Possible explanation why $\tau_{B^\pm}\sim\tau_{B^0}$ but $\tau_{D^\pm}\sim 2\tau_{D^0}$

Hui-Shi Dong,[2] Xin-Heng Guo,[3,4] Xue-Qian Li,[1,2] and Rui Zhang[2]
[1]*CCAST (World Laboratory), P.O. Box 8730, Beijing 100080, China*
[2]*Department of Physics, Nankai University, Tianjin 300071, China*
[3]*Department of Physics and Mathematical Physics and Special Research Center for Subatomic Structure of Matter, University of Adelaide, SA 5005, Australia*
[4]*Institute of High Energy Physics, Academia Sinica, Beijing 100039, China*
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Data show that $\tau_{B^\pm}\sim\tau_{B^0}$, but $\tau_{D^\pm}\sim 2\tau_{D^0}$. The naive interpretation which attributes $\tau_{D^\pm}\sim 2\tau_{D^0}$ to a destructive interference between two quark diagrams for $D^\pm$ decays definitely fails in the B case. We investigate the suggestion of Close and Lipkin that the phases for producing radially excited states ψ_{2s} in the decay products of B mesons can possess an opposite sign to the integrals for ψ_{1s} decay products. Their contributions can partially compensate each other to result in $\tau_{B^\pm}\sim\tau_{B^0}$. Since D mesons are much lighter than B mesons, such possibilities do not exist in D decays. [S0556-2821(98)05711-0]

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I. INTRODUCTION

The naive explanation for $\tau_{D^\pm}\sim 2\tau_{D^0}$ [1] is that a destructive interference between two quark diagrams for $D^\pm$ [2] reduces the strength of decay amplitudes and thereby elongates the life of $D^\pm$. More explicitly, if the lifetime of a meson is mainly determined by the Cabibbo favored decay modes, for D^+ there is only one topology $D^+\to\bar{K}^0M^+$ (where M generically refers to π,ρ etc. and K to strange mesons), whereas for D^0 there are two channels $D^0\to\bar{K}^0M^0$ and $D^0\to K^-M^+$. For the D^+ decays, the two quark diagrams shown in Figs. 1(a) and 1(b) interfere, while for D^0, the two diagrams 1(c) and 1(d) correspond to two different modes, and therefore do not interfere. For the B decays, similar diagrams exist and there could be also destructive interference in B^- decays. However, the experimental data show that $\tau_{B^\pm}\sim\tau_{B^0}$ [1].

The explanation for the lifetime differences in the D and B cases involves nonperturbative QCD phenomena. Actually some authors [3,4] proposed the so-called Pauli interference (PI) mechanism as a correction to the ''pure'' spectator mechanism for taking into account the light degrees of freedom. The PI effects only exist in $D^\pm$ and $B^\pm$ decays but not in D^0 and B^0 decays. Based on QCD, Bigi *et al.* [4] introduced a virtual gluon so that one of the quarks produced by the weak decay of the heavy quark interferes with the spectator quark. In this mechanism, the PI term modifies the ''pure'' spectator diagram and it is found that such interference is destructive and is proportional to Γ_0/m_Q^2 ($Q=b$ or c). This mechanism partly explains why $\tau_{D^\pm}\sim 2\tau_{D^0}$ and $\tau_{B^\pm}\sim\tau_{B^0}$.

In the present work we try to investigate the lifetime differences in another way which is based on the idea of Close and Lipkin. Recently Close and Lipkin [5] have analyzed the data on low-lying exclusive quasi-two-body final states in both D and B decays. They noted that in D decays the sign of interference in exclusive channels is still amiguous, while in B decays there is a clear and uniform tendency towards constructive interference between the color-favored and color-suppressed exclusive channels where all final-state mesons have nodeless wave functions. They noted that in B decays, in order that $\tau_{B^\pm}\sim\tau_{B^0}$ [1], this interference must be compensated in as yet unmeasured channels. They suggested that the sign of interference may be changed in channels where excited states of the decay products, whose wave functions contain nodes, are involved. It is the motivation of the present paper to include the contributions from the excited states of the B-decay products so that constructive interference is obtained in $B^\pm$ decays. Such excited states only exist in B decays but not in D decays because of the phase-space requirement. It will be shown that in our model the lifetime differences in B and D mesons can also be explained.

