially recovered for sufficiently thick plate (excluding the regions close to the free surface), when the effect of the corner singularities is negligible. In all other cases, non-zero displacements and strains exist at the crack tip. The magnitudes of the displacements and strains are related to the three-dimensional effects (3D corner singularity and plate thickness) as well as to the intensity of the remote loading (K_I^{∞}) and elastic properties (E and ν). Nevertheless, the plane stress solution cannot be recovered at the crack tip even for very thin plates.

4. Experimental study

In this section, we present the outcomes of an experimental study for edge cracked semi-circular specimens subjected to symmetric (mode I) three-point bend loading. The specimens were fabricated from 50 mm thick sheets of Polymethylmethacrylate (PMMA) with Young's modulus $E=2970$ Mpa and Poisson's ratio $\nu=0.35$.

The dimensions of the specimens (see Fig. 5a) are as follows: the radius of the semi-circular specimen $R=150$ mm, crack length $a=75$ mm and specimen thickness $2h=50$ mm. To achieve a sharp crack tip, initially a saw blade of 1 mm thickness was used to create a notch with a depth of approximately 70 mm. Then, this notch was extended with a very thin fret saw blade with a thickness of 0.3 mm to finally produce a sharp edge crack of 75 mm in total length. The bottom-support distances were set at $S1=S2=100$ mm from the centre line. The surfaces of the specimens were sprayed with a black paint. The spraying process produced a fine speckle pattern needed to facilitate the image processing for the calculation of the displacements and strains (see Fig. 5a). The maximum load applied on the top of the specimen was limited to $P=3.5$ kN to avoid brittle fracture of the specimens, which was expected at approximately $P_{max}=9.2$ kN (this corresponds to fracture toughness of PMMA of $K_{IC}=1.6$ MPa m$^{1/2}$).

The Vic-3D system (Correlated Solutions, Inc.) was employed in these experiments to accurately evaluate the transverse displacement field at the free surface of the fabricated specimens. The Vic-3D system is based on the image correlation technique (Sutton et al., 2009) and consists of two digital cameras with a range of lenses with different focal lengths to achieve a high resolution and accuracy (see Fig. 5b). Images acquired by the digital cameras during the mechanical loading were analysed using the Vic-3D software to provide the three-dimensional displacement field very close to the crack tip.

Fig. 6 shows a contour plot of (a) in-plane strain ε_{yy} component and (b) transverse displacements, u_z, around the crack tip when the semi-circular specimen is stressed in pure mode I. In the area adjacent to the crack edge the field components cannot be evaluated with DIC because the surface displacements are discontinuous or undefined. The size of this area is approximately half of the subset size (Sutton et al., 2009). This area is highlighted in grey colour in Fig. 6. From Fig. 6b, it can be seen that there exists a small circular area around the crack tip, where the transverse displacements are almost constant as predicted by theoretical calculations, see Eq. (4b). The radius of the circle is approximately one tenth of the half plate thickness ($0.1h$), which is consistent with results from past and present numerical studies, see Fig. 3.

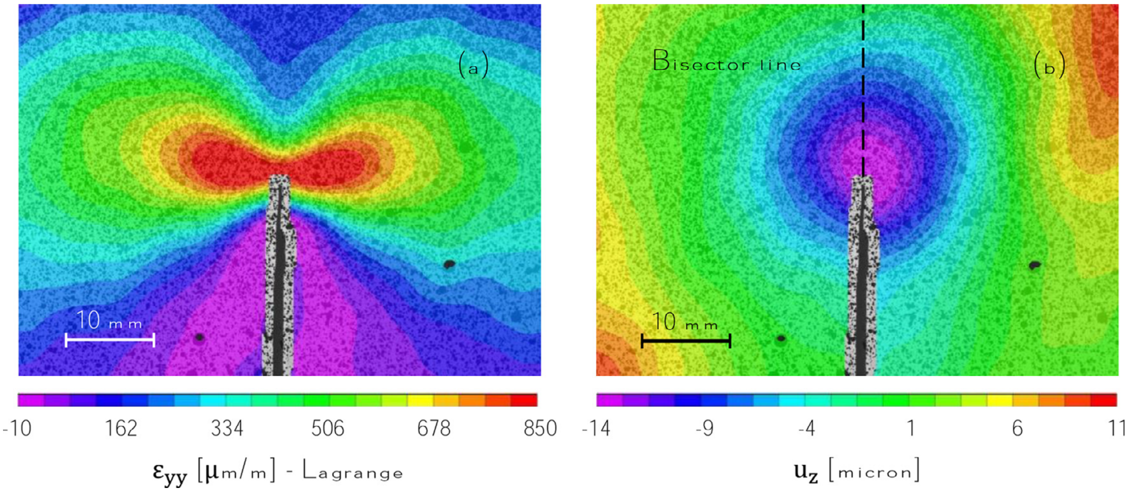


Fig. 6. Plots of (a) in-plane strain ε_{yy} contour, and (b) transverse displacement u_z contour in front of the crack tip of the semi-circular specimen.

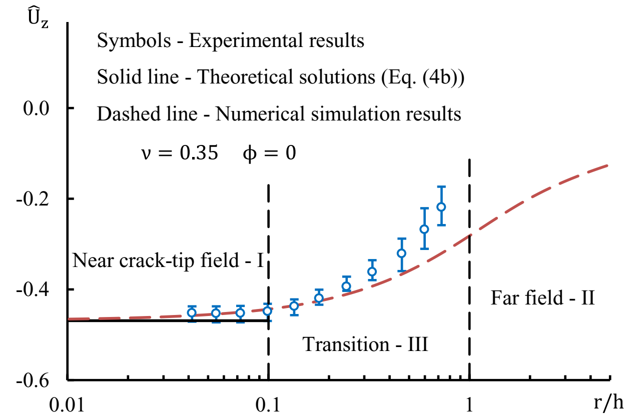


Fig. 7. Distribution of the dimensionless transverse displacement $\hat{U}_z$ along the bisector line $\phi = 0$, at the plate free surface for the selected Poisson's ratio $\nu = 0.35$.

Further, we extracted the transverse displacement component u_z, along the bisector (or symmetry) line (see Fig. 6b). The dimensionless transverse displacement $\hat{U}_z$ can then be simply derived through substituting the experimental results into Eq. (3). The value of the applied stress intensity factor, K_I^∞, is estimated using results of 2D numerical simulations by Ayatollahi et al. (2011) as

$$K_I = \frac{P}{4Rh}\sqrt{\pi a}Y_I(a/R,\ S1/R, S2/R) \qquad (6)$$

where P is the applied load, Y_I is a dimensionless geometry factor, which equals to 5.23 for the present geometry of the specimens.

Fig. 7 summarises the main theoretical and experimental results obtained in this paper. It can be seen from this figure that the experimental data agrees very well with the numerical calculations as well as with the proposed simplified Eq. (4b). The discrepancies between numerical and experimental values in region II (transition zone) can be attributed to the ignorance of higher order terms in the numerical solution (and simplified equation) as explained in Section 2. This good correlation between experimental and theoretical results opens a new opportunity for the evaluation of stress intensity factor from the transverse surface displacements in the close vicinity of the crack tip affected by three-dimensional effects.

5. Conclusion

In this work we attempted to develop a new experimental procedure for the evaluation of mode I stress intensity factor from transverse displacements in the area in the very close proximity to the crack tip. Numerical studies were conducted to establish a link between the applied stress intensity factor and the displacement field in the area of interest. This displacement field does not follow either plane stress or plane strain assumption and is excluded from analysis in the existing experimental techniques. Simple fitting equation has been proposed, which links the value of the transverse displacements with the plate thickness, applied (or remote) stress intensity factor, Young's modulus, Poisson's ratio and the strength (power) of corner singularity, which affects the displacement field close to the crack front. The experimental study did confirm that the transverse displacement field follows the theoretical solution, and this laid a foundation for the development of a new experimental procedure. It has to be noted that the correlation between experimental and numerical results in the near crack tip region became possible with the development of advanced experimental techniques, such as 3D Digital Image Correlation method, which provide a very high resolution combined with the high accuracy of the field measurements. Of course all these measurements and evaluations are possible if the size of the plastic or process zone is much less than the characteristic size of the near crack-tip area ($0.1h$). Therefore, for plastic materials the application of the current experimental procedure can be limited by small values of the applied loading.

The transverse displacement field near the crack tip for mode I was found to be almost constant and independent of the angular coordinate. This implies that the accuracy of the determination of the stress intensity factor from the near crack tip region could be essentially much better in comparison with the existing experimental techniques, which rely on a curve fitting of the experimental data to the classical plane stress asymptotic expansion of strains or displacements near crack tip. These techniques normally require an incorporation of the higher order (or non-singular) terms of the asymptotic expansion, which are all function of the angular position, to achieve a high accuracy of the evaluation. In the present experimental procedure the effect of these non-singular terms on the displacement field near the crack tip (and accuracy of the stress intensity factor evaluation) can be minimised by selecting the appropriate (small) radius of the data extraction region as discussed earlier. This means that the accuracy of the present method could be much better in comparison with the existing experimental techniques specifically when the region controlled by the asymptotic expansion is small or significantly affected by the general stress state in the plate component. Finally, the new experimental procedure can be applied to other specimen geometries and loading conditions without changing the theoretical framework.

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Chapter 6

An Experimental Method for Evaluating Mode II Stress Intensity Factor from Near Crack Tip Field

Statement of Authorship

Title of Paper	An experimental method for evaluating mode II stress intensity factor from near crack tip field
Publication Status	Published
Publication Details	Published in *International Journal of Fracture*, 2015.

Principal Author

Name of Principal Author	Zhuang He		
Contribution to the Paper	Created FE models, conducted experiment, performed analyses, interpreted data and wrote manuscript		
Overall percentage (%)	50		
Certification:	This paper reports on original research I conducted during the period of my Higher Degree by Research candidature and is not subject to any obligations or contractual agreements with a third party that would constrain its inclusion in this thesis. I am the primary author of this paper.		
Signature		Date	21/12/2015

Co-Author Contributions

By signing the Statement of Authorship, each author certifies that:

1. the candidate's stated contribution to the publication is accurate (as detailed above);
2. permission is granted for the candidate in include the publication in the thesis; and
3. the sum of all co-author contributions is equal to 100% less the candidate's stated contribution.

Name of Co-Author	Andrei Kotousov		
Contribution to the Paper	Supervised work development, participated in work discussions and co-wrote manuscript		
Signature		Date	21/12/2015

Name of Co-Author	Andrea Fanciulli		
Contribution to the Paper	Assisted in experimental data extraction and analyses		
Signature		Date	20/12/2015

Name of Co-Author	Filippo Berto		
Contribution to the Paper	Participated in manuscript review and evaluation		
Signature		Date	16/12/2015

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International Journal of Fracture, *197*(1), 119-126.

Chapter 7

On Evaluation of Stress Intensity Factor from Far and Near Vertex Fields

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Publication Details	Submitted for publication in *Experimental Mechanics*, 2015.

Principal Author

Name of Principal Author	Zhuang He		
Contribution to the Paper	Created FE models, conducted experiment, performed analyses and assisted in data interpretation, co-wrote manuscript		
Overall percentage (%)	60		
Certification:	This paper reports on original research I conducted during the period of my Higher Degree by Research candidature and is not subject to any obligations or contractual agreements with a third party that would constrain its inclusion in this thesis. I am the primary author of this paper.		
Signature		Date	21/12/2015

Co-Author Contributions

By signing the Statement of Authorship, each author certifies that:

1. the candidate's stated contribution to the publication is accurate (as detailed above);
2. permission is granted for the candidate in include the publication in the thesis; and
3. the sum of all co-author contributions is equal to 100% less the candidate's stated contribution.

