

CLASSICAL SOLUTION TO THE BUCKLING
OF A THIN CYLINDRICAL SHELL

by

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SIGNED STATEMENT

This thesis contains no material which has been accepted for the award of any other degree or diploma in any University. To the best of my knowledge and belief the thesis contains no material previously published or written by any other person, except where due reference is made in the text of the thesis.

(N.S. KOSHITSKY)

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