Maier, Holger R.; Lence, B. J.; Tolson, B. A.; Foschi, R. O.
First-order reliability method for estimating reliability, vulnerability, and resilience, Water Resources Research, 2001; 37 (3):779-790.

http://hdl.handle.net/2440/1069

First-order reliability method for estimating reliability, vulnerability, and resilience

Holger R. Maier

Centre for Applied Modelling in Water Engineering, Department of Civil and Environmental Engineering
Adelaide University, Adelaide, South Australia, Australia
Department of Civil Engineering, University of British Columbia, Vancouver, British Columbia, Canada

Barbara J. Lence, Bryan A. Tolson, and Ricardo O. Foschi

Department of Civil Engineering, University of British Columbia, Vancouver, British Columbia, Canada

Abstract. Reliability, vulnerability, and resilience provide measures of the frequency, magnitude, and duration of the failure of water resources systems, respectively. Traditionally, these measures have been estimated using simulation. However, this can be computationally intensive, particularly when complex system-response models are used, when many estimates of the performance measures are required, and when persistence among the data needs to be taken into account. In this paper, an efficient method for estimating reliability, vulnerability, and resilience, which is based on the First-Order Reliability Method (FORM), is developed and demonstrated for the case study of managing water quality in the Willamette River, Oregon. Reliability, vulnerability, and resilience are determined for different dissolved oxygen (DO) standards. DO is simulated using a QUAL2EU water quality response model that has recently been developed for the Oregon Department of Environmental Quality (ODEQ) as part of the Willamette River Basin Water Quality Study (WRBWQS). The results obtained indicate that FORM can be used to efficiently estimate reliability, vulnerability, and resilience.

1. Introduction

The risk-based performance measures reliability, vulnerability, and resilience were first introduced to the water resources community by *Hashimoto et al.* [1982], although similar concepts (e.g., frequency, magnitude, and duration of failure) had previously been used to assess water supply systems [*Fiering*, 1969] and to describe natural hazards [e.g., *Kates*, 1970]. *Hashimoto et al.* [1982] define reliability as the frequency that a system is in a satisfactory state, vulnerability as the likely magnitude of a failure, if one occurs, and resilience (or resiliency) as the inverse of the expected value of the length of time a system's output remains unsatisfactory after a failure. These definitions are adopted in this paper. These criteria or variations thereof [e.g., *Burn et al.*, 1991; *Moy et al.*, 1986] have been used to assess reservoir operating policies [e.g., *Burn et al.*, 1991; *Hashimoto et al.*, 1982; *Moy et al.*, 1986], to measure the performance of water distribution systems [e.g., *Zongxue et al.*, 1998] and to characterize regional droughts [e.g., *Correia et al.*, 1986].

In all of the aforementioned applications, estimates of reliability, vulnerability, and resilience are obtained by simulation. In some cases [e.g., *Moy et al.*, 1986; *Zongxue et al.*, 1998], the number of time steps used is <400, as limited deterministic data sets are used. Since reasonable estimates under stochastic inputs require several thousand realizations [*Melching*, 1992], synthetic data generation is used by *Hashimoto et al.* [1982] and *Burn et al.* [1991] to obtain time series of sufficient length. The major disadvantage of an approach using long data series is its

computational inefficiency. This becomes especially important if estimates of reliability, resilience, and vulnerability are used to optimize decisions, as many estimates of these criteria may be required, depending on the optimization algorithm used.

In this paper, an approach using the First-Order Reliability Method (FORM) is developed as an alternative to simulation for obtaining probabilistic estimates of reliability, vulnerability, and resilience. The approach is illustrated for an example water quality case study based on the Willamette River, Oregon. The remainder of the paper is organized as follows. In section 2, details of FORM are given, and the relative advantages and disadvantages of the method are discussed. In section 3, the approach that uses FORM to estimate reliability, vulnerability, and resilience is outlined, and the case study is introduced in section 4. The results of the case study are presented and discussed in section 5, and conclusions are given in section 6.

2. Reliability Analysis

2.1. Introduction

The performance of any engineered system can be expressed in terms of its load (demand) and resistance (capacity). Use of the load resistance analogy for water resources problems has been discussed by a number of authors, including *Duckstein and Bernier* [1986] and *Kundzewicz* [1989]. For example, in the water supply case, water demand corresponds to system load and supply capacity to system resistance, whereas in the water quality case, pollution load and a given water quality standard correspond to the system's load and resistance, respectively.

If $\mathbf{X} = (X_1, X_2, \ldots, X_n)^T$ is the vector of random variables that influences a system's load (L) and resistance (R), the performance function, $G(\mathbf{X})$, is commonly written as
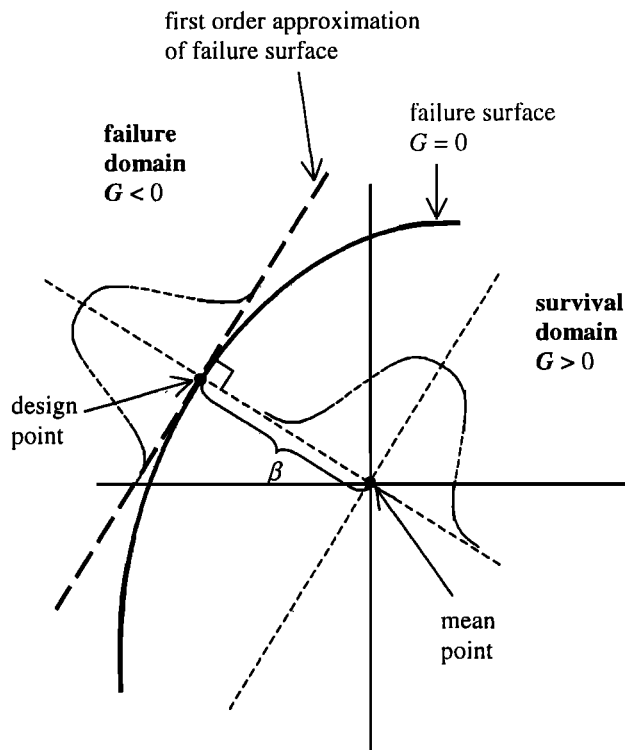
first order approximation
of failure surface

Figure 1. FORM approximation of the failure surface in standard normal space.

$$G(\mathbf{X}) = R - L. \tag{1}$$

The failure (limit state) surface, $G = 0$, separates all combinations of X that lie in the failure domain (F) from those in the survival domain (S). Consequently, the probability of failure, p_f, is given as

$$p_f = Pr\{\mathbf{X} \in F\} = Pr\{G(\mathbf{X}) < 0\} = \int_{G(X)<0} f_{\mathbf{x}}(\mathbf{x})\, dx, \tag{2}$$

where $f_{\mathbf{x}}(\mathbf{x})$ is the joint probability density function (PDF) of $\mathbf{X}$.

In most realistic applications, the integral in (2) is difficult to compute. Approximate solutions can be obtained by using a variety of techniques including Monte Carlo Simulation (MCS), Mean-value First-Order Second-Moment analysis (MFOSM), the First-Order Reliability Method (FORM), also known as Advanced First-Order Second-Moment analysis (AFOSM), and the Second-Order Reliability Method (SORM). This paper concentrates on FORM, although the advantages and disadvantages of FORM in comparison with SORM and MCS also are discussed. Detailed descriptions of the MCS and MFOSM approaches, as well as comparative studies between MFOSM and FORM (AFOSM), are given by *Tung* [1990] and *Melching and Anmangandla* [1992].

2.2. First-Order Reliability Method (FORM)

FORM was originally developed to assess the reliability of structures [*Hasofer and Lind*, 1974; *Rackwitz*, 1976]. More recently, FORM has been used in water resources engineering. It has been applied primarily to groundwater problems [e.g., *Jang et al.*, 1994; *Sitar et al.*, 1987; *Skaggs and Barry*, 1997], although there have been some surface water applications. For example, *Tung* [1990] compared the performance of MFOSM, FORM,

and MCS for evaluating the probability of violating various dissolved oxygen (DO) standards for a hypothetical case study. A similar study was carried out by *Melching and Anmangandla* [1992], who used the hypothetical DO case studies of *Burges and Lettenmaier* [1975] and *Tung and Hathhorn* [1988]. In both papers, the performance of FORM was very similar to that of MCS. However, MFOSM did not perform as well, especially at the extremes of the range of DO standards investigated. *Melching et al.* [1990] used FORM to determine the uncertainty of the peak discharge predictions obtained from a rainfall-runoff model for the Vermillion River watershed, Illinois. *Melching* [1992] carried out a comparison between MFOSM, FORM, and MCS for the same case study. There was good agreement between FORM and MCS for a wide range of storm magnitudes and types. MFOSM did not perform as well in cases where nonlinearities were significant.

