Quark propagator in a covariant gauge

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Using mean–field improved gauge field configurations, we compare the results obtained for the quark propagator from Wilson fermions and Overlap fermions on a $12^3 \times 24$ lattice at a spacing of $a = 0.125(2)$ fm.

1. INTRODUCTION

In QCD, asymptotic freedom states that at short distances the effective coupling constant vanishes. In that regime the interaction between quarks and gluons is largely reduced. The dynamics of these particles can be studied through their propagators in momentum space. The quark propagator is really a description of how the quark propagates in the QCD vacuum and is one of the most fundamental building blocks of QCD. By studying the momentum dependent quark mass function in the infrared region, the scalar part of the propagator, we can gain some insights into the mechanism of chiral symmetry breaking. Chiral symmetry is dynamically broken in the QCD vacuum. This gives rise to mass generation in the infrared.

In the deep infrared region, artifacts associated with the finite size of the lattice spacing become small. This is the most interesting region as non-perturbative physics lies here. However, the ultraviolet behaviour of the propagator at large momenta will in general strongly deviate from the correct continuum behaviour. This behaviour will be action dependent.

In this brief report we compare the quark propagators of Wilson and overlap fermions. Additional details may be found in Ref. [1].

1.1. The Quark Propagator

In the continuum, it is possible to study dynamical chiral symmetry breaking (DCSB) using the renormalized quark Dyson–Schwinger equation (quark–DSE) in Euclidean space. In this equation, information about the renormalized dressed gluon propagator and dressed quark–gluon vertex is embodied. The quark propagator has the general form

$$S(p) = \frac{1}{i\gamma \cdot pA(p^2;\zeta^2) + B(p^2;\zeta^2)} = \frac{Z(p^2;\zeta^2)}{i\gamma \cdot p + M(p^2;\zeta^2)}, \qquad (1)$$

where $M = B/A$ and $Z = 1/A$. Here, the parameter ζ represents the renormalization point. The functions $A(p^2;\zeta^2)$ and $B(p^2;\zeta^2)$ carry all the effects of vector and scalar quark dressing induced by the quark interactions with the gluon field.

Through this simple form we can extract the quark mass function $M(p)$ and renormalization function, $Z(p)$.

2. QUARK PROPAGATOR ON THE LATTICE

On the lattice we expect the bare quark propagators, in momentum space, to have a similar form as in the continuum [2–4]. Hence, the dimensionless inverse lattice bare quark propagator takes the general form of

$$S^{-1}(p) \equiv i\left(\sum_\mu C_\mu(p)\gamma_\mu\right) + B(p) \equiv \frac{i\left(\sum_\mu \mathcal{C}_\mu(p)\gamma_\mu\right) + \mathcal{B}(p)}{\mathcal{C}^2(p) + \mathcal{B}^2(p)}, \qquad (2)$$

with $\mathcal{C}^2(p) = \sum_\mu(\mathcal{C}_\mu(p))^2$. The discrete momentum values for a t–antiperiodic lattice of size $N_i^3 \times N_t$, with $n_i = 1,..,N_i$ and $n_t = 1,..,N_t$, are given by $p_i = \frac{2\pi}{N_i a}\left(n_i - \frac{N_i}{2}\right)$ and $p_t = \frac{2\pi}{N_t a}\left(n_t - \frac{1}{2} - \frac{N_t}{2}\right)$. The quark propagator is

$$S(p) \equiv -i\left(\sum_\mu \mathcal{C}_\mu(p)\gamma_\mu\right) + \mathcal{B}(p). \quad (3)$$

Taking the trace we obtain

$$\begin{aligned} \mathcal{C}_\mu(p) &= \frac{i}{4N_c}\mathrm{Tr}[\gamma_\mu S(p)], \text{ and} \\ \mathcal{B}(p) &= \frac{1}{4N_c}\mathrm{Tr}[S(p)], \end{aligned} \quad (4)$$

which we use to construct the $C_\mu(p)$ and $B(p)$

$$C_\mu(p) = \frac{\mathcal{C}_\mu(p)}{\mathcal{D}(p)}, \quad \text{and} \quad B(p) = \frac{\mathcal{B}(p)}{\mathcal{D}(p)}, \quad (5)$$

where $\mathcal{D}(p) = \mathcal{C}^2(p) + \mathcal{B}^2(p)$. At tree–level the lattice quark propagator takes the same form as in Eq. (1) and we know that the free propagator $(S^{(0)}(p))^{-1} \equiv (Z^{(0)}(p))^{-1}\left[i\not{k} + M^{(0)}(p)\right]$, where $k_\mu \longrightarrow p_\mu$ as $p_\mu \longrightarrow 0$. It is then possible to extract the momentum directly from the lattice by calculating

$$q_\mu \equiv C_\mu^{(0)}(p) = \frac{\mathcal{C}_\mu^{(0)}(p)}{(\mathcal{C}^{(0)}(p))^2 + (\mathcal{B}^{(0)}(p))^2}. \quad (6)$$

