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Weak semileptonic decays of heavy baryons containing two heavy quarks

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In the heavy quark limit a heavy baryon which contains two heavy quarks is believed to be composed of a heavy diquark and a light quark. Based on this picture, we evaluate the weak semileptonic decay rates of such baryons. The transition form factors between two heavy baryons are associated with those between two heavy mesons by applying superflavor symmetry. The effective vertices of the W boson and two heavy diquarks are obtained in terms of the Bethe-Salpeter equation. Numerical predictions on these semileptonic decay widths are presented and they will be tested in future experiments. [S0556-2821(98)00721-8]

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I. INTRODUCTION

Heavy flavor physics has been an interesting subject for many years. The meson case has been studied much more intensively both in experiments and in theory than the baryon case. The existence of three valence quarks in a baryon makes the theoretical study much more complicated. Recently more and more data for heavy baryons which contain one heavy quark have been accumulated [1] and in the near future we may expect even more data from the CERN e^+e^- collider LEP and other experimental groups. Although we do not have any data for doubly heavy baryons containing two heavy quarks at present, it would be interesting to make predictions on their properties which will be tested in future experiments. In our previous paper [2] we studied the production of a pair of doubly heavy baryons in electron-position collisions. It is the aim of the present work to study their weak semileptonic decays.

The basic problem is how to deal with the transition form factors between such doubly heavy baryons where one heavy flavor transits (explicitly $b\to c$) with another heavy quark, the light flavor remaining unchanged. Since it is determined by nonperturbative QCD effects, the solution is by no means trivial. The heavy quark effective theory (HQET) provides a way to appropriately simplify the evaluation of the hadronic matrix elements [3] because by applying the HQET we are able to find relations among the form factors, and consequently reduce the independent number of these form factors. It is well known that in the heavy quark limit the extra symmetries $SU(2)_f\times SU(2)_s$ manifest and the nonperturbative effects are attributed to the well-defined Isgur-Wise function $\xi(v\cdot v')$, where v and v' are the four-velocities of the concerned heavy quarks.

There have been some theoretical papers which study the properties of doubly heavy baryons. It has been pointed out that in the heavy quark limit the two heavy quarks QQ' ($Q=b$ or c) inside a doubly heavy baryon bind into a $\bar{3}$ heavy diquark which acts as a color source and is pointlike to the light degrees of freedom [2,4–7]. The leftover light quark moves in the color field induced by the heavy diquark. The size of the heavy diquark is much smaller compared with the QCD scale. Based on this picture, White and Savage analyzed semileptonic decays between doubly heavy baryons where the weak transition matrix elements were given in terms of the overlap of the Coulomb wave functions of heavy diquarks and the Isgur-Wise function which entered through the application of the superflavor symmetry [8,9]. In the same direction, Sanchis-Lozano compared semileptonic decays of doubly heavy baryons with the analogous decays of heavy quarkonia [5,6]. Furthermore, based on the potential model analysis, it was shown that in the preasymptotic quark mass region the spin symmetry is still a good approximation for doubly heavy baryons [6].

The ground state of heavy diquark composed of Q and Q' can be a spin-1 ($V_{QQ'}$) or spin-0 ($S_{QQ'}$) object. Because of the Pauli principle, when $Q=Q'$, the cc or bb diquark can only be in the spin-1 state while for bc diquark its spin may be either 0 or 1. Therefore, from cc or bb diquark we can construct a heavy baryon either with spin-$\frac{3}{2}$ ($\Sigma_{3/2}^{QQ}$) or with spin-$\frac{1}{2}$ ($\Sigma_{1/2}^{QQ}$). On the other hand, from bc diquark we may have a spin-$\frac{1}{2}$ baryon which is constructed from S_{bc} ($\gamma_{1/2}^{bc}$) or from V_{bc} ($\Sigma_{1/2}^{bc}$), and also a spin-$\frac{3}{2}$ baryon from V_{bc} ($\Sigma_{3/2}^{bc}$). In the above discussion, we have followed the notation of White and Savage [4]. In the present paper we will study the weak transition hadronic matrix elements between these heavy baryons and then give the predictions for their semileptonic decay widths.

Due to the analogue of a heavy meson and a heavy baryon with a heavy diquark, superflavor symmetry is applicable to associate the transition matrix elements between two doubly heavy baryons with those between two heavy mesons. Superflavor symmetry was first established by Georgi and Wise [8] for interchanging a heavy quark and a heavy scalar object. Later Carone [9] generalized it to the symmetry of interchanging a heavy quark and a heavy axial vector object. Since the heavy diquark is not really pointlike with respect to weak transitions, we need to derive the explicit expressions of the effective vertices $DD'W^{\pm}$ $(D=S$ or $V)$ by taking into account the inner structure of heavy diquarks. Obviously these vertices are associated with the bound state properties of heavy diquarks D and D'. Therefore, some nonperturbative model has to be adopted. As in our previous work [2] we apply the Bethe-Salpeter (BS) equation model to analyze the weak transition matrix elements between two heavy diquarks, unlike the approach of White and Savage and Sanchis-Lozano, while we will also apply the superflavor symmetry with which different matrix elements between heavy mesons or doubly heavy baryons are expressed in terms of the same Isgur-Wise function.