Name of Co-Author	Andrei Kotousov		
Contribution to the Paper	Supervised work development, participated in work discussions and data interpretation, wrote manuscript		
Signature		Date	21/12/2015

On Evaluation of Stress Intensity Factor from Far and Near Vertex Fields

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Abstract

Experimental approaches to the evaluation of stress intensity factor of through-cracked plate components are currently based on the classical plane stress (or 2D) asymptotic power series expansion (Williams' solution). Besides the plasticity effects, the practical evaluation has to take into account a finite domain of convergence of the power series expansion as well as three-dimensional (3D) effects, which prevail in the close vicinity of the crack tip. In this paper we demonstrate and confirm that attempts to fit Williams' solution to experimental data in the near crack tip region could lead to significant errors. In addition, it is verified that the stress intensity factor can be evaluated from the out-of-plane (transverse) displacements in the region controlled by 3D effects, and in particular, by 3D corner singularity. Under mode I loading, the transverse displacement field in this region is uniform and largely unaffected by the higher order terms of the asymptotic expansion, which make this approach particularly attractive for experimental techniques.

Keywords: *Stress intensity factor; Three-dimensional effects; Digital image correlation; Williams' expansion; Plates; Linear Elastic Fracture Mechanics*

1 Introduction

The stress intensity factor (SIF) concept represents a cornerstone of the contemporary Linear Elastic Fracture Mechanics. It is not surprising that many analytical, numerical and experimental approaches were developed in the past in order to evaluate SIFs for various problems. For relatively simple geometries and loading conditions, the SIF can be directly calculated, based on the available theoretical equations presented in many handbooks (Rooke and Cartwright 1976; Tada et al., 1985; Murakami, 1987). For more complicated geometries and loading conditions, numerical approaches, such as FEM, can provide reliable estimation of SIF (Lim et al., 1992; Courtin et al., 2005; Pak and Kim, 2010). However, in many practical situations the theoretical and numerical solutions will require careful, detailed validation, which can only be achieved with the application of suitable experimental approaches (Theocaris, 1984).

The experimental approaches for the evaluation of SIFs can rely on the measurement of strain or displacement fields near the tip of a crack with strain gauges (Wei and Zhao, 1997; Dorogoy and Rittel, 2008) or laser interferometry (Humbert et al., 2000; Ravi-Chandar, 2008). Optical methods such as caustics (Baik et al., 1995; Yazdanmehr and Soltani, 2014) and photo elasticity (Ayatollahi and Nejati, 2011) have also been used extensively for estimating the stress intensity factors. The latter method is based on the property of birefringence, as exhibited by certain transparent materials. In the last couple of decades, the technique of digital image correlation (Sutton et al., 2008; 2009) has been increasing in popularity, which facilitated by the recent advances in computer technology and digital imaging. In contrast to other experimental techniques, DIC has many attractive features, including: (1) full field measurement compared with strain gages, which are limited to finite points; (2) the DIC technique can be applied to any type of materials (both opaque and transparent); (3) it needs no or very little surface preparation, experimental setup is less complex compared with caustics and photo elasticity. Moreover, the technique is suitable for measurements at various scale levels (McCormick and Lord, 2010; Brynk et al., 2012). It is also adopted for the purpose of the current study.

Consider a through-crack in a plate made of sufficiently brittle material. Further we assume that the process zone is very small in comparison with all the other characteristic dimensions. Near the crack tip on the free surface of the plate, which is available for experimental measurements, several characteristic areas can be identified (see Fig. 1). Two areas are related to the plate thickness (2h): the near crack-tip area affected by 3D corner singularity, for which characteristic size is approximately five percent of the plate thickness (or ~ 0.1h) (He et al., 2015b), and the region of 3D effects, which is confined within the half plate thickness (h) in the radial direction (Rosakis and Ravi-Chandar, 1986; Kotousov, 2010; Berto et al., 2011a). In this region the displacement and stress fields are essentially three-dimensional.

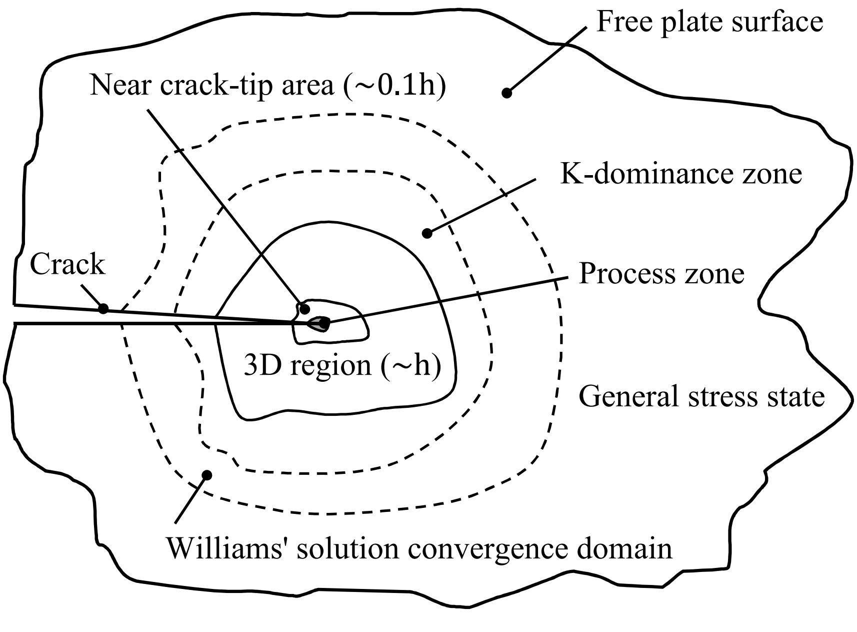

Figure 1: Characteristic surface regions around the vertex point (not to scale), 2h is the plate thickness.

As an example, Fig. 2 shows one of the 3D effects: the non-uniform distribution of the local SIF along the crack front when a crack in a thin elastic plate is subjected to remote mode I loading with intensity K_I^{ap}. The straight crack front is parallel to the z-axis, where $z = 0$ corresponds to the mid-plane and $z = h$ represents the free surface of the plate. Here, we refer to a study by He et al. (2015a); however, the same results were presented in many other articles (She and Guo, 2007; Berto et al., 2011b). The sharp decrease of the local SIF near the free surface is often attributed to the presence of 3D corner singularity, which replaces the common square root singularity at the vertex points. Therefore, the traditional evaluation of the SIF from the experimental data collected over the surface area near the vertex point could lead to large errors. A comprehensive overview of 3D effects near crack and sharp notches can be found in a number of recent papers including Kotousov et al. (2013) and Pook (2013).

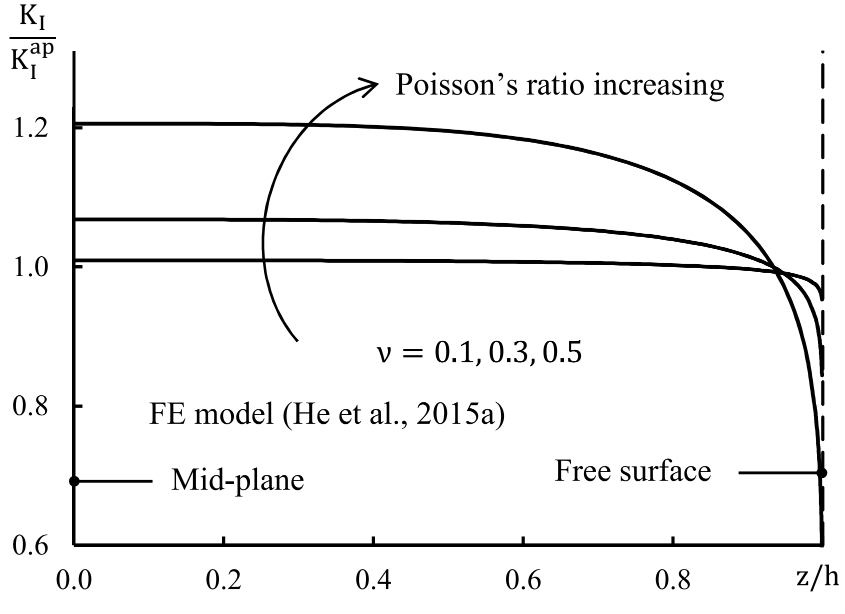

Figure 2: Stress intensity factor along the crack front for models stressed in mode I with different Poisson's ratio.

Two other characteristic regions (in Fig. 1) are largely determined by the in-plane geometry of the problem as well as the boundary and loading conditions. In the first region, which is the Williams' solution convergence domain, the stress and displacement fields can be accurately approximated by retaining a sufficient number of terms in the classical plane stress power series expansion (or Williams' solution). For example, for a crack in an infinite linear elastic plate analytical and numerical investigations have identified the existence of a circle of convergence of Williams' solution centred at the crack tip whose radius is equal to the crack length (Hello et al., 2012). This power series solution converges in the open disk limited by this circle while the convergence of the solution in Laurent series was demonstrated in the complementary open annulus.

The second region is the so-called K-dominance region, where the stress and strain fields are dominated by the leading (singular or first) term of the classical asymptotic expansion. In certain situations, specifically for thick plate components, K-dominance and Williams' solution convergence regions may not exist or can be significantly affected by 3D stress states. In this case the stress intensity factor derived for the corresponding plane elastic problem cannot be viewed as a single parameter controlling fracture initiation. Therefore, the classical LEFM criterion is, strictly speaking, applicable to analysis of brittle fracture when K-dominance zone is sufficiently large and fully encapsulate the 3D region. Extensive numerical studies indicated that the characteristic size of K-dominance zone has to be larger than the plate thickness (2h). In this case the 3D stress state near the crack front is not affected by the way in which the boundary conditions were prescribed (Kotousov, 2010). This is an additional condition of the validity of LEFM, which is often disregarded in experimental and theoretical studies.

Currently, all experimental techniques for the evaluation of stress intensity factors are based on a fitting of experimentally obtained displacement or strain data into the truncated plane stress series solution (Williams' solution). It will be demonstrated in this paper that for an accurate evaluation of SIF, the 3D region has to be excluded from the experimental data extraction area. It will be also verified that the accurate evaluation of SIFs is possible from the out-of-plane displacement data collected within the 3D region, specifically from the surface area encapsulating the crack tip with a radius less than five percent of the plate thickness, see Fig. 1, which we call the near crack tip area. The latter approach has several advantages in comparison with the existing experimental techniques. For example, it does not require any fitting procedure and the evaluation of SIF can be conducted based on a single measurement of the transverse displacement component anywhere in the area sufficiently close the crack tip.

Currently, all experimental techniques for the evaluation of stress intensity factors are based on a fitting of experimentally obtained displacement or strain data into the truncated plane stress series solution (Williams' solution). It will be demonstrated in this paper that for an accurate evaluation of SIF, the 3D region has to be excluded from the experimental data extraction area. This was first highlighted by Rosakis and Ravi-Chandar (1983) who evaluated the SIF of a through-crack as a function of the distance from the crack tip (vertex) to the data extraction region using the optical shadow spot measurement method. It will also be verified that the accurate evaluation of SIFs is possible from the out-of-plane displacement data collected within the 3D region, specifically from the surface area encapsulating the crack tip with a radius less than five percent of the plate thickness, see Fig. 1, which we call the near crack tip area. The latter approach has several advantages in comparison with the existing experimental techniques. For example, it does not require any fitting procedure and the evaluation of SIF can be conducted based on a single measurement of the transverse displacement component anywhere in the area sufficiently close the crack tip.