The effective Hamiltonian of nonleptonic decays in the D case [6,7] is

$$H_{\rm eff}=\frac{G_F}{\sqrt{2}}V_{cs}^*V_{ud}[c_1\bar{s}\gamma_\mu(1-\gamma_5)c\bar{u}\gamma^\mu(1-\gamma_5)d+c_2\bar{s}\gamma_\mu(1-\gamma_5)d\bar{u}\gamma^\mu(1-\gamma_5)c], \quad (1)$$

where $c_1=(c_++c_-)/2$ and $c_2=(c_+-c_-)/2$. By the renormalization-group equation (RGE) we have

$$c_-=\left(\frac{\alpha_s(m_c^2)}{\alpha_s(m_b^2)}\right)^{12/25}\left(\frac{\alpha_s(m_b^2)}{\alpha_s(M_W^2)}\right)^{12/23};\quad c_+=\frac{1}{\sqrt{c_-}}. \quad (2)$$

With the Fiertz transformation, the coefficients c_1 and c_2 in Eq. (1) should be replaced by a_1 and a_2 with

$$a_1=c_1+\xi c_2,\quad \text{and}\quad a_2=c_2+\xi c_1, \quad (3)$$

where ξ is $1/N_c$ if the factorization assumption holds perfectly, otherwise $\xi=(1+\delta)/N_c$ where δ denotes a color-octet contribution proportional to $\langle\lambda^a\lambda^a\rangle$ [8,3,9]. Recently, Blok and Shifman gave a more theoretical estimation [10], but they also pointed out that the obtained value is not accurate for practical calculations. Generally, δ is a negative number ranging between 0 and -1, so that ξ takes values

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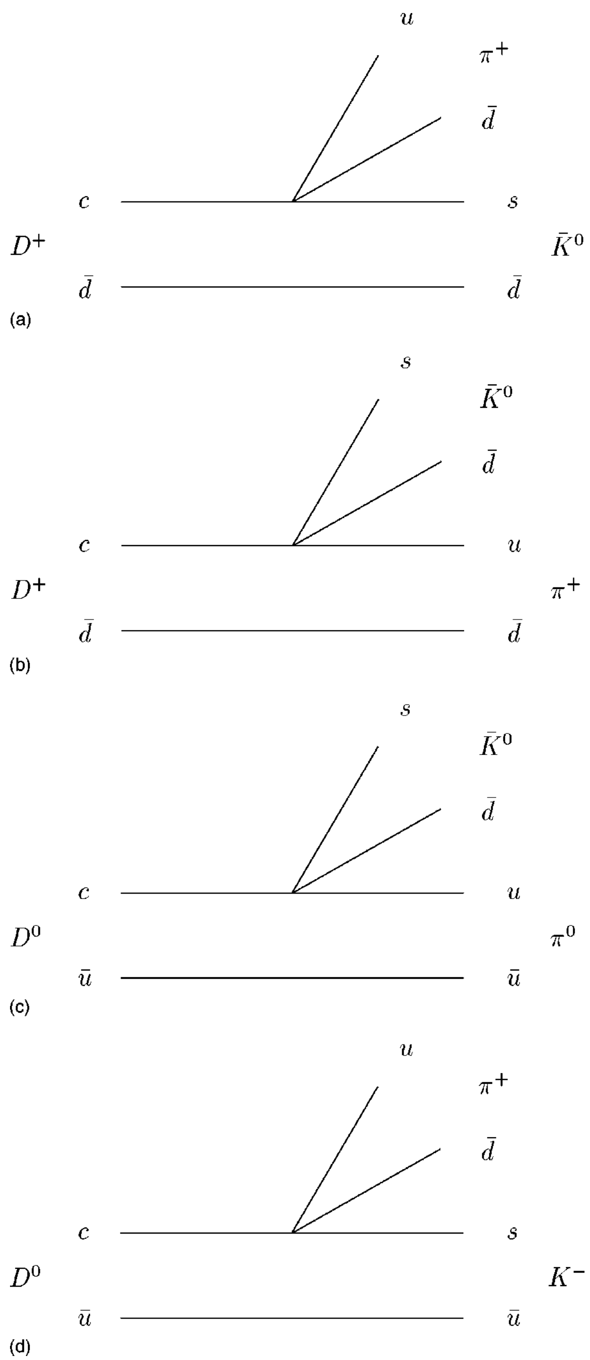

FIG. 1. (a)–(d) The quark diagrams for the nonleptonic decays of B and D mesons (here we take $D\to K\pi$ as an example).

between 0 and $1/N_c$. Later, in our numerical calculations we will take δ as 0, -0.5, and -1, respectively.

For the B case, we have a Hamiltonian similar to Eq. (1),

$$H_{\rm eff}=\frac{G_F}{\sqrt{2}}V_{cb}V_{ud}^{*}[c_1^{(B)}\overline{c}\gamma_\mu(1-\gamma_5)b\overline{d}\gamma^\mu(1-\gamma_5)u + c_2^{(B)}\overline{c}\gamma_\mu(1-\gamma_5)u\overline{d}\gamma^\mu(1-\gamma_5)b], \tag{4}$$

and coefficients $c_1^{(B)}$ and $c_2^{(B)}$,

$$c_-^{(B)}=\left(\frac{\alpha_s(m_b^2)}{\alpha_s(M_W^2)}\right)^{12/23}; \quad c_+^{(B)}=\frac{1}{\sqrt{c_-^B}}, \tag{5}$$

whereas $a_1^{(B)}$, $a_2^{(B)}$ have similar forms in analog to that for the charm case.