This paper is organized as follows. In Sec. II we give a detailed derivation of the transition form factors between two heavy diquarks with a virtual W boson being emitted. Consequently we obtain the effective currents for heavy diquark weak transitions. Then in Sec. III we apply superflavor symmetry to give the formulation for the weak matrix elements between heavy baryons and the semileptonic decay widths. The numerical results will be presented in Sec. IV. Finally the last section is devoted to summary and discussions.

II. DERIVATION OF THE HEAVY DIQUARK TRANSITION FORM FACTORS

Since in the heavy quark limit the two heavy quarks in a baryon constitute a heavy diquark, in the decay process this diquark may be treated as a color-triplet quasi-particle. It is noted that for applying HQET to associate a baryon case to a meson case, the diquark should be of pointlike structure. The reason for this is that all nonperturbative effects are attributed into a well-defined Isgur-Wise function, therefore the necessary condition is that the diquark is seen by the light quark as a pointlike color source. However, it by no means demands that in the weak transition the weak current sees a pointlike structureless object. On the contrary, there is a complicated structure due to the bound state effects of the diquark. The structure effects of the heavy diquark should be described by the bound state equation. Hence we have to adopt a plausible method to deal with the diquark structure effects which are governed by the nonperturbative QCD. In this section we solve the BS equation [10] to obtain the bound state wave function of the heavy diquark and then give the transition form factors between such heavy diquarks in the weak decay processes.

Since the bound state BS wave functions and the transition form factors between two heavy diquarks are obtained in the same framework, in our formulation one does not need to invoke some phenomenological inputs except the commonly accepted parameters such as α_s and κ in the Cornell potential model.

The BS equation for a heavy diquark can be written in the following form:

$$\chi_P(p)=S_1(\lambda_1 P+p)\int G(P,p,q)\chi_P(q)\frac{d^4q}{(2\pi)^4}S_2(\lambda_2 P-p), \tag{1}$$

where $S_j(j=1,2)$ are the propagators of heavy quark 1 and quark 2 in the diquark, respectively, and $G(P,p,q)$ is the BS equation kernel defined as the sum of all the irreducible diagrams concerning the interaction between the two quarks of the diquark, $\lambda_1=m_1/(m_1+m_2)$, $\lambda_2=m_2/(m_1+m_2)$, and m_1, m_2 are the quark masses. P is the total momentum of the diquark and can be expressed as $P=Mv$ where M is the mass of the diquark and v is its four-velocity.

Using the relation

$$S_j(p)=i\left[\frac{\Lambda_j^+(p_t)}{p_l-W_j+i\epsilon}+\frac{\Lambda_j^-(p_t)}{p_l+W_j-i\epsilon}\right]\not{v} \quad (j=1,2), \tag{2}$$

where $p_l=p\cdot v$, $p_t=p-p_l v$, $W_j=\sqrt{|p_t|^2+m_j^2}$, and $\Lambda_j^{\pm}(p_t)=[W_j\pm\not{v}(-\not{p}_t+m_j)]/2W_j$, Eq. (1) can be expressed explicitly as

$$\begin{aligned}\chi_P^{++}(p)=&\frac{-\Lambda_1^+(p_t)\not{v}}{\lambda_1 M+p_l-W_1+i\epsilon}\\&\times\int G(P,p,q)[\chi^{++}(q)+\chi^{--}(q)]\\&\times\frac{d^4q}{(2\pi)^4}\frac{\not{v}\Lambda_2^+(-p_t)}{p_l+W_2-\lambda_2 M-i\epsilon},\end{aligned} \tag{3}$$

$$\begin{aligned}\chi_P^{--}(p)=&\frac{-\Lambda_1^-(p_t)\not{v}}{\lambda_1 M+p_l+W_1-i\epsilon}\\&\times\int G(P,p,q)[\chi^{++}(q)+\chi^{--}(q)]\frac{d^4q}{(2\pi)^4}\\&\times\frac{\not{v}\Lambda_2^-(-p_t)}{p_l-W_2-\lambda_2 M+i\epsilon},\end{aligned} \tag{4}$$

where $\chi_P^{\pm\pm}(p)=\Lambda_1^{\pm}(p_t)\chi_P(p)\Lambda_2^{\pm}(-p_t)$.

In the heavy quark limit it can be shown that $\Lambda_1^+(p_t)\approx(1+\not{v})/2$, $\Lambda_2^+(-p_t)\approx(1+\not{v})/2$, and χ_P^{--} is small and negligible. In the following we will only consider the large component χ_P^{++}.

So for a scalar or an axial vector diquark, the BS wave function can be written in the form

$$\chi_P^S(p)=\frac{1+\not{v}}{2}\sqrt{2M}\,\phi(p), \quad \chi_P^V(p)=\frac{1+\not{v}}{2}\sqrt{2M}\,\gamma_5\not{\eta}\,\phi(p).$$

The superscript S and V denote the scalar and axial vector diquark, respectively, and η is the polarization vector of the axial vector diquark.