2 Details of Numerical and Experimental Studies

2.1 Specimen Geometry

A three point bend fracture specimen is selected for the purpose of the current study, see Fig. 3. The material is Polymethylmethacrylate (PMMA) with Young's modulus E = 2.97 GPa and Poisson's ratio $\nu = 0.35$, this material is often utilised in brittle fracture examinations. The thickness of the specimen (2h) is 50 mm, which offers a sufficient surface area for near crack tip measurements, specifically in the region affected by 3D corner singularity, ~0.1h from the crack tip. An edge crack with crack length a = 50 mm was fabricated in the rectangular plate with the total length of L = 430 mm and width of w = 100 mm. This particular geometry has been previously thoroughly investigated with 2D FEA in Liao et al. (2015). The outcomes of this analysis will be utilised in the evaluation of SIF from the asymptotic power series solution and in the verification of 3D models to be presented next.

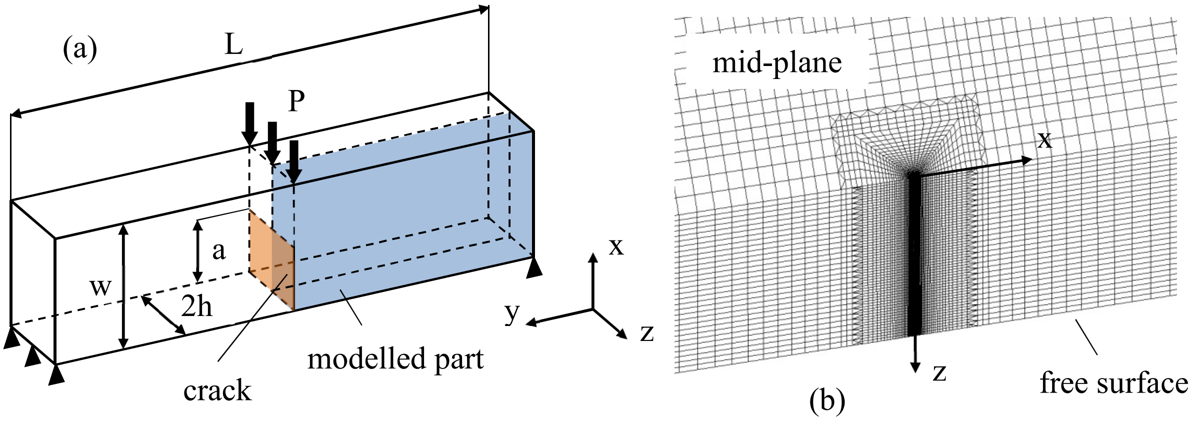

Figure 3: (a) Schematic diagram of three points bend specimen and (b) mesh details of the 3D FE model, the crack front lies along the z axis.

2.2 Details of Numerical Modelling

For investigation of 3D effects on the surface displacement field near the vertex, a 3D finite element model of the fracture specimen was developed. Fig. 3a shows the geometry and boundary conditions implemented in the FEA. One quarter of the specimen is modelled due to the double-plane symmetry of the specimen, boundary and loading conditions; this part is highlighted in blue colour in Fig. 3a. The near crack front mesh is shown in Fig. 3b. An initial arrangement of 15-node trapezoidal elements was developed around the crack front, which was surrounded by a radial array of 20-node brick elements. A uniform distribution of 60 layers was considered along the thickness direction. Similar analysis with non-uniform layers can be found elsewhere (Berto et al., 2012; Kotousov et al., 2012). The mesh has a total of 1261728 nodes and 311532 elements. The FE simulations were also carried out using the ANSYS 14.5 software package.

2.3 Details of Specimen Preparation and Experimental Procedure

The crack was created using a very thin fret saw blade. Initially a notch (with slightly less than the final specified length) was machined using water-jet cutting. Then the notch was sharpened with a thin fret saw blade of thickness 0.3 mm to finally produce a crack of 50 mm in total length. The specimen was placed on two bottom supports (symmetric with respect to the centre line) with a fixed distance of 400 mm and was loaded by a vertical load P at the centre. The three points bend test was performed in a screw-driven tensile test machine, interfaced to a computer for loading and displacement control. The maximum load applied on the top of the specimen was limited to $P = 1.24$ kN to avoid brittle fracture of the specimen, which is expected at about $P_{max} = 2.39$ kN. This value corresponds to a typical fracture toughness of PMMA, which was reported to be around $K_C = 1.6$ MPa $m^{1/2}$ (Ayatollahi et al. 2011).

2.4 Measurement of Displacement by DIC

The experimental set up is shown in Fig. 4a. The specimen for digital image correlation experiment was prepared by spraying the lateral surface with random black dots, which created a fine speckle pattern. This speckle pattern is required for the image correlation procedure for the calculation of surface displacement and strain fields.

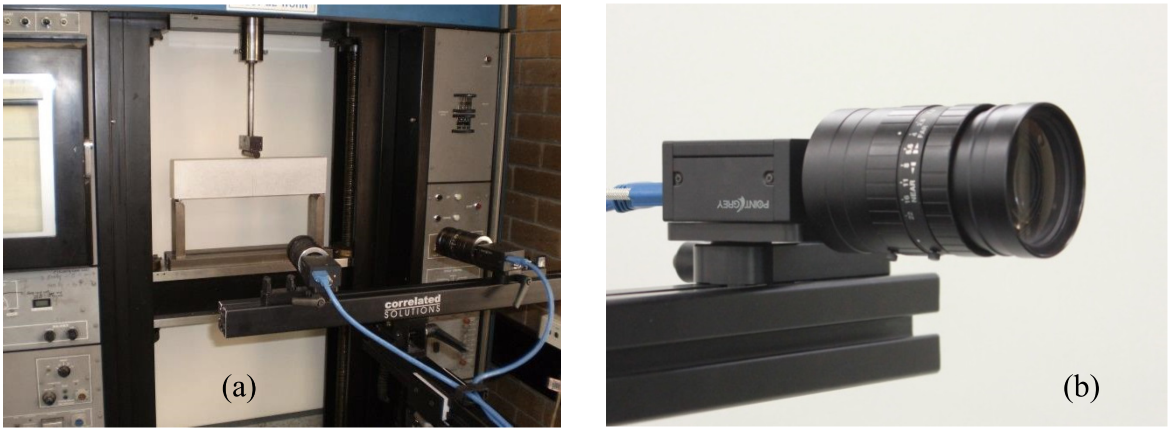

Figure 4: (a) Vic-3D (Correlated Solutions, Inc.) set up for the measurement of surface displacements; (b) Digital camera of the Vic-3D.

The Vic-3D system (Correlated Solutions, Columbia, USA) was utilised for the measurement of the full-field displacement and strain at the free surface of the specimen facing the cameras. The 3D system incorporates the image correlation software (Sutton et al., 2009) and two digital cameras with a pair of FUJINON (HF75SA-1) lenses with adjustable focal length (see Fig. 4b) to achieve the desirable resolution and accuracy. A white light source illuminated the specimen's surface during testing. A number of images of the displacement field were acquired by the digital cameras over a short period of time immediately after the maximum loading has been achieved. The displacement field images were averaged to reduce the statistical error and were correlated with the reference image (image of the unloaded specimen) to obtain the displacement and strain fields in the area of interest. The captured images had a spatial resolution of 1920×1200 pixels with an 8-bit depth. Each 1 mm of real distance corresponds to 15.29 pixels. The accuracy of the DIC algorithm was approximately equal to 0.02 pixels. In this study, a subset size of 29×29 pixels with a step size (distance between the pixels in each subset) of 7 pixels was utilised. The displacement measurement accuracy of approximately ± 0.003 mm was achieved with the current settings.

2.5 Extraction of SIF from Far Crack Tip Field

The evaluation of the stress intensity factor from the in-plane displacement data, obtained from the DIC measurements, was based on the asymptotic series solution first derived by Williams (1957) (Williams' solution). The displacement field components the in x and y directions at the crack tip can be written as:

$$\mathrm{u_x}(\mathrm{r}, \phi) = \sum_{\mathrm{n}=0}^{\infty} \frac{\mathrm{r}^{\frac{\mathrm{n}}{2}}}{2\mu} \mathrm{a}_{\mathrm{n}}^{\infty} \left[\left(\kappa + \frac{\mathrm{n}}{2} + (-1)^{\mathrm{n}} \right) \cos \frac{\mathrm{n}}{2} \phi - \frac{\mathrm{n}}{2} \cos \left(\frac{\mathrm{n}}{2} - 2 \right) \phi \right] \qquad (1a)$$

$$\mathrm{u_y}(\mathrm{r}, \phi) = \sum_{\mathrm{n}=0}^{\infty} \frac{\mathrm{r}^{\frac{\mathrm{n}}{2}}}{2\mu} \mathrm{a}_{\mathrm{n}}^{\infty} \left[\left(\kappa - \frac{\mathrm{n}}{2} - (-1)^{\mathrm{n}} \right) \sin \frac{\mathrm{n}}{2} \phi + \frac{\mathrm{n}}{2} \sin \left(\frac{\mathrm{n}}{2} - 2 \right) \phi \right] \qquad (1b)$$

here, r and ϕ are polar coordinates, the crack tip is located at the origin of the coordinate system, and the crack edges coincide with the line $\phi = \pi$, $\mu = \mathrm{E}/2(1 + \nu)$ is the shear modulus, E is the elastic modulus, ν is Poisson’s ratio, $\kappa = (3 - \nu)/(1 + \nu)$ is Kolosov’s constant for plane stress conditions. $\mathrm{a_n}$ are coefficients, which can be determined from the measurements by using a fitting procedure or found numerically. The coefficient at the first (leading) power term, $\mathrm{a_1}$, is related to the mode I stress intensity factor as $\mathrm{a_1} = \mathrm{K_I}/\sqrt{2\pi}$.

For the single-edge cracked specimen of width $w = 1$, length $L = 4w$ and crack length $a = 0.5w$ subjected to three points bend loading (force $P = 1$), Liao et al. (2015) derived the coefficients of Williams' asymptotic solution by using the weak form quadrature element method combined with the subregion generalised variational principle. The first five coefficients (a_1 to a_5) of Williams' solution for the current fracture specimen are listed in Table 1. Note that all units are self-consistent, so the material properties such as Young's modulus and Poisson's ratio do not affect the calculations.

Table 1: Coefficients a_1 to a_5 of Williams' solution for a single-edge cracked specimen, subjected to three points bend loading

a_1	a_2	a_3	a_4	a_5
4.2186	0.2625	-5.3566	0.9571	-3.2333

2.6 Extraction of SIF from near Crack Tip Field

For the evaluation of the stress intensity factor from the out-of-plane displacement data in the near crack tip region, see Fig. 1, a theoretical solution proposed by He et al. (2015b) has been employed. The authors have previously demonstrated that the out-of-plane surface displacements very near the crack tip, specifically within $0.1h$-distance of the crack tip, are independent of the angular position ϕ, and the transverse displacement field is essentially uniform in this area. Based on dimensionless considerations, the out-of-plane (transverse) displacement on the plate surface can be approximately expressed as:

$$u_z(r, \phi, \pm h) \approx \frac{-1.34 \cdot \nu}{E} K_I^{ap} \sqrt{h} + u_{0z} \tag{2}$$

where the numerical coefficient in the nominator (-1.34) was obtained from a careful 3D FE analysis and u_{0z} is the out-of-plane displacement due to remote loading parallel to the crack.

