DENTAL-ARCH MORPHOLOGY OF AUSTRALIAN ABORIGINES:
a metric study of arch size and shape

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DENTAL-ARCH MORPHOLOGY

OF

AUSTRALIAN ABORIGINES

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SUMMARY

The size and shape morphology of dental arches was studied by means of measurements obtained from 276 serial dental casts representing 28 male and 16 female Australian Aborigines. The age range of the subjects over the observation period was from about seven to seventeen years, corresponding to a change in dental status from the early mixed dentition to eruption of all the permanent teeth except third molars.

The casts were photographed and the reference points of tooth landmarks were recorded in Cartesian coordinates. Ten polynomial equations were fitted to each set of dental-arch reference points and a selection was made to determine the equation best describing the shape of each dental arch. Arch areas, perimeters, polar vectors and polar overjets were derived from the shape-representing polynomial equations and average values were computed for males and for females categorized in four groups determined by tooth emergence status. Polynomial curves and dental-arch heptagons constructed from the polar vectors were used to describe arch size and shape diagrammatically for the four groups. Areas were used as measures of arch size and polar vectors as measures of arch shape.

The arches of males were found to be larger in area and perimeter than those of females at corresponding stages of dental development. They were also broader and more pronounced in overjet. The sex difference in size was more noticeable in the maxillary canine region. Changes in size and shape of dental arches for males and females from the early mixed dentition stage to the early permanent dentition stage involved decreases in area and perimeter. The arches became broader posteriorly and somewhat narrower anteriorly. There was a reduction in depth and an increase in overjet.

It is suggested that shape-representing polynomials may have useful applications in clinical orthodontics for diagnosis and treatment planning. Further research on a simpler methodology is suggested to give a wider scope for the application of polynomials in dentistry.

SIGNED STATEMENT

This thesis contains no material which has been accepted for the award of any other degree or diploma in any University. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person, except when due reference is made in the text of the thesis.

PETER CHUNG KWONG CHENG

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CHAPTER 1

INTRODUCTION

Investigations of the morphology of human dental arches can be broadly grouped into anthropological studies and odontological studies. Usually the objectives of anthropological studies are to describe the size and shape of the dental arches and to compare findings for various groups of people. Odontological studies on the other hand are more concerned with 'normal' arch form, with age changes in the size and shape of dental arches during growth and development of the dento-facial complex, with the relation of arch form and occlusion of the teeth, and with other features of dental-arch morphology and occlusion of interest to orthodontists and prosthodontists.

Most commonly the size of dental arches is described by measurements of breadth, depth and perimeter. However, the multiplicity of reference points from which measurements have been made by various investigators makes comparisons between the studies very difficult. There is no standard measurement method.

Similarly, there is no standard method for describing the shape of dental arches. The subjective classification of Hrdlička

(1916) is unreliable. Classification of dental-arch shape by this method may differ between observers and even within the same observer when repeating the observations. The dental-arch index of De Terra (1905) is also unreliable. One of the deficiencies pointed out by Moorrees (1957) and by Barrett and Brown (1968) is that visually recognizable differences in arch shape can be distinguished between subjects with identical arch indices. Other methods of dental-arch shape description make use of mathematical equations or geometrical constructions. However, these do not always give an adequate description for comparison of dental-arch shape between individuals. The main problem is to derive a simple, quantitative measure of arch shape.

AIMS OF THE INVESTIGATION

The present investigation was undertaken with the principal aim of studying variations in the size and shape of the dental arches and age changes in these features in a group of Australian Aborigines. Specific aims of the investigation were:

(1) to develop a method of dental arch measurement and analysis based on coordinate geometry;

(2) to obtain coordinate measurements from serial dental casts of Australian Aborigines; and

(3) to analyse the measurements in terms of group statistics for

subject observations categorized by sex and by dental groups determined by tooth emergence status.

CHAPTER 2

REVIEW OF LITERATURE

John Hunter (1771) was one of the first to study growth of the jaws. He stated that the jaw increased in size at all points up to twelve months after birth, but subsequently did not increase in length between the symphysis and the sixth tooth. He also referred to the common occurrence of irregularities in the position of permanent teeth in the upper jaw which he stated was due to the larger size of the permanent teeth compared with the deciduous teeth.

De Terra (1905) suggested the use of the dental-arch index as a numerical measure of dental-arch shape for the study of racial differences. The index expresses the relation between arch breadth and depth as a percentage.

Hrdlička (1916) suggested a classification of the dental arches into six different types, namely: hemispheric - resembling one-half of a circle; parabolic - shaped like one-half of an oval; ovoid - parabolic but converging towards the molar region; hyperbolic - relatively broader than parabolic; elliptic - relatively narrower than parabolic; and U-shaped - in the form of a capital letter U.

Williams (1917) described the 'normal' dental arch as having the six anterior teeth arranged in an arc of a circle. His observations also led him to suggest that in 'normal' dental arches there was a constant ratio between arch breadths in the canine and molar regions.

Sved (1917, 1952) agreed with the findings of Williams and suggested a method with applications in prosthetic dentistry and orthodontics to determine an arch shape suitable for the proposed treatment.

Campbell (1925) described various features of the dentition and palate of the Australian Aboriginal from an investigation of 630 skull specimens. He found that average values of dental-arch breadth increased with increasing age but dental-arch depth decreased with age. He also noted that edge-to-edge bite was generally found in adult Aborigines, and he suggested that the change from the juvenile overbite incisor relation was associated with tooth wear accompanied by changes in the relative positions of the anterior teeth of the maxillary and mandibular dental arches.

Ramsay-Smith (1926) discussed the problem of describing the various forms of alveolar arches and pointed out that indices used in anthropological descriptions, such as the 'palatal index', merely give the proportion percentage between length and breadth of the jaws but no idea of the shape of the arches. He illustrated alveolar arches of skull specimens of Australian, New Caledonian, Maori and Veddah people which could be described as square-form, U-shape, horse-shoe shape or parabolic.

Izard (1927) put forward the hypothesis that the relation between the breadth of the arch and the 'width of the face' was constant; also the depth of the arch and the 'depth of the face'. He claimed that the 'normal' arch form could be reduced to variations of the ellipse. He derived a method to determine the shape of 'normal' arch in terms of facial dimensions.

Chapman (1935) studied changes in the dental arches of eight children from birth to adulthood using serial dental casts collected by himself and others. He observed that there was no increase in dental-arch breadth between age two and five years, but thereafter until age twelve years there was an increase, greater in the maxillary arch.

Goldstein and Stanton (1935) studied 547 sets of dental casts of normal and abnormal occlusions obtained from 300 American children of age one to eleven years. The study was partly longitudinal. They found that dental-arch breadth showed a general increase from age three years to nine years. The increase in intercanine breadth during the eruption of permanent incisors was most apparent. The increase in intermolar breadth was greater in the mandible than the maxilla. Dental-arch breadth was larger in boys at almost all ages and with greater fluctuation in the rate of increase, whereas the arch breadth of girls increased steadily from two to nine years of age and at a faster rate. The rate of intermolar breadth increase was greater in the mandible than the maxilla. The arch depth of the deciduous dental arch showed a very slight decrease between two and nine years of age. However, the reduction of maxillary dental-arch depth was compensated by maxillary growth after the eighth year. Dental arches of girls showed a more regular mode of growth and greater increase in arch depth than those of boys. The authors expressed dental-arch form by the index of arch breadth over arch depth. The maxillary and the mandibular arches were classified on the basis of index ratings into three subdivisions: dolichuranic (narrow arch), mesuranic (medium arch) and brachuranic (broad arch). Allowance was made for a merging in forms between the subdivisions.

Cohen (1940) carried out a longitudinal study of changes in dental arches of fifteen boys and thirteen girls with 'fairly normal' arches over a period of ten years, from age three-and-a-half to thirteen-and-a-half years. He concluded that the greatest growth of the dental arch occurred in the canine region during eruption of the permanent incisors. Maximum lateral growth in the maxillary arch was observed from age five to eight years and the growth slowed down by twelve years. Lateral growth in the mandibular arch reached its maximum at eight-and-a-half years.

Little growth occurred subsequently. He also reported that there was very little lateral growth of the arches between the permanent first molars and only slight lateral growth between the premolars. There was a decrease in the length of the posterior segment of the dental arch measured from the mesial surface of the canine to the distal surface of the second premolar. Decrease of this dimension in the mandibular dental arch was observed to be greater than in the maxillary arch. Dental arches of girls were found to be wider posteriorly and narrower anteriorly than those of boys. Cohen pointed out that there were wide variations in dental arch development and that individuals often departed from the pattern he described.

Siepel (1946) studied positional variations of the teeth of 500 Swedish children and young adults. The sample consisted of about equal numbers of males and females. Three groups of subjects were studied - age four years, age thirteen years and age twenty-one years. Measurements were taken directly from the oral cavity of the individuals. Seipel found that between age four and thirteen years the dental-arch breadth measured as the distance between the first premolars increased slightly more in males than in females.

Numata (1947) was a pioneer in describing dental-arch shape and dimensions by means of polar and rectangular coordinate systems.

He defined the abscissa as a line passing through the tip of the mesiobuccal cusp of the right and left permanent first molars, and the ordinate as a line perpendicular to the abscissa through the mid-point between the two central incisors. He recorded the position of the reference points in polar coordinates by a specially designed protractor, and he transformed the values of the polar coordinates to a rectangular coordinate system. He measured dental arches of various forms - namely, parabolic, V-shaped, squarish and circular - and derived dental-arch dimensions from these measurements for analysis and comparison.

MacConaill and Scher (1949) studied the dental-arch form of twenty-five sets of dental casts of a group of males and females, and also two standard denture models. Dental-arch breadths were measured, and from these measurements mean values of dental-arch breadths between corresponding occlusal points were calculated. The average maxillary and mandibular human dental-arch forms and those of the denture models were plotted from the mean values and were found to conform closely to catenary curves - formed by varying the distances of the two suspended ends of a piece of fine rolled-gold chain of 20 cm length. MacConaill and Scher suggested that the ideal human dental-arch form would conform to the catenary curve.

Kato and Odagiri (1950) studied 127 maxillary dental casts using the rectangular coordinate system as suggested by Numata (1947). They classified the casts into the same four groups as used by Numata. Quadratic equations were derived to represent the shape of each group of dental arches. Kato and associates (1964) reported a similar study of the 'normal' dental arches of a mixed group of 205 males and females, thirteen to forty-six years of age. They converted the values of the x and y coordinates to percentages of the linear distance between the reference points of the left and right permanent first molars in the same dental arch. In this manner they studied the relative position of the teeth in each dental arch. Quadratic equations representing the polygonal lines of the dental arch curves were derived. They found that the forms of the maxillary and mandibular dental-arch curves were similar. They also found that there were no significant age or sex differences in the relative positions of the teeth within the maxillary or the mandibular arches, but there were significant differences in the relative positions of the teeth between the maxillary and the mandibular arches.

Baume (1950) reported the findings of a longitudinal study of developmental changes in dental arches during the period of deciduous dentition, the eruption of permanent molars, and the replacement of deciduous teeth by their successors. Measurements were taken from study casts. This review is only concerned with findings on changes of dental arch dimensions. From dental casts of thirty children aged three to five-and-half years Baume found that arch depth and arch breadth remained unchanged from age four years to the time of eruption of the permanent molars. From serial casts of sixty children before, during and after eruption of the permanent incisors he reported that the mean increase in intercanine breadth was greater in the maxillary dental arches than the mandibular arches, and that a greater increase was found in the previously non-spacing deciduous dental arches. Maximum lateral growth of the maxillary arch was observed during eruption of the central incisors, and in the mandibular arch during eruption of the lateral incisors. The average amount of forward growth of the arches was 1.0 mm greater in the maxilla than the mandible and no difference was shown between the previously spacing or non-spacing arches.

Woods (1950) studied growth changes from series of lateral and frontal cephalometric radiographs of twenty-eight individuals (fourteen boys and fourteen girls). The records were obtained half-yearly from age three to five years and yearly thereafter to the age of fifteen years. Most of this group of children had normal occlusions or Angle Class I malocclusions. Only one child had been treated orthodontically. The remainder of the group consisted of three children with Class II and one with Class III malocclusions. He found that the maxillary intercanine breadth increased with age except for a decrease between seven to twelve years, whilst the mandibular intercanine breadth remained the same from age three to fifteen years except for a decrease between age six to eleven years. The maxillary intermolar breadth increased until these teeth came into occlusion and the rate of increase slowed down. The mandibular intermolar breadth decreased gradually until the teeth were in occlusion. Then the breadth increased or decreased at a constant rate which varied with the individual. He also observed that the female arches were generally smaller than the male arches in all dimensions.

Speck (1950) conducted a longitudinal study of the developmental changes in mandibular dental arches using 170 dental casts of fifty-three boys and girls over the period of development between the completion of deciduous dentition and the replacement of those teeth by their successors. The subjects were selected by the criterion of a 'good' occlusion in the permanent dentition. He used a photographic method to transform the occlusal surface of the casts into two dimensions. Measurements were obtained from the photographs. Direct measurement of the casts provided a check on the accuracy of the indirect measurements. Speck found in most instances that the arch perimeter of the permanent dentition anterior to the mesial contact areas of the permanent first molars was less than that of the deciduous dentition. Similarly, when the arch perimeter mesial to the permanent first molars between the mixed and the permanent dentitions was compared, he observed that the arch perimeter of the permanent dentition was less.

Brown and Daugaard-Jensen (1951) studied the dentition changes of forty individuals from the early teens to the early twenties. Sixteen of the subjects had received orthodontic treatment. This review is only concerned with the non-treated group of twenty-four individuals. Two sets of casts were obtained from each subject. The first set was obtained at an average age of twelve years and ten months, and the other at twenty-one years and six months. No sex distinction was made in the study. They found that as age increased the majority showed a slight decrease in intercanine and intermolar breadth, in arch depth and in overbite. The authors also observed that there was a tendency for space closure and increase in crowding of the anterior teeth. The casts did not reveal changes in molar relation, but a definite tendency for the canine relation to change from an Angle Class II to a Class I position with increasing age was noted.

Sillman (1951) studied dental arch changes of two male and two female children with good occlusion selected from a group of sixty 'White' children. Serial dental casts of the children were available from birth to twelve years. He measured and described various dimensional changes of the dental arches of the subjects. Sillman suggested that a 'dimension R' could be used as 'an index of the foundation or basal structure of the maxillary dental arch'. He found that the curves plotted for each of his subjects for changes of 'dimension R' were of similar 'character and magnitude' but the timing of the changes differed from one subject to the other. Sillman (1964) reported another longitudinal study of dental arches from 750 serial dental casts of sixty-five 'normal White' children. Among them the dental records for forty individuals had been followed from birth to twenty or twenty-five years of age. The rest of the children had a shorter series of casts. Malocclusion individuals were included in the study. Sillman found that the intercanine breadth increased from birth to thirteen years in the maxilla and to twelve years in the mandible for boys. In girls the increase sometimes extended to age sixteen years and there were no significant changes after that. The intermolar breadth increased more in the maxilla than in the mandible from the deciduous dentition stage to age fourteen years in boys and sixteen years in girls. The posterior breadth of the mandibular arch was larger than that of the maxillary arch at all ages. There was an overall decrease in the arch depth - more in the mandibular than the maxillary arches - from age three to twenty-five years for both sexes. However, in boys there were increases in the maxillary arch depth between age six to ten years. Sillman also described and graphically represented the changes in mean patterns of dental arches, using the intercanine line as the base line for superimposition. Sillman (1965) studied the dental development of the oral structures of two males and two females. The subjects were selected from a group of sixty-five 'normal White' persons of New York City, and their dental records were followed from birth to age twenty-five years. Only one male and one female of the selected four individuals had good occlusion. Dental dimensions were measured from dental casts and the results were tabulated. Sillman used the 'standard score method' to describe the size of the dental arches, and suggested that specific information concerning good and poor dental development could thus be obtained.

Burson (1952) studied changes in the mandibular intercanine arch breadths of twenty-four children. There were twelve boys - six with Angle Class I and six with Class II occlusion - and twelve girls - seven with Class I and five with Class II occlusion. Measurements were made on the 239 serial casts of the selected group of children which were collected at the Child Research Council in Denver, Colorado. The periods of observation covered the time from three years before the eruption of the mandibular permanent central incisors to two years after the eruption of the permanent canines. The greatest increase in intercanine arch breadth occurred during the 'spurt', a period of accelerated increase. The time of the 'spurt' was observed to have a wide individual variation but the average was between age five and eight years. When dental age was used for data grouping there was less variability in the timing of the 'spurt'. No correlation was found between the amount of increase in mandibular intercanine arch breadth during the 'spurt' and the total amount of increase during the entire period of observation. However, it was found that children of Angle Class II type malocclusion presented with a wider variation in the total increment in the intercanine arch breadth than those with Class I malocclusion. Burson concluded: '(1) that no general rule concerning timing, duration, or amount of increase in mandibular bicanine dimension may safely be applied to a given individual; and (2) that future increases or decreases in this dimension are not easily predictable in attempting to evaluate an orthodontic problem for such an individual in this age range.'

Barrow and White (1952) analyzed the developmental changes of fifty-one children from 528 sets of serial dental casts. The children were from the University of Michigan Elementary and High Schools at Ann Arbor, Michigan. The period under study was from the 'completed primary dentition' to the eruption of all permanent teeth except the third molars. No sex grouping was presented in the findings. The dental-arch form was classified subjectively as tapered, trapezoid, square, ovoid, U-shaped and hyperboloid.

It was found that 65 per cent of the deciduous dental arches were trapezoidal in form, 30 per cent were ovoid and 5 per cent were tapered. There was little change in form from the deciduous to the early permanent dentition. From age three to five years there was little change in the intercanine arch breadth. The intercanine breadth increased very rapidly from age five to eight or nine years, slightly more in the maxillary than the mandibular arch. The intercanine breadth decreased gradually after age fourteen years. Dental-arch breadth of both arches, measured between the deciduous second molars, increased from five to ten years of age. From seven to eleven years the intermolar breadth increased both in the maxillary and the mandibular arches, but decreased from age eleven to fifteen years. Barrow and White believed that the greater decrease in the mandibular arch was due to the greater mesial movement of the mandibular permanent first molars after exfoliation of the deciduous molars. From age fifteen to seventeen years a continuous decrease in the intermolar breadth was observed in most of the subjects. Dental-arch depth of both arches decreased from four-and-a-half to six years of age. From six to twelve years the maxillary arch depth increased but the mandibular arch decreased. Between twelve to thirteen-and-a-half years both the maxillary and the mandibular arch depths decreased. The total change in arch depth from four-and-a-half to thirteen-and-a-half years was an increase of 0.2 mm in the maxilla and a decrease of 2.2 mm in the mandible.

Henriques (1953) correlated palatal and facial growth during the period of changing dentition in a cross-sectional study of 600 children of age seven to twelve years. Measurements were obtained directly in the mouth. The palatal measurements, which are of concern here, comprised seven breadth and two depth measurements. Henriques found a general progressive increase in palatal breadth with age, while the palatal arch depth anterior to the permanent first molar remained unchanged. The mean increase in intercanine breadth between ages seven and twelve years was greater for girls than boys, but the mean increase in intermolar breadth was greater for boys than girls.

Björk (1953) studied cephalometric radiographs of 243 Swedish males at the age of twelve years and again at twenty years. He found that age changes for both overjet and overbite showed pronounced individual differences in direction and magnitude. The overjet generally decreased between twelve and twenty years of age, due to a tendency for a forward displacement of the mandibular incisors in relation to the maxillary incisors. Generally the overbite also decreased with advancing age. However, there were many individual variations. Björk (1955) studied the association of bite development, dental-arch shape and body build of the group of twenty-year-old Swedish males. He observed that skeletal sturdiness was associated with dental arches large in all dimensions, with large teeth, with broad maxillary arches compared to mandibular arches, with a tendency to buccal occlusion and with early eruption of permanent teeth. Slender skeletal constitution, on the other hand, appeared to be associated with a tendency to cross-bite (lingual occlusion), where the maxillary arch was narrower than the mandibular arch. No correlation was found between skeletal factors and the degree of spacing or crowding of the teeth and no significant relation was found between dental-arch shape and the 'muscle factor'.

Dockrell, Clinch and Scott (1954) studied 189 children, 107 boys and 82 girls, aged from twelve-and-a-half to sixteen-and-a-half years. They lived in the Aran Islands situated in the Atlantic Ocean thirty miles west of Galway city. Records of full-face and profile photographs, anthropometric measurements, and alginate impressions of the dentition for dental casts were taken. The study was planned to be the first stage of a serial investigation of the children. Two measurements were made on each of the intercanine and the intermolar arch breadths. The internal measurements were taken from the 'gum margin opposite the lingual ridge' and the external measurements from the 'widest part of the labial surface' of the tooth. They found that in the maxillary arch the internal and external intercanine breadths increased from age three to sixteen years for both sexes with the exception of a decrease in internal intercanine breadth in girls between ages eleven and sixteen years. From ages five to sixteen years the maxillary intermolar breadth measurements increased for both sexes, though changes of the internal intermolar breadths for boys were not apparent. In the mandibular arch the intercanine breadths decreased from age three to sixteen years for both sexes with the exception of an increase in external measurements for boys. However, from age five to sixteen years the mandibular intermolar breadths increased for both sexes and more so for girls. The overbite and overjet of the deciduous dentition decreased with increasing age, but with the eruption of the permanent incisors the overbite and overjet increased with age.

Meredith and Cox (1954) investigated sex differences in intermolar breadths and the relationship of arch breadths between maxillary and mandibular arches. They measured dental casts of a group of 'White' children, forty-four boys and fifty girls, at age nine years. It was found that the average dental-arch breadth for boys was greater than for girls, and that the dental-arch breadth of the maxilla was greater than that of the mandible. Meredith and Hopp (1956) in a longitudinal study of seventy-seven North American White children, forty boys and thirty-seven girls, measured dental casts obtained from these subjects at ages four, six and eight years. They found that the interbuccal breadths of the deciduous second molars increased with age, that the maxillary arch was wider than the mandibular arch and that the dental arches of males were wider than those of females.