It is noted that in the case of D meson decays a_1 is positive and a_2 is negative. From the data of D physics the value of a_2 is about -0.5 [11]. In D^+ decays, the a_1 term corresponds to the external W emission, while a_2 corresponds to the internal W emission; naturally a destructive interference would occur between the two quark diagrams.

In the following we will express the corresponding transition amplitudes as A_1 and A_2 which are proportional to a_1 and a_2, respectively, thus $A_1=\kappa_1 a_1$ and $A_2=\kappa_2 a_2$ where κ_1 and κ_2 are the hadronic transition matrix elements.

Then we have the amplitude square as

$$|\langle\overline{K^0}\pi^+|H_{\rm eff}|D^+\rangle|^2=|A_1|^2+|A_2|^2+2{\rm Re}(A_1A_2^*). \tag{6}$$

In addition to a common phase factor such as the Cabibbo-Kabayashi-Maskawa (CKM) phase, both A_1 and A_2 are real. Thus if $A_1\cdot A_2$ is negative, this is a destructive interference. Otherwise we have constructive interference.

In contrast, for D^0 decays,

$$\langle K^-\pi^+|H_{\rm eff}|D^0\rangle\propto a_1 \quad \text{and} \quad \langle\overline{K^0}\pi^0|H_{\rm eff}|D^0\rangle\propto a_2.$$

We can roughly assume

$$\langle K^-\pi^+|H_{\rm eff}|D^0\rangle\approx A_1 \quad \text{and} \quad \langle\overline{K^0}\pi^0|H_{\rm eff}|D^0\rangle\approx A_2.$$

Thus if we only consider the CKM favored channels which dominate the lifetime of D mesons, we have

$$\Gamma(D^+)=[|A_1|^2+|A_2|^2+2\,{\rm Re}(A_1A_2^*)]\times{\rm LIPS}, \tag{7}$$

$$\Gamma(D^0)=(|A_1|^2+|A_2|^2)\times{\rm LIPS}, \tag{8}$$

where LIPS is the Lorentz-invariant-phase-space of the final products. If $A_2\sim-0.26A_1$, one can numerically obtain $\Gamma(D^0)\sim 2\Gamma(D^+)$ (or $\tau_{(D^\pm)}\sim 2\tau_{D^0}$). Of course, the other channels (Cabibbo-suppressed) and semileptonic decays all contribute to the lifetime, so this obtained number is not rigorous. However, since the Cabibbo favored channels dominate, one can expect that a solution for A_1 and A_2 does not deviate much from the aforementioned value.

Taking $\alpha_s(M_Z^2)=0.118$ [1], one can obtain a ratio of A_1/A_2 for D decays to be roughly consistent with the required value. From recent work, the hadronic matrix elements can be evaluated more easily in terms of the heavy-quark effective theory [12].

In the same scenario and from Eqs. (4), (5), a_1 and a_2 are still of opposite sign in B decays. It is a consequence of the renormalization-group equation (RGE) which is proved to be valid for perturbative QCD. If so, one could expect a result similar to the D case that $\tau_{B^-}\sim 2\tau_{B^0}$. However, this does not coincide with the data for B decays.

The $B^{(\pm)}$ lifetime is very close to that of B^0 as $\tau_{B(\pm)}\sim(1.62\pm0.06)\times10^{-12}$ s and $\tau_{B^0}\sim(1.56\pm0.06)\times10^{-12}$ s [1]. There could be small measurement uncertainty as $\tau_{(B^\pm)}\sim 1.47\times10^{-12}$ s, $\tau_{(B^0)}\sim 1.25\times10^{-12}$ s, by the ALEPH Collaboration [13,15] and $\tau_{(B^\pm)}\sim 1.72\times10^{-12}$ s, $\tau_{(B^0)}\sim 1.63\times10^{-12}$ s, by the DELPHI Collaboration [14].

Similar quark diagrams exist in B decays; namely there are both external and internal W emissions for $B^-\to D^0\pi^-$

which destructively interfere, but for B^0, $B^0\to D^+\pi^-$ and $B^0\to D^0\pi^0$ corresponding to external and internal W emissions, respectively, do not interfere. Thus if that is the case, one would wonder why $\tau_{B^{\pm}}$ is so close to τ_{B^0}.

To fit the data of B decays, one needs to take a positive value for a_2 [11]. This contradicts the result of RGE which is obviously correct by the perturbative QCD theory and there is no doubt of application of perturbative QCD at the m_b energy region.

However, one can notice that even though A_1, A_2 are proportional to a_1, a_2, respectively, they also possess certain factors corresponding to the hadronic matrix elements. These hadronic matrix elements involve some overlapping integrations of the decay parent and daughter wave functions. If the integrations can contribute a negative sign, the interference between two diagrams would become constructive and it may be equivalent to an ''effective'' positive a_2 value.