It can be stated that the above equation, in particular, indicates that the plane strain assumption ($u_z = 0$), which is often applied in theoretical studies, is not valid in the near crack tip area. This was discussed exhaustively in the literature over the past decade. In addition, it was also demonstrated (He et al., 2015b) that the higher terms of the asymptotic expansion ($a_n, n > 1$) do not affect the magnitude of the transverse displacement very near the vertex point.

Similar conclusions have been derived by Yang and Freund (1984), who applied Kane-Mindlin theory to investigate the lateral displacements near the crack tip of a semi-infinite through-the thickness crack subjected to remote K_I^{ap}. This approximate theory postulates that the transverse extensional strain is uniform in the thickness direction. Based on this assumption and utilising Fourier transform methods and the Wiener-Hopf technique, Yang and Freund obtained the following solution for the out-of-plane displacements:

$$u_z(r, \phi, \pm h) = \frac{1}{2}\left(\frac{2}{1-\nu}\right)^{\frac{1}{4}} \frac{\nu}{E} K_I^{ap} \sqrt{h} \left\{ -0.676 - 0.493 \frac{r}{\kappa} \cos \phi \right. \\ \left. + \frac{1}{2}\left(\frac{r}{\kappa}\right)^{3/2} \left(\cos\frac{\phi}{2} + \frac{1}{3}\cos\frac{3\phi}{2}\right) \right\} + O\left(\left(\frac{r}{\kappa}\right)^2\right) \qquad (3)$$

where $\kappa = \frac{1}{h}\sqrt{\frac{1-\nu}{6}}$ and $u_{0z} = 0$ in this case. This analytical solution (3) generally confirms the main outcomes of the numerical simulations, in particular, that the out-of-plane displacement near the crack tip is proportional to the applied stress intensity factor, K_I^{ap}, inverse proportional to Young's modulus E and near proportional to Poisson's ratio, ν, as $(1 - \nu)^{1/4}$ does not change significantly over the whole range of Poisson's ratios. At r/h less than 0.1, the contribution of the second order term of the power series solution (1) into the value of the out-of-plane displacements is less than five percent. However, Kane and Mindlin theory predicts the value of the out-of-plane displacement approximately three times less than the numerical results. This can be explained by the approximate nature of this analytical theory, which is based on a quite radical assumption that the transverse extensional strain is uniform in the thickness direction. In addition, this theory disregards the contribution of 3D corner singularity into the displacement field near the crack tip as described in the Introduction.

3 Results and Discussion

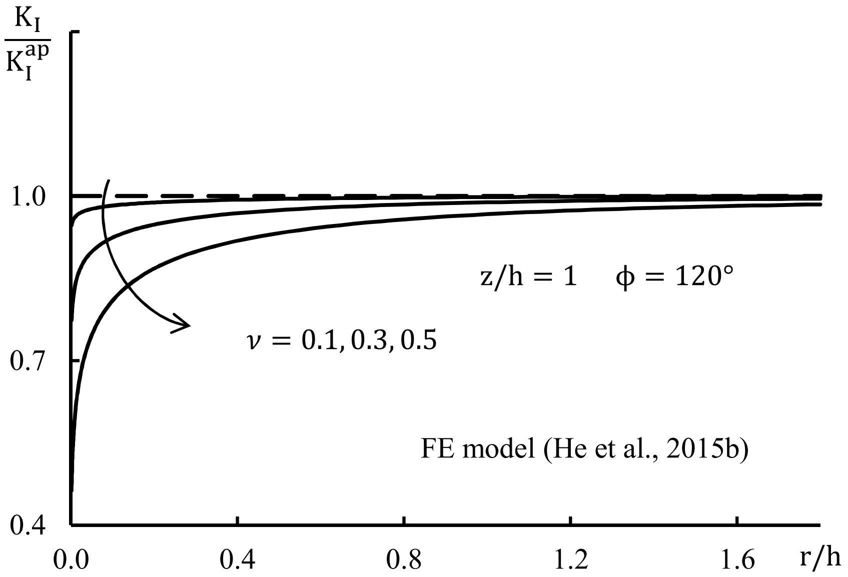

Figure 5: Distribution of stress intensity factor as a function of r/h for different Poisson’s ratio.

Furthermore, we investigate the effect of the distance of the data points from the crack tip on the accuracy of the evaluation of the SIF from Williams' solution (1). We also demonstrate that the out-of-plane displacement data collected within the near crack tip area is consistent with the theoretical predictions (2) and, subsequently, can be effectively utilised for the evaluation of SIF.

The 3D surface displacement field near the crack tip was obtained with the 3D linear elastic FE analysis as described in He et al. (2015b). The extracted in-plane components were then used for the evaluation of SIF, K_I, from the 2D power series solution (1). Fig. 5 shows the ratio of the calculated and applied stress intensity factors as a function of the distance from the crack tip to the data points at various values of Poisson's ratio.

It can be concluded that relatively far from the crack tip, say at distances further than a half-plate thickness ($r > h$), the surface displacements follow the theoretical plane stress solution (1). Therefore, outside this 3D region, the measurements of the surface displacements, in conjunction with Williams' solution, can provide an accurate evaluation of the SIF. As the distance from the crack tip to the data points decreases, the error in the SIF evaluation increases and becomes unacceptably large in the near crack area, see Fig. 5.

The above observations and conclusions were fully confirmed with the experimental measurements of the displacement field with the 3D DIC technique, as described in the previous sections. Fig. 6 shows the typical 3D contour plot of the out-of-plane and in-plane displacement components, later components of the displacement field were analysed based on the Williams' solution (1), for which coefficients were presented in Table 1. The first three terms were sufficient for the solution to converge. The calculated SIF, based on experimental measurements together with the corresponding FEA, as described in Section 2, are shown in Fig. 7 for various distances r/h and angular positions ϕ.

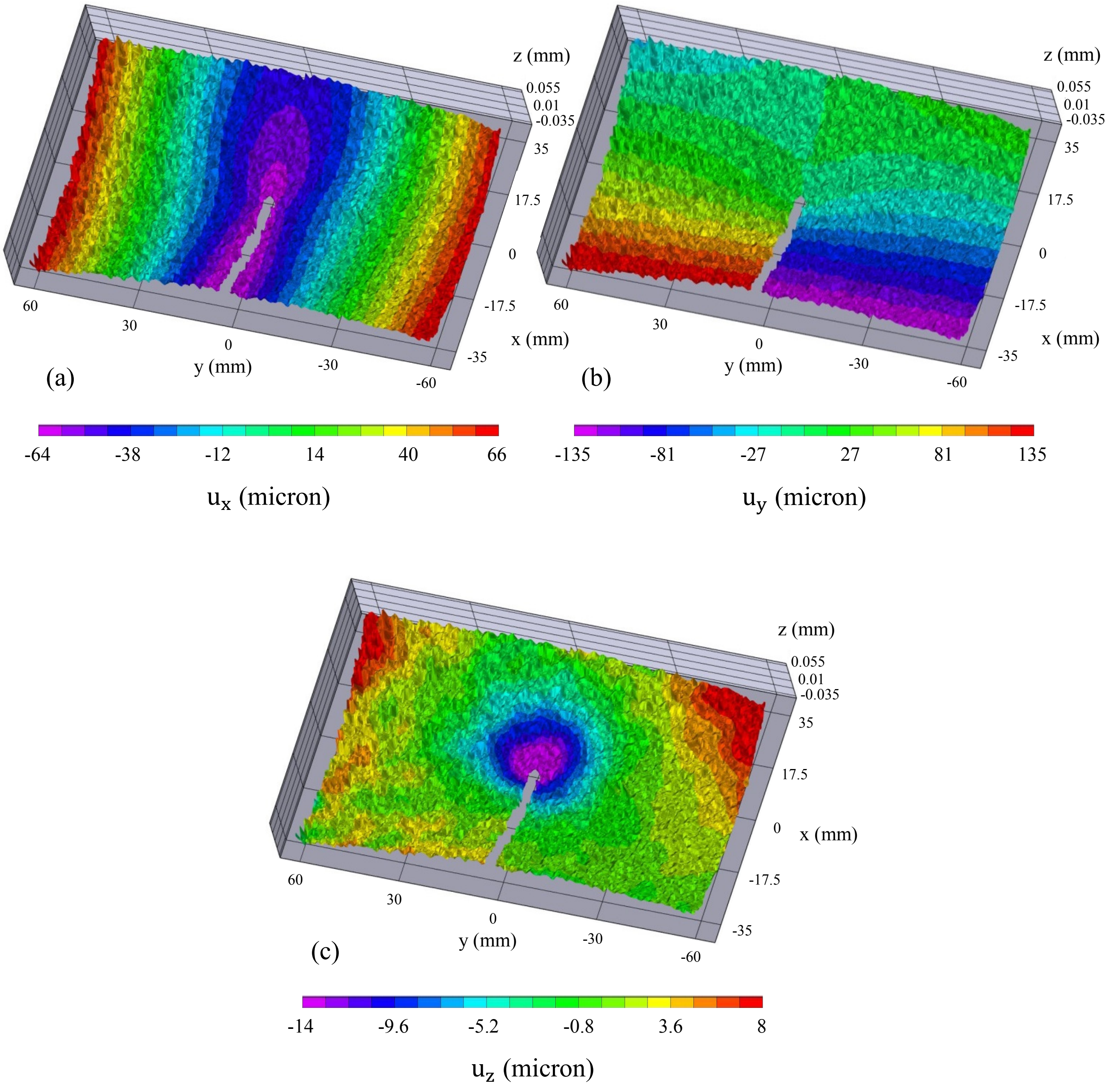

Figure 6: 3D contour plots of (a) in-plane displacement u_x (b) in-plane displacement u_y and (c) out-of-plane displacement u_z in front of the crack tip. The rigid body translation and rigid body rotation were eliminated.

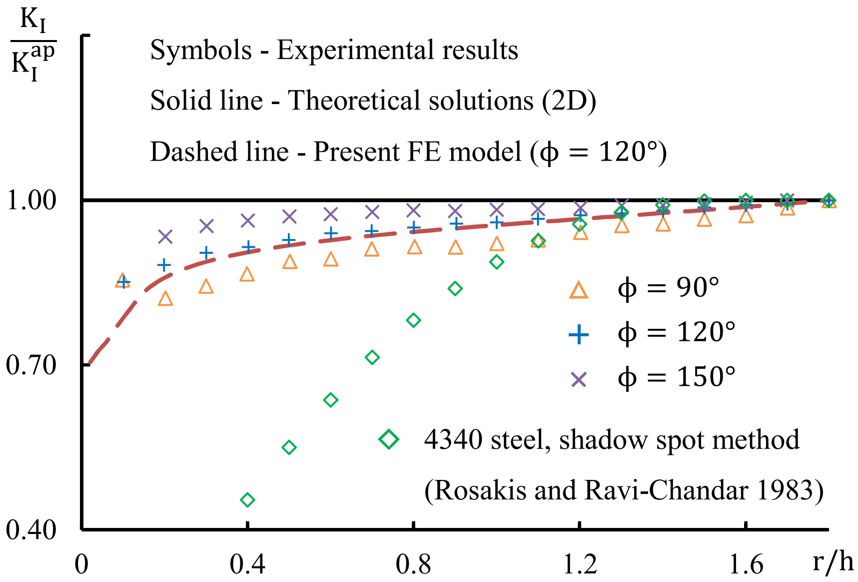

Figure 7: Stress intensity factor (calculated based on u_y).