Results are displayed in Fig. 1.

2.1. Mass and Renormalization Functions

The dimensionless inverse lattice bare quark propagator takes the form

$$\begin{aligned} S^{-1}(p) &\equiv ia\not{q}A(p) + B(p) \\ &= [Z^{\mathrm{L}}(p)]^{-1}\left(ia\not{q} + M^{\mathrm{L}}(p)\right), \end{aligned} \quad (7)$$

where $M^{\mathrm{L}}(p) = B(p)/A(p)$ is the lattice quark mass function and $Z^{\mathrm{L}}(p) = 1/A(p)$ the lattice renormalization function. The functions $A(p)$ and $B(p)$ may be written as:

$$A(p) = \frac{\mathcal{A}(p)}{\mathcal{D}(p)}, \quad \text{and} \quad B(p) = \frac{\mathcal{B}(p)}{\mathcal{D}(p)}, \quad (8)$$

where $\mathcal{D}(p) = \mathcal{A}^2(p)q^2 + \mathcal{B}^2(p)$. Hence an equivalent definition for the quark propagator is $S(p) \equiv -ia\not{q}\mathcal{A}(p) + \mathcal{B}(p)$. Extracting the functions $\mathcal{A}(p)$ and $\mathcal{B}(p)$ is done via:

$$\begin{aligned} \mathcal{A}(p) &= \frac{i}{4N_c a q^2}\mathrm{Tr}[\not{q}S(p)], \quad \text{and} \\ \mathcal{B}(p) &= \frac{1}{4N_c}\mathrm{Tr}[S(p)]. \end{aligned} \quad (9)$$

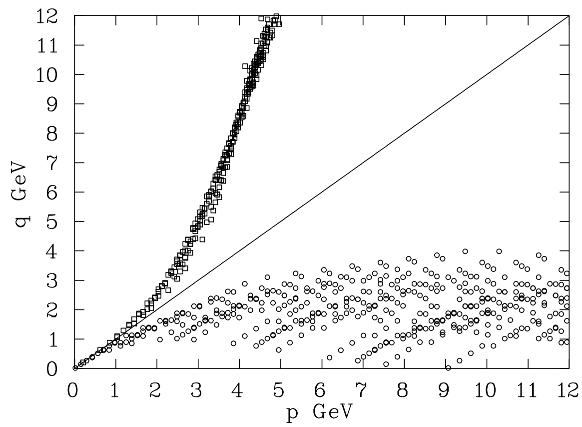

Figure 1. The lattice momentum q versus the discrete momentum p, both in GeV. Overlap (□) and Wilson fermion (○) momenta are indicated.

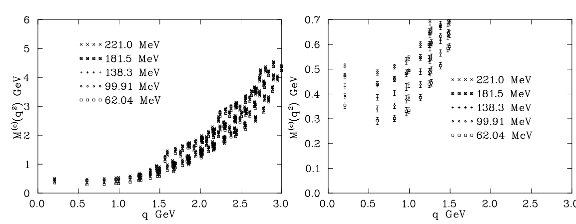

Figure 2. The uncorrected mass function $M(q^2)$ for Wilson fermions.

In Fig. 2 we show the uncorrected mass function for Wilson fermions, where we can see a large divergence in the ultraviolet region.