Now we assume the kernel G to have the form [2,11]

$$-iG=1\otimes 1V_1+\not{v}\otimes\not{v}V_2 \tag{5}$$

and

$$V_1(p,q)=\frac{8\pi\beta_1\kappa}{[(p_t-q_t)^2+\mu^2]^2}-(2\pi)^3\delta^3(p_t-q_t)\times\int\frac{8\pi\beta_1\kappa}{(k^2+\mu^2)^2}\frac{d^3k}{(2\pi)^3}$$

and

$$V_2(p,q)=-\frac{16\pi\beta_2\alpha_s}{3(|p_t-q_t|^2+\mu^2)},$$

where V_1 and V_2 are the parts of the kernel associated with the scalar confinement and one-gluon-exchange diagram, respectively [11]. The parameters β_1 and β_2 are different for various color states. For mesons, $\beta_1=1$, $\beta_2=1$, while for color-triplet diquarks, β_2 is directly associated to the color factor caused by the single-gluon exchange, so it should be 0.5. In contrast, β_1 which is related to the linear confinement cannot be determined yet and we just take it as a free parameter within a range of 0–1 in numerical evaluations. As a matter of fact, later we pick up two typical values 0.5 and 1 for β_1 for demonstrating the influence of the color factor. In fact, the final results are not sensitive to its value, so our predictions made with the value within a certain range can give rise to a reasonable order of magnitude, although not to a precise number. The parameters κ and α_s are well determined by fitting experimental data of heavy meson spectra. From the heavy meson experimental data, $\kappa=0.18$, $\alpha_s=0.4$ [12]. After substituting the form of the kernel Eq. (5) into Eq. (1) we have the following form of the BS equation:

$$\tilde{\phi}(p_t)=\frac{-1}{M-W_1-W_2}\int(V_1-V_2)\tilde{\phi}(q_t)\frac{d^3q_t}{(2\pi)^3}, \tag{6}$$

where $\tilde{\phi}(p_t)=\int\phi(p)(dp_l/2\pi)$. The above equation can be solved numerically and by applying the relation between $\phi(p_l,p_t)$ and $\tilde{\phi}(p_t)$ we finally obtain the numerical solution of the BS equation. This solution will be applied to calculate the weak transition matrix elements of heavy diquarks.

The weak transition form factors of heavy diquarks are closely associated with their inner structure. Namely, to evaluate a transition $b\to c$ which are constituent quarks of the initial and final diquarks, some Q^2-dependent form factors naturally emerge.

The form factors are process dependent. For the semileptonic decay $D_{bQ'}(v)\to D'_{cQ'}(v')+l+\bar{\nu}$ with the light quark being a spectator, the fundamental vertex J_μ corresponds to a radiation of a virtual W boson, so that

$$J^\mu=\frac{g_w}{2\sqrt{2}}V^*_{cb}\bar{c}\gamma^\mu(1-\gamma_5)b, \tag{7}$$

where g_w is the weak coupling constant.

The effective currents L_μ in the expressions of the heavy diquark transition matrix elements are calculated by means of the BS equation for heavy diquarks.

For scalar or axial vector diquark transitions, one has the following four types:

$$\langle S'(v')|J_\mu|S(v)\rangle=2\sqrt{MM'}[f_1(v\cdot v')v'_\mu+f_2(v\cdot v')v_\mu], \tag{8}$$

$$\begin{aligned}\langle V'(v',\eta')|J^\mu|V(v,\eta)\rangle=2\sqrt{MM'}[&f_3(v\cdot v')\,\eta'\cdot\eta v'_\mu+f_4(v\cdot v')\,\eta'\cdot\eta v_\mu+f_5(v\cdot v')\,\eta\cdot v'\,\eta'\cdot v v'_\mu+f_6(v\cdot v')\,\eta\cdot v'\,\eta'\cdot v v_\mu\\&+f_7(v\cdot v')\,\eta\cdot v'\,\eta'_\mu+f_8(v\cdot v')\,\eta'\cdot v\,\eta_\mu+f_9(v\cdot v')i\epsilon_{\mu\nu\rho\sigma}\eta'^\nu\eta^\rho v'^\sigma+f_{10}(v\cdot v')i\epsilon_{\mu\nu\rho\sigma}\eta'^\nu\eta^\rho v^\sigma],\end{aligned} \tag{9}$$

$$\langle V'(\eta',v')|J_\mu|S(v)\rangle=2\sqrt{MM'}[f_{11}\eta'_\mu+f_{12}\eta'\cdot v v'_\mu+f_{13}\eta'\cdot v v_\mu+f_{14}i\epsilon_{\mu\nu\rho\sigma}\eta'^\nu v'^\rho v^\sigma], \tag{10}$$

$$\langle S'(v')|J_\mu|V(\eta,v)\rangle=2\sqrt{MM'}[f_{15}\eta_\mu+f_{16}\eta\cdot v' v'_\mu+f_{17}\eta\cdot v' v_\mu+f_{18}i\epsilon_{\mu\nu\rho\sigma}\eta^\nu v^\rho v'^\sigma]. \tag{11}$$

