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## Simple quark model with chiral phenomenology

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We propose a new approach to the determination of hadronic observables in which the essential features of chiral symmetry are combined with conventional constituent quark models. To illustrate the approach, we consider the simple quark model in the limit of SU(3) flavor symmetry at the strange quark mass. The comparison with data is made after an analytic continuation which ensures the correct leading nonanalytic behavior of chiral perturbation theory. The approach not only gives an excellent fit for the octet baryon magnetic moments but the prediction for the  $\Delta^{++}$  magnetic moment is also in good agreement with current measurements.

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Quark models have traditionally suffered from two major shortcomings [1–3]. First they have omitted the effects of the pion and kaon clouds which give rise to nonanalytic behavior in the quark mass. Second, implicit in the simple constituent quark picture is the idea that the contribution made by a quark to a hadronic observable is independent of its environment. For example, the contribution of the  $u$  quark to the magnetic moment of the neutron ( $udd$ ) and the  $\Xi^0(uss)$  is usually taken to be the same. However, for the quark masses considered in lattice QCD calculations, these environment effects are easily observed [3]. Perhaps the clearest indication of a problem is the enormous violation of charge symmetry in the constituent quark masses quoted by the Particle Data Group [4], with  $M_u = 338$  MeV and  $M_d = 322$  MeV, differing by an unacceptable 5%.

Recent studies of the variation of hadron properties as a function of (current) quark mass within lattice QCD [5–7] have led to new insights into hadron structure, which suggest a relatively simple approach to overcoming these problems, while avoiding the complexities of fully-fledged chiral quark models [8,9]. In particular, these studies have revealed the following behavior with quark mass [10]:

(i) In the region of current quark masses  $m > 60$  MeV or so ( $m_\pi$  greater than typically 400–500 MeV) hadron properties are smooth, slowly varying functions of something like a constituent quark mass,  $M \sim M_0 + c m$  (with  $c \sim 1$ ).

(ii) Indeed,  $M_N \sim 3M$ ,  $M_{\rho,\omega} \sim 2M$  and magnetic moments behave like  $1/M$ .

(iii) As  $m$  decreases below 60 MeV or so, chiral symmetry leads to rapid, nonanalytic variation.

The speed with which rapid chiral variations are suppressed above 60 MeV or so suggests that this is the region in which constituent quark models should be most appropriate. The connection to the physical world, including quantitative fits to experimental data, should be undertaken after chiral extrapolation in a manner consistent with the general constraints of chiral perturbation theory ( $\chi$ PT).

With this in mind, we explore the utility of employing a quark model in the region of heavier quark mass and then extrapolating to the physical world with an analytic continuation of  $\chi$ PT. This extrapolation function builds in the leading nonanalytic (LNA) behavior of  $\chi$ PT and therefore explicitly incorporates the pion and kaon contributions in extrapolating to the chiral regime. As a first example we illustrate this procedure with the simple SU(6) constituent quark model (CQM) in the calculation of octet baryon magnetic moments. We begin with the CQM in the limit of SU(3)-flavor symmetry where all three quarks have the same mass—taken to be the strange quark mass. These baryon magnetic moments, determined at large quark masses where constituent quark degrees of freedom are manifest [10], are then analytically continued to the physical mass regime. The resulting description of the experimental octet baryon magnetic moments is excellent and when applied to the charged  $\Delta$  baryons the model also produces values in good agreement with current data.

In what follows we first present the extrapolation technique used to link the baryon moments calculated near the limit of SU(3)-flavor symmetry to the physical world. We then present the details of the model (referred to as AccessQM)<sup>1</sup> and apply it to the baryon octet.

The extrapolation technique utilized here to link baryon magnetic moments at large quark masses with the physical mass regime has only recently been exploited in lattice QCD. Here we extend the previous approach [5,6] incorporating pion cloud contributions

$$\mu(m_\pi) = \frac{\mu_0}{1 - \frac{\chi_\pi}{\mu_0} m_\pi + \beta m_\pi^2} \quad (1)$$

to include the kaon cloud. In order to place the leading

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<sup>1</sup>The name indicates the mathematical origins of the model: Analytic Continuation of the Chiral Expansion for the SU(6) Simple Quark Model.

TABLE I. The baryon chiral coefficients,  $\chi_i$ , for the spin-1/2 octet. Coefficients are from Ref. [11], with one-loop corrected values for  $D=0.61$  and  $F=0.40$ . Note here we have suppressed the kaon loop contribution in the calculation of the  $\chi_K$  by using  $f_K = 1.2 f_\pi$ .