The hadronization process is very nonperturbative and we cannot evaluate it accurately, so that we attribute the nonperturbative effects into the parameters of meson wave functions which exist in the overlapping integration. To evaluate such overlapping integrations, one needs to invoke some concrete models and later we employ the nonrelativistic quark model. Since the decaying B meson is a pseudoscalar at ψ_{1s} radial ground state, if the decay product is at ψ_{1s} state, the overlapping integration would certainly be positive, however, if the decay products can be radially excited states ψ_{2s}, the integration can turn sign (see next section for details). Because the D meson is much lighter than the B meson, ψ_{2s} states do not seem to exist as decay products of D, but definitely there should be ψ_{2s} excited states showing up as decay products of the B meson. This change may modify the whole picture and finally leads to the consequence that $\tau_{B(\pm)}\sim\tau_{B^0}$. Later our numerical results will show that the involvement of the ψ_{2s}-decay products can indeed do the job. In the next section, we give the formulation in every detail and in Sec. III, we present our numerical results, while the last section is devoted to conclusion and discussion.

II. FORMULATION

A. The transition amplitudes

As usual, we ignore the W exchange and annihilation diagrams because the two fast quarks would pick up a quark pair from the vacuum and speed them up [8]. Even though the factorization approach is not very reliable in evaluating the internal W emission diagrams, we may use a phenomenological parameter δ to compensate it. Therefore by the vacuum saturation

$$\begin{aligned}\langle K^-\pi^+|a_1(\bar{s}c)(\bar{u}d)+a_2(\bar{s}d)(\bar{u}c)|D^0\rangle &= a_1\langle\pi^+|(\bar{u}d)|0\rangle\langle K^-|(\bar{s}c)|D^0\rangle + a_2\langle K^-\pi^+|(\bar{s}d)|0\rangle\langle 0|(\bar{u}c)|D^0\rangle\\ &= a_1 f_\pi p_\pi^\mu\langle K^-|(\bar{s}c)|D^0\rangle + a_2 f_D p_D^\mu\langle K^-\pi^+|(\bar{s}d)|0\rangle,\end{aligned}\tag{9}$$

where $(\bar{q}q')\equiv\bar{q}\gamma_\mu(1-\gamma_5)q'$. The second term corresponds to a W-annihilation diagram and obviously is much smaller than the first one as it is proportional to $f_D(m_K^2-m_\pi^2)$. As argued in the literature this term is negligible and we will omit such contributions in later calculations. Then we also have

$$\begin{aligned}&\langle\bar{K^0}\pi^0|a_1(\bar{s}c)(\bar{u}d)+a_2(\bar{s}d)(\bar{u}c)|D^0\rangle\\ &\quad= a_2 f_K p_K^\mu\langle\pi^0|(\bar{u}c)|D^0\rangle,\end{aligned}\tag{10}$$

and

$$\begin{aligned}&\langle\bar{K^0}\pi^+|a_1(\bar{s}c)(\bar{u}d)+a_2(\bar{s}d)(\bar{u}c)|D^+\rangle\\ &\quad= a_1 f_\pi p_\pi^\mu\langle\bar{K^0}|(\bar{s}c)|D^+\rangle\\ &\qquad+ a_2 f_K p_K^\mu\langle\pi^+|(\bar{u}c)|D^+\rangle.\end{aligned}\tag{11}$$

Instead, for $P\to PV$,

$$\begin{aligned}&\langle K^-\rho^+|a_1(\bar{s}c)(\bar{u}d)+a_2(\bar{s}d)(\bar{u}c)|D^0\rangle\\ &\quad= a_1 f_\rho m_\rho\epsilon^{*\mu}\langle K^-|(\bar{s}c)|D^0\rangle,\end{aligned}\tag{12}$$

$$\begin{aligned}&\langle K^{-*}\pi^+|a_1(\bar{s}c)(\bar{u}d)+a_2(\bar{s}d)(\bar{u}c)|D^0\rangle\\ &\quad= a_1 f_\pi p_\pi^\mu\langle K^{-*}|(\bar{s}c)|D^0\rangle,\end{aligned}\tag{13}$$

$$\begin{aligned}&\langle\bar{K^0}^*\pi^0|a_1(\bar{s}c)(\bar{u}d)+a_2(\bar{s}d)(\bar{u}c)|D^0\rangle\\ &\quad= a_2 f_{K^*} m_{K^*}\epsilon^{*\mu}\langle\pi^0|(\bar{u}c)|D^0\rangle,\end{aligned}\tag{14}$$

$$\begin{aligned}&\langle\bar{K^0}\rho^0|a_1(\bar{s}c)(\bar{u}d)+a_2(\bar{s}d)(\bar{u}c)|D^0\rangle\\ &\quad= a_2 f_K p_K^\mu\langle\rho^0|(\bar{u}c)|D^0\rangle,\end{aligned}\tag{15}$$