An outline of the principles underlying FORM are given below. Detailed descriptions are given by *Madsen et al.* [1986], *Sitar et al.* [1987], *Melching* [1992], and *Skaggs and Barry* [1997]. As mentioned in section 2.1, the objective of FORM is to obtain an estimate of the integral in (2) and hence the probability of failure. A "reliability index," β, is computed which is then used to obtain the probability of failure by

$$p_f = \Phi(-\beta), \tag{3}$$

where $\Phi(\ )$ is the standard normal cumulative distribution function (CDF). In the n-dimensional space of the n random variables, β can be interpreted as the minimum distance between the point defined by the values of the n variable means (mean point) and the failure surface (Figure 1). Consequently, β may be thought of as a safety margin, as it indicates how far the system is from failure when it is in its mean state. The point on the failure surface closest to the mean point generally is referred to as the design point, $\mathbf{X}^*$, which may be thought of as the most likely failure point. In other words, the design point yields the highest risk of failure among all points on the failure surface.

Determination of the design point, and hence β, is a constrained nonlinear minimization problem. Suitable optimization techniques include the Rackwitz-Fiessler method [*Madsen et al.*, 1986], the generalized reduced gradient algorithm [see *Cheng*, 1982], and the Lagrange Multiplier method [see *Shinozuka*, 1983].

Equation (3) is exact only if (1) the elements of X are uncorrelated normal variables with a mean of zero and a standard deviation of one and (2) the failure surface is a hyperplane. These conditions are rarely met in realistic applications. The approach taken in FORM to deal with the first of these problems is to transform all random variables ($X_1, X_2, \ldots, X_n$) to the space of uncorrelated standard normal variables ($Z_1, Z_2, \ldots, Z_n$). Generally, the method of *Der Kiureghian and Liu* [1986] is used to perform this transformation, as it accounts for the correlation structure among the variables. The second condition cannot be accounted for exactly by FORM, and the failure surface is approximated by its tangent hyperplane at the design point in standard normal space, $\mathbf{Z}^*$, using first-order Taylor Series expansion (Figure 1). Consequently, the probability of failure obtained using FORM is only an approximation, unless the performance function is linear. The degree of nonlinearity in the performance function, and hence the accuracy of FORM, is problem dependent. SORM is identical to FORM with the exception that a second-order approx-

imation of the failure surface at the design point is used. A detailed description of SORM is given by *Madsen et al.* [1986].

2.3. FORM, SORM, and MCS

The major disadvantage of MCS is its high computational cost. The number of realizations required to estimate the probability of failure accurately depends on the unknown failure probability itself. Generally, of the order of 10,000 realizations are needed to obtain accurate estimates of small probabilities of failure (≤ 0.01) [see *Cheng*, 1982; *Melching*, 1992]. It should be noted that a number of variants of conventional MCS have been developed in order to increase its computational efficiency [see *Tung*, 1990]. For example, in importance sampling MCS [*Mazumdar*, 1975] a distribution with reduced variance is fitted around the neighborhood of failure, not around the mean point as in conventional MCS. Consequently, computational efficiency can be greatly increased, as more failures are obtained with a smaller number of realizations.

In most applications, FORM only needs a small number of iterations for convergence, making it more computationally efficient than MCS. This is particularly so when the failure probabilities are low. However, it should be noted that when FORM is used, the number of evaluations of the performance function per iteration equals $2n + 1$, as the performance function and its gradient have to be calculated at each step. Consequently, the relative advantage of FORM diminishes as the number of random variables increases. For example, *Jang et al.* [1994] found that for a two-dimensional groundwater contaminant transport model where the number of random variables was greater than 100, FORM was computationally more expensive than MCS. However, the computational cost of FORM can be reduced significantly by using sensitivity methods [e.g., *Ahlfeld et al.*, 1988], rather than divided differences, to compute the gradient [*Skaggs and Barry*, 1997]. When SORM is used, $\sim 8n$ additional evaluations of the performance function need to be carried out per iteration [*Skaggs and Barry*, 1997]. As a result, *Skaggs and Barry* [1997] suggest that the computational efficiency of SORM is no greater than that of MCS when the number of random variables is large (~ 100).

The probability estimated by MCS generally closely approximates the exact value, provided the number of iterations is sufficiently large [*Melching*, 1992]. Testing for convergence by applying MCS with different numbers of realizations can be used to assess the accuracy of MCS. In contrast, as discussed in section 2.2, the accuracy of FORM and SORM depends on the shape of the failure surface and thus is problem dependent. As such, the accuracy of FORM and SORM can only be assessed in comparison with MCS. However, the first- and second-order approximations given by FORM and SORM, respectively, generally give good results in standard normal space, as the probability density decays exponentially with distance from the origin (Figure 1). As a result, most of the probability content in the unsafe region is in the vicinity of the design point, where the first- and second-order expansions are good approximations to the failure surface [*Sitar et al.*, 1987].

Apart from its computational efficiency, FORM also provides a measure of the sensitivity of the probability of failure to the input parameters, X, and their statistical moments in the vicinity of the design point with little or no additional computational cost [*Sitar et al.*, 1987; *Skaggs and Barry*, 1997]. Such information is also available when MCS is used if, for each realization, the random inputs and resulting output are recorded and a postsimulation analysis of variance (ANOVA) or
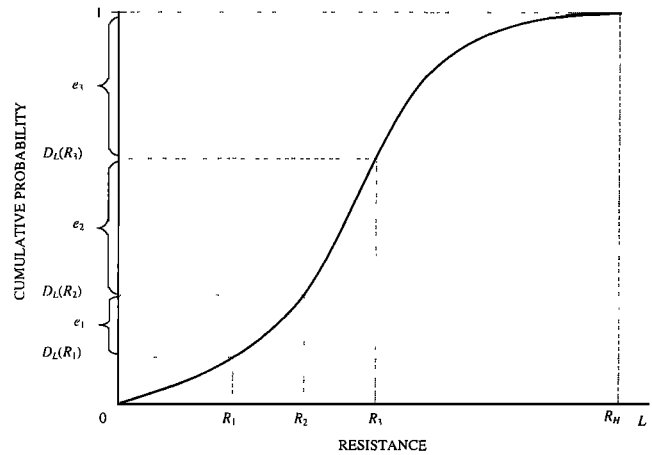
Figure 2. Schematic representation of multiple failure states on a cumulative probability density function.

rank correlation is performed. An advantage the FORM/SORM approach has over MCS is that it determines the combination of model parameters that are most likely to result in failure (i.e., the design point). In addition to the fact that the accuracy of the probability estimates obtained are problem dependent when FORM is used, the requirement that the performance function divide the parameter space into distinct failure and survival regions also can present difficulties in certain applications [see *Skaggs and Barry*, 1997].

3. FORM-Based Estimators of Reliability, Vulnerability, and Resilience

3.1. Reliability

Reliability is a measure of the probability of system survival. *Hashimoto et al.* [1982] define the reliability of a system, α, at time t as

$$\alpha = Pr\{X_t \in S\}, \tag{4}$$

which is the complement of the probability of failure. Using (3), reliability can thus be estimated as

$$\alpha = 1 - p_f = 1 - \Phi(-\beta) = \Phi(\beta). \tag{5}$$

It should be noted that the above relation is only exact if the failure surface is a hyperplane. Otherwise, it is only an approximation as discussed in section 2.2.

3.2. Vulnerability

Vulnerability is a measure of the magnitude of a system's failure. *Hashimoto et al.* [1982] define vulnerability, v, as follows:

$$v = \sum_{j \in D} w_j e_j, \tag{6}$$

where e_j is the probability that the system performance variable, L, is in discrete failure state j, and w_j is a numerical indicator of the severity of failure state j (Figure 2). If the discrete failure states are bounded by a hierarchy of failure levels, $R_1 \leq R_2 \leq R_3 \leq \ldots \leq R_H$, e_j is given by (Figure 2)

$$e_j = Pr\{R_j < L \leq R_{j+1}\} = D_L(R_{j+1}) - D_L(R_j), \tag{7}$$

where D_L is the CDF of the load, L. Consequently, vulnerability is given by

$$v = \sum_{j \in F} w_j [D_L(R_{j+1}) - D_L(R_j)]. \tag{8}$$

Using FORM, the probability of failure is given by

$$p_f = \Phi(-\beta) = Pr\{G(X_t) < 0\} = Pr\{L > R\}$$

$$= 1 - Pr\{L \leq R\} = 1 - D_L(R). \tag{9}$$

Consequently,

$$D_L(R) = 1 - \Phi(-\beta) = \Phi(\beta). \tag{10}$$

Combining (8) and (10), vulnerability can be expressed in terms of the reliability index, β, as follows:

$$v = \sum_{j \in F} w_j[\Phi(\beta_{j+1}) - \Phi(\beta_j)] = \sum_{j \in F} w_j[\Phi(-\beta_j) - \Phi(-\beta_{j+1})], \tag{11}$$

where β_j is the reliability index for resistance level R_j. As pointed out by Melching et al. [1990] and Skaggs and Barry [1997], (10) can also be used to obtain points on the CDF of the system performance variable, L, by repeating the reliability analysis for a range of values of system resistance, R.

