THE ROLE OF MONETARY SHOCKS IN THE U.S. BUSINESS CYCLE

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DECLARATION

Except where appropriately acknowledged this thesis is my own work, has been expressed in my own words and has not previously been submitted for assessment.

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ABSTRACT

The purpose of this study is to illustrate how the basic Real Business Cycle (RBC) model can be modified to incorporate money in an attempt to construct monetary business cycle models of the U.S. economy. This is done for one case where money enters the model as direct lump-sum transfers to households and for the other case where money injections enter the economy through the financial system. Interestingly, the two channels generate very different responses to a money growth shock. In the first case, a positive money growth shock increases nominal interest rates and depresses economic activity, which is called the *anticipated inflation effect*. However, the popular consensus among economists is that nominal interest rates fall after a positive monetary shock. This motivates the construction of our second model where it is conjectured that the banking sector plays an important role in the monetary transmission mechanism and money is injected into the model through financial intermediaries. It is observed in this model that a positive monetary shock reduces interest rates and stimulates economic activity, which is called the *liquidity effect*. Furthermore, the statistics generated by the models show that monetary shocks have no effect on real variables when money enters as direct lump-sum transfers to households. On the contrary, such shocks have significant real impact when money enters through the financial system. Taken together, this implies that how money enters into the model significantly matters for the impact of monetary shocks and such shocks entering through financial intermediaries may be important in determining the cyclical fluctuations of the U.S. economy.

Keywords: Business cycle, money growth shock, monetary transmission mechanism, financial intermediaries, anticipated inflation effect, liquidity effect.

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1 Introduction

As economies grow over time, they exhibit short-run fluctuations in various economic aggregates. Business cycle research focuses on the causes and consequences of this periodic expansion and contraction in economic activity. However, during the last century, exploration of the causes of business cycles has itself undergone periods of fluctuation. During the early years of the 20^{th} century and before the Great Depression, studies focused on real theories. With the onset of the Great Depression and the publication of '*The General Theory of Employment, Interest, and Money*' by John Maynard Keynes and the subsequent rise in Keynesian Macroeconomics, economists were influenced to be more sceptical about real factors and focused more on monetary factors. This led to the development of macro-econometric models which were widely used by policymakers in the 1960s

The idea that economic fluctuations are driven by demand shocks was further strengthened by the empirical findings of Friedman & Schwartz (1963) who documented strong association between periods of economic decline and declines in the stock of money. Their findings influenced the construction of equilibrium business cycle models where unanticipated changes in the money supply played an important role in generating fluctuations (Lucas Jr 1972).

The fatal breakdown of macro-econometric models in the 1970s left many economists and policymakers in despair and the subsequent rational expectations revolution (Lucas Jr 1976) cautioned against using historically observed data to draw conclusions about the effects of potential economic policy changes that have not been previously used in the prevailing economic environment. This led to a desperate search for a general equilibrium model with rational forward looking agents who understood the cross equation restrictions inherent in their economic environment and led to the subsequent development of Real Business Cycle (RBC) theory based on ground-breaking work by Kydland & Prescott (1982) and Long Jr & Plosser (1983).

The core of the RBC methodology is the neoclassical growth model that resembles a stable economy, assumed to be following its long-term growth trend. When hit by exogenous shocks that affect its environment, the model generates fluctuations that resemble business cycles. The model is termed real because it ignores nominal factors such as money and bonds. It is a dynamic general equilibrium model in that it studies an economy that evolves over time. However, a remarkable feature of the business cycle in many industrialized countries is the striking association between movements in monetary aggregates and aggregate output. In fact, the strength of this association has been sufficiently persuasive in the U.S. that M2 has long been included in the Commerce Department's Index of Leading Economic Indicators (LEI). Although correlation does not imply causality, this coherence is interpreted by authors as an evidence that monetary forces are important for fluctuations in economic aggregates. Allocations in equilibrium RBC models are Pareto optimal due to the presence of a complete set of contingent claims markets and perfectly flexible prices. Hence, equilibrium RBC models implicitly assume that monetary shocks do not have significant impact on real variables. However, the absence of money in the real business cycle model has been a source of discomfort for many macroeconomists and influenced studies focusing on the effect of money on output by Bernanke (1986) and Eichenbaum & Singleton (1986) among others.

The purpose of this study is two-fold. First, it attempts to illustrate how the basic neoclassical business cycle model can be modified to incorporate money in an attempt to construct monetary business cycle models of the US economy. Traditional RBC research have focused on technology shocks calculated from the Solow residuals as the main driver of economic fluctuations. This study, however, explores whether and how monetary forces can be an important cause of business cycle fluctuations over and above technology shocks in a world where agents are assumed to behave rationally. Second, it attempts to show that how money enters an economy matters for the impact of monetary shocks. In this regard, this thesis offers a quantitative assessment of how the fluctuations and comovements that we observe in the data compare with those displayed by the artificial economies that are constructed.

The paper is organized as follows. Section 2 provides a review of literature focusing on money and the role of monetary shocks in business cycles. Section 3 describes the sources and explains the measurement of data such that it matches with the structure and assumptions of the model. Section 4 provides some stylized facts of the US business cycle where business cycles are defined, following Lucas Jr (1977), as recurrent deviations of aggregate variables from their long-term trend and measured by the Hodrick-Prescott filter (Hodrick & Prescott 1997). In subsequent sections, we consider two different artificial economies. In the first model environment, section 5, agents simply hold money because cash is required to purchase consumption goods. In this model, monetary shocks enter as direct lump-sum transfers to households. In the second model, section 6, we add a financial intermediary to the previous model and allow monetary shocks to enter into the economy through the financial system rather than directly through households. In our analysis, we will focus on the extent to which these basic neoclassical models with money can explain the observed features of business cycles. Section 7 then provides a brief discussion and finally section 8 concludes.

2 Literature review

Almost all economists believe that money is neutral in the long-run as long-run effects of money fall almost entirely on prices with little impact on real variables. However, many also believe that monetary factors can have effects on real variables in the short-run. The most influential evidence that money does matter for business cycle fluctuations is the comprehensive historical research by Friedman & Schwartz (1963). Based on almost 100 years of data from the United States, they documented that faster money growth tends to be followed by increases in output above the trend and slowdowns in money growth tend to be followed by declines in output. However, evidence based on timing patterns and simple correlations may not indicate the true causal role of money. Tobin (1970) was the first to formally model the idea that timing evidence as empirical proof of propositions about causation does not imply that money 'causes' output and the causality could in fact run in the opposite direction. With the development of time-series econometrics, identified vector autoregressions (VARs) have been employed to estimate the impact of monetary policy. The consensus from the empirical literature on the short-run effects of money is that exogenous monetary policy shocks produce hump-shaped movements in real economic activity with the peak effects occurring after a lag of several quarters.[1]

The neoclassical growth model proposed by Solow (1956) is a non-monetary model. Although transactions take place, there is no medium of exchange and hence no role for money. Employing the neoclassical framework to analyze monetary issues require a role for money to be specified so that agents will wish to hold positive amounts of money in equilibrium. This leads to the fundamental question of how we should model the demand for money.

Sidrauski (1967) introduced money by treating it symmetrically with other goods in assuming that holdings of real cash balances generate a flow of services per unit of time and incorporated real money balances into the utility function. This came to be known as the *money-in-the-utility function* model. In this model the growth rate of money and hence the inflation rate have no effect on steady state values of real variables and the model displays what is called *superneutrality.* Since inflation reduces real money balances, an increase in the rate of monetary expansion generates a welfare loss. In

[1]For instance, see Christiano et al. (2005).

terms of the dynamics of the model, the real variables of the economy are not affected by money growth shocks for the case of log-separable utility function. Modeling money in this fashion, however, implies that there is no clear purpose of money in this model other than giving utility from its possession. For instance, no trade ever takes place and money is never used in model economies with only one good and identical agents.

Clower (1967) identified that the role of money is indistinguishable from that of any other commodity when money is treated symmetrically and argued that a precise distinction between money and non-money commodities is required for a theory of monetary phenomena. He puts forth the role of money as a medium of exchange by requiring explicitly that money be used for certain types of transactions. This idea was later developed formally by Lucas Jr (1980) where each household consists of two members - a shopper and a worker. The shopper spends each day shopping at different stores while the worker works at the same store. A *cash-in-advance* constraint requires households to bring in money from the previous period which they use in the current period to make purchases.

Clower (1967, pp.5) also stated that "Money buys goods and goods buy money; but goods do not buy goods". Motivated by this real world phenomenon we intend to model money in this study as a medium of exchange requiring explicitly that money be used for the purchase of consumption goods. The requirement that money be used to purchase goods is simply imposed. Nothing in the model explains why money is used but rather it is a social convention. If, for some reason, everyone else uses money for transactions, then it is in one's own interest to use money as well.

Early attempts to juxtapose the long-run neutrality of money and the short-run effects of money were made by Friedman (1968) and Lucas Jr (1972). Friedman (1968) distinguished between actual and perceived real wages and argued that actual real wages are important for firms hiring decisions whereas perceived real wages are important for workers labor-supply decisions. Lucas Jr (1972) constructed Friedman's idea by creating information problems for rational economic agents. He showed that monetary shocks could result in real fluctuations if they created confusion among economic agents as to whether changes in observed prices reflect changes in relative prices or changes in the aggregate price level. The paper is considered to be a ground-breaking work as it is the first equilibrium business cycle model in which agents have rational expectations, all markets clear, and monetary shocks are impulses leading to aggregate fluctuations.

A second way of exploring the effects of monetary shocks on real activity is to introduce nominal rigidities. For instance, Cho & Cooley (1995) examined the quantitative implications of multi-period wage contracts for business cycle fluctuations. First, they showed that monetary shocks, propagated by nominal contracts, are not the major source of business cycle fluctuations. Second, they further showed that monetary shocks combined with technology shocks do not account for business cycle fluctuations as monetary shocks seem too strong in their results.

In stark contrast, Christiano et al. (2005) showed that a model embodying moderate amounts of nominal rigidity accounts well for their estimate of the dynamic response of the U.S. economy to a monetary policy shock. The impulse responses of key macroeconomic variables were estimated using structural vector autoregression (VAR). A key finding is that stickiness in nominal wages is crucial for the model's performance.

It is worthy of notice that the effect of a positive money growth shock results in two opposing effects. One is known as the *liquidity effect* in which the extra money pushes down interest rates and stimulates economic activity. The other is known as the *anticipated inflation effect* in which people expect more increases in money growth and higher inflation in the future. According to Fisherian fundamentals[2], this results in higher nominal interest rates and thus depresses economic activity. A conventional view held by most economists and monetary policymakers is that central banks can reduce short-term nominal interest rates by employing policies that lead to faster growth in the money supply and by doing so can lead to a persistent increase in the level of employment and output.[3]

Christiano (1991), Christiano & Eichenbaum (1992) and Fuerst (1992) among others introduced the *liquidity effect* by distinguishing between households, firms and financial intermediaries. Households in these models allocate resources between bank deposits and money balances used to finance consumption. Financial intermediaries lend out their deposits to firms who either borrow to finance purchases of only labor services as in Christiano & Eichenbaum (1992) and Christiano (1991) or to finance purchases of both labor services and capital goods as in Fuerst (1992). Money is injected into the economy through financial intermediaries. A key feature of these models which lets them generate a substantial *liquidity effect* is the assumed sluggishness of household saving decisions. Thus a monetary shock affects households and firms asymmetrically as firms have to absorb a disproportionately larger share of a money injection which causes nominal interest rates to decline. Firms then borrow more and increase production as their costs are now lower. This puts upward pressure on employment and economic

[2]Fisher (1930).

[3]See Christiano (1991).

activity. Christiano (1991) further analyzed the implications of sluggish investment. An important point to note is that some of these models allow current period wage income to be used for current consumption (Christiano & Eichenbaum 1992) while others do not (Christiano 1991, Fuerst 1992).

It is well perceived that the question of why money matters and how monetary shocks generate real effects are critical for any normative analysis of monetary policy since designing good policy requires understanding of how monetary policy affects the real economy. Therefore, we believe that there is strong motivation to focus our research on construction of monetary business cycle models in order to gain deeper understanding about the monetary transmission mechanism. In this regard this study is an addition to research focused on the role of monetary shocks.

3 Data

In order to analyze the performance of our artificial economies, quarterly data is required to represent the equivalent of the variables in the model. The variables are output, consumption, investment, labor, price level, inflation, money supply and nominal interest rate. The purpose of this section is to discuss how we have matched our data measurements to the structure of the models which will be discussed in section 5 and section 6. However, for now, it is important to note that the models we discuss in this study are one-sector models.

The data series of this study is from 1960(1) to 2012(2) and is chosen based on data availability.[4] All the data, except for total hours worked, is obtained from the Federal Reserve Bank of St. Louis. For more details, see Appendix D at the end.

Gross Domestic Product (GDP) is the market value of all final goods and services produced in an economy during a given period of time and thus represents output in our models. GDP is generally calculated as the sum of consumption, investment, government expenditure and net exports. However, the model economies to be discussed are very abstract as it contains no government sector, no household production sector, no foreign sector and no explicit treatment of inventories. Accordingly, we need to calculate our consumption and investment so that their sum is equal to GDP.

In our models, there is only one consumption good available and therefore following Farmer & Guo (1995) the final consumption expenditure from all sectors in the economy represents the consumption variable which also includes government expenditure.

[4] Number in parenthesis refers to the quarter.

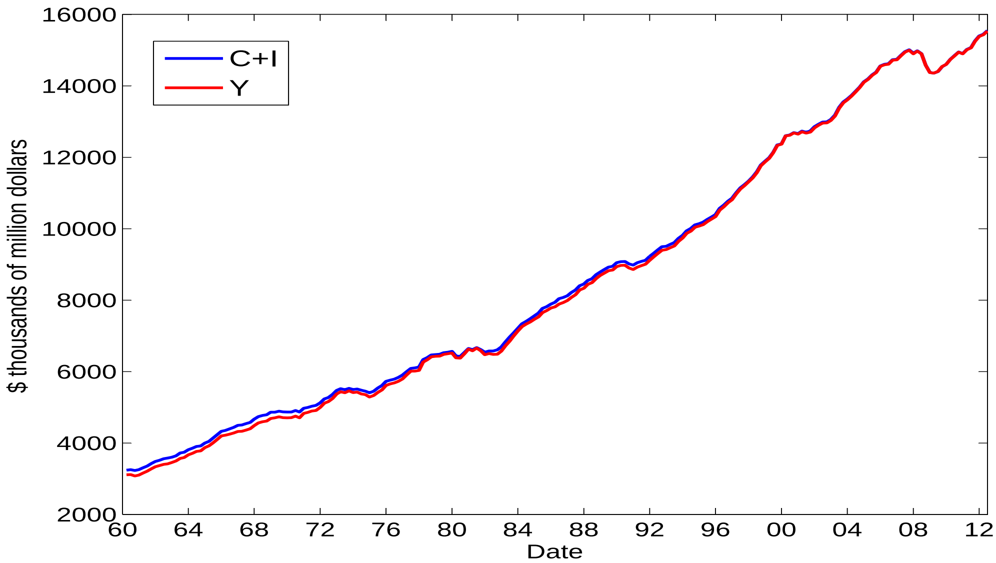


Figure 1: Output Construction (Y vs. C+I)

Cooley (1997) stated that investment for a one sector economy should correspond to the sum of gross fixed capital formation from all sectors, consumption of consumer durables, changes in inventories and net exports. Consumption of durable goods are included in investment rather than in consumption expenditure because they are seen as additions to the household's stock of capital. Net exports are also included because there is no foreign sector. However, due to the unavailability of suitable data, our investment measure does not include consumer durables and the resulting addition to output. Consumption of durable goods is a part of our measure for consumption expenditure. Figure 1 above shows the sum of consumption and investment against output which shows that the two measures are very close to each other.

Labor input is a multi-faceted concept and can cover broad definitions. Our interest is in the intensive margin of labor input and hence it is represented by total hours worked. Since there is no household production sector or farm sector, this is a measure of hours worked by all labor engaged in the production of goods and services in the non-farm business sector. An intensive margin such as total hours worked is presumed to be a better measure of labor input than an extensive margin such as civilian employment because it captures changes in weekly hours, changes in the proportion of part-time workers, overtime hours and annual leave.

Consumer Price Index (CPI) is the price measure used to represent the price level.

Inflation is then calculated from the data for CPI as the change in the natural logarithm of CPI, i.e., as ΔLN(CPI).

M1 and M2 are the two monetary aggregates used as a measure to represent money supply in our models. In the model in section 5, money stock is simply cash balances held for the purchase of consumption goods and so we use M1 as its empirical counterpart. However, in section 6, money represents both cash and bank deposits[5] and accordingly we use M2 which is a broader concept of money.

Finally, since the nominal interest rate in our models are either return on government bond holdings or return on risk-free deposits at the bank, 3-month treasury bill rate in the secondary market is used as its empirical counterpart.

Having constructed the data, we now turn to present some stylized facts of the US business cycle.

4 Stylized Facts of U.S. Business Cycles

In order to evaluate the performance of business cycle models we need to extract the cyclical component from the actual data. In this section we want to accomplish two things. First we want to discuss how to extract business cycle component from the data. Second, having de-trended the data, we want to take the business cycle component and document the main stylized facts of the U.S. business cycle.

4.1 The Hodrick-Prescott Filter

Most economic aggregates grow over time while exhibiting transitory fluctuations around the growth trend. The idea is to characterize an observed time series, y_t, as the sum of a cyclical component, y_t^c, and a growth component, y_t^g. The statistical measurement of business cycles involve making the series stationary by removing the secular trend. We intend to employ a widely used technique for representing growth and business cycle components, known as the Hodrick-Prescott (HP) filter, introduced by Hodrick & Prescott (1997). The HP filter is derived by solving the following minimization problem:

$$\underset{\{y_t^g\}_{t=0}^{\infty}}{min} \sum_{t=1}^{\infty} \{(y_t - y_t^g)^2 + \lambda[(y_{t+1}^g - y_t^g) - (y_t^g - y_{t-1}^g)]^2\}$$

[5]In this model deposits should not be interpreted as checking deposits but rather as less liquid deposits that earn higher interest rates.

The first term is the sum of the squared deviations from trend and therefore measures the degree of fit between y_t and y_t^g. The second term measures the degree of smoothness in y_t^g. For quarterly data, the standard value chosen for the smoothing parameter λ is 1600. When $\lambda = 0$ the trend coincides with the original series, while with $\lambda = \infty$ the solution to this problem is a linear trend.

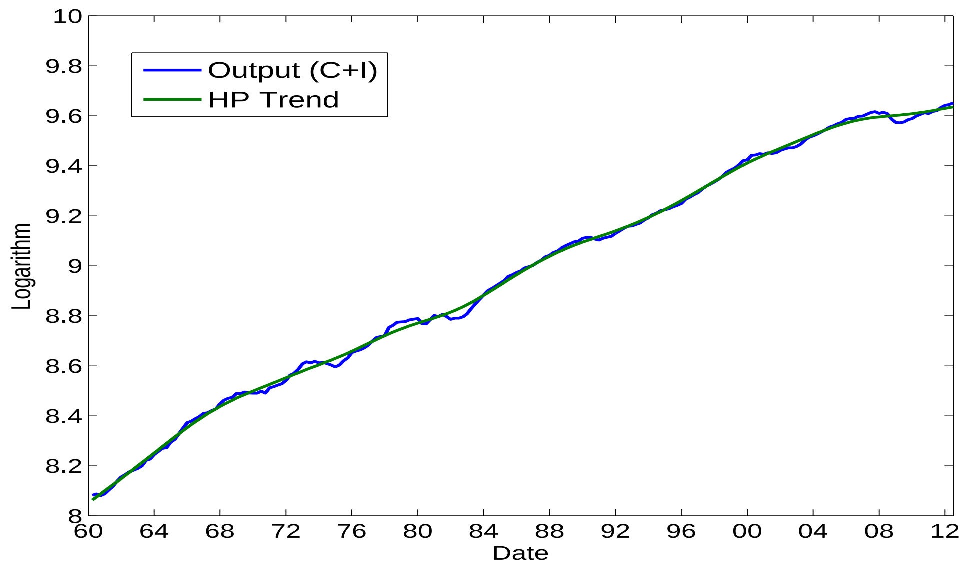
Figure 2 in the next page shows how cyclical component of output is constructed. In panel (a), the logarithm of current output is the series exhibiting more variability whereas trend output is the smoother one. The HP filtered cyclical component of output is the series in panel (b) defined by the difference from the series in the first panel, i.e. $y_t^c = y_t - y_t^g$.[6]

It is important to note that there has been controversies regarding the appropriateness of the use of HP filter for business cycle research. Prescott (1986) points out that the HP filter is a high-pass filter as it is designed to eliminate stochastic components with periodicities greater than 32 quarters. This implies that in using the HP filter we are necessarily defining business cycles as fluctuations in economic time-series with periodicity of 8 years or less. Low frequency movements of the data, which are omitted by this subjective definition, may have important implications for business cycle research.

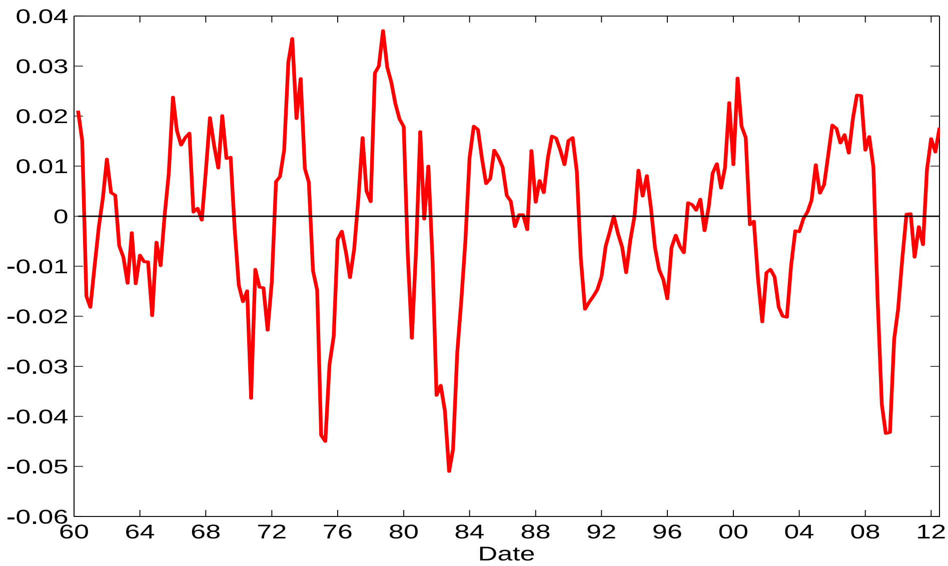
4.2 Features of U.S. business cycles

In this section the business cycle facts of the U.S. economy are represented by calculating several statistics and displaying the cyclical components from the HP filtered time series data. We report the amplitude of the fluctuations in aggregate variables in order to assess their relative magnitudes, measure the correlation of aggregate variables with real output to capture the extent to which variables display co-movement, and finally measure the cross-correlation over time to indicate whether there is any evidence that variables lead or lag one another. Table 1 shows the summary statistics whereas Table 8 in Appendix A shows a more elaborate table with up to 5 leads and lags. Entries in column x are the contemporaneous cross-correlation coefficients between the cyclical component of the series and the cyclical component of output. Entries in columns x(-1) and x(+1) are the non-contemporaneous cross-correlation coefficients at one lag and one lead respectively. Figure 9 in Appendix B compares the cyclical component of the relevant data against the cyclical component of GDP.

[6]This difference is multiplied by 100 so that cyclical output is a percentage.



(a) Output and HP Trend



(b) Cyclical Component

Figure 2: HP filtering

Table 1: U.S. data

Variable	**Std. Dev.**	**Rel SD.**	**x(-1)**	**x**	**x(+1)**
Output	1.65	1	0.83	1	0.83
Consumption	0.97	0.59	0.75	0.81	0.71
Investment	8.8	5.33	0.67	0.89	0.7
Total hours worked	1.9	1.15	0.43	0.66	0.83
CPI	1.24	0.75	-0.54	-0.41	-0.28
Inflation: $\Delta LN(CPI)$	0.48	0.29	0.2	0.32	0.33
M1	2.6	1.58	0.14	0.09	-0.01
M2	1.7	1.03	0.27	0.16	0.05
3-month T-bill rate	1.23	0.75	0.2	0.37	0.43

From observing the relationship between output (GDP) and other data sets, the following characteristics are regarded as the most significant features of the US business cycle for the period $1960(1) - 2012(2)$:

- Consumption is less volatile than output and is pro-cyclical.

- Investment is more than five times as volatile as output and is pro-cyclical.

- Total hours worked is slightly more volatile than output and is pro-cyclical.

- Pro-cyclical and leading money: There is a slight contemporaneous positive correlation between the nominal money stock (measured as either M1 or M2) and real output. More importantly, there is also a pronounced phase shift in the correlation between output and money stock. The cross-correlation of output with the monetary aggregates show that output is more highly correlated with lagged values of the aggregates, implying that money peaks before output.