2.2. Tree–Level Correction

The tree-level multiplicative correction is given by

$$A^{(c)}(p) = \frac{A(p)}{A^{(0)}(p)}1\ , \ B^{(c)}(p) = \frac{B(p)}{B^{(0)}(p)}m_q, \quad (10)$$

where $A^{(0)}(p)$ and $B^{(0)}(p)$ are obtained from $S^{(0)}(p)$. Then tree-level corrected mass and renormalization functions are constructed using Eq.(10)

$$M^{(c)}(q^2) = \left(\frac{B^{(c)}(p)}{A^{(c)}(p)}\right) \; , Z^{(c)}(q^2) = \frac{1}{A^{(c)}(p)}. \quad (11)$$

3. FERMIONS ON THE LATTICE

3.1. Wilson Fermions

The Wilson fermion [5] action on the lattice is defined as,

$$S_W[U, \overline{\psi}, \psi] = \sum_{xy} \overline{\psi}(x) D_W(x, y) \psi(y),$$

with $D_W(x, y)$ the usual Wilson fermion operator. The lattice bare mass is related to the hopping parameter via $\kappa = 1/(2m_q a + 8r)$ and $m_q \equiv (1/2a)(1/\kappa - 1/\kappa_c)$.

3.2. Overlap Fermions

The overlap fermion [6] realizes exact chiral symmetry on the lattice

$$D(\mu) \;\; = \;\; \frac{1}{2}\left[1 + \mu + (1-\mu)\frac{D_W}{\sqrt{D_W^\dagger D_W}}\right]$$

with $0 \leq \mu \leq 1$ describing fermions with a positive mass from 0 to ∞. The hopping parameter is given by $\kappa = 1/(-2m + 8r)$ and $m \equiv (1/2\kappa_c - 1/2\kappa)$. To describe a single massless Dirac fermion for $D(0)$ we must have $0 \leq m \leq 2$ at tree–level. The overlap propagator is given by

$$\tilde{D}^{-1}(\mu) = (1-\mu)^{-1}[D^{-1}(\mu) - 1], \quad (12)$$

and is related to the continuum propagator by $D_c^{-1}(m_q) = \mathcal{Z}_\psi \tilde{D}^{-1}(\mu)$, and similarly for the quark mass $m_q = \mathcal{Z}_m^{-1}\mu$.

At tree-level [7] it is found that $\mathcal{Z}_\psi^{(0)} = \mathcal{Z}_m^{-1} = 2m$, and the free propagator becomes

$$\left[D_c^{(0)}(m_q)\right]^{-1} = \left[\mathcal{Z}_\psi^{(0)} \tilde{D}^{(0)}(\mu)\right]^{-1},$$

and when the interactions are turned on we have

$$[D_c(m_q)]^{-1} = \left[\mathcal{Z}_\psi \tilde{D}(\mu)\right]^{-1},$$

where $\mathcal{Z}_\psi = \mathcal{Z}_m^{-1}$.

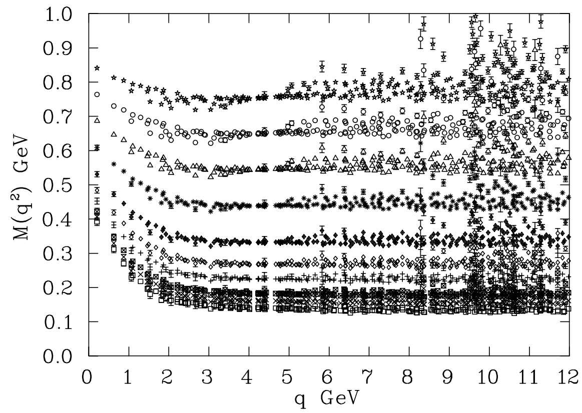

Figure 3. The mass function $M(q^2)$ for overlap fermions (full data) for the ten different masses.

4. NUMERICAL RESULTS

Using fifty improved gauge field configurations, on a $12^3 \times 24$ lattice with $a = 0.125(1)$ fm, the overlap quark propagator is calculated for ten different masses, $m_q = \mathcal{Z}_\psi \mu = \{126, 147, 168, 210, 252, 315, 420, 524, 629, 734\}$ MeV. Results are illustrated in Figs. 3 and 4.

For Wilson fermions, using a hundred configurations, five masses are considered, namely $m_q = \{221, 181.5, 138.3, 99.91, 62.04\}$ MeV. Results are displayed in Figs. 5 and 6.

A linear extrapolation is used to compare the two actions. In Fig. 7 we show the linearly extrapolated quark mass function for both actions plotted versus the discrete momentum p. When plotting M and Z versus the lattice momentum, q, the points are pushed away from the origin in the case of overlap fermions as opposed to Wilson fermions where the points are pulled towards the origin. The overlap action (plotted as $\Box$) produces in the deep infrared $M(0) = 297(11)$ MeV. Comparing with the Wilson fermion action ($\times$) we see a clear superiority of the overlap action over the Wilson action. Wilson fermions show a significant dip between 0.8 GeV and 2.0 GeV.