3.3. Resilience

A number of alternative concepts of resilience have been proposed in the literature [e.g., Fiering, 1982; Holling, 1996]. In water resources engineering, resilience generally has been used as a measure of how quickly a system recovers from failure, once failure has occurred. Hashimoto et al. [1982] give two equivalent definitions of resilience, γ. One is a function of the expected value, ($E[$ $]$), of the length of time a system's output remains unsatisfactory after a failure, T_f (see (12)). The other is based on the probability that the system will recover from failure in a single time step (see (13)).

$$\gamma = \frac{1}{E[T_f]} \tag{12}$$

$$\gamma = Pr\{X_{t+1} \in S | X_t \in F\} = \frac{Pr\{X_t \in F \text{ and } X_{t+1} \in S\}}{Pr\{X_t \in F\}}. \tag{13}$$

In water resources applications, (12) has generally been used to obtain estimates of resilience. This is undertaken by examining a time series (real or synthetic) of the system performance variable and counting the number of consecutive time steps the system remains in failure, once failure has occurred. However, Kundzewicz [1989] and Tickle and Goulter [1994] show that crossing theory (also known as renewal theory or the theory of run durations), which has been used in a number of hydrologic applications [e.g., Rosjberg, 1977; Sen, 1976], can be used to obtain estimates of resilience in accordance with (13). FORM also can be used to obtain estimates of resilience based on the conditional probability definition of resilience (13), as outlined below.

In many instances in structural engineering, there is a need to consider multiple failure modes. For example, a beam may fail in bending or in shear, or a retaining wall may fail by overturning or by sliding. If there are two failure modes, the probability of failure is given by

$$p_f = p_{f1} + p_{f2} - p_{f12} = Pr\{G_1 < 0\} + Pr\{G_2 < 0\} - Pr\{G_1 < 0 \text{ and } G_2 < 0\}, \tag{14}$$

where p_{f1} and p_{f2} are the probabilities of failure due to failure modes 1 and 2, respectively, p_{f12} is the joint probability of failure for failure modes 1 and 2 and $G_1 = G(X_1)$ and $G_2 = G(X_2)$ are the performance functions for failure modes 1 and 2, respectively. The failure probabilities for the individual failure modes (p_{f1} and p_{f2}) can be obtained using (3). The joint probability of failure, p_{f12}, is given by Madsen et al. [1986] as

$$p_{f12} = \Phi(-\beta_1, -\beta_2; \rho_{12}) = \Phi(-\beta_1)\Phi(-\beta_2)$$

$$+ \int_0^{\rho_{12}} \psi(-\beta_1, -\beta_2; y) \, dy, \tag{15}$$

where $\Phi(\ , \ ; \rho)$ is the CDF for a bivariate normal vector with zero mean values, unit variances, and correlation coefficient ρ and $\psi(\ , \ ; \rho)$ is the corresponding PDF. It should be clarified that the bivariate normal distribution is used because all variables are converted to standard normal space. Although this conversion changes the correlation matrix values between the original variables, the new correlation matrix is estimated using the method developed by Der Kiureghiam and Liu [1986]. The new correlation matrix must then be diagonalized to uncorrelate the standard normal variables. The integral in (15) is generally obtained numerically. The correlation coefficient needed to evaluate this integral, ρ_{12}, is calculated using [Madsen et al., 1986]

$$\rho_{12} = \frac{Z_1^{*T} Z_2^*}{|Z_1^*||Z_2^*|} = \frac{1}{\beta_1 \beta_2} Z_1^{*T} Z_2^*, \tag{16}$$

where Z_1^* and Z_2^* are the design points in standard normal space for failure modes 1 and 2, respectively.

If we define the performance functions as

$$G_1 = R_t - L_t \tag{17}$$

$$G_2 = L_{t+1} - R_{t+1}, \tag{18}$$

the corresponding individual and joint failure probabilities are given by

$$p_{f1} = Pr\{X_t \in F\} = \Phi(-\beta_1) \tag{19}$$

$$p_{f2} = Pr\{X_{t+1} \in S\} = \Phi(-\beta_2) \tag{20}$$

$$p_{f12} = Pr\{X_t \in F \text{ and } X_{t+1} \in S\} = \Phi(-\beta_1, -\beta_2, \rho_{12}). \tag{21}$$

It should be noted that the conventional definition of the performance function (see (1)) is used in (17). However, the order of L and R is reversed in (18), so that the probability of failure, as defined in (2), is actually the probability that the system will return to a nonfailure state (see (20)). Combining (13), (19), and (21), resilience is given by

$$\gamma = \frac{\Phi(-\beta_1, -\beta_2; \rho_{12})}{\Phi(-\beta_1)}. \tag{22}$$

If system load and resistance are stationary processes, $L_t = L_{t+1}$, $R_t = R_{t+1}$, and $p_{f1} = 1 - p_{f2}$. Persistence in the time series is accounted for by the lag 1 autocorrelations and cross-correlations between the elements of X_t and X_{t+1}. As presented by Hashimoto et al. [1982], if X_t and X_{t+1} are statistically independent, resilience is equivalent to reliability, and is given by

$$\gamma = \frac{\Phi(-\beta_1, -\beta_2; 0)}{\Phi(-\beta_1)} = \frac{\Phi(-\beta_1)\Phi(-\beta_2)}{\Phi(-\beta_1)} = \Phi(-\beta_2). \quad (23)$$

3.4. Application to Water-Quality Systems

There have only been limited applications of reliability, vulnerability, and resilience to water quality problems [Bain and Loucks, 1999; Hägglöf, 1996]. However, the need to consider the frequency, magnitude, and length of violations of water quality standards has been recognized for some time. Loucks and Lynn [1966] suggest that the specification of a rigid water quality standard, which assumes that water quality is satisfactory above a certain level and unsatisfactory below it, is inadequate, as it does not reflect the stochastic nature of water quality systems. They propose that a more realistic approach would be to specify water quality standards in terms of the maximum allowable probability for the event that a water quality parameter drops below or exceeds a specified level (depending on the parameter in question) for a given length of time. Hathhorn and Tung [1988] emphasize the need to consider the relative severity of water quality standard violations, thus reflecting the different levels of tolerance aquatic biota have to various pollution levels. Of course the setting of water quality standards is very dependent on the nature of the parameter in question. For example, for acutely toxic constituents, an instantaneous standard would be best, whereas for a chronic constituent, a long-term average would be most appropriate.

As mentioned in section 2.1, in water quality systems, resistance generally is expressed as the water quality standard, and load is expressed as the ambient water quality under a given set of emission levels and environmental conditions. In some cases (e.g., ammonia management) the conventional form of the performance function (1) can be used, as failure $(G < 0)$ occurs when load, i.e., the ambient water quality, is greater than resistance, i.e., the water quality standard (see (24)). In other cases (e.g., dissolved oxygen management), failure $(G < 0)$ occurs when load, i.e., the ambient water quality, is less than resistance, i.e., the water quality standard (see (25)).

$$G(X) = Q_s - Q_a(X) = R - L \quad (24)$$

$$G(X) = Q_a(X) - Q_s = L - R, \quad (25)$$

where $Q_a(X)$ is the ambient water quality, which is generally estimated using a water quality response model, and Q_s is a fixed water quality standard at a critical location.

When response models are used to obtain ambient water quality values, they are subject to inherent, model, and parameter uncertainties [see Burges and Lettenmaier, 1975; Loucks and Lynn, 1966; Tung and Hatthhorn, 1988]. In order to incorporate these uncertainties into FORM, they must be expressed as random variables. This requires information about the mean, standard deviation, and distribution of each of the random variables, as well as the correlation structure among them. This information cannot always be obtained in its entirety, necessitating expensive data collection programs or that assumptions be made based on experience or studies conducted elsewhere. Consequently, it is desirable to keep the number of random variables to a minimum, while ensuring that all significant sources of uncertainty are included. Another reason for restricting the number of random variables is the fact that computational time is a function of the number of random variables, as discussed in section 2.3.