- Counter-cyclical prices: Price level, measured as Consumer Price Index (CPI), shows that prices are counter-cyclical.

- Inflation is positively correlated with output. It also tends to lag the GDP cycle by three quarters.

- There is a positive correlation between output and nominal interest rates (3-month T-bill rate).

These are the primary facts that characterize the business cycle of the US economy. Now we proceed to describe the models used in this study.

5 Baseline Cash-in-Advance Model

This section describes our first model where, following Cooley & Hansen (1989), we introduce the cash-in-advance motive for holding money into the basic indivisible labor real business cycle model. A specific characteristic of the cash-in-advance model is that it requires agents to have carried over money from the previous period which they use in the current period to make purchases. In our model, the use of money carried over from the previous period is restricted to the purchase of consumption goods. However, investment goods do not require the use of money and can be purchased using current period's income. We are interested in exploring both the qualitative and quantitative effects of monetary shocks for fluctuations of economic aggregates over the business cycle.

5.1 The Structure

The economy is assumed to be populated by a continuum of agents indexed by i of unit mass so that per capita variables are equal to the aggregate of the same variable. An infinitely lived agent maximizes the discounted expected utility function,

$$E_0 \sum_{t=0}^{\infty} \beta^t (ln\ c_t^i + B h_t^i),$$

where β is the discount factor, c_t^i is time t consumption, h_t^i is time t labor supply and B is the marginal disutility attached to an extra unit of work.

Following Hansen (1985), the utility function has indivisible labor where each family signs a contract to provide a fixed amount of labor with a certain probability. Hansen (1985) shows that introducing labor contracts in which wages are paid to all agents but only a fraction of the agents end up working smooths out the goods consumption set and makes it convex over goods consumption and expected hours worked. Adding such an unemployment insurance allows the optimization problem to have a well-defined solution.

Production is assumed to be characterized by constant returns to scale and occurs through a Cobb-Douglas aggregate production function

$$Y_t = \lambda_t K_t^{\theta} H_t^{1-\theta},$$

where Y_t is output, H_t is aggregate labor, K_t is capital, $0 < \theta < 1$ is the capital share parameter and λ_t is total factor productivity. A constant capital share (and thus labor share) is supported by empirical studies.[7] Technology evolves exogenously according to

$$ln\ \lambda_t = (1-\gamma) ln\ \overline{\lambda} + \gamma ln\ \lambda_{t-1} + \epsilon_t^{\lambda},$$

[7] See Gollin (2002).

where the error term is independently and identically distributed as $\epsilon_t^\lambda \sim N(0, \sigma^2_{\epsilon_\lambda})$, $0 < \gamma < 1$ and the stationary state value of the level of technology is $\overline{\lambda} = 1$.

An agent i carries over an amount of money from the previous period, m^i_{t-1}, and receives a nominal transfer from or pays a tax to the government equal to T_t. The cash-in-advance constraint on consumption purchases implies that

$$p_t c^i_t \leq m^i_{t-1} + (1 + i_{t-1})b_{t-1} + T_t - b_t,$$

where $p_t c^i_t$ is consumption expenditure in period t, $(1 + i_{t-1})b_{t-1}$ is the principal plus interest on government bond holdings and b_t is bond acquired in current period and carried into the next period.

In addition to the cash-in-advance constraint, individual i faces the flow budget constraint,

$$c^i_t + k^i_{t+1} + \frac{m^i_t}{p_t} + \frac{b_t}{p_t} \leq w_t h^i_t + r_t k^i_t + (1 - \delta)k^i_t + \frac{m^i_{t-1}}{p_t} + \frac{T_t}{p_t} + \frac{(1 + i_{t-1})b_{t-1}}{p_t},$$

where the right-hand side shows the sum of labor income, capital income, the amount of undepreciated capital at the end of the period and the real value of money held at the beginning of the period (including the transfer/tax from the government and the return on bond holdings) while the left-hand side shows the sum of consumption, capital to be taken to the next period and the real value of money and bonds to be held at the beginning of next period.

Real government spending, G_t, and the per capita money stock, M_t, are assumed to follow a stochastic process. In addition, the government must satisfy a budget constraint in each period which is given as follows,

$$p_t G_t + T_t = M_{t+1} - M_t + B_t - (1 + i_{t-1})B_{t-1},$$

where B_t is the nominal government debt and the initial stock of government debt, B_0, is given[8].

Since we are not studying the impact of government spending shocks here, we set $G_t = 0$. *Ricardian equivalence theorem* states that a change in the timing of taxes/transfers by the government is neutral. This means that in equilibrium a change in current taxes/transfers, which is exactly offset in present value terms by an equal and opposite change in future taxes/transfers, has no effect on the real interest rate or on the

[8]Note that there is a unit mass of individuals so the per capita variables are equal to the aggregate of the same variable.

consumption of individual consumers. By the virtue of this theorem, we assume that $B_t = 0 \bigvee t : 0 \to \infty$. This assumption implies that no bonds are held in this economy in equilibrium and that $T_t = M_{t+1} - M_t$. The money stock is assumed to grow at the rate g_t where g_t evolves stochastically according to the AR(1) process,

$$ln\ g_t = (1 - \pi)ln\ \overline{g} + \pi ln\ g_{t-1} + \epsilon_t^g.$$

The inclusion of the term $(1-\pi)ln\overline{g}$ causes this money growth process to have a stationary state value of $\overline{g}$ and the error term is independently and identically distributed as $\epsilon_t^g \sim N(0, \sigma_{\epsilon_g}^2)$.

Note that in order to solve this model, the cash-in-advance constraint must be binding in every period. Cooley & Hansen (1989) have shown that this condition is met as long as the expected gross growth rate of money, g_t, is greater than the discount factor, β.

5.2 The Full Model

When the stationary state gross growth rate of money is anything but 1, money stock will be either growing or shrinking over time and we will not be able to find stationary states for the nominal variables. Since we are developing a model similar to Cooley & Hansen (1989), we will follow their procedure and normalize the nominal variables in each period by dividing them by M_t, and define $\hat{p}_t = \frac{p_t}{M_t}$, $\hat{m}_t^i = \frac{m_t^i}{M_t}$ and $\frac{M_t}{M_t} = 1$.

The presence of money creates a friction in the economy so that the competitive equilibrium is no longer Pareto optimal and therefore we cannot simply solve a social planner's problem to find the equilibrium allocations. Instead, we will use the recursive competitive equilibrium concept. All individuals are identical by assumption and all will end up doing the same thing. However, if each individual chooses to do something else it will not have an effect on aggregate outcome. The equilibrium is found by solving the following maximization problem,

$$max\ E_0 \sum_{t=0}^{\infty} \beta^t (ln\ c_t^i + Bh_t^i),$$

subject to the budget constraints

$$c_t^i = \frac{\hat{m}_{t-1}^i + (g_t - 1)}{g_t \hat{p}_t}$$

and

$$c_t^i + k_{t+1}^i + \frac{\hat{m}_t^i}{\hat{p}_t} = w_t h_t^i + r_t k_t^i + (1 - \delta)k_t^i + \frac{\hat{m}_{t-1}^i + (g_t - 1)}{g_t \hat{p}_t}.$$

To find the first order conditions of optimality we form the Lagrangian,

$$L = \max_{c_t^i, k_{t+1}^i, h_t^i, \hat{m}_t^i} E_0 \sum_{t=0}^{\infty} \beta^t \{ln\ c_t^i + Bh_t^i + \phi_t(\hat{p}_t c_t^i - \frac{\hat{m}_{t-1}^i + g_t - 1}{g_t}) +$$
$$\psi_t(k_{t+1}^i + \frac{\hat{m}_t^i}{\hat{p}_t} - w_t h_t^i - r_t k_t^i - (1-\delta)k_t^i)\}$$

where ϕ_t is the Lagrange multiplier on the cash-in-advance constraint and ψ_t is the multiplier on the flow budget constraint.

The first order conditions are:

$$\frac{\partial L}{\partial c_t^i} = \frac{1}{c_t^i} + \phi_t \hat{p}_t = 0,$$
$$\frac{\partial L}{\partial h_t^i} = B - \psi_t w_t = 0,$$
$$\frac{\partial L}{\partial k_{t+1}^i} = \psi_t - \beta E_t \psi_{t+1}[(1-\delta) + r_{t+1}] = 0,$$

and

$$\frac{\partial L}{\partial \hat{m}_t^i} = \psi_t \frac{1}{\hat{p}_t} - \beta E_t \phi_{t+1} \frac{1}{g_{t+1}} = 0$$

The first order conditions with respect to c_t^i and h_t^i gives us expressions for the two Lagrangian multipliers,

$$\phi_t = -\frac{1}{\hat{p}_t c_t^i}$$

and

$$\psi_t = \frac{B}{w_t}.$$

We use these expressions to remove the multipliers from the other two first order conditions which then together with our two budget constraints give the following four optimality conditions:

$$\frac{1}{\beta} = E_t \frac{w_t}{w_{t+1}}[(1-\delta) + r_{t+1}], \tag{5.2.1}$$

$$\frac{B}{w_t \hat{p}_t} = -\beta E_t \frac{1}{\hat{p}_{t+1} c_{t+1}^i g_{t+1}}, \tag{5.2.2}$$

$$\hat{p}_t c_t^i = \frac{\hat{m}_{t-1}^i + g_t - 1}{g_t} \tag{5.2.3}$$

and

$$k_{t+1}^i + \frac{\hat{m}_t^i}{\hat{p}_t} = w_t h_t^i + r_t k_t^i + (1-\delta)k_t^i. \tag{5.2.4}$$

Moreover, assuming perfect competition in factor markets gives us the following two conditions,

$$w_t = (1-\theta)\lambda_t(\frac{K_t}{H_t})^\theta \tag{5.2.5}$$

and

$$r_t = \theta\lambda_t(\frac{K_t}{H_t})^{\theta-1}. \tag{5.2.6}$$

5.3 The Stationary State

Here we obtain the stationary states of the model.[9] The model works in such a way that in the absence of technology and money growth shocks, the optimal choice of the real variables and the nominal variables (normalized by dividing them by M_t) will converge to steady states or constant values. Taking aggregation into account, the equations of the model in a stationary state are

$$\frac{1}{\beta} = (1-\delta) + \overline{r}, \tag{5.3.1}$$

$$\frac{B}{\overline{w}} = -\frac{\beta}{\overline{g}\overline{C}}, \tag{5.3.2}$$

$$\overline{p}\overline{C} = 1, \tag{5.3.3}$$

$$\frac{1}{\overline{p}} = (\overline{r}-\delta)\overline{K} + \overline{w}\overline{H}, \tag{5.3.4}$$

$$\overline{w} = (1-\theta)(\frac{\overline{K}}{\overline{H}})^\theta \tag{5.3.5}$$

and

$$\overline{r} = \theta(\frac{\overline{K}}{\overline{H}})^{\theta-1}. \tag{5.3.6}$$

Equation 5.3.1 gives us the real rental on capital as

$$\overline{r} = \frac{1}{\beta} - (1-\delta).$$

Combining equations 5.3.5 and 5.3.6 we get

$$\overline{w} = (1-\theta)[\frac{\overline{r}}{\theta}]^{\frac{\theta}{\theta-1}}. \tag{5.3.7}$$

[9]Variables with a bar on top represent the steady states of the corresponding variables.

From equation 5.3.2 consumption is

$$\overline{C} = -\frac{\beta\overline{w}}{\overline{g}B},$$

and from equation 5.3.3 the relative price level is

$$\overline{p} = \frac{1}{\overline{C}}. \tag{5.3.8}$$

Using the factor market condition, equation 5.3.6, we get

$$\overline{H} = (\frac{\overline{r}}{\theta})^{\frac{1}{1-\theta}}\overline{K}. \tag{5.3.9}$$

Using $\overline{w}$, $\overline{p}$ and $\overline{H}$ from equation 5.3.7, 5.3.8 and 5.3.9 respectively to remove the corresponding variables in equation 5.3.4, we get

$$\overline{K} = \frac{\overline{C}}{\frac{\overline{r}}{\theta} - \delta}.$$

Output can then be found as

$$\overline{Y} = \overline{C} + \delta\overline{K}.$$

We use the parameter values as discussed in the calibration section in 5.4. Using these parameter values, the stationary states for the variables are shown in Table 2.

Table 2: Stationary state values of variables for the baseline cash-in-advance model

$\overline{r}$	$\overline{w}$	$\overline{C}$	$\overline{p}$	$\overline{K}$	$\overline{H}$	$\overline{Y}$
0.0454	1.828	$\frac{0.7328}{\overline{g}}$	$1.3647\overline{g}$	$\frac{6.655}{\overline{g}}$	$\frac{0.3266}{\overline{g}}$	$\frac{0.8991}{\overline{g}}$

5.4 Calibration

The term calibration can take two distinct definitions. One definition, known as strict calibration, refers to the use of economic theory as the basis of restricting our model's structure and at the same time using the implications of that structure in the measurement of data. The other definition, known as classic calibration, refers to choosing parameters such that they are consistent with empirical studies on micro level data about the economy.[10] In this paper, we will use both strict and classic calibration and our measurement of data in Section 3 contains techniques of calibration procedure according to these definitions. A list of values for the parameters are given in Table 3.

[10]See Cooley (1997) for more detail on calibration.

Table 3: Parameter Values

Parameter	Value	Description
β	0.98	Household's discount factor
δ	0.025	Depreciation rate of capital
θ	0.336	Capital share in Cobb-Douglas production function
A	1.63	Preference weight on leisure
B	-2.4447	Marginal disutility of labor
γ	0.979	AR(1) coefficient in TFP process
π	0.66	AR(1) coefficient in money growth process
σ_ϵ^λ	0.0072	Standard deviation of TFP shock
σ_ϵ^g	0.0098	Standard deviation of money growth shock

The discount factor (β) and the depreciation rate (δ), following King & Rebelo (1999), are chosen to be 0.98 and 0.025 respectively. Quarterly depreciation rate on capital, δ, is derived in King & Rebelo (1999) from a conventional depreciation rate of 10% per annum. Next, following Gollin (2002), we use a capital share (θ) of 0.336 which implies a labor share $(1-\theta)$ of 0.664.

We justify the use of steady state value of labor as follows. Federal Reserve Bank of St. Louis reports *average hours of work per week* between 1947 and 1969 while it reports *average annual hours worked per employed person* between 1971 and 2011. Using these data, average weekly hours between 1960 and 1969 is 40.5 hours while between 1970 and 2011 is 35 hours. Burnside & Eichenbaum (1996) used 15 hours as the daily time endowment as sleep was not included. Multiplying 15 by 7 gives the weekly time endowment of 105 hours. To find $\overline{H}$, we divide average weekly hours by the weekly time endowment and this gives a steady state value which is approximately equal to 1/3.

In order to simulate our models so that we can compare them to the U.S. economy, we need to obtain measures for the technology shocks and money growth shocks that feed into our model. Once the shocks are obtained, we feed them in the recursive equilibrium law of motion. We set all the variables to zero for 1959(4) and feed our first shock in the next quarter, 1960(1). The simulated series are then de-trended by the HP filter to remove any growing trend in the data before it is compared to actual data. Following King & Rebelo (1999), the $AR(1)$ coefficient for technology shock, γ, is chosen to be 0.979 and the standard deviation, σ_ϵ^λ, is set to 0.0072.

Next we use data on $M1$ to estimate the $AR(1)$ process for money growth rate and the regression (standard error in parenthesis) over the sample period $1960(1) - 2012(4)$ produces the following equation:

$$\Delta ln(M1)_t = \underset{(0.000985)}{0.004733} + \underset{(0.0532)}{0.656}\, \Delta ln(M1)_{t-1}, \qquad \hat{\sigma_\epsilon^g} = 0.0098.$$

The results of this regression lead us to set π equal to 0.66 and σ_ϵ^g equal to 0.0098. The implied average growth rate of money, $\overline{g}$, is 1.38 per cent per quarter. To ensure that the gross growth rate of money always exceeds the discount factor, as required for the cash-in-advance constraint to bind, we draw money growth shocks from a log normal distribution which implies that $ln(g_t)$ will never become negative.

Following Hansen (1985), the utility function has indivisible labor which implies that the elasticity of substitution between leisure in different periods for the representative agent is infinite. This follows from the assumption that all variation in the labor input reflects adjustment along the extensive margin. B represents the marginal disutility received from working an extra unit of time according to 'Hansen-Rogerson' preferences. Calculation of B is now discussed below.

With divisible labor model, utility is concave with respect to both consumption and leisure and is given by

$$u(c_t, l_t) = ln(c_t) + Aln(1 - h_t),$$

where A is the preference weight on leisure.

Hansen (1985) shows that with indivisible labor and the employment lottery assumption, the expected utility in period t is equal to

$$u(c_t, \alpha_t) = ln(c_t) + h_t \frac{Aln(1 - h_0)}{h_0} + A(1 - \frac{h_t}{h_0})ln(1),$$

where $\frac{h_t}{h_0} = \alpha_t$ is the probability that a particular household will be chosen to provide labor and h_0 is the amount of fixed labor to be provided by a household if chosen to work. This simplifies to

$$u(c_t, h_t) = ln(c_t) + Bh_t,$$

where $B = \frac{Aln(1-h_0)}{h_0}$.

The parameter A is chosen so that households spend one-third of their time working. Hansen (1985) chose a value of $A = 2$ whereas for our choice of parameters, a value of

$A = 1.63$ results in about one-third of the available time spent working. The parameter B is then chosen by setting the expression of hours worked in steady state for the divisible and indivisible labor models equal to each other. This gives B equal to -2.4447. [11]

5.5 Impulse Responses for Cash-in-Advance Model

In this section we use the recursive equilibrium laws of motion to examine the model's implications to a technology shock and money growth shock. Since each endogenous variable responds to a change in the stochastic component, it is important to see how the model responds to their fluctuations. The linear laws of motion are given by

$$x_t = Px_{t-1} + Qz_t,$$

$$y_t = Rx_{t-1} + Sz_t$$

and

$$z_t = Nz_{t-1} + \epsilon_t.$$

where x_t is the vector of state variables, y_t is the vector of control variables and z_t is the vector of exogenous stochastic variables.

We define the state variables as $x_t = [\widetilde{K_{t+1}}]$, the control variables as $y_t = [\tilde{r}_t, \tilde{w}_t, \tilde{H}_t, \tilde{p}_t]$ and the stochastic variables as $z_t = [\tilde{\lambda}_t, \tilde{g}_t]$ where a tilde over a variable stands for log deviation of the variable from its steady state. The first order conditions that are found above are non-linear and so it is usually difficult, if not impossible, to solve the problem analytically. Linear models are relatively easier to solve. However, the problem is to convert a non-linear model into a linear approximation which can then be solved and used to analyze the underlying non-linear system. A standard method for linear approximation is to log-linearize a model around its stationary state. Solution technique for finding the policy matrices follow method of undetermined coefficients and are discussed in length in the appendix.[12] In all cases, the response is to a single positive shock of 1 standard deviation that occurs in period 2.

[11] Hours worked in steady state for the economy with divisible labor is given by $\overline{H} = \frac{1}{1+\frac{A}{1-\theta}[1-\frac{\beta\delta\theta}{1-\beta(1-\delta)}]}$ and for the economy with indivisible labor by $\overline{H} = -\frac{1-\theta}{\frac{Aln(1-h_0)}{h_0}[1-\frac{\beta\delta\theta}{1-\beta(1-\delta)}]}$.

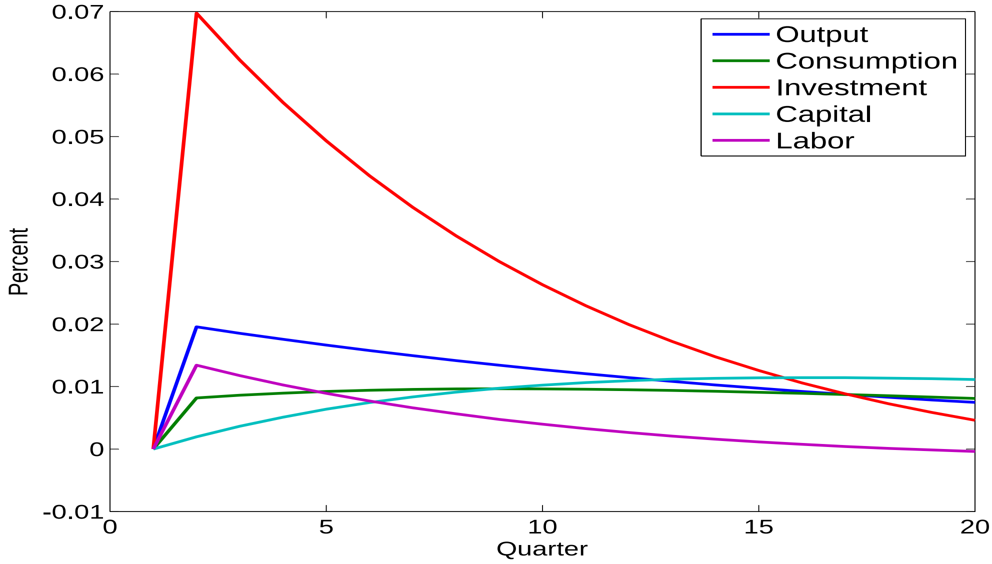
[12] See Uhlig(1999) for an exposition.

5.5.1 Response to a technology shock

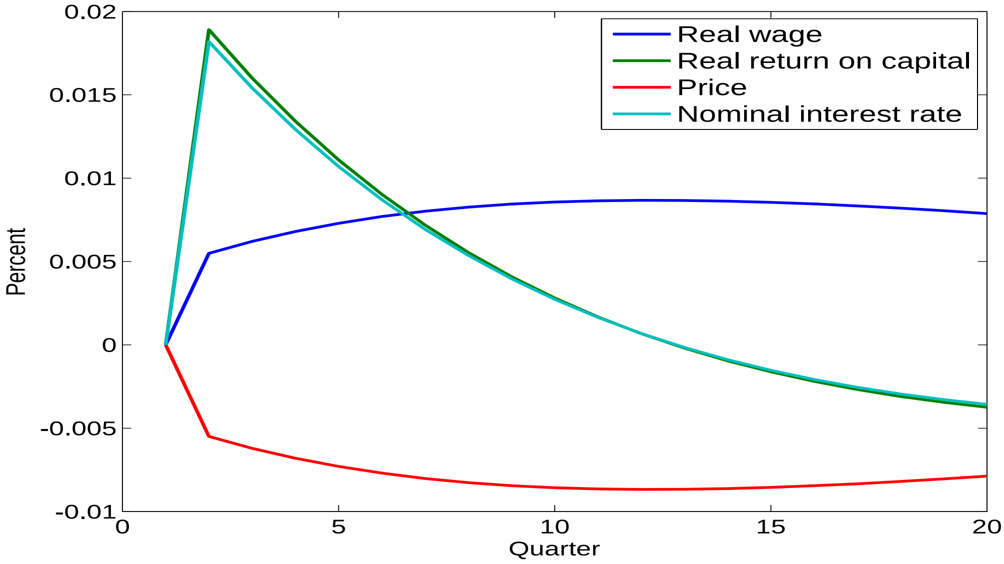
Figure 3 shows the response of the baseline cash-in-advance model to a positive 1 per cent technology shock. It is worth noting that the response of the economy to a technology shock is almost the same as that of the indivisible labor model in Hansen (1985). With a positive technology shock, the individual's lifetime income is higher and so his/her lifetime consumption will be higher as well. The individual wants to consume more in every period and this gain in lifetime consumption will be spread out over time due to the assumed concavity of the utility function with respect to consumption which implies that the individual wants to 'smooth' his/her consumption rather than having them wildly fluctuate. This leads to the inter-temporal substitution of labor. By increasing current output through working harder and smoothing consumption, the individual builds up capital stock so that in the future less labor needs to be used in production and more leisure can be enjoyed. Saving is supported by a rise in the marginal product of capital and the associated rise in real rental on capital. It is worthy of note that the price level goes down after a positive technology shock. This is because a fixed money stock is chasing an increasing amount of goods.