The linearly extrapolated renormalization function is shown in Fig. 8. In the deep infrared

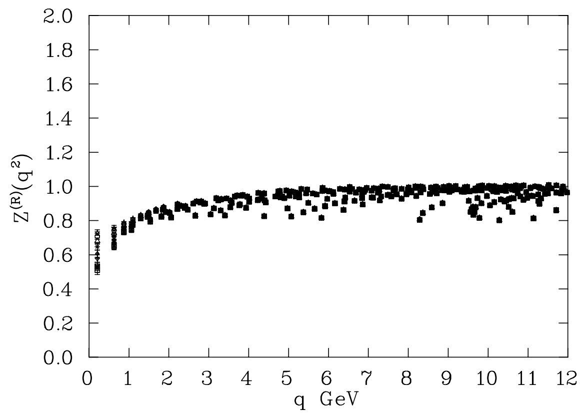

Figure 4. The renormalization function for overlap fermions (full data) for the ten different masses. The function is renormalized at $\zeta = 8.2$ GeV.

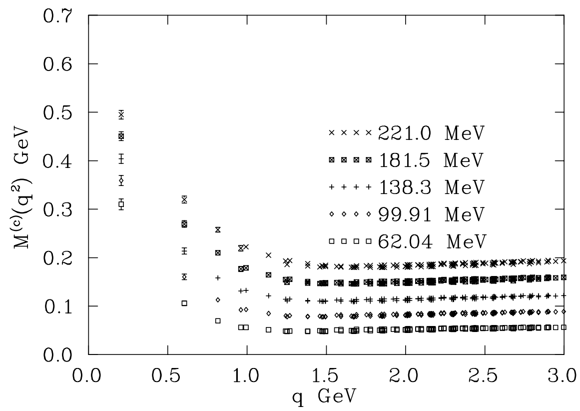

Figure 5. The tree–level corrected mass function $M^{(c)}(q^2)$ for Wilson fermions (half cut data).

for the overlap action we have $Z(0) = 0.48(2)$ with the renormalization point located at $\zeta = 3.9$ GeV.

5. SUMMARY

Tree-level correction represents a crucial step and a powerful tool for correcting the Wilson action results. The division method produces a smooth quark mass function throughout the momentum spectrum. We are able to gain some insights into the analytic structure of the overlap quark propagator: $B^{(0)}(p) = \mathcal{Z}_\psi^{(0)}\mu$ and $A^{(0)}(p) = 1$. The overlap produces very good results for $M(p)$ and $Z(p)$. No tree-level correction is required for overlap fermions.

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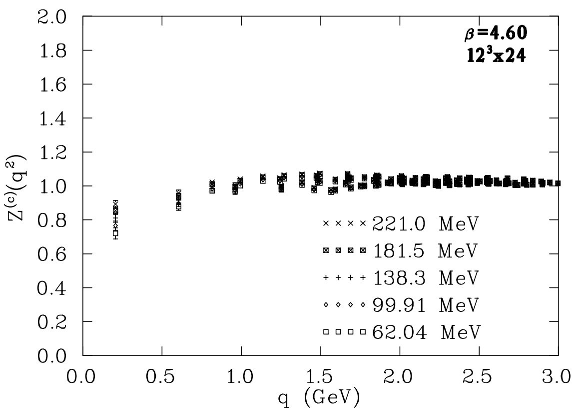


Figure 6. The tree–level corrected renormalization function for Wilson fermions (half cut data).

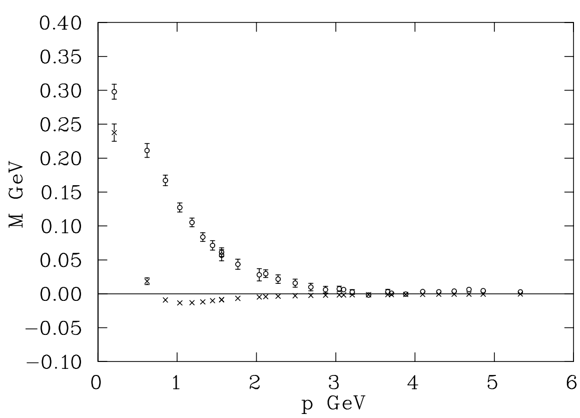


Figure 7. Comparison of the linearly extrapolated mass function M for both Wilson fermions ($\times$) and overlap fermions ($\Box$), (cylinder cut data).

