



# Inference for General Random Effects Models

Colleen Hunt

School of Applied Mathematics

Master of Science by research, The University of Adelaide, October 13, 2003

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
1.1	Mixed effects models . . . . .	1
1.2	Part One . . . . .	3
1.3	Part Two . . . . .	4
<b>2</b>	<b>A Test for Homogeneity in one-way variance component models</b>	<b>6</b>
2.1	Introduction . . . . .	7
2.2	A Score test . . . . .	9
2.3	An approximate likelihood function . . . . .	14
2.4	Approximate score tests . . . . .	17
2.4.1	All parameters assumed known . . . . .	17
2.4.2	Using the estimation of parameters $(\sigma^2, \sigma_\eta^2, \mu)$ . . . . .	19
2.5	Simulations . . . . .	21
2.6	Power of the test . . . . .	24
2.7	Examples . . . . .	39
2.7.1	CD4 Count Data . . . . .	39
2.7.2	Blood Pressure Data . . . . .	49
<b>3</b>	<b>Laplace approximations</b>	<b>55</b>
3.1	Example: 2-way table . . . . .	56

3.1.1	Other representations . . . . .	59
<b>4</b>	<b>Approximate likelihood functions</b>	<b>61</b>
4.1	An approximation to the likelihood function for general random effects models . . . . .	61
4.1.1	Models with nested random effects . . . . .	65
4.2	Approximations for models with two random effects: explicit expressions . . . . .	67
4.2.1	Two-way crossed with no interaction . . . . .	67
4.2.2	Two way crossed with interaction . . . . .	74
4.2.3	Models with two nested random effects . . . . .	75
4.2.4	Expansion around the true values . . . . .	78
4.3	Using approximations to the log-likelihood in estimation of effects in mixed models . . . . .	79
<b>5</b>	<b>Simulations of the likelihood function</b>	<b>81</b>
5.1	Two-way linear crossed model . . . . .	82
5.2	A Poisson model . . . . .	83
5.3	A nested model . . . . .	84
5.4	The Salamander mating data . . . . .	86
<b>A</b>	<b>Splus functions</b>	<b>93</b>
A.1	Score test statistics . . . . .	93
A.2	Approximate likelihood functions . . . . .	98

# List of Tables

2.1	Means and standard deviations of the score statistic under the null hypothesis $H_0 : \kappa = 0$ for simulated data with $\kappa = 0$ , with variance components ranging from 0.001 to 5. . . . .	26
2.2	Number of rejected points from 100 repetitions of the score test statistic for simulated data with $\kappa = 0$ . . . . .	27
2.3	Means and standard deviations of the score statistic under the null hypothesis $H_0 : \kappa = 0$ for simulated data with variance components $\sigma_\eta^2 = \sigma^2 = 1$ and various $\kappa$ . . . . .	28
2.4	Means and standard deviations of the score statistic under the null hypothesis $H_0 : \kappa = 0$ for simulated data with $\kappa = 0.5$ , with variance components ranging from 0.01 to 0.5 . . . . .	30
2.5	Number of rejected points from 100 repetitions of the score test statistic for simulated data with $\kappa = 0.5$ . . . . .	31
2.6	Number of rejected points from 100 repetitions of the score test statistic for simulated data with $\kappa = 1$ . . . . .	32
2.7	Results of regression on raw data, log and square root transformations	47
2.8	Score test <b>mevar2a</b> results for the CD4 cell count data . . . . .	48
2.9	Score test results for the CD4 cell count data versions <b>mevar2</b> and <b>mevar1</b> . . . . .	48

2.10	Score test results for the CD4 cell count data versions <b>mevar2</b> and <b>mevar1</b> using variance component estimates from the two-stage regression. . . . .	50
2.11	<b>mevar2a</b> Score test statistic for the blood pressure data . . . . .	50
5.1	Estimates from a linear model with two random effects . . . . .	83
5.2	Estimates of the variance components for simulated Poisson data with various starting values . . . . .	84
5.3	Variance component estimates for the exponential nested model for both a second-order Laplace approximation and a fourth-order approximation. . . . .	85
5.4	Estimates of parameters for the Salamander mating experiment . . . . .	87
5.5	Variance component estimates for the logit model for both a second-order Laplace approximation and a fourth-order approximation. . . . .	88