5.5.2 Response to a money growth shock

We can see the response of this economy to a positive 1 per cent money growth shock delineated in Figure 4. The very clear reaction is that of prices which respond very quickly to the one time money growth shock resulting in inflation in this economy. In this model, inflation occurs through lump-sum transfers of money directly to consumers. These transfers reduce the return on money held over from the previous period and reduce incentives to use money. Since it is a money growth shock and not just a money supply shock, a surprise increase in money leads people to expect more such increases in the future and so more inflation, which is captured by the autoregressive process for money growth. This results in borrowers and lenders adding an inflation premium to interest rates and consequently nominal interest rates are pushed up. The rise in interest rate implies that the opportunity cost of holding money is now higher. Thereby, in increasing the expected inflation rate and the nominal interest rate, a positive money growth shock acts as a tax, called *inflation tax*, on consumption. Basic economic intuition tells us that when the opportunity cost of some activity increases, people do less of that activity. Accordingly, the inflation tax on consumption induces individuals to substitute leisure for consumption. This would then lower the labor supply in the aggregate economy and

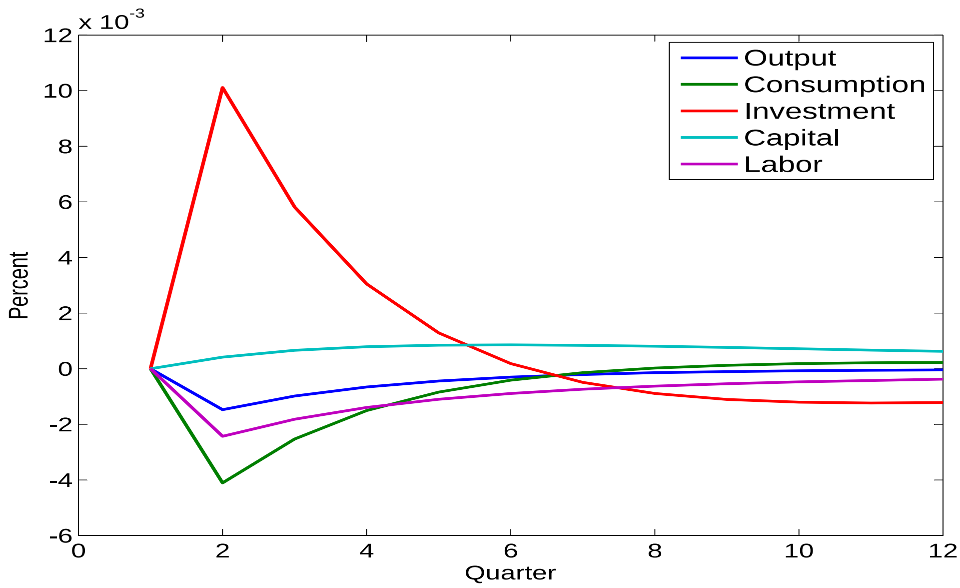


(a)

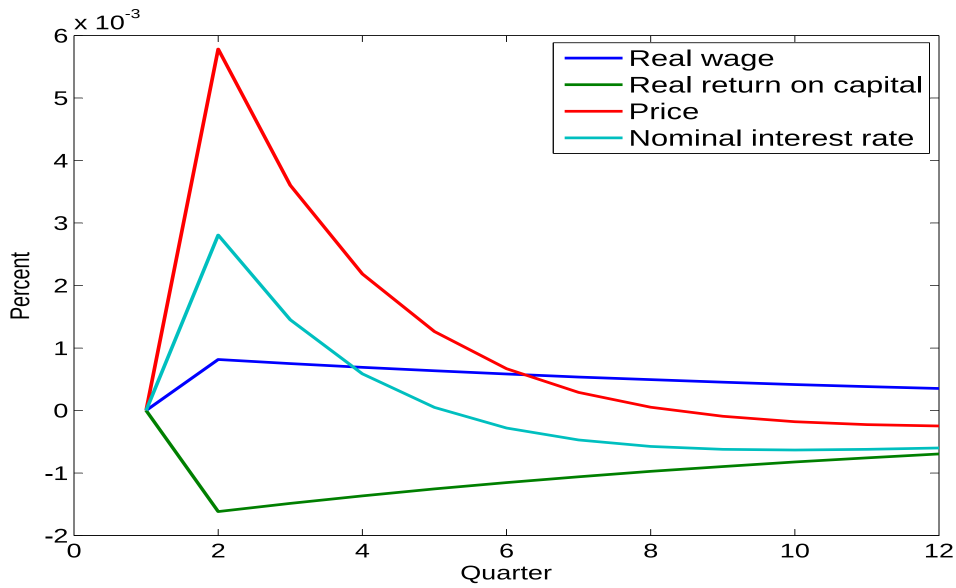


(b)

Figure 3: Response of cash-in-advance model to a technology shock



(a)



(b)

Figure 4: Response of cash-in-advance model to a money growth shock

depress economic activity causing a fall in output as seen in the figure. This phenomenon is known as the *anticipated inflation effect* of a money growth shock.

However, most economists believe that central banks can reduce short-term nominal interest rates through expansionary monetary policy. In this regard, the cash-in-advance model where money is injected into the economy through lump-sum transfers to households only display the *anticipated inflation effect* and totally misses out the *liquidity effect*.

Note that since money growth is less serially correlated than technology (the coefficient on the first lag term is 0.66 for the money growth process and 0.979 for the technology process as shown in the calibration section), the shock dies off much faster than in the case of a technology shock. Moreover, as can be seen from the figure, a money growth shock has almost no effect on output or hours but has a more sizeable, although still quite small, effect on consumption and investment.

5.6 Assessing the baseline Cash-in-Advance Model

In this section we discuss the outcome of simulating the artificial economy and the key summary statistics are shown in Table 4.[13] 100 simulations of 210 periods in length (same as the number of periods in time series from the actual economy) are computed and each simulated time series has been filtered using the HP filter. The statistics reported in the table are averages of those computed for each of the 100 simulations. In Appendix A, Table 9 displays elaborate results for the economy with only a technology shock while Table 10 presents the results with both technology and money growth shocks. In Appendix B, Figures 10 and 11 show the series of the model against the corresponding actual series.

The introduction of money in Hansen's RBC model enables us to study the behavior of the nominal variables with only technology shock and the performance of the model economy is poor in this dimension. Even though the price level is counter-cyclical as in the U.S. economy, it is less variable than the actual price level measured as CPI. Also, the inflation rate in the model is negatively correlated whereas it is positively correlated in the data. Finally, with only technology shock operating, nominal interest rate[14] shows higher volatility and is very highly positive correlated with output.

[13] Relative standard deviation is shown in parenthesis.

[14] Calculated using the Fisher equation (Fisher 1930) as the sum of the real interest rate and the inflation rate

Table 4: Summary Statistics: Actual data vs. Technology shock vs. Both shocks

	Actual data		Technology shock		Both shocks	
Variable	**Std. Dev.**	**Corr. Coef.**	**Std. Dev.**	**Corr. Coef.**	**Std. Dev.**	**Corr. Coef.**
Output	1.65(1)	1	1.77(1)	1	1.74(1)	1
Consumption	0.97(0.59)	0.81	0.63(0.36)	0.877	0.88(0.51)	0.679
Investment	8.8(5.33)	0.89	7.27(4.11)	0.983	7.34(4.22)	0.921
Labor	1.9(1.15)	0.66	1.26(0.71)	0.970	1.26(0.72)	0.964
Price level	1.24(0.75)	-0.41	0.63(0.36)	-0.876	2.81(1.61)	-0.222
Inflation	0.48(0.29)	0.32	0.37(0.21)	-0.424	1.53(0.88)	-0.171
Nominal interest	1.23(0.75)	0.37	1.81(1.02)	0.945	2.37(1.36)	0.723
Money	2.6(1.58)	0.09			2.59(1.49)	-0.0184

Comparing the results in the last two blocks in Table 4, it can be easily seen that the behavior of the real variables are almost similar when money growth shocks are added. There is almost no change in the behavior of output, investment and hours worked while consumption is slightly more volatile and slightly less positively correlated. However, adding money growth shocks through an autoregressive stochastic process affects the statistical properties of the nominal variables. Nominal interest rate displays even higher variability when money growth shocks are added and is slightly less positively correlated but it is still higher than the correlation observed in the data. Both the price level and inflation rate are more variable and in fact displays more variability than what is observed in the data. Also, inflation is less negatively correlated with output in this second economy.

The results for the real variables from comparing the two tables clearly show that monetary shocks entering the economy as direct lump-sum transfer to households do not have real effects. This is consistent with what is shown in Cooley & Hansen (1989). Furthermore, monetary shocks have implications for nominal variables in this model that are inconsistent with what we observe in the data. The cash-in-advance model is thought to be one of the workhorses in monetary theory. However, what is missing in the baseline model is some form of money transmission mechanism by which monetary shocks can be shown to have significant real consequences at business cycle frequencies. In section 6, we will explore a model in which such a mechanism potentially exists.

6 Working Capital Model

Modern economies are characterized by the presence of financial intermediaries that receive funds from people and firms and use these funds to buy bonds or stocks or to make loans to other people or firms. Financial intermediaries, mainly banks, play an important role in the monetary transmission mechanism and an analysis of the impact of monetary shocks cannot ignore them. This section describes our second model where we follow McCandless (2008) and add a financial intermediary to the cash-in-advance model of the previous section. This financial intermediary is modeled as a perfectly competitive banking sector which takes deposits of money from households and lends it to firms. Firms need to cover the wage bill before the goods are sold and hence they borrow to pay for labor services. Monetary policy in this model works through the banking sector where the central bank can make lump-sum monetary transfers to or withdrawals from the financial system. What most central banks do, including the Fed, is to set short-term interest rates (for example, federal funds rate in the U.S.) through open market operations. While central banks do not make direct injections of money into the financial system, however, injecting money into the economy in this way is a delicate way of modeling monetary policy. This is because using interest rate rules effectively means that a central bank is changing the amount of money going into the financial system. However, unlike the baseline model in the previous section, no money transfers go to households. We intend to show that how money enters an economy matters significantly for its impact on real as well as nominal variables.

6.1 The Structure

The economy in this model has four types of agents: households, firms, financial intermediaries and a monetary authority.

6.1.1 Households

The household maximizes a utility function of the form

$$E_0 \sum_{t=0}^{\infty} \beta^t (ln\ c_t^i + Bh_t^i),$$

subject to a cash-in-advance constraint,

$$P_t c_t^i \leq m_{t-1}^i - N_t^i + \eta w_t h_t^i,$$

and a flow budget constraint,

$$c_t^i + k_{t+1}^i + \frac{m_t^i}{P_t} \leq w_t h_t^i + r_t k_t^i + (1-\delta)k_t^i + \frac{m_{t-1}^i}{P_t} - \frac{N_t^i}{P_t} + \frac{r_t^n N_t^i}{P_t},$$

where N_t^i is family i's period t nominal lending to the financial intermediary, r_t^n is the gross interest rate paid by the financial intermediary on deposits, η is the fraction of period t wage income spent or deposited in a financial intermediary by household i and all the other variables are the same as before. We will consider two cases, one where wages cannot be spent until the next period ($\eta = 0$) and the other where wages can only be spent in the current period ($\eta = 1$) and try to understand how the dynamics of the model varies in these two cases. Unlike McCandless (2008), we do not model η in this thesis and pick these two values rather arbitrarily to compare the two extreme cases. McCandless (2008) models η by stating that there is a time cost of spending current income in the current period which depends on η. Leisure is then time available minus time spent working and time used to spend wage income quickly. Another important point to note is that the interest rate r_t^n received by households is simultaneously both real and nominal. It is nominal because it is paid in money and at the same time it is real because the deposits are made and paid back during the same period.

6.1.2 Firms

A perfectly competitive representative firm hires labor and rents capital in order to maximize profit in each period and the production function, as before, is given by

$$Y_t = \lambda_t K_t^\theta H_t^{1-\theta}.$$

Technology evolves exogenously as before according to

$$ln\lambda_t = (1-\gamma)ln\overline{\lambda} + \gamma ln\lambda_{t-1} + \epsilon_t^\lambda,$$

where the error term is independently and identically distributed as $\epsilon_t^\lambda \sim N(0, \sigma_{\epsilon_\lambda}^2)$, $0 < \gamma < 1$ and the stationary state value of the level of technology is $\overline{\lambda} = 1$.

6.1.3 Financial Intermediaries

We model the financial intermediary as a perfectly competitive banking sector with no operation costs that takes deposits from households and makes risk-less loans to firms. The loans are risk-less because they are made after observing the shocks. Importantly, the monetary authority operates its monetary policy in this model through the financial intermediary as stochastic injections or withdrawals of money from the financial system.

Since the financial intermediary is assumed to be perfectly competitive, all of what it earns on loans is paid out to the depositor and the zero profit condition is

$$r_t^f(N_t + (g_t - 1)M_{t-1}) = r_t^n N_t, \tag{6.1.1}$$

where g_t is the gross growth rate of money in period t and r_t^f is the gross interest rate paid by the firm on the working capital that it borrows from the bank. Again, r_t^f is both real and nominal since the loans are intra-period loans.

The financial market clears in every period which means that all of the funds that households have lent to the financial intermediary plus net financial injections or withdrawals from the monetary authority are lent by the financial intermediary to firms. This market clearing condition is given by

$$(N_t + (g_t - 1)M_{t-1}) = P_t w_t H_t. \tag{6.1.2}$$

6.1.4 Monetary Policy

Monetary authority is assumed to follow a very simple form of monetary policy,

$$M_t = g_t M_{t-1}. \tag{6.1.3}$$

Here g_t is assumed, as in the baseline cash-in-advance model, to follow the law of motion,

$$ln\ g_t = (1-\pi)ln\ \overline{g} + \pi ln\ g_{t-1} + \epsilon_t^g, \tag{6.1.4}$$

where again the error term is independently and identically distributed as $\epsilon_t^g \sim N(0, \sigma_{\epsilon_g}^2)$ and $0 < \pi < 1$.

Monetary policy in this model with financial intermediaries might seem strange as it is simply a stochastic process for money growth. Central banks in most industrialized countries today probably follow some form of Taylor rule[15]. In this regard, money growth shocks can be understood as a surprise change in policy.

[15]See Taylor (1993) for an exposition.

6.2 The Optimization Problem

6.2.1 The Representative Household's Problem

The representative agent maximizes the following discounted expected lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t (ln\ c_t^i + Bh_t^i),$$

subject to a cash-in-advance constraint,

$$P_t c_t^i = m_{t-1}^i - N_t^i + \eta w_t P_t h_t^i,$$

and a flow budget constraint,

$$\frac{m_t^i}{P_t} + k_{t+1}^i = (1-\eta) w_t h_t^i + r_t k_t^i + (1-\delta) k_t^i + \frac{r_t^n N_t^i}{P_t}.$$

We limit ourselves to cases where $g_t \geq \beta$ as required for the cash-in-advance constraint to hold with equality in every period. To find the first order necessary conditions, we form the following Lagrangian,

$$L = \max_{c_t^i, k_{t+1}^i, h_t^i, m_t^i, N_t^i} E_0 \sum_{t=0}^{\infty} \beta^t \{ln\ c_t^i + Bh_t^i + \phi_t (P_t c_t^i - m_{t-1}^i + N_t^i - \eta w_t P_t h_t^i) + \psi_t (\frac{m_t^i}{P_t} + k_{t+1}^i$$

$$-(1-\eta) w_t h_t^i - r_t k_t^i - (1-\delta) k_t^i - \frac{r_t^n N_t^i}{P_t})\}$$

where again ϕ_t is the Lagrange multiplier on the cash-in-advance constraint and ψ_t is the multiplier on the flow budget constraint.

The first-order necessary conditions are

$$\frac{\partial L}{\partial c_t^i} = \frac{1}{c_t^i} + \phi_t P_t = 0, \tag{6.2.1}$$

$$\frac{\partial L}{\partial h_t^i} = B - \phi_t \eta w_t P_t - \psi_t (1-\eta) w_t = 0, \tag{6.2.2}$$

$$\frac{\partial L}{\partial k_{t+1}^i} = \psi_t - \beta E_t \{(1-\delta) + r_{t+1}\} \psi_{t+1} = 0, \tag{6.2.3}$$

$$\frac{\partial L}{\partial m_t^i} = \psi_t \frac{1}{P_t} - \beta E_t \phi_{t+1} = 0, \tag{6.2.4}$$

$$\frac{\partial L}{\partial N_t^i} = \phi_t - r_t^n \frac{1}{P_t} \psi_t = 0. \tag{6.2.5}$$

From equation 6.2.1 we get,

$$\phi_t = -\frac{1}{P_t c_t^i}$$

and using equation 6.2.5 and ϕ_t from above we get,

$$\psi_t = -\frac{1}{r_t^n c_t^i}.$$

We use ϕ_t and ψ_t to remove the multipliers from equations 6.2.2, 6.2.3 and 6.2.4. Taking aggregation into account, these three respective equations plus the two budget constraints of the household gives the following five equations on the household side of the model:

$$-BC_t = [\eta + (1-\eta)\frac{1}{r_t^n}]w_t, \tag{6.2.6}$$

$$\frac{1}{r_t^n C_t} = \beta E_t \frac{1}{r_{t+1}^n C_{t+1}}[r_{t+1} + (1-\delta)], \tag{6.2.7}$$

$$\frac{1}{r_t^n C_t} = \beta E_t \frac{1}{C_{t+1}} \frac{P_t}{P_{t+1}}, \tag{6.2.8}$$

$$P_t C_t - M_{t-1} + N_t - \eta w_t P_t H_t = 0, \tag{6.2.9}$$

$$\frac{M_t}{P_t} + K_{t+1} - (1-\eta)w_t H_t - r_t K_t - (1-\delta)K_t - \frac{r_t^n N_t}{P_t} = 0. \tag{6.2.10}$$

6.2.2 The Representative Firm's Problem

The representative firm's problem is to maximize profit in each period and the optimization problem can be written as:

$$L = \max_{K_t, H_t} \lambda_t K_t^{\theta} H_t^{1-\theta} - r_t^f w_t H_t - r_t K_t.$$

The first-order necessary conditions are,

$$\frac{\partial L}{\partial K_t} = \theta \lambda_t K_t^{\theta-1} H_t^{1-\theta} = r_t, \tag{6.2.11}$$

$$\frac{\partial L}{\partial H_t} = (1-\theta)\lambda_t K_t^{\theta} H_t^{-\theta} = r_t^f w_t. \tag{6.2.12}$$

Equation 6.2.11 says that the marginal product of capital is equal to the real rental on capital and equation 6.2.12 says that the marginal product of labor needs to cover the real wage and the financing costs.

6.3 The Competitive Equilibrium

The competitive equilibrium is a sequence of allocations $\{C_t, K_{t+1}, H_t, M_t, N_t\}_{t=0}^{\infty}$ *and prices* $\{w_t, r_t, P_t, r_t^f, r_t^n\}_{t=0}^{\infty}$ *such that given the sequence of productivity* $\{\lambda_t\}_{t=0}^{\infty}$ *and money growth rates* $\{g_t\}_{t=0}^{\infty}$*,*

1. *households satisfy their optimal policies,*
2. *firms satisfy their optimal policies,*
3. *prices clear the loan, labor, capital and goods market*

for all dates $t = 0, 1, 2, ..., \infty$*.*

Equation 6.2.6 is a leisure-consumption trade off. When $\eta = 0$, this equation becomes

$$-BC_t = \frac{1}{r_t^n} w_t. \tag{6.3.1}$$

We know from equation 6.2.8 that

$$r_t^n = \frac{1}{E_t \frac{\beta P_t C_t}{P_{t+1} C_{t+1}}}. \tag{6.3.2}$$

Substituting r_t^n from above into equation 6.3.1 we get,

$$-B = \beta E_t \frac{w_t P_t}{P_{t+1}} \frac{1}{C_{t+1}},$$

which is an inter-temporal leisure consumption trade-off. If the household chooses to work an extra unit of time, the utility cost is $-B$ utils. By working an extra unit of time, it earns a nominal wage of $w_t P_t$ which can be used to purchase $\frac{w_t P_t}{P_{t+1}}$ units of consumption goods in period $t+1$ giving extra utility of $\beta E_t \frac{w_t P_t}{P_{t+1}} \frac{1}{C_{t+1}}$ utils in period t.

On the other hand, when $\eta = 1$, equation 6.2.6 becomes

$$-B = w_t \frac{1}{C_t},$$

which is now an intra-temporal leisure consumption trade-off with its regular interpretation. These leisure-consumption trade-offs have important implications for the dynamics of the model in the case of a money growth shock and will be discussed further when we will talk about impulse responses in later sections.

Equation 6.2.7 gives inter-temporal consumption trade-off for saving through capital. Once we substitute for r_t^n in equation 6.2.7 with that from equation 6.3.2 and rearrange, we get

$$P_t \beta E_t \frac{1}{P_{t+1}} \frac{1}{C_{t+1}} = \beta^2 E_t \{P_{t+1}(1 + r_{t+1} - \delta)\} \frac{1}{P_{t+2}} \frac{1}{C_{t+2}}.$$

Suppose the household gives up P_t dollars at date t to obtain one extra unit of capital which it carries into the next period. This means that the household takes P_t dollars less into period $t+1$. The period t utility cost of taking a dollar less into period $t+1$ is the reduction in period $t+1$ utility that results from having to reduce period $t+1$ consumption by $\frac{1}{P_{t+1}}$ which is $\beta E_t \frac{1}{P_{t+1}} \frac{1}{C_{t+1}}$. The extra capital obtained in period t does not generate extra income until period $t+1$. At the end of period $t+1$, household's investment generates $P_{t+1}(1 + r_{t+1} - \delta)$ dollars. The value to an individual of an extra dollar at the end of period $t+1$ is $\beta^2 E_t \frac{1}{P_{t+2}} \frac{1}{C_{t+2}}$ utils. Hence marginal benefit from purchasing an extra unit of capital in period t is $\{P_{t+1}(1 + r_{t+1} - \delta)\} \beta^2 E_t \frac{1}{P_{t+2}} \frac{1}{C_{t+2}}$ utils in period t terms.

Equation 6.2.8 gives inter-temporal consumption trade-off for saving through bank deposit which can be rearranged as

$$\frac{1}{P_t} \frac{1}{C_t} = \beta E_t r_t^n \frac{1}{P_{t+1}} \frac{1}{C_{t+1}}.$$

Suppose the household decides to put an extra dollar into the bank instead of holding it for consumption. This will reduce period t consumption by $\frac{1}{P_t}$ units and the utility cost from having to reduce period t consumption by $\frac{1}{P_t}$ units is $\frac{1}{P_t} \frac{1}{C_t}$ utils. The extra dollar put into the bank generates a gross return of r_t^n dollars at the end of period t which can be used to purchase consumption goods in period $t+1$. Therefore, the value to an individual of an extra dollar at the end of period t is $\beta E_t \frac{1}{P_{t+1}} \frac{1}{C_{t+1}}$ utils.

Using these weights to value the costs and benefits of a change from the household's optimal plan leads to the above two inter-temporal conditions. Had these conditions not hold, households could increase utility by changing investment decisions or by changing decisions about how much to put into the bank.

6.4 The Stationary State

All real variables are constant over time in a stationary state. The nominal variables can either grow or decline, however, the real values of the nominal variables have the same value across time. $\overline{\frac{M}{P}}$ is defined as stationary state real money balances of households and $\overline{\frac{N}{P}}$ as stationary state real household lending to the financial intermediary.

Including three first-order necessary conditions from household optimization, two budget constraints on the household side, two optimality conditions from firm's profit maximization, a production function, and finally two equations from the credit market, we have ten equations all together. In stationary state, these equations are:

$$\frac{1}{\beta} = (1-\delta) + \overline{r}, \tag{6.4.1}$$

$$\overline{r^n} = \frac{\overline{\pi}}{\beta}, \tag{6.4.2}$$

$$-B\overline{C} = [\eta + (1-\eta)\frac{1}{\overline{r^n}}]\overline{w}, \tag{6.4.3}$$

$$\overline{C} - \frac{\overline{\frac{M}{P}}}{\overline{g}} + \overline{\frac{N}{P}} - \eta\overline{w}\overline{H} = 0, \tag{6.4.4}$$

$$\overline{\frac{M}{P}} - (\overline{r} - \delta)\overline{K} - (1-\eta)\overline{w}\overline{H} - \overline{r^n}\overline{\frac{N}{P}} = 0, \tag{6.4.5}$$

$$\overline{Y} = \overline{\lambda}\ \overline{K}^{\theta}\overline{H}^{1-\theta}, \tag{6.4.6}$$

$$\overline{r} = \theta\overline{\lambda}\ \overline{K}^{\theta-1}\overline{H}^{1-\theta}, \tag{6.4.7}$$

$$\overline{r^f}\overline{w} = (1-\theta)\overline{\lambda}\ \overline{K}^{\theta-1}\overline{H}^{1-\theta}, \tag{6.4.8}$$

$$\overline{r^f}[\overline{\frac{N}{P}} + (\overline{g}-1)\frac{\overline{\frac{M}{P}}}{\overline{g}}] = \overline{r^n}\overline{\frac{N}{P}}, \tag{6.4.9}$$

$$\overline{\frac{N}{P}} + (\overline{g}-1)\frac{\overline{\frac{M}{P}}}{\overline{g}} = \overline{w}\overline{H}. \tag{6.4.10}$$

Equation 6.4.1 implies that in the stationary state, the real return on capital is

$$\overline{r} = \frac{1}{\beta} - 1 + \delta.$$

From equation 6.4.2 the stationary state nominal return on lending to the financial intermediary is,

$$\overline{r^n} = \frac{\overline{\pi}}{\beta} = \frac{\overline{g}}{\beta}.$$

The stationary state gross growth rate of money is $\overline{g}$ and it equals the stationary state inflation rate, $\overline{\pi}$, so that the real money stock, $\frac{M_t}{P_t}$, stays constant for all t.