Hayashi (1956) applied a rectangular coordinate system to the analysis of dental-arch shape. He proposed the formula $y = ax^n$ to represent the curve of the dental arch, where the x-axis was the 'normal drawn at the median incisal point' and the y-axis was the 'median sagittal line'. By altering the values of 'a' and 'n' the shape of the curve could be changed to suit the required arch form for description. He tested his formula by applying it to Hrdlička's six typical dental-arch shapes, and to the dental arches of a random sample of fifty patients at the prosthetic department of the Tokyo Medical and Dental University. He found deviations between the theoretical and the actual curves of the dental arches, and derived a compensating formula to improve the fit of the curve. The concluding formula which he suggested was $y = ax^n \pm e^{\alpha(x-\beta)}$.

Scott (1957) applied MacConaill and Scher's (1949) method of describing 'normal' dental arches by catenary curves. He proposed the use of a 'catenometer' for arch shape description. The 'catenometer' consisted of a horizontal bar with two movable stops or clips from which a chain of fixed length was suspended. By adjusting the movable stops on the horizontal bar the various forms of curves could be expressed by reading off from the scale which showed the distance apart of the two stops.

Moorrees (1957) studied dental casts obtained from 156 Aleuts who were investigated in 1948. Adapting the method of Hrdlička he analysed dental-arch shape by subjective classification into six groups: hyperbolic, parabolic, ovoid, trapezoid, omega-shaped and the unclassifiable forms. Most of the dental arches were recorded as parabolic in shape - 77 per cent in the maxilla and 84 per cent in the mandible. The shape of dental arches was also described by the dental arch index. Moorrees commented that neither of the methods he used was satisfactory for the description of dental arch shapes. Dental-arch breadth and depth were measured by odontometric and anthropometric methods.

Moorrees (1959) conducted a longitudinal study of growth and developmental changes of the dentition of 184 North American White children. Dental casts of these children were collected by Dr. H.L. Stuart of the Harvard University, and Dr. R.H. Stucklen of Wilmington, Delaware. The Stuart series consisted of 132 children, fifty-nine boys and seventy-three girls, with dental casts obtained yearly or half-yearly from age about two years to eighteen years, except for a discontinuity of four years between twelve and sixteen years of age of the children because of World War II. The Stucklen series consisted of fifty-two children, twenty-five boys and twenty-seven girls with dental casts obtained yearly near the birth date of the children, from about five to eighteen years of age. A total of only twenty-eight children in the groups had malocclusions but many had missing teeth due to extractions. Concerning dimensional changes of dental arches, Moorrees found that the average dental-arch depth decreased with increasing age except during the eruption of the permanent incisors. The maxillary and mandibular arch depths decreased from age three to eighteen years for both sexes and the total decrease appeared to be more in the mandible and in girls. The dental-arch breadth increased irregularly with age and it was generally associated with the eruption of the permanent incisors, canines and premolars. He found that the mean increment of intercanine breadth from age five to eighteen years was greater in the maxilla and in the external (intercuspal) measurements. Similarly, the mean intermolar breadths increased from age six to eighteen years. At the same time the arch breadth increase was found to be greater in boys than girls and more in the maxilla than the mandible. From age five to eighteen years the maxillary dental-arch perimeter increased more in boys than girls, whilst the mandibular dental-arch perimeter decreased more in girls. Moorrees pointed out that there were wide individual variations in size and directional changes, especially during eruption of permanent teeth. These changes defied any generalization on changes of the dental arches.

Wada (1959) used a rectangular coordinate system as proposed by Numata (1947) to study 'normal' dental arches of 205 Japanese, 80 males and 125 females, aged between thirteen and forty-six years. He analysed the positional differences of the teeth between the upper and the lower jaws, and between ages and sexes.

Van Reenen (1964) made a cross-sectional study of the dental arches of 521 subjects, 277 males and 244 females. These subjects consisted of 406 Bushmen, 52 non-Europeans and 63 Europeans. Ages of the subjects were estimated according to the stage of dental eruption and general physical appearance. A full complement of teeth was present in 80 per cent of the old-aged farm Bushmen. Measurements were made from the dental casts of Bushmen for the external and internal arch breadths, arch depth and the buccolingual width of the incisors. Mean values of the maxillary and mandibular measurements and the similar previous findings from a selected group of Bantu and Whites were plotted on the same graph. He found that the external arch breadth measurements of the groups formed a curve which was catenary in shape. The internal measurements of the arch breadth also formed a curve but with two components - 'one extending through the first two centimetres and other to the posterior and terminal three centimetres of the arch length'. Arch indices were also calculated from measurements of eighty-four palates. The mean value obtained was 107.1.

Lu (1964, 1966) proposed the use of a degree four orthogonal polynomial to represent dental arches and to derive indices for classification of dental-arch form. The general polynomial equation proposed was:

$$y = b_o \mathcal{E}_o + \sum_{i=2}^{p} b_i \mathcal{E}_i + \sum_{j=1}^{q} b_j \mathcal{E}_j$$

where i = 2, 4 and j = 1, 3 and the $\mathcal{E}_i$'s or $\mathcal{E}_j$'s were the orthogonal polynomials in x. The 'taperedness' and 'squareness' of the symmetrical curve were measured by the quadratic and quartic terms, whilst the linear and cubic terms measured the 'lopsidedness' and 'tiltedness' of the curve asymmetry. Various indices of symmetry and asymmetry of the curve were derived. Lu suggested that these indices could be used to classify dental-arch form into various isomorphic groups.

Mills (1964) studied the dental arches with neutrocclusion of molars of 230 males of mixed European ancestry from the United States Naval Academy aged seventeen to twenty-one years. Measurements were made directly from the oral cavity. The mean intercanine arch breadth measurements in the maxilla and the mandible were 35.13 ± 0.20 mm and 26.00 ± 0.15 mm respectively. Other measurements were made of arch breadth at the premolar regions, of arch length, of incisor crown diameter, and of malalignment score. Mills found that there was a significant association between malalignment of teeth and arch breadth.

Barrett, Brown and Macdonald (1965) studied dental casts obtained from a group of adolescent and adult Australian Aborigines, 102 males and 102 females. Dental-arch dimensions of the subjects were compared with those of Swedes and Aleuts and were found to be larger. In general, dental-arch dimensions were greater for males than females during adolescence. Between adolescence and adulthood the dental-arch breadth was similar in the maxilla but was greater for adults in the mandible; whilst the arch depth was smaller for adults in both arches. They also found no significant sex differences in arch depth for adults. Barrett and Brown (1968) used the same material to study the shape of the dental arches by deriving dental-arch indices for odontometric and anthropometric measurements. They found that dental-arch indices differed little between adolescent males and females, but the values found for adult males were significantly greater than those for females, which indicated that adult males had dental arches broader in relation to the depth than females. The mean values of the indices were significantly greater in the adult group than the adolescent group in both sexes. Barrett and Brown suggested that age-group and sex differences in dental-arch indices were mainly due to differences in arch depth and a greater reduction in arch depth between adolescence and adulthood in males than females. No marked associations were found between breadth and depth measurements within arches, but these measurements were in close association between opposing arches which indicated the conformity of maxillary and mandibular arches within individuals. They concluded that the description of dental-arch shape by means of dental-arch indices was not satisfactory and suggested that a coordinate system of measurement and analysis could be a better approach for arch shape description and comparison.

Mills and Hamilton (1965) conducted a cross-sectional study of 326 maxillary dental casts with full complement of teeth obtained from the United States Naval Academy, Annapolis, Maryland. They determined the perimeter of the dental arch from the distal surface of the permanent first molar to the distal surface of its antimere by the parabolic formula $2\sqrt{y^2 + \frac{4x^2}{3}}$, where y was half of the intermolar breadth, and x was the arch depth measured from the mid-point of the intermolar line to the top of the interdental papilla between the central incisors. They used a small digital computer to solve the formula and compared the results obtained from direct measurement by orthodontic wire around the perimeter of the dental arch. No differences between the means obtained by either method was found, but the error for the measured-wire method was six times greater than the other. They concluded that the mathematical method was the more efficient of the two.

Burdi and Lillie (1966) undertook a cross-sectional study of the maxillary dental arch shape of human embryos aged six-and-a-half through twelve weeks. Histological slides made from the embryos were studied by light microscopy. Graphic reconstructions of the dental-arch shapes were made and photographed. They found that the maxillary dental arches of six-and-a-half to eight weeks embryos were wide and flat anteroposteriorly. By seven-and-a-half to nine weeks the dental arches increased in depth to form C-shaped arches. The embryonic dental-arch shape did not conform to the catenary curve until age nine-and-a-half to twelve weeks. Burdi (1968) studied mandibular arches by the same method. Similar changes were found to occur, except that the dental-arch shape conformed to the catenary curve at approximately eight-and-a-half weeks.

Fisk (1966) investigated the mandibular arch of twenty males aged nine to sixteen years. The subjects were selected from the serial control sample of the Burlington (Ontario) Orthodontic Research Centre. Selection was based on 'normal' mandibular development and with 'regular' or 'mild irregular' teeth in the incisor region at age sixteen years. He found that there was a general decrease in arch perimeter and that the decrease exceeded 5 mm in most of the subjects. A positive 'leeway', that is tooth widths less than the available arch space, was observed at age nine years, but two-thirds of the subjects showed negative 'leeway' at age sixteen years. Most of the subjects showed an increase in incisor irregularity with increasing age. Spontaneous correction of rotated incisors was not observed at age sixteen years.

Moss and Chase (1966) studied deciduous dental arch dimensions of twenty-two Liberian Negro children and compared their findings with those for American 'White' children. Measurements by vernier calipers were obtained from photographs of the casts and the arch circumference outlines were traced for the study of occlusion. They found that the deciduous dental-arch dimensions of the

Liberian Negro were significantly greater than those of the White children. There were no differences in the amount of incisal overbite or overjet. Diastemal spaces in the Negro series were found to be greater than in the Whites, but the differences were not analysed statistically. They also noted that it was not usual to have Angle Class II occlusion (distocclusion) in the Negro series. They agreed with Begg (1954) that dynamic factors in functional occlusion were of great significance.

Larnach and Macintosh (1966) studied 188 Australian Aboriginal skulls, 52 females and 66 males, recovered from the coast of New South Wales. Among other anthropometric measurements dental-arch indices were calculated from measurements of the 'alveolo-maxillary lengths (alveolon to prosthion) and breadths (between the two ectomolaria)'. Dental-arch indices were used to classify palates as small, medium or large.

Lear (1967, 1968, 1969) and Lear and Moorrees (1969) reported studies on buccolingual muscle forces and changes in dental-arch form. Seven male university students, aged eighteen to thirty-two years, with 'normal' occlusion and a full complement of teeth (except the third molars in some instances) were selected for investigation. Buccal and lingual forces at the premolar regions during customary oral activities were measured by specially designed strain gauge transducers for a period of twenty-four hours.

Changes in dental-arch form were assessed by superimposing tracings of the buccal surface outlines of the teeth from occlusal radiographs of the dental casts recorded before and one year after the investigation. Lear and Moorrees found in most of the subjects that the lingual forces were greater than those of the buccal. However, there was no clear-cut relationship between arch shape and the balance of the buccal and lingual muscular forces. They also found that there was no correlation between head posture during sleep and symmetry of arch shape. Attention was drawn to the need for further investigation of the threshold levels of force magnitude, force duration, rate of force application and removal, and lateral force components from occlusal contacts for a better understanding of the relationship between arch form and function.

Bowden and Goose (1968) studied 123 Liverpool families to determine the role of inheritance in the determination of palatal arch dimensions. The children selected for study were aged fourteen years or over. They found that environmental factors had a great influence in the determination of palatal arch width, whilst the hereditary factors gave only small additive effects.

Savara and Sanin (1969) reported a method for recording morphological landmarks 'independently' from dental casts, photographs or Xerox prints by the use of a modified comparator and a decimal converter connected to an IBM key punch. Measurements of the dental-arch dimensions and the size and irregularity in position of teeth could be derived by computer facilities. They suggested that the magnitude of errors was small in the proposed method. Savara (1972) reported the use of computer facilities and the Cartesian coordinate system to construct three-dimensional models of facial bones and dental arches for growth studies. He presented a brief account of the dentofacial research using computer facilities conducted by himself and his associates. Savara suggested that dental-arch shape and dental-arch perimeter could be determined by fitting a fourth-degree polynomial equation to the mesiodistal landmarks of the dental casts.

Lavelle and Foster (1969) undertook a cross-sectional study of age changes in human dental arches from 1,020 sets of dental casts of 'fairly normal occlusion' taken from a group of British children, thirty boys and thirty girls, aged four to twenty years. At age four years the dental arches of girls were wider than those of boys, but the reverse was found at age twenty years. Hence age changes of dental-arch breadth were greater in boys than in girls. In absolute terms the dental-arch depth in boys was greater than in girls, but the sex difference in age changes of dental-arch depth were not significant. They also reported that the dental-arch dimensions increased up to nine years of age in the incisor region, but little change was noticed thereafter. Foster, Hamilton and Lavelle (1969) studied dental-arch dimensions of one hundred

British 'White' children, an equal number of boys and girls, in the age range from two-and-a-half to three years. Measurements were obtained from dental casts. They found that all dimensions of the dental arches of boys were larger than those of girls except for the distance between the distal surfaces of the mandibular central incisors. Differences in measurements between sexes were often statistically significant. They also found that there was a positive correlation between tooth size and dental-arch size. Lavelle and Plant (1969) compared the left and right side of the dental-arch dimensions of eighty sets of casts of 'fairly normal occlusion' taken from an equal number of males and females of the West Midland population at the age of eighteen years. Mesiodistal crown diameters of the teeth were measured by a travelling microscope. They observed that the dimensions of the teeth and the arch length were greater on the right than the left side, though the differences were not statistically significant. However, overcrowding on the right side of the arches was found to be significantly greater in both sexes. Lavelle and associates (1970) studied age changes of dental-arch dimensions using canonical analysis of discriminants. Dental casts from 280 subjects aged between three to fifteen years were studied in seven groups according to age. Each group consisted of twenty males and twenty females. Measurements were made by dial calipers. It was found that maximal changes in overall arch dimensions were between five and seven years and between eleven and thirteen years. They suggested that these changes were related to the period of change from the deciduous to the permanent dentition.

Currier (1969) analysed dental-arch shape from radiographs of dental casts obtained from twenty-five Caucasian adults with 'ideal' occlusion. He used a computer programme for the fitting of curves of the ellipse and parabola to the outer, middle and inner curves of the dental arches. The abscissa of the rectangular coordinate system was a line connecting the distal contact points of the right and left permanent first molars, and the origin of the coordinate system was given by the intersection of the abscissa with a line connecting the buccal eminence on the left permanent first and second molars. He reported that the ellipse fitted the outer curve of the dental arches in both the maxilla and mandible, while the parabola fitted the middle curve of both arches.

Grewe, Meskin and Kenny (1969) studied variability of the intercanine breadth of 584 male and 649 female Navajo Indians whose ages ranged from six to twenty-one years. They found that the maxillary intercanine breadth increased from age six to eleven years in females and from age six to eighteen years in males, though a greater portion of the increase for males was between age six to twelve years. For the mandibular intercanine breadth the maximum mean values were found to be at age eight years for females and at age twelve years for males. At ages sixteen, seventeen and eighteen years the maxillary intercanine breadths for males were greater than for females of similar ages, but the mandibular intercanine breadths did not vary significantly at the 1 per cent level between the sexes of the same age. The relative variability was found to be least in the maxillary arches of females and most in the mandibular arches of males. The findings were similar to those for Chippewa Indian children of corresponding age and sex. Grewe and associates suggested that similar dental-arch dimensions could be observed in different tribes of one sub-type of a major racial group. In a separate investigation Grewe (1969) measured intercanine breadths using dental casts obtained from 299 females and 307 males of part-Chippewa Indian children aged between six and eighteen years. These children were classified according to age, sex, and 'their percentage of Chippewa Indian ancestry (0-49 per cent, 50-74 per cent and 75-100 per cent)'. They found no significant difference in the maxillary intercanine measurements between the 0-49 per cent and 50-74 per cent Indian groups, and in the mandibular intercanine breadths of the females. In males the intercanine breadths were found to be greater in the 75-100 per cent Indian group than those in the 0-49 per cent groups. The maximum mandibular intercanine breadth for females was at age eight years; and for males at age nine years. At age eight years the mandibular intercanine breadth of the females was greater than that of the males, but at age fifteen years the same dimension was significantly larger in males than in females. The maximum maxillary intercanine breadth in females was at age eleven years; and in males at age thirteen years. This same dimension was significantly greater in males than females at different levels of significance for ages eight, thirteen, fifteen, sixteen and seventeen years. Intercanine breadths of the 75-100 per cent Indian group were found to be significantly greater than those of the Caucasian group at corresponding ages.

Huddart, Clarke and Thacker (1971) conducted a pilot study on the maxillary arch dimensions of thirty 'normal' children at birth. Photocopies of the maxillary arch were obtained in horizontal and transverse views. They used a 'D-mac pencil follower trace analyser' to convert the lines of the diagram into a series of dots in the coordinate system. The data points were punched on to paper tapes which were fed to a computer for analysis. They pointed out that the method was fundamentally sound and the system was efficient in data analysis.

Walker (1972) described procedures for using computer facilities and coordinate systems to obtain mathematical models of lateral or frontal cephalograms of the skull, and of the dentition from the occlusal aspect. He pointed out that by using these mathematical models separately, morphological changes due to growth or due to clinical treatment could be analysed; and using them in combination, a three-dimensional model of the head, face and jaws could be formed. He reported that applications of the two-dimensional models had been used for clinical and teaching purposes.

Biggerstaff (1972) studied tooth occlusion using a Cartesian coordinate system and computer facilities to build a two-dimensional mathematical model. He analysed the magnitude and direction of the cusp-fossa and cusp-ridge relationships between selected occlusal landmarks of the posterior teeth in the maxilla and in the mandible. From 199 sets of casts duplicated from records of the semi-longitudinal twin study at the Forsyth Dental Center, sixty sets of casts, forty-seven with good occlusion (Angle Class I) and thirteen with full-cusp Angle Class II, Division I malocclusion, were photographed and analysed. He observed differences between the total population data, the good occlusion data, and the malocclusion data in terms of magnitude and direction of deviation from the 'ideal' occluding position for all of the occluding relationships. However, the data differences did not show a general pattern between the good occlusion and the malocclusion groups. He also observed that there was a sidedness to occlusion, slightly Angle Class III on the left and slightly Class II on the right. He pointed out that mathematical model used in the investigation had no clinical application but it could form a framework for a more complex three-dimensional model in future occlusion studies.

Brader (1972) studied dental-arch form in relation to intraoral forces. He suggested the use of a closed curve with trifocal elliptic properties to represent 'superior dental arch form'.

He also proposed a formula expressing the theoretical relation between natural dental-arch forms and the intraoral muscular forces acting on them.

CHAPTER 3

MATERIAL AND METHODS

MATERIAL AVAILABLE

The material available for study consisted of dental casts obtained by others in a longitudinal growth study of a group of Australian Aborigines living under settlement conditions at Yuendumu, 185 miles north-west of Alice Springs in the Northern Territory of Australia. The location of the settlement is shown on the map in Figure 1.

The total collection of 1,736 sets of casts, representing 245 males and 205 females, was assembled over a period of twenty years by workers engaged in the growth study (Barrett, 1957a, 1965; Barrett, Brown and Fanning, 1965).

YUENDUMU ABORIGINES

Yuendumu was established in 1946 as a ration depot for Aborigines who had abandoned their tribal way of life. It is a Commonwealth Government Settlement on an Aboriginal Reserve of 850 square miles under the control of the Welfare Branch of the Northern Territory Administration. There are about 1,000

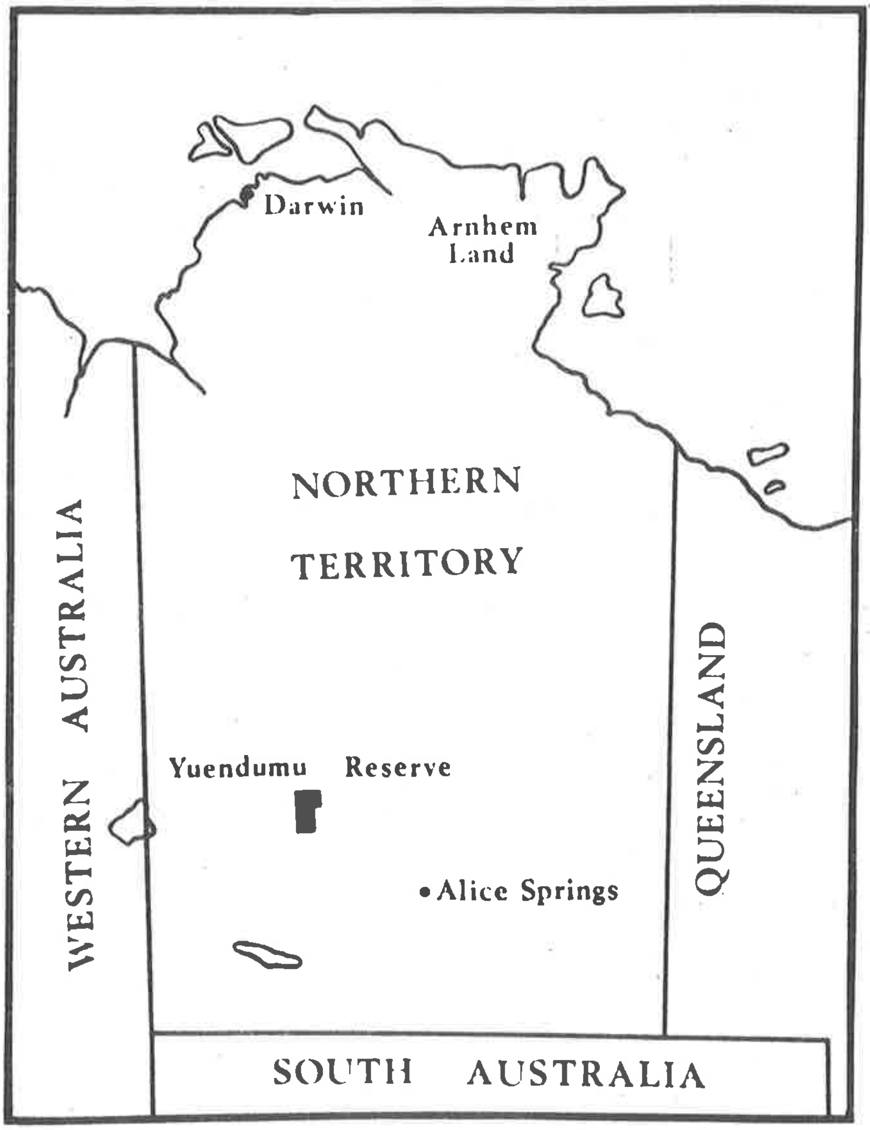

Figure 1. Map showing location of Yuendumu Settlement

Aborigines living at Yuendumu at the present time. Approximately 90 per cent of the population are Wailbri people and the remainder belong to the Pintubi tribe.