On the other hand, the effective matrix elements of heavy diquark transitions can be expressed by the BS wave functions as the following:

$$\langle D'(v')|J_\mu|D(v)\rangle=\int\mathrm{Tr}\left[\overline{\chi}^{D'}_{P'}(p')\Gamma\chi^D_P(p)S^{-1}(p_2)(2\pi)^4\delta^4(p_2-p'_2)\frac{d^4p}{(2\pi)^4}\frac{d^4p'}{(2\pi)^4}\right], \tag{12}$$

where $S(p_2)$ is the propagator of quark 2, Γ is the vertex of J_μ, $\overline{\chi_P^S}=\sqrt{2M}\phi^*(p)[(1+\not v)/2]$ and $\overline{\chi_P^V}=-\sqrt{2M}\phi^*(p)\not\eta\gamma_5[(1+\not v)/2]$ for scalar and axial vector diquarks, respectively. p_i' and $p_i(i=1,2)$ are

$$\begin{aligned} p_1'&=\lambda_1' M' v' + p', \qquad & p_2'&=\lambda_2' M' v' - p',\\ p_1&=\lambda_1 M v + p, \qquad & p_2&=\lambda_2 M v - p, \end{aligned} \tag{13}$$

where M,M' are the masses of initial and final diquarks.

Therefore, the form factors $f_i\,(i=1,\ldots,18)$ in Eqs. (8)–(11) can be expressed as an integral of the two diquarks' wave functions along with specific coefficients. The numerical values for the coefficients f_i $(1,\ldots,18)$ in Eqs. (8)–(11) are derived by the combination of Eq. (12) and Eqs. (8)–(11). The effective currents L_μ inducing weak transitions between heavy diquarks can be expressed as

$$L_\mu=\sum_{i=1}^{18} f_i J_\lambda^{(i)}, \tag{14}$$

where the explicit expressions for $J_\lambda^{(i)}$ are given in the following:

$$\begin{aligned}
&J_\lambda^{(1)}=S'^\dagger(v')v_\lambda' S(v), \quad J_\lambda^{(2)}=S'^\dagger(v')v_\lambda S(v),\\
&J_\lambda^{(3)}=V'^{\mu\dagger}(v')v_\lambda' V_\mu(v), \quad J_\lambda^{(4)}=V'^{\mu\dagger}(v')v_\lambda V_\mu(v),\\
&J_\lambda^{(5)}=[V'^{\mu\dagger}(v')v_\mu]v_\lambda'[v_\nu' V^\nu(v)], \quad J_\lambda^{(6)}=[V'^{\mu\dagger}(v')v_\mu]v_\lambda[v_\nu' V^\nu(v)],\\
&J_\lambda^{(7)}=V_\lambda'^\dagger(v')[v_\nu' V^\nu(v)], \quad J_\lambda^{(8)}=[V'^{\mu\dagger}(v')v_\mu]V_\lambda(v),\\
&J_\lambda^{(9)}=i\epsilon_{\lambda\delta\rho\sigma}V'^{\delta\dagger}(v')v'^\sigma V^\rho(v), \quad J_\lambda^{(10)}=i\epsilon_{\lambda\delta\rho\sigma}V'^{\delta\dagger}(v')v^\sigma V^\rho(v),\\
&J_\lambda^{(11)}=V_\lambda'^\dagger(v')S(v), \quad J_\lambda^{(12)}=[V'^{\mu\dagger}(v')v_\mu]v_\lambda' S(v),\\
&J_\lambda^{(13)}=[V'^{\mu\dagger}(v')v_\mu]v_\lambda S(v), \quad J_\lambda^{(14)}=i\epsilon_{\lambda\delta\rho\sigma}V'^{\delta\dagger}(v')v'^\rho v^\sigma S(v),\\
&J_\lambda^{(15)}=S'^\dagger(v')V_\lambda(v), \quad J_\lambda^{(16)}=S'^\dagger(v')v_\lambda' v'^\mu V_\mu(v),\\
&J_\lambda^{(17)}=S'^\dagger(v')v_\lambda v'^\mu V_\mu(v), \quad J_\lambda^{(18)}=i\epsilon_{\lambda\delta\rho\sigma}S'^\dagger(v')v^\rho v'^\sigma V^\delta(v),
\end{aligned} \tag{15}$$

where $S(v)$, $S'(v')$, $V^\mu(v)$, and $V'^\mu(v')$ stand for the initial scalar, final scalar, initial axial vector, and final axial vector diquark fields in the baryons, respectively. For instance, the terms in L_μ which contribute to $\Sigma_{1/2}^{bb}\to\Sigma_{1/2}^{bc}$ are $\Sigma_{i=3}^{10} f_i J_\lambda^{(i)}$. In next section we will apply the effective currents to calculate the hadronic transition matrix elements with the aid of superflavor symmetry. From the heavy meson experimental data, $\kappa=0.18\ \mathrm{GeV}^2$, $\alpha_s=0.4$, $m_b=4.8$ GeV, $m_c=1.45$ GeV. From the BS equation, the numerical results of M (the heavy diquark mass) corresponding to the various quarks $m_i(i=1,2)$ and β_1 are listed in Table I.