4. Case Study: Willamette River Basin

4.1. Background

The Willamette River basin is located in northwestern Oregon and includes the state's three largest cities, Portland, Salem, and Eugene. The mainstem of the Willamette River is 300 km long, and its flow is regulated by a number of storage and reregulation reservoirs [Leland et al., 1997]. The river may be divided into four distinct regions, based on their hydraulic and physical characteristics [Tetra Tech, 1995b]. Reach I, which extends from the mouth of the river to the Willamette Falls (River Kilometre, RK, 0-42) is influenced by tides and the Columbia River; reach II, which extends from the Willamette Falls to above Newberg (RK 42-96), is deep and slow-moving; reach III, which extends from above Newberg to Corvallis (RK 96-208), is shallow and fast-moving; and reach IV, which consists of the portion of the river upstream of Corvallis (RK 208-300), also is shallow and fast-moving [Tetra Tech, 1995b].

Water pollution has been an issue in the Willamette River for a number of decades. Prior to the introduction of secondary treatment requirements in the 1970s, the river experienced severe water quality problems as a result of the discharge of oxygen demanding substances from municipal and industrial point source dischargers [Tetra Tech, 1993]. Since that time, there have been substantial improvements in water quality, and the current health of the river is marginal to good [Leland et al., 1997]. However, pressure on water quality in the Willamette River is likely to increase in the future, as the Willamette basin is the fastest growing and most economically developed region of Oregon [Leland et al., 1997]. Consequently, a DO model has been developed to help managers prevent the potential deterioration of the water quality in the river due to increased waste discharges [Tetra Tech, 1995b].

The mainstem of the Willamette River receives carbonaceous biochemical oxygen demanding (CBOD) effluent from 51 waste dischargers [Tetra Tech, 1995a]. At present, water quality standards are defined in terms of a DO concentration that must be exceeded during all flows greater than or equal to critical environmental conditions (e.g., the minimum 7-day average flow that occurs once every 10 years) [Oregon Department of Environmental Quality (ODEQ), 1995]. However, the choice of an appropriate level of protection is difficult, as there is a continuum of risk that is not well defined. For example, at DO concentrations between saturation and 3 mg L^{-1}, salmonids experience chronic effects of varying severity, including reductions in swimming speed, growth rate, and food conversion efficiency, whereas DO concentrations below 3 mg L^{-1} generally are acutely lethal [ODEQ, 1995]. In addition, impacts are more severe if exposure to low concentrations of DO occurs more frequently and for longer periods of time [ODEQ, 1995]. Consequently, use of the risk-based performance indicators reliability, vulnerability, and resilience may provide a better representation of the actual impact of varying DO regimes on aquatic biota.

In this paper, the FORM-based method for estimating reliability, vulnerability, and resilience presented in section 3 is applied to the Willamette River. The effect of various uniformly decreasing CBOD wasteloads on the reliability, vulnerability, and resilience of the system in terms of violating different DO standards is investigated at the mouth (RK 0). The latter is chosen as the critical location as DO concentrations generally are lowest at this point in the river [Tetra Tech, 1995b].

Table 1. Details of Flow and Temperature Data Used

Variable	USGS Station Number	Time Step	Years of Record
Flow in Willamette River at confluence of Coast and Middle Fork	14157500 + 14152000	Day	1966–1986, 1988–1991, 1993–1997
Flow in McKenzie River	14166000 − (14157500 + 14152000)	Day	1966–1986, 1988–1991, 1993–1997
Flow in Santiam River	14189000	Day	1966–1986, 1988–1991, 1993–1997
Flow in Clackamas River	14210000	Day	1966–1986, 1988–1991, 1993–1997
Temperature at Salem	14191000	Day	1977–1978, 1980–1981, 1983, 1985–1987

4.2. Method

FORM and MCS are implemented using RELAN [*Foschi and Folz*, 1990], a general reliability analysis software package that was developed at the University of British Columbia in Vancouver, Canada. The Rackwitz-Fiessler algorithm is utilized to find the design point when the FORM option of RELAN is used. RELAN accepts a range of probability distributions for the random variables and also can perform reliability analyses by SORM, Response Surface Methodologies, and Adaptive Sampling Simulation.

The formulation of the performance function given in (25) is used for reasons discussed in section 3.4. The ambient DO concentrations needed to evaluate the performance function are estimated using the QUAL2EU water quality response model (Version 3.22) [*Brown and Barnwell*, 1987] developed by Tetra Tech for the ODEQ [*Tetra Tech*, 1993, 1995a] (see section 4.3). The QUAL2EU and RELAN programs source codes, both in FORTRAN, are slightly modified and linked together such that the random variable values generated by RELAN are input to QUAL2EU and the resulting DO concentration at RK 0 is output to RELAN for evaluation of the performance function. Details of the random variables included in the DO model are given in section 4.4. In order to ensure that the reliability estimates obtained using FORM are accurate for the case study considered, the FORM-based reliabilities are compared with those obtained using MCS for a number of DO standards. Reliability, vulnerability, and resilience estimates are then obtained for various CBOD wasteloads and DO standards. Details of the critical DO concentrations and the wasteloads used are given in sections 4.7 and 4.8, respectively.

4.3. Dissolved Oxygen Response Model

As mentioned in section 4.2, the QUAL2EU model developed by *Tetra Tech* [1993, 1995a] is used in this study. The model is one-dimensional, steady state, and includes sediment oxygen demand (SOD) and average daily phytoplankton growth effects on DO. It consists of 141 model segments, each of which is subdivided into computational elements of 0.16 km in length, and incorporates inflows from 14 tributaries and 51 point source wasteload dischargers. The model is calibrated using data from August 1992, is verified using data from August 1994, and is considered to be valid for the summer low-flow season, July to September, which is the critical period for DO. Some of the limitations of the model include that it does not incorporate the effect of periphyton production on DO, it does not account for tidal mixing with the Columbia River, and it does not consider nonpoint or diffuse sources of nutrients or oxygen demanding substances.

4.4. Random Variables

In this study, only natural and parameter uncertainties are considered. It is assumed that the QUAL2EU model adequately simulates all of the processes affecting DO concentrations in the Willamette River, although this is not strictly correct (see section 4.3). Since the number of random variables included should be kept to a minimum, only the naturally varying model inputs and uncertain parameters that are considered to have a significant effect on the output of the DO model, and for which sufficient information characterizing their uncertainty exists, are used as random variables.

4.4.1. Natural variability. In this paper, the natural variability in flow and temperature are considered as random vari-

Table 2. Statistics of the 7-day Moving Average for Flow and Temperature Data Used in the Reliability and Vulnerability Calculations

Variable	Mean (Parameter 1)[a]	Standard Deviation (Parameter 2)[a]	Lower Bound	Distribution Type
Flow in Willamette River at confluence of Coast and Middle Fork	62.3 m³ s⁻¹	13.7 m³ s⁻¹	2.8 m³ s⁻¹	lognormal
Flow in McKenzie River	68.3 m³ s⁻¹	12.5 m³ s⁻¹	2.8 m³ s⁻¹	lognormal
Flow in Santiam River	(6.1)[b]	(41.8)[b]	2.8 m³ s⁻¹	Weibull
Flow in Clackamas River	26.2 m³ s⁻¹	4.8 m³ s⁻¹	2.8 m³ s⁻¹	lognormal
Temperature at Salem	21.2°C	1.2°C	1.7°C	lognormal

[a]Parameter 1 and 2 are for other distributions that are not normal or lognormal and are specified below for the relevant distributions.

[b]Santiam River data (in m³ s⁻¹) fit to a Weibull distribution with location, shape, and scale parameters equal to 0, 6.1, and 41.8, respectively.

Table 3. Correlation Coefficients for the 7-day Moving Average for Flow and Temperature Data Used in the Reliability and Vulnerability Calculations

| Variable Number | Description | Variable Number | | | | |
		1	2	3	4	5
1	Flow in Willamette River at confluence of Coast and Middle Fork	1	0.0	0.0	0.0	0.0
2	Flow in McKenzie River	0.0	1	0.0	0.0	0.0
3	Flow in Santiam River	0.0	0.0	1	0.7	−0.8
4	Flow in Clackamas River	0.0	0.0	0.7	1	−0.7
5	Temperature at Salem	0.0	0.0	−0.8	−0.7	1

ables, as they have been found to be important in previous studies, and sufficient data are available to characterize their variability. It should be noted that additional uncertainties such as those associated with wasteload levels [see *Loucks and Lynn*, 1966] may also have a significant impact on the results obtained. However, insufficient data are available to characterize this uncertainty for the case study considered.