A brief general account of the present-day settlement at Yuendumu has been given by Barrett and Williamson (1972). The ecological surroundings of the people in the early days of the settlement have been described by Cleland and Tindale (1954) and the food and eating habits by Campbell and Barrett (1953) and by Meggitt (1957, 1962). The social organization of the Wailbri people and their kinship systems and marriage practices have been studied by Meggitt (1962, 1965). Family data have been studied by Tindale (1953) and by Fleming, Barrett and Fleming (1971). Physical characteristics and growth patterns have been studied by Abbie and Abey (1953a, 1953b, 1955), Abbie (1957, 1961), Brown and Barrett (1971, 1972a, 1972b), and Barrett and Brown (1971). There have been many studies of the dental and craniofacial features of the Yuendumu population: tooth formation by Fanning and Moorrees (1969), Fanning and Brown (1971); tooth emergence by Barrett (1957b), Barrett, Brown and Cellier (1964), Barrett and Brown (1966); tooth size by Barrett, Brown and Luke (1963), Barrett, Brown and Macdonald (1963a), Barrett, Brown, Arato and Ozols (1964); tooth occlusion by Barrett (1958), Beyron (1964), Barrett (1969), Brown (1969); dental arches by Barrett, Brown and Macdonald (1965), Barrett and Brown (1968); craniofacial morphology by Brown (1962, 1967), Barrett, Brown and Macdonald (1963b), Brown and Barrett (1964), Brown, Barrett and Darroch (1965a, 1965b), Gresham, Brown and Barrett (1965), Barrett, Brown and McNulty (1968), McNulty, Barrett and Brown (1968), McNulty (1968), McNulty and Morris (1970), Grave (1971), Brown, Barrett and Grave (1971); dental diseases and the prevalence of malocclusion by Barrett (1953, 1956), Cran (1955, 1957, 1959, 1960), Björk and Helm (1969), Barrett and Williamson (1972).

The Aborigines at Yuendumu are now living under conditions markedly different from the hunting and food gathering way of life of their predecessors. Most of the families live in houses but some still prefer native camps or wurlies. There is a well-staffed hospital and child welfare centre at Yuendumu. Children from about age five attend school. Many of the adults are employed in settlement jobs. In recent years the money income of the community has increased because of higher wages and social services allowances. Food and clothing are purchased from a local store. Although there is a communal dining hall where cheap meals are available, few make use of this amenity. Most of the Aborigines prefer to purchase foodstuffs and prepare their own meals in their houses or camps.

The community water supply has about 0.8 parts per million fluoride. This has limited the dental caries occurrence that could be expected to result from consumption of European food with a large content of refined carbohydrates. The methods of food preparation used by the Aborigines are crude. For example, dampers are cooked in the ashes of a fire on the ground. Meals are taken seated on the ground. With the unintentional inclusion of abrasive material in the food - grit and ashes - tooth attrition occurs. But it is much less than can be seen in museum skulls of Aborigines who lived under native conditions.

The influence of settlement living conditions on teeth and dental arches will be referred to in later discussion of the findings of the investigation.

SELECTION OF MATERIAL

The serial dental casts of 28 males and 16 females were studied in the present investigation. The subjects represented by these casts met the following criteria: (1) they were of pure Australian Aboriginal ancestry; (2) their birth dates were recorded; (3) their records covered the age period about seven to seventeen years; and (4) their dental-cast series included at least one set of casts obtained about the time of emergence of the maxillary lateral incisor and another set obtained after the exfoliation of the last of the deciduous teeth and before the emergence of the third molars. The subjects were the same as those selected by Grave (1971) for his study of growth and ossification because they had no physical deformities and their serial hand and wrist roentgenograms and lateral cephalometric roentgenograms were sufficient in number for his investigation.

The 44 subjects were represented by 340 sets of dental casts. All of the casts were photographed preliminary to obtaining measurements but only 267 sets were measured. These 267 sets were selected because they were obtained within the period after emergence of the maxillary lateral incisors and before emergence of the third molars - a period of dental development selected for the present investigation. The other 73 sets were not measured because 69 sets of them were obtained either before or after the selected period of dental development, and the remaining 4 sets, though within the period, had missing permanent first molars or central incisors. The presence of these teeth was essential to provide reference points for the determination of polynomial equations used in the investigation to define arch shape.

DENTAL GROUPINGS

The dental casts of the series for each subject were categorized in four dental groups determined by tooth emergence status. Table 1 shows the dental criteria used to group the subjects. These dental groups form part of a more extensive system used in other investigations of the longitudinal material (see Appendix 1).

Table 2 shows the means and ranges of ages for subject observations in the four dental groups. Because the mean values were calculated by considering each observation on a subject as an entity they do not represent ages at which the subjects attained the various stages of dental development. In most instances a subject was observed on more than one occasion while still categorized in a particular dental group. Therefore the mean ages shown in Table 2 are greater than ages of attainment.

SUBJECTS AND MATERIAL STUDIED

Table 3 shows the distribution of subjects studied - by sex and by numbers of dental casts photographed and measured. Table 4 shows the distribution of dental casts measured - by sex and by dental groups.

Table 1. Dental groupings determined by tooth emergence status

Group	Criteria
A	Mixed dentition - at least one permanent maxillary lateral incisor present
B	Mixed dentition - some but not all primary canines or molars exfoliated but no maxillary second premolars present
C	Mixed dentition - some primary teeth present and at least one maxillary second premolar present
D	Permanent dentition - excluding third molars

Groups A, B, C and D correspond to codes 33, 41, 42 and 51 respectively of a more extensive dental grouping system (see Appendix 1).

Table 2. Means and age ranges of subject observations in the four dental groups

Group	Sex	Age in decimal years	
		Mean	Range
A	M	9.0	7.3 - 10.6
	F	8.4	7.6 - 9.6
B	M	10.8	9.1 - 12.7
	F	10.3	9.1 - 12.6
C	M	12.0	10.0 - 13.7
	F	11.0	9.9 - 13.6
D	M	13.6	10.1 - 17.3
	F	13.0	10.4 - 16.1

Table 3. Distribution of subjects studied - by sex and by number of dental casts photographed and measured

	Number of subjects	Number of dental casts	
		Photographed	Measured
Males	28	226	172
Females	16	114	95
Total	44	340	267

Table 4. Distribution of dental casts measured - by sex and by dental groups

	Numbers of dental casts by dental groups				
	A	B	C	D	Total
Males	32	35	12	93	172
Females	18	22	6	49	95
Total	50	57	18	142	267

DENTAL STATUS OF SUBJECTS

The oral and dental health of 309 children and young adults at Yuendumu was surveyed in 1970 by Barrett and Williamson (1972). Included in the survey were 38 of the 44 subjects studied in the present investigation.

For the sample of 38 the mean number of DMF (decayed, missing, filled) permanent teeth was 1.6. The low caries prevalence is emphasised by the distribution of subjects according to the number of DMF teeth: 14 with none; 6 with 1; 9 with 2; 4 with 3; 3 with 4; 1 with 5; and 1 with 6.

Reference has already been made to the presence of 0.8 parts per million of fluoride in the community water supply. As well as its limiting effect on dental caries occurrence this concentration of fluoride has led to endemic dental fluorosis in the community (Williamson and Barrett, 1972). Ratings by the classification of Dean (1934) were found in 1970 for the sample of 38 to be as follows: 19 subjects with 'very mild'; 10 with 'mild'; and 9 with 'moderate' dental fluorosis.

The extent of gingival and periodontal disease was assessed in the 1970 survey by means of periodontal index ratings (Russell, 1956). Index values for the sample of 38 ranged from 0.5 to 2.8 with an average value of 2.0. Food debris and dental plaque were found on the teeth of all subjects. Mild gingivitis was universally present.

Tooth attrition as it affected the occlusal surfaces of the permanent first molar teeth was assessed in the 1970 survey by means of attrition index ratings (Davies and Pedersen, 1955). Attrition index values for the sample of 38 ranged from 0.5 to 2.0 with an average value of 1.4.

The latest set of dental casts in the series for each of the 44 subjects of the present study was examined to determine the antero-posterior relation of the maxillary and mandibular permanent first molars as classified by Angle (1907). It was found that 36 (81.8 per cent) presented a Class I molar relation; 3 (6.8 per cent) a Class II relation; and 5 (11.5 per cent) a Class I relation on one side of the arch and a Class II relation on the other side. Ten subjects (22.7 per cent) were considered to have malocclusion, mainly due to tooth crowding.

INDIRECT MEASUREMENT METHOD

Coordinate measurements of reference points defining maxillary and mandibular arch shape were obtained from each set of casts. Polynomial expressions approximating the shapes were computed from these values and subsequently used to derive measurements of various arch dimensions. The use of register points common to the maxillary and mandibular casts when related in the position of maximum intercuspation enabled superimposition of curves derived from the polynomials and calculation of values representing the inter-arch relation.

This indirect method of obtaining measurements from dental casts has several advantages. First, there is a limited number of measurements to be obtained initially, that is, the coordinate measurements. And special care is paid to ensure the accuracy of these few measurements. Values for many additional variables can be calculated to an accuracy of the same order as that of the coordinate measurements. Secondly, the additional variables can be defined objectively in mathematical terms related to the definitions of the reference points used to obtain the coordinate measurements. Thirdly, when the sample size is large the indirect method saves considerable time.

COORDINATE MEASUREMENTS

In general outline the procedures for acquisition of coordinate data from the dental casts consisted of preparation of the dental casts, photographing the casts, and obtaining coordinate measurements by means of a record reader. Definitions of the terms used and details of the procedures follow.

DEFINITIONS

Reference points

The reference points were selected morphological features used to describe the location of teeth. They are illustrated in Figures 2 and 3 and specified in the curve definitions which follow.

Vestibular curve

The vestibular curve was defined as a curve passing through the mid-points of the incisal edges of the central and lateral incisors, the tips of the canines, the tips of the buccal cusps of the premolars, and the tips of the mesiobuccal cusps of the maxillary molars or the distobuccal cusps of the mandibular molars. The vestibular curves are illustrated in Figure 4.

Sulcus curve

The sulcus curves were separate right and left curves passing through the mid-points of the central developmental grooves of the premolars and the central pits of the central fossae of the molars in the maxillary dental arch. The sulcus curves are illustrated in Figure 4.

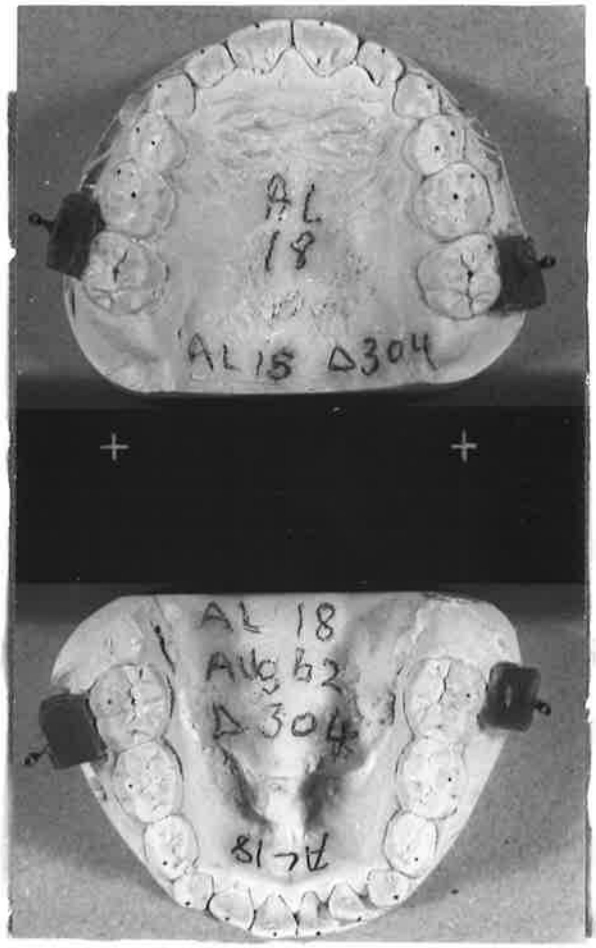

Figure 2. Early mixed dentition - reference points, register points and scale points

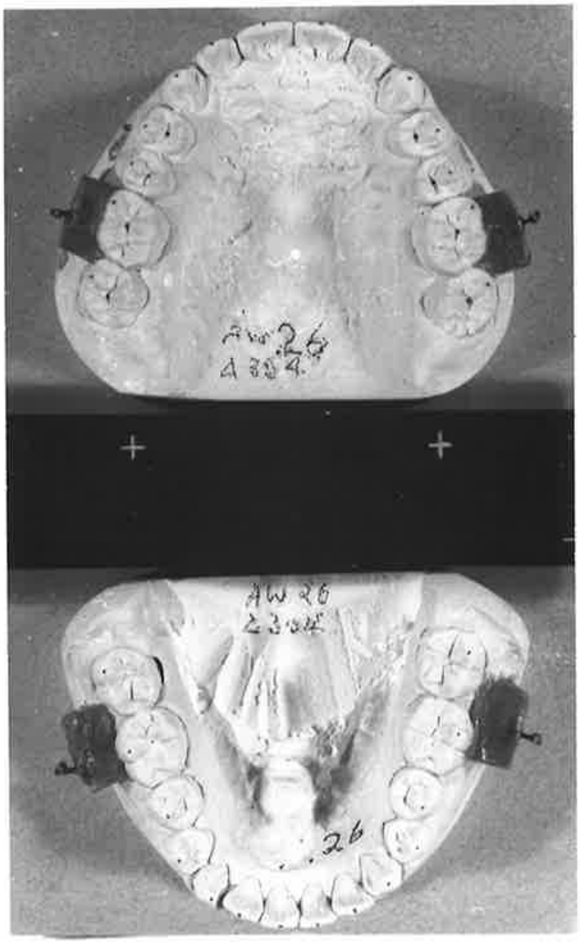

Figure 3. Early permanent dentition - reference points, register points and scale points

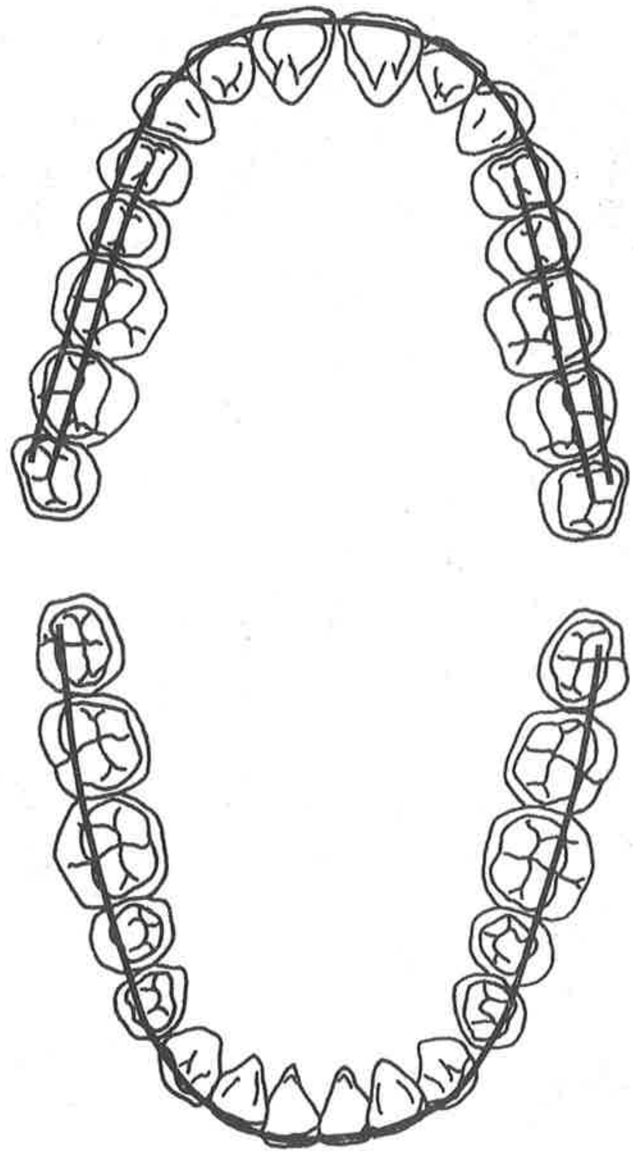

Figure 4. Vestibular and sulcus curves

Register points

The register points were points common to the maxillary and mandibular casts when related in the position of maximum intercuspation (Figures 2 and 3). Register points were established to enable superimposition of maxillary and mandibular curves derived by computer from coordinates of the reference points (Figure 5).

Scale points

The scale points were located by two white crosses 5 cm apart on a black plastic strip placed at the same level as the occlusal surface of the cast when the cast was photographed (Figures 2 and 3). Scale points were included in the photographs of the casts to enable computer rescaling of the coordinate values read from the reference and register points.

Occlusal plane

The occlusal plane was defined as a plane which included a point on the median line of the maxillary arch at the level of the incisal edges of the central incisors as well as the central fossa points of the maxillary right and left permanent first molars (Figure 6).

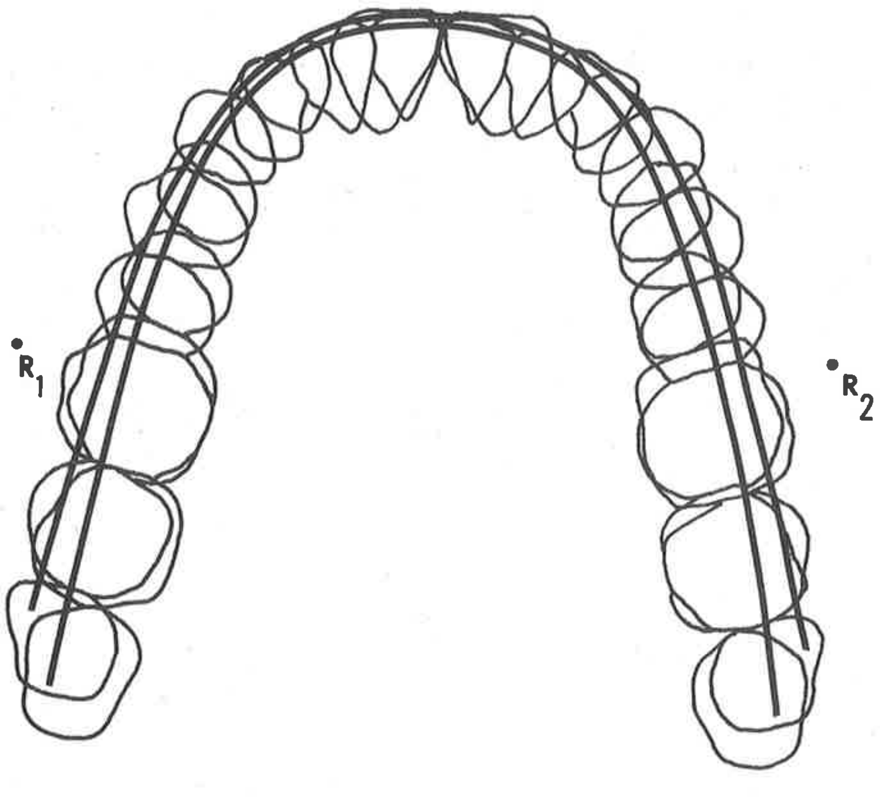

Figure 5. Superimposed maxillary (outer) and mandibular (inner) vestibular curves and register points (R_1 and R_2)

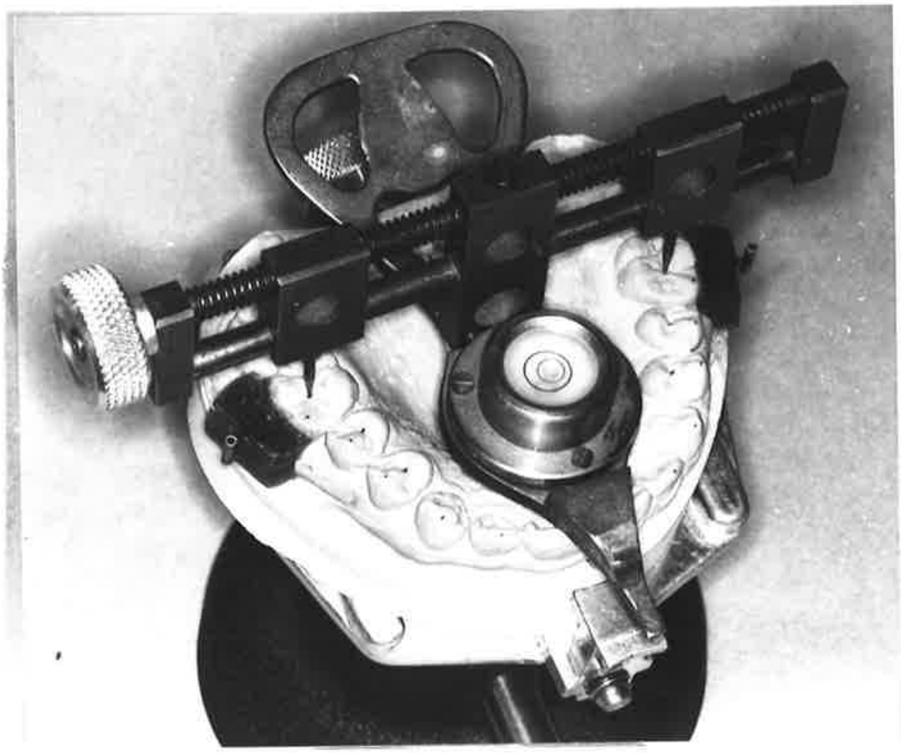

Figure 6. Tripod device for levelling the maxillary occlusal plane.