# List of Figures

2.1	Distributions of the score test statistic for simulated data with $\kappa = 0$ , $\sigma_\eta^2 = 0.5, \sigma^2 = 0.1$ , 100 repetitions . . . . .	29
2.2	Distributions of the score test statistic for simulated data with $\kappa = 1$ , $\sigma_\eta^2 = 0.5, \sigma^2 = 0.1$ , 100 repetitions . . . . .	33
2.3	Comparisons of <b>mevar1</b> and <b>mevar2</b> distributions data simulated with $\kappa = 0, \sigma_\eta^2 = 2, \sigma^2 = 1$ . . . . .	34
2.4	Distributions of the score test statistic <b>mevar2</b> $\sigma_\eta^2 = 0.5, \sigma^2 = 0.1, \mu = 10$ and $\kappa$ ranging from 0 to 0.5. . . . .	35
2.5	Distributions of the score test statistic <b>mevar2</b> $\sigma_\eta^2 = \sigma^2 = 1$ and $\mu = 10$ . . . . .	36
2.6	Distributions of the score test statistic with simulated data with $\kappa = 0.2, 0.6, \sigma_\eta^2 = \sigma^2 = 1$ . . . . .	37
2.7	Power of the score test . . . . .	38
2.8	Kernel density estimates of whole CD4 data . . . . .	41
2.9	Kernel density estimates of CD4 data showing AIDS and NoAIDS separately . . . . .	42
2.10	Individual trajectories of CD4 cell counts over time t in months . . . . .	43
2.11	Means versus standard deviations for transformation of the AIDS data . . . . .	52
2.12	Means versus standard deviations for transformation of the NoAIDS data . . . . .	53

2.13	Individual trajectories of the CD4 data for AIDS and NoAIDS cohorts	54
5.1	Profile plots for Poisson simulated data with $\sigma_A^2 = 0.1$ and $\sigma_B^2 = 0.1$	89
5.2	Approximate log likelihood for Poisson simulation model with $\sigma_A^2 = 0.1$ and $\sigma_B^2 = 0.1$ .	90
5.3	Profiles and 3D perspective plots showing the approximation to the log likelihood for data simulated from the nested model with exponential mean and $\sigma_A^2 = 1, \sigma_B^2 = 1$ .	91
5.4	Contours of approximate log-likelihood functions for Salamander data.	92

## **Abstract**

This thesis describes methods associated with general random effects models. It is divided into two parts. Part one describes a technique for investigating mean-variance relationships in random effects models. A simple one-way random effects model is proposed as a basis for deriving a score test for homogeneity of variance in one-way random effects models. An arbitrary mean-variance relationship is captured by a single parameter which allows for the possibility of detecting situations where the variance changes systematically with the mean. Part two derives an approximation to the likelihood function using a Laplace expansion to the fourth order. This approximation may be applied to general models with multiple crossed and/or nested effects. The score test of homogeneity and the approximate likelihood function are examined using simulations and simple data analyses.



## Declaration

*This work contains no material which has been accepted for the award of any other degree or diploma in any University or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.*

*I give consent to this copy of my thesis, when deposited in the University Library, being available for loan and photocopying.*

## Acknowledgements

- I thank my supervisor Associate Professor Patty Solomon for suggesting the topic of research and for providing guidance and help in the development of the research.
- I am indebted to Dr Bill Venables for his significant contribution towards this thesis and for always having time for me.
- I wish to thank Dr Brian Cullis, the staff of BiometricsSA and especially Assoc. Prof. Ari Verbyla for being there and helping me through some difficult times.
- This work was carried out in the Department of Statistics. The Department was merged to form what is now the School of Applied Mathematics. The Department of Statistics provided a positive environment for statistical research.
- This research was carried out under a Postgraduate Research Scholarship funded through a grant from the Australian Research Council.

*No animals were harmed throughout the making of this thesis.*