The QUAL2EU model uses inputs from the headwaters of the Willamette River and from 14 of its tributaries. On inspection of the flow records for each of these, only the headwater flows (at the confluence of the Coast and Middle Fork of the river) and the flows from the McKenzie, Santiam, and Clackamas Rivers are used as random variables, as their combined flows are approximately 1 order of magnitude larger than the flows in the remaining tributaries. The flow data used are available at www.usgs.gov and Table 1 summarizes the USGS flow measurement stations, time step for the data, and years of record used. The headwater flow is determined by adding flows at two U.S. Geological Survey (USGS) gauging stations while the flow in the McKenzie River is determined by a flow balance between two mainstem USGS gauging stations as by *Tetra Tech* [1993, 1995a]. Although QUAL2EU is capable of modeling temperature, this option is not utilized in the DO model developed by *Tetra Tech* [1993, 1995a]. Instead, temperatures are assumed to increase uniformly from the headwaters to the mouth of the river. In this study, this relationship is maintained while randomly varying temperature at one location. This approach is reasonable because it is consistent with *Tetra Tech* [1993, 1995a], and temperature data in the Willamette are sparse. The temperature data used are obtained from the USGS, and the information related to the USGS temperature measurement station considered is also summarized in Table

1. Salem is chosen as the location at which the temperature variable is based, as it is the site with the best temperature record.

All data analyses are carried out using only values from July to September, as this is the time of year for which the QUAL2EU model is calibrated. For the reliability and vulnerability calculations, the 7-day moving average is obtained for all data, and the statistics for the random variables are obtained using the annual extreme low-flow values. Seven-day moving average temperatures occurring on the same day as the annual extreme low-flow values are used as the raw temperature data. The mean, standard deviation, and distribution type for the flow and temperature data considered are shown in Table 2. The correlations between the random variables are shown in Table 3. Only correlations >0.7 are used in this study, since preliminary results showed that ignoring correlations less than this had a negligible impact on the results. For the resilience calculations, the statistics for the random variables are obtained using 13-day independent averages of flow and temperature and are summarized in Table 4. The reason for using 13-day averages of flow and temperature for resilience are discussed in section 4.5. The cross correlations and autocorrelations used for the resilience calculations are given in Table 5. As shown in Tables 2 and 4, the variables are bounded to ensure that the inputs to the QUAL2EU model are realistic. It should be noted that RELAN automatically adjusts the PDF for each random variable so that the total probability is equal to one.

4.4.2. Parameter uncertainty. The parameter uncertainties considered in this study include those associated with the reaeration coefficient (K_a) and the SOD value. The former is included as it has been found to be the parameter that has the

Table 4. Statistics of the 13-day Independent Averages for Flow and Temperature Data Used in the Resilience Calculations

Variable	Mean (Parameter 1)[a]	Standard Deviation (Parameter 2)[a]	Distribution Shift[b]	Lower Bound	Distribution Type
Flow in Willamette River at confluence of Coast and Middle Fork	60.5 m^3 s^{-1}	33.2 m^3 s^{-1}	25.3 m^3 s^{-1}	2.8 m^3 s^{-1}	lognormal
Flow in McKenzie River	(24.0)[c]	(3.1)[c]	—	2.8 m^3 s^{-1}	gamma
Flow in Santiam River	47.8 m^3 s^{-1}	31.6 m^3 s^{-1}	13.4 m^3 s^{-1}	2.8 m^3 s^{-1}	lognormal
Flow in Clackamas River	13.8 m^3 s^{-1}	8.4 m^3 s^{-1}	14.9 m^3 s^{-1}	2.8 m^3 s^{-1}	lognormal
Temperature at Salem	(9.6)[d]	(19.4)[d]	—	1.7°C	Weibull

[a]Parameter 1 and 2 are for other distributions that are not normal or lognormal and are outlined below for the relevant distributions.
[b]The distribution shift is a constant added to the random values generated by the respective distributions and parameters.
[c]The McKenzie River flow data (m^3 s^{-1}) is fitted with a Gamma distribution with $\alpha = 24.0$ and $\beta = 3.1$.
[d]The Salem temperature data (°C) is fitted with a Weibull distribution with location, shape, and scale parameters equal to 0, 9.6, and 19.4, respectively.

Table 5. Correlation Coefficients for the 13-day Independent Averages for Flow and Temperature Data Used in the Resilience Calculations

Variable Number	Description	Variable Number									
		1	2	3	4	5	6	7	8	9	10
1	flow in Willamette River (t) at confluence of Coast and Middle Fork	1	0.0	0.7	0.0	−0.8	0.8	0.0	0.7	0.0	−0.8
2	flow in McKenzie River (t)	0.0	1	0.0	0.0	0.0	0.0	0.8	0.0	0.0	0.0
3	flow in Santiam River (t)	0.7	0.0	1	0.0	−0.8	0.7	0.0	0.7	0.0	−0.7
4	flow in Clackamas River (t)	0.0	0.0	0.0	1	0.0	0.0	0.0	0.0	0.7	0.0
5	temperature at Salem (t)	−0.8	0.0	−0.8	0.0	1	−0.7	0.0	−0.8	0.0	0.7
6	flow in Willamette River ($t + 1$)	0.8	0.0	0.7	0.0	−0.7	1	0.0	0.7	0.0	−0.8
7	flow in McKenzie River ($t + 1$)	0.0	0.8	0.0	0.0	0.0	0.0	1	0.0	0.0	0.0
8	flow in Santiam River ($t + 1$)	0.7	0.0	0.7	0.0	−0.8	0.7	0.0	1	0.0	−0.8
9	flow in Clackamas River ($t + 1$)	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.0	1	0.0
10	temperature at Salem ($t + 1$)	−0.8	0.0	−0.7	0.0	0.7	−0.8	0.0	−0.8	0.0	1

most significant impact on predicted DO concentrations in a number of hypothetical and actual case studies [e.g., *Chadderton et al.*, 1982; *Melching and Yoon*, 1996]. The latter is considered as it has been found to have a marked effect on the outputs obtained from the QUAL2EU model for the Willamette River [*Tetra Tech*, 1995a]. It should be noted that other reaction coefficients may also have a significant impact on predicted DO concentrations if assumed uncertain but are not considered here because of insufficient data.

In the QUAL2EU model used, K_a values are calculated as a function of depth and velocity using the *O'Connor and Dobbins* [1958] equation. Four random variables are used to characterize the K_a uncertainty in this research since the original modelers delineated four distinct hydraulic reaches of the river (section 4.1). As no site specific information is available regarding the accuracy of the O'Connor and Dobbins equation, the database of 371 measured K_a values and the corresponding stream characteristics developed by *Melching and Flores* [1999] is utilized to estimate the accuracy of this equation. From the database, for streams with similar flow regimes as the Willamette River, the depth and velocity data are used to calculate the O'Connor and Dobbins estimate of K_a. These estimates are then compared with the measured K_a values to estimate the error of the O'Connor and Dobbins equation. As given by *Melching and Flores* [1999], the estimated and measured K_a values are transformed logarithmically ($\log_{10}$) before the error estimates are generated. Analysis of the database shows that there is insufficient information to divide the error data into four groups that matched the hydraulic characteristics of each reach of the Willamette River. Therefore each source of K_a uncertainty is characterized by the same statistics and probability distribution. Further details on the development of these

error statistics are given by *Tolson* [2000]. The four random errors associated with using the O'Connor and Dobbins equation are assumed to be spatially independent and their statistics are summarized in Table 6. The sampling statistics for SOD obtained by *Tetra Tech* [1995a] are used in this study to characterize the SOD uncertainty with two spatially independent random variables and are also summarized in Table 6. As no information on the autocorrelation structure of SOD and K_a is available, the autocorrelations are assumed to be 0. However, it is likely that these parameter autocorrelations are greater than 0. For example, if there is a high error in the O'Connor and Dobbins estimate of K_a in one time step, then it is likely that the error in the next time step is also high. Future work should be done to estimate the actual values of these auto-correlations.

In summary, the random variables considered in this study are four tributary flows, one temperature, two SOD coefficients, and four K_a coefficients. For the reliability and vulnerability estimations then, there are 11 random variables in total considered in the analysis. For the resilience estimation, since two time steps are considered, this set of random variables must be generated twice, and thus a total of 22 random variables are used in the analysis.