PREPARATION OF DENTAL CASTS

The dental casts were carefully examined and prepared to eliminate any artefacts introduced during the casting procedures and to ensure that the maxillary and the mandibular casts could be brought together into maximum intercuspation without any interference.

The reference points previously specified were marked on the casts using a sharp soft pencil. Care was taken to ensure that the dots were small and clearly identifiable (Figures 2 and 3).

Blocks of soft red modelling wax were attached to the casts for the insertion of the registration markers. A sheet of wax was softened and rolled into a stick of about 1 cm in diameter. The wax stick was then cut into pieces about 2 cm in length. While still soft the wax pieces were moulded into blocks of squarish cross-section. Two wax blocks were softened and attached to the maxillary cast at the buccal surface of each permanent first molar. The mandibular cast was then brought into position of maximum intercuspation with the maxillary cast and two additional wax blocks were attached in positions corresponding to those on the maxillary cast. Care was taken that the maxillary and mandibular was blocks were close together but not touching when the casts were occluded (Figure 7).

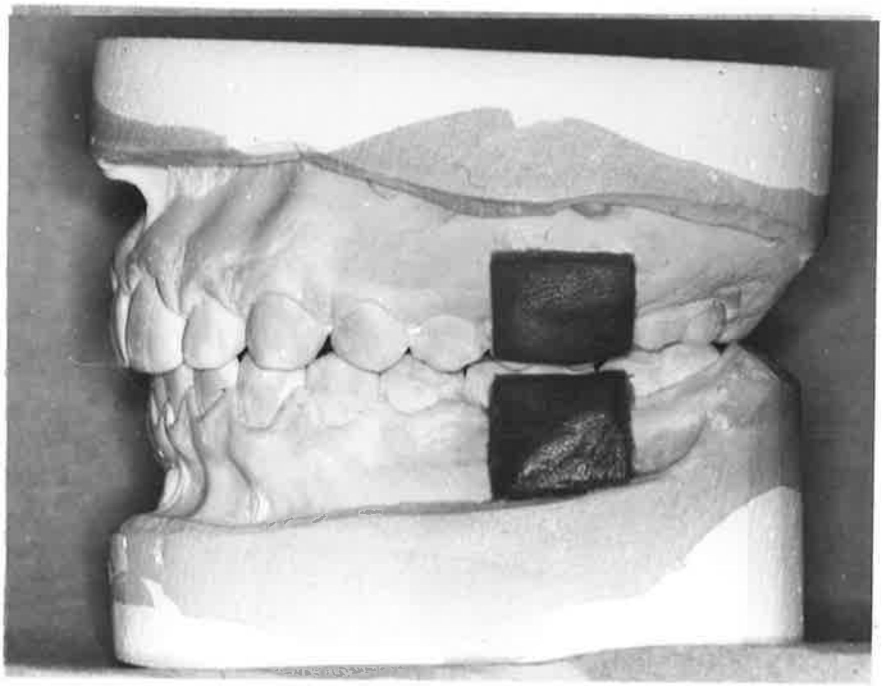

Figure 7. Wax blocks for the attachment of registration markers

To establish register points for the casts two pairs of registration markers made of orthodontic molar buccal tubes (0.25 in. x 0.036 in. internal diameter) were used (Figure 8). Each tube was soldered at right angles to a piece of stainless steel round wire (0.036 in. diameter). A short length of wire soldered to the free end of the wire formed a stem for attachment to the wax blocks.

The maxillary and the mandibular casts were occluded in position of maximum intercuspation and held together by hand. Two registration markers were aligned by inserting a special locking device (Figure 8) formed from stainless steel wire (0.036 in. diameter). The stems were heated and inserted into the wax blocks, one attached to the maxillary cast and the other to the mandibular cast. While the wax was setting the long axis of the aligned tubes was held perpendicular to the occlusal plane of the casts. The aligned markers were held in position until the wax had set. A similar procedure was followed to attach another pair of markers on the opposite side of the casts (Figure 9). The locking device was released and the casts were separated. The centres of the holes of the marker tubes constituted the register points.

PHOTOGRAPHING DENTAL CASTS

The objective was to photograph each cast under carefully controlled standard conditions. Prior to photography the cast was

A

B

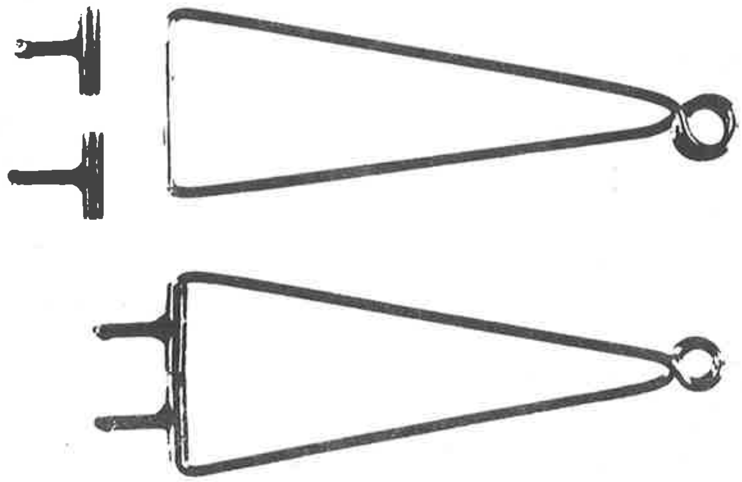

Figure 8. Registration markers and locking device used to hold the markers during attachment to wax blocks.
A - the three components; B - the assembled components.

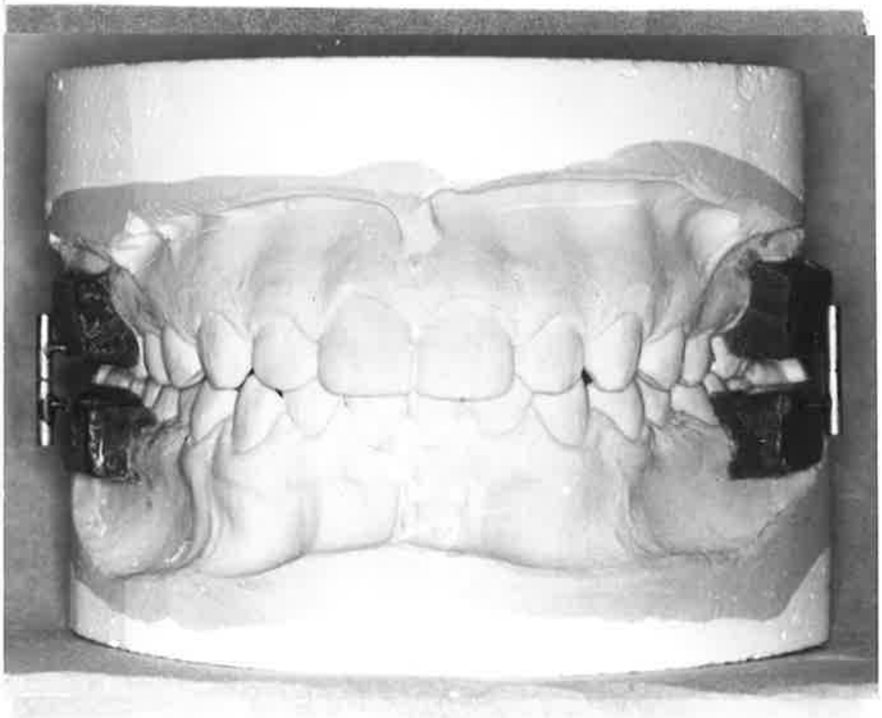

Figure 9. Dental casts with registration markers attached

aligned so that the occlusal plane was parallel with and at a standard distance from the film plane of the camera and was positioned centrally in relation to the optical axis of the camera lens.

The photographic apparatus is shown in Figure 10. It consisted of a rigidly constructed framework supporting a machined, mild steel, reference base and a vertical extension with camera stage on which a 6 cm by 6 cm single lens reflex camera* was attached by means of a rack and pinion bellows extension. The level of the reference base was adjustable and illumination was provided by two desk lamps attached to the framework.

The reference base of the photographic apparatus was levelled with the aid of a tripod device, and thus became the reference plane. The levelling tripod consisted of two metal points, connected by a screw mechanism so that the distance between the points could be varied, and an extension forming the third leg of the tripod. A spirit level formed part of the device (Figure 6).

The occlusal plane was adjusted parallel with the reference plane.

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* Hasselblad 500C with 120 mm f5.6 Zeiss S-planar lens.

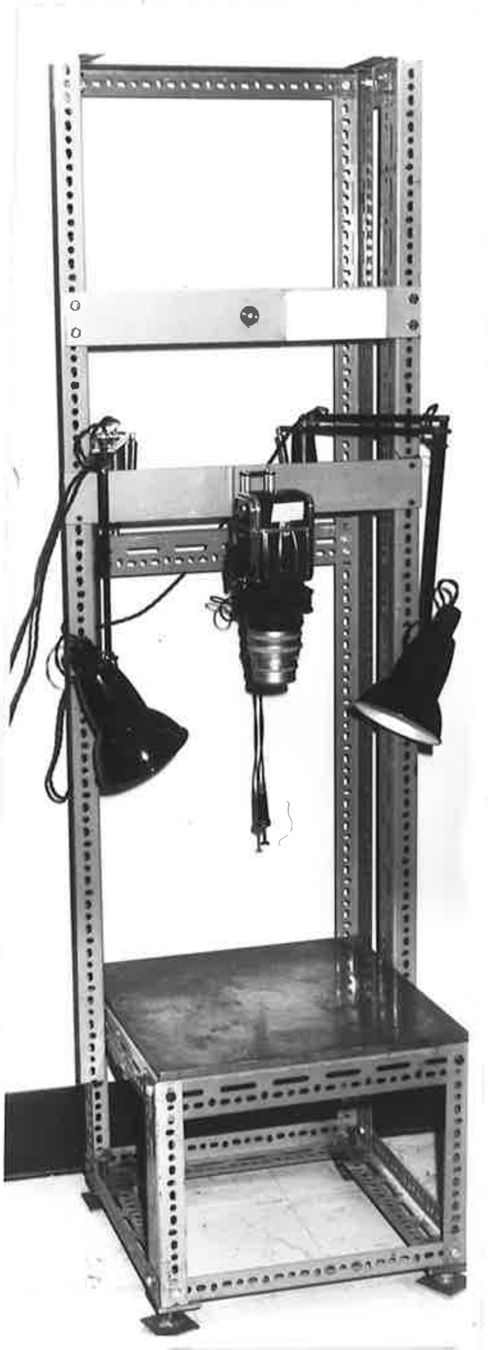

Figure 10. Photographic apparatus

The maxillary cast was attached to a cast surveyor model clamp* and the levelling device adjusted so that the legs of the tripod rested on the occlusal plane reference points. The model clamp was placed on the reference base of the photographic stand and adjusted by means of its universal joint mechanism so that the occlusal plane was level (Figure 11).

The mandibular cast was attached to a second model clamp and placed on the maxillary cast in the intercuspal relation. The inverted mandibular model clamp was adjusted by means of its universal joint so that its base was level, again using the levelling tripod (Figure 12).

With its clamp, the cast to be photographed was placed on the level platform of a rise and fall device and its height adjusted by means of a rack and pinion so that the incisor reference point of the maxillary cast or the reference point of the permanent first molar of the mandibular cast was level with the upper surface of the open frame of the device. The scale plastic strip was placed to the front of the open frame.

A label identifying the cast was placed on the plastic strip (Figure 13). The maxillary cast of the set was photographed first,

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* New Manufacturing Company, Hartford, Connecticut.

Figure 11. Model clamp and tripod device used to level the maxillary occlusal plane

Figure 12. Method used to level the base of the model clamp attached to the mandibular cast

Figure 13. Device for adjusting height of occlusal plane prior to photography

followed by the mandibular cast. The right side of the cast was always placed to face left.

MEASUREMENTS

The negative photographic film strip was mounted in a projector forming part of the record reader*. Images of the pictures were projected on the screen for recording reference points, register points and scale points.

A semi-automatic record reader with decimal converter was used to obtain coordinate measurements of each of the reference points, register points and scale points in turn (Figure 14). Readings of x and y coordinates were obtained from the image of the photograph of the casts by manually aligning two hairlines to intersect over each point in specified sequence. Activating a button switch automatically punched out the coordinates on to data cards ready for computer entry.

Orientation of the photographic images in a standard relation to the coordinate axes of the record reader was not necessary because the set of point coordinates was subsequently aligned to a specified standard by computer. However, for ease of data

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* Oscar F/DLF Strip Chart and Film Digitizing System, Computer Industries Inc., Graphic Systems Division, Van Nuys, California.

Figure 14. Semi-automatic record reader

collection the screen images of the casts were orientated to have the anterior arc of the maxillary dental arch upwards and that of the mandibular arch downwards.

The coordinates of the required data points were read and punched on to cards in the following sequence: the maxillary right register point, the right scale point, the left scale point, the left register point, the maxillary reference points along the vestibular curve from the last molar on the right to that on the left, and along the sulcus curves from the last molar on the right to that on the left, the mandibular reference points from the last molar on the right to that on the left, the right mandibular register point, the right scale point, the left scale point and the left register point (Figure 15).

To provide a check on the accuracy of the coordinate measurements repeat readings were obtained from the cast photographs. It was necessary to rewind the film before the second determination could be made.

COMPUTING PROCEDURES

In general outline the computing procedures consisted of checking and refining the raw data, approximating polynomial curves, deriving metric variables, and comparing various dental-arch

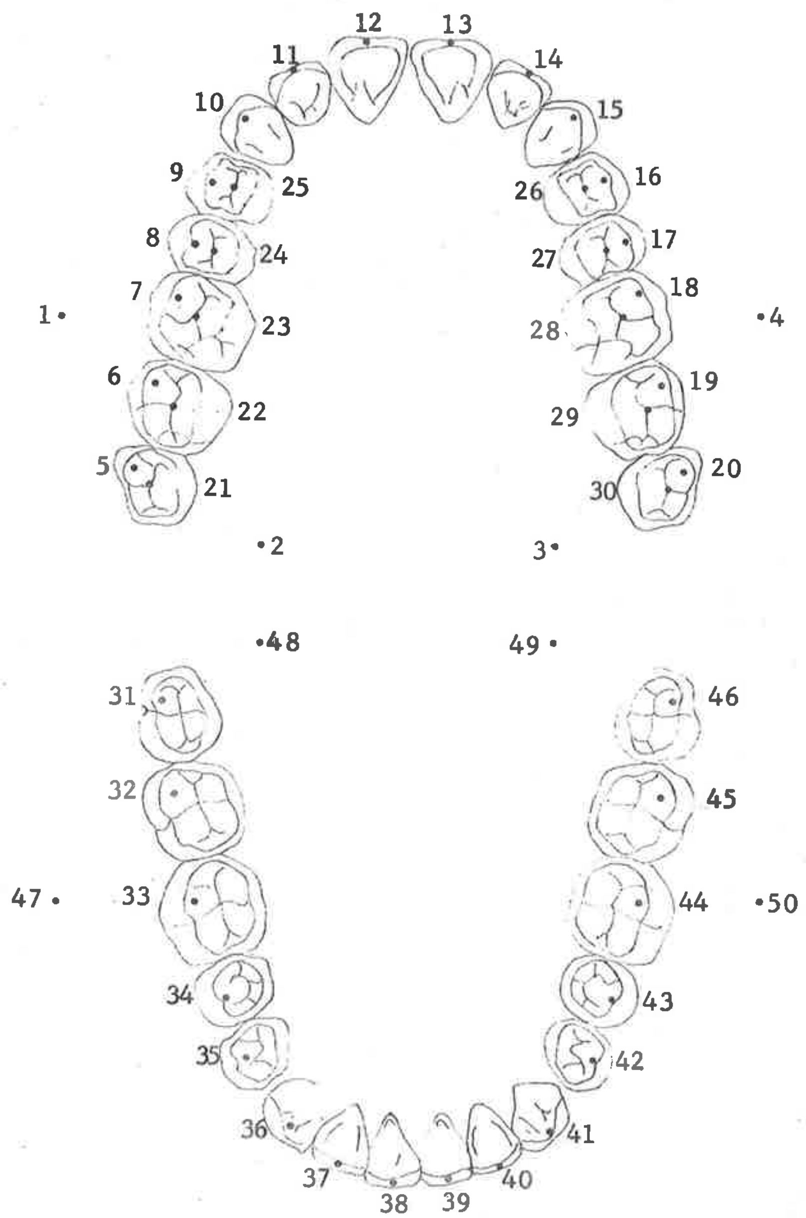

Figure 15. **Data points recording sequence**

dimensions. Definitions of the terms used and details of procedures follow.

DEFINITIONS

Intermolar line

The intermolar line was defined as the line joining the right and left permanent first molar reference points - the central fossa points for the maxillary arch and the distobuccal cusp points for the mandibular arch.

Abscissa and ordinate

The abscissa was defined as the line coincident with the intermolar line; and the ordinate as the line at right angles to the abscissa and passing through the mid-point of the intermolar line (see Appendix 2).

Dental-arch breadth

Dental-arch breadth was defined as the distance between the points of intersection of the vestibular curve on the abscissa, that is, the x-intercepts of the curve (Figure 16).

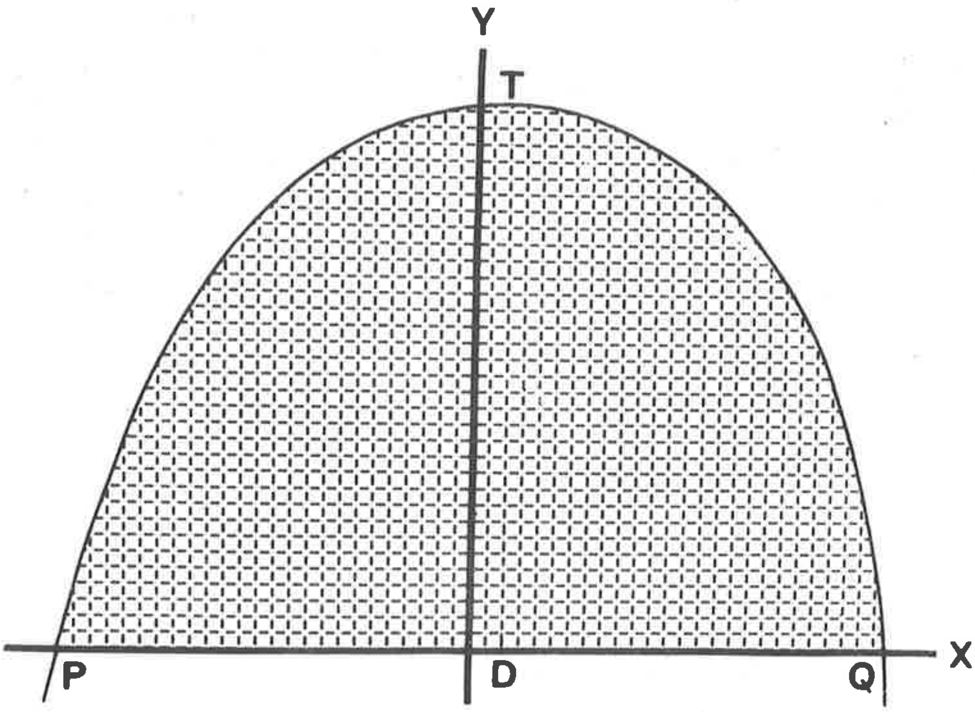

Figure 16. Dental-arch dimensions: arch breadth, PQ; arch depth, TD; arch perimeter, PTQ; and arch area, shaded area

Dental-arch depth

Dental-arch depth was defined as the distance between the abscissa and the point of intersection of the vestibular curve on the ordinate, that is, the value of the y-intercept of the curve (Figure 16).

Dental-arch perimeter

Dental-arch perimeter was defined as the length of the vestibular curve between the points of intersection of the vestibular curve and the abscissa (Figure 16).

Dental-arch area

Dental-arch area was defined as the area of the dental arch included within the vestibular curve and the abscissa (Figure 16).

Polar vectors

Polar vectors were defined as linear distances from the origin of the coordinate axes to the points of intersection of the vestibular curve with lines at 0°, 30°, 60°, 90°, 120°, 150° and 180°. The seven polar vectors were designated respectively as the left molar, left premolar, left canine vectors, the incisor vector, and the right canine, right premolar and right molar vectors (Figure 17).