III. FORMULATION FOR THE TRANSITION MATRIX ELEMENTS AND DECAY WIDTHS

(i) The transition amplitudes. For semileptonic decays, the process can be described as a transition of a heavy baryon into another heavy baryon radiating a virtual W boson which turns into a lepton pair $l\bar{\nu}$ $(\bar{l}\nu)$. In the process, the factorization is perfect, so that the total transition amplitude can be written as

$$T\approx\langle \mathrm{J}'|J_\alpha|\mathrm{J}\rangle l^\alpha\left(\frac{i}{M_W^2}\right), \tag{16}$$

where J, $\mathrm{J}'=\gamma_{1/2}$, $\Sigma_{1/2}$, and $\Sigma_{3/2}$ and the contribution from the leptonic current is

TABLE I. Values of heavy diquark masses.

β_1	0.5	1	0.5	1	0.5	1
m_1(GeV)	4.8	4.8	4.8	4.8	1.45	1.45
m_2(GeV)	4.8	4.8	1.45	1.45	1.45	1.45
M(GeV)	9.68	9.74	6.46	6.58	3.27	3.33

$$l^{\alpha}\equiv\frac{g_w}{2\sqrt{2}}\overline{u}_l(p_l)\gamma^{\alpha}(1-\gamma_5)v_{(\nu)}(p_\nu)\quad \text{for}\quad b\to cl\overline{\nu}_l.$$

At the concerned decay energy scale, the W-boson propagator $i/(q^2-M_W^2)(-g_{\mu\nu}+q_\mu q_\nu/M_W^2)$ can be approximated as $-ig_{\mu\nu}/M_W^2$. In our calculations, we neglect the lepton masses, and because τ-lepton production is hard to measure, we only discuss the cases of $e^-\overline{\nu}_e$ and $\mu^-\overline{\nu}_\mu$ radiation.

Thus we need to derive the forms of the hadronic matrix elements $\langle \mathrm{J}'|J_\mu|\mathrm{J}\rangle$. The effective currents L_μ are derived in Sec. II, so we obtain the hadronic transition matrix elements $\langle \mathrm{J}'|J_\mu|\mathrm{J}\rangle$ by calculating $\langle \mathrm{J}'|L_\mu|\mathrm{J}\rangle$ in the diquark-quark picture. The scalar or axial vector diquark is treated as a pointlike object of color $\overline{3}$ and spin 0 or 1 with definite form factors which are reflected in the coefficients f_i's of the effective currents L_μ, and combines with the light quark to constitute a baryon of spin 1/2 or 3/2. Thus we can use superflavor symmetry to evaluate the transition matrix elements at the hadron level. In this scenario, there is only one uncertain function which is determined by nonperturbative QCD, i.e., the Isgur-Wise function $\xi(v\cdot v')$, unlike the case for transitions between light baryons where there are many form factors. Therefore, here we may expect to reduce the uncertainty and improve the prediction power, which is the advantage of employing superflavor symmetry.

In the scenario of the superflavor symmetry [8], the wave function for a baryon consisting of a scalar diquark would be

$$\tilde{\Psi}_\gamma=\begin{pmatrix} u_\gamma^T C/\sqrt{2M_S} \\ 0\end{pmatrix}, \tag{17}$$

where M_S is the mass of the scalar diquark and C is the charge-conjugation operator satisfying $C^{-1}\gamma_\mu^T C=-\gamma_\mu$. For the spin-1 diquark case [9],

$$\tilde{\Psi}_{\Sigma_{1/2}}(v)=\frac{1}{\sqrt{6M_V}}\begin{pmatrix} 0 \\ u_\Sigma^T C\sigma^{\mu\beta}v_\beta\gamma_5\end{pmatrix} \tag{18}$$

and

$$\tilde{\Psi}_{\Sigma_{3/2}}=\frac{1}{\sqrt{2M_V}}\begin{pmatrix} 0 \\ \psi_\Sigma^{\mu T}C\end{pmatrix}, \tag{19}$$

where M_V is the mass of the spin-1 diquark.

Thus the hadronic transition matrix element can be obtained as

$$\begin{aligned}T_\mu&\equiv\langle \mathrm{J}'(v')|L_\mu|\mathrm{J}(v)\rangle\\ &=-\xi(v\cdot v')\mathrm{Tr}\left[\bar{\tilde{\Psi}}'_{\mathrm{J}'}(v')\sum_i f_i\Gamma_i\tilde{\Psi}_{\mathrm{J}}(v)\right],\end{aligned} \tag{20}$$

where Γ_i's are the corresponding vertices in the effective current L_μ. In Ref. [8], the authors presented some transition matrix elements with certain effective currents. Here our effective currents correspond to the weak interaction.[1] There are a total of twelve different weak transition matrix elements $T_{i\mu}(i=1,\ldots,12)$ (see Table II in Sec. IV for the twelve transitions). The explicit expressions for various hadronic transition matrix elements $T_{i\mu}(i=1,\ldots,12)$ can be obtained straightforwardly after tedious derivations. Take the first transition in Table II, $\Sigma_{1/2}^{bb}\to\Sigma_{1/2}^{bc}$, as an example. $T_{1\mu}$ has the following expression:

$$\begin{aligned}T_{1\mu}=\langle\Sigma_{1/2}^{bc}|L_\mu|\Sigma_{1/2}^{bb}\rangle=\frac{1}{3}\xi(v\cdot v')(\{(-f_3v'_\mu-f_4v_\mu)(2+v\cdot v')+(f_5v'_\mu+f_6v_\mu)[1-(v\cdot v')^2]\\ -(f_7v'_\mu+f_8v_\mu)(1+v\cdot v')\}\overline{u'_\Sigma}u_\Sigma+(f_7+f_8)(1+v\cdot v')\overline{u'_\Sigma}\gamma_\mu u_\Sigma+i(f_9v^\rho v'^\sigma-f_{10}v'^\rho v^\sigma)\epsilon_{\mu\delta\rho\sigma}\overline{u'_\Sigma}\gamma^\delta u_\Sigma\\ -i(f_9v'^\sigma+f_{10}v^\sigma)\epsilon_{\mu\delta\rho\sigma}\overline{u'_\Sigma}\gamma^\delta\gamma^\rho u_\Sigma).\end{aligned} \tag{21}$$

(ii) The amplitude square. To calculate the cross section, we need to take square of the amplitudes $\Sigma_{\mathrm{spins}}|T_{i\lambda}l^\lambda|^2$ $(i=1,\ldots,12)$. In the derivations we take the lepton mass to be zero. Again taking the semileptonic decay $\Sigma_{1/2}^{bb}\to\Sigma_{1/2}^{bc}l\overline{\nu}$ as an example one has

$$\begin{aligned}\sum_{\mathrm{spins}}|T_{1\lambda}l^\lambda|^2=\frac{8}{9}|\xi(v\cdot v')|^2\mathrm{Tr}[A_\lambda A_{\lambda'}\overline{u'}u\overline{u}u'+B^2\overline{u'}\gamma_\lambda u\overline{u}\gamma_{\lambda'}u'+(A_\lambda B\overline{u'}u\overline{u}\gamma_{\lambda'}u'+\mathrm{C.T.})\\ +C^{\rho\sigma}C^{\rho'\sigma'}\epsilon_{\lambda\delta\rho\sigma}\epsilon_{\lambda'\delta'\rho'\sigma'}\overline{u'}\gamma^\delta u\overline{u}\gamma^{\delta'}u'-(C^{\rho\sigma}D^{\sigma'}\epsilon_{\lambda\delta\rho\sigma}\epsilon_{\lambda'\delta'\rho'\sigma'}\overline{u'}\gamma^\delta u\overline{u}\gamma^{\rho'}\gamma^{\delta'}u'+\mathrm{C.T.})\\ +D^\sigma D^{\sigma'}\epsilon_{\lambda\delta\rho\sigma}\epsilon_{\lambda'\delta'\rho'\sigma'}\overline{u'}\gamma^\delta\gamma^\rho u\overline{u}\gamma^{\rho'}\gamma^{\delta'}u'][p_3^\lambda p_4^{\lambda'}+p_3^{\lambda'}p_4^\lambda-(p_3\cdot p_4)g^{\lambda\lambda'}],\end{aligned} \tag{22}$$

where C.T. means the conjugate term and

[1]For evaluating a radiative decay, one can have similar effective currents with only small changes from that given for weak interactions.

$$
\begin{aligned}
A_\lambda &= (-f_3 v'_\lambda - f_4 v_\lambda)(2+v\cdot v') + (f_5 v'_\lambda + f_6 v_\lambda)[1-(v\cdot v')^2] - (f_7 v'_\lambda + f_8 v_\lambda)(1+v\cdot v'),\\
B &= (f_7+f_8)(1+v\cdot v'),\\
C^{\rho\sigma} &= f_9 v^\rho v'^\sigma - f_{10} v'^\rho v^\sigma,\\
D^\sigma &= f_9 v'^\sigma + f_{10} v^\sigma.
\end{aligned}
\tag{23}
$$

(iii) The integration over the final state phase space. To obtain the partial decay width, one needs to integrate out the phase space of the three-body final state. In the limit of $m_l \sim 0$, the integration becomes much simplified.

It is easy to notice that the amplitude square can be written in the general form

$$
\begin{aligned}
\sum_{\text{spins}} |T_{i\lambda} l^\lambda|^2 \frac{1}{M_W^4} \equiv{}& F_1(p_1\cdot p_2)(p_3\cdot p_4)\\
&+F_2(p_1\cdot p_3)(p_2\cdot p_4) + F_3(p_3\cdot p_1)\\
&\times(p_4\cdot p_1) + F_4(p_3\cdot p_2)(p_4\cdot p_2)\\
&+F_5(p_2\cdot p_3)(p_1\cdot p_4)\\
&+F_6(p_3\cdot p_4),
\end{aligned}
\tag{24}
$$

where p_3 and p_4 are the momenta of emitted lepton and neutrino, $p_1 = mv$ is the decaying baryon momentum which can also be $m(1,\vec{0})$, $p_2 = m'v'$ is the momentum of the decay product which should be integrated over, m and m' are the masses of initial and final baryons, respectively, and $T_{i\lambda}$ $(i=1,\ldots,18)$ have been derived with the help of superflavor symmetry.