4.5. Resilience Estimation

The resilience estimate obtained for this case study refers to the probability that given a set of inputs leading to system failure at steady state in the previous time step, the inputs in the next time step will result in the system recovering from failure at steady state. The time step by time step evaluation of resilience is conducted for the Willamette River over the entire low-flow season. This resilience estimate is with respect to the

Table 6. O'Connor and Dobbins Prediction Error Statistics [*Tolson*, 2000] and Sampling Statistics for SOD[a] [*Tetra Tech*, 1995a]

Parameter	River Reach	Mean	Standard Deviation	Lower Bound	Distribution Type
SOD	tidal (RK 0-RK 42)	2.12 g m^{-2} d^{-1}	0.60 g m^{-2} d^{-1}	0 g m^{-2} d^{-1}	normal
SOD	Newberg (RK 42-RK 81)	1.98 g m^{-2} d^{-1}	0.52 g m^{-2} d^{-1}	0 g m^{-2} d^{-1}	normal
K_a[b]	same statistics for all four reaches[c]	−0.111	0.155	⋯[d]	normal

[a]Prediction error statistics are given by *Tolson* [2000]. Sampling statistics are given by *Tetra Tech* [1995a].

[b]K_a error data generated by: error = $\log_{10}$(predicted K_a) = Log_{10}(measured K_a). Original measured K_a data are to the base e at 20°C in units of days^{-1}.

[c]See section 4.1 for details of each reach.

[d]The anti-log transformation back to units of days^{-1} ensures that K_a is always greater than 0.

Table 7. Information About the Failure States Used in the Vulnerability Calculation[a]

DO Standard, mg L^{-1}	DO Range, mg L^{-1}	Effects	Severity Index, w_t
5, 6	>standard	none to minor	0
6	5–6	some avoidance behavior, reduced cruising speed, reduced appetite	1
5, 6	3–5	increased avoidance behavior, reduced food conversion efficiency,	2.7
5, 6	0–3	acutely lethal	15.2

[a]Given by *ODEQ* [1995].

steady state system inputs and not strictly with respect to the speed of system recovery as measured by the DO at Portland. For example, owing to the spatial differences in the model inputs, the DO at Portland may actually recover from failure before steady state is reached.

The conditional definition of resilience used in this study does not require a time series of inputs to be generated. Instead, it only requires estimates of the lag 1 correlation between consecutive time steps. This definition allows the resilience of the system to be estimated by generating repeated events of two time steps in duration. The implementation of the resilience approach is limited by the water quality response model used and the system being modeled. For example, when using the steady state DO model for the Willamette River, the resilience time step should equal the travel time of the river, as the DO model does not respond to more rapid changes in the system.

The travel time for the Willamette River during an average annual 7-day moving average low-flow event is ~13 days. Therefore the statistics used for the resilience estimation are based on 13-day independent averages of flow and temperature from July 1 to September 29 of each year. These 13-day averages, input to QUAL2EU, are assumed to produce approximately similar DO estimates to those that would result from a dynamic estimate of DO as a function of a 13-day time series of inputs. Owing to the length of this averaging period this assumption may not hold at all times for the Willamette River. However, in general, the validity of this assumption should increase as the travel time in the modeled system decreases.

4.6. Comparison With Monte Carlo Simulation

As mentioned in section 2.2, FORM has already been found to be a suitable tool for evaluating the probability of violating DO standards in a number of hypothetical cases [e.g., *Tung*, 1990]. However, only few such comparisons have been extended to actual case studies. Consequently, the reliability estimates obtained using FORM are compared with those obtained using MCS for the case study considered. The use of 5000 Monte Carlo realizations is considered sufficient for this purpose [see *Tetra Tech*, 1995a]. A wide range of DO standards is examined to ensure that the results obtained span the full complement of possible failure probabilities. The DO standards used range from 3.0 to 8.0 mg L^{-1} at 0.5 mg L^{-1} increments. The analyses are carried out at RK 0 using a 60% CBOD treatment level (see section 4.8).

4.7. Critical Dissolved Oxygen Levels

The current DO standard at RK 0 is 5 mg L^{-1} [*ODEQ*, 1995] and is used for the reliability and resilience calculations. In addition, a standard of 6 mg L^{-1} is considered. The critical DO

concentrations used to bound the various failure states, the potential physical impacts of being in these failure states and their numerical indicators of severity used for the vulnerability calculations for both of the standards considered are summarized in Table 7. It should be noted that the failure states pertain to the adult life stages of cold water fish, as the reach of the river investigated is generally only used for anadromous fish passage [*ODEQ*, 1995]. The numerical indicators of severity are assumed to be zero at DO concentrations above the adopted standard, as failure is defined in terms of violation of a particular DO standard. This is despite the fact that some deleterious effects can occur at higher DO concentrations. The set of numerical indicators of severity in Table 7 are chosen arbitrarily for illustration purposes, as there is no information on the quantitative impacts of being in the various failure states.
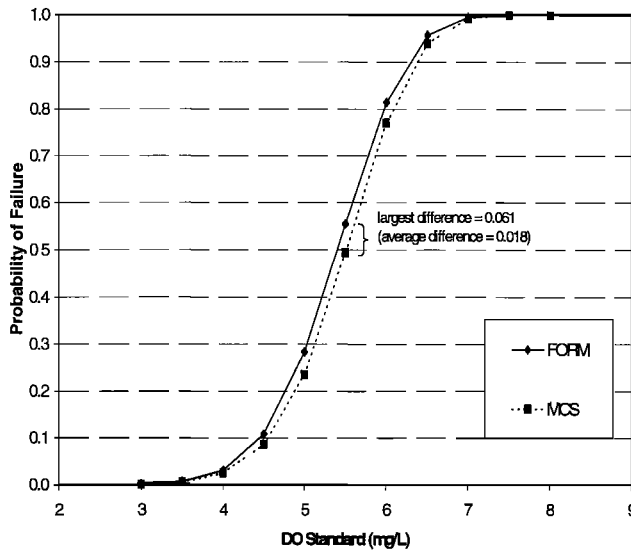
4.8. Wasteloads

The QUAL2EU model developed by *Tetra Tech* [1995a] uses the current wasteloads emitted by each of the 51 dischargers included in the model. Instead of using the current wasteload levels as a basis for varying the wasteloads, the average raw CBOD wasteload estimates available for 17 of the most important dischargers for the summer low-flow season are used (Steve Schnurbusch and Mark Hamlin, ODEQ, personal communication, 2000). These 17 dischargers account for ~95% of the point source wasteload in the QUAL2EU model developed by *Tetra Tech* [1995a]. Therefore the effect on reliability, vulnerability, and resilience of CBOD waste treatment levels of 35, 50, 60, 70, 80, 90, and 95% for each of the 17 dischargers is investigated. The wasteloads for the remaining 34 dischargers are left at their original levels, as specified by *Tetra Tech* [1995a], throughout the analyses.

5. Results and Discussion

A plot of the cumulative probabilities of failure for achieving different DO standards obtained using MCS and FORM is shown in Figure 3. It can be seen that the probabilities of failure obtained using both methods are similar. If it is assumed that the results obtained when MCS is used are accurate, FORM slightly overpredicts the actual probabilities of failure. This is in agreement with the results obtained by *Tung* [1990], who carried out a similar comparison for a hypothetical case study using the Streeter-Phelps equation. The absolute differences between the failure probabilities obtained using FORM and MCS vary from 0.00 to 6.1%. This is comparable with the range of 0.2–5.6% obtained by *Tung* [1990]. Consequently, FORM appears to be a suitable tool for predicting the probabilities of failure for the system investigated.

For the case study considered, the computational efficiency

Figure 3. Cumulative probability density function obtained using FORM and MCS for current wasteload levels and a range of DO standards at RK 0.

of FORM is much greater than that of MCS. Preliminary testing with the RELAN program shows that the computational execution time required by MCS and FORM is essentially equal for the same number of evaluations of the performance function. Therefore the efficiency of each method can be compared in terms of the number of performance function evaluations required. The number of FORM iterations required for convergence range from 2 to 5, which corresponds to 46 and 115 evaluations of the performance function, respectively, when eleven random variables are used. In comparison, when MCS is used, 5000 realizations of the performance function are required for convergence. Consequently, the computational efficiency of FORM is of the order of 10–100 times greater than that of MCS.

Plots of the reliability, vulnerability, and resilience of the system for the 5 and 6 mg L^{-1} standards are shown in Figure 4. As expected, reliability increases as the level of CBOD removal increases for each standard and under a standard of 5 mg L^{-1} higher reliability levels are achieved. The differences in the reliabilities between standards are a maximum at the 60% CBOD removal level and a minimum at the 95% CBOD removal level and are equal to 0.54 and 0.24, respectively.