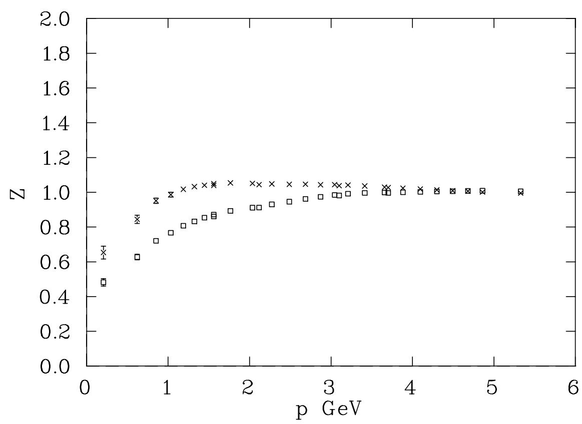
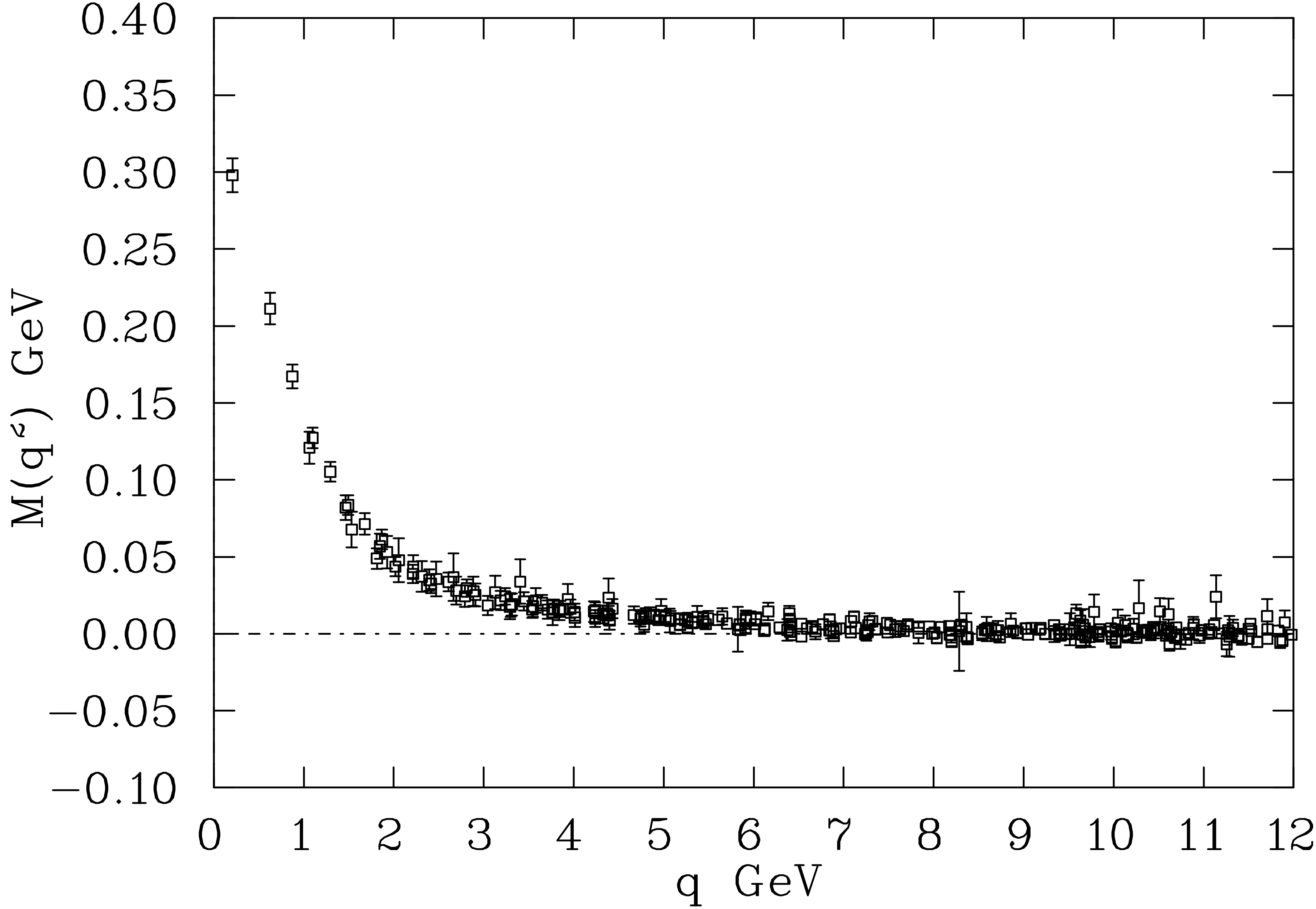