Equation 6.4.3 when combined with equation 6.4.2 gives the stationary state consumption as

$$\overline{C} = -\frac{[\eta + (1-\eta)\frac{\beta}{\overline{g}}]\overline{w}}{B}. \tag{6.4.11}$$

Using equation 6.4.7 and 6.4.8, we get

$$\overline{K} = \overline{H}(\frac{\theta}{\overline{r}})^{\frac{1}{1-\theta}} \tag{6.4.12}$$

and

$$\overline{r^f}\overline{w} = (1-\theta)(\frac{\theta}{\overline{r}})^{\frac{\theta}{1-\theta}}. \tag{6.4.13}$$

Recall that the stochastic process for technology was chosen so that $\overline{\lambda} = 1$.

Next, using the production function in steady state, equation 6.4.6, and $\overline{K}$ from equation 6.4.12, we get

$$\overline{Y} = \overline{H}(\frac{\theta}{\overline{r}})^{\frac{\theta}{1-\theta}}. \tag{6.4.14}$$

We eliminate wages from equation 6.4.13 using $\overline{w}$ from equation 6.4.11 and get

$$\overline{r^f} = -(1-\theta)(\frac{\theta}{\overline{r}})^{\frac{\theta}{1-\theta}}(\frac{\eta + (1-\eta)\frac{\beta}{\overline{g}}}{B\overline{C}}). \tag{6.4.15}$$

The two steady state equations from the financial market, equations 6.4.9 and 6.4.10, can be written as

$$(\overline{r^n} - \overline{r^f})\overline{\frac{N}{P}} = \overline{r^f}(1 - \frac{1}{\overline{g}})\overline{\frac{M}{P}} \tag{6.4.16}$$

and

$$\overline{\frac{N}{P}} + (1 - \frac{1}{\overline{g}})\overline{\frac{M}{P}} = -(\frac{1}{\eta + (1-\eta)\frac{\beta}{\overline{g}}})B\overline{C}\overline{H}. \tag{6.4.17}$$

Table 5: Stationary state values of variables for working capital model

η	$\overline{r}$	$\overline{r^n}$	$\overline{r^f}$	$\overline{\frac{M}{P}}$	$\overline{\frac{N}{P}}$	$\overline{C}$	$\overline{Y}$	$\overline{w}$	$\overline{H}$	$\overline{K}$
0	0.0454	1.0377	0.9991	1.3096	0.5664	0.7213	0.8850	1.8299	0.3214	6.5495
1	0.0454	1.0377	1.0161	0.7361	0.5779	0.7361	0.8894	1.7993	0.3230	6.5821

The cash-in-advance constraint, equation 6.4.4, gives

$$\overline{C} = \frac{\overline{\frac{M}{P}}}{\overline{g}} - \frac{\overline{N}}{P} - (\frac{\eta}{\eta + (1-\eta)\frac{\beta}{\overline{g}}})B\overline{C}\overline{H}. \tag{6.4.18}$$

Finally, in stationary state the household's flow budget constraint, equation 6.4.5, can be written after replacing $\overline{w}$, $\overline{r^n}$ and $\overline{K}$ by what they equal, as

$$\frac{\overline{M}}{P} = \frac{\overline{g}}{\beta}\frac{\overline{N}}{P} + [(\overline{r}-\delta)(\frac{\theta}{\overline{r}})^{\frac{1}{1-\theta}} - \frac{1-\eta}{\eta + (1-\eta)\frac{\beta}{\overline{g}}})B\overline{C}]\overline{H}. \tag{6.4.19}$$

The set of equations 6.4.15 through 6.4.19 is a system of five equations in five unknowns: $\overline{\frac{M}{P}}$, $\overline{\frac{N}{P}}$, $\overline{r^f}$, $\overline{C}$ and $\overline{H}$. The standard values of the model's parameters are the same as those discussed in the calibration section. However, the stationary state gross growth rate of money, $\overline{g}$, is now equal to 1.01697 and this value comes from estimating the AR(1) process for money growth shock using data for M2. Since money in this model comprises of both cash and bank deposits, we think M2 would be a better aggregate in calculating the money growth shocks. The regression over the sample period of our study gives us the following equation:

$$\Delta ln(M2)_t = \underset{(0.001004)}{0.006054} + \underset{(0.053371)}{0.638}\,\Delta ln(M2)_{t-1}, \qquad \hat{\sigma_\epsilon^g} = 0.0066.$$

Since $\overline{g}$ is not equal to 1, this system of five equations cannot be worked out analytically and we use the MATLAB routine *fsolve* to calculate the stationary state values. Once we know the value of these five unknowns, we can use these to find the rest. For instance, equation 6.4.13 together with the value $\overline{r^f}$ can be used to find $\overline{w}$ and equation 6.4.14 together with $\overline{H}$ can be used to find $\overline{Y}$. Table 5 gives the stationary state values of the variables in this model for the case when $\eta = 0$ and when $\eta = 1$.

6.5 Impulse Responses for Working Capital Model

The recursive equilibrium laws of motion for examining the models implications to a technology shock and money growth shock are similar to the ones in the previous section and are given, as before, by

$$x_t = Px_{t-1} + Qz_t,$$
$$y_t = Rx_{t-1} + Sz_t$$

and

$$z_t = Nz_{t-1} + \epsilon_t$$

where again x_t is the vector of state variables, y_t is the vector of control variables and z_t is the vector of exogenous stochastic variables. We define the state variables as $x_t = [\tilde{K_{t+1}}, \tilde{M_t}, \tilde{P_t}]$, the control variables as $y_t = [\tilde{r_t}, \tilde{w_t}, \tilde{Y_t}, \tilde{C_t}, \tilde{H_t}, \tilde{N_t}, \tilde{r_t^n}, \tilde{r_t^f}]$ and the stochastic variables as $z_t = [\tilde{\lambda_t}, \tilde{g_t}]$. The policy matrices P, Q, R and S are solved using Uhlig's method of log linearization and details are left in Appendix C. The responses shown by the impulses are to a single positive shock of 1 standard deviation that occurs in period 2.

6.5.1 Response to a technology shock

Figures 7 and 8 in Appendix B shows the responses of the model economy to a technology shock for the case when $\eta = 0$ and $\eta = 1$ respectively. Note that the responses of the real variables to a technology shock are very similar to the basic indivisible labor RBC model and also to the baseline cash-in-advance model of the previous section and so is not discussed in length here.[16] This implies that how money enters into the model does not matter for the impact of technology shock on real variables. For the nominal variables, prices decline as a fixed money stock is chasing after an increasing amount of goods and nominal deposits in the financial intermediary initially rise. Furthermore, technology shock has trivial impact on interest rate received by households on bank deposits. It is also worthy of note that the responses to a technology shock are qualitatively similar for both $\eta = 0$ and $\eta = 1$ as can be seen from comparing these two figures.

[16] Refer to section 5.5.1 for a description of the impact of technology shock on real variables.

6.5.2 Response to a money growth shock when $\eta = 0$

Figure 5 shows the response of the working capital model to a money growth shock for the case where current period wage income cannot be used for current consumption purchases. Unlike in the baseline cash-in-advance model, new issues of money are injected directly into financial intermediaries as additional loanable funds. Hence, a money growth shock generates a wedge between the interest rate received by households and that paid by firms. The intuition for understanding this is straightforward. In order to induce firms to absorb more cash for employment purposes, financial intermediaries must lower the interest rate charged on loans to firms. Lower real borrowing costs for firms increase their demand for labor. Consequently, real wages increase and additional labor is supplied by households which then stimulates economic activity.

Looking at the household sector of the economy, we know that at the beginning of each period households are holding money that they are carrying over from the previous period and they lend some of this money to financial intermediaries. With a positive money growth shock and expansion of lending activity, banks earn a higher revenue. Since banks are assumed to be perfectly competitive, the extra revenue is distributed to households as higher interest rates on their deposits. Consequently, households' opportunity cost of holding cash balances for consumption increases. As such, households hold less money for consumption purposes and put more money into banks. Since current period wage income are not allowed to be used to purchase consumption goods, consumption falls unequivocally as there is no intra-temporal leisure consumption trade-off of the kind to be discussed in the next subsection (6.5.3). Also, with additional labor, the marginal product of capital increases and the capital stock grows until the marginal product of capital again equals $\overline{r}$, its steady state value.

6.5.3 Response to a money growth shock when $\eta = 1$

The dynamics of the model is interestingly different when we allow current period wage income to be used for consumption purchases and Figure 6 shows the model's response. Now, the household side of the economy shows a somewhat different story. In order to understand the dynamics consider first holding wages and labor fixed. When a positive money growth shock hits the economy, interest rate received by households on bank deposits go up and households substitute current consumption for more bank deposits (i.e. future consumption). This is the same as with the case when $\eta = 0$. Now allow hours worked to respond. As households reduce their current consumption, the marginal

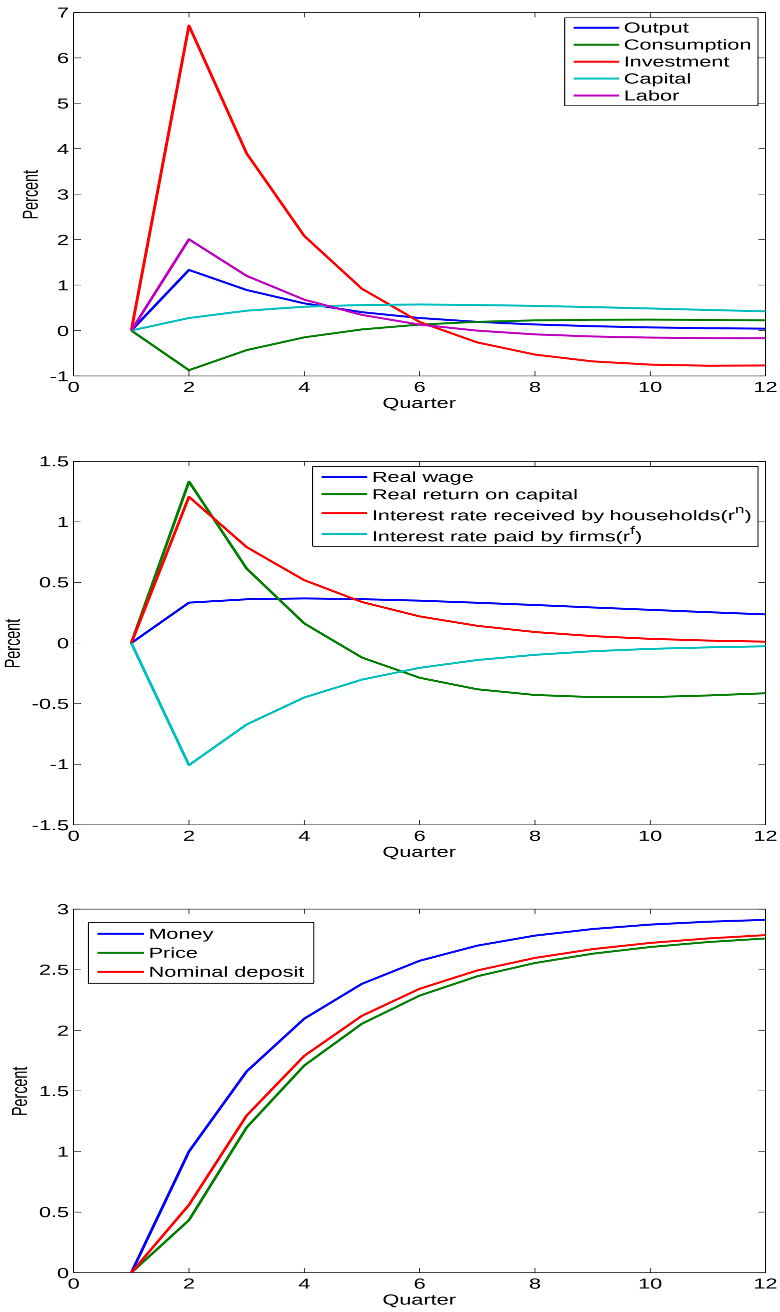


Figure 5: Response of the working capital model to a money growth shock ($\eta = 0$)

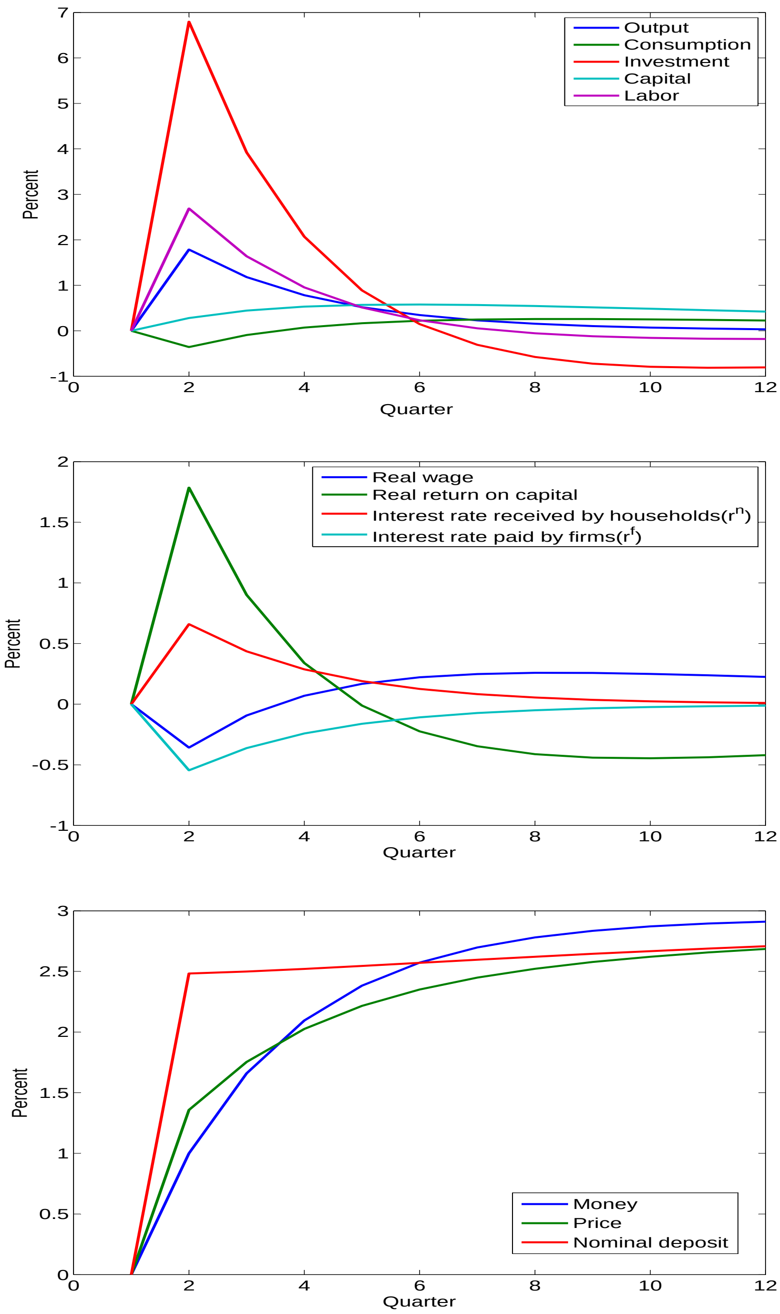


Figure 6: Response of the working capital model to a money growth shock ($\eta = 1$)

utility of current consumption increases and so households become more willing to trade leisure for consumption. This is the intra-temporal aspect of a money growth shock and it gives rise to a further increase in hours worked on top of that induced by a higher demand for labor from firms.

Now allow wages to adjust. On the one hand, when $\eta = 0$, a positive money growth shock tends to increase real wages. However, when $\eta = 1$, we see that initially wages fall overall. This is because of the intra-temporal leisure consumption trade-off that is absent when $\eta = 0$. When $\eta = 1$, a higher marginal utility of consumption makes households more willing to substitute leisure for consumption. This gives rise to a further increase in hours supplied on top of that induced by a higher demand for labor from firms. Taken altogether, labor used in production increases and so the marginal product of labor declines sufficiently such that overall equilibrium wages fall. At the same time, as more labor is in use, the marginal product of capital increases such that the equilibrium real rate of return on capital increases.

As time progresses, the interest rate on bank deposits fall and nominal deposits continue to increase but at a slower rate. At some point, households are no longer willing to forgo current consumption for more bank deposits and at this point consumption increases. Of course, as consumption rises, the marginal utility of consumption falls and households become more willing to trade consumption for leisure causing labor supply to decrease. As labor is drawn out of production, the remaining supply of labor yields a high marginal product causing equilibrium wages to rise and at the same time the large capital stock has less labor to work with and so the marginal product of capital falls causing the real return on capital to decline as well.

6.6 Assessing the Working Capital Model when $\eta = 0$

In order to evaluate the performance of the working capital model presented in this section, the statistics generated by the model are compared against those observed in the U.S. economy presented in section 4. Table 6 below displays some key summary statistics. Detailed tables are left in Appendix A where Table 11 displays summary statistics for an economy with only technology shock operating and Table 12 presents results with both technology and money growth shocks. Figures 12 and 13 in Appendix B display the series of the model plotted against the corresponding actual series.

It is immediately obvious from looking at Table 6 that injecting money into the model through financial intermediaries enable money growth shocks to have significant

Table 6: Summary Statistics ($\eta = 0$): Actual data vs. Technology shock vs. Both shocks

	Actual data		Technology shock		Both shocks	
Variable	**Std. Dev.**	**Corr. Coef.**	**Std. Dev.**	**Corr. Coef.**	**Std. Dev.**	**Corr. Coef.**
Output	1.65(1)	1	1.6(1)	1	1.88(1)	1
Consumption	0.97(0.59)	0.81	0.55(0.34)	0.862	0.9(0.48)	0.112
Investment	8.8(5.33)	0.89	6.69(4.18)	0.982	10.49(5.58)	0.924
Labor	1.9(1.15)	0.66	1.03(0.64)	0.969	1.78(0.95)	0.889
Price level	1.24(0.75)	-0.41	0.96(0.60)	-0.986	1.79(0.95)	-0.452
Inflation	0.48(0.29)	0.32	0.7(0.44)	-0.447	0.91(0.48)	-0.036
Nominal interest rate	1.23(0.75)	0.37	0.06(0.04)	0.987	0.87(0.46)	0.547
Money	1.7(1.03)	0.16			1.69(0.9)	0.127

real effects. Output, consumption, investment and labor are all more volatile in the second economy where both technology and money growth shocks are operating. This is a major qualification that we would like to point out in this study. How money enters a model matters significantly for its impact on real variables. When money entered the model directly as lump-sum transfers to households as in Section 5, we showed that money growth shocks had no significant impact on real variables.

The results from comparing the standard deviations of the variables in the model economy with those observed in the U.S. economy shows that with both technology and money growth shocks, the model does a reasonably good job in replicating the variability of output, consumption, investment and labor. The standard deviation of output is 14% larger compared to actual data; that of consumption and labor is 7.2% and 6.3% smaller respectively whereas that of investment is 19% larger than its empirical counterpart. In terms of nominal variables, prices and inflation show higher volatility whereas nominal interest rate shows lower volatility than that of actual data. In fact, inflation shows 90% more volatility which is somewhat disconcerting.

When comparing the contemporaneous correlation of the variables with output, the addition of money growth shock hurts in one dimension but helps in others. Inflation, although not pro-cyclical as in the data, is less negatively correlated and almost acyclical which is a move in the desired direction. Moreover, prices are very highly negatively correlated with only technology shock. With both shocks operating, Table 6 shows that prices are less negatively correlated and almost aligned to what is observed in the data. On the contrary, the contemporaneous correlation of consumption with output deteriorates as it is now less positively correlated than in the data but this might be anticipated given a fall in consumption after a positive money growth shock.

The nominal asset through which agents can transfer wealth inter-temporally in this model is the nominal bank deposit. Since in-period uncertainty is revealed before the loans take place, firms are always assumed to pay back the loans and hence banks never default on its borrowing from households. Nominal deposits are risk-less and hence we compare the nominal interest rate received by households with the 3-month T-bill rate for the U.S. economy. With only technology shocks, nominal interest rates on bank deposits show very little volatility and a very high contemporaneous correlation with output. With both shocks operating volatility increases, although it is still lower than in the data. Moreover, nominal interest rates are now less positively correlated with output and hence more aligned to the data.

Another important characteristic to note is the behavior of the money stock itself in this model. Table 6 shows us that money is slightly positively correlated with current output and this is keeping with what is observed in the data. More importantly, as discussed in Section 4.2, money supply is a leading variable as observed from the phase shift in correlation with output in Table 1. Our model economy with financial intermediaries where monetary shocks enter the economy via this financial sector is well able to replicate this phenomenon and Table 12 in the appendix shows us that indeed money peaks before output in the model as output is more highly correlated with lagged values of money.

A question of related interest is whether money growth shocks alone can generate realistic business cycle fluctuations. Table 13 shows the simulation results for the model with only money growth shock. From looking at the table we can see that money growth shocks alone are not capable of generating realistic business cycles. Contemporaneous correlation of consumption with output is very highly negative which is grossly counterfactual as we know that consumption is pro-cyclical. Moreover, price level in the model is acyclical whereas we have seen that prices are counter-cyclical in the data. These and other statistics from Table 13 shows that monetary shocks are not the sole major driver of business cycles.

6.7 Assessing the Working Capital Model when $\eta = 1$

Table 7 shows summary statistics from simulations of this economy when current wages can be used for current consumption and Tables 14 and 15 in Appendix A show more elaborate results. Figures 14 and 15 in Appendix B display the series of the model against the corresponding actual series. Once again, from comparing the results shown in Table 7 it is evident that money growth shocks have real effects as the variables in the economy with both shocks display higher volatility. Volatility of hours worked is now even higher and this can be understood from the intra-temporal leisure consumption trade-off where households now provide even higher labor supply with money growth shocks when $\eta = 1$. Indeed, hours worked are more volatile than output which is in line with what is observed in the US data.

Table 7: Summary Statistics ($\eta = 1$): Actual data vs. Technology shock vs. Both shocks

	Actual data		Technology shock		Both shocks	
Variable	**Std. Dev.**	**Corr. Coef.**	**Std. Dev.**	**Corr. Coef.**	**Std. Dev.**	**Corr. Coef.**
Output	1.65(1)	1	1.69(1)	1	2.17(1)	1
Consumption	0.97(0.59)	0.81	0.6(0.36)	0.881	0.72(0.33)	0.374
Investment	8.8(5.33)	0.89	6.96(4.12)	0.982	10.97(5.06)	0.962
Labor	1.9(1.15)	0.66	1.18(0.70)	0.972	2.32(1.07)	0.914
Price level	1.24(0.75)	-0.41	0.6(0.36)	-0.882	1.72(0.79)	-0.026
Inflation	0.48(0.29)	0.32	0.36(0.21)	-0.582	0.94(0.43)	0.334
Nominal interest rate	1.23(0.75)	0.37	0(0)	0.939	0.48(0.22)	0.604
Money	1.7(1.03)	0.16			1.75(0.81)	0.128

The performance of the model economy improves in the sense that when current wages are allowed to be used for current consumption, consumption is more positively correlated, even though still lower than in the data. Another improvement is that inflation is now pro-cyclical and the magnitude of the positive correlation perfectly matches with what is observed in the data. However, inflation is still more volatile as before. Money, as in the case when $\eta = 0$, is again seen to be pro-cyclical and is also a leading variable.

However, in other dimensions the performance of the model economy is poor. When current wages can be used for current consumption, the addition of money growth shock to technology shock results in prices being acyclical which is at odds to what is observed in the U.S. data. Moreover, the volatility of nominal interest rates are even lower when $\eta = 1$ than when $\eta = 0$ when compared with the volatility of 3-month T-bill rate.

Finally, Table 16 shows the outcome from simulating the artificial economy with only money growth shock and the results confirm what we have stated in the previous section that monetary shocks alone cannot generate realistic business cycle fluctuations.

7 Discussion

In this paper we have illustrated how the basic neoclassical model can be modified to incorporate money in an attempt to replicate the cyclical fluctuations of the U.S. economy. A neoclassical model with stochastic perturbations to technology and money growth rate has been built upon in this regard. We have done this for one case where money enters the model as direct lump-sum transfers to households and in the other case where new money injections enter the economy through the financial system. What is particularly interesting about the two channels studied in this paper is that the models generate very different responses to a money growth shock. In the first case, a positive money growth shock results in *anticipated inflation effect* with interest rates rising and hours and output falling. It is noted that this phenomenon is in contrast to the popular consensus that expansionary monetary policy lowers nominal interest rates. Next, following McCandless (2008), we have studied a model with financial intermediaries where money is injected through the financial system. In this case, a positive money growth shock results in lower lending rates thus persuading firms to borrow more and to expand their scale of operation. Accordingly, hours worked and output both increases with a positive money growth shock illustrating the *liquidity effect* at work.