	$p$	$n$	$\Lambda$	$\Sigma^+$	$\Sigma^0$	$\Sigma^-$	$\Xi^0$	$\Xi^-$
$\chi_\pi$	-4.41	4.41	0	-2.46	0	2.46	-0.191	0.191
$\chi_K$	-1.71	-0.133	1.47	-3.06	-1.47	0.133	3.06	1.71

nonanalytic (LNA) kaon contribution in the denominator, we replace  $\chi_K m_K$  by  $\chi_K(m_K - m_K^{(0)})$ , where  $m_K^{(0)}$  is the kaon mass in the SU(2) chiral limit

$$\mu(m_\pi) = \frac{\mu_0}{1 - \frac{\chi_\pi}{\mu_0} m_\pi - \frac{\chi_K}{\mu_0} (m_K - m_K^{(0)}) + \beta m_\pi^2}. \quad (2)$$

We stress that  $\chi_\pi$  and  $\chi_K$  are model independent constants fixed by chiral perturbation theory (see Table I) and only  $\mu_0$  and  $\beta$  are fit parameters. Further, using the Gell Mann–Oakes–Renner relation for the pion and kaon masses, one has

$$m_K^2 = m_K^{(0)2} + \frac{1}{2} m_\pi^2, \quad (3)$$

for fixed strange quark mass, with

$$m_K^{(0)} = \sqrt{(m_K^{\text{phys}})^2 - \frac{1}{2} (m_\pi^{\text{phys}})^2}. \quad (4)$$

Motivated by the success of recent studies of the behavior of hadron properties calculated using lattice QCD as a function of quark mass, we now consider an amalgamation of the CQM with the techniques of chiral extrapolation developed there. In particular, we take as the *input* for the extrapolation to the chiral limit, the CQM for the baryon magnetic moments in the SU(3) limit. At sufficiently large quark mass ( $m_u = m_d = m_s$  near the physical strange quark mass) chiral loop contributions should be suppressed. Since the extrapolation function involves two parameters, we need two input values for each baryon and these are obtained by uniformly shifting the masses of the  $u$  and  $d$  quarks slightly below and then slightly above the physical strange quark mass

$$\begin{aligned} M_1 &= M_s - \Delta M, \\ M_2 &= M_s + \Delta M, \end{aligned} \quad (5)$$

where we consistently use a capital  $M$  for a constituent quark mass and  $m$  for a current quark mass, throughout this Rapid Communication.

In Eqs. (5)  $\Delta M$  and  $M_s$  are input parameters. The magnetic moments of the baryons are simply related to the constituent quark masses via

$$\begin{aligned} \mu_p &= (4\mu_u - \mu_d)/3, & \mu_n &= (4\mu_d - \mu_u)/3, \\ \mu_{\Sigma^+} &= (4\mu_u - \mu_s)/3, & \mu_{\Sigma^-} &= (4\mu_d - \mu_s)/3, \\ \mu_{\Xi^0} &= (4\mu_s - \mu_u)/3, & \mu_{\Xi^-} &= (4\mu_s - \mu_d)/3, \\ \mu_\Lambda &= \mu_s, \end{aligned}$$

with

$$\mu_u = \frac{2}{3} \frac{M_N}{M_u} \mu_N, \quad \mu_d = -\frac{1}{3} \frac{M_N}{M_d} \mu_N, \quad \mu_s = -\frac{1}{3} \frac{M_N}{M_s} \mu_N,$$

where  $M_N$  is the nucleon mass and  $M_u = M_d = M_i$ , as discussed above.

To fit Eq. (2), which is a function of  $m_\pi$ , to the two magnetic moments obtained with  $M_u = M_d = M_i (i=1,2)$ , we relate the pion mass to the constituent quark mass via the current quark mass. Chiral symmetry provides

$$\frac{m_q}{m_q^{\text{phys}}} = \frac{m_\pi^2}{(m_\pi^{\text{phys}})^2}, \quad (6)$$

where  $m_q^{\text{phys}}$  is the quark mass associated with the physical pion mass,  $m_\pi^{\text{phys}}$ . From lattice studies, we know that this relation holds well over a remarkably large regime of pion masses, up to  $m_\pi \sim 1$  GeV. The link between constituent and current quark masses is provided by

$$M = M_\chi + c m_q, \quad (7)$$

where  $M_\chi$  is the constituent quark mass in the chiral limit and  $c$  is of order 1. Using Eq. (6) this leads to

$$M = M_\chi + \frac{c m_q^{\text{phys}}}{(m_\pi^{\text{phys}})^2} m_\pi^2. \quad (8)$$

