GENERATING INTENSIONAL LOGICS

The application of paraconsistent logics to investigate certain areas of the boundaries of mathematics

By

Peter Stanley Lavers, B.Sc.(Ma.) (Hons.), Dip.Ed.

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Philosophy Department
University of Adelaide

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ERRATA

p.8. Delete lines 18-20 (after definition 2.1.7).

Substitute 'a' for 'the' in line 14 (twice).

p.14. Delete line 6 ((b) of theorem 2.1.13).

p.16. Line 2, delete the phrase 'and because W is closed under these too'.

p.29. Line 19, replace 'substitution instances' by 'theorems'.

p.38. Line 2, replace 'an admissible' by 'a derived'.

p.147. Line 11, insert the following after 'intensional bases,': 'the substitution of equivalents rule (x) (AvB) ≡ A v (x) B (where x is not free in A) to the substitution of equivalents rules permitted in obtaining generalised disjunctive and conjunctive normal forms (definition 2.1.9)'.

p.148. In the statement of the rules (&$\vdash$) and ($\vdash$v) replace 'A&B' by

'{A&B}' and 'AvB' by '{AvB}', respectively.
{A&B} {BvA}

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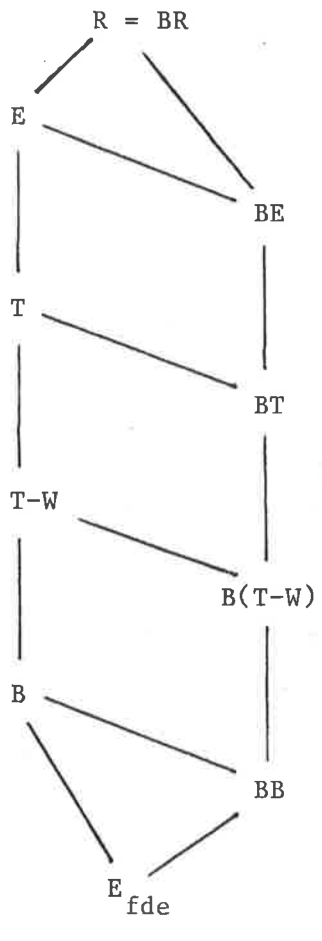
SUMMARY

I use a natural generalisation of Anderson and Belnap's definition of variable sharing, based on similar intuitions, in order to characterise intensional logics which contain theorems of arbitrary degree. This characterisation limits the interplay between the intensional → and extensional connectives &, v to this generalised notion of variable sharing. Thus the extensional/intensional interplay is completely perspicuous, as well as remaining faithful to the intuitions underlying the Tautological Entailments - contrary to the case of the standard relevant logics and their axiomatic formulations.

This new method for characterising logics delivers a very broad class of logics, because the method can be applied to any intensional base (i.e. implication-negation logic) and extensional base (usually just adjunction). This class includes some of the standard relevant logics (notably R) but not others (such as those weaker than E).

I prove that many of the logics so characterised have a corresponding axiomatic formulation, which just involves adding purely extensional and/or purely intensional axioms and rules to those of E_{fde}. Thus the extensional/intensional interplay is grounded, in the axiomatic formulation, in the axioms and rules of E_{fde}. This adds weight to the claim that such interplay is an intuitive generalisation of variable sharing. For those logics which can be formulated as the axioms and rules of B plus purely intensional and/or purely extensional axioms and rules (which includes the standard relevant logics), the process amounts to weakening the axioms $(A\rightarrow B) \& (A\rightarrow C) \underset{\cdot}{\rightarrow} A \rightarrow (B \& C)$

and $(A \to C) \& (B \to C) \to (A \vee B) \to C$ to the rules $\vdash A \to B$ and $\vdash A \to C \Rightarrow \vdash A \to (B \& C)$, and $\vdash A \to C$ and $\vdash B \to C \Rightarrow \vdash (A \vee B) \to C$. Thus obtaining a logic with the same purely intensional and purely extensional basis, but which mediates the extensional/intensional interplay via our generalised notion of variable sharing. Letting BL represent the logic obtained when the process is applied to L, the following relationships hold:



I set out the algebraic semantics for BB and its extensions and use these to prove that BB and many of its extensions are decidable, and that BB and a couple of its extensions are both prime and negation-consistent (thus satisfying γ).

I then set out a relational semantics for BB and its extensions, which leans heavily on those for B and on the recipe of ESL. A notion of theoryhood is introduced which is a useful analytical tool for understanding the relational semantics.

I also point out some errors in ALG II and give alternative proofs which provide a partial resurrection of the required results.

Finally, I conclude with some ruminations and open problems.

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university.

To the best of my knowledge and belief, the thesis contains no material previously published or written by another person except when due reference is made in the text of the thesis.

I consent to the thesis being made available for photocopy and loan.

Peter Lavers

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