It can be noted that the experimental data and corresponding numerical results (from the present FE analysis) correlate very well. Another interesting feature of the obtained results is that the error in the evaluation of the SIF decreases at larger angles, ϕ. Furthermore, experimental results obtained by Rosakis and Ravi-Chandar using the optical shadow spot measurement method are plotted in Fig. 7 (note that the actual experimental data were extracted from Yang and Freund, 1984), and large errors in SIF evaluation within the 3D region can also be observed. These errors are larger than those obtained using the DIC technique. All the results indicate that in order to avoid large errors in the evaluation of the SIF, the experimental measurements have to be taken outside the 3D region.

Finally, in Fig. 8 we plotted experimental results obtained for the normalised out-of-plane displacement components for the present specimen geometry (three point bend fracture specimen) as well as the earlier results for a semi-circular specimen. The experimental results indicate that the magnitude of the out-of-plane displacement field in the close vicinity of the vertex is essentially constant, and it does not vary with the angular position, $u_z(r, \phi, \pm h) = \text{const}$. Another observation is that the theoretical equation (2) is applicable to describe the magnitude of the displacements for various geometries of fracture samples. This indirectly verifies the outcomes of numerical simulations to show that the higher order terms of asymptotic expansion do not significantly affect the transverse field near the vertex point. These findings form the basis for a new experimental technique for the evaluation of SIF from the out-of-plane displacements in the near crack tip field. In this technique, one of the parameters in Equation (2) can be identified if the other three are known. In particular, the SIF can be based on the measurements of the transverse displacement near the vertex point, provided that the elastic properties and the plate thickness are known.

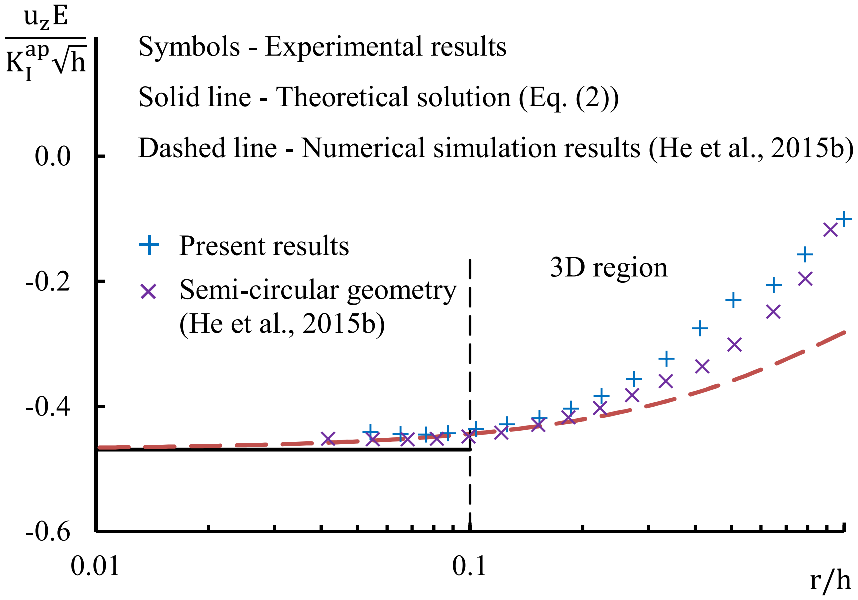

Figure 8: Normalised out-of-plane (transverse) displacements on the plate free surface ($z/h = 1$) as a function of distance from the vertex (crack tip) point r/h.

4 Conclusions

The presented outcomes of the numerical simulations and experimental studies indicate that, in order to achieve an accurate evaluation of SIF based on the classical 2D asymptotic power series expansion, the experimental measurements have to be extracted outside the 3D region, see Fig. 1. This surface region is confined within a half plate thickness from the vertex in the radial direction. The error in the experimental evaluation of the SIF increases if the measurements are taken closer to the crack tip in this region. These conclusions are based on particular specimen geometry and locations of data points; however, it is believed that these represent the general tendencies, which are common for all fracture geometries.

It was demonstrated earlier that the out-of-plane displacement in the near crack-tip region is a linear function of the applied stress intensity factor and Poisson's ratio, inverse proportional to Young's modulus and square root of the plate thickness, Eq. (2). The experimental verification was conducted for two different specimens to demonstrate the independence of the magnitude of the out-of-plane displacement near the crack tip from the geometry of the fracture specimen as well as the higher order terms of the asymptotic expansion (1).

The obtained results provide a new way to evaluate SIF from the measurements of the out-of-plane displacements in the close vicinity of the crack tip. Of course, the evaluation is possible when the material is sufficiently brittle or loads are small and the process zone or zone of plastic deformations is much smaller than all the other characteristic dimensions. The new approach can have several advantages. It does not require any fitting procedure, which is normally needed for the calculation of the coefficients in the asymptotical power series expansion for finite geometries. The measurements can be conducted anywhere in the near crack tip region as the magnitude of the out-of-plane (transverse) displacements in this region is essentially constant. The latter also implies that plane stress or plane strain assumptions provide incorrect results for the out-of-plane displacements and stress components in the close vicinity of the tip of a crack or in the 3D region. The characteristic size of the 3D region is not related to the plasticity effects and is of an order of the plate thickness. This imposes an additional condition on the validity of the classical LEFM, which is often disregarded in Fracture Mechanics texts.

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Chapter 8

Conclusions and Future Work

8.1 Summary

As highlighted in the Introduction, despite considerable advances made in fracture mechanics over the past few decades, there are still many research areas which need further understanding and development (Erdogan 2000). A careful literature review conducted at the beginning of candidature has identified 3D crack problems as a key area where further intensive research is required. Of course, this represents a very wide area of Fracture and Applied Mechanics. Consequently, the current research aimed to elucidate the role of some 3D effects on deformation, fracture and fatigue phenomena. In particular, it investigated the effects of 3D corner singularity on stress intensities and the displacement field near the crack front, fatigue crack front shape evolution and fatigue crack growth rates. A significant part of this work was devoted to the development of a new experimental technique for the evaluation of mode I and mode II stress intensity factors from the measurements of the near crack tip displacement field, which is affected by 3D stress states. To the best of my knowledge, this has never been attempted previously worldwide and represents one of the main outcomes of this study. The purpose of the following sections is to provide a brief summary of the major outcomes of this thesis. Recommendations for future work are also outlined.

8.2 Development of A Simplified Method for Evaluating Fatigue Crack Front Shapes

Over the past few years there have been a number of very careful numerical studies focusing on the evaluation of fatigue crack front shapes (Branco & Antunes 2008; Ševčík et al. 2012). However, the application of the direct numerical techniques to fatigue phenomena is a very tedious and time consuming process and often these procedures can be quite ambiguous (Lin & Smith 1999). One major outcome of this PhD project was the development and validation of a simplified method for the evaluation of fatigue crack front shapes under mode I loading. The method is based on a number of simplifications and utilises an analytical model developed earlier (Codrington & Kotousov 2009) for plasticity-induced crack closure in plates of finite thickness, as well as linear-elastic 3D finite element modelling. The application of the analytical model for calculation of the effective stress intensity factor, based on the classic plasticity-induced crack closure, allows for a significant reduction in the complexity of the evaluation procedure. By using the developed method, the influence of the curved crack front on the fatigue crack growth and effective stress intensity factor was further analysed. The results showed that the mid-section effective stress intensity factor calculated for a straight front can provide a good approximation of the effective stress intensity factor range in the case of stable fatigue crack propagation. Another important conclusion is that for materials with, or conditions resulting in, a high sensitivity to the effective stress intensity range, the difference in fatigue life predictions based on the two-dimensional and three-dimensional considerations can be very significant (the maximum difference between the normalised load ratio parameters was found to be 50% in the present study). This also highlights the significance of this 3D effect in fatigue evaluation of plate components.

8.3 Investigation of the Effect of 3D Corner Singularity on Stress Intensities near Plate Free Surfaces

The stress intensity factor distribution along the front of a through-thickness crack is influenced significantly by the presence of the 3D corner singularities (Pook 2013). All existing 3D FE studies indicated that for mode I, the SIF rapidly decreases near the free surface and for mode II, it increases sharply (She & Guo 2007; Manrique et al. 2012). However, it is unclear what the limiting values of SIF near the free surface are and whether these values are infinite or bounded at the vertex point. An extensive finite element study was conducted to investigate the influence of 3D corner singularities on the stress intensity for a straight through-thickness crack in sufficiently large elastic plate subjected to mode I/II loading. A theoretical equation was proposed, which can accurately describe the SIF behaviour near the corner (vertex) point. It is demonstrated that the asymptotic behaviour of SIF near the free surface is governed by the difference in the strength of the corner and edge singularities. The predicted unbounded growth of SIF near the free surface under mode II loading can explain instabilities associated with the propagation of a through-thickness crack under shear loading. The theoretical analysis also implies that the brittle fracture is likely to be initiated in the middle section of the straight crack front if the crack is subjected to mode I loading and near the free surfaces if it is subjected to mode II loading. However, it is still unclear how significant these effects in practical situations are. Subsequently, further experimental studies are expected to shed more light on this problem.

8.4 Development of an Experimental Method for Evaluating SIF from Near Crack Tip Displacement Field

All current experimental methods of determining stress intensity factors are based on the assumption that the state of stress near the crack tip is plane stress (McNeill et al. 1987; Zhang & He 2012). Therefore, these methods rely on strain or displacement measurements made outside the 3D region, which is confined within a half plate thickness from the crack tip in the radial direction (Rosakis & Ravi-Chandar 1986; Kotousov 2010; Berto et al. 2011). One major outcome of this project was the development and validation of a new experimental approach for evaluating mode I/II stress intensity factors from out-of-plane displacements in the area adjoined to the crack tip, which is controlled by 3D effects. Extensive numerical studies were conducted to establish a link between the applied stress intensity factors and the displacement field in the area very close to the crack tip. This displacement field follows neither the plane stress nor the plane strain assumptions. Simple fitting equations were proposed, which link the value of the out-of-plane displacement component with the applied stress intensity factor, plate thickness, properties of the material, as well as strengths of out-of-plane singularity and corner singularities. For validation, the measured experimental data were compared with those obtained from theoretical solutions and finite element computations. A good correlation was obtained, which confirmed that the developed approach has sufficient accuracy for engineering applications. It is also found that the out-of-plane displacement field near the crack tip (for both mode I and II conditions) is not significantly affected by the higher order non-singular terms of crack tip asymptotic expansion. This implies that the accuracy of the determination of stress intensity factors from the near crack tip region could essentially be much better in comparison with the existing experimental techniques, which rely on curve fitting of the leading (singular) and higher order non-singular terms in the asymptotic expansion.

8.5 Recommendations for Future Work

In the present study, we developed a simplified approach for evaluating the fatigue crack front shapes. However, this new approach was limited to through-the-thickness cracks subjected to constant amplitude loading only. Thus, further research may focus on extending this approach to variable loading and evaluating the crack front shapes in the presence of modes II and III. It is expected that these developments will contribute significantly to the evaluation of the fatigue life of structural components. The understanding of how the front shape changes with the loading can also assist in failure investigations. The present study can be considered as an initial step leading to a wider area of research.