and

$$\begin{aligned}&\langle\bar{K^0}^*\pi^+|a_1(\bar{s}c)(\bar{u}d)+a_2(\bar{s}d)(\bar{u}c)|D^+\rangle\\ &\quad= a_1 f_\pi p_\pi^\mu\langle\bar{K^0}^*|(\bar{s}c)|D^+\rangle\\ &\qquad+ a_2 f_{K^*}\epsilon_{K^*}^{*\mu} m_{K^*}\langle\pi^+|(\bar{u}c)|D^+\rangle,\end{aligned}\tag{16}$$

$$\begin{aligned}&\langle\bar{K^0}\rho^+|a_1(\bar{s}c)(\bar{u}d)+a_2(\bar{s}d)(\bar{u}c)|D^+\rangle\\ &\quad= a_1 f_\rho\epsilon_\rho^{*\mu} m_\rho\langle\bar{K^0}|(\bar{s}c)|D^+\rangle\\ &\qquad+ a_2 f_K p_K^\mu\langle\rho^+|(\bar{u}c)|D^+\rangle.\end{aligned}\tag{17}$$

For $P\to VV$,

$$\begin{aligned}&\langle\bar{K^0}^*\rho^0|a_1(\bar{s}c)(\bar{u}d)+a_2(\bar{s}d)(\bar{u}c)|D^0\rangle\\ &\quad= a_2 f_{K^*} m_{K^*}\epsilon^{*\mu}\langle\rho^0|(\bar{u}c)|D^0\rangle,\end{aligned}\tag{18}$$

$$\langle K^{-*}\rho^+|a_1(\overline{s}c)(\overline{u}d)+a_2(\overline{s}d)(\overline{u}c)|D^0\rangle$$
$$=a_1 f_\rho m_\rho \epsilon^{*\mu}\langle K^{-*}|(\overline{s}c)|D^0\rangle, \quad (19)$$

and

$$\langle \overline{K^0}{}^*\rho^+|a_1(\overline{s}c)(\overline{u}d)+a_2(\overline{s}d)(\overline{u}c)|D^+\rangle$$
$$=a_1 f_\rho \epsilon^\mu_\rho m_\rho \langle \overline{K^0}{}^*|(\overline{s}c)|D^+\rangle$$
$$+a_2 f_{K*}\epsilon^{*\mu}_{K*} m_{K*}\langle \rho^+|(\overline{u}c)|D^+\rangle. \quad (20)$$

The above formulas indicate that the external and internal W emissions in D^+ decays interfere. For the B case, we can have similar expressions with an effective Hamiltonian Eq. (4) and corresponding coefficients a_1^B, a_2^B in Eq. (5).

B. The matrix elements

It is noted that in the scenario of factorization, the hadronic matrix elements are related to a weak transition [16], for $P \to P$,

$$\langle X|j_\mu|I\rangle=\left(P_I+P_X-\frac{M_I^2-M_X^2}{q^2}q\right)_\mu F_1(q^2)$$
$$+\frac{M_I^2-M_X^2}{q^2}q_\mu F_0(q^2), \quad (21)$$

with $q\equiv P_I-P_X$ and $F_1(0)=F_0(0)$. For $P\to V$, we have

$$\langle X^*|j_\mu|I\rangle=\frac{2}{M_I+M_{X*}}\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}P_I^\rho P_{X*}^\sigma V(q^2)+i\frac{\epsilon^*\cdot q}{q^2}2M_{X*}q_\mu A_0(q^2)$$
$$+i\left\{\epsilon^*_\mu(M_I+M_{X*})A_1(q^2)-\left(\frac{\epsilon^*\cdot q}{M_I+M_{X*}}\right)(P_I+P_{X*})_\mu A_2(q^2)-\frac{\epsilon^*\cdot q}{q^2}2M_{X*}q_\mu A_3(q^2)\right\}, \quad (22)$$

with $A_3(0)=A_0(0)$ and here

$$A_3(q^2)=\frac{M_I+M_{X*}}{2M_{X*}}A_1(q^2)-\frac{M_I-M_{X*}}{2M_{X*}}A_2(q^2). \quad (23)$$

So our task is to calculate the form factors. Taking the nearest pole approximation

$$F_1(q^2)\approx\frac{h_1}{1-q^2/M_1^2} \quad \text{for } P_I\to P_X, \quad (24)$$

$$V(q^2)\approx\frac{h_V}{1-q^2/M_2^2}, \quad A_0(q^2)=\frac{h_{A_0}}{1-q^2/M_3^2}$$
$$\text{for } P_I\to P_{X*}, \quad (25)$$

where M_1, M_2, M_3 are masses of mesons corresponding to the nearest poles which can be found in the data book. With this approximation, to evaluate the form factors, one only needs to calculate the constant parameters $h_0=h_1$, h_V, h_{A_1}, h_{A_2}, and $h_{A_3}=h_{A_0}$, which turn out to be the values of the form factors at the unphysical kinematic region $q^2=0$ and we will use the nonrelativistic quark model to calculate them. Moreover, for the case of a pseudoscalar B or D meson transiting to a vector meson, we use the helicity amplitude method [17] which can much simplify our calculations.