Thus after a simple manipulation, the final form of the decay width can be written (in the following expression the spin factor $2s+1=2$ for the spin-1/2 baryon decay while for spin-3/2 baryon decay $2s+1=4$) as

$$
\begin{aligned}
\Gamma ={}& \frac{m'}{16(2s+1)\pi^3}\int_0^{(m-m')^2} ds_2\\
&\times\Bigg\{ \frac{F_1}{16m^2} s_2[m^2+m'^2-s_2] + \frac{F_2+F_5}{96m^2}[(m^2-m'^2+s_2)\\
&\times(m^2-m'^2-s_2)+s_2(m^2+m'^2-s_2)]\\
&+\frac{F_3}{48}\left[\frac{(m^2+s_2-m'^2)^2}{2m^2}+s_2\right]\\
&+\frac{F_4}{48m^2}[(m^2-m'^2-s_2)^2+m'^2 s_2] + \frac{F_6}{8m^2}s_2\Bigg\}\\
&\times\lambda^{1/2}(m^2,m'^2,s_2),
\end{aligned}
\tag{25}
$$

where

$$
\lambda(a,b,c) \equiv a^2+b^2+c^2-2ab-2bc-2ca,
$$

and F_1 through F_6 are given in the expressions of $\Sigma|T_{i\alpha} l^\alpha|^2/M_W^4$ by rearranging the corresponding terms so that they are expressed in terms of the form factors $f_i(i=1,\ldots,18)$ which have been calculated in the BS approach. The concrete forms are obtained by running the REDUCE computer programs and it is a very lengthy and tedious procedure. The relations between $F_i(i=1,\ldots,6)$ and $f_i(i=1,\ldots,18)$ are very complicated and we will not list them here.

IV. NUMERICAL RESULTS

In the relations between $F_i(i=1,\ldots,6)$ and $f_i(i=1,\ldots,18)$ there is an uncertain function, the Isgur-Wise function. Its behavior is controlled by nonperturbative QCD effects which have to be dealt with in some phenomenological model. Because in the heavy quark limit the spin of the heavy quark has no effects on the dynamics inside the hadron one expects that the Isgur-Wise function is totally determined by the light degrees of freedom. Therefore, the Isgur-Wise function between the transition of heavy baryons consisting of two heavy quarks should be the same as that of the corresponding heavy mesons. Actually this is the plausibility of applying superflavor symmetry. Hence we can simply use the form of the Isgur-Wise function for $B \to D$ in our numerical calculations for the decay width of heavy baryons which contain two heavy quarks. There are some model calculations for the Isgur-Wise function for $B\to D$ [13–15]. In the following numerical calculations we will use the following simple form given in Ref. [13]:

$$
\xi(\omega) = \frac{1}{1-\omega^2/\omega_0^2},
\tag{26}
$$

where the constant is taken to be $\omega_0 = 1.24$. It is noted that different forms of the Isgur-Wise function will give somewhat different predictions. However, our numerical computations show that with various Isgur-Wise function forms the order is not changed. The numerical results for the semileptonic decay widths for different processes are listed in Table II. From the numerical results in Table II we can see that the decay widths are around the order 10^{-13}–10^{-14} GeV. It can also be seen that the results are insensitive to the parameter β_1.

V. SUMMARY AND DISCUSSIONS

In the present work we discussed the weak transitions between heavy baryons which consist of two heavy quarks in the heavy quark limit. We restricted our analysis to the ground states of such baryons. When the mass of the heavy

TABLE II. Semileptonic decay widths of doubly heavy baryons (GeV).

β_1	1	0.5
$\Sigma_{1/2}^{bb} \to \Sigma_{1/2}^{bc}$	2.85×10^{-13}	2.78×10^{-13}
$\Sigma_{1/2}^{bb} \to \gamma_{1/2}^{bc}$	4.28×10^{-14}	4.81×10^{-14}
$\Sigma_{1/2}^{bb} \to \Sigma_{3/2}^{bc}$	2.72×10^{-13}	2.69×10^{-13}
$\Sigma_{3/2}^{bb} \to \Sigma_{3/2}^{bc}$	1.29×10^{-13}	1.41×10^{-13}
$\Sigma_{3/2}^{bb} \to \Sigma_{1/2}^{bc}$	5.20×10^{-13}	5.15×10^{-13}
$\Sigma_{3/2}^{bb} \to \gamma_{1/2}^{bc}$	8.57×10^{-14}	9.61×10^{-14}
$\Sigma_{3/2}^{bc} \to \Sigma_{3/2}^{cc}$	1.72×10^{-13}	1.68×10^{-13}
$\Sigma_{3/2}^{bc} \to \Sigma_{1/2}^{cc}$	2.75×10^{-13}	2.81×10^{-13}
$\Sigma_{1/2}^{bc} \to \Sigma_{3/2}^{cc}$	1.41×10^{-13}	1.43×10^{-13}
$\Sigma_{1/2}^{bc} \to \Sigma_{1/2}^{cc}$	8.93×10^{-14}	9.32×10^{-14}
$\gamma_{1/2}^{bc} \to \Sigma_{3/2}^{cc}$	2.88×10^{-13}	2.91×10^{-13}
$\gamma_{1/2}^{bc} \to \Sigma_{1/2}^{cc}$	7.76×10^{-14}	7.82×10^{-14}