In this thesis, the influence of 3D stress states (in particular the vertex singularities) on the stress intensities and out-of-plane displacement fields was thoroughly investigated for a straight through-thickness crack in a sufficiently large elastic plate subjected to mode I/II loading. The approach and results can (and have to) be generalised for curved crack fronts, which can be formed during fatigue loading, as described above. The important and new area of research is the 3D investigations of the stress state near the vertex point for sharp corners and bi-material wedges. For these problems, an application of analytical techniques is quite challenging. It seems that only the theoretical values for the strength of the singularities at the vertex point can be found using analytical techniques. At the same time, the developed numerical models are capable of providing an accurate evaluation of the stress field, which can be validated using analytical results. These could result in better understanding of the stress state of crack and notched components. Further research can also incorporate rounded notches. The fillet radius leads to the finite stresses; however, the stress distribution can still be controlled by stress asymptotic.

The new experimental approach first developed in the present thesis (which was published in a number of papers) needs a more comprehensive validation using different samples and loading conditions. It has the potential to become a standard tool for the experimental evaluation of stress intensity factors. Further research can also be directed towards mode III loading. The evaluation of plasticity and the finite crack tip radius effect may represent another important task, which was not fully addressed in this thesis. Nevertheless, the obvious extensions of this approach are the notched components. The methodology utilised in the current thesis (numerical simulations, development of simplified equations and experimental validation with different samples and loading conditions) can be applied in this case as well.

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Appendix A

On Influence of Non-Singular Stress States on Brittle Fracture

Statement of Authorship

Title of Paper	On influence of non-singular stress states on brittle fracture
Publication Status	Published
Publication Details	Published in *International Journal of Fracture*, vol. 185, no. 1, pp. 201-208, 2014.

Principal Author

Name of Principal Author	David Gardeazabal		
Contribution to the Paper	Assisted in FE model creation and analyses, wrote the manuscript		
Overall percentage (%)	50		
Signature	NA	Date	

Co-Author Contributions

By signing the Statement of Authorship, each author certifies that:

1. the candidate’s stated contribution to the publication is accurate (as detailed above);
2. permission is granted for the candidate in include the publication in the thesis; and
3. the sum of all co-author contributions is equal to 100% less the candidate’s stated contribution.

Name of Co-Author	Zhuang He		
Contribution to the Paper	Created FE models, performed analyses, interpreted data and co-wrote manuscript		
Certification:	This paper reports on original research I conducted during the period of my Higher Degree by Research candidature and is not subject to any obligations or contractual agreements with a third party that would constrain its inclusion in this thesis.		
Signature		Date	21/12/2015

Name of Co-Author	Andrei Kotousov		
Contribution to the Paper	Supervised work development, participated in work discussions and co-wrote manuscript		
Signature		Date	21/12/2015

Gardeazabal, D., He, Z. & Kotousov, A. (2014). On influence of non-singular stress states on brittle fracture.
International Journal of Fracture, *185*(1), 201-208.

Appendix B

Application of Digital Image Correlation Technique for Investigation of the Displacement and Strain Field within a Sharp Notch

Statement of Authorship

Title of Paper	Application of digital image correlation technique for investigation of the displacement and strain fields within a sharp notch
Publication Status	Published
Publication Details	Published in *Theoretical and Applied Fracture Mechanics*, 2015.

Principal Author

Name of Principal Author	Andrei Kotousov		
Contribution to the Paper	Supervised work development, participated in work discussions and wrote manuscript		
Overall percentage (%)	50		
Signature		Date	21/12/2015

Co-Author Contributions

By signing the Statement of Authorship, each author certifies that:

1. the candidate's stated contribution to the publication is accurate (as detailed above);
2. permission is granted for the candidate in include the publication in the thesis; and
3. the sum of all co-author contributions is equal to 100% less the candidate's stated contribution.

Name of Co-Author	Zhuang He		
Contribution to the Paper	Conducted experiment, performed all analyses, interpreted data and co-wrote manuscript		
Certification:	This paper reports on original research I conducted during the period of my Higher Degree by Research candidature and is not subject to any obligations or contractual agreements with a third party that would constrain its inclusion in this thesis.		
Signature		Date	21/12/2015

Name of Co-Author	Andrea Fanciulli		
Contribution to the Paper	Assisted in experimental data extraction and FE model creation		
Signature		Date	20/12/2015

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Application of digital image correlation technique for investigation of the displacement and strain fields within a sharp notch

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ABSTRACT

Prof. Lazzarin received the recognition for his distinguished contributions into the analysis of stresses in structural components weakened by sharp and blunt notches. He has been instrumental in developing the strain energy density approach for the evaluation of fracture initiation and characterisation of fatigue life. Some of the latest works of Prof. Paolo Lazzarin were devoted to the investigation of 3D effects in structures weakened by various stress concentrators. In this paper we continue to elucidate the role of 3D effects and apply the advanced digital image correlation technique to investigate experimentally the displacement and strain fields near a sharp notch front. The outcomes of this study are compared with the previous analytical and numerical results. In particular, we demonstrate the variation of the strain field across the thickness near the notch front, which cannot be described with the classical solutions of the plane theory of elasticity.

1. Introduction

In this section we provide a brief summary of main contributions into the linear-elastic analysis of stresses and brittle fracture of structural components weakened by sharp notches. In this summary it was impossible to review or mention all significant achievements made by many researchers in the past sixty years [1]. Therefore, this review will be mainly focused on relatively recent developments, namely, the investigation of 3D effects associated with sharp notches. These effects were a subject of intensive analytical and numerical investigations over the past two decades, which were facilitated by advances in computer power and numerical algorithms [2–5]. These investigations were motivated by the essential discrepancies between the classical results of plane (or 2D) theories of elasticity and the corresponding 3D elastic solutions [1,6]. Many recent experimental studies have also confirmed a significant role of 3D effects in fracture and fatigue phenomena [6–8].

1.1. Linear-elastic stresses at sharp notches

The possibility of the presence of a singular (unbounded) stress at the tip of a sharp notch in a liner-elastic plate was first demonstrated by Williams [9]. He utilised the classical plane (2D) theory of elasticity and an eigenvalue expansion approach to represent the stress and displacement fields in power series in the vicinity of the tip of a sharp notch. It was demonstrated that an application of the appropriate boundary conditions on radial edges results into a set of eigenvalue equations for the determination of the degree of singularity of the stress and strain components. This degree of singularity depends on the type (or mode) of loading as well as the notch opening angle only, and is independent of the overall geometry of the plate component and the magnitude of the applied loading. The normalised intensities of the asymptotic stress fields or coefficients of the singular terms in the asymptotic expansion are known as the notch stress intensity factors. These factors can be viewed as an extension of the conventional stress intensity factor concept adopted in the classical Fracture Mechanics [10,11].

The characterisation of singular stress states and its intensities at points where there is an abrupt change in geometry, material properties or boundary conditions leading to unbounded stresses was a subject of intensive investigations in Fracture Mechanics. The early contributions, however, focused mostly on the evaluation of the degree of singularity as a function of the local notch geometry and combination of material properties [12–15]. Meanwhile, the practical applications of the theoretical concepts and results in the design, fracture and fatigue evaluations of joints and structural components began two decades later [16–18].

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1.2. Fracture and fatigue evaluation of notched components

Most fracture and fatigue initiation criteria in elastic brittle homogeneous and isotropic structures weakened by sharp notches can be roughly divided into two groups: energy-based, and stress-based [3]. The latter group of criteria normally postulate the existence of critical or threshold value of the notch stress intensity factor, which is considered as a material property [16–18]. However, these values are different for different notch opening angles. This situation significantly complicates the practical application of the notch stress intensity factor concept, and, therefore, this concept hasn't had the similar development and appreciation by the fracture community as it had for crack problems. For practical evaluations one would normally require an incorporation of the notch stress intensity factors of different fracture modes and different notch angles into a single parameter. This requirement led to an adoption of additional assumptions and, a subsequent development of various stress-based fracture criteria. These stress-based criteria were applied to the evaluation of fracture and fatigue initiation of components as well as failure assessments of joints and bonds [16–23].

Energy-based criteria utilise the critical energy release rate (ERR) concept, which was originally proposed by Irwin [24]. According to this criterion a crack initiates when the ERR reaches a critical value, which is considered to be a material property. For linear-elastic materials and zeroth notch opening angle (crack) the stress- and energy-based criteria are equivalent and predict the same conditions of failure initiation. However, an application of the energy concept to notched components is not trivial and again has to rely on additional hypothesis or postulates in order to reduce the number of material constant needed for fracture characterisation [25].

Among energy-based fracture criteria, the strain energy density criterion, suggested by Prof. Lazzarin and his colleagues, has many advantages [25,26]. It is based on the evaluation of the averaged strain energy density over a control volume, and therefore, it has a simple physical interpretation. Further, this criterion allows a direct comparison of fracture initiation conditions of notched components with different notch opening angles overcoming the problem of combining the notch stress intensity factors of different fracture modes and different notch angles into a single parameter. In the latest works these concept was successfully developed for a number of applications [25–31].

1.3. 3D effects near sharp notch fronts

In problems with sharp notches the 2D theories of elasticity can often lead to peculiar results due, in part, to the fact that these are approximate theories even when the governing equations of these theories are solved exactly. From a large number of 3D numerical simulations conducted over the past two decades the following characteristic regions can be identified: region I is the half-spherical region controlled by the 3D corner singularity $(R \sim 0.1h)$ [2], region II is the cylindrical region $(r \sim h)$ where the stress state does not follow either plane stress or plane strain assumptions [32,33]. Other two regions can also exist in many practical problems: III is the K-dominance zone as it is defined in the classical Fracture Mechanics and IV general state, as illustrated in Fig. 1.

One of the characteristic features of 3D solution of elastic problems with sharp notches is the existence of coupled modes, (which are illustrated in Fig. 2 for a particular case of the notch opening angle $\alpha = 0$ or crack) [34,35]. These coupled modes cannot be described within the plane stress/strain theories of elasticity. The coupled mode in shear loading was called the out-of-plane mode, or mode O, to distinguish it from the conventional mode III. It was also demonstrated that the out-of-plane mode is generated due to the three-dimensional effects linked to Poisson's ratio of the material, and it has the same characteristic equation for the degree of stress singularities as the conventional mode III. It is a localised fracture mode, which disappears with the distance from the notch front and it is significantly affected by Poisson's ratio in contract to fracture mode III. A similar coupled mode is generated in the case of existence of mode III loading. In this case the characteristic equation describing the degree of singularity of the coupled mode is the same as for mode II. A large computational effort was recently directed to the characterisation of these coupled modes for various structural components [36,37]. However, this area lacks of experimental studies to verify experimentally all theoretical findings, and, subsequently, one of the objectives of this work is to fill this gap.

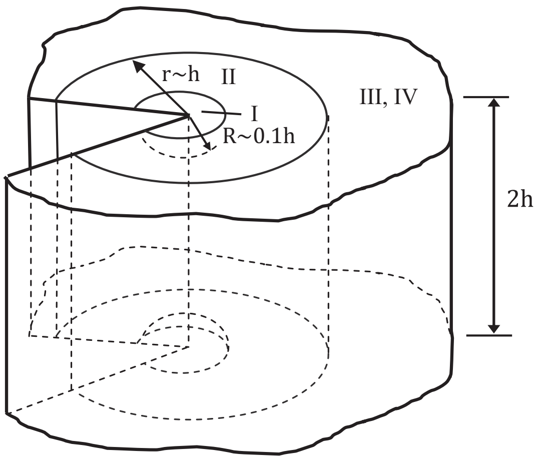

Fig. 1. Characteristic regions around the front of a sharp notch.

In the case when the 3D region is fully encapsulated by the K-dominance zone, then based on the dimensionless considerations the effect of the plate thickness on the intensities of the coupled modes and 3D corner singularity can be explicitly derived. For example, this effect predicts an increase of the intensity of the coupled mode generated by mode II loading with an increase of the plate thickness. An opposite dependence can be obtained for the coupled mode generated by mode III loading [32,38]. These dimensionless considerations and the predicted thickness effect were also validated in a number of careful numerical studies but yet to be validated experimentally.