The parameters are related to an overlapping integral over the wave functions of initial pseudoscalar and final pseudoscalar or vector mesons. To carry out the integration, one needs to invoke concrete models and the most popular one is to take the wave function of harmonic oscillation potential as the orbital part. In the Bauer-Stech-Wirbel approach [16] the following wave function model is employed:

$$R_m(p_T,x)=N_m\sqrt{x(1-x)}\exp(-p_T^2/2\omega^2)$$
$$\times\exp\left(\frac{-m^2}{2a^2}\left(x-\frac{1}{2}\frac{m_{q_1}^2-m_{q_2}^2}{2m^2}\right)^2\right), \quad (26)$$

where N_m is the normalization factor while Guo and Huang [16] used the following wave function form in the light-cone formalism:

$$R_m(x,k_\perp)=A\exp\left(-b^2\left(\frac{k_\perp^2+m_1^2}{x_1}+\frac{k_\perp^2+m_2^2}{x_2}\right)\right). \quad (27)$$

These wave functions apply in the infinite-momentum frame. Here instead, we choose the wave function at the rest frame of the decaying meson [18]. Everything in the picture is nonrelativistic, but it is accurate enough for the qualitative conclusion and we will discuss it in the final section.

Here we only list the radial wave functions of ψ_{1s} and ψ_{2s} and the others can be found in Ref. [19]:

$$\psi_{1s}=\left(\frac{4\beta^3}{\sqrt{\pi}}\right)^{1/2}\exp\left(-\frac{1}{2}\beta^2r^2\right)\sqrt{m}Y_{00}(\theta,\phi), \quad (28)$$

where adding a factor $\sqrt{m}$ is for proper normalization, and

$$\psi_{2s}=\left(\frac{4\beta^3}{6\sqrt{\pi}}\right)^{1/2}(3-2\beta^2r^2)\exp\left(-\frac{1}{2}\beta^2r^2\right)Y_{00}(\theta,\phi)\sqrt{m}, \quad (29)$$

where β is the only free parameter to be fixed by data. Here $r\equiv|\vec{r}_1-\vec{r}_2|$ in the potential picture. To convert into the momentum space, we have

$$h_1=h_0=\frac{2m_I}{m_I^2-M_X^2}\int d^3p_1\phi_X^*(\vec{p}_{1'})\phi_I(\vec{p}_1)\left(\frac{p_{1'}^3}{p_{1'}^0+m_{1'}}+\frac{p_1^3}{p_1^0+m_1}\right)\sqrt{\frac{(p_{1'}^0+m_{1'})(p_1^0+m_1)}{p_{1'}^0p_1^0}}, \tag{30}$$

and

$$h_V=\frac{i}{m_I-m_{X*}}\int d^3p_1\phi_X^*(\vec{p}_{1'})\phi_I(\vec{p}_1)\left(\frac{p_{1'}^3}{p_{1'}^0+m_{1'}}-\frac{p_1^3}{p_1^0+m_1}\right)\sqrt{\frac{(p_{1'}^0+m_{1'})(p_1^0+m_1)}{p_{1'}^0p_1^0}}, \tag{31}$$

$$h_{A_1}=\frac{i}{m_I+m_{X*}}\int d^3p_1\phi_X^*(\vec{p}_{1'})\phi_I(\vec{p}_1)\left(1-\frac{p_{1'}^3p_1^3}{(p_{1'}^0+m_{1'})(p_1^0+m_1)}\right)\sqrt{\frac{(p_{1'}^0+m_{1'})(p_1^0+m_1)}{p_{1'}^0p_1^0}}, \tag{32}$$

$$h_{A_2}=\frac{2(m_I+m_{X*})^2}{3m_I^2+m_{X*}^2}h_{A_1}-\frac{i4m_Im_{X*}}{(m_I-m_{X*})(3m_I^2+m_{X*}^2)}\int d^3p_1\phi_X^*(\vec{p}_{1'})\phi_I(\vec{p}_1)\left[\frac{p_{1'}^3}{p_{1'}^0+m_{1'}}+\frac{p_1^3}{p_1^0+m_1}\right]\sqrt{\frac{(p_{1'}^0+m_{1'})(p_1^0+m_1)}{p_{1'}^0p_1^0}}, \tag{33}$$

where the $\phi_{(X,X*)}$ are wave functions of $\psi_{(1s,2s)}$ in the momentum space, i.e., the Fourier transformed Eqs. (28) and (29), in the expressions, $\vec{p}_1$ and $\vec{p}_{1'}$ denote the 3-momenta of the quarks which take part in the reaction in the initial and final mesons, while $m_1, m_{1'}$ are their masses, respectively. p^3 and p^0 correspond to the third and the zeroth components of the concerned 4-momenta. In the helicity-coupling picture, all momenta of the mesons are along $\hat{z}$, so

