ON A SOLUTION OF THE U(N) ⊃ O(N) STATE LABELLING PROBLEM, FOR TWO-ROWED REPRESENTATIONS

by

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TABLE OF CONTENTS

ABSTRACT

Considerations of the number of labelling operators in a general state labelling problem imply that for two-rowed irreducible representations of U(N), labelled $[p,q,o,..,o]$, just one additional commuting labelling operator, Λ, is required for the solution of the $U(N) \supset O(N)$ state labelling problem. A detailed investigation of the $U(N) \supset O(N)$ reduction via tensor representations leads to a proposal of an integral label λ, implying a non-orthogonal labelling scheme. A simple $U(N) \supset O(N)$ branching theorem for two-rowed representations is formulated in terms of λ.

The additional labelling operator Λ, with eigenvalue λ, is to be defined implicitly by an equation of the form $f(\Lambda,\Phi,\cdots) = o$, where f is a polynomial in Λ, the additional O(N) invariants Φ in the representation subduced by U(N) $[p,q,o,..,o]$, and other known labels and invariants. The O(N) invariants, which are cubic and quartic in the generators of U(N), are discussed. Techniques developed by Green and Bracken[1] for the $U(3) \supset SO(3)$ problem are used to evaluate the single independent cubic invariant, and it is shown how this is used in obtaining the desired implicit operational definition of Λ.

A cubic characteristic polynomial identity, satisfied by the generators of U(N) in two-rowed representations, is also found using these techniques.

Some possible physical applications of the $U(N) \supset O(N)$ state labelling problem are briefly discussed.

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university. To the best of the author's knowledge and belief, the thesis contains no material previously published or written by another person, except when due reference is made in the text.

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