We have assessed the quantitative importance of monetary shocks for business cycle fluctuations in these two environments. In doing so, we have added money growth shocks to technology shocks and unconditional moments are then generated to provide a basis for comparison with the empirical counterparts. By comparing the results for the real variables in the baseline cash-in-advance model, it is observed that monetary shocks entering the economy as direct lump-sum transfers to households do not have real effects. Moreover, the explicit monetary environment has enabled us to study the implications for nominal variables which are inconsistent with what we observe in the data.

What is absent in the baseline model is some form of monetary transmission mechanism. Moreover, traditional monetary policy is thought to follow some sort of interest rate rule which operates through the financial system and involves short term interest rates (e.g. the *federal funds rate* on overnight interbank loans). Motivated by this real world feature, we have added a perfectly competitive banking sector which acts as an intermediary between borrowers (firms) and lenders (households). The results from simulating this model environment shows that monetary shocks have significant real effects at business cycle frequencies. All the real variables are more volatile when monetary shocks are added.

We have analyzed two situations, one where households can use current period wage income for current consumption ($\eta = 1$) and the other where they cannot ($\eta = 0$). In either case simulation results show that monetary shocks dominate technology shocks in this environment. This can be inferred from looking at moments and for the ease of the exposition let us consider the case where $\eta = 1$. The contemporaneous correlation of consumption with output (0.374) is lower than what is observed in the data (0.81). This can be thought to occur because of the dominance of monetary shocks that drive consumption down as observed from the impulse responses. Moreover, prices are acyclical whereas they are counter-cyclical in the data. Counter-cyclical prices give some support for the relative importance of technology shocks in causing business cycles and our finding that prices are acyclical again shows the dominance of monetary shocks. However, we do have some ideas that could reduce the apparent strong dominance of monetary shocks in this model and help in aligning the results with the observed data.

The financial market is frictionless according to the way we have modeled it. Households' decisions about how much to lend to the financial intermediary are made after observing both technology and money growth shocks. After a positive money growth shock we have observed that interest rates received by households increase. Consequently, this induces households to deposit more into banks. As a result banks have an even larger pot of money to lend out to the firms (both the new injections from the monetary authority and higher deposits from households). The assumption that households can continuously revise their consumption and savings decisions is probably too strong when compared to the real world scenario. Presumably, there are costs associated with continual updating. For instance, there are penalties that the intermediaries charge on early withdrawals and interest rates earned in the first period in which new deposits have been made are generally lower. Accordingly, we could modify the working capital model in one of two ways that might help in reducing the impact of monetary shocks and make it more realistic. Either we could assume that households have less-than-perfect flexibility in responding to a monetary shock and assume that portfolio decisions must be made before observing the current period shock. This class of models, called *limited participation* models, have been studied by Christiano (1991), Christiano & Eichenbaum (1992) and Fuerst (1992) among others. In *limited participation* models, with household deposit decisions fixed in the previous period, bank lending of working capital to firms would not expand by as much as in our version after a positive monetary shock. Consequently, lending rates would decline to a lesser extent and hours worked and output would not increase by as much as we have observed. Otherwise, following Cooley & Quadrini (1999), we could assume that households have perfect flexibility but there is an adjustment cost associated with changing portfolio decisions which would similarly help to minimize the impact of monetary shocks.

Moving forward the decision about how much to lend to the financial intermediary, as in *limited participation* models, would help in making the model more realistic in one other dimension as well. For the model economy where we have allowed current period wage income to be used for current consumption, transactions happen very fast. When $\eta = 1$, this means that wages paid at the beginning of the period can be used to make deposits at the bank which are then lent to finance wages in the current period. As McCandless (2008) points out, even for a quarterly economy, one would think that such a turnover is too fast. Allowing for only 'limited participation' by households where they make their deposit decisions a period ahead would mean that current period deposits are used to finance the following period's wages.

8 Conclusion

This study has delineated how the basic real business cycle model can be tailored to analyze the role of monetary shocks in business cycles. By developing linear approximations to the models studied, we have been able to show the response of the economy to unanticipated changes in the growth rate of money supply. For the standard values of the model's parameters, the statistics generated by the working capital model shows that it is capable of depicting some of the business cycle features of the U.S. economy. This indicates that money growth shocks may be important in determining the cyclical fluctuations at business cycle frequencies. However, as discussed in the previous section, the relatively stronger impact of monetary shocks in our model environment implies that inclusion of the basic mechanism alone does not provide the perfect representation.

The framework of the working capital model can be adopted to study a range of different monetary policy issues. The analysis of this thesis has dealt with only a closed economy. Once the interdependencies of an economy that engages in substantial international trade with the rest of the world are recognized, monetary policy can have additional effects. For instance, domestic output and prices will depend on exchange rates which, in turn, may depend on monetary policy.

Finally, we conclude by reemphasizing that the question of how monetary shocks generate real effects are critical for any normative analysis of monetary policy and as such monetary versions of real business cycle models have huge potential. Having constructed a model where monetary shocks can have positive effects on real variables, this study provides future opportunities for further research about what would be the best policy option for a central bank to follow.

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Tables

Table 8: U.S. economy statistics

			Cross-Correlation of Output with:										
Variable	**Std. Dev.**	**Rel. SD**	**x(-5)**	**x(-4)**	**x(-3)**	**x(-2)**	**x(-1)**	**x**	**x(+1)**	**x(+2)**	**x(+3)**	**x(+4)**	**x(+5)**
Output	1.65	1	0.02	0.21	0.41	0.62	0.83	1	0.83	0.62	0.41	0.21	0.02
Consumption	0.97	0.59	0.09	0.27	0.42	0.61	0.75	0.81	0.71	0.58	0.43	0.3	0.16
Investment	8.8	5.33	-0.04	0.1	0.28	0.47	0.67	0.89	0.7	0.48	0.27	0.07	-0.1
Total hours worked	1.9	1.15	-0.25	-0.14	0.01	0.19	0.43	0.66	0.83	0.85	0.78	0.65	0.48
CPI	1.24	0.75	-0.52	-0.61	-0.64	-0.62	-0.54	-0.41	-0.28	-0.15	-0.007	0.13	0.23
Inflation: $\Delta LN(CPI)$	0.48	0.29	-0.29	-0.22	-0.07	0.05	0.2	0.32	0.33	0.34	0.37	0.34	0.27
M1	2.6	1.58	0.19	0.21	0.2	0.19	0.14	0.09	-0.01	-0.07	-0.09	-0.1	-0.1
M2	1.7	1.03	0.23	0.30	0.32	0.32	0.27	0.16	0.05	-0.02	-0.06	-0.12	-0.15
3-month T-bill rate	1.23	0.75	-0.45	-0.34	-0.21	-0.05	0.2	0.37	0.43	0.46	0.43	0.40	0.36

Table 9: Simulated statistics from cash-in-advance model with only technology shock and constant money growth

			Cross-Correlation of Output with:										
Variable	**Std. Dev.**	**Rel. SD**	**x(-5)**	**x(-4)**	**x(-3)**	**x(-2)**	**x(-1)**	**x**	**x(+1)**	**x(+2)**	**x(+3)**	**x(+4)**	**x(+5)**
Output	1.77	1	-0.0465	0.0735	0.2394	0.4503	0.7058	1	0.7058	0.4503	0.2394	0.0735	-0.0465
Consumption	0.63	0.36	0.2767	0.3926	0.5214	0.6531	0.7766	0.8765	0.523	0.2336	0.0092	-0.155	-0.2646
Investment	7.27	4.11	-0.168	-0.0539	0.1158	0.3436	0.6333	0.9828	0.7297	0.5032	0.311	0.1552	0.0391
Labor	1.26	0.71	-0.2057	-0.0943	0.0754	0.307	0.6056	0.9702	0.732	0.5164	0.3315	0.1801	0.0661
Price level	0.63	0.36	-0.2763	-0.3924	-0.5213	-0.6529	-0.7764	-0.8762	-0.523	-0.2338	-0.0095	0.1546	0.2643
Inflation	0.37	0.21	0.1415	0.1573	0.1621	0.1453	0.1154	-0.424	-0.3455	-0.2735	-0.1951	-0.1272	-0.0799
Nominal interest rate	1.81	1.02	-0.2234	-0.1166	0.0510	0.2805	0.5785	0.9446	0.5961	0.4259	0.2799	0.1598	0.0708

Table 10: Simulated statistics from cash-in-advance model with both technology and money growth shocks

			Cross-Correlation of Output with:										
Variable	**Std. Dev.**	**Rel. SD**	**x(-5)**	**x(-4)**	**x(-3)**	**x(-2)**	**x(-1)**	**x**	**x(+1)**	**x(+2)**	**x(+3)**	**x(+4)**	**x(+5)**
Output	1.74	1	-0.0343	0.086	0.2405	0.4391	0.6908	1	0.6908	0.4391	0.2405	0.086	-0.0343
Consumption	0.88	0.51	0.1984	0.2815	0.3673	0.4616	0.5669	0.6793	0.3967	0.181	0.0172	-0.104	-0.1921
Investment	7.34	4.22	-0.1483	-0.0384	0.1136	0.3182	0.5848	0.9208	0.6752	0.4673	0.2996	0.1659	0.0582
Labor	1.26	0.71	-0.1873	-0.0758	0.0804	0.2964	0.5875	0.9639	0.7066	0.493	0.3204	0.1821	0.0696
Price level	2.81	1.61	-0.1038	-0.1311	-0.1564	-0.1821	-0.2066	-0.222	-0.0934	-0.0054	0.0551	0.0929	0.132
Inflation	1.53	0.88	0.0433	0.0407	0.0353	0.0327	0.021	-0.1714	-0.1245	-0.0785	-0.0496	-0.0194	0.0006
Nominal interest rate	2.37	1.36	-0.1639	-0.0832	0.0428	0.2167	0.4381	0.7230	0.3903	0.2881	0.1953	0.1120	0.0493
Money	2.59	1.49	-0.0468	-0.0479	-0.0458	-0.0412	-0.0312	-0.0092	0.0341	0.056	0.0657	0.0651	0.0571

Table 11: Simulated statistics from working capital model with only technology shock ($\eta = 0$)

			Cross-Correlation of Output with:										
Variable	**Std. Dev.**	**Rel. SD**	**x(-5)**	**x(-4)**	**x(-3)**	**x(-2)**	**x(-1)**	**x**	**x(+1)**	**x(+2)**	**x(+3)**	**x(+4)**	**x(+5)**
Output	1.6	1	-0.0504	0.0743	0.2338	0.4404	0.6935	1	0.6935	0.4404	0.2338	0.0743	-0.0504
Consumption	0.55	0.34	0.2885	0.4076	0.5296	0.6534	0.7667	0.8622	0.4997	0.2141	-0.0053	-0.1638	-0.2748
Investment	6.69	4.18	-0.1693	-0.0514	0.1107	0.3333	0.6198	0.9815	0.716	0.4911	0.3032	0.1542	0.0339
Labor	1.03	0.64	-0.212	-0.0966	0.0658	0.2927	0.5893	0.9687	0.7192	0.5066	0.3265	0.1822	0.0633
Price level	0.96	0.6	-0.0602	-0.1868	-0.3389	-0.5245	-0.7393	-0.9859	-0.6514	-0.3798	-0.1625	0.0017	0.1259
Inflation	0.7	0.44	0.1728	0.2063	0.2501	0.2872	0.3247	-0.4468	-0.3662	-0.2946	-0.2238	-0.1705	-0.123
Nominal interest rate	0.06	0.04	-0.1564	-0.0369	0.1256	0.347	0.6303	0.9867	0.7153	0.4869	0.2962	0.1456	0.0238

Table 12: Simulated statistics from working capital model with both technology and money growth shocks ($\eta = 0$)

			Cross-Correlation of Output with:										
Variable	**Std. Dev.**	**Rel. SD**	**x(-5)**	**x(-4)**	**x(-3)**	**x(-2)**	**x(-1)**	**x**	**x(+1)**	**x(+2)**	**x(+3)**	**x(+4)**	**x(+5)**
Output	1.88	1	-0.0799	0.0295	0.1738	0.3756	0.6457	1	0.6457	0.3756	0.1738	0.0295	-0.0799
Consumption	0.9	0.48	0.2811	0.337	0.3793	0.3764	0.3004	0.1121	0.0712	0.011	-0.0557	-0.1072	-0.1426
Investment	10.49	5.58	-0.1836	-0.0979	0.0262	0.2217	0.5114	0.9243	0.5979	0.3601	0.1905	0.0707	-0.0228
Labor	1.78	0.95	-0.2052	-0.1272	-0.0105	0.1806	0.4693	0.8887	0.5753	0.3501	0.1914	0.0792	-0.0082
Price level	1.79	0.95	0.1517	0.1177	0.0574	-0.0427	-0.2034	-0.452	-0.4308	-0.3583	-0.2664	-0.1785	-0.0957
Inflation	0.91	0.48	0.0693	0.1179	0.1939	0.3099	0.4791	-0.0361	-0.1371	-0.1738	-0.1686	-0.1598	-0.1386
Nominal interest rate	0.87	0.46	-0.1001	-0.0637	-0.0096	0.0955	0.2713	0.547	0.28	0.1159	0.0229	-0.0324	-0.069
Money	1.69	0.9	0.1875	0.2262	0.2521	0.2593	0.2267	0.1265	-0.084	-0.1846	-0.2194	-0.2193	-0.1991

Table 13: Simulated statistics from working capital model with only money growth shock ($\eta = 0$)

			Cross-Correlation of Output with:										
Variable	**Std. Dev.**	**Rel. SD**	**x(-5)**	**x(-4)**	**x(-3)**	**x(-2)**	**x(-1)**	**x**	**x(+1)**	**x(+2)**	**x(+3)**	**x(+4)**	**x(+5)**
Output	0.94	1	-0.17	-0.1235	-0.023	0.165	0.4784	1	0.4784	0.165	-0.023	-0.1235	-0.17
Consumption	0.69	0.73	0.3135	0.3072	0.2369	0.0565	-0.2945	-0.9334	-0.538	-0.2874	-0.1231	-0.0222	0.0377
Investment	7.99	8.5	-0.2268	-0.1945	-0.1037	0.0839	0.4147	0.9888	0.5071	0.2147	0.0331	-0.0696	-0.1223
Labor	1.44	1.53	-0.2354	-0.2056	-0.1166	0.0713	0.4063	0.9877	0.5125	0.2213	0.0405	-0.0618	-0.115
Price level	1.46	1.55	0.3984	0.4665	0.5012	0.4738	0.3334	0.0051	-0.3267	-0.4692	-0.4992	-0.4635	-0.3957
Inflation	0.55	0.59	-0.1718	-0.0867	0.0795	0.3691	0.8538	0.8625	0.375	0.0838	-0.0837	-0.1693	-0.2031
Nominal interest rate	0.86	0.91	-0.1823	-0.1387	-0.0401	0.1482	0.4661	0.9996	0.4855	0.1756	-0.0114	-0.1124	-0.1603
Money	1.63	1.73	0.3502	0.4304	0.4902	0.509	0.4442	0.2395	-0.2008	-0.4114	-0.4849	-0.4749	-0.4214

Table 14: Simulated statistics from working capital model with only technology shock ($\eta = 1$)

			Cross-Correlation of Output with:										
Variable	**Std. Dev.**	**Rel. SD**	**x(-5)**	**x(-4)**	**x(-3)**	**x(-2)**	**x(-1)**	**x**	**x(+1)**	**x(+2)**	**x(+3)**	**x(+4)**	**x(+5)**
Output	1.69	1	-0.0644	0.0532	0.2203	0.4317	0.6935	1	0.6935	0.4317	0.2203	0.0532	-0.0644
Consumption	0.6	0.36	0.2476	0.3649	0.4987	0.6352	0.7683	0.8814	0.5159	0.2208	-0.0036	-0.1671	-0.2721
Investment	6.96	4.12	-0.1788	-0.068	0.1014	0.3283	0.6217	0.9823	0.7165	0.4837	0.2899	0.1324	0.0172
Labor	1.18	0.7	-0.2162	-0.1085	0.061	0.2918	0.5953	0.9715	0.7207	0.4979	0.3113	0.1571	0.044
Price level	0.6	0.36	-0.2475	-0.3648	-0.4987	-0.6352	-0.7683	-0.8815	-0.5159	-0.2208	0.0035	0.1671	0.272
Inflation	0.36	0.21	0.1891	0.2133	0.2155	0.2062	0.1714	-0.5821	-0.4727	-0.3631	-0.2681	-0.1751	-0.1073
Nominal interest rate	0	0	-0.2843	-0.1834	-0.0161	0.2193	0.5367	0.9388	0.7191	0.5192	0.3479	0.2029	0.0941

Table 15: Simulated statistics from working capital model with both technology and money growth shocks ($\eta = 1$)

			Cross-Correlation of Output with:										
Variable	**Std. Dev.**	**Rel. SD**	**x(-5)**	**x(-4)**	**x(-3)**	**x(-2)**	**x(-1)**	**x**	**x(+1)**	**x(+2)**	**x(+3)**	**x(+4)**	**x(+5)**
Output	2.17	1	-0.0926	0.0101	0.1564	0.3569	0.6301	1	0.6301	0.3569	0.1564	0.0101	-0.0926
Consumption	0.72	0.33	0.3248	0.4078	0.4765	0.5189	0.4955	0.3739	0.1921	0.0418	-0.0749	-0.1587	-0.2112
Investment	10.97	5.06	-0.192	-0.1054	0.0311	0.2338	0.5319	0.9623	0.6195	0.3715	0.1902	0.0574	-0.0368
Labor	2.32	1.07	-0.2026	-0.1249	0.0022	0.1937	0.4849	0.9138	0.5761	0.3377	0.1679	0.0464	-0.0397
Price level	1.72	0.79	0.1047	0.1171	0.1171	0.0969	0.0554	-0.0255	-0.2072	-0.2738	-0.2745	-0.2389	-0.1861
Inflation	0.94	0.43	-0.0223	0.0012	0.0367	0.0749	0.1465	0.3336	0.1261	0.0063	-0.0613	-0.093	-0.1096
Nominal interest rate	0.48	0.22	-0.1075	-0.0637	0.0095	0.1211	0.309	0.6043	0.306	0.1192	0.003	-0.0669	-0.1082
Money	1.75	0.81	0.2349	0.2814	0.3097	0.3076	0.2577	0.1283	-0.1264	-0.2536	-0.3016	-0.3003	-0.2697

Table 16: Simulated statistics from working capital model with only money growth shock ($\eta = 1$)

			Cross-Correlation of Output with:										
Variable	**Std. Dev.**	**Rel. SD**	**x(-5)**	**x(-4)**	**x(-3)**	**x(-2)**	**x(-1)**	**x**	**x(+1)**	**x(+2)**	**x(+3)**	**x(+4)**	**x(+5)**
Output	1.29	1	-0.1538	-0.1	0.0003	0.1904	0.4983	1	0.4983	0.1904	0.0003	-0.1	-0.1538
Consumption	0.36	0.28	0.4236	0.4461	0.4096	0.2516	-0.0893	-0.7385	-0.5332	-0.3857	-0.2706	-0.1908	-0.1275
Investment	8,2	6.36	-0.2128	-0.1717	-0.0794	0.112	0.4391	0.9893	0.5267	0.2378	0.0547	-0.0456	-0.104
Labor	1.96	1.52	-0.2095	-0.1673	-0.0745	0.1174	0.4436	0.9923	0.525	0.2343	0.0504	-0.05	-0.1078
Price level	1.6	1.24	0.3157	0.3857	0.4451	0.4867	0.4831	0.4065	-0.0967	-0.3572	-0.4692	-0.4858	-0.4534
Inflation	0.85	0.66	-0.1246	-0.1038	-0.0694	0.0149	0.1489	0.9299	0.4863	0.2133	0.0382	-0.054	-0.1059
Nominal interest rate	0.48	0.37	-0.1577	-0.1046	-0.0047	0.1875	0.495	1	0.5007	0.194	0.0043	-0.0961	-0.1503
Money	1.74	1.35	0.3786	0.4485	0.4963	0.502	0.4274	0.2204	-0.2011	-0.4103	-0.489	-0.487	-0.4434

B

Figures

Figure 7: Response of the working capital model to a technology shock ($\eta = 0$)

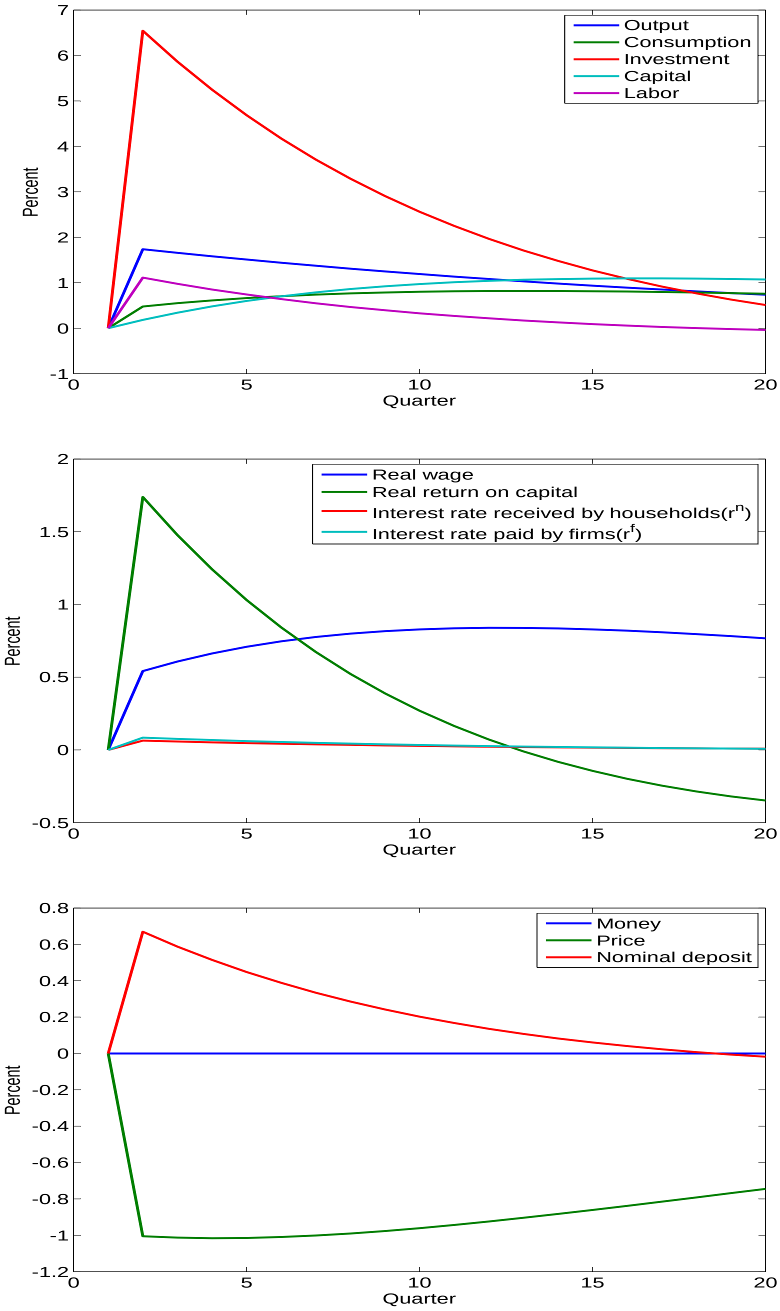


Figure 8: Response of the working capital model to a technology shock ($\eta = 1$)