$$p_I\equiv|\vec{p}_I|,\quad p_{X(X*)}^3\equiv\pm|\vec{p}_{X(X*)}|,$$

but the quark momenta can be along any directions. In the center of mass (CM) frame of the decaying meson $\vec{p}_I=0$ and $|\vec{p}_{X(X*)}|=(m_I^2-m_{X(X*)}^2)/2m_I$ as $q^2=0$, thus one has

$$p_1+p_2\equiv p_I=(M,\vec{0}),$$

and

$$p_{1'}+p_{2'}\equiv p_{X(X*)}=(p_{X(X*)}^0,0,0,p_{X(X*)}^3).$$

The resultant formulas look quite different from that given in Ref. [16], but as a matter of fact, as $|\vec{p}|\gg M$, they coincide with each other. Substituting all the information into Eqs. (21) and (22), we can have the final numerical results.

III. THE NUMERICAL RESULTS

In the whole calculations, only β is a free parameter and one can fix it by the energy-minimum condition

$$\frac{\partial E}{\partial\beta}=\frac{\partial\langle H\rangle}{\beta}=0.$$

Then one obtains

$$\beta_{1s}=\left(\frac{4\mu}{3\sqrt{\pi}a^2}\right)^{1/3}, \tag{34}$$

$$\beta_{2s}=\left(\frac{6\mu}{7\sqrt{\pi}a^2}\right)^{1/3}, \tag{35}$$

where μ is the reduced mass and a is an average radius of the meson. There can be an uncertainty for a and μ, it does not affect our qualitative conclusion even though indeed the numerical results can be declined by a few tens of percents. (see below).

Even though f_D is not well measured yet, there are some reasonable estimated values, so we take $f_D=0.15$ GeV and $f_B=0.125$ GeV [20]. Numerically we use

$$f_\pi=0.132,\quad f_K=0.161,\quad f_\rho=0.212,\quad f_{K*}=0.221$$

in GeV.

By the well-measured value $\alpha_s(M_Z^2)=0.118$ [1], we have $\alpha_s(m_b=5\ \text{GeV})=0.203$, $\alpha_s(m_c=1.5\ \text{GeV})=0.265$, and

$$c_1^{(D)}=1.26,\ c_2^{(D)}=-0.51,$$

$$c_1^{(B)}=1.10,\ c_2^{(B)}=-0.23.$$

Our result is fully consistent with Ref. [23] obtained in terms of RGE.

It is also noted that since m_c is not very large, one can expect that the real values of $c_{1,2}^{(D)}$ may deviate from that predicted by the perturbative QCD calculation, for example, it is claimed that a set of $c_1^{(D)}=1.26\pm0.04$ and $c_2^{(D)}=-0.51\pm0.05$ can fit data better. However, below we will rely on the perturbative QCD and use the values obtained by RGE.

The corresponding $a_1^{(B,D)}$ and $a_2^{(B,D)}$ would depend on ξ of Eq. (3). For the radially excited ψ_{2s} states, we will take $M_D(2s)\approx2.4$ GeV and $M_\pi(2s)=1.0$ GeV, $M_K(2s)=1.4$ GeV. By Eq. (34), we fix

$$\beta_B=0.5,\quad \beta_D(1s)=0.45,\quad \beta_D(2s)=0.39,$$

$$\beta_\pi(1s)=0.3,\quad \beta_\pi(2s)=0.26,$$

$$\beta_K(1s)=0.4,\quad \beta_K(2s)=0.34$$

in GeV. All the parameters are obtained according to Eqs. (34) and (35).

Numerically, we have

$$\frac{\Gamma_{D^+}}{\Gamma_{D^0}}=\begin{cases}0.9, & \delta=0,\\ 0.70, & \delta=-0.5,\\ 0.56, & \delta=-1.\end{cases}\tag{36}$$

It seems that the $\delta=-1$ solution suits the data on D decays better than other δ values and this conclusion was also predicted by Stech *et al.* a long while ago [24].

For the B case, without considering the ψ_{2s} excited-state contribution, we have

$$\frac{\Gamma_{B^-}}{\Gamma_{B^0}}=\begin{cases}1.28, & \delta=0,\\ 0.90, & \delta=-0.5,\\ 0.59, & \delta=-1.\end{cases}\tag{37}$$

If one looks at $\delta=-1$ which is consistent with that obtained in D decays, the ratio is close to 0.5 as expected (see the introduction). When we take into account the contributions from the ψ_{2s} excited states, the whole result is modified as

$$\frac{\Gamma_{B^-}}{\Gamma_{B^0}}=\begin{cases}1.02, & \delta=0,\\ 0.99, & \delta=-0.5,\\ 0.98, & \delta=-1,\end{cases}\tag{38}$$

this result is very consistent with the data on the lifetimes of both D and B mesons. We will discuss this result in the next section.