Figure 8. Comparison of the linearly extrapolated renormalization function for Wilson fermions ($\times$) and overlap fermions ($\Box$), (cylinder cut data).



M(q²) GeV
0.40
0.35
0.30
0.25
0.20
0.15
0.10
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0.00
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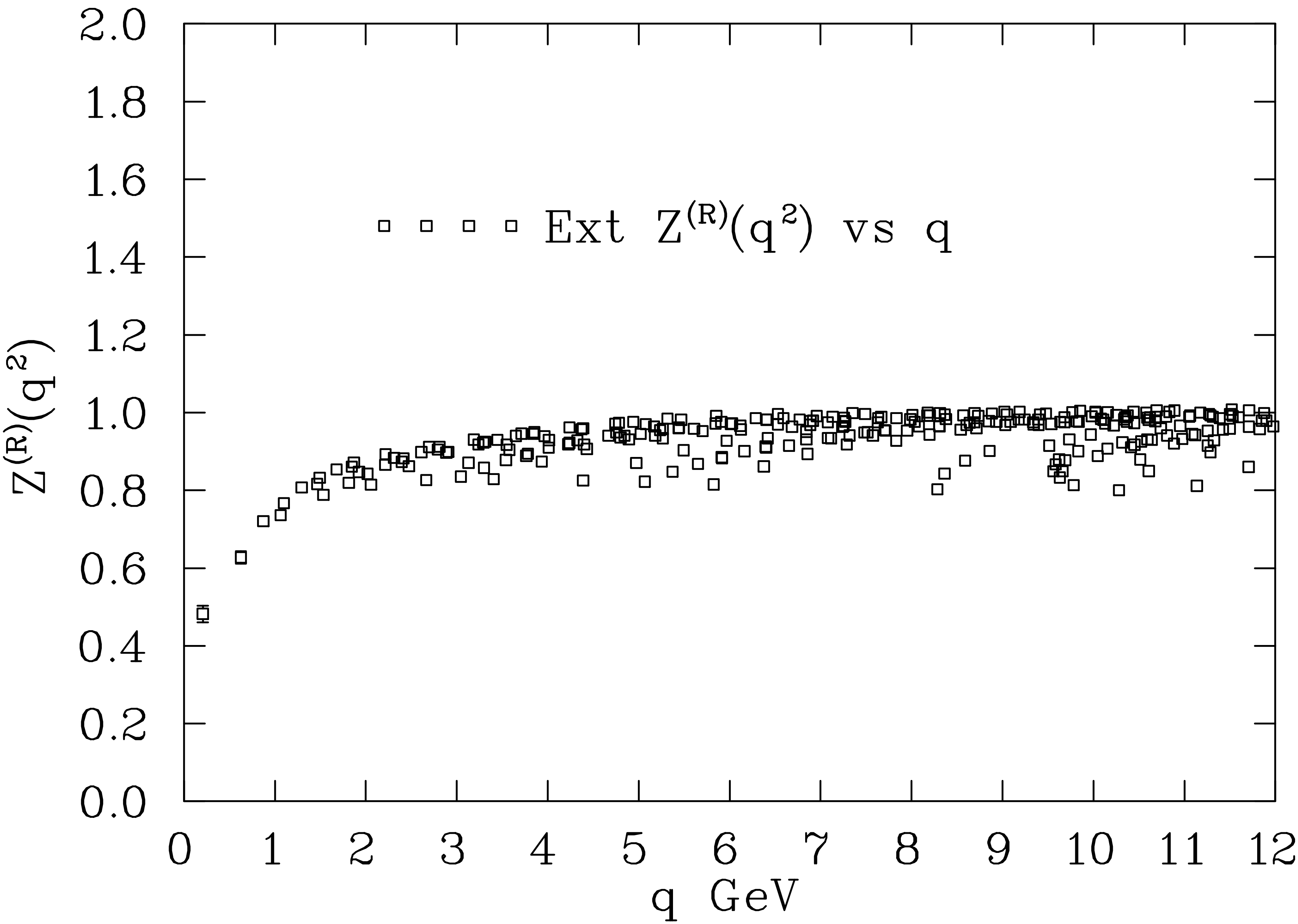
−0.10
0
1
2
3
4
5
6
7
8
9
10
11
12
q GeV