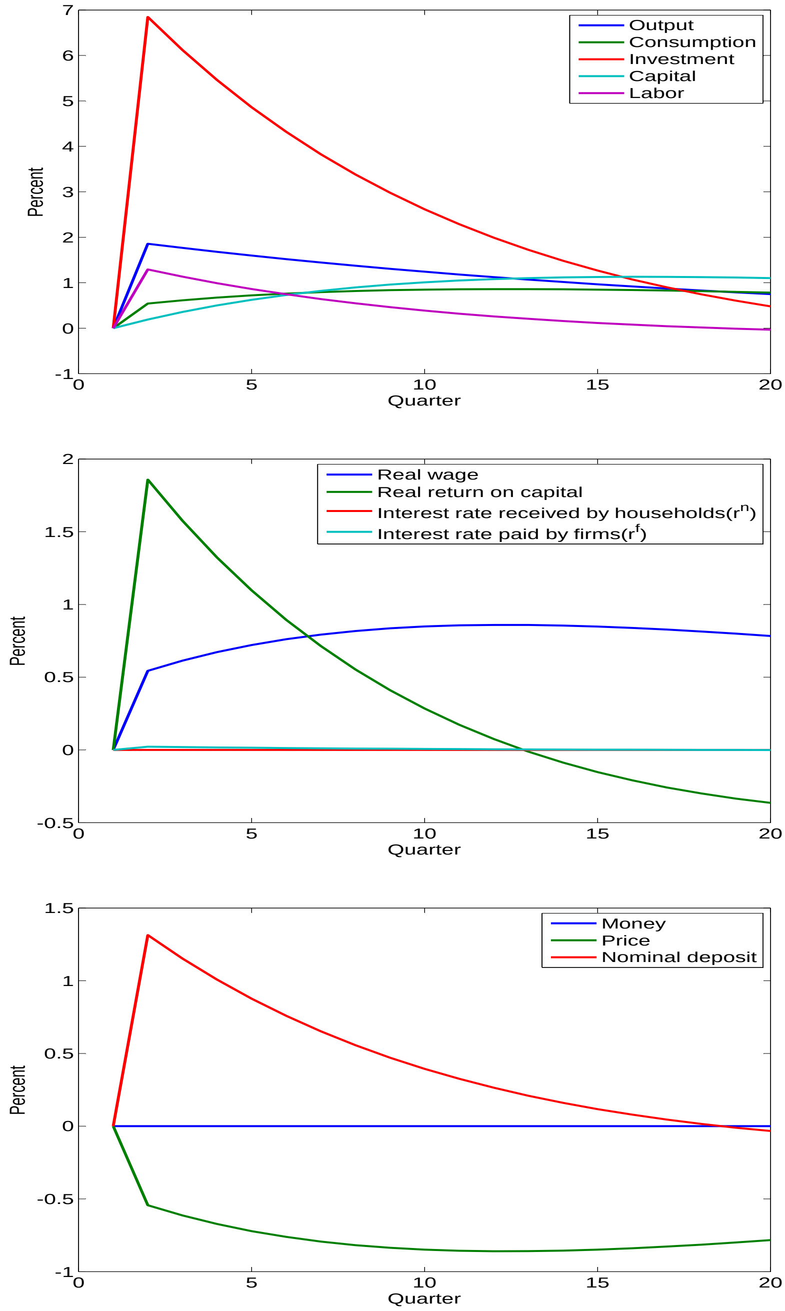


Figure 9: Cyclical Comparisons

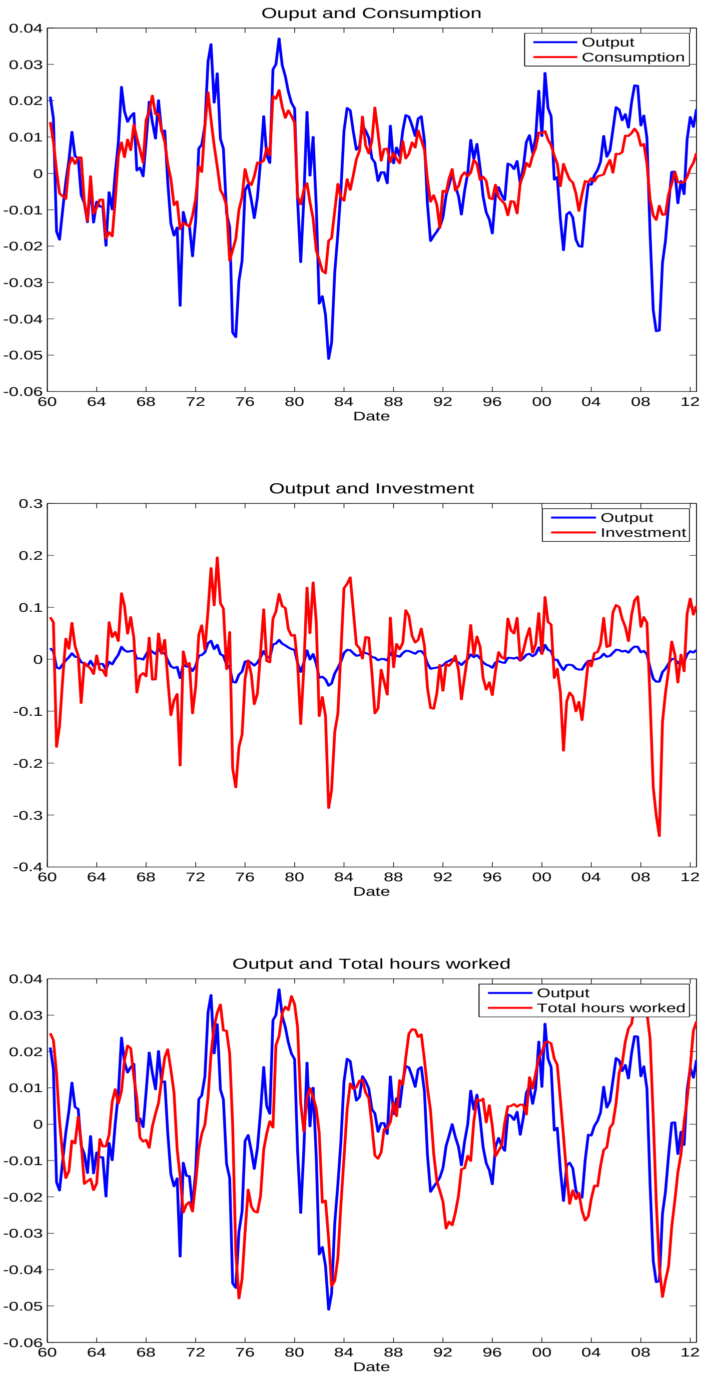
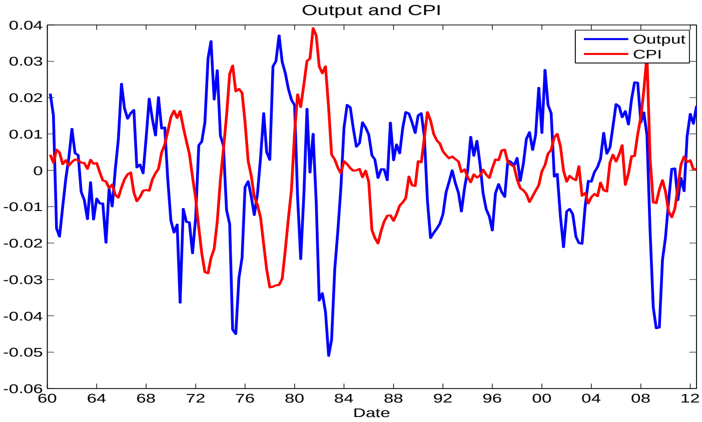
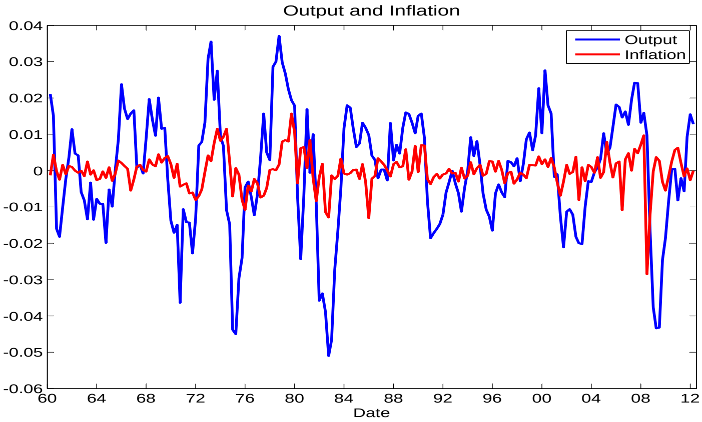


Figure 9 cont.





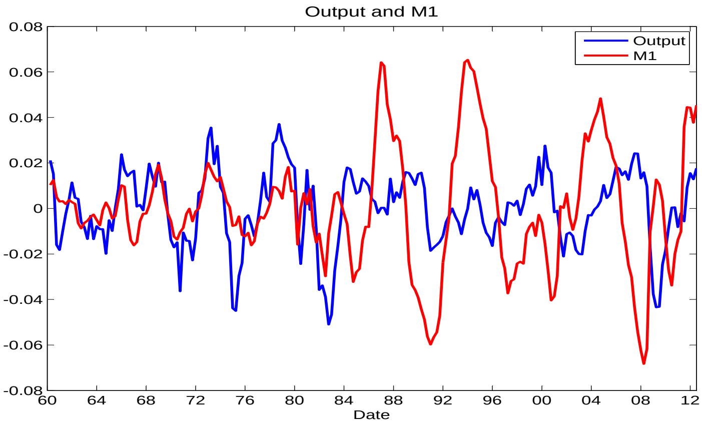
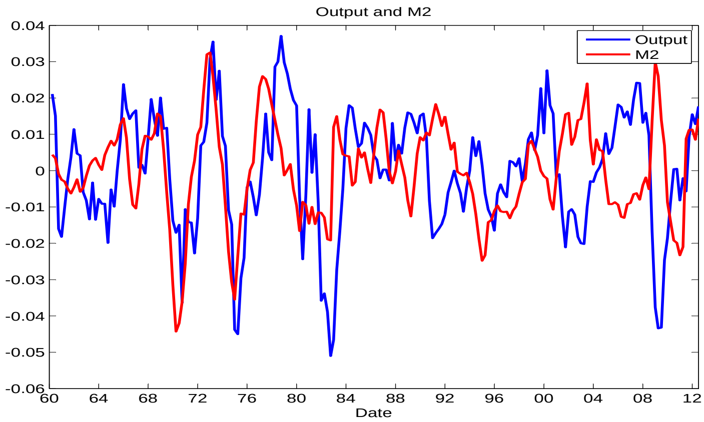


Figure 9 cont.



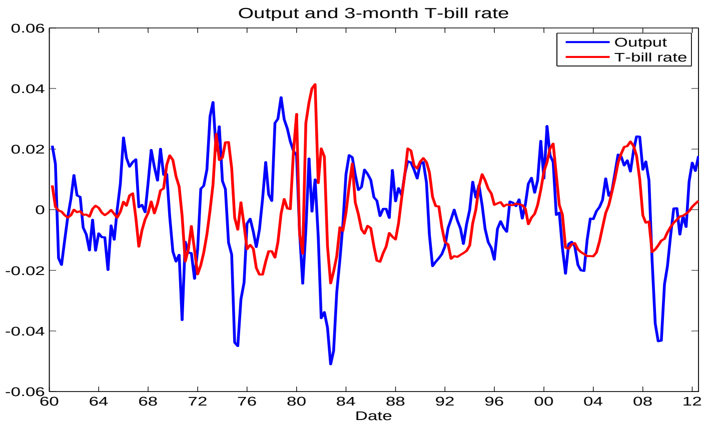


Figure 10: Cash-in-Advance Model with only technology shock

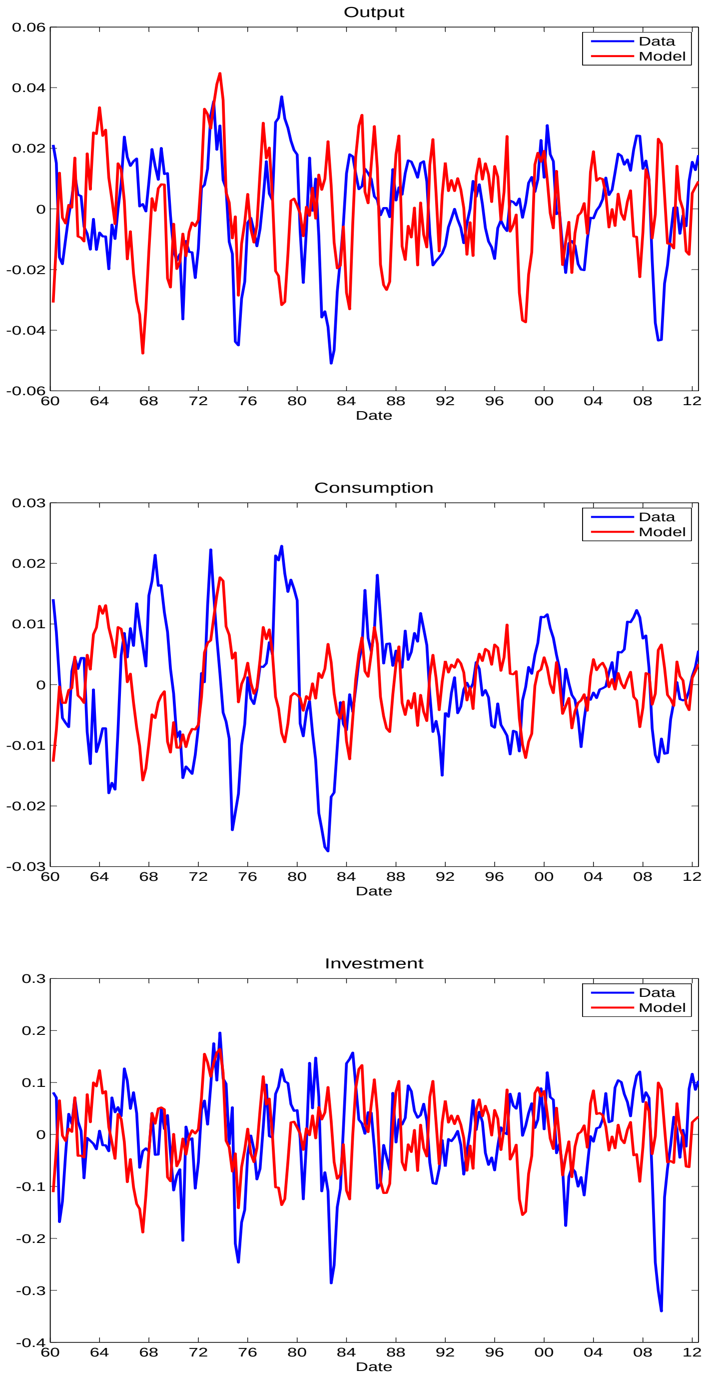
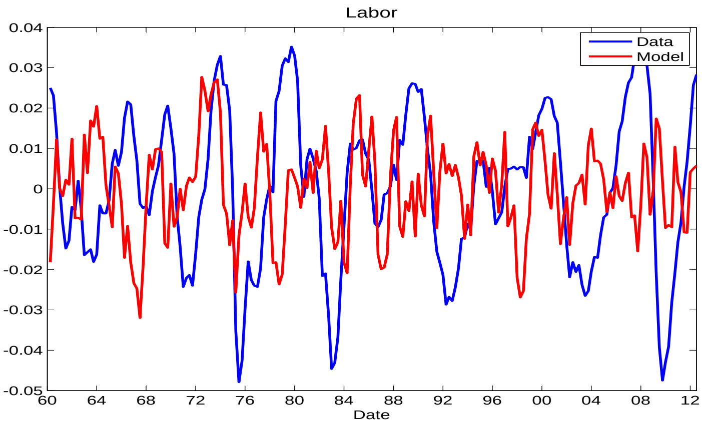


Figure 10 cont.



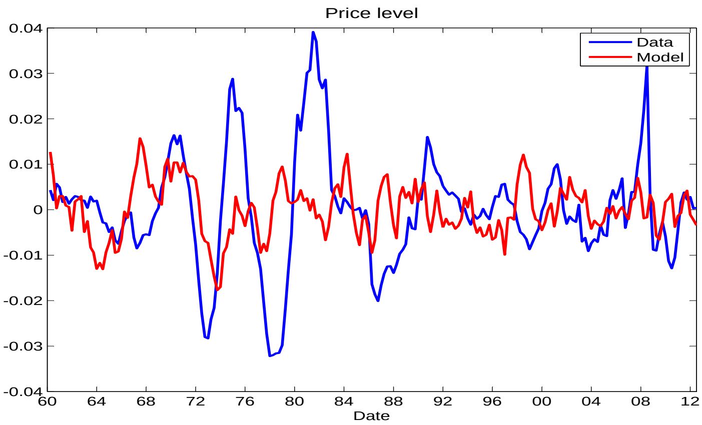
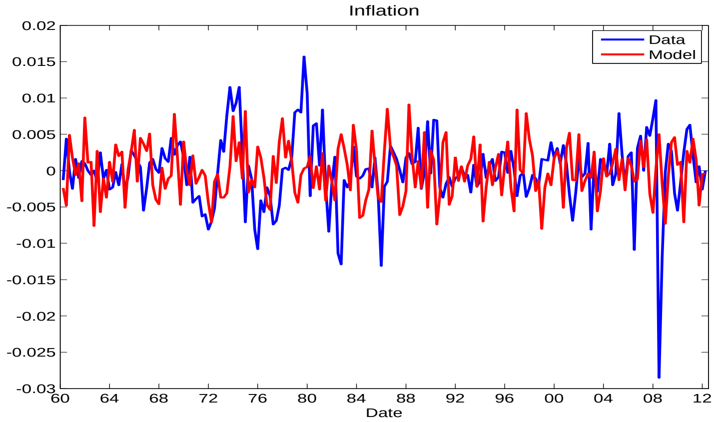


Figure 10 cont.



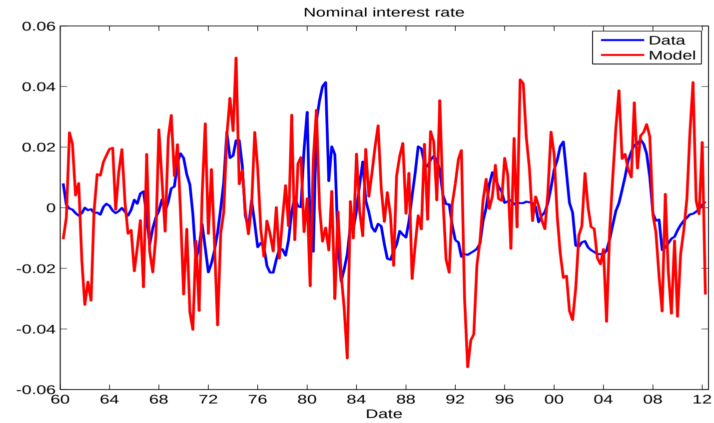


Figure 11: Cash-in-Advance Model with both technology and money growth shock

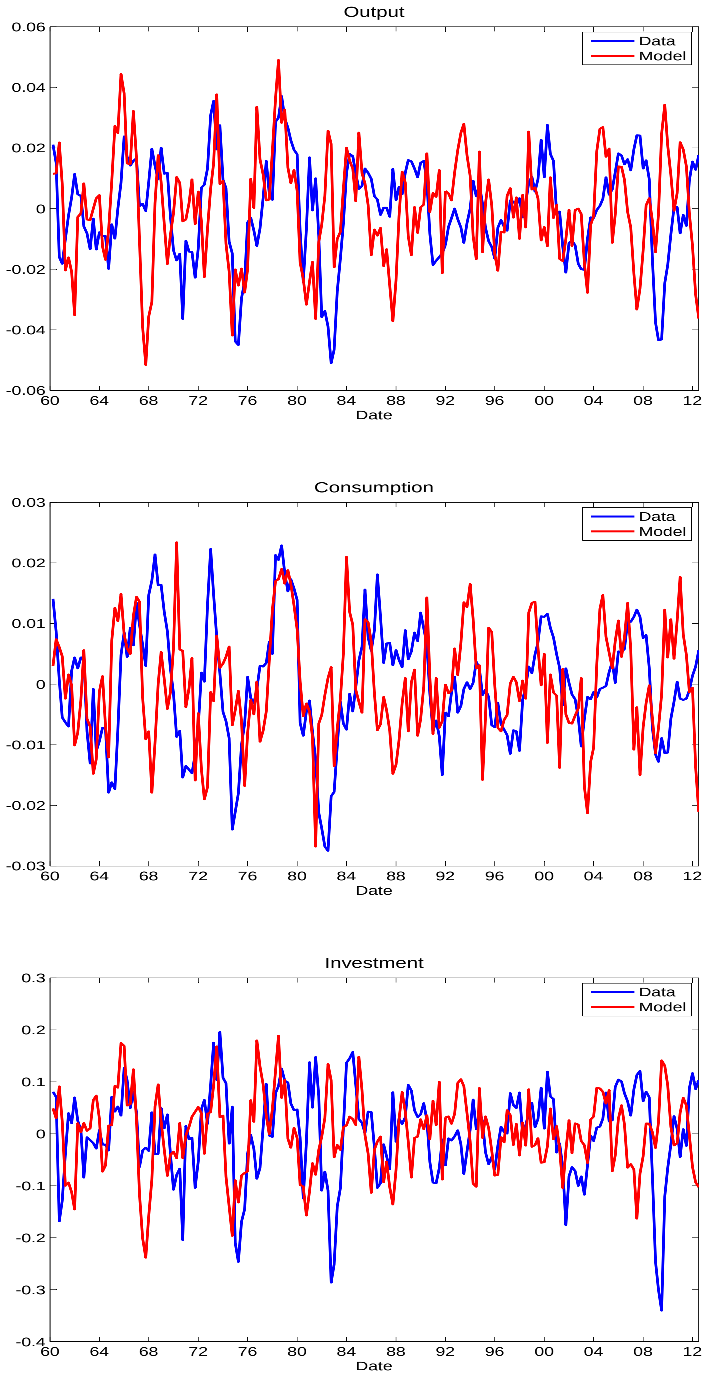


Figure 11 cont.

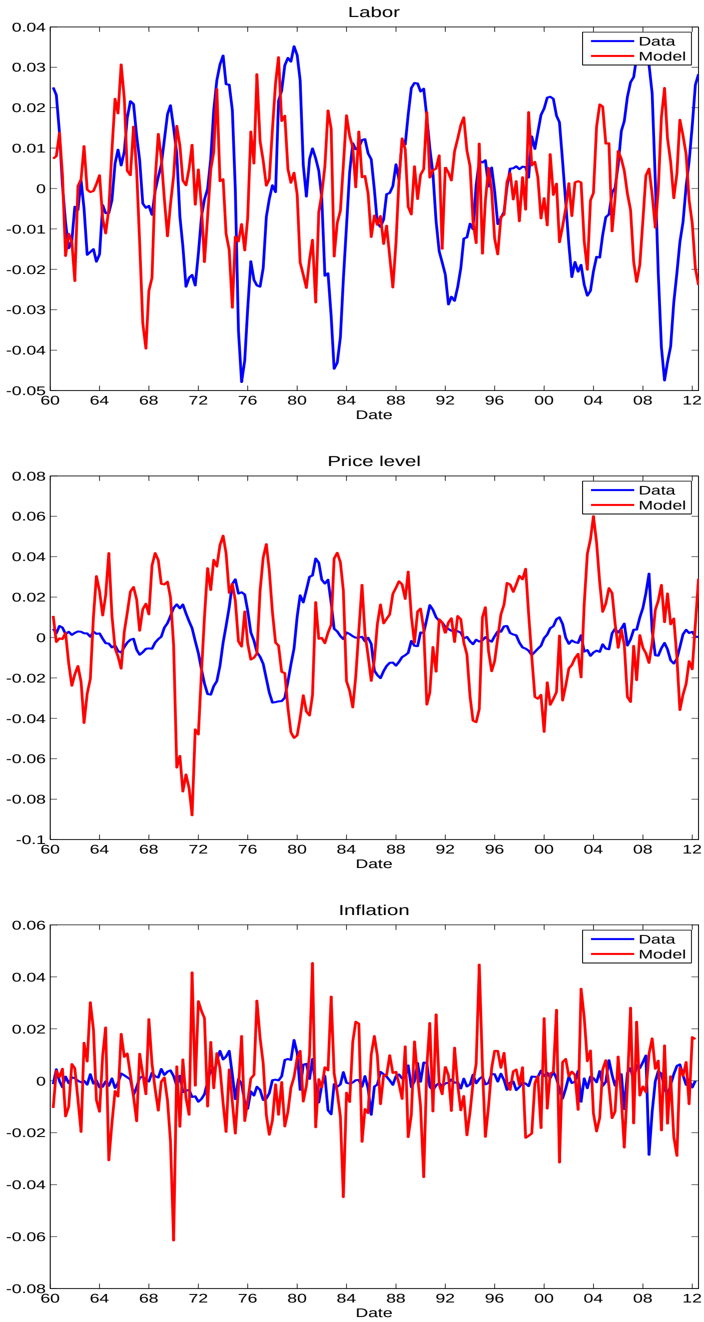
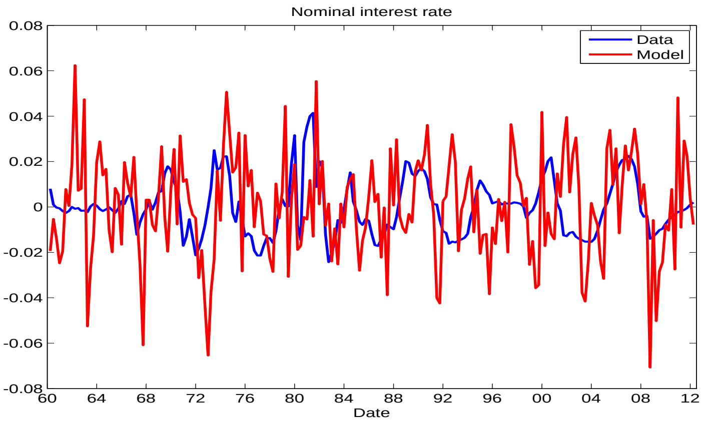


Figure 11 cont.