1.4. Objectives of this paper

Over the past two decades the digital image correlation (DIC) method is gaining in popularity, which is facilitated by the recent advances in computer power, digital imaging and processing. DIC is a full-field image analysis method, based on grey value digital images that can determine the surface displacements and deformations. This method is very attractive as it needs no or very little surface preparation. The experimental setup is far simpler as compared with caustics and photo-elasticity methods. Moreover, it is suitable for measurements at various scale lengths [39,40]. This method will be utilised in the current work to investigate experimentally the strain and displacement fields in sharp notches subjected to mode I and II loading. In particular, we demonstrate the disappearance of the in-plane singularity for sharp notches subjected to mode II loading. Further, we are focusing on the 3D region (see Fig. 1), where the displacement and strain fields are essentially three-dimensional. We investigate with DIC the variation of the

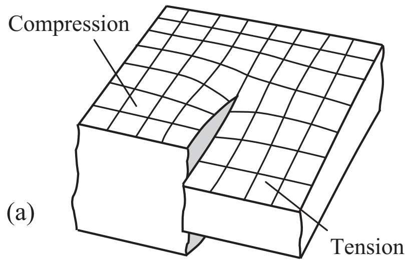

Fig. 2. Illustration of coupled fracture modes due to Poisson's effect and the redistribution of stresses close to the free surfaces for a notch (crack) subjected to (a) shear and (b) anti-plane loading.

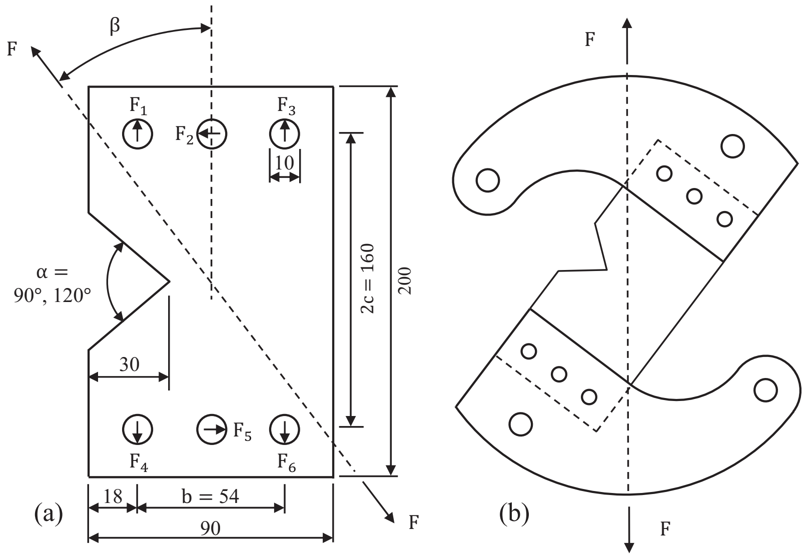

Fig. 3. (a) Configuration of the single-edge-notch specimens (all dimensions in mm) and (b) loading device.

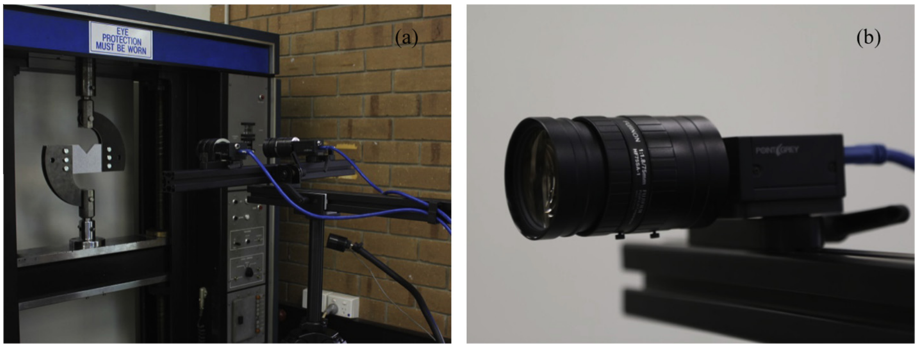

Fig. 4. (a) Experimental set up for the measurement of in-plane strain at the free surface of the specimen; (b) digital camera of the Vic-3D adopted in this work.

strain field across the thickness near the notch front and compare these measurements with a careful 3D finite element analysis.

2. Test samples

2.1. Specimen preparation

The sharply notched specimens were manufactured from Polymethylmethacrylate (PMMA) with Young's modulus $E = 2.97$ GPa and Poisson's ratio $\nu = 0.35$, which was widely used in the past to investigate brittle fracture phenomena. Fig. 3a illustrates the principal dimensions of the test samples. The test samples had two different notch angles $\alpha = 90^{\circ}$ and 120° to investigate the effect of the notch opening angle on the displacement and strain fields.

The "loading device" (the term used by Richard and Benitz) is shown in Fig. 3b, which is similar to that developed in [41] for the generation of mixed mode loading conditions. It is capable of producing pure mode I and pure mode II conditions by changing angle β (see Fig. 3a) between the longitudinal axis of the specimens and the load direction. The specimens have circular holes while the loading set up has elongated holes: (1) external holes are elongated in the direction parallel to the notch bisector line so that forces will always be normal to the notch bisector line; (2) middle holes are elongated in the direction perpendicular to the notch bisector line

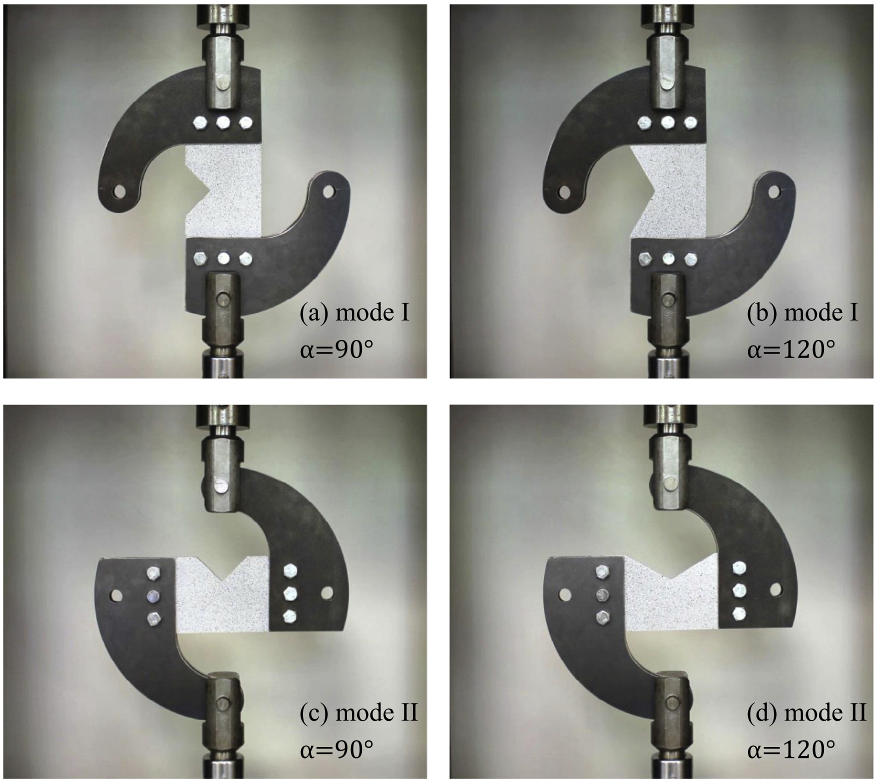

Fig. 5. Specimens (with different notch angles) loaded in different modes: (a) $\alpha = 90^\circ$ loaded in mode I; (b) $\alpha = 120^\circ$ loaded in mode I; (c) $\alpha = 90^\circ$ loaded in mode II; (d) $\alpha = 120^\circ$ loaded in mode II.

so that only forces parallel to the notch can be transmitted from the loading device to the test specimen. The test specimens for the investigation with DIC technique were prepared by painting the surface with a thin layer of white paint and then spraying black paint, which created a random speckle pattern to facilitate the image correlation process for the calculation of surface displacement and strain fields [42].

2.2. Experimental procedure

The experimental set up is shown in Fig. 4a. The tests were performed on a screw-driven (INSTRON) tensile testing machine, which had the maximum load capacity of 100 kN. All tests were carried out in air and at room temperature. The Vic-3D system (Correlated Solutions, Inc., Columbia, USA) was employed in this work to evaluate the three-dimensional strain and displacement fields around sharp notches subjected to pure mode I and pure II loading. The 3D system incorporates the image correlation software [42] and two 2.8 Megapixel digital cameras with a pair of FUJINON (HF75SA-1) lenses with adjustable focal lengths to achieve the required resolution and accuracy (see Fig. 4b). A standard LED lighting system was also utilised to illuminate the sample surfaces. Forty images acquired by the digital cameras in a short period immediately after loading were averaged to reduce the statistical error and correlated with the reference image of the unloaded sample to obtain the three-dimensional strain and displacement fields of the specified regions.

3. Results and discussion

3.1. Strength of in-plane singularities

In the beginning we investigated the strain field at the free surface of the specimens. Four different combinations of specimen geometries and loading conditions were considered as shown in Fig. 5, namely for notch angle (a) $\alpha = 60^\circ$ loaded in mode I, (b) $\alpha = 120^\circ$ loaded in mode I, (c) $\alpha = 90^\circ$ loaded in mode II and (d) $\alpha = 100^\circ$ loaded in mode II. In all these cases, the specimen thickness was chosen as $2h = 15$ mm and the maximum load applied on the loading device (see Fig. 3) was limited to a constant value of $F = 3.5$ kN (which is smaller than the critical fracture load) to avoid brittle fracture of the specimens.

Fig. 6 shows the contour maps of in-plane normal strain ε_y around the notch tip obtained from digital image correlation analysis for specimens with notch angle (a) $\alpha = 90^\circ$ and (b) $\alpha = 120^\circ$ loaded in mode I, and in-plane shear strain ε_{xy} (tensorial shear strain) fields for specimens with notch angle (c) $\alpha = 90^\circ$ and (d) $\alpha = 110^\circ$ subjected to pure mode II. The presented results are obtained by using the subset size of 21×21 pixels with a step size (distance between the pixels in each subset) of 5 pixels. In the area adjacent to the notch faces the strain field components cannot be evaluated with DIC because they are discontinuous or undefined. The size of this area will always be half of the subset size. This area is highlighted in grey colour in Fig. 6. It can be observed from Fig. 6 that the absolute values of in-plane strain increase for point close to the notch tip except in case (d) $\alpha = 120^\circ$ loaded in mode II, where the absolute value of shear strain ε_{xy} gradually decreases for point towards to the notch tip, which demonstrates the disappearance of in-plane singularities at $\alpha > \alpha_{cr}$, see Fig. 7 ahead.