In general, the information provided by the vulnerability and reliability trade-off curves is similar. As expected, vulnerability decreases as reliability increases. However, the change in vulnerability resulting from an increase in the DO standard from 5 to 6 mg L^{-1} is somewhat less pronounced than the associated relative decrease in reliability. The reason for this is that the severity of being in the failure state between DO concentrations of 5 and 6 mg L^{-1} is much less than that associated with failure states at lower DO concentrations. However, it should be noted that the differences in the information provided is a function of the index of severity assigned to each failure state. For example, if the same index of severity is used for each failure state, the information provided by reliability and vulnerability is identical.

The shapes of the resilience curves when DO standards of 5 and 6 mg L^{-1} are considered provides somewhat similar information as the corresponding reliability and vulnerability

curves. For example, optimal values for reliability, vulnerability, and resilience all occur at the maximum CBOD removal levels. The resilience curves for the two standards considered differ from each other in magnitude (i.e., resilience is higher across all CBOD removal levels for a standard of 5 mg L^{-1}) and in the shape of the curve at higher CBOD removal levels. For a standard of 5 mg L^{-1}, the maximum resilience is nearly achieved at the 80% CBOD removal level and further increases in CBOD removal levels result in minimal improvements to the system resilience. In contrast, at a standard of 6 mg L^{-1}, resilience significantly improves with increases in the CBOD removal levels above 80%.

6. Conclusions

The FORM-based approach developed in this paper appears to be an efficient means of estimating reliability, vulnerability, and resilience for CBOD-DO management problems, without the need for MCS. Provided the number of random
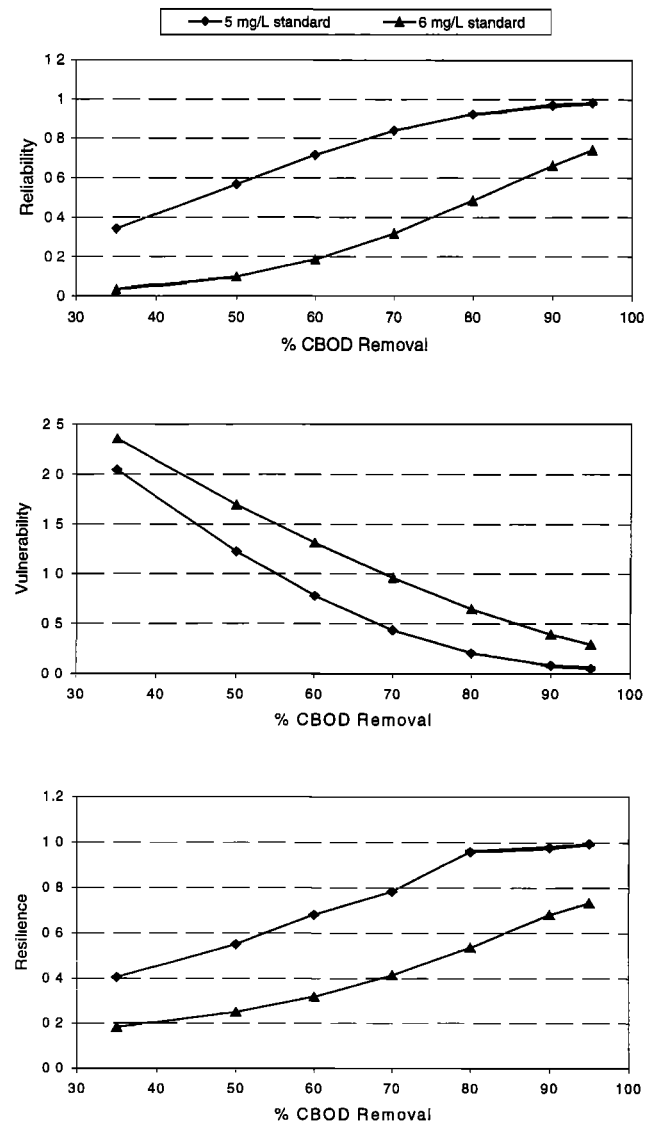
Figure 4. Trade-off curves between reliability, vulnerability, and resilience at RK 0 for the DO standards and wasteload management regimes considered.

variables is moderate, the FORM-based approach is likely to be more attractive than MCS when the system response model is computationally intensive, when many estimates of the performance measures are needed (e.g., for optimization approaches that employ iterative search techniques), and when the time steps of the data used are small. In the latter case, the generation of the synthetic data needed for each MCS realization is complicated by persistence among the data. These features are important in many water quality and quantity management problems. Moreover, the estimates of reliability, vulnerability, and resilience may be combined to evaluate other performance measures, which can be functions of vulnerability and resilience [see *Dracup et al.*, 1980] or reliability and resilience [*Duckstein and Bernier*, 1986].

The failure surface generated for the Willamette River case study using the QUAL2EU water quality response model is sufficiently linear, so that FORM is an adequate estimator of the failure probabilities for DO standards ranging from 3 to 8 mg L^{-1}. The trade-off curves developed show that reliability, vulnerability, and resilience vary over the range of DO standards considered.

The results of this case study are based on evaluating the three performance measures for the DO level at RK 0. However, it should be noted that the output from the DO model may not be valid below RK 16, as the model does not take into account tidal mixing with the Columbia River [*Tetra Tech*, 1993]. Furthermore, the current DO standard on the Willamette River varies along its length [*ODEQ*, 1995]. Therefore further analyses for this system should examine other potential critical points within the river. In addition, for a comprehensive assessment of the reliability, vulnerability, and resilience of the system in terms of overall water quality, other criteria [see *Costanza et al.*, 1998; *Xu et al.*, 1999] and the effects of nonpoint pollution sources [*Leland et al.*, 1997] would also have to be considered.

In water resources applications, the distribution of resilience has generally not been considered. However, knowledge of the probabilities associated with failure periods of various lengths is useful in certain situations. For example, *Loucks and Lynn* [1966] suggest that water quality standards should be specified in terms of the maximum allowable probability level associated with a particular length of violation for a given water quality standard. *Kundzewicz* [1989] and *Tickle and Goulter* [1994] derive probability distributions for resilience based on the assumption that the model variables follow a first-order Markov Process. If sufficient data are available, the probability distribution of resilience also can be estimated by carrying out a frequency analysis on the durations of individual failure periods for a given level of resistance [see *Weisman*, 1978]. In future work the use of FORM for estimating the probabilities associated with the occurrence of failure periods of various lengths will be investigated.

Risk-based performance measures should be considered in addition to traditional water quality management goals such as minimizing social and financial costs. In a classical optimization framework, the management solutions obtained and the insights gained by incorporating the risk-based performance measures as objectives or constraints may depend on the problem investigated. For example, there may be cases where a strict threshold value for a given measure must be observed, for example, a limit on the length of time that an acute water quality standard may be violated, or cases where the cumulative effects on water quality need to be minimized. As the

risk-based performance measures are time-dependent, the optimization formulations that include them may be difficult to solve with classical optimization techniques. Heuristic iterative search techniques that use FORM to estimate the performance measures at each iteration may be effective for solving such problems and these approaches, as well as different optimization-model formulations for determining water quality management solutions, will be investigated in future work.

References

Ahlfeld, D. P., J. M. Mulvey, G. F. Pinder, and E. F. Wood, Contaminated groundwater remediation design using simulation, optimization, and sensitivity theory, 1, Model development, *Water Resour. Res.*, *24*(3), 431–441, 1988.

Bain, M. B., and D. P. Loucks, Linking hydrology and ecology in simulation for impact assessments, paper presented at 26th Annual Water-resources Planning and Management Conference, Am. Soc. of Civil Eng., Tempe, Ariz., June 6–9, 1999.

Brown, L. C., and T. O. Barnwell Jr., The enhanced stream water-quality models QUAL2E and QUAL2E-UNCAS: documentation and user manual, rep. EPA/600/3-87/007, U.S. Environ. Protect. Agency, Athens, Georgia, 1987.

Burges, S. J., and D. P. Lettenmaier, Probabilistic methods in stream quality management, *Water Resour. Bull.*, *11*(1), 115–130, 1975.

Burn, D. H., H. D. Venema, and S. P. Simonovic, Risk-based performance criteria for real-time reservoir operation, *Can. J. Civil Eng.*, *18*(1), 36–42, 1991.

Chadderton, R. A., A. C. Miller, and A. J. McDonnell, Uncertainty analysis of dissolved oxygen model, *J. Environ. Eng. Div.*, *108*(EE5), 1003–1013, 1982.