The link between  $M_i$  of Eqs. (5) and  $m_\pi$  is thus provided by

$$m_{\pi i}^2 = (m_\pi^{\text{phys}})^2 \frac{M_i - (M_s - c m_s^{\text{phys}})}{c m_q^{\text{phys}}} \quad (i=1,2), \quad (9)$$

where  $M_s - c m_s^{\text{phys}} = M_\chi$  encapsulates information on the constituent quark mass in the chiral limit, and  $c m_s^{\text{phys}}$  provides information on the strange current quark mass. We use the ratio

$$\chi_{sq} = \frac{m_s^{\text{phys}}}{m_q^{\text{phys}}} = 24.4 \pm 1.5, \quad (10)$$

provided by  $\chi$ PT [12] to express the light current quark mass,  $m_q^{\text{phys}}$ , in terms of the strange current quark mass,  $m_s^{\text{phys}}$ , in Eq. (9).

In summary the AccessQM requires three input parameters.

(1)  $M_s$ —the strange constituent quark mass, to determine the SU(3)-flavor limit.

(2)  $c m_s^{\text{phys}}$ —the strange current quark mass, if  $c \sim 1$ .

(3)  $\Delta M$ —the spacing about the SU(3)-flavor limit, needed to determine the shift of the two magnetic moments away from this limit.

The exactness of the SU(3) symmetry is determined through the value of  $\Delta M$ . In the limit  $\Delta M \rightarrow 0$ , one is effectively using the magnitude and slope predicted by the CQM in fitting the extrapolation function. Our conclusions are not sensitive to the choice of this parameter over the range (0,50] MeV and we fix  $\Delta M$  at 20 MeV.

We now fit Eq. (2) to the two baryon magnetic moments given by the constituent quark model, one either side of the SU(3) limit, allowing an extrapolation back to the physical mass regime. This fit is easily accomplished as we have two equations, given by the extrapolation function evaluated at each  $m_{\pi i}$  (i.e.,  $M_i$ ) and two unknowns  $\mu_0$  and  $\beta$ —the two fit parameters. We then simply solve for  $\mu_0$  and  $\beta$  simultaneously, where the positive root provides the smooth, nonsingular extrapolation.

There are a number of approaches one could take with which to report the theoretical predictions made by the AccessQM. One would be to simply substitute in the expected values for  $M_s$  and  $c m_s^{\text{phys}}$  then report the predicted magnetic moments of the octet. However as these quantities are only approximately known, we choose to do an optimization over the octet. We choose to minimize the rms deviation between the theoretical and experimental magnetic moments of the octet and we denote this optimization function by<sup>2</sup>

$$\chi^2 = \frac{1}{7} \sum_{i=1}^7 (\mu_i - \mu_i^{\text{exp}})^2. \quad (11)$$

The values returned for  $M_s$  and  $c m_s^{\text{phys}}$  are 565 MeV and 144 MeV, respectively, with  $\chi = 0.051 \mu_N$ . This provides some *a posteriori* justification for the approach taken within the AccessQM, as the values obtained for  $M_s$  and  $m_s^{\text{phys}}$  (taking the preferred value of  $c=1$ ) lie well within the range of their expected values. Table II provides a summary of the AccessQM predictions compared to experiment.

To do a direct comparison between the CQM and the AccessQM we perform an analogous optimization for the CQM. We choose to optimize the three constituent masses  $M_s$ ,  $M_u$ , and  $M_d$  subject to the charge symmetry constraint that  $M_u$  and  $M_d$  should be equal to within one percent [13]. We find  $M_u = 345$  MeV,  $M_d = 342$  MeV, and  $M_s = 538$  MeV with a rms deviation of  $\chi = 0.122 \mu_N$ . The resulting magnetic moments are given in Table II. We see that the AccessQM provides more than a factor of 2 reduction in the rms deviation of theory from experiment. This gives a good indication for the need to incorporate meson cloud effects into conventional constituent quark models.

Figures 1 and 2 show the behavior of the analytic continuation of  $\chi$ PT fitted to the two magnetic moments, one either side of the SU(3)-flavor limit. These baryon magnetic moments near the SU(3)-limit are indicated by a dot (•) and

<sup>2</sup>We omit the  $\Sigma^0$  moment from the  $\chi^2$  as it has not been experimentally measured.

TABLE II. The optimized AccessQM and CQM predictions for the magnetic moments of the spin-1/2 baryon octet. The values for the experimental magnetic moments are taken from Ref. [3]. Note the quoted value for  $c m_q^{\text{phys}}$  is just  $c m_s^{\text{phys}} / \chi_{sq}$ .