IV. CONCLUSION AND DISCUSSION

Since B and D mesons all contain a heavy quark and a light one, we have every reason to believe that they have similar characteristics. Indeed a symmetry between b and c quarks (B and D mesons) [12] is confirmed by phenomenology. However the obvious discrepancy that $\tau_{D^\pm}\sim 2\tau_{D^0}$, while $\tau_{B^\pm}\sim\tau_{B^0}$ implies some distinction between B and D mesons.

There have been alternative ways to interpret the lifetime difference of B and D. For example, Bander, Silverman, and Soni [21] suggested the reaction $D^0\to s+\bar{d}+gluon$ as a source for the difference in the lifetimes of D^0 and $D^\pm$ and in another way, one can suppose that the factorization factor δ can be different for B and D or the signs of a_2 can change, etc. However, if we consider similarities between B and D, it is natural to accept an assumption that δ would not be too declined in the B and D cases. In the literature [24] of D physics, δ is very close to -1 and our results confirm this allegation. Cheng found [22] that $r_2=-0.67,-(0.9-1.1)$ for $D\to\bar{K}\pi,\bar{K}^*\pi$, respectively, where our $\delta=(N_c/2)r_2$, it indicates that $\delta\sim-1$. But to fit B-decay data, Cheng concluded $r_2=+0.36$ which drastically deviates from the parameter for D decays, so one would ask how it could be so?

Instead, we accept the assumption that a symmetry between b and c holds and $c_1^{(D,B)}$, $c_2^{(D,B)}$ can be derived with the RGE. Meanwhile we also notice that since B mesons are much heavier than D mesons, there can be radially excited states ψ_{2s}^D and ψ_{2s}^π as decay products in B decays, but not for D decays. The ψ_{2s} states may cause the hadronic matrix elements to be in opposite sign to the ψ_{1s} final states and it would result in a change making $\tau_{B^\pm}\sim\tau_{B^0}$. Obviously, it is determined by an overlapping integral between wave functions of the final and initial mesons. Our numerical results show that the integrals for ψ_{2s} and ψ_{1s} can have opposite signs depending on the parameter β. Our β values are reasonably determined by data, even though not very accurate. We show that $\delta\sim-1$, taking into account the contribution from $\psi_{2s}^{D,\pi}$ as well as $\psi_{1s}^{D,\pi}$, approximately

$$\tau_{B^\pm}\sim\tau_{B^0},\qquad \tau_{D^\pm}\sim 2\tau_{D^0}.$$

Our mechanism is in parallel to the PI effects discussed by some authors [3,4]. It is based on the common knowledge that as long as all the exclusive channels (in fact, the main ones) are summed up, the total width should be obtained, i.e., equivalent to the inclusive evaluation. Thus in our picture an interference between the decay products of the b (c) quark and the light one is automatically considered via the a_1 and a_2 interference.

Since, indeed, we only consider the most Cabibbo-favorable channels to estimate the lifetimes, there can be contributions from the rare decays and the numerical results can deviate a bit, but in general the same mechanism proposed by Close and Lipkin can apply. Hence the rule is the same for all channels, namely ψ_{2s} always contributes as well as ψ_{1s}, our results seem sufficiently convincing. As a matter of fact, the ψ_{2s} is still light enough and there is large phase-space available for B, but in contrast, not for the D meson.

For evaluating the hadronic matrix elements, we use the nonrelativistic quark model. Even though the model is approximate, our qualitative conclusion does not change.

Surely, we can make the ratios of lifetimes for D and B mesons perfectly coincide with the data by carefully adjusting the β values in the wave functions. However, since there are many uncertain factors such as the contributions of the rare decays, the nonrelativistic form of the wave functions, and the factorization factor δ, etc., which make a very accurate evaluation impossible, adjusting the β value to fit data seems not necessary. In fact, as the most important point, one can draw a qualitative conclusion confidently that the contribution of ψ_{2s} is important to B decays, namely the puzzle of the lifetimes of B and D mesons can be reasonably explained away by its participation.

It is important to notice that not only the lifetimes of B mesons are contrary to our knowledge based on the perturbative QCD and D physics, if the ψ_{2s} contribution is not taken into account, but also similar puzzles exist at many channels of B-meson decays. It is that the value of a_2 is not universal [11] and its sign is also uncertain. It is hard to understand. We hope that by taking into account the ψ_{2s} contributions, all the discrepancies may get a reasonable explanation. Because the relatively heavy ψ_{2s} is still light compared to the B meson and does not affect its phase-space integration very much, maybe in measurements of exclusive channels, a certain ψ_{2s} with the same quantum numbers as the ψ_{1s} gets mixed in and is not well tagged out. It causes the superficial discrepancy. To carefully and thoroughly investigate the influence and effects of possible ψ_{2s}-decay products in B decays is the goal of our next work.

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