Cheng, S.-T., Overtopping risk evaluation for an existing dam, Ph.D. thesis, Dep. of Civil Eng., Univ. of Ill. at Urbana-Champaign, 1982.

Correia, F. N., M. A. Santos, and R. R. Rodrigues, Risk, resilience and vulnerability in regional drought studies, in *Systems Analysis Applied to Water and Related Land Resources: Proceedings of the IFAC Conference*, vol. 4, Pergamon Press, Tarrytown, N.Y., 1986.

Costanza, R., M. Mageau, B. Norton, and B. C. Patten, Predictors of ecosystem health, in *Ecosystem Health*, pp. 240–250, edited by D. Rapport et al., Blackwell Sci., Malden, Mass., 1998.

Der Kiureghiam, A., and P.-L. Liu, Structural reliability under incomplete probability information, *J. Eng. Mech.*, *112*(1), 85–104, 1986.

Dracup, J. A., K. S. Lee, and E. G. J. Paulson, On the definition of droughts, *Water Resour. Res.*, *16*(2), 297–302, 1980.

Duckstein, L., and J. Bernier, System framework for engineering risk analysis, in *Risk-Based Decision Making in Water Resources*, Am. Soc. of Civ. Eng., New York, 1986.

Fiering, M. B., Forecasts with varying reliability, *J. Sanit. Eng. Div. Am. Soc. Civ. Eng.*, *95*(4), 629–644, 1969.

Fiering, M. B., Alternative indices of resilience, *Water Resour. Res.*, *18*(1), 33–39, 1982.

Foschi, R. O., and B. Folz, RELAN: Reliability Analysis User's Manual, Univ. of B. C., Vancouver, B. C., Canada, 1990.

Hägglöf, K., Optimization under uncertainty with applications in telecommunication and water-quality management, thesis 592, Dep. of Math., Inst. of Technol., Linköping Univ., Linköping, 1996.

Hashimoto, T., J. R. Stedinger, and D. P. Loucks, Reliability, resiliency, and vulnerability criteria for water resource system performance evaluation, *Water Resour. Res.*, *18*(1), 14–20, 1982.

Hasofer, A. M., and N. S. Lind, Exact invariant second-moment code format, *J. Eng. Mech.*, *100*(1), 111–121, 1974.

Hathhorn, W. E., and Y.-K. Tung, Assessing the risk of violating stream water-quality standards, *J. Environ. Manage.*, *26*, 321–338, 1988.

Holling, C. S., Engineering resilience versus ecological resilience, in *Engineering Within Ecological Constraints*, edited by P. C. Schulze, pp. 31–43, Nat. Acad. Press, Washington, DC, 1996.

Jang, Y. S., N. Sitar, and A. Der Kiureghian, Reliability analysis of contaminant transport in saturated porous media, *Water Resources Research*, *30*(8), 2435–2448, 1994.

Kates, R. W., Natural hazard in human ecological perspective: Hypotheses and models, Univ. of Toronto, Toronto, Ont., Can., 1970.

Kundzewicz, Z., W., Renewal theory criteria of evaluation of water-resource systems. Reliability and resilience, *Nordic Hydrol.*, *20*(4–5), 215–230, 1989.

Leland, D., S. Anderson, and D. J. Sterling, The Willamette—A river in peril, *J. Am. Water Works Assoc.*, *89*(11), 73–83, 1997.

Loucks, D. P., and W. R. Lynn, Probabilistic models for predicting stream quality, *Water Resour. Res.*, *2*(3), 593–605, 1966.

Madsen, H. O., S. Krenk, and N. C. Lind, *Methods of Structural Safety*, 403 pp., Prentice-Hall, Englewood Cliffs, N.J., 1986.

Mazumdar, M., Importance sampling in reliability estimation, in *Reliability and Fault Tree Analysis, Theoretical and Applied Aspects of System Reliability and Safety Assessment*, 927 pp, Soc. for Ind. and Appl. Math., Philadelphia, Penn., 1975.

Melching, C. S., An improved first-order reliability approach for assessing uncertainties in hydrologic modeling, *J. Hydrol.*, *132*, 157–177, 1992.

Melching, C. S., and S. Anmangandla, Improved first-order uncertainty method for water quality modeling, *J. Environ. Eng.*, *118*(5), 791–805, 1992.

Melching, C. S., and H. E. Flores, Reaeration equations derived from U.S. Geological Survey database, *J. Environ. Eng.*, *125*(5), 407–413, 1999.

Melching, C. S., and C. G. Yoon, Key sources of uncertainty in QUAL2E model of Passaic River, *J. Water Resour. Plann. Manage.*, *122*(2), 105–113, 1996.

Melching, C. S., B. C. Yen, and H. G. Wenzel Jr., A reliability estimation in modeling watershed runoff with uncertainties, *Water Resour. Res.*, *26*(10), 2275–2286, 1990.

Moy, W.-S., J. L. Cohon, and C. S. ReVelle, A programming model for analysis of the reliability, resilience, and vulnerability of a water-supply reservoir, *Water Resour. Res.*, *22*(4), 489–498, 1986.

O'Connor, D. J., and W. E. Dobbins, Mechanisms of reaeration in natural streams, *Trans. Am. Soc. Civ. Eng.*, *123*, 641–684, 1958.

Oregon Department of Environmental Quality, (ODEQ), Final issue paper: Dissolved oxygen—1992–1994 water quality standards review, Portland, Oreg., 1995.

Rackwitz, R., Practical probabilistic approaches to design, *Bull. 112*, Com. Eur. du Beton, Paris, 1976.

Rosjberg, D., Crossing and extremes in dependent annual series, *Nord. Hydrol.*, *8*, 257–266, 1977.

Sen, Z., Wet and dry periods of annual flow series, *J. Hydraul. Div.*, *102*(HY10), 1503–1514, 1976.

Shinozuka, M., Basic analysis of structural safety, *J. Struct. Eng.*, *109*, 721–740, 1983.

Sitar, N., J. D. Cawlfield, and A. Der Kiureghian, First-order reliability approach to stochastic analysis of subsurface flow and contaminant transport, *Water Resour. Res.*, *23*(5), 794–804, 1987.

Skaggs, T. H., and D. A. Barry, The first-order reliability method of predicting cumulative mass flux in heterogeneous porous formations, *Water Resour. Res.*, *33*(6), 1485–1494, 1997.

Tetra Tech, Willamette River basin water quality study: Willamette River dissolved oxygen modeling component, *TC 8983-03*, Redmond, Wash., 1993.

Tetra Tech, Willamette River basin water quality study, phase II: Steady-state model refinement component—QUAL2E-UNCAS dissolved oxygen model calibration and verification, rep., Redmond, Wash., 1995a.

Tetra Tech, Willamette River basin water-quality study: summary and synthesis of study findings, rep., Bellevue, Wash., 1995b.

Tickle, K. S., and I. C. Goulter, Statistical properties of reliability and resiliency measures, in *Stochastic and Statistical Methods in Hydrology and Environmental Engineering*, vol. 4, pp. 209–220, edited by K. W. Hipel and L. Fang, Kluwer Acad., Norwell, Mass., 1994.

Tolson, B. A., Genetic algorithms for multi-objective optimization in water-quality management under uncertainty, Master of Applied Science thesis, Dep. of Civ. Eng., Univ. of B. C., Vancouver, B. C., Canada, 2000.

Tung, Y.-K., Evaluating the probability of violating dissolved oxygen standard, *Ecol. Model.*, *51*, 193–204, 1990.

Tung, Y.-K., and W. E. Hathhorn, Assessment of probability distribution of dissolved oxygen deficit, *J. Environ. Eng.*, *114*(6), 1421–1435, 1988.

Weisman, R. N., Characterizing low flows using threshold levels, *J. Irrig. Drain. Div.*, *104*(IR2), 231–235, 1978.

Xu, F. L., S. E. Jorgensen, and S. Tao, Ecological indicators for assessing freshwater ecosystem health, *Ecol. Model.*, *116*(1), 77–106, 1999.

Zongxue, X., K. Jinno, A. Kawamura, S. Takesaki, and K. Ito, Performance risk analysis for Fukuoka water-supply system, *Water Resour. Manage.*, *12*(1), 13–30, 1998.

R. O. Foschi, B. J. Lence, and B. A. Tolson, Department of Civil Engineering, University of British Columbia, 2324 Main Mall, Vancouver, B.C., Canada V6T 1Z4.

H. R. Maier, Centre for Applied Modelling in Water Engineering, Department of Civil and Environmental Engineering, Adelaide University, Adelaide, SA 5005 Australia. (hmaier@civeng. adelaide.edu.au)