FREQUENCY RESPONSE MATCHING METHODS

FOR

THE DESIGN OF DIGITAL CONTROL SYSTEMS

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This thesis embodies the results of supervised project work making up two thirds of the work for the degree.

EXAMINER'S REPORT OF M.ENG.SC. THESIS;

J. SHI, "Frequency response matching methods for the design of digital control systems"

ERRATA OR OMISSIONS

p.3-8: the relation for the zero Z_1 in terms of other parameters is omitted. It is

$$Z_1 = \frac{I + \tan(\alpha+90^o)(I^2+R^2-R)}{I-\tan(\alpha+90^o)(1-R)}$$

p.3-10: Fourth line, (correction)

$$b = (C-A)(D+1)-BC$$

p3-12: The term

$$\frac{C^2 + (D-1)^2}{4D}$$

under the square root sign in both expressions should be positive, not negative.

pA-10: Eqn D-16.
The numerator term should be A-B, not A+B.

pA-15 to pA-28: The omission of values for the normalised parameters t_s/T (C/L step time-response) and $\omega_c T$ (O/L frequency-response) render the sets of specifications in each of these domains incomplete.

I recommend that the tables for damping ratio ξ=0.7 be recalculated to include these values, and the results be included with an errata sheet in the copies of the thesis.

pA-16: The variables t_s and ω_c should be defined under the appropriate heading.

DEDICATED

TO

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ABSTRACT

The frequency response matching technique for the synthesis of digital control systems has been investigated. The basic philosophy of this technique is to design a digital controller so that the frequency response of the designed closed-loop system matches a specified frequency response model. The approaches described in the current literature include Rattan's complex-curve fitting method and Shieh's dominant data matching method.

Two new design methods are proposed in this thesis. The first one is the *iterative complex-curve fitting* (ICCF) design method based on Rattan's algorithm. With its iterative calculations, the new method improves the frequency matching accuracy significantly by eliminating in Rattan's algorithm the frequency dependent weighting factor which severely degrades the matching accuracy at high sampling frequencies. The second one is the *simplex optimization-based* (SIM) design method. With the aid of non-linear constraints on the controller parameters, this method provides a good compromise between the desired system frequency response and the required controller characteristics to avoid problems such as an excessively high controller gain or an oscillatory control signal.

The non-linearity of the Shieh's method in the case of the design of a controller with an integrator is removed by choosing the appropriate controller form and dominant frequency points. As a result, the relevant computational algorithm is considerably simplified.

The determination of a frequency response model from design specifications given in the time, frequency and z- domains is discussed. It is shown that the choice of the model may be critical to the succcess of the frequency matching, in particular when there is discrepancy between the primary frequency range of the system under design and that of the model. To help select an appropriate z-transfer function as a model, an

easy-to-use approach is developed which is based on a comprehensive investigation on the dynamic performances of second-order discrete systems.

A number of design studies is conducted in order to assess the frequency matching design methods. The frequency matching accuracies and time responses of the designed systems form the basis for the comparative evaluation. The results show that ICCF and SIM methods proposed in this thesis are superior to current methods. The effect of the discrepancy between the primary frequency ranges on the matching accuracy and the convergency of optimization algorithms are illustrated as well.

The ***hybrid frequency response*** is defined for the system containing both discrete- and continuous- time components. Unlike the commonly-used ***discrete frequency response*** , which is derived from the system z-transfer function and provides no information about the time response between sampling instants, the hybrid frequency response includes the characteristics of the inter-sampling time response.

DECLARATION

I hereby declare that, (a) the thesis contains no material which has been accepted for the award of any other degree or diploma in any university and that, to the best of my knowledge and belief, the thesis contains no material previously published or written by another person, except where due reference is made in the text of the thesis; and (b) I consent to the thesis being made available for photocopying and loan if applicable.

Jianfei Shi

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LIST OF SYMBOLS AND ABBREVIATIONS

This list summarizes symbols and abbreviations commonly used in this study. The page number indicates the place where the symbol or abbreviation is defined.

Symbol		Page
A	coefficient of a second-order discrete transfer function	3-1
B	coefficient of a second-order discrete transfer function	3-1
C	coefficient of a second-order discrete transfer function	3-1
D	coefficient of a second-order discrete transfer function	3-1
$D(\ )$	transform or frequency response of a digital controller	2-2
E	squared-matching-error function	2-10
or	objective function	2-23
$E(s)$	transfer function of an error signal	2-2
$E(z)$	z-transfer function of an error signal	2-3
$F(s)$	transfer function of a control element in the system feedback loop	2-2
$G(s)$	transfer function of a continuous-time plant	2-2
$G_hG(\ )$	transform or frequency response of a plant with ZOH	2-3
GM	gain margin	3-2
$H(\ )$	transform or frequency response of a closed-loop system	2-3
I	imaginary part of a complex number	2-5
$M(\ )$	transform or frequency response of a closed-loop model	2-3
$M_Q(\ )$	transform or frequency response of a open-loop model	2-3
M_p	maximum overshoot	3-2
M_r	resonance peak value	3-2
$N(\ )$	numerator polynomial	2-9
P	open-loop or closed-loop poles	3-6
$P(\ )$	denominator polynomial	2-9
PM	phase margin	3-2
$Q(\ )$	transform of frequency response of an open-loop system	2-3
R	real part of a complex number	2-5
$R(\ )$	transform of an input signal	2-2
T	sampling period	1-3
T_m	sampling period of a frequency response model	2-14

Symbol		Page
$U(z)$	z-transfer function of the output signal of a digital controller	2-3
WIAE	weighted integral absolute error	4-12
$Y(\ )$	transform of the output signal of a closed-loop system	2-2
Z	zero of a control system	3-6
α	reflection coefficient in the simplex method	2-26
or	angle to define the relative location of a zero to a pair of complex poles	3-5
β	contraction coefficient in the simplex method	2-29
γ	expansion coefficient in the simplex method	2-28
ε	weighted-squared-error function	2-10
ϑ	magnitude of a complex number in decibels	2-23
λ	auxiliary optimization variable	2-24
μ	parameters of the constraints for a controller	2-24
ν	parameters of the constraints for a controller	2-24
ξ	system damping ratio	3-7
τ	magnitude of a complex number	3-10
ψ	phase angle of a complex number	2-23
ω	frequency variable in rad/s	2-3
ω'	weighted frequency variable	4-12
ω_b	bandwidth ($-3\ db$) of a closed-loop frequency response	2-8
ω_c	gain cross-over frequency	3-9
ω_g	phase cross-over frequency	3-10
ω_n	undamped natual frequency of a closed-loop system	3-7
ω_o	system oscillatory frequency	3-7
ω_p	primary frequency range of a discrete system	2-4
ω_{pm}	primary frequency range of a frequency response model	4-36
ω_r	resonant frequency	3-2
ω_s	sampling frequency	2-3