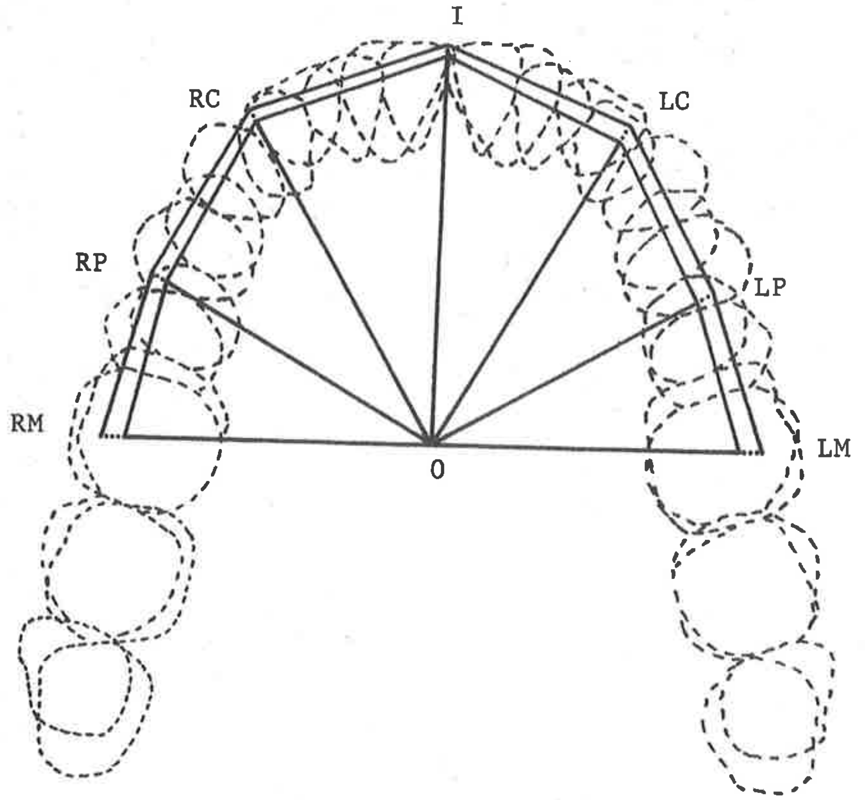

Figure 17. Maxillary and mandibular polar vectors, dental-arch heptagons and polar overjets

Dental-arch heptagon

Dental-arch heptagon was defined as the polygonal representation of the shape of the dental arch formed by lines joining the points specified for the polar vectors (Figure 17).

Polar overjets

Polar overjets were defined as the differences between maxillary polar vectors and their corresponding mandibular polar vectors when the arches were opposed in the intercuspal or functional relation (Figure 17).

DATA CHECKS

The punched cards output from the record reader were first checked by computer to ensure that they were in proper sequence. Each set of data in turn was transformed by translation and rotation so that the mid-point of the intermolar line coincided with the origin of the new coordinate axes and the intermolar line coincided with the abscissa. With reference to the known distance between the scale points the data were rescaled to their natural values.

The two sets of transformed and rescaled data obtained by duplicate readings were compared to reveal discrepancies. Mean values were calculated for each of the point coordinates if the differences between the original and the repeat observations in their transformed and rescaled values did not excees a specified, acceptable level of discrepancy. Otherwise, the computer rejected the observations. It was then necessary to perform another double determination for all the reference points in that particular set of records for another data check. The mean values constituted the refined data for subsequent calculations and they were stored on magnetic tape.

POLYNOMIAL CURVES

The general form of the polynomial equation used in the present investigation is as follow:

$$y = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots\ldots + a_{n-1}x^{n-1} + a_nx^n$$

where n is the number denoting the degree of the polynomial.

The following is a brief outline of the method used to select polynomial equations approximating the dental-arch curves. Chapter 4 gives a fuller account of the use of polynomial equations for this purpose and details the results of a pilot study on methodology.

Ten curves ranging from degree one to degree ten were derived from each set of coordinate points by applying the least squares method of curve fitting. Only ten curves were computed because polynomials of higher degree than ten generally did not produce smooth curves to fit the twelve to fourteen coordinate points of a dental arch.

The y-intercepts of each of these ten curves were then compared with the y-intercept of a straight line passing through the central incisor reference point coordinates. The curve which gave the minimum but positive figures in the comparison was selected as the curve best describing the shape of the dental arch as specified by the coordinate points.

METRIC VARIABLES

Having obtained shape-representing polynomial equations specific for each of the dental arches studied the next procedure was to compute the metric variables which were defined previously (see page 72). The following is an outline of the mathematical principles involved in the computations.

Dental-arch breadth

Newton's method of root finding was used to determine the values of the x-intercepts near the permanent first molars. Calculations were made according to the formula:

$$x_2 = x_1 - \frac{f\,(x_1)}{f'(x_1)}$$

where x_1 = a value close to the true value of one of the roots of x of the given equation; and

x_2 = the calculated value which was closer than x_1 to the true value of the same root of x of the given equation.

For the present study, the values of x_1 were taken to be the same as the x-coordinates of the central fossa reference points of the maxillary permanent first molars, or the tip of the distobuccal cusp of the mandibular permanent first molars.

By substituting x_2 into the formula for x_1 a more accurate new value of the required root of x was determined. The required x-intercepts were determined by an iterative procedure.

The value of the dental-arch breadth was given by the sum of the absolute values of the x-intercepts obtained.

However, when the permanent second molar reference points were not available for some of the maxillary dental arches the polynomial equations selected to describe their shapes might not intersect the abscissa; or might produce x-intercept values too large or too small to be acceptable. Under such circumstances the x-intercepts of the curves were not determined by the usual procedures. Instead, the values were taken as the x-intercepts of the lines tangent to the curves at the points where the x-coordinate values were equivalent to the x-coordinates of the central fossa reference points of the maxillary permanent first molars. Newton's method of root finding was applied without iteration to obtain new values of x which were the x-intercept values of the tangents to the curves at the selected points.

Dental-arch depth

Dental-arch depth (the y-intercept of the curve) was equivalent to the value of y when x = 0 in the equation. Thus, the dental-arch depth was taken to be the same as the value of the constant term in the polynomial equation.

Dental-arch perimeter

To calculate the perimeter, the curve between the known x-intercepts was divided into segments of small increments of x. From one of the values of the x-intercepts and the known small increment of x, the increment of the y value could be obtained. Then based on Pythagoras theorem the length of the small segment of the curve was given by the square root of the sum of the squares of the x and y increments. The perimeter was then determined as the sum total of the lengths of the small segments of the curve between the x-intercepts.

Dental-arch area

Determination of the area of the dental arch was based on the following mathematical equation:

$$\text{Area} = \int_p^q f(x)\,dx$$

where f(x) was the polynomial equation and p and q were the x-intercepts of the required portion of the curve.

Polar vectors

The values of the incisor vector were equivalent to the y-intercepts of the polynomial equations; and the molar vectors the x-intercepts. The canine and premolar vectors were obtained by (1) locating the coordinate points on the curve which formed the ratio y:x = tan 60° or tan 120° for the canine vectors and y:x = tan 30° or tan 150° for the premolar vectors, and (2) calculating the square root of the sum of squares of the values x and y at each of the located points.

Polar overjets

Values of the polar overjets were obtained by computing the differences between the maxillary polar vectors and the corresponding mandibular polar vectors with the arches in the intercuspal or functional relation.

STATISTICAL METHODS

For each of the variables studied its mean and standard deviation were computed. Differences between the means of any two groups were assessed for their statistical significance by the 'Student's' t test. Standard statistical methods were used (Bailey, 1959).

ERRORS AND LIMITATIONS OF METHOD

Errors of method were of two types: observer errors and systematic errors. The following section defines the terms used and gives details of procedures for minimizing the errors. Reference is made to other limitations of the method.

DEFINITIONS

Linear discrepancy

Linear discrepancy was defined as the linear distance between the coordinate measurements of a point on repeated determinations.

Tolerance value

Tolerance value was defined as the permissible variation in linear discrepancy. For the present investigation the tolerance value was set at ± 0.5 mm (see Appendix 3).

OBSERVER ERRORS

Identifiable sources of observer errors were: (1) bias in marking reference points, (2) errors in relating maxillary and mandibular casts in the position of maximum intercuspation, (3) frank measurement mistakes, and (4) inconsistencies in locating reference points during measurement.

In some of the dental casts where tooth attrition had led to a wearing of cusp summits there was difficulty in locating all of the reference points with the same degree of confidence. When marking the reference points with pencil a judgement had to be made on their locations within wear facets. Hence there was the possibility of bias. The magnitude of the observer error from this source was not estimated.

Since there were no interocclusal records of the intercuspal relations of the dental casts, a judgement had to be made on their occlusion in position of maximum intercuspation prior to the establishment of registration markers on the casts. Hence there was the possibility of errors. However, these errors were considered to be small because (1) the dental arches being intact provided a positive guide to the position of maximum intercuspation, (2) reference to other sets of casts in the series for the same subject could be made if any uncertainty was encountered, and (3) reference to the lateral cephalograms of the subject could be made.

Double determinations of coordinate values of each set of reference points were obtained. Differences between the sets of coordinate data obtained by the same observer on different occasions were used to determine the linear discrepancies. A computer programme checked the coordinate data. Any double set of data with one or more reference points having a linear discrepancy which exceeded the tolerance value was discarded and double determinations for all the reference points in that set of records were repeated for a new analysis. By this means any frank mistakes in the reading of reference points were eliminated. Thus the accuracy of the recorded locations of the reference points was improved.

Mean coordinate values of each reference point were derived by averaging separately the abscissa and the ordinate values of the first and second locations of the reference point on repeated determinations. The mean coordinate values thus obtained were accepted as the best estimates of the locations of the reference points marked on the casts.

SYSTEMATIC ERRORS

Systematic errors arose from the limitations of instruments and measuring methods. Standardized equipment and technical procedures were used in the present investigation to minimize these errors. The following outlines the procedures for minimizing errors in the use of registration markers for locating the maximum intercuspal relation of the maxillary and the mandibular casts, in the photography of the dental casts, and in the use of the record reader for measuring the coordinate points.

Malalignment of the registration markers prior to photography would affect the study of the maximum intercuspal relation of the maxillary and mandibular casts. To minimize errors from this source care was taken to properly align and retain the markers in their correct positions until the molten wax around the marker stems had set (see page 57). After the casts had been separated care was taken to avoid any disturbance to the markers prior to photography (see Appendix 4).

Photography of the dental casts was carried out under standardized condition. Photographic distortion was considered to be negligible because the Zeiss S-planar lens used for the photography was capable of producing a distortion-free image even at close object to film distances.

The record reader used in the present investigation was a sensitive and reliable instrument for reading coordinate points. The same field area of the screen was used during the whole operation of data reading and a new calibration to the same scale was made at every seating. Occasionally the reader generated errors. However, these were detected either immediately by the unusual operating sound of the record reader when the coordinate values were digitized or by the process of computer checking the double determinations of the coordinate values. If errors were found double determinations of the coordinate points were repeated for another data check.

OTHER LIMITATIONS OF THE METHOD

Limitations of the method other than observer errors and systematic errors have been discussed in sections dealing with the abscissa and ordinate (see Appendix 2), and dental-arch breadth (see page 78). The following Chapter on polynomial equations also discusses some limitations of the use of polynomial equations for dental-arch description.

CHAPTER 4

POLYNOMIAL EQUATIONS

A brief outline of the method used in the present study to select polynomial equations approximating the dental-arch curves has already been given (p. 77). The following is a fuller account of the use of polynomial equations to represent dental arches for analyses of their size and shape. Results of a pilot study on methodology are presented.

SHAPE DESCRIPTION

The use of a mathematical equation to represent a shape is not uncommon. To students of mathematics the equation $x^2 + y^2 = r^2$ describes a circle. But to others not conversant with mathematics the equation means very little. However, the shape it represents becomes obvious when the equation is transformed into its graphical form with the whole circle plotted on paper. Similarly, a polynomial equation derived from the reference points of a dental arch does not convey to the reader the shape it describes because it is not presented in a common recognizable form. But when its curve is plotted the shape is revealed.

The polynomial equation in its mathematical form is not without values in its contribution to the description of dental-arch shape. Two characteristics of the curve it describes can be recognized without plotting the equation, namely, the symmetry and the depth of the dental arch.

A symmetrical curve is represented by a polynomial equation with no odd powers of x. The graph of such an equation will have the right half of the curve exactly the mirror image of the left half. Figure 18 shows a curve plotted from the polynomial equation $y = 99 - 20x^2 + x^4$, with only even powers of x. The curve is symmetrical with reference to the ordinate and the depth of the curve is the y-intercept, equivalent to the value of the constant in the equation.

An asymmetrical curve is represented by a polynomial equation with odd as well as even powers of x. The curve will skew to one side. Figure 19 illustrates a curve plotted from the polynomial equation $y = 99 - x - 20x^2 + x^4$, with both odd and even powers of x. The peak of the curve is to right side of the ordinate. The ordinate value of the highest point of the curve is slightly greater than the value of the y-intercept. Provided the skewness is not too great, the y-intercept or the constant in the equation gives a value very similar to the ordinate value of the highest point of the curve.

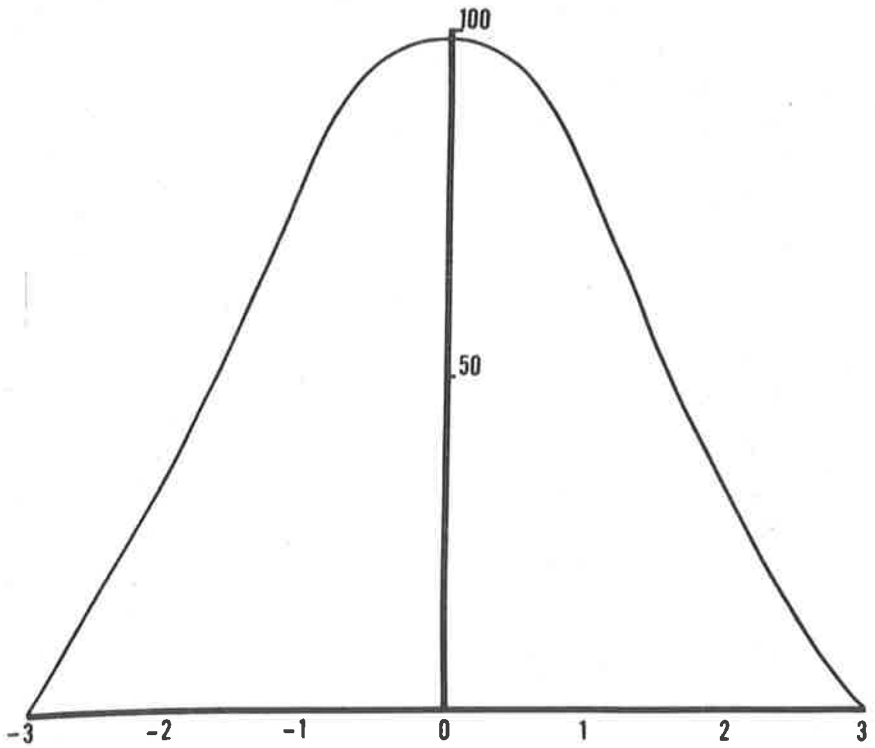

Figure 18. Curve plotted from a polynomial equation with only even powers of x

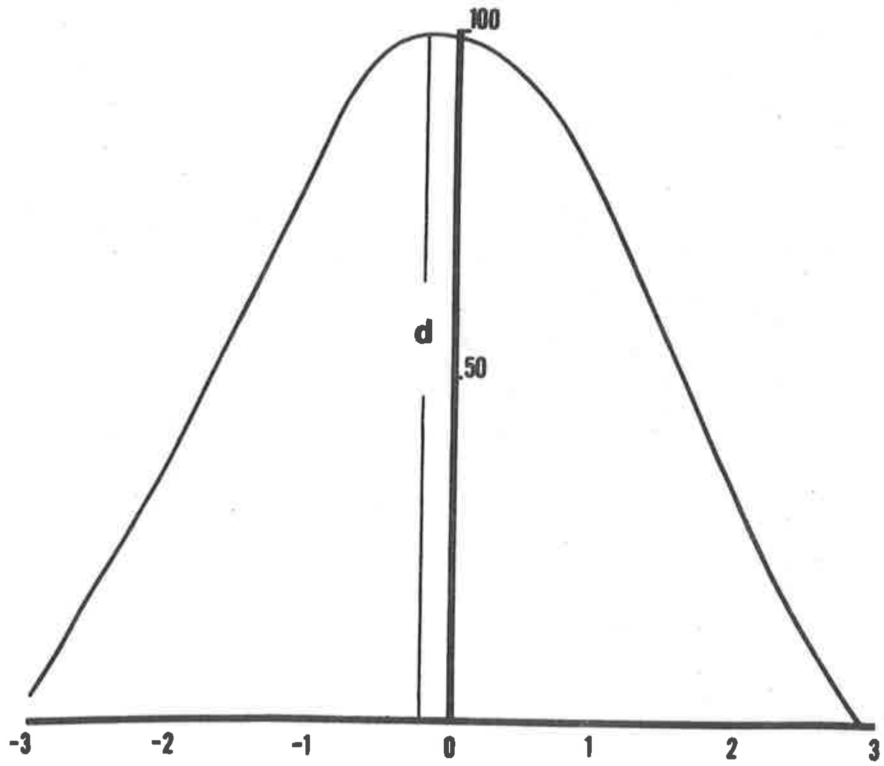

Figure 19. Curve plotted from a polynomial equation with both odd and even powers of x

ANALYSES

Mathematical treatment of polynomial equations describing dental-arch curves can be performed in many ways. Lu (1964, 1966) suggested a method for the analysis of the various terms of x in a polynomial equation to determine the degree of symmetry or asymmetry of the curve. Minor changes in the formulae suggested by Lu would enable a similar analysis to be applied to polynomials of higher order than the fourth degree which he proposed.

Polynomials derived from mean values of the point coordinates of groups of subjects could be useful for comparison of dental arches between different groups, to determine age differences or sex differences, for example. By superimposition of the curves derived from polynomial equations, growth changes of the same dental arch or the relative growth changes between the maxillary and mandibular dental arches could be studied. Dental-arch dimensions - for example, breadth, depth, polar vector, perimeter and area of the dental arch - could also be derived mathematically from the equation. However, the accuracy of these derived values would depend on the method of data acquisition and also on the specification of the polynomial equation which is chosen to represent the dental-arch curve.

PILOT STUDY

The review of literature revealed that polynomial equations of various degrees have been used to describe dental-arch shapes. Kato and associates (1950, 1964) described dental arches by quadratic polynomial equations; Hayashi (1956) used the equation, $y = ax^n \pm e^{\alpha(x-\beta)}$; Lu (1964, 1966) suggested that polynomials of degree four were sufficient for dental arch description; and Currier (1969) used parabolic and elliptic equations.

Following the suggestion of Lu, polynomials of degree four were used in the pilot study of the present investigation (Cheng, 1970). Five sets of dental casts from a male Australian Aboriginal at ages nine, twelve, thirteen, fifteen and sixteen years were analysed. Curves plotted from the degree four polynomials for this subject are shown in Figure 20. By visual examination it was apparent that the curves did not adequately represent the shapes in every case. For example, the maxillary arch curve at age nine years had a slight depression towards the apex of the curve and did not conform closely to the coordinate points of the central and lateral incisors. These findings led to further study of the polynomial equations.

The five sets of maxillary and mandibular coordinate points from the pilot study were used. Ten polynomial equations, ranging from degree one to degree ten, were derived from each set of

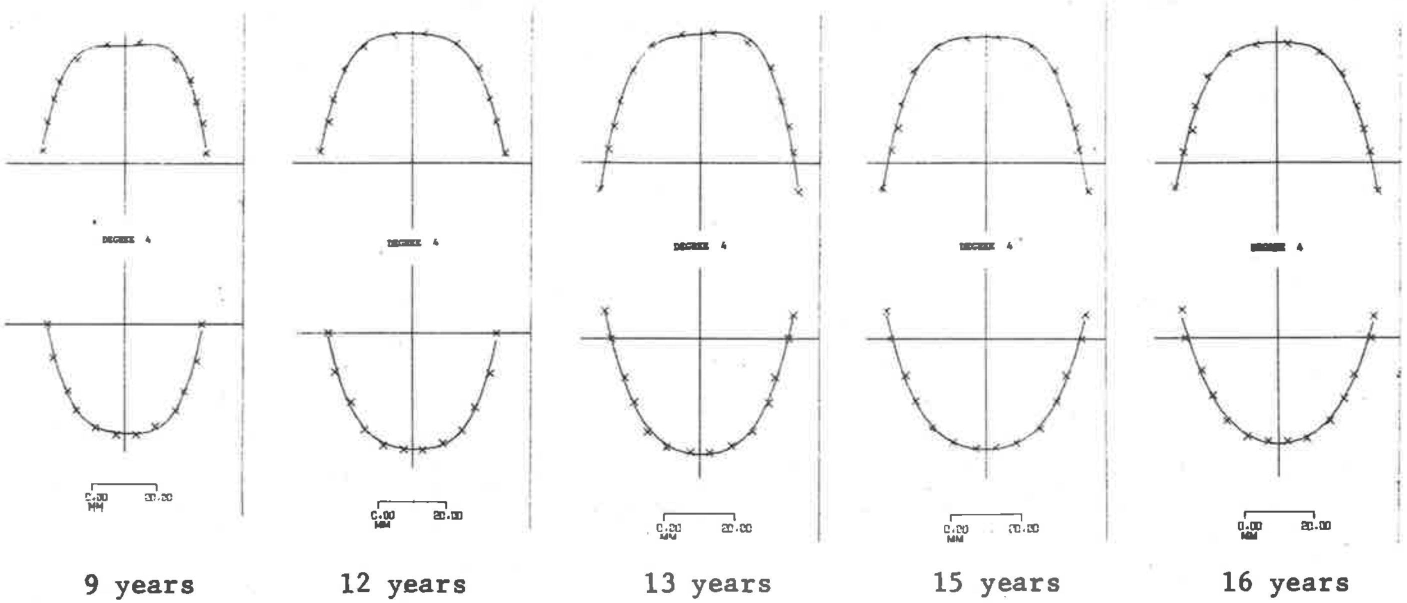

Figure 20. Curves plotted from degree four polynomial equations representing dental arches of one subject at ages nine, twelve, thirteen, fifteen and sixteen years

coordinates. Curves of these equations were plotted together with the coordinate points as illustrated in Figures 21 to 25.