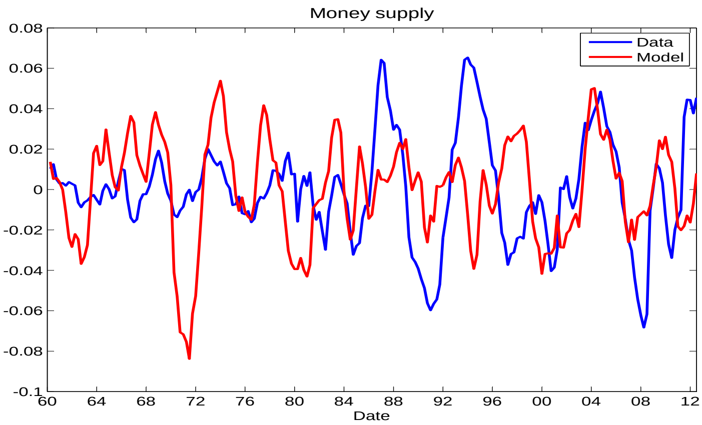


Figure 12: Working Capital Model with only technology shock ($\eta = 0$)

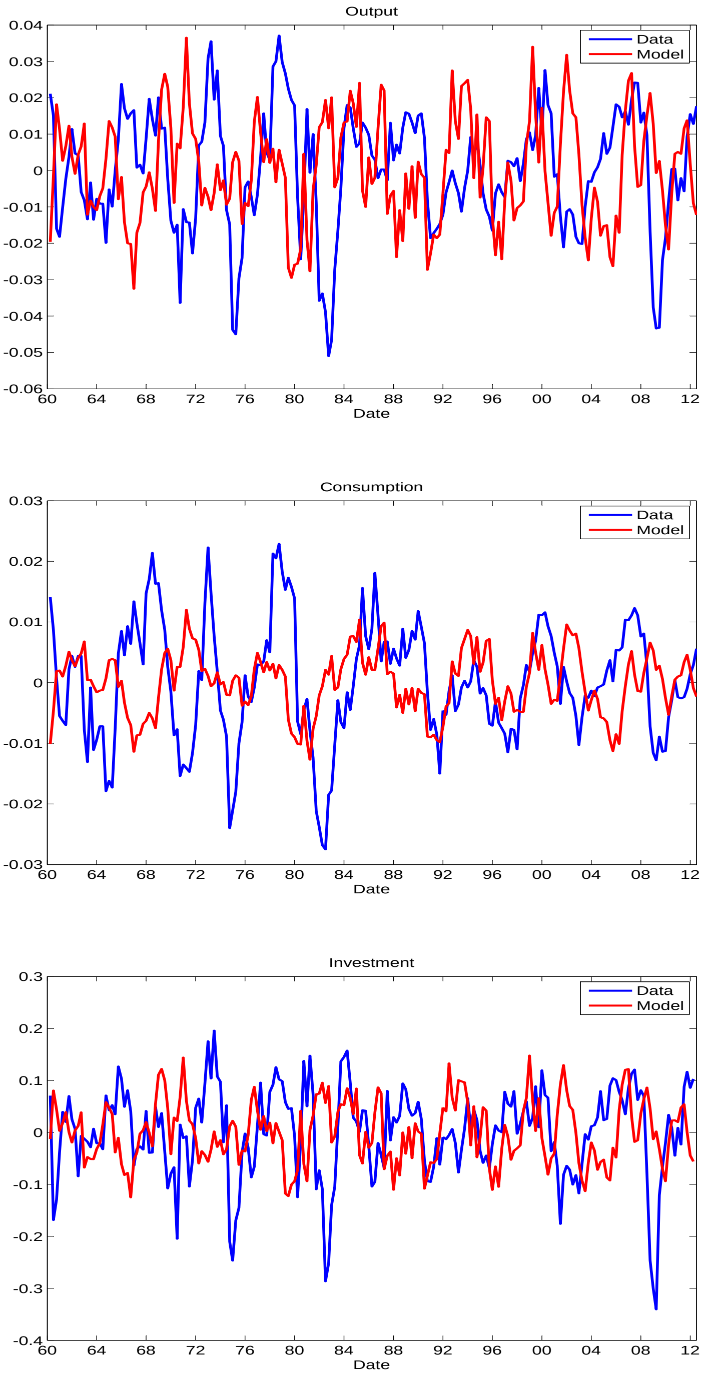
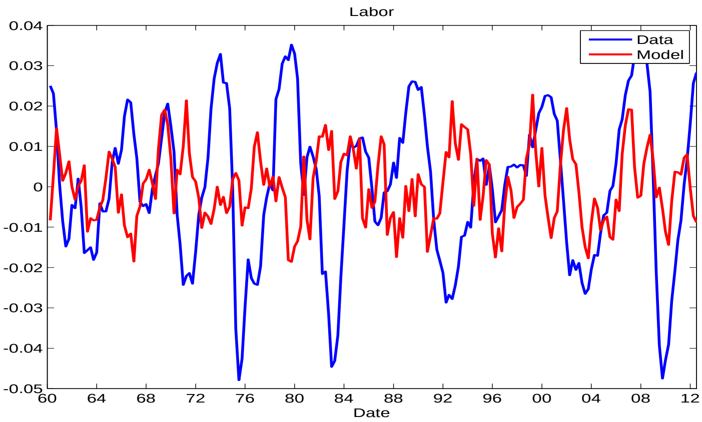


Figure 12 cont.



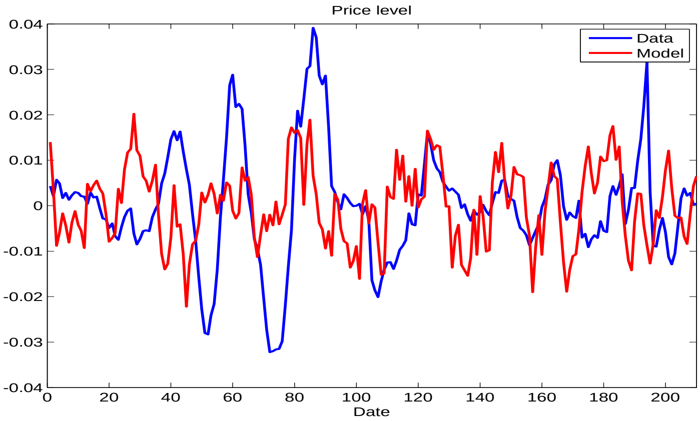
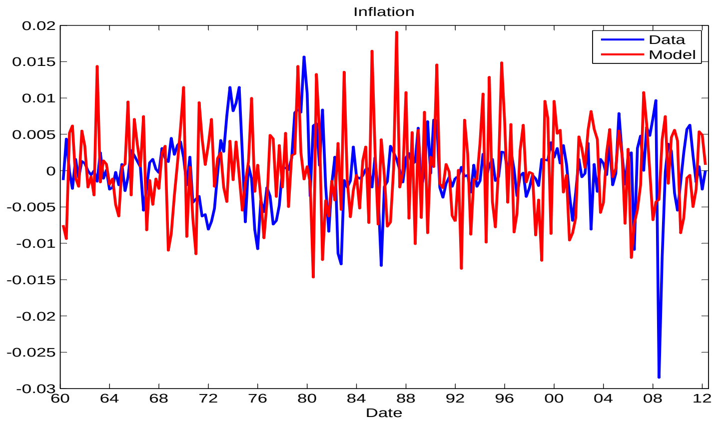


Figure 12 cont.



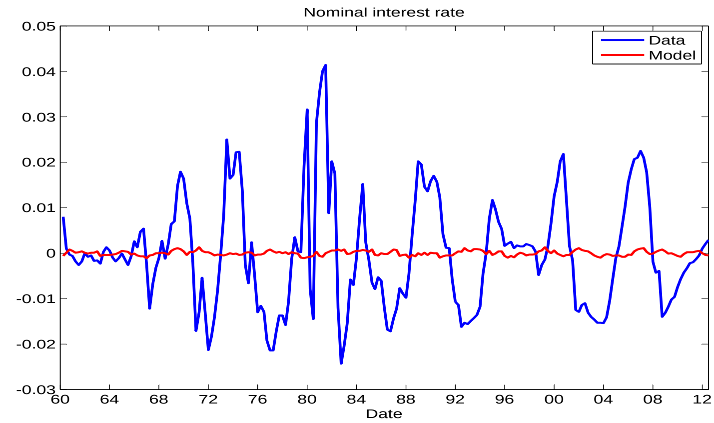


Figure 13: Working Capital Model with both technology and money growth shock ($\eta = 0$)

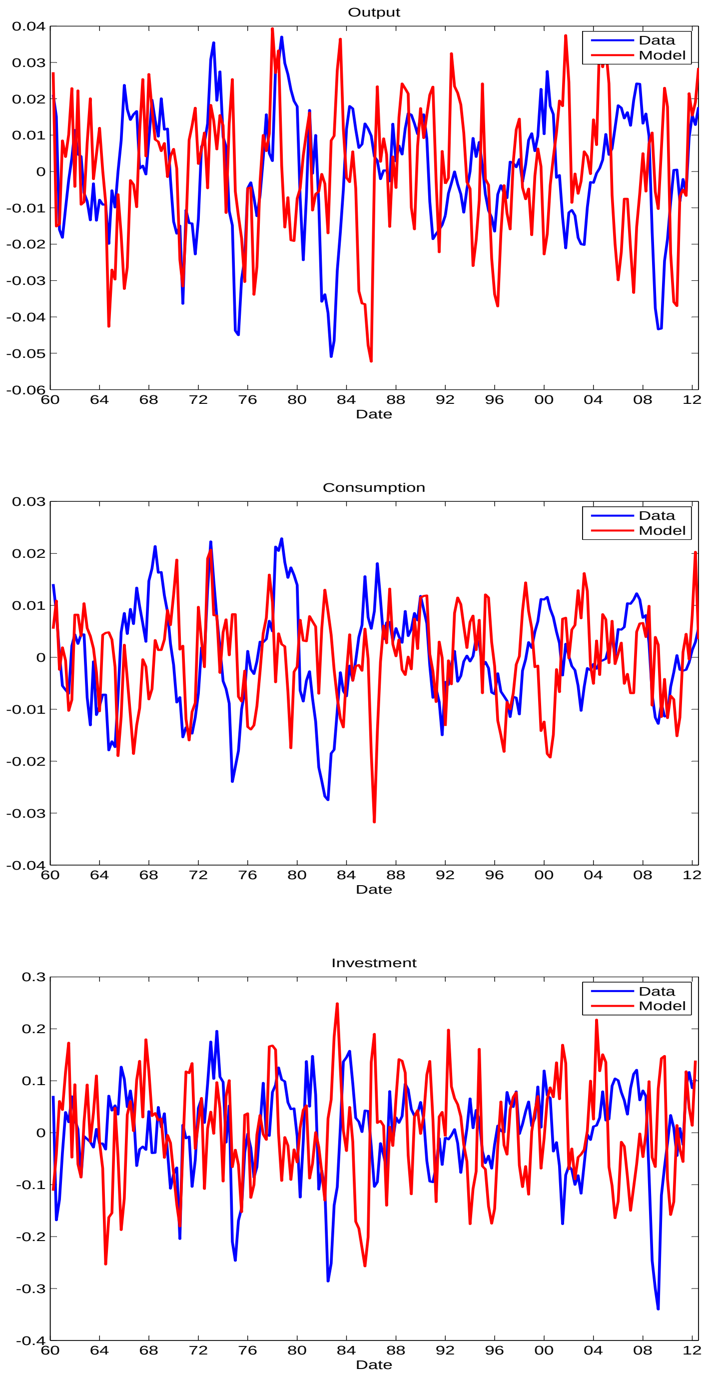
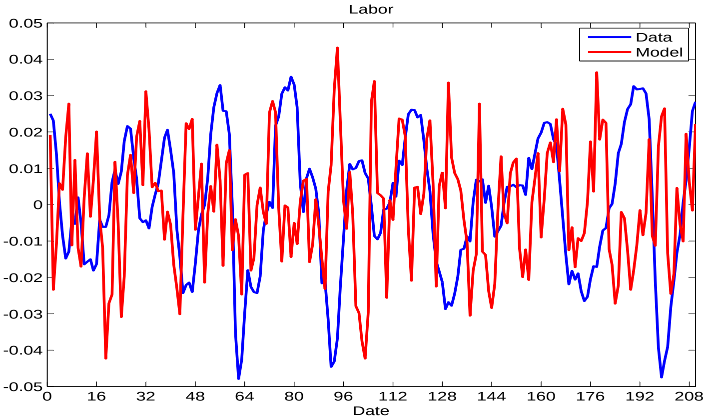
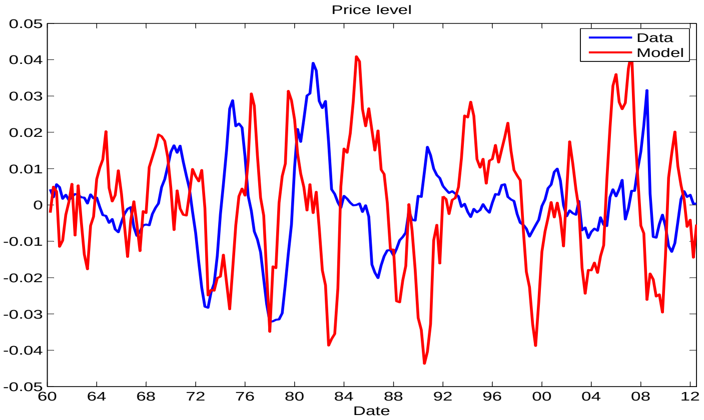


Figure 13 cont.





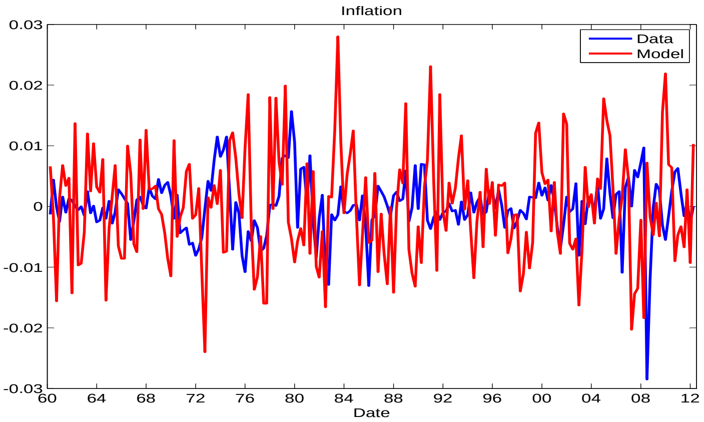
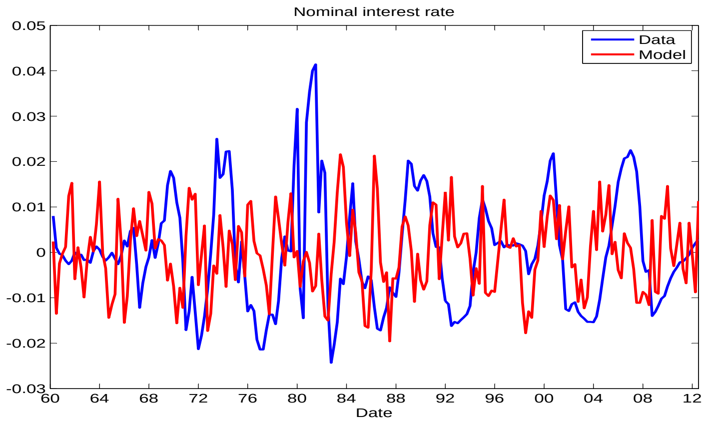


Figure 13 cont.



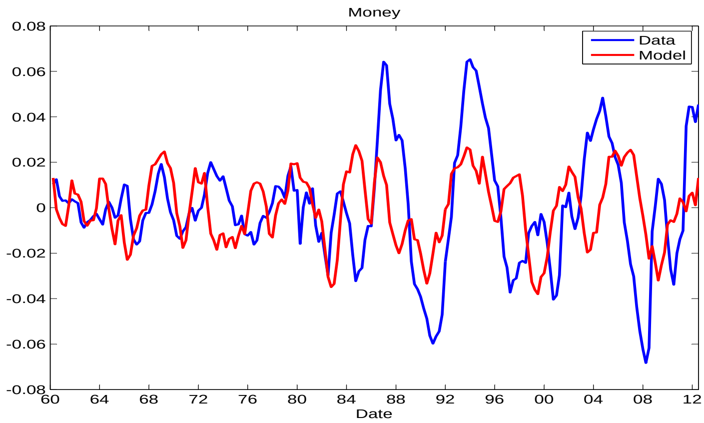


Figure 14: Working Capital Model with only technology shock ($\eta = 1$)

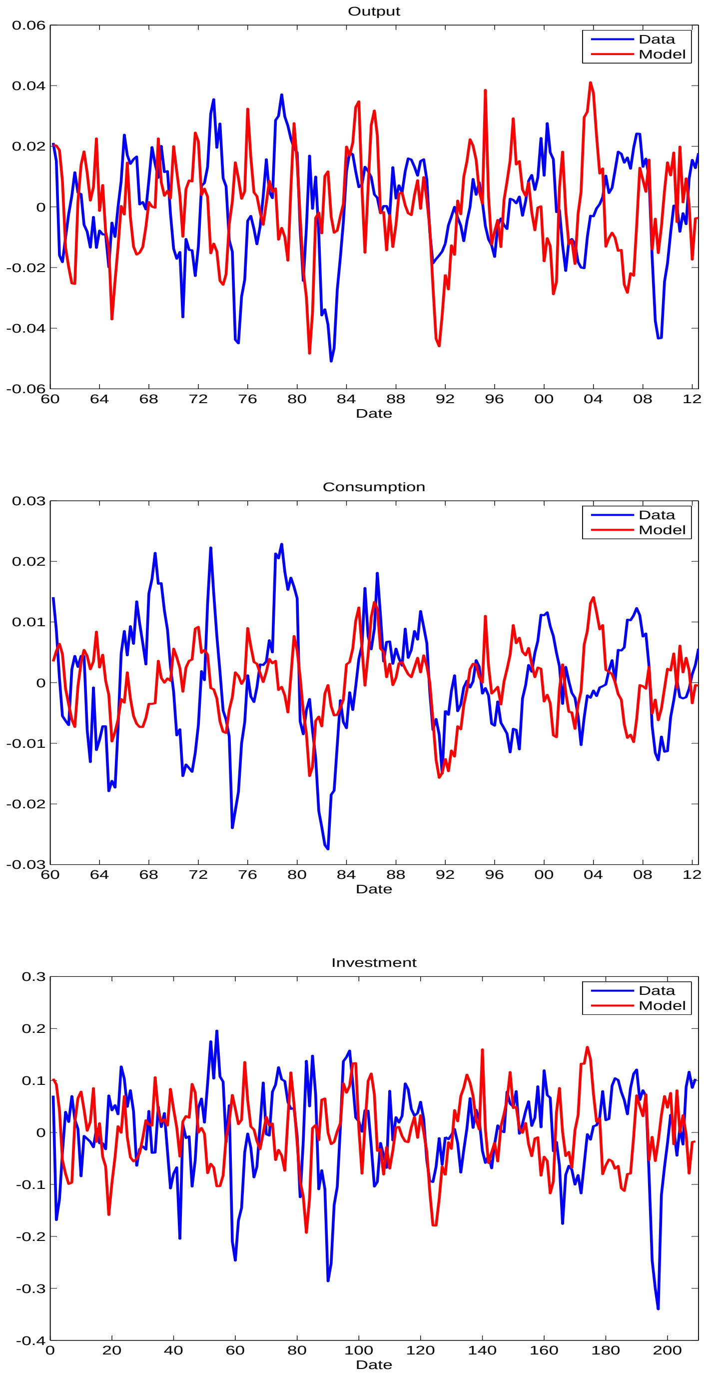
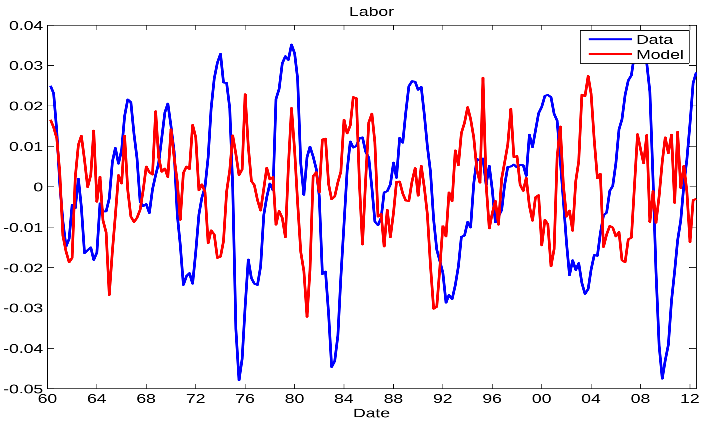


Figure 14 cont.



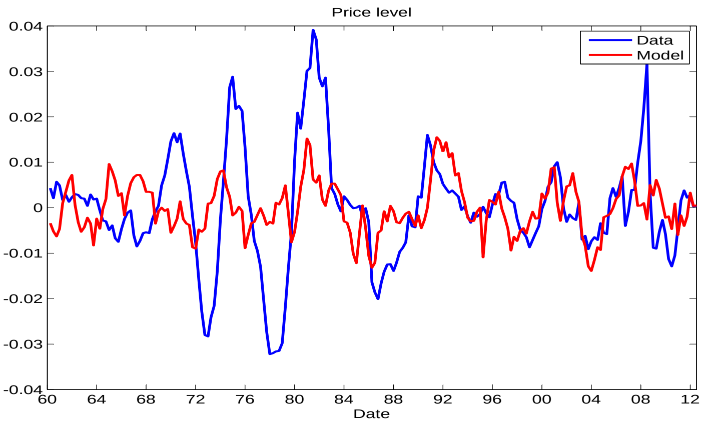
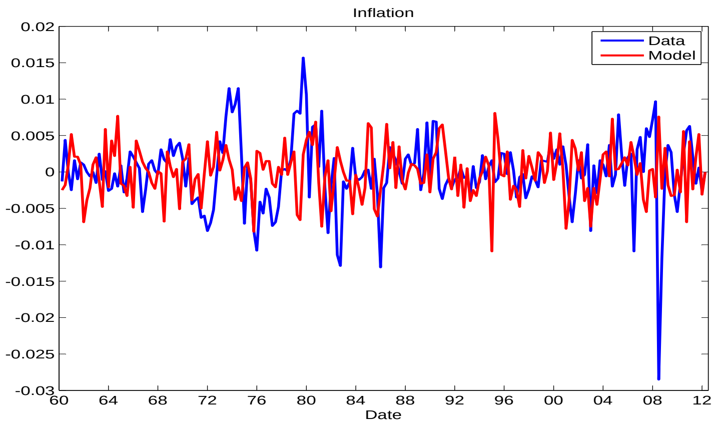


Figure 14 cont.



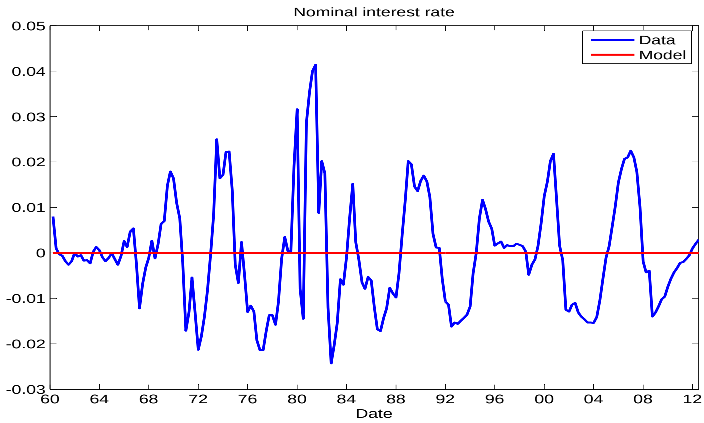


Figure 15: Working Capital Model with both technology and money growth shock ($\eta = 1$)

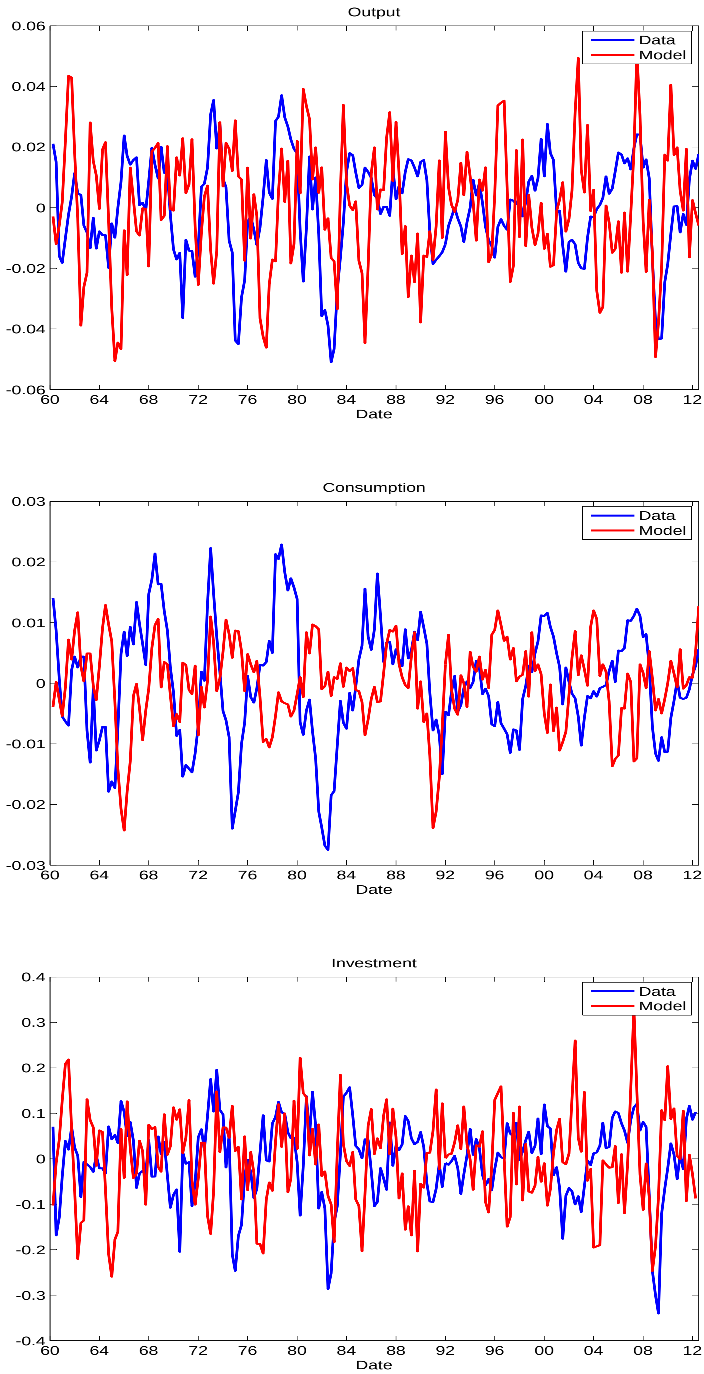


Figure 15 cont.