Further, we extracted the corresponding strain components along the notch bisector line from the obtained in-plane strain fields to perform a least squares fitting for determining the strength of in-plane singularities. Material points from the near notch tip region are not taken into account, due to the 3D effects as explained in the Introduction. The corresponding values are compared with the previously published results, and the comparison is presented in Fig. 7. It is found that the maximum error between the present results and those from Ref. [9] are less than

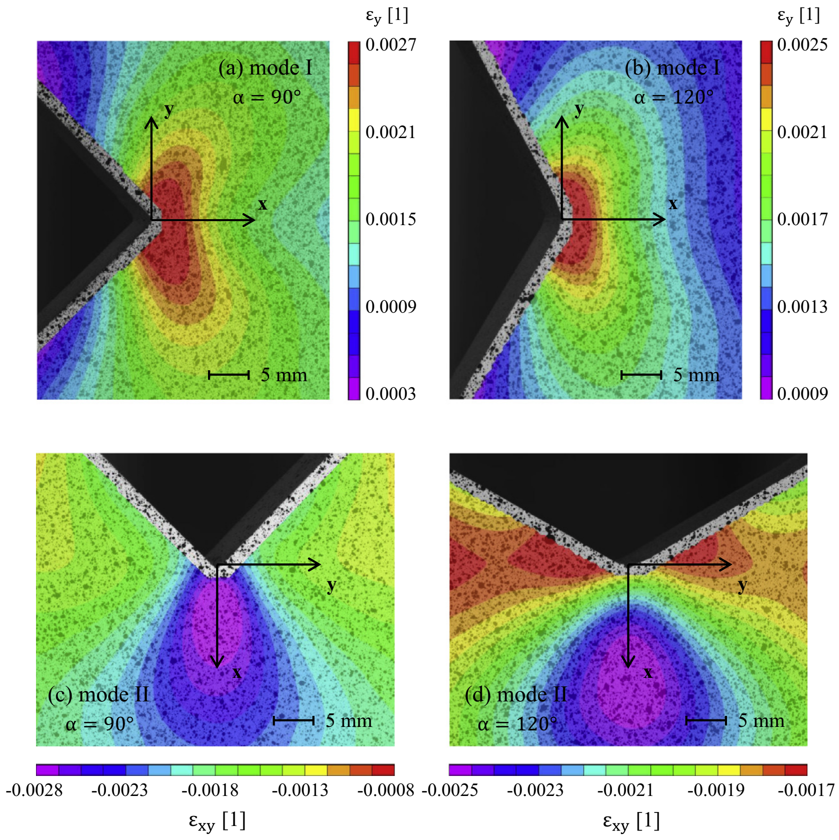

Fig. 6. Contour plot of: in-plane normal strain ε_y for notches (a) $\alpha = 90^\circ$; (b) $\alpha = 120^\circ$ loaded in mode I; in-plane shear strain ε_{xy} for notches; (c) $\alpha = 90^\circ$; (d) $\alpha = 120^\circ$ loaded in mode II.

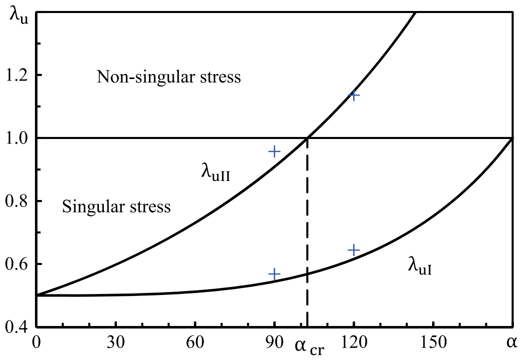

Fig. 7. Power of the displacement λ_u as functions of the notch opening angle α.

5%, which can be considered as a good agreement. In addition, we further confirmed the disappearance of in-plane singularity for sharp notches ($\alpha = 120^\circ$) subjected to pure mode II loading as mentioned above.

3.2. Strain distribution across the thickness

Next, we investigated with the digital image correlation technique the variation of the strain field across the thickness near the notch front. The test specimen with thickness $2h = 20$ mm and sharp notch angle $\alpha = 120^\circ$ was subjected to pure mode I loading (see Fig. 8a). The maximum load applied on the loading device was limited to a constant value of $F = 4$ kN. This time, we let both cameras focused to one of the notch opening faces. A square area (20×20 mm), as shown in Fig. 8a within the dashed line, was chosen to be the region of interest. Fig. 8b shows the contour plot of in-plane strain $\varepsilon_{y'}$ on notch opening face. From Fig. 8b, it can be clearly seen that, similar to the variation of stress intensity factors, the in-plane strain $\varepsilon_{y'}$ is not constant along the thickness direction as it is adopted by plane stress theories of elasticity, specifically near the notch front.

A finite element analysis using ANSYS 14.5 software also had been performed in order to compare the numerical predictions with the experimental results. Fig. 9 shows the model and provides details of mesh around the notch tip used in FE analysis, where an initial arrangement of 15-node trapezoidal elements with four mid-side nodes at the quarter points were used around the notch front, surrounded by a radial array of 20-node brick elements. In order to accurately capture the strain and displacement fields close to the notch tip, a higher mesh density was applied close to the notch tip. The straight notch front is located at the origin of coordinate system ($x = y = 0$) along the z-axis, where, $z = 0$ corresponds to the mid-plane and $z = \pm h$ represents the free surface of the specimen. The symmetries of the specimen were also used to simplify the specimen geometry and boundary conditions.

The point loads, as shown in Fig. 3a, are related with the applied uniaxial load F according to the following expressions [43]:

$$F_1 = F_6 = F\left(\frac{1}{2}\cos\beta + \frac{c}{b}\sin\beta\right) \tag{1a}$$

$$F_2 = F_5 = F\sin\beta \tag{1b}$$

$$F_3 = F_4 = F\left(\frac{1}{2}\cos\beta - \frac{c}{b}\sin\beta\right) \tag{1c}$$

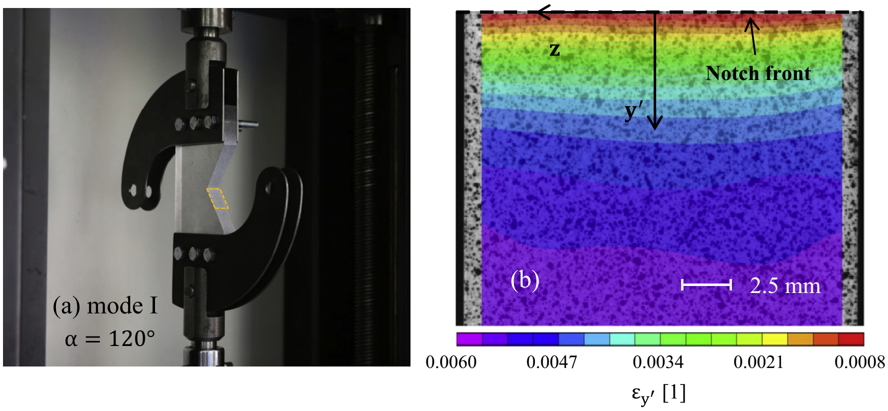

Fig. 8. (a) Specimen with notch angle $\alpha = 120^\circ$ loaded in mode I; (b) contour plot of in plane strain $\varepsilon_{y'}$ on notch opening face.

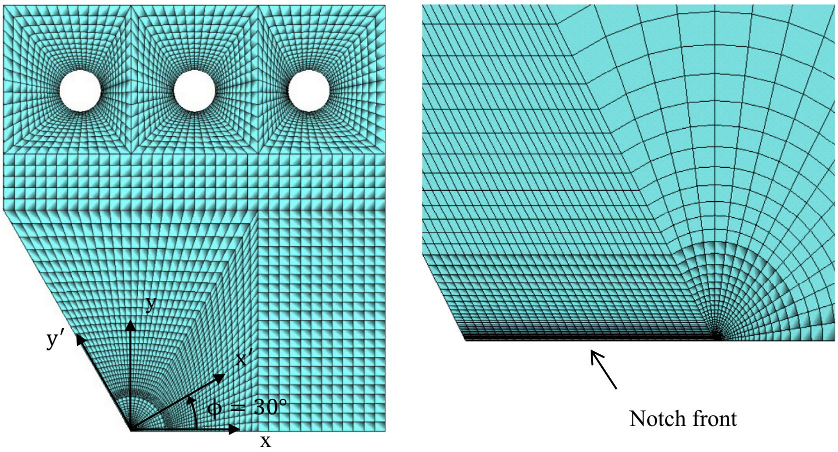

Fig. 9. Finite element mesh for single-edge-notch model with notch angle $\alpha = 120^\circ$.

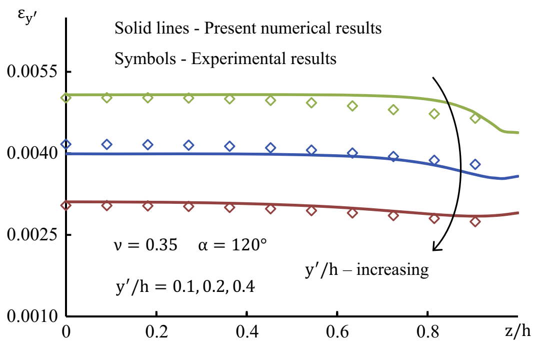

Fig. 10. Plot of $\varepsilon_{y'}$ across the plate thickness (z) at different distances y' from the notch front.

Fig. 10 shows the variation of the strain $\varepsilon_{y'}$ field across the thickness (near the notch front) obtained from the DIC measurements and the FE analysis, which is up to 15% for the considered distances from the notch front. A good agreement was observed between experimental and numerical results. The difference between the numerical and experimental results does not exceed 5%. A larger variation across the thickness is expected as predicted by FE for shorter distances from the notch front, lower notch opening angles or larger thicknesses of the test specimens [37,38]. The current experimental results have also confirmed the outcomes of many numerical studies that 3D region (region II) is limited by the half of the plate thickness in the radial direction [32,33].

4. Conclusions

Over the past two decades a large number of computational investigations were undertaken to investigate the 3D effects near sharp and blunt notches. These investigations revealed many important features of the stress distribution near the front of notches. These include the existence of the coupled modes associated with mode II and III loading, the thickness effect, which leads to the increase or decrease of the intensities of 3D corner singularities and coupled modes with an increase or decrease of the plate thickness. Another interesting effect is the existence of the coupled mode in pure shear (mode II) loading at the notch opening angles above the critical ($\alpha > 102^\circ$, see Fig. 7) [9,32]. However, all these theoretically predicted effects and phenomena have not been confirmed and investigated experimentally.

Subsequently, this paper was focused on the application of the Digital Image Correlation technique for the investigation of the displacement and strain fields within a sharp notch. The first set of experiments focused on the identification of the degree of stress singularity for different notch opening angles and loading modes (mode I and II). The outcomes of this experimental study demonstrated that the difference with the theoretically predicted values is less than 5% [9,35,36]. The similar outcomes were previously reported in many other experimental investigations utilising different measurement methods [44,45].

The second set of tests for the first time attempted to investigate the through-the-thickness strain field near the front of a sharp notch in an elastic plate subjected to mode I loading. A number of experimental studies (including the current numerical results) have indicated that the stress intensity for mode I is higher at the mid-plane and decreases near the free surfaces [32–38]. This was confirmed experimentally by the direct measurements of the transverse normal strain on radial faces of the test specimens.

With the current thickness of the specimen it was impossible to investigate the intensity or effect of corner singularities [2] and the presence of the coupled fracture modes in the area adjacent to the notch faces, as the strain field components cannot be evaluated with DIC in this area because they are discontinuous or undefined [42]. The investigation of these singular strain and stress states will require a thicker specimens as the area of interest is encapsulated by the distance $\sim 0.1h$ [32]. With 50 mm specimen thickness we were able to experimentally investigate the strain and displacement fields controlled by 3D effects and, in particular, by the 3D corner singularity [46]. Finally, the main conclusion of this paper is that with 3D DIC technique it is possible to accurately evaluate many of 3D features of the elastic solutions predicted theoretically over the past two decades.

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