Visual examination of the maxillary arch curves of the subject at nine years (Figure 21) showed that the equation of degree one did not contribute meaningful information to the study of dental-arch shape. Curves from polynomials of degree four, five, six and seven inadequately represented the arch shape because their y-intercepts deviated too greatly from an acceptable estimate of the arch depth. The y-intercepts of the curves of degree four and five were less than an acceptable estimate of arch depth. On the other hand, the y-intercepts of the curves of degree six and seven were greater. It appeared that the curves best describing the maxillary arch shape of the subject at age nine years were from polynomials of degree eight or nine. The polynomial of degree ten inadequately represented the arch shape even though the curve more closely fitted the points than the other curves of the series. The peaks and troughs between some of the points, as if the curve had been forced beyond stability into a state of oscillation, obviously did not represent the dental-arch shape.

Visual examination of the maxillary arch curves of the subject at ages twelve, thirteen, fifteen and sixteen years suggested that the curves best describing arch shape were those from polynomial equations of degree five, seven, ten and nine respectively.

It was noted that the peaks of the arch curves derived from various degrees of polynomials sometimes varied in position from one

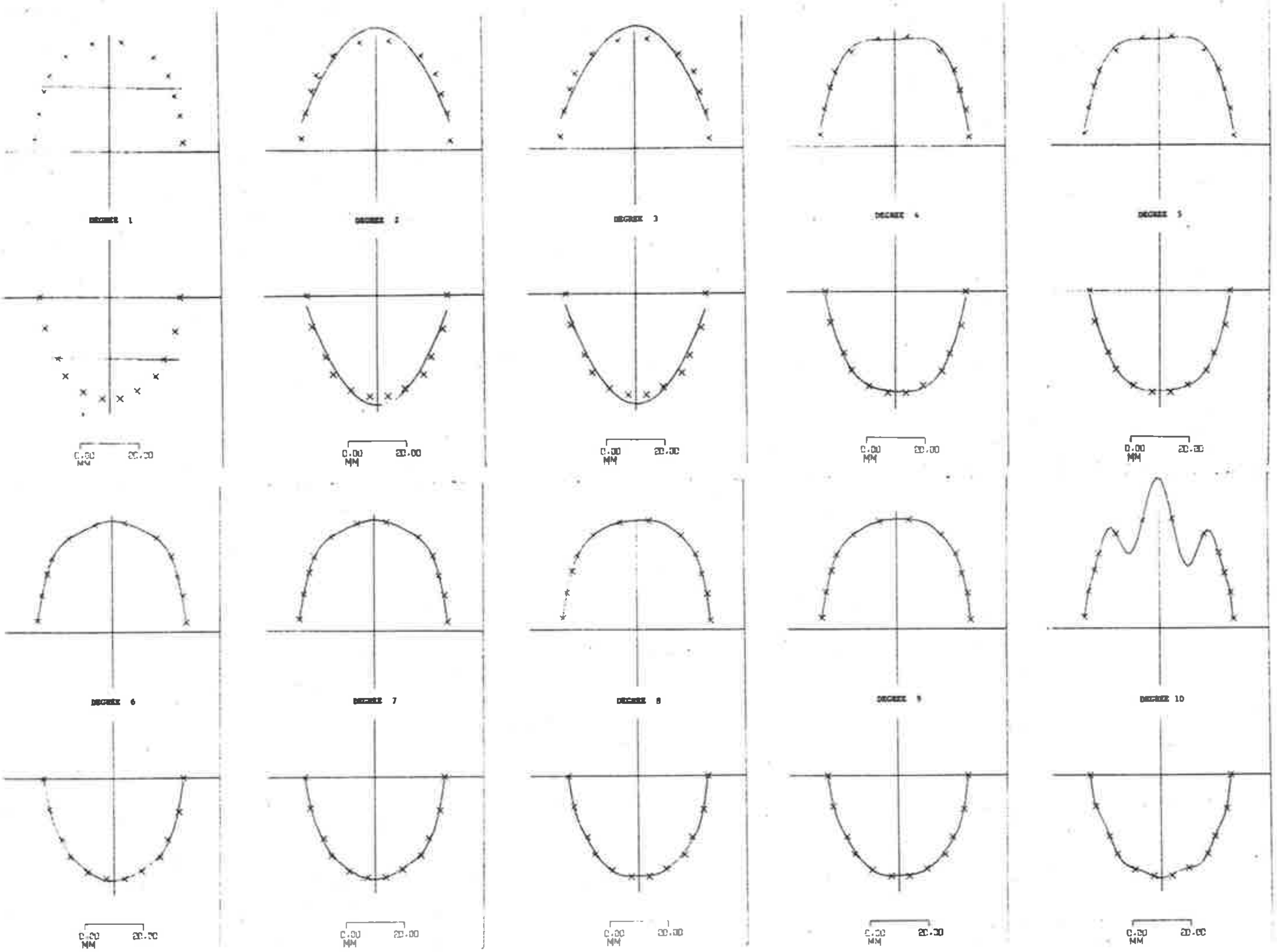

Figure 21. Curves plotted from polynomial equations ranging from degree one to degree ten for one subject at age nine years

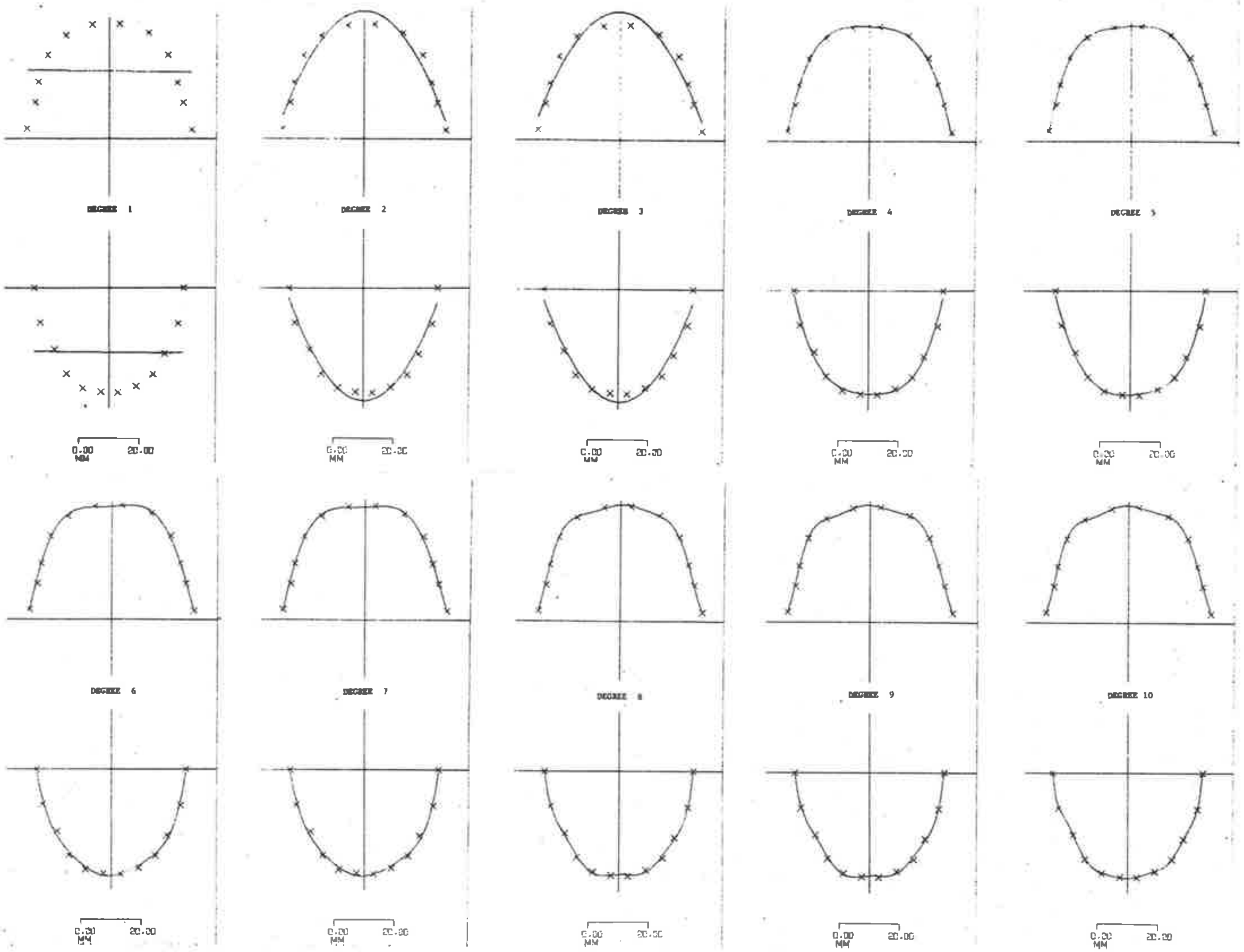

Figure 22. Curves plotted from polynomial equations ranging from degree one to degree ten for one subject at age twelve years

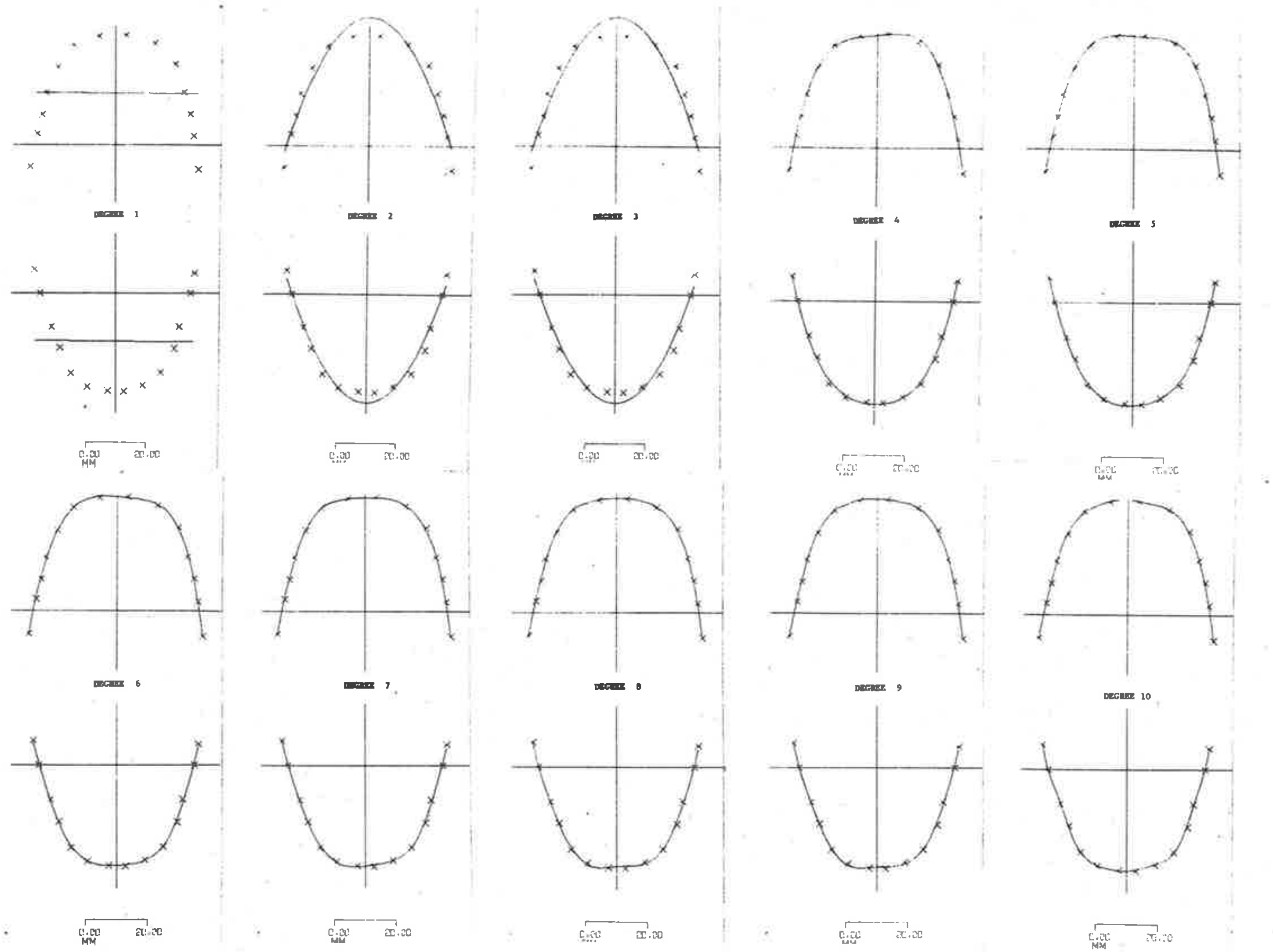

Figure 23. Curves plotted from polynomial equations ranging from degree one to degree ten for one subject at age thirteen years

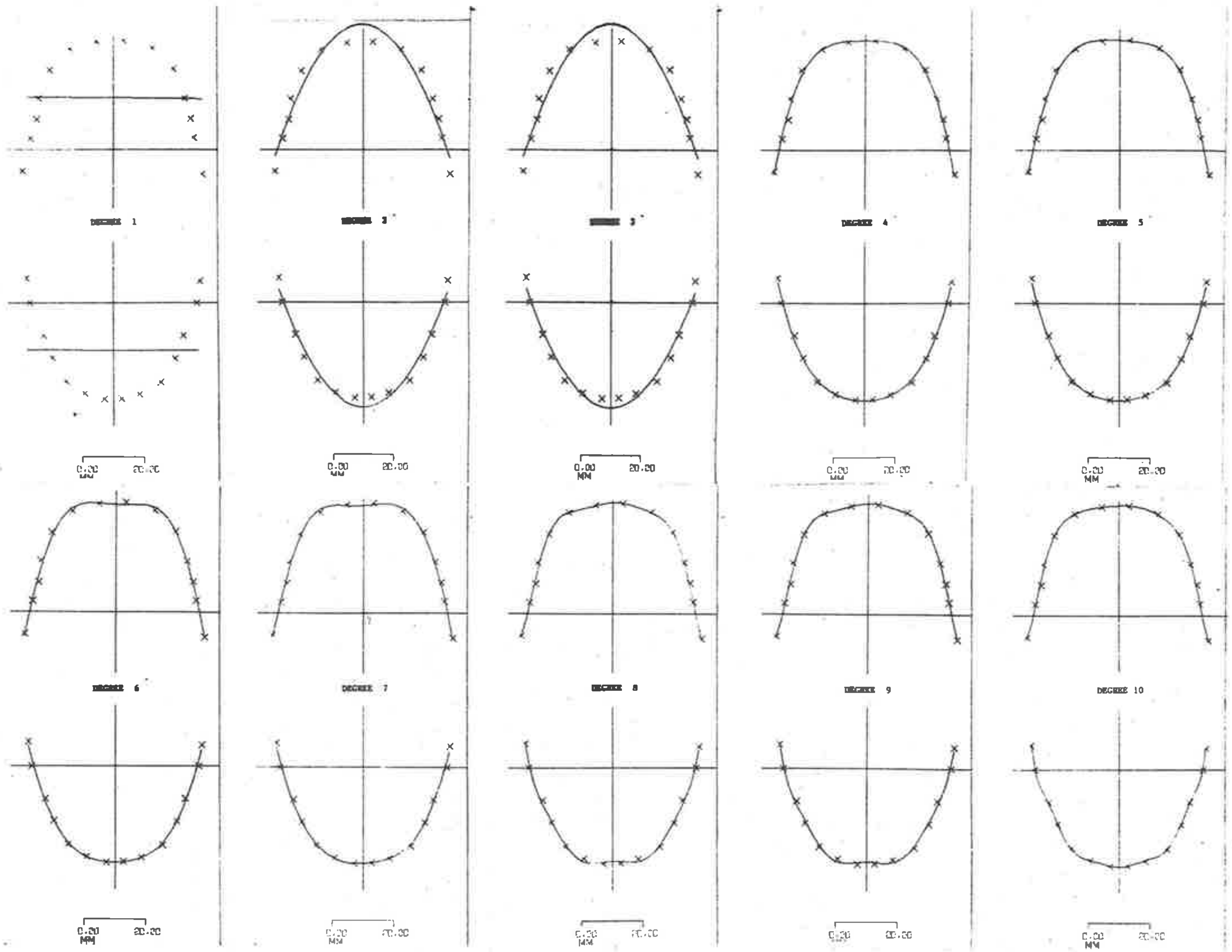

Figure 24. Curves plotted from polynomial equations ranging from degree one to degree ten for one subject at age fifteen years

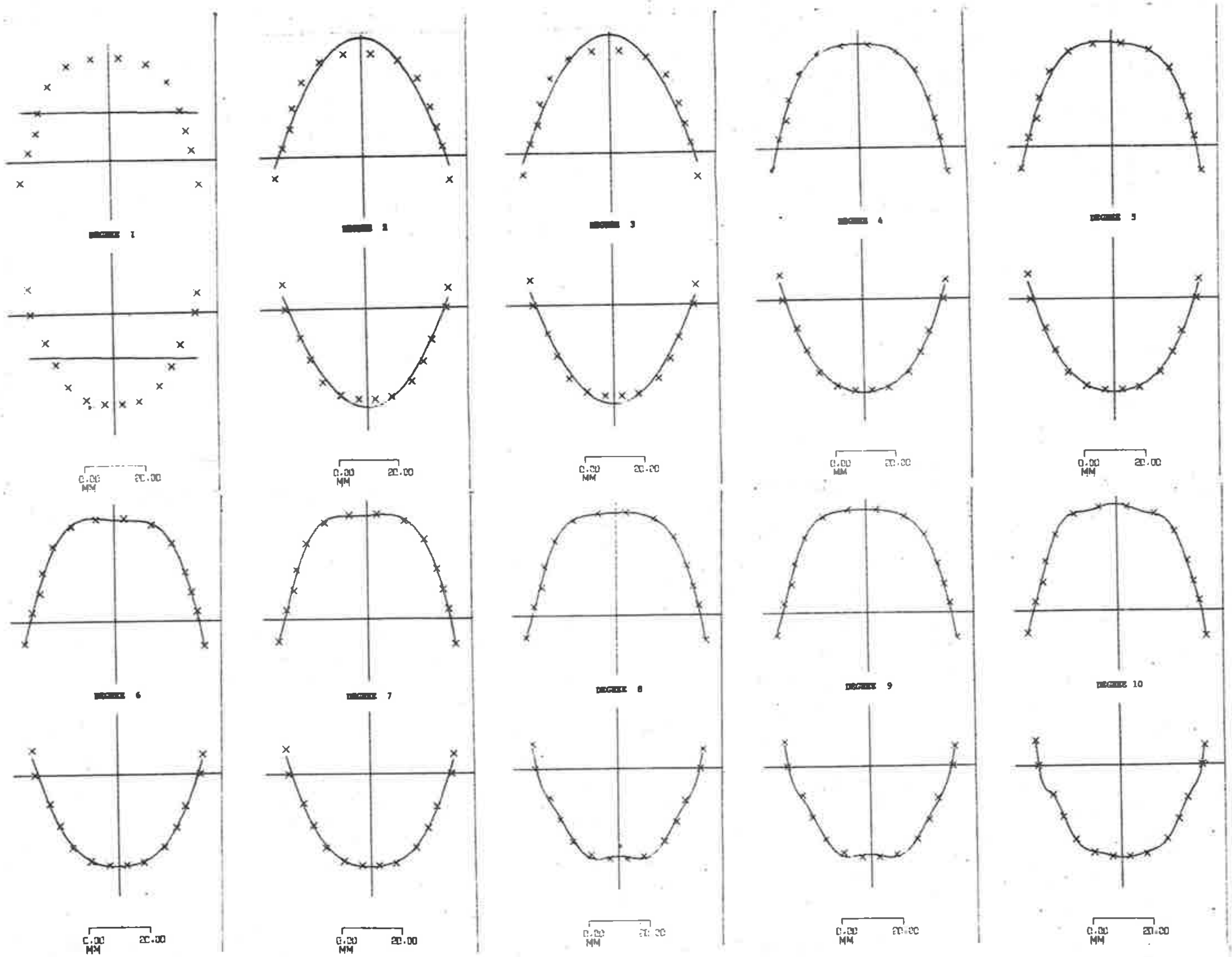

Figure 25. Curves plotted from polynomial equations ranging from degree one to degree ten for one subject at age sixteen years

side of the ordinate to the other. This is well-illustrated in Figure 21 which shows that the maxillary curve of degree four polynomial skewed to one side whilst that of degree five polynomial skewed to the other.

These observations led to the conclusion that a particular form of polynomial, that is, one of degree four or degree five, and so on, would not have universal application in description of dental-arch shape. Various dental arches might require polynomials of different degree.

The polynomial best describing a dental arch could be determined by subjective assessment and selection from a series of curves plotted in the same way as described above. However, this would be time consuming and costly. The best fitting curve by the mathematical least squares criterion would be the polynomial of degree n - 1, where n is the number of points. Although such a curve would pass through all the points it would invariably oscillate beyond the assumed smooth curve between the reference points, thereby falsely representing the dental-arch shape. Information on the extent by which one curve differed in shape from the other could be obtained by comparing the area square of the curves between two selected abscissa values adjacent to the x-intercepts of the curves. However, trials with this method did not help in selecting the curve of best shape to describe the dental arch.

To derive an objective method of defining polynomial equations which best describe the shape of particular dental arches it was necessary to study the problem further.