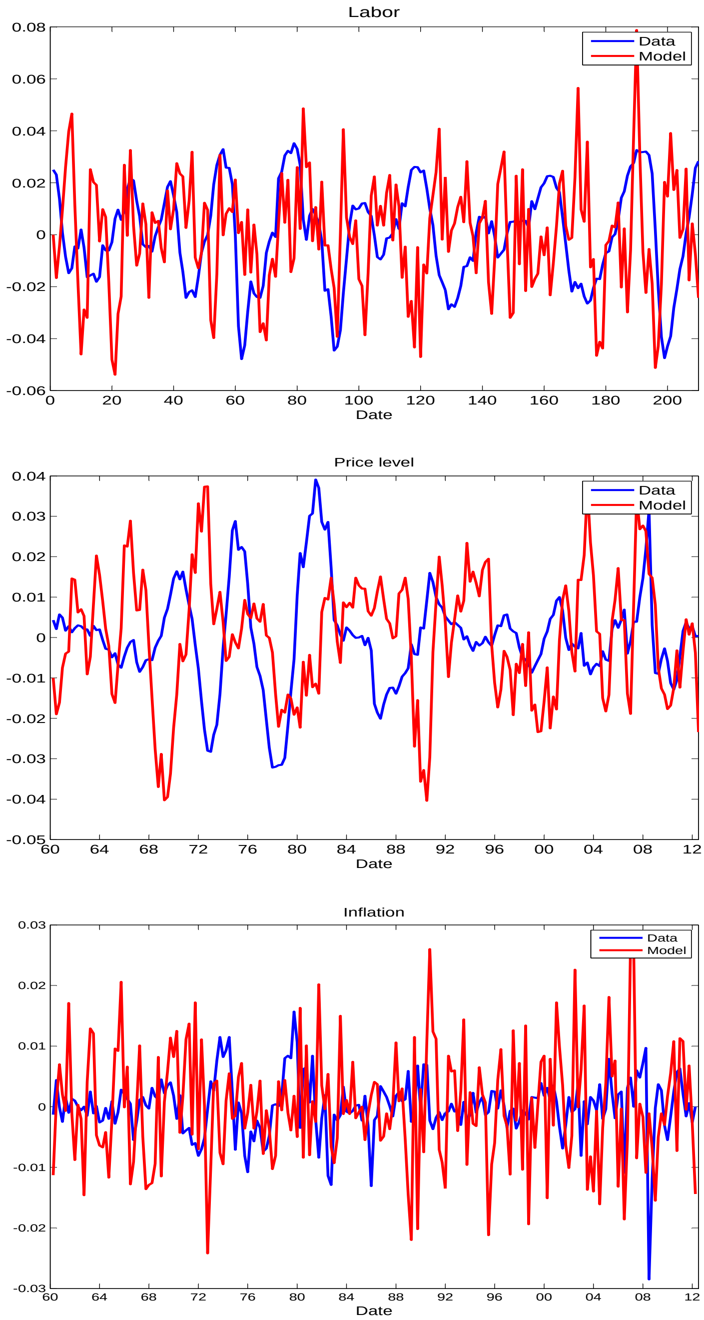
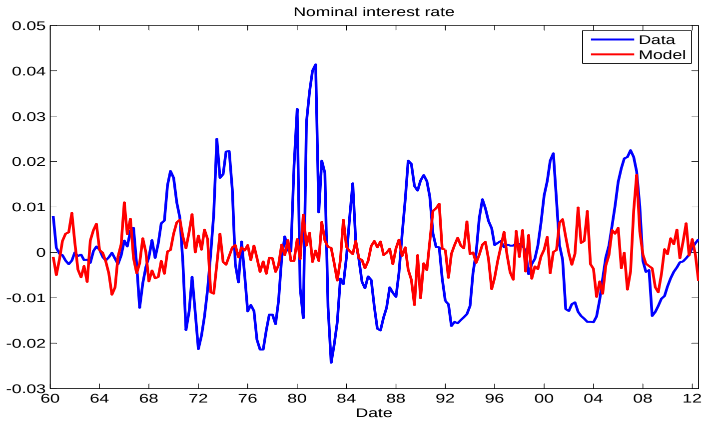
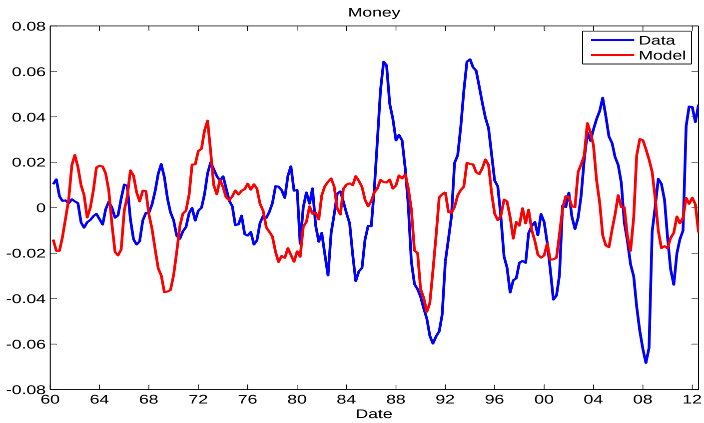


Figure 15 cont.





C

The Log Linearization

C.1 The Baseline Cash-in-Advance Model

Consider a variable X_t. Following Uhlig (1999), let us define $\tilde{X}_t = lnX_t - ln\overline{X}$. The tilde variable is the log difference of the original variable from the value $\overline{X}$. The original variable can be written as

$$X_t = \overline{X}e^{\tilde{X}_t}$$

since

$$\overline{X}e^{\tilde{X}_t} = \overline{X}e^{lnX_t - ln\overline{X}} = \overline{X}e^{\frac{lnX_t}{\overline{X}}} = \overline{X}\frac{X_t}{\overline{X}} = X_t.$$

Accordingly, taking the log linearization of the two first order conditions, equations 5.2.1 and 5.2.2, give

$$-\tilde{w}_t = \beta E_t[\overline{r}(\tilde{r}_{t+1} - \tilde{w}_{t+1}) - (1-\delta)\tilde{w}_{t+1}] \tag{C.1.1}$$

and

$$-\frac{B}{\overline{C}\overline{w}}[\tilde{p}_t + \tilde{w}_t] = \beta E_t[\frac{1}{\overline{g}}\tilde{g}_{t+1}]. \tag{C.1.2}$$

The log-linear version of the cash-in-advance constraint, equation 5.2.3, in aggregate gives

$$\tilde{p}_t + \tilde{C}_t = 0. \tag{C.1.3}$$

The real budget constraint, equation 5.2.4, is written as

$$\overline{k}\tilde{k}_{t+1} + \frac{\overline{m}}{\overline{p}}[\tilde{m}_t - \tilde{p}_t] = \overline{w}\overline{h}[\tilde{w}_t + \tilde{h}_t] + \overline{r}\overline{k}[\tilde{r}_t + \tilde{k}_t] + (1-\delta)\overline{k}\tilde{k}_t. \tag{C.1.4}$$

Log-linear versions of the competitive factor market conditions, equations 5.2.5 and 5.2.6, are

$$\overline{w}\tilde{w}_t = (1-\theta)\overline{K}^{\theta}\overline{H}^{-\theta}[\tilde{\lambda}_t + \theta[\tilde{K}_t - \tilde{H}_t]] \tag{C.1.5}$$

and

$$\overline{r}\tilde{r}_t = \theta\overline{K}^{\theta-1}\overline{H}^{1-\theta}[\tilde{\lambda}_t + (\theta-1)[\tilde{K}_t - \tilde{H}_t]]. \tag{C.1.6}$$

The stochastic processes for the technology and money growth shocks are

$$\tilde{\lambda}_t = \gamma\tilde{\lambda}_{t-1} + \epsilon_t^{\lambda} \tag{C.1.7}$$

and

$$\tilde{g}_t = \pi \tilde{g}_{t-1} + \epsilon_t^g. \tag{C.1.8}$$

It is possible to remove the expectations from $C.1.2$ by using the money growth process and get

$$-\frac{B}{\overline{C}\overline{w}}[\tilde{p}_t + \tilde{w}_t] = \frac{\beta\pi}{\overline{g}}\tilde{g}_t. \tag{C.1.9}$$

We replace the individual variables with the aggregate variables that they equal in equilibrium. We also remove the money stock variable since aggregate money must always equal 1 which implies $\tilde{m}_t = 0$. Finally, we have a system of four equations without expectations,

$$0 = \overline{K}\tilde{K}_{t+1} - \frac{1}{\overline{p}}\tilde{p}_t - \overline{w}\overline{H}\tilde{w}_t - \overline{r}\overline{K}\tilde{r}_t - \overline{r}\overline{K}\tilde{K}_t - (1-\delta)\overline{K}\tilde{K}_t,$$

$$0 = \tilde{r}_t - \tilde{\lambda}_t - (\theta - 1)\tilde{K}_t + (\theta - 1)\tilde{H}_t,$$

$$0 = \tilde{w}_t - \tilde{\lambda}_t - \theta\tilde{K}_t + \theta\tilde{H}_t,$$

and

$$0 = \tilde{p}_t + \tilde{w}_t - \pi\tilde{g}_t,$$

one equation with expectations,

$$0 = \tilde{w}_t + \beta\overline{r}E_t\tilde{r}_{t+1} - E_t\tilde{w}_{t+1},$$

the stochastic technology shock,

$$\tilde{\lambda}_t = \gamma\tilde{\lambda}_{t-1} + \epsilon_t^{\lambda},$$

and the stochastic money growth shock,

$$\tilde{g}_t = \pi\tilde{g}_{t-1} + \epsilon_t^g.$$

C.1.1 Solving the Log-Linear System

This subsection provides a brief overview of the way the log-linear version of the model is solved once it has been divided into a set of equations with expectations and a set without expectations. The division of the model is important because we try to keep the dimension of the second (the expectational) equation small and to have the matrix C of full rank and hence invertible. The model is divided into the sets of matrix equations

$$0 = Ax_t + Bx_{t-1} + Cy_t + Dz_t,$$

$$0 = E_t[Fx_{t+1} + Gx_t + Hx_{t-1} + Jyt + 1 + Kyt + Lz_{t+1} + Mz_t$$

and a stochastic process

$$z_{t+1} = Nz_t + \epsilon_{t+1},$$

where $x_t = [\tilde{K}_{t+1}]'$, $y_t = [\tilde{r}_t \ \tilde{w}_t \ \tilde{N}_t \ \tilde{p}_t]'$ and $z_t = [\tilde{\lambda}_t \ \tilde{g}_t]'$, and where

$$A = \begin{bmatrix} \overline{K} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$B = \begin{bmatrix} -(\overline{r} + 1 - \delta)\overline{K} \\ (1-\theta) \\ -\theta \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} -\overline{r}\overline{K} & -\overline{w}\overline{N} & -\overline{w}\overline{N} & -\frac{1}{\overline{p}} \\ 1 & 0 & (\theta-1) & 0 \\ 0 & 1 & \theta & 0 \\ 0 & -1 & 0 & -1 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 \\ -1 & 0 \\ -1 & 0 \\ 0 & \pi \end{bmatrix}$$

$$F = [0]$$

$$G = [0]$$

$$H = [0]$$

$$J = [\beta\overline{r} \ \ -1 \ 0 \ 0]$$

$$K = [0 \ 1 \ 0 \ 0]$$

$$L = [0 \ 0]$$

$$M = [0 \ 0]$$

$$N = \begin{bmatrix} \gamma & 0 \\ 0 & \pi \end{bmatrix}.$$

The linear laws of motion that we are looking for are given by the matrices P, Q, R and S where

$$x_t = Px_{t-1} + Qz_t$$

and

$$y_t = Rx_{t-1} + Sz_t$$

C.2 The Working Capital Model

The log-linear version of the three optimality conditions(6.2.6, 6.2.7 and 6.2.8), the cash-in-advance constraint(6.2.9) and the real flow budget constraint(6.2.10) on the household side of the economy are given by

$$0 = \{\eta\overline{w} + (1-\eta)\frac{\overline{w}}{\overline{r^n}}\}\tilde{w}_t - (1-\eta)\frac{\overline{w}}{\overline{r^n}}\tilde{r_t^n} + B\overline{C}\tilde{C}_t, \tag{C.2.1}$$

$$0 = \tilde{C}_t - E_t\tilde{C_{t+1}} + \tilde{r_t^n} - E_t\tilde{r_{t+1}^n} + \beta\overline{r}E_t\tilde{r_{t+1}}, \tag{C.2.2}$$

$$0 = \tilde{r_t^n} + \tilde{C}_t + \tilde{P}_t - E_t\tilde{C_{t+1}} - E_t\tilde{P_{t+1}}, \tag{C.2.3}$$

$$0 = \overline{C}\tilde{C}_t - \frac{\overline{\frac{M}{P}}}{g}\tilde{M_{t-1}} + (\frac{\overline{\frac{M}{P}}}{g} - \frac{\overline{N}}{P})\tilde{P}_t + \frac{\overline{N}}{P}\tilde{N}_t + \frac{\eta B\overline{CH}}{\eta + (1-\eta)\frac{\beta}{g}}\tilde{w}_t + \frac{\eta B\overline{CH}}{\eta + (1-\eta)\frac{\beta}{g}}\tilde{H}_t \tag{C.2.4}$$

and

$$0 = \frac{\overline{M}}{P}\tilde{M}_t + (\overline{r^n}\frac{\overline{N}}{P} - \frac{\overline{M}}{P})\tilde{P}_t + \overline{K}\tilde{K_{t+1}} - (1-\eta)\overline{w}\overline{H}\tilde{w}_t - (1-\eta)\overline{w}\overline{H}\tilde{H}_t - \overline{r}\overline{K}\tilde{r}_t -$$

$$(\overline{r} + 1 - \delta)\overline{K}\tilde{K}_t - \overline{r^n}\frac{\overline{N}}{P}\tilde{r_t^n} - \overline{r^n}\frac{\overline{N}}{P}\tilde{N}_t. \tag{C.2.5}$$

Log-linear versions of the two competitive factor market conditions(6.2.11 and 6.2.12) are

$$0 = \tilde{r}_t - \tilde{\lambda_t} - (\theta - 1)\tilde{K}_t - (1-\theta)\tilde{H}_t \tag{C.2.6}$$

and

$$0 = \tilde{w}_t + \tilde{r^f_t} - \tilde{\lambda}_t - \theta\tilde{K}_t + \theta\tilde{H}_t. \tag{C.2.7}$$

Log-linearizing the production function gives

$$0 = \tilde{Y}_t - \tilde{\lambda}_t - \theta\tilde{K}_t - (1-\theta)\tilde{H}_t. \tag{C.2.8}$$

Log-linearization of the two equations on the financial market, equations 6.1.1 and 6.1.2, are as follows

$$0 = \overline{r^f}\{\frac{\overline{N}}{P} + \frac{\overline{M}}{P}(1-\frac{1}{g})\}\tilde{r^f_t} + (\overline{r^f} - \overline{r^n})\frac{\overline{N}}{P}\tilde{N}_t - \{(\overline{r^f} - \overline{r^n})\frac{\overline{N}}{P} + \overline{r^f}\frac{\overline{M}}{P}(1-\frac{1}{g})\}\tilde{P}_t +$$

$$\overline{r^f}\frac{\overline{M}}{P}\tilde{g}_t + \overline{r^f}\frac{\overline{M}}{P}(1-\frac{1}{g})\tilde{M_{t-1}} - \overline{r^n}\frac{\overline{N}}{P}\tilde{r^n_t} \tag{C.2.9}$$

and

$$0 = \frac{\overline{N}}{P}\tilde{N}_t + \frac{\overline{M}}{P}(1-\frac{1}{g})\tilde{M_{t-1}} - \{\frac{\overline{N}}{P} + \frac{\overline{M}}{P}(1-\frac{1}{g})\}\tilde{P}_t + \frac{\overline{M}}{P}\tilde{g}_t - \overline{w}\overline{H}\tilde{w}_t - \overline{w}\overline{H}\tilde{H}_t. \tag{C.2.10}$$

Finally, log-linearization of the money supply process, the stochastic process for technology shock and the stochastic process for money growth shock are given by

$$0 = \tilde{M}_t - \tilde{g}_t - \tilde{M_{t-1}}, \tag{C.2.11}$$

$$\tilde{\lambda}_t = \gamma\tilde{\lambda}_{t-1} + \epsilon^{\lambda}_t \tag{C.2.12}$$

and

$$\tilde{g}_t = \pi\tilde{g}_{t-1} + \epsilon^g_t. \tag{C.2.13}$$

C.2.1 Solving the log-linear system

The model is divided as before into the sets of matrix equations

$$0 = Ax_t + Bx_{t-1} + Cy_t + Dz_t,$$

$$0 = E_t[Fx_{t+1} + Gx_t + Hx_{t-1} + Jyt + 1 + Kyt + Lz_{t+1} + Mz_t$$

and a stochastic process

$$z_{t+1} = Nz_t + \epsilon_{t+1},$$

where $x_t = [\tilde{K}_{t+1}, \tilde{M}_t, \tilde{P}_t]'$, $y_t = [\tilde{r}_t, \tilde{w}_t, \tilde{Y}_t, \tilde{C}_t, \tilde{H}_t, \tilde{N}_t, \tilde{r^n_t}, \tilde{r^f_t}]'$ and $z_t = [\tilde{\lambda}_t, \tilde{g}_t]'$, and where

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \overline{C} + \frac{\eta B \overline{CH}}{\eta + (1-\eta)\frac{\beta}{g}} \\ \overline{K} & \frac{\overline{M}}{P} & \overline{r^n}\frac{\overline{N}}{P} - \frac{\overline{M}}{P} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\{(\overline{r^f} - \overline{r^n})\frac{\overline{N}}{P} + \overline{r^f}\frac{\overline{M}}{P}(1 - \frac{1}{g})\} \\ 0 & 0 & -\{\frac{\overline{N}}{P} + \frac{\overline{M}}{P}(1 - \frac{1}{g})\} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\frac{\frac{\overline{M}}{P}}{g} & 0 \\ -(\overline{r} + 1 - \delta)\overline{K} & 0 & 0 \\ -\theta & 0 & 0 \\ -(\theta - 1) & 0 & 0 \\ -\theta & 0 & 0 \\ 0 & \overline{r^f}\frac{\overline{M}}{P}(1 - \frac{1}{g}) & 0 \\ 0 & \frac{\overline{M}}{P}(1 - \frac{1}{g}) & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & \eta\overline{w} + (1-\eta)\frac{\overline{w}}{\overline{r^n}} & 0 & B\overline{C} & 0 & 0 & -(1-\eta)\frac{\overline{w}}{\overline{r^n}} & \\ 0 & \frac{\eta B \overline{CH}}{\eta + (1-\eta)\frac{\beta}{g}} & 0 & \overline{C} & \frac{\eta B \overline{CH}}{\eta + (1-\eta)\frac{\beta}{g}} & \frac{\overline{N}}{P} & 0 & 0 \\ -\overline{r}\overline{K} & -(1-\eta)\overline{w}\overline{H} & 0 & 0 & -(1-\eta)\overline{w}\overline{H} & -\overline{r^n}\frac{\overline{N}}{P} & -\overline{r^n}\frac{\overline{N}}{P} & 0 \\ 0 & 1 & 0 & 0 & \theta & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & -(1-\theta) & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -(1-\theta) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & (\overline{r^f} - \overline{r^n})\frac{\overline{N}}{P} & -\overline{r^n}\frac{\overline{N}}{P} & \overline{r^f}\{\frac{\overline{N}}{P} + \frac{\overline{M}}{P}(1 - \frac{1}{g})\} \\ 0 & -\overline{w}\overline{H} & 0 & 0 & -\overline{w}\overline{H} & \frac{\overline{N}}{P} & 0 & 0 \end{bmatrix}$$

$$
D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -1 & 0 \\ -1 & 0 \\ -1 & 0 \\ 0 & \overline{r^f}\overline{\frac{M}{P}} \\ 0 & \overline{\frac{M}{P}} \end{bmatrix}
$$

$$
F = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
$$

$$
G = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}
$$

$$
H = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}
$$

$$
J = \begin{bmatrix} 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ \beta\overline{r} & 0 & 0 & -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
$$

$$
K = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
$$

$$
L = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}
$$

$$
M = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix}
$$

$$
N = \begin{bmatrix} \gamma & 0 \\ 0 & \pi \end{bmatrix}.
$$

The linear laws of motion that we are looking for are the same as before and are given by the matrices P, Q, R and S where

$$x_t = Px_{t-1} + Qz_t$$

and

$$y_t = Rx_{t-1} + Sz_t.$$

D

Data Sources

This appendix provides detailed information about the U.S. quarterly time series data used in this thesis. The time period covered is 1960(1) to 2012(2).

Output: Gross Domestic Product, Billions of Chained 2009 Dollars, Quarterly, Seasonally Adjusted; NIPA Table 1.1.6 (line1).

Consumption: Personal Consumption Expenditures, Billions of Chained 2009 Dollars, Quarterly, Seasonally Adjusted; NIPA Table 1.1.6 (line2) + Government consumption expenditures and gross investment, Billions of Chained 2009 Dollars, Quarterly, Seasonally Adjusted; NIPA Table 1.1.6 (line21).

Investment: Gross Private Domestic Investment, Billions of Chained 2009 Dollars, Quarterly, Seasonally Adjusted; NIPA Table 1.1.6 (line7) + Exports of Goods and Services, Billions of Chained 2009 Dollars, Quarterly, Seasonally Adjusted; NIPA Table 1.1.6 (line15) + Imports of Goods and Services, Billions of Chained 2009 Dollars, Quarterly, Seasonally Adjusted; NIPA Table 1.1.6 (line18).

Labor: Total nonfarm business hours (all persons); Valerie Ramey's website (http://weber.ucsd.edu/ṽramey/research.html#data).

Average Hours of Work per Week: Average hours worked per week by the employed labor force 'at work', Total, Household Survey, Quarterly; Source: National Bureau of Economic Research, Release: NBER Macrohistory Database, NBER Indicator: m08354.

Average Annual Hours Worked per Employed Person, Annual, Source: U.S. Department of Labor: Bureau of Labor Statistics, Release: International Comparisons of GDP per Capita and per Hour.

Price Level: Consumer Price Index: Total All Items, Index 2005=1, Quarterly, Seasonally Adjusted; Source: OECD, Release: Main Economic Indicators, OECD descriptor ID: CPALTT01, OECD unit ID: IXOBSA, OECD country ID: USA.

Nominal Interest Rate: 3-Month Treasury Bill: Secondary Market Rate, Quarterly, Source: Board of Governors of the Federal Reserve System, Release: H.15 Selected Interest Rates.

Money Supply: M1, Quarterly, Seasonally Adjusted; Source: OECD, Release: Main Economic Indicators, OECD descriptor ID: MANMM101, OECD unit ID: STSA, OECD country ID: USA
and

M2, Quarterly, Seasonally Adjusted; Source: OECD, Release: Main Economic Indicators, OECD descriptor ID: MABMM201, OECD unit ID: STSA, OECD country ID: USA.