Analytical Modelling of Two-Phase Multi-Component Flow in Porous Media with Dissipative and Non-equilibrium Effects

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To

my husband, Ali

who has been a constant source of support and encouragement during the challenges of my study and life.

and my mother, Zahra, who has always loved me unconditionally.

I am truly thankful for having you in my life.

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Abstract

Hereby I present a PhD thesis by publications. The thesis includes five journal papers, of which two have already been published and three have been submitted for publication and are presently under review. The journals include high-impact-factor ones (Water Resources Research, Applied Mathematics Letters and Transport in Porous Media), and also Journal of Petroleum Science and Engineering, which is a major academic journal in petroleum industry.

The thesis develops a *new version of so-called splitting theory*. The current 2006-version of the theory encompasses analytical modelling of thermodynamically-equilibrium conservation law systems for two-phase multicomponent flow in porous media. The theory allows the derivation of numerous analytical solutions. The thesis generalizes the splitting method and applies it for flow systems with dissipation, non-equilibrium phase transitions and chemical reactions. It is shown how the general $n \times n$ system is split into an $(n\text{-}1)\times(n\text{-}1)$ auxiliary system and one scalar lifting equation. The auxiliary system contains thermodynamic parameters only, while the lifting equation contains transport properties and solves for phase saturation.

First application of the splitting method is developed for low-salinity water-flooding. Two major effects are accounted for: the wettability alternation and the induction of fines migration, straining and attachment. One-dimensional (1D) problems of sequential injection of high-salinity water slug, low salinity water slug and high-salinity water chase drive corresponds to one of the most promising modern processes of Enhanced Oil Recovery, which currently is under intensive investigation in major world oil companies. Both auxiliary and lifting problems allow for exact solutions. The exact analytical solution consists of implicit formulae for profiles of

phase saturations, salinity and fine particle concentrations. The exact solution allows for deriving explicit formulae in oil recovery. The solution permits the comparative study of the impact of both effects, which are the wettability alternation and the induction of fines migration, on incremental recovery. It was found out that both effects are significant for typical values of the physics constants. The exact solution allows for multi-variant study to optimize the injected water composition in a concrete oilfield.

The *second application* of the splitting method corresponds to 1D displacement of oil by a low-salinity polymer slug followed by a low-salinity water slug and, finally, high salinity water chase drive. This problem corresponds to the Enhanced Oil Recovery Method that merges two traditional methods of polymer- and low-salinity water-floods. The exact analytical solutions are the result of the splitting system. The method was also generalized for the case of several low-salinity slugs and Non-Newtonian properties of the polymer solution. The exact solution yields explicit formulae for propagation of saturation and concentration shocks, dynamics of different flow zones and explicit formulae for incremental oil recovery. The analytical model developed allows optimizing polymer concentration and its slug size, salinity concentration and sizes of slugs for secondary and tertiary oil recovery.

The *third application* of the new splitting method is oil displacement by suspensions and colloids of solid micro particles. The injection of one suspension or colloid with multiple particle capture mechanisms is assumed. The novelty of this work is considering numerous particle capture mechanisms and kinetic equations for the capture rates, which do not have a conservation law type. However, the system is susceptible for splitting by the introduction of Lagrangian co-ordinate and using it instead of time as an independent variable in the general system of Partial Differential Equations (PDEs). Introduction of the concentration potential linked with retention concentrations yields an exact solution for auxiliary problem. The exact formulae allow predicting the profiles and breakthrough histories for the suspended and retained concentrations and phase saturations. It also allows the calculation of penetration depth.

The analytical models derived in the thesis are applicable also in numerous environmental and chemical engineering processes, including the disposal of industrial wastes in aquifers with propagation of contaminants and pollutants, industrial water treatment, injection of hot- or low-salinity water into aquifers and water injection into geothermal reservoirs.

Declaration

I certify that this work contains no material which has been accepted for the award of any other degree or diploma in my name, in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission in my name, for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide and where applicable, any partner institution responsible for the joint-award of this degree. I give consent to this copy of my thesis when deposited in the University Library, being made available for loan and photocopying, subject to the provisions of the Copyright Act 1968. The author acknowledges that copyright of published works contained within this thesis resides with the copyright holder(s) of those works.

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Thesis by Publication

Published Journal Papers

Borazjani, S., Bedrikovetsky, P., Farajzadeh, R.: Exact Solution for Non-Self-Similar Wave-Interaction Problem during Two-Phase Four-Component Flow in Porous Media, Abstract Applied Analysis, 2014, 13 (2014)

Borazjani, S., Roberts, A.J., Bedrikovetsky, P.: Splitting in Systems of PDEs for Two-Phase Multicomponent Flow in Porous Media, Applied Mathematics Letter, 53: 25-32 (2016)

Borazjani, S., Bedrikovetsky, P., Farajzadeh, R.: Analytical Solutions of Oil Displacement by Polymer Slug with Varying Salinity, Journal of Petroleum Science and Engineering, 140, 28-40 (2016)

Submitted Journal Papers

Borazjani, S., Behr, A., Genolet, L., Van Der Net, A., Bedrikovetsky, P. Effects of fines migration on low-salinity water-flooding: analytical modelling, submitted to Journal of Transport in Porous Media

Borazjani, S., Bedrikovetsky, P. Exact solutions for two-phase colloidal-suspension transport in porous media, submitted to Water Resources Research

Borazjani, S., Bedrikovetsky, P. Exact Solutions for 1-D Polymer Flooding Accounting for Mechanical Entrapment, submitted to Water Resources Research

International conference papers and presentations

Zeinijahromi, A., **Borazjani, S.**, Rodrigues, T., Bedrikovetsky, P. Low salinity fines-assisted water-flood: analytical modelling and reservoir simulation, presented at SPE Asia Pacific Oil & Gas Conference and Exhibition, Society of Petroleum Engineers, Adelaide, Australia (2014). A full volume Conference paper, 19 pages

Borazjani, S., Farajzadeh, R., Roberts, A., Bedrikovetsky, P. Exact non-self-similar solutions for two-phase four-component flows in porous media, presented at 7th International Conference on Porous Media, Padova, Italy (2015)