Careful examinations of the curve series of the pilot study (Figures 21 to 25) revealed that the curves selected subjectively as best representing the various sets of coordinate points had their y-intercepts greater but not very much greater than the y-intercept of a straight line joining the point coordinates of their corresponding central incisors (Figure 26 illustrates the relationship of the curve and the straight line through the central incisor points). The difference between these two y-intercept values, ID, is hereafter known as the 'y-intercept difference value'. This observation formed the basis of an assumption that the curve of choice would give a value of the y-intercept greater but not very much greater than that of the y-intercept of a straight line passing through the coordinate points of the central incisors.

The above assumption was subjected to further investigation for verification. The y-intercept difference values for the maxillary arches of the five series of curves from the pilot study were obtained. Their values are tabulated in Table 5. The curve having the least positive y-intercept difference value was chosen from the series of curves to represent arch shape. The polynomial equations selected by this criterion to represent the maxillary dental arches at ages nine, twelve, thirteen, fifteen and sixteen

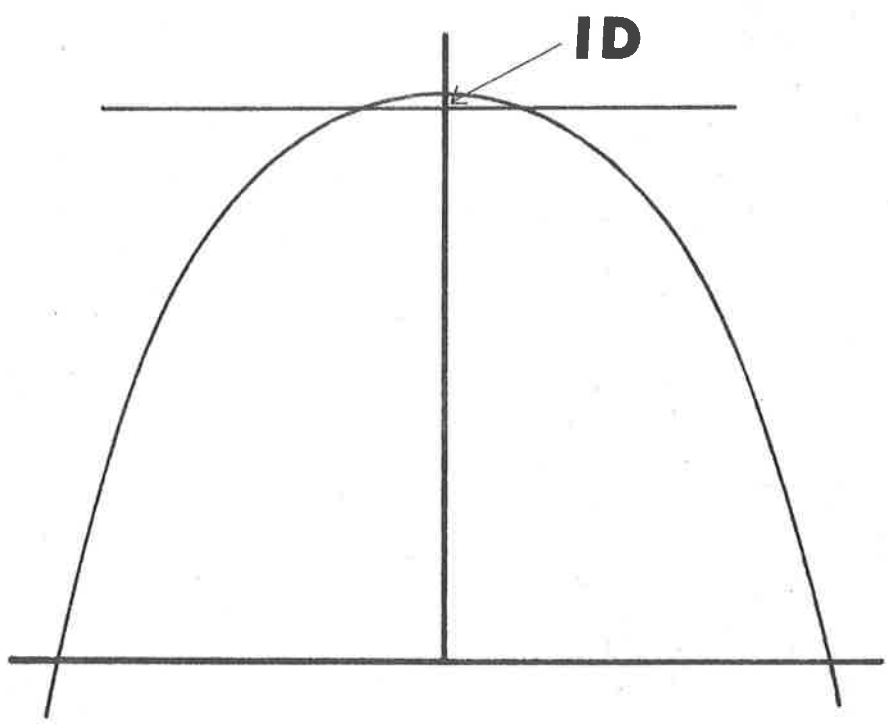

Figure 26. Dental-arch curve and a straight line passing through the central incisor points - ID is the y-intercept difference value

Table 5. The y-intercept difference values computed from polynomials of degree one to degree ten for one subject at ages nine, twelve, thirteen, fifteen and sixteen years

Degree of polynomial	Y-intercept difference values				
	Age 9	Age 12	Age 13	Age 15	Age 16
1	-15.4441	-15.5285	-22.5341	-18.7205	-17.4409
2	4.7046	4.5748	6.3045	5.9133	5.7450
3	4.7317	4.5754	6.3453	5.9146	5.8452
4	-0.8544	0.2318	-0.1647	0.2256	0.8413
5	-0.8451	0.2328	-0.2863	0.1956	0.7882
6	0.9110	-0.3054	0.0340	-0.6164	-0.0435
7	0.9066	-0.3062	0.0923	-0.5811	-0.0166
8	0.2791	0.7377	0.2669	0.6021	0.4230
9	0.2856	0.8483	0.2880	0.5952	0.4224
10	13.9775	0.8355	0.4281	0.2078	1.2308

years were of degree eight, four, six, five and nine respectively. The curves of these equations were examined together with the coordinate points (Figures 21 to 25). It was noted that these curves conformed closely to the shape of the dental arches they described. Some coincided with the curves selected subjectively; others differed little from the subjective choice. Therefore the above criterion for the selection of polynomial to best describe the shape of the dental arches was adopted for the present study.

It is necessary to point out that this method for the selection of polynomial equations also has its limitations. Theoretically, the above procedures may result in curves well describing the dental arches anteriorly but not so well posteriorly. In practice, however, this deficiency occurred only occasionally - usually at the premolar regions of the mandibular arches when high degree polynomial equations were selected.

CHAPTER 5

RESULTS AND DISCUSSION

Results of the investigation of dental-arch size and shape derived from measurements of 267 sets of serial dental casts representing 44 subjects at different ages are presented under the following main headings: (1) Cartesian coordinates; (2) polar coordinates; (3) polar overjets; (4) dental-arch areas; and (5) dental-arch perimeters. The findings are summarized and a general discussion is given at the end of the chapter.

1. CARTESIAN COORDINATES

The refined data were analysed by sex and by dental groups to provide basic descriptive statistics for the Cartesian coordinates of the reference points defining the vestibular dental-arch curves.

1.1 Right side compared with left side values

Tables 6 and 7 show the sample numbers, means, standard deviations and differences between absolute values of right and left side Cartesian coordinates for the vestibular dental-arch curve reference points for males and females respectively.

Table 6a. Cartesian coordinates for the vestibular dental-arch curve reference points of Group A males - sample numbers, means, standard deviations and differences between right and left side values.

GROUP A	Tooth	Side	N	Abscissa			Ordinate		
				Mean	Dif.	S.D.	Mean	Dif.	S.D.
MAXILLA	1	R	18	- 4.3	0.5	0.8	34.7	0.0	2.0
		L	18	4.8		0.9	34.7		2.1
	2	R	17	-12.3	0.9*	1.1	30.9	-0.4	1.4
		L	17	13.2		1.2	30.5		1.8
	3/C	R	18	-17.6	0.6	1.3	25.8	-0.1	1.2
		L	18	18.2		0.9	25.7		1.2
	4/D	R	18	-20.7	0.6	1.0	19.2	-0.1	0.8
		L	18	21.3		1.0	19.1		1.1
	5/E	R	18	-23.1	0.3	1.2	13.3	-0.2	0.7
		L	18	23.4		1.2	13.1		0.6
	6	R	18	-25.6	0.1	1.1	3.7	-0.1	0.4
		L	18	25.7		1.1	3.6		0.4
	7	R	-						
		L							
MANDIBLE	1	R	18	- 3.1	-0.4	0.8	-32.3	0.0	1.9
		L	18	2.7		0.7	-32.3		2.0
	2	R	18	- 8.7	-0.3	0.9	-30.4	-0.2	2.2
		L	18	8.4		1.0	-30.2		2.2
	3/C	R	18	-13.9	-0.1	1.4	-28.0	-0.1	1.7
		L	18	13.8		1.0	-27.9		1.5
	4/D	R	18	-16.6	0.2	1.1	-22.1	-0.2	1.3
		L	18	16.8		1.0	-21.9		1.0
	5/E	R	18	-20.9	0.2	1.0	-11.5	0.0	0.6
		L	18	21.1		1.0	-11.5		0.6
	6	R	18	-23.2	0.0	0.9	0.0	0.0	0.0
		L	18	23.2		0.9	0.0		0.0
	7	R	-						
		L	-						

* $p < 0.05$

Table 6b. Cartesian coordinates for the vestibular dental-arch curve reference points of Group B males - sample numbers, means, standard deviations and differences between right and left side values.

GROUP B	Tooth	Side	N	Abscissa Mean	Abscissa Dif.	Abscissa S.D.	Ordinate Mean	Ordinate Dif.	Ordinate S.D.
MAXILLA	1	R	22	- 4.3	0.3	0.7	35.7	0.1	2.0
		L	22	4.6		0.8	35.8		1.8
	2	R	22	-12.2	0.6*	0.8	31.7	-0.1	1.5
		L	22	12.8		0.9	31.6		1.6
	3/C	R	21	-17.6	0.7*	0.9	26.2	-0.2	1.3
		L	21	18.3		0.8	26.0		1.0
	4/D	R	21	-20.9	1.0†	1.2	19.1	0.0	1.1
		L	21	21.9		0.6	19.1		1.0
	5/E	R	22	-23.5	0.5	1.0	13.4	-0.1	0.5
		L	22	24.0		1.0	13.3		0.7
	6	R	22	-26.2	0.2	1.0	3.8	-0.2	0.5
		L	22	26.4		1.1	3.6		0.4
	7	R	1	-31.2	-2.2	0.0	- 7.5	0.4	0.0
		L	4	29.0		1.6	- 7.9		0.6
MANDIBLE	1	R	22	- 3.1	-0.3	0.9	-32.6	-0.1	1.4
		L	22	2.8		0.8	-32.5		1.6
	2	R	22	- 8.8	-0.1	0.7	-30.6	0.0	1.7
		L	22	8.7		0.9	-30.6		1.7
	3/C	R	21	-14.4	-0.4	1.2	-27.1	0.4	1.6
		L	21	14.0		1.1	-27.5		1.2
	4/D	R	19	-17.5	0.2	0.8	-21.3	0.3	1.2
		L	20	17.7		1.3	-21.6		1.0
	5/E	R	22	-21.3	0.0	1.1	-11.4	0.1	0.6
		L	22	21.3		0.9	-11.5		0.6
	6	R	22	-23.8	0.0	0.9	0.0	0.0	0.0
		L	22	23.8		0.9	0.0		0.0
	7	R	4	-26.5	-0.2	1.9	11.2	-0.1	0.5
		L	5	26.3		1.0	11.1		0.5

* $p < 0.05$ † $p < 0.01$

Table 6c. Cartesian coordinates for the vestibular dental-arch curve reference points of Group C males - sample numbers, means, standard deviations and differences between right and left side values.

GROUP C	Tooth	Side	N	Abscissa Mean	Abscissa Dif.	Abscissa S.D.	Ordinate Mean	Ordinate Dif.	Ordinate S.D.
MAXILLA	1	R	6	- 4.7	-0.7	0.6	34.4	0.1	1.0
		L	6	4.0		0.5	34.5		1.1
	2	R	6	-12.3	-0.4	0.9	30.5	-0.1	0.9
		L	6	11.9		0.5	30.4		0.9
	3/C	R	5	-17.6	-0.5	0.8	25.2	-0.3	0.8
		L	5	17.1		0.4	24.9		0.7
	4/D	R	6	-21.4	-0.2	0.7	17.2	0.0	1.0
		L	6	21.2		0.4	17.2		1.0
	5/E	R	6	-23.3	-0.3	1.5	10.4	0.7	1.5
		L	6	23.0		0.8	11.1		1.6
	6	R	6	-26.0	0.1	0.8	3.9	0.0	0.3
		L	6	26.1		0.6	3.9		0.2
	7	R	2	-26.7	0.2	0.0	- 8.4	-0.1	0.0
		L	3	26.9		1.1	- 8.3		0.5
MANDIBLE	1	R	6	- 3.9	-2.0†	0.9	-31.3	0.0	1.9
		L	6	1.9		0.7	-31.3		1.9
	2	R	6	- 9.3	-1.7†	0.2	-29.2	0.8	2.1
		L	6	7.6		1.1	-30.0		2.3
	3/C	R	6	-15.2	-1.4	1.3	-26.2	0.4	2.5
		L	6	13.8		1.2	-26.6		1.8
	4/D	R	6	-17.9	-1.2†	0.6	-19.3	0.7	1.7
		L	6	16.7		0.6	-20.0		1.1
	5/E	R	5	-20.5	1.1	1.4	-10.7	0.4	0.6
		L	6	21.6		1.9	-11.1		1.0
	6	R	6	-23.6	0.0	0.5	0.0	0.0	0.0
		L	6	23.6		0.5	0.0		0.0
	7	R	3	-25.1	0.4	1.0	12.0	-0.3	1.1
		L	3	25.5		2.4	11.7		0.2

† $p < 0.01$

Table 6d. Cartesian coordinates for the vestibular dental-arch curve reference points of Group D males - sample numbers, means, standard deviations and differences between right and left side values.

GROUP D	Tooth	Side	N	Abscissa			Ordinate		
				Mean	Dif.	S.D.	Mean	Dif.	S.D.
MAXILLA	1	R	49	- 4.6	-0.5†	0.7	34.2	0.0	2.3
		L	49	4.1		0.7	34.2		2.2
	2	R	49	-12.5	-0.1	0.7	30.5	0.2	1.6
		L	49	12.4		0.5	30.7		1.7
	3/C	R	49	-18.5	-0.2	1.0	25.1	0.0	1.5
		L	49	18.3		1.1	25.1		1.5
	4/D	R	49	-22.1	0.0	1.0	16.8	0.0	0.9
		L	49	22.1		0.7	16.8		1.2
	5/E	R	49	-24.0	0.0	1.1	9.9	-0.1	0.6
		L	49	24.0		1.1	9.8		0.6
	6	R	49	-26.7	-0.1	1.0	3.7	0.1	0.4
		L	49	26.6		1.1	3.8		0.4
	7	R	48	-28.7	-0.1	1.7	- 8.2	-0.2	0.7
		L	48	28.6		1.7	- 8.0		0.6
MANDIBLE	1	R	49	- 3.1	-0.4†	0.7	-30.4	-0.1	1.8
		L	49	2.7		0.6	-30.3		1.8
	2	R	49	- 9.0	-0.3*	0.6	-28.5	0.0	2.0
		L	49	8.7		0.7	-28.5		1.7
	3/C	R	49	-14.7	-0.2	1.0	-25.3	-0.1	1.6
		L	49	14.5		0.8	-25.5		1.6
	4/D	R	49	-18.4	-0.2	1.0	-18.3	-0.2	1.0
		L	49	18.2		0.9	-18.1		0.9
	5/E	R	49	-20.4	-0.3	1.1	-11.2	-0.2	1.0
		L	49	20.1		1.1	-11.0		0.7
	6	R	49	-24.1	0.0	1.1	0.0	0.0	0.0
		L	49	24.1		1.1	0.0		0.0
	7	R	47	-26.3	0.2	1.7	11.5	0.3*	0.6
		L	47	26.5		1.5	11.8		0.7

* $p < 0.05$ † $p < 0.01$

Table 7a. Cartesian coordinates for the vestibular dental-arch curve reference points of Group A females - sample numbers, means, standard deviations and differences between right and left side values.

GROUP A	Tooth	Side	N	Abscissa Mean	Abscissa Dif.	Abscissa S.D.	Ordinate Mean	Ordinate Dif.	Ordinate S.D.
MAXILLA	1	R	32	- 4.5	0.3	0.8	35.6	0.0	1.9
		L	32	4.8		0.6	35.6		1.9
	2	R	31	-13.1	0.5*	1.0	31.5	0.0	1.5
		L	31	13.6		1.0	31.5		1.5
	3/C	R	32	-18.6	0.3	0.9	25.7	0.1	1.1
		L	32	18.9		0.9	25.8		1.2
	4/D	R	32	-21.9	0.3	1.0	19.5	-0.1	1.0
		L	32	22.2		1.1	19.4		1.1
	5/E	R	32	-24.3	0.3	1.0	13.3	0.0	0.5
		L	32	24.6		1.1	13.3		0.7
	6	R	32	-27.0	0.0	1.1	3.5	0.1	0.4
		L	32	27.0		1.1	3.6		0.4
	7	R	-						
		L	-						
MANDIBLE	1	R	32	- 3.1	-0.5*	0.6	-32.9	-0.1	1.6
		L	32	2.6		1.2	-32.8		1.7
	2	R	32	- 9.2	-0.3	0.8	-31.1	-0.1	1.8
		L	32	8.9		0.7	-31.0		1.8
	3/C	R	32	-14.6	-0.4	0.9	-27.8	-0.3	1.5
		L	32	14.2		1.0	-27.5		1.5
	4/D	R	32	-17.4	-0.4	1.0	-21.8	-0.2	1.3
		L	32	17.0		1.1	-21.6		1.3
	5/E	R	32	-21.9	-0.1	1.0	-11.4	0.1	0.8
		L	32	21.8		1.1	-11.5		0.7
	6	R	32	-24.4	0.0	1.1	0.0	0.0	0.0
		L	32	24.4		1.1	0.0		0.0
	7	R	-						
		L	-						

* $p < 0.05$

Table 7b. Cartesian coordinates for the vestibular dental-arch curve reference points of Group B females - sample numbers, means, standard deviations and differences between right and left side values.

GROUP B	Tooth	Side	N	Abscissa			Ordinate		
				Mean	Dif.	S.D.	Mean	Dif.	S.D.
MAXILLA	1	R	35	- 4.4	0.4*	0.8	36.4	-0.1	2.0
		L	35	4.8		0.7	36.3		2.0
	2	R	35	-12.9	0.5*	1.0	32.6	-0.1	1.8
		L	35	13.4		0.8	32.5		1.8
	3/C	R	34	-18.3	0.7*	1.5	26.4	-0.1	1.5
		L	34	19.0		1.5	26.3		1.6
	4/D	R	35	22.5	0.2	1.1	19.1	0.0	1.0
		L	35	22.7		1.2	19.1		1.1
	5/E	R	35	-24.9	0.3	1.2	13.2	0.0	0.8
		L	35	25.2		1.2	13.2		0.7
	6	R	35	-27.7	0.1	1.0	3.6	0.0	0.5
		L	35	27.8		1.0	3.6		0.4
	7	R	6	-28.8	-0.1	0.4	- 8.4	0.2	0.8
		L	5	28.7		0.3	- 8.6		0.5
MANDIBLE	1	R	35	- 3.3	-0.9†	0.5	-33.2	0.2	1.7
		L	35	2.4		1.2	-33.0		1.8
	2	R	35	- 9.2	-0.5†	0.7	-31.2	0.1	2.0
		L	35	8.7		0.9	-31.3		1.9
	3/C	R	33	-14.9	-0.6*	0.8	-27.6	0.0	1.6
		L	34	14.3		1.2	-27.6		1.8
	4/D	R	33	-18.2	-0.5	1.0	-21.3	-0.2	1.4
		L	34	17.7		1.3	-21.1		1.7
	5/E	R	35	-22.4	-0.4	0.9	-11.2	0.1	0.7
		L	35	22.0		1.2	-11.3		0.6
	6	R	35	-24.9	0.0	1.1	0.0	0.0	0.0
		L	35	24.9		1.1	0.0		0.0
	7	R	9	-26.9	-0.5	1.2	11.4	0.2	0.7
		L	8	26.4		1.5	11.6		0.8

* $p < 0.05$ † $p < 0.01$

Table 7c. Cartesian coordinates for the vestibular dental-arch curve reference points of Group C females - sample numbers, means, standard deviations and differences between right and left side values.

GROUP C	Tooth	Side	N	Abscissa			Ordinate		
				Mean	Dif.	S.D.	Mean	Dif.	S.D.
MAXILLA	1	R	12	- 4.5	-0.2	0.7	35.0	-0.1	2.5
		L	12	4.3		0.7	34.9		2.5
	2	R	12	-13.1	0.1	1.1	31.6	-0.1	2.0
		L	12	13.2		0.9	31.5		2.0
	3/C	R	12	-19.0	0.2	1.1	26.0	-0.1	1.9
		L	11	19.2		1.5	25.9		2.0
	4/D	R	12	-23.1	0.0	0.8	17.8	-0.1	1.3
		L	12	23.1		1.3	17.7		1.4
	5/E	R	11	-25.5	0.4	0.9	10.3	0.9	1.8
		L	12	25.9		2.0	11.2		2.0
	6	R	12	-28.1	0.1	0.9	3.5	0.4	0.5
		L	12	28.2		1.1	3.9		0.5
	7	R	8	-29.0	0.1	2.1	- 8.9	-0.5	0.6
		L	9	29.1		1.3	- 8.4		0.8
MANDIBLE	1	R	12	- 3.6	1.3†	0.5	-32.0	0.1	1.2
		L	12	2.3		0.7	-32.1		1.5
	2	R	12	- 9.6	-1.3†	0.6	-30.2	0.3	1.6
		L	12	8.3		0.9	-30.5		1.4
	3/C	R	12	-15.2	-1.1†	0.7	-26.8	0.3	1.5
		L	12	14.1		0.9	-27.1		1.0
	4/D	R	12	-19.0	-1.0	0.8	-19.9	0.5	1.3
		L	12	18.0		1.6	-20.4		1.2
	5/E	R	12	-21.6	0.6	1.4	-11.6	-0.4	1.1
		L	12	22.2		0.8	-11.2		0.9
	6	R	12	-25.2	0.0	0.9	0.0	0.0	0.0
		L	12	25.2		0.9	0.0		0.0
	7	R	7	-27.0	-0.2	1.8	11.9	-0.4	0.9
		L	7	26.8		1.7	11.5		0.5

† $p < 0.01$

Table 7d. Cartesian coordinates for the vestibular dental-arch curve reference points of Group D females - sample numbers, means, standard deviations and differences between right and left side values.