Ext $Z^{(R)}(q^2)$ vs q
$Z^{(R)}(q^2)$
q GeV
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
0
1
2
3
4
5
6
7
8
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11
12

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1.1. Spacing

We normally recommend the use of 1.0 (single) line spacing. However, when typing complicated mathematical text LaTeX automatically increases the space between text lines in order to prevent sub- and superscript fonts overlapping one another and making your printed matter illegible.

1.2. Fonts

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Tables should be made with LaTeX; illustrations should be originals or sharp prints. They should be arranged throughout the text and preferably be included *on the same page as they are first discussed*. They should have a self-contained caption and be positioned in flush-left alignment with the text margin within the column. If they do not fit into one column they may be placed across both columns (using `\begin{table*}` or `\begin{figure*}` so that they appear at the top of a page).

3.1. Tables

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Horizontal lines should be placed above and below table headings, above the subheadings and at the end of the table above any notes. Vertical lines should be avoided.

If a table is too long to fit onto one page, the table number and headings should be repeated above the continuation of the table. For this you have to reset the table counter with `\addtocounter{table}{-1}`. Alternatively, the table can be turned by 90° ('landscape mode') and spread over two consecutive pages (first an even-numbered, then an odd-numbered one) created by means of `\begin{table}[h]` without a caption. To do this, you prepare the table as a separate LaTeX document and attach the tables to the empty pages with a few spots of suitable glue.

3.2. Useful table packages

Modern LaTeX comes with several packages for tables that provide additional functionality. Below we mention a few. See the documentation of the individual packages for more details. The

Table 2: The next-to-leading order (NLO) results *without* the pion field.

Λ (MeV)	140	150	175	200	225	250	Exp.	v_{18} [?]
r_d (fm)	1.973	1.972	1.974	1.978	1.983	1.987	1.966(7)	1.967
Q_d (fm^2)	0.259	0.268	0.287	0.302	0.312	0.319	0.286	0.270
P_D (%)	2.32	2.83	4.34	6.14	8.09	9.90	–	5.76
μ_d	0.867	0.864	0.855	0.845	0.834	0.823	0.8574	0.847
$\mathcal{M}_{\mathrm{M1}}$ (fm)	3.995	3.989	3.973	3.955	3.936	3.918	–	3.979
$\mathcal{M}_{\mathrm{GT}}$ (fm)	4.887	4.881	4.864	4.846	4.827	4.810	–	4.859
$\delta_{\mathrm{1B}}^{\mathrm{VP}}$ (%)	−0.45	−0.45	−0.45	−0.45	−0.45	−0.44	–	−0.45
$\delta_{\mathrm{1B}}^{\mathrm{C2:C}}$ (%)	0.03	0.03	0.03	0.03	0.03	0.03	–	0.03
$\delta_{\mathrm{1B}}^{\mathrm{C2:N}}$ (%)	−0.19	−0.19	−0.18	−0.15	−0.12	−0.10	–	−0.21

The experimental values are given in ref. [4].

Table 1
The next-to-leading order (NLO) results *without* the pion field.

Λ (MeV)	140	150	175	200
r_d (fm)	1.973	1.972	1.974	1.978
Q_d (fm^2)	0.259	0.268	0.287	0.302
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μ_d	0.867	0.864	0.855	0.845
$\mathcal{M}_{\rm M1}$ (fm)	3.995	3.989	3.973	3.955
$\mathcal{M}_{\rm GT}$ (fm)	4.887	4.881	4.864	4.846
$\delta_{\rm 1B}^{\rm VP}$ (%)	−0.45	−0.45	−0.45	−0.45
$\delta_{\rm 1B}^{\rm C2:C}$ (%)	0.03	0.03	0.03	0.03
$\delta_{\rm 1B}^{\rm C2:N}$ (%)	−0.19	−0.19	−0.18	−0.15

The experimental values are given in ref. [4].

packages can be found in LaTeX's `tools` directory.