Baryon	Quark model	AccessQM	Experiment
$p$	2.724	2.765	2.793
$n$	-1.826	-1.930	-1.913
$\Lambda$	-0.582	-0.591	-0.613(4)
$\Sigma^+$	2.613	2.532	2.458(10)
$\Sigma^-$	-1.027	-1.079	-1.160(25)
$\Xi^0$	-1.381	-1.247	-1.250(14)
$\Xi^-$	-0.471	-0.583	-0.6507(25)
$M_s$	538	565	$\approx 550$
$c m_s^{\text{phys}}$	( $M_u=346$ )	144	75 to 170 (at 2 GeV <sup>2</sup> )
$c m_q^{\text{phys}}$	( $M_d=342$ )	5.90	2 to 7 (at 2 GeV <sup>2</sup> )
$\chi (\mu_N)$	0.122	0.051	

experimental values for the baryon magnetic moments are given, at the physical pion mass, by an asterisk (\*). To obtain the AccessQM magnetic moment prediction one simply reads off the value of the extrapolation function at the physical pion mass. Note, we have also included a fit where only the chiral behavior associated with the pion-cloud is considered—this is indicated by the dashed line. These results are obtained by using the same values for the input parameters, only this time setting the respective  $\chi_K$ 's to zero in Eq. (2) prior to determining  $\mu_0$  and  $\beta$ . It is evident, from Figs. 1 and 2, that the role of the kaon-cloud is slight over the octet. Indeed, it only plays a significant role for the  $\Lambda$ ,  $\Xi^0$ , and  $\Xi^-$  magnetic moments.

A longstanding problem of the CQM is its prediction of the  $\Xi^-/\Lambda$  magnetic moment ratio. The CQM predicts this ratio is given by

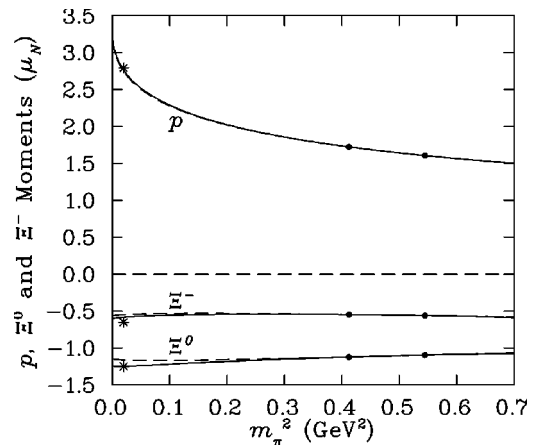


FIG. 1. The extrapolation function fit for the  $p$ ,  $\Xi^-$ , and  $\Xi^0$  magnetic moments. The experimental values are shown as asterisks (\*) and the magnetic moments either side of the SU(3)-flavor limit as dots (•). The extrapolation including only the chiral behavior associated with the pion cloud is shown by the dashed lines.

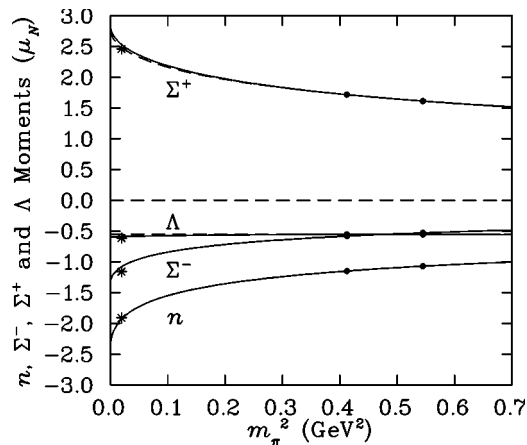


FIG. 2. The extrapolation function fit for the  $n$ ,  $\Sigma^-$ ,  $\Sigma^+$ , and  $\Delta$  magnetic moments. The experimental values are shown as asterisks (\*) and the magnetic moments either side of the SU(3)-flavor limit as dots (•). The extrapolation including only the chiral behavior associated with the pion cloud is shown by the dashed lines. Note that the addition of the kaon-cloud results in negligible changes for the neutron and  $\Sigma^-$  magnetic moments.

$$\frac{\mu_{\Xi^-}}{\mu_{\Lambda}} = \frac{1}{3} \left( 4 - \frac{\mu_d}{\mu_s} \right), \quad (12)$$

which becomes

$$\frac{\mu_{\Xi^-}}{\mu_{\Lambda}} = \frac{1}{3} \left( 4 - \frac{M_s}{M_d} \right), \quad (13)$$

and is therefore less than 1, as  $M_d < M_s$ . Experiment places this ratio at 1.06(1). The addition of the meson cloud gives us the opportunity of solving this problem, and indeed we find from Table II a ratio of 0.99 for the AccessQM, compared with 0.81 for the CQM. While this is a significant improvement, there is a residual disagreement, suggesting the need to replace the very simplest CQM with something a little more sophisticated [14].