quark is much larger than the QCD scale $\Lambda_{\rm QCD}$ the two heavy quarks bind into a heavy diquark and the three-body system is simplified into a two-body system of a heavy diquark and a light quark. The heavy diquark is a pointlike [for instance, in the Coulomb potential model, the radius of the heavy diquark is of the order $1/\alpha_s(m_Q)m_Q$, much smaller than $1/\Lambda_{\rm QCD}$ in the heavy quark limit] spin-0 or spin-1 object to the light quark which is blind to the spin and flavor of the heavy diquark. Therefore, we can apply superflavor symmetry which relates the heavy quark, heavy scalar diquark, and heavy axial vector diquark. Thus the matrix elements between doubly heavy baryons and those between heavy mesons are related to each other and can be described by the same Isgur-Wise function. To deal with the weak transitions between heavy diquarks we work in the BS equation approach in the heavy quark limit. We obtain numerical solutions of the BS equation by assuming a kernel which contains linear scalar confinement and one-gluon-exchange vector terms. The numerical solutions are used to obtain the effective currents between two heavy diquarks. These effective currents are expressed in terms of the coefficients $f_i(i=1,\ldots,18)$ which can be solved numerically from the BS equation for heavy diquarks. Then the weak transition matrix elements between heavy baryons are expressed in terms of the Isgur-Wise function and $f_i(i=1,\ldots,18)$. Consequently we give the predictions for the semileptonic decay widths for all the possible twelve decay channels between two heavy baryons. The decay widths are around the order 10^{-13}–10^{-14} GeV. These predictions will be tested in future experiments.

There are some uncertainties in our work which arise from both the approximations and the nonperturbative QCD models we have used. First, we have been working in the heavy quark limit in which physics is greatly simplified and apart from the form factors between heavy diquark weak transitions we have only one unknown function, the Isgur-Wise function from the application of superflavor symmetry. In reality, however, the masses of heavy quarks are not infinitely large. Therefore, if one wishes to make a precise comparison of the theoretically calculated numbers with data, the $1/m_Q$ and, especially $1/m_c$ corrections must be taken into account. These corrections could be of the order $\Lambda_{\rm QCD}/m_c\sim0.15$ if we ignore the effects of the form factors appearing at the $1/m_Q$ order in the expansions in HQET for heavy baryons.

Secondly, when we consider the heavy diquark, we also work in the heavy quark limit, i.e., we neglect $|p_t|/m_Q$ terms. However, the heavy diquark is different than a single heavy hadron. The residual momentum in a heavy diquark is not simply $O(\Lambda_{\rm QCD})$ [6]. In the Coulomb potential model, $|p_t|/m_Q\sim\alpha_s(m_Q)$ [16] while in other potential models $|p_t|/m_Q$ may have a different behavior with respect to m_Q. In our BS equation approach, we find that when $|p_t|$ becomes large the BS wave function is strongly suppressed and the average value of $|p_t|/m_Q$ is smaller than 0.25. Therefore, the ignorance of $|p_t|/m_Q$ terms in our approach may cause about 25% corrections.

In addition, in our numerical calculations we do not distinguish the masses of heavy spin-0 and spin-1 diquarks. Since we are working in the preasymptotic mass region, this difference should be small (as discussed in Ref. [6], this difference is of the order $1/m_Q$ in the Coulomb plus linear potential model).

In addition to the above approximations when we calculate the effective currents between two heavy diquarks we work in the BS equation approach in which the most uncertain point is the kernel which depends on nonperturbative QCD effects. Motivated by the potential model we use a simple form which has linear scalar confinement and one-gluon-exchange terms. Furthermore, we use an instantaneous approximation in the kernel. In the confinement part the parameter β_1 is not fixed and we pick up two typical values of 0.5 and 1. Fortunately, the decay widths are insensitive to this parameter. Although we cannot estimate the corrections to these assumptions about nonperturbative QCD effects such as other phenomenological nonperturbative QCD models do, we believe that it should give reasonable results based on the success of potential models and the successful applications of the BS equation approach in other cases.

The Isgur-Wise function is another uncertain point since it is also controlled by nonperturbative QCD dynamics between the heavy diquark and light quark and thus its evaluation is model dependent. To get the numerical results we use the simple form obtained in Ref. [13]. Different forms for the Isgur-Wise function may result in different decay widths. However, since the Isgur-Wise function changes slowly in the decay region we are concerned with and different model predictions for it do not differ much [14,15] the uncertainty from the Isgur-Wise function should not change the order of our predictions.

From the above discussion, we expect that we have given reasonable predictions for semileptonic decays of doubly heavy baryons. On the other hand, since the predictions are model dependent, future experiments will test the reliability of our model.

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