`array` Various extensions to LaTeX's `array` and `tabular` environments.

`longtable` Automatically break tables over several pages. Put the table in the `longtable` environment instead of the `table` environment.

`dcolumn` Define your own type of column. Among others, this is one way to obtain alignment on the decimal point.

`tabularx` Smart column width calculation within a specified table width.

`rotating` Print a page with a wide table or figure in landscape orientation using the `sidewaystable` or `sidewaysfigure` environments, and many other rotating tricks. Use the package with the `figuresright` option to make all tables and figures rotate in clockwise. Use the starred form of the `sideways` environments to obtain full-width tables or figures in a two-column article.

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Line drawings may consist of laser-printed graphics or professionally drawn figures attached to the manuscript page. All figures should be clearly displayed by leaving at least one line of spacing above and below them. When placing a figure at the top of a page, the top of the figure should align with the bottom of the first text line of the other column.

Do not use too light or too dark shading in your figures; too dark a shading may become too dense while a very light shading made of tiny points may fade away during reproduction.

All notations and lettering should be no less than 2 mm high. The use of heavy black, bold lettering should be avoided as this will look unpleasantly dark when printed.

3.4. PostScript figures

Instead of providing separate drawings or prints of the figures you may also use PostScript files which are included into your LaTeX file and printed together with the text. Use one of the packages from LaTeX's `graphics` directory: `graphics`, `graphicx` or `epsfig`, with the `\usepackage` command, and then use the appropriate commands (`\includegraphics` or `\epsfig`) to include your PostScript file.

The simplest command is:
`\includegraphics{file}`, which inserts the PostScript file `file` at its own size. The starred version of this command:
`\includegraphics*{file}`, does the same, but clips the figure to its bounding box.

With the `graphicx` package one may specify a series of options as a key–value list, e.g.:

```
\includegraphics[width=15pc]{file}
\includegraphics[height=5pc]{file}
\includegraphics[scale=0.6]{file}
\includegraphics[angle=90,width=20pc]{file}
```

See the file `grfguide`, section “Including Graphics Files”, of the `graphics` distribution for all options and a detailed description.

The `epsfig` package mimicks the commands familiar from the package with the same name in LaTeX2.09. A PostScript file `file` is included with the command `\psfig{file=file}`.

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Photographs must always be sharp originals (*not screened versions*) and rich in contrast. They will undergo the same reduction as the text and should be pasted on your page in the same way as line drawings.

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Sharp originals (*not transparencies or slides*) should be submitted close to the size expected in publication. Charges for the processing and printing of colour will be passed on to the author(s) of the paper. As costs involved are per page, care should be taken in the selection of size and shape so that two or more illustrations may be fitted together on one page. Please contact the Author Support Department at Elsevier (E-mail: `authorsupport@elsevier.nl`) for a price quotation and layout instructions before producing your paper in its final form.

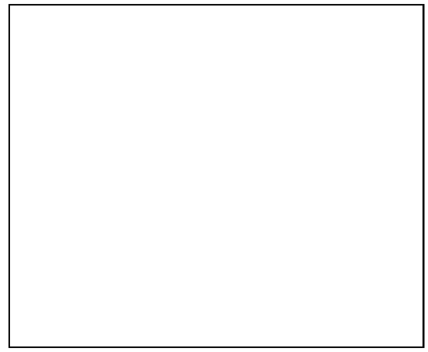

Figure 2. Remember to keep details clear and large enough.

4. EQUATIONS

Equations should be flush-left with the text margin; LaTeX ensures that the equation is preceded and followed by one line of white space. LaTeX provides the document class option `fleqn` to get the flush-left effect.

$$H_{\alpha\beta}(\omega) = E_\alpha^{(0)}(\omega)\delta_{\alpha\beta} + \langle\alpha|W_\pi|\beta\rangle \tag{1}$$

You need not put in equation numbers, since this is taken care of automatically. The equation numbers are always consecutive and are printed in parentheses flush with the right-hand margin of the text and level with the last line of the equation. For multi-line equations, use the `eqnarray` environment.

For complex mathematics, use the $\mathcal{AMS}$math package. This package sets the math indentation to a positive value. To keep the equations flush left, either load the `espcrc` package *after* the $\mathcal{AMS}$math package or set the command

`\mathindent=0pt` in the preamble of your article.

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Above we have listed some references according to the sequential numeric system [1–4].