We have shown that using the very simplest CQM, with all three quark masses near the physical strange quark mass, and extrapolating to the physical mass regime using an analytic continuation of  $\chi$ PT which ensures the correct LNA behavior in the chiral limit, does offer a considerable improvement to the theoretical predictions of the spin-1/2 baryon octet magnetic moments, as compared to those of the CQM alone. The results indicate the importance of incorporating the meson cloud contribution in any calculation of baryon magnetic moments as well as the need to accommodate the hadronic environment of the constituent quarks.

This work serves to introduce the idea that one should merge the general class of constituent quark models with known chiral properties of hadronic observables. While the results presented here display great promise, there is a demonstrated need for further refinement. For example, one could explore the possibility that decuplet baryon intermediate states make an important contribution in the chiral extrapolation. For the decuplet itself there is limited experimental data, but for the  $\Delta^{++}$ , which is known to lie in the range  $[3.7, 7.5] \mu_N$  [4], with the latest experimental measurement yielding  $4.52 \pm 0.50 \pm 0.45$  [15], the application of the model outlined here yields a value of  $5.39 \mu_N$ —in good agreement with the data. The predictions for the remaining charged  $\Delta$  baryons are  $\mu_{\Delta^+} = 2.58 \mu_N$  and  $\mu_{\Delta^-} = -2.99 \mu_N$ . It will also be possible to tune these models to data at larger quark masses from the new generation of lattice QCD simulations now underway. Finally, there is also an important opportunity to refine the constituent quark model itself, where spin-dependent interactions [16,17] between quarks can give rise to important contributions [18,19]. It is our hope that the ideas presented here will lead to a new appreciation of the role of the constituent quark model in modern hadron phenomenology in which there is no longer a conflict with the constraints of chiral perturbation theory.

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- [1] D.B. Leinweber, A.W. Thomas, and R.D. Young, Phys. Rev. Lett. **86**, 5011 (2001).  
[2] D.B. Leinweber and A.W. Thomas, Phys. Rev. D **62**, 074505 (2000).  
[3] D.B. Leinweber, R.M. Woloshyn, and T. Draper, Phys. Rev. D **43**, 1659 (1991).  
[4] Particle Data Group, D.E. Groom, *et al.*, Eur. Phys. J. C **15**, 1 (2000).  
[5] D.B. Leinweber, D.H. Lu, and A.W. Thomas, Phys. Rev. D **60**, 034014 (1999).  
[6] E.J. Hackett-Jones, D.B. Leinweber, and A.W. Thomas, Phys. Lett. B **489**, 143 (2000).  
[7] W. Detmold, W. Melnitchouk, J.W. Negele, D.B. Renner, and A.W. Thomas, Phys. Rev. Lett. **87**, 172001 (2001).  
[8] S. Theberge and A.W. Thomas, Nucl. Phys. **A393**, 252 (1983).  
[9] A. Manohar and H. Georgi, Nucl. Phys. **B234**, 189 (1984).  
[10] W. Detmold, D.B. Leinweber, W. Melnitchouk, A.W. Thomas, and S.V. Wright, Pramana, J. Phys. **57**, 251 (2001).  
[11] E. Jenkins, M. Luke, A.V. Manohar, and M.J. Savage, Phys. Lett. B **302**, 482 (1993).  
[12] H. Leutwyler, Phys. Lett. B **378**, 313 (1996).  
[13] G.A. Miller, B.M. Nefkens, and I. Slaus, Phys. Rep. **194**, 1 (1990).  
[14] G. Morpurgo, hep-ph/0107049.  
[15] A. Bosshard *et al.*, Phys. Rev. D **44**, 1962 (1991).  
[16] A. DeRújula, H. Georgi, and S.L. Glashow, Phys. Rev. D **12**, 147 (1975).  
[17] M. A. Preston and R. K. Bhaduri, *Structure of the Nucleus* (Addison-Wesley, 1975), p. 463.  
[18] S. Capstick, S. Godfrey, N. Isgur, and J. Paton, Phys. Lett. B **175**, 457 (1986).  
[19] N. Isgur and G. Karl, Phys. Rev. D **18**, 4187 (1978); **19**, 2653 (1979).