GROUP D	Tooth	Side	N	Abscissa			Ordinate		
				Mean	Dif.	S.D.	Mean	Dif.	S.D.
MAXILLA	1	R	93	- 4.5	-0.2*	0.6	34.3	-0.1	2.0
		L	93	4.3		0.5	34.2		2.1
	2	R	93	-13.0	-0.1	0.7	31.1	-0.1	1.9
		L	93	12.9		0.8	31.0		1.7
	3/C	R	93	-19.3	-0.1	1.2	25.4	0.0	1.4
		L	93	19.2		1.1	25.4		1.5
	4/D	R	93	-23.2	-0.2	1.0	16.6	0.1	0.9
		L	93	23.0		1.0	16.7		1.0
	5/E	R	92	-25.3	-0.1	1.0	9.5	0.1	0.6
		L	91	25.2		1.0	9.6		0.6
	6	R	93	-28.1	-0.1	1.1	3.5	0.1	0.4
		L	93	28.0		1.1	3.6		0.4
	7	R	90	-29.9	-0.2	1.5	- 8.5	-0.2*	0.7
		L	90	29.7		1.3	- 8.3		0.6
MANDIBLE	1	R	93	- 3.3	-1.0†	0.5	-30.4	0.0	1.5
		L	93	2.3		0.6	-30.4		1.5
	2	R	93	- 9.2	0.9†	0.8	-28.6	0.1	1.7
		L	93	8.3		0.8	-28.7		1.7
	3/C	R	93	-14.9	-0.8†	0.8	-24.9	0.1	1.2
		L	93	14.1		1.0	-25.0		1.3
	4/D	R	93	-18.9	-0.6†	0.9	-18.0	0.1	0.8
		L	93	18.3		1.1	-18.1		1.0
	5/E	R	93	-20.8	-0.4*	1.0	-10.9	0.1	0.7
		L	93	20.4		1.4	-11.0		0.8
	6	R	93	-25.1	0.0	1.1	0.0	0.0	0.0
		L	93	25.1		1.1	0.0		0.0
	7	R	92	-27.4	-0.2	1.5	11.9	0.1	0.8
		L	91	27.2		1.5	12.0		0.7

* $p < 0.05$ † $p < 0.01$

Only one of the differences between maxillary right and left side abscissa values was greater than 1.0 mm and few were statistically significant. More of the mandibular side differences in abscissa values were greater than 1.0 mm and more were statistically significant.

In the maxillary arch the left side abscissa values tended to be greater than the right side for groups A, B and C in males, and for Groups A and B in females; whereas the right side abscissa values tended to be greater for group D males and groups C and D females. In the mandibular arch the right side values tended to be greater than the left side values for both males and females. Generally, the side differences in abscissa values were greater and more were statistically significant in the anterior than in the posterior tooth regions.

The side differences in ordinate values were small. Few of the differences were greater than 0.5 mm and only two were statistically significant.

1.2 <u>Coordinate values and plotted curves</u>

Degree four polynomial equations were computed in the first instance and arch-shape curves for the various groups were plotted. A visual qualitative examination of the curves revealed subjectively that many were unacceptable representations of the arch shapes. Degree five polynomials were then computed and it was found that the curves were more acceptable.

The point coordinates presented in Tables 6 and 7 and approximations of the dental-arch curves by means of polynomial equations of degree five are shown diagrammatically in Figure 27. Maxillary central fossa points and approximations of the dental-arch sulcus curves are also shown. The polynomial equations from which the vestibular curves were derived are presented in Table 8. Term 1 is the constant term of the polynomial equation. Term 2 is the coefficient of x; term 3 of x^2; term 4 of x^3; term 5 of x^4; and term 6 of x^5.

For many of the dental-arch data sets the curves from degree five polynomials closely approximated the coordinate points. However, arch asymmetry was exaggerated in some instances - the maxillary curve for group B females for example. In other instances the curves did not approximate the incisor points closely - both curves for group C females for example.

The plotted curves described the maxillary dental-arch shapes remarkably well in groups A and D; similarly the mandibular dental-arch shapes in all instances except group C females. However, the maxillary curve in groups B and C males and both curves in group C females did not approximate the coordinate

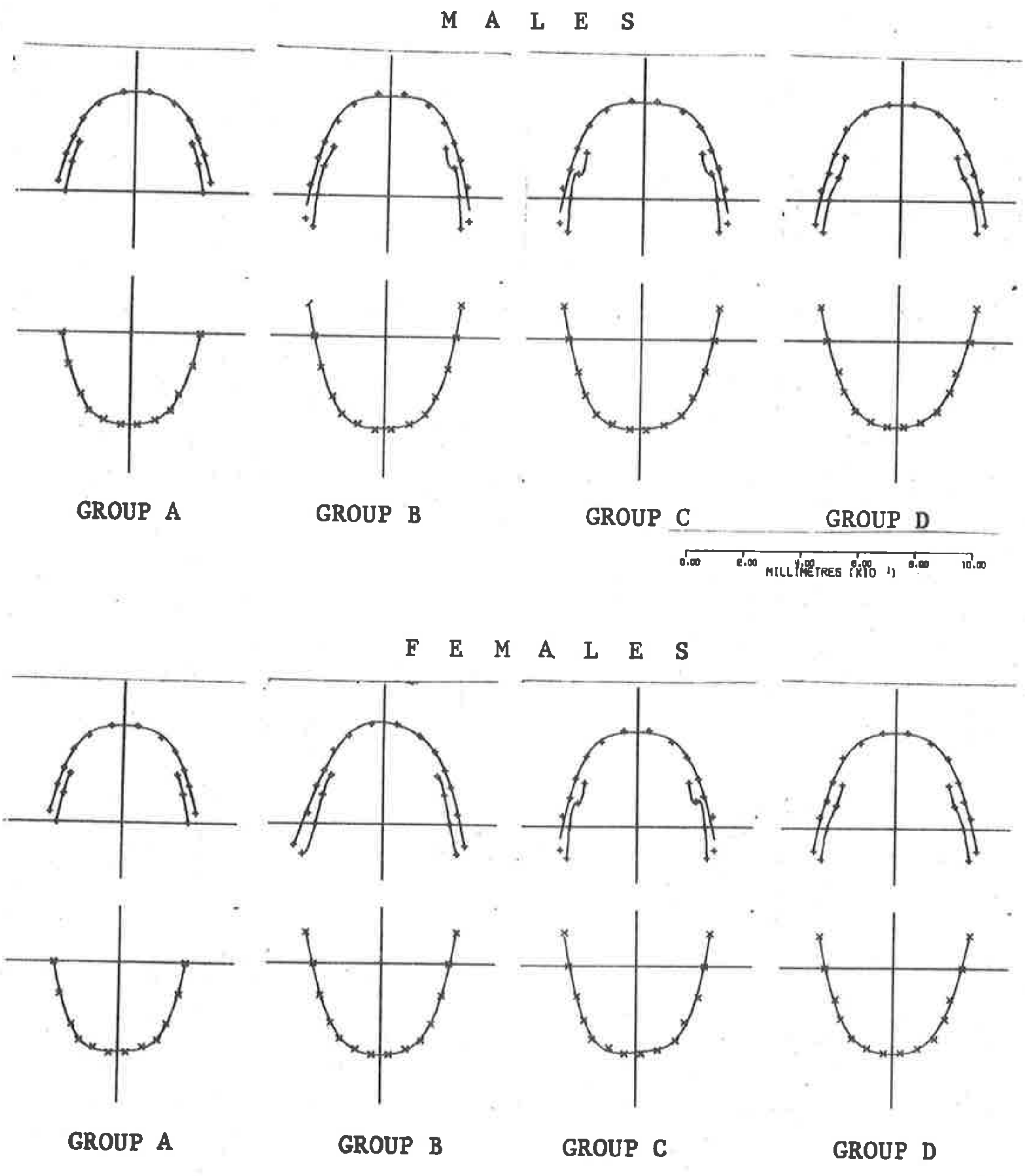

Figure 27. Degree five polynomial curves representing dental arches of males and females by dental groups

Table 8. Polynomial equations approximating maxillary and mandibular dental arches of males and females - by dental groups

Group	Arch	Term 1	Term 2	Term 3	Term 4	Term 5	Term 6
MALES							
A	Max	3.56 x 10	2.83×10^{-3}	-1.38×10^{-2}	8.88×10^{-5}	-4.08×10^{-5}	-1.23×10^{-7}
	Man	-3.28 x 10	1.74×10^{-2}	1.41×10^{-2}	6.94×10^{-5}	6.78×10^{-5}	-1.71×10^{-7}
B	Max	3.55 x 10	1.15×10^{-2}	-3.03×10^{-3}	5.55×10^{-5}	-5.58×10^{-5}	-9.03×10^{-8}
	Man	-3.27 x 10	5.06×10^{-2}	9.16×10^{-3}	-1.79×10^{-4}	7.29×10^{-5}	2.43×10^{-7}
C	Max	3.43 x 10	-3.41×10^{-2}	-2.45×10^{-3}	1.46×10^{-4}	-5.19×10^{-5}	-9.52×10^{-8}
	Man	-3.18 x 10	6.48×10^{-2}	9.22×10^{-3}	-2.39×10^{-4}	6.81×10^{-5}	2.19×10^{-7}
D	Max	3.43 x 10	-1.37×10^{-2}	-1.06×10^{-2}	4.13×10^{-5}	-4.06×10^{-5}	3.84×10^{-8}
	Man	-3.08 x 10	3.86×10^{-2}	2.25×10^{-2}	-6.00×10^{-5}	4.57×10^{-5}	3.53×10^{-8}
FEMALES							
A	Max	3.47 x 10	-1.42×10^{-2}	-1.09×10^{-2}	2.14×10^{-4}	-5.41×10^{-5}	-2.69×10^{-7}
	Man	-3.19 x 10	3.53×10^{-2}	5.24×10^{-3}	-2.23×10^{-4}	9.83×10^{-5}	2.84×10^{-7}
B	Max	3.65 x 10	-1.08×10^{-1}	-2.56×10^{-2}	7.98×10^{-4}	-2.84×10^{-5}	-9.52×10^{-7}
	Man	-3.28 x 10	2.41×10^{-2}	1.69×10^{-2}	-2.10×10^{-4}	6.78×10^{-5}	3.04×10^{-7}
C	Max	3.40 x 10	-1.60×10^{-2}	-1.02×10^{-2}	-6.51×10^{-5}	-6.05×10^{-5}	1.53×10^{-7}
	Man	-3.08 x 10	1.01×10^{-1}	2.89×10^{-3}	-4.46×10^{-4}	9.77×10^{-5}	3.33×10^{-7}
D	Max	3.43 x 10	-3.34×10^{-3}	-1.41×10^{-2}	1.36×10^{-5}	-4.46×10^{-5}	-1.42×10^{-8}
	Man	-3.07 x 10	-8.64×10^{-3}	1.86×10^{-2}	1.86×10^{-4}	6.03×10^{-5}	-2.89×10^{-7}

points as closely as the others, especially with respect to the incisor points.

The dental-arch sulcus curves in general were well represented - except those in group B and group C males and in group C females. The exceptions showed irregularities in the curves which resulted from using polynomials of too high a degree. In each instance the data points for the sulcus curve were fitted with the highest possible degree of polynomial equations.

Using polynomials with the same number of terms simplifies presentation as shown in Table 8. However, the tabulation does not give meaningful information to the reader except for the constant terms of the equation which signify the depths of the dental arches (see p. 88). The constant terms are directly comparable but the whole equations are not.

1.3 Sex differences

Sex differences in abscissa and ordinate values by dental groups are shown in Table 9. In view of the small differences between right and left side values and to simplify presentation of the findings the analyses were based on the coordinate points of the left side only. Left side values were chosen rather than right side values because the abscissae of the former were all positive. In order to facilitate reading and comprehension, the absolute values of the ordinates were used for comparison.

Table 9. Sex differences in absolute values of abscissae and ordinates for the vestibular dental-arch curve reference points of the left side in millimetres - by dental groups

Tooth	Arch	Group A		Group B		Group C		Group D	
		Abscissa	Ordinate	Abscissa	Ordinate	Abscissa	Ordinate	Abscissa	Ordinate
1	Max	0.0	0.9	0.2	0.5	0.3	0.4	0.2	0.0
	Man	-0.1	0.5	-0.4	0.5	0.4	0.8	-0.4†	0.1
2	Max	0.4	1.0	0.6*	0.9	1.3†	1.1	0.5†	0.3
	Man	0.5	0.8	0.0	0.7	0.7	0.5	-0.4†	0.2
3/C	Max	0.7*	0.1	0.7*	0.3	2.1†	1.0	0.9†	0.3
	Man	0.4	-0.4	0.3	0.1	0.3	0.5	-0.4*	-0.2
4/D	Max	0.9†	0.3	0.8†	0.0	1.9†	0.5	0.9†	-0.1
	Man	0.2	-0.3	0.0	-0.5	1.3*	0.4	0.1	0.0
5/E	Max	1.2†	0.2	1.2†	-0.1	2.9†	0.1	1.2†	-0.2*
	Man	0.7*	0.0	0.7*	-0.2	0.6	0.1	0.3	0.0
6	Max	1.3†	0.0	1.4†	0.0	2.1†	0.0	1.4†	-0.2*
	Man	1.2†	0.0	1.1†	0.0	1.6†	0.0	1.0†	0.0
7	Max			-0.3	-0.7	2.2*	0.1	1.1†	0.3†
	Man			0.1	0.5	1.3	-0.2	0.7*	0.2

* $p < 0.05$ † $p < 0.01$

Maxillary abscissa values were greater for males than females with only one exception, the permanent second molar region in group B. With few exceptions, mainly in the central incisor region, the abscissa differences were statistically significant.

Similarly, the maxillary ordinate values were generally greater for males than females, except for the second premolar and permanent second molar regions in group B and the premolar regions and the permanent first molar region in group D. Ordinate value differences were not significant except for some of those in group D.

The mandibular abscissa values were greater for males than females except for the central incisor regions for most of the groups and for the anterior tooth regions for group D. Abscissa differences were statistically significant mainly in the permanent molar and premolar regions, and in the anterior tooth regions for group D.

Mandibular ordinate values were also generally greater in males than in females except in some canine and premolar regions. However, none of the ordinate differences were statistically significant.

1.4 Differences between dental groups

Dental group differences for males and females are shown in Tables 10 and 11 respectively. Left side absolute values of the Cartesian coordinates are compared.

Comparison of male maxillary abscissa values between groups A and B, or between groups B and C, or between groups A and D showed that the abscissa values of the latter group of any two dental groups compared were generally less in the incisor regions and greater in the posterior tooth regions. However, comparison between groups C and D showed that most of the maxillary abscissa values were less in group D.

Unlike the abscissa values the maxillary ordinate values for males were generally less in the latter group when any two groups were compared, except in four instances - the incisor and canine regions between groups A and B, and the permanent first molar region between groups B and C.

The trend of increase or decrease in the mandibular abscissa and ordinate values for males was similar to that in the maxillary values with only a few exceptions, mainly in the canine and premolar regions.

Abscissa and ordinate value differences between dental groups were statistically significant mainly for some ordinate values in

Table 10. Dental group differences in absolute values of abscissae and ordinates for the vestibular dental-arch curve reference points of the left side in millimetres - for males

		MALES							
		Groups compared							
Tooth	Arch	A & B		B & C		C & D		A & D	
		Abscissa	Ordinate	Abscissa	Ordinate	Abscissa	Ordinate	Abscissa	Ordinate
1	Max	0.0	0.7	-0.5	-1.4	0.0	-0.7	-0.5†	-1.4†
	Man	-0.2	0.2	-0.1	-0.9	0.0	-1.7†	-0.3	-2.4†
2	Max	-0.2	1.0*	-0.2	-1.0	-0.3	-0.5	-0.7†	-0.5
	Man	-0.2	0.3	-0.4	-0.8	0.0	-1.8†	-0.6†	-2.3†
3/C	Max	0.1	0.5	0.2	-0.4	0.0	-0.5	0.3	-0.4
	Man	0.1	0.1	-0.2	-0.5	0.0	-2.1†	-0.1	-2.5†
4/D	Max	0.5	-0.3	0.4	-1.4†	-0.1	-1.0*	0.8†	-2.7†
	Man	0.7*	-0.5	0.3	-0.7	0.3	-2.3†	1.3†	-3.5†
5/E	Max	0.6*	-0.1	0.7	-2.0†	-0.7	-1.6*	0.6†	-3.7†
	Man	0.2	-0.2	0.2	-0.1	-1.8†	-0.2	-1.4†	-0.5†
6	Max	0.8†	0.0	0.4	0.3	-0.2	-0.3	1.0†	0.0
	Man	0.5	0.0	0.3	0.0	-0.1	0.0	0.7†	0.0
7	Max			0.4	-0.2	0.6	-0.1		
	Man			0.4	-0.1	0.4	0.5*		

* $p < 0.05$ † $p < 0.01$

Table 11. Dental group differences in absolute values of abscissae and ordinates for the vestibular dental-arch curve, reference points of the left side in millimetres - for females

FEMALES

Tooth	Arch	A & B Abscissa	A & B Ordinate	B & C Abscissa	B & C Ordinate	C & D Abscissa	C & D Ordinate	A & D Abscissa	A & D Ordinate
1	Max	-0.2	1.1	-0.6	-1.3	0.1	-0.3	-0.7*	-0.5
	Man	0.1	0.2	-0.9	-1.2	0.8*	-1.0	0.0	-2.0†
2	Max	-0.4	1.1*	-0.9†	-1.2*	0.5	0.3	-0.8†	0.2
	Man	0.3	-0.4	-1.1	-0.6	1.1	-1.5	0.3	-1.7†
3/C	Max	0.1	0.3	-1.2†	-1.1*	1.2†	0.2	0.1	-0.6
	Man	0.2	-0.4	-0.2	-0.9	0.7	-1.4	0.7†	-2.7†
4/D	Max	0.6	0.0	-0.7*	-1.9†	0.9†	-0.4	0.8†	-2.3†
	Man	0.9*	-0.3	-1.0*	-1.6*	1.5†	-1.9*	1.4†	-3.8†
5/E	Max	0.6	0.2	-1.0*	-2.2*	1.0*	-1.3	0.6	-3.3†
	Man	0.2	0.0	0.3	-0.4	-1.5	-0.1	-1.0†	-0.5*
6	Max	0.7	0.0	-0.3	0.3	0.5	-0.1	0.9†	0.2
	Man	0.6	0.0	-0.2	0.0	0.5	0.0	0.9†	0.0
7	Max			-2.1	0.4	1.7	-0.3		
	Man			-0.8	0.6	1.0	0.1		

* $p < 0.05$ † $p < 0.01$

the premolar regions, for most of the mandibular ordinate values between groups C and D, and for most of the abscissa and ordinate values between groups A and D.

For females a similar trend of increase or decrease in coordinate values between the groups was found with two main exceptions - the maxillary abscissa values for group C were generally less than those for group B or group D. Differences of abscissa and ordinate values were statistically significant, mainly for the maxillary values between groups B and C and for the maxillary and mandibular values between groups A and D.

2. POLAR COORDINATES

Polar coordinates were derived for the maxillary and mandibular arches of each subject from polynomial equations selected by the method described previously (see p. 78). Seven length measurements, termed polar vectors, were calculated at 30° intervals from 0° to 180°. These polar vectors were analysed by sex and by dental group to provide statistics describing the dental arches.

2.1 Right side compared with left side values

Tables 12 and 13 show the sample numbers, means, standard deviations and differences between right and left side polar

Table 12. Polar vector values for males - sample numbers, and means, standard deviations and differences between right and left side values in millimetres

			MALES															
	Polar vector		Group A				Group B				Group C				Group D			
		Side	N	Mean	Dif.	S.D.	N	Mean	Dif.	S.D.	N	Mean	Dif.	S.D.	N	Mean	Dif.	S.D.
MAXILLA	Incisor		32	35.9		1.9	35	36.7		2.0	12	35.6		2.7	93	34.6		2.1
	Canine	R	32	32.8	0.4	1.4	35	33.2	0.5	1.8	12	33.0	0.0	1.9	93	32.7	-0.1	1.7
		L	32	33.2		1.3	35	33.7		1.8	12	33.0		2.1	93	32.6		1.6
	Premolar	R	32	27.8	0.2	1.1	35	28.3	0.3	1.0	12	28.2	0.0	0.9	93	27.9	0.0	1.0
		L	32	28.0		1.3	35	28.6		1.2	12	28.2		0.9	93	27.9		1.0
	Molar	R	32	27.8	0.0	1.3	35	28.5	0.1	1.3	12	28.5	0.5	1.4	93	28.3	-0.1	1.1
		L	32	27.8		1.3	35	28.6		1.1	12	29.0		2.1	93	28.2		1.1
MANDIBLE	Incisor		32	33.1		1.7	35	33.3		1.8	12	32.3		1.4	93	30.8		1.5
	Canine	R	32	30.4	-0.5	1.2	35	30.6	-0.6	1.5	12	30.4	-0.7	1.3	93	28.9	-0.6†	1.3
		L	32	29.9		1.5	35	30.0		1.9	12	29.7		1.7	93	28.3		1.7
	Premolar	R	32	24.8	-0.1	0.9	35	25.2	-0.3	1.0	12	24.9	0.1	1.0	93	24.1	-0.2	1.4
		L	32	24.7		1.1	35	24.9		1.2	12	25.0		0.9	93	23.9		1.1
	Molar	R	32	24.7	0.1	1.2	35	25.2	0.0	1.1	12	25.4	0.1	0.9	93	25.2	0.1	1.1
		L	32	24.8		1.1	35	25.2		1.0	12	25.5		0.9	93	25.3		1.1

† $p < 0.01$

Table 13. Polar vector values for females - sample numbers, and means, standard deviations and differences between right and left side values in millimetres

F E M A L E S

	Polar vector	Side	N	Mean	Dif.	S.D.	N	Mean	Dif.	S.D.	N	Mean	Dif.	S.D.	N	Mean	Dif.	S.D.
MAXILLA	Incisor		18	35.2		2.2	22	36.1		2.0	6	35.0		0.8	49	34.6		2.2
	Canine	R	18	31.8	0.5	1.4	22	31.9	0.9*	1.5	6	31.3	-0.4	1.0	49	31.6	0.2	1.3
		L	18	32.3		1.5	22	32.8		1.2	6	30.9		0.8	49	31.8		1.3
	Premolar	R	18	26.6	0.2	1.0	22	27.0	0.5	1.0	6	26.2	0.1	1.0	49	26.9	0.0	0.8
		L	18	26.8		1.1	22	27.5		1.0	6	26.3		0.5	49	26.9		1.0
	Molar	R	18	26.3	0.0	1.2	22	27.1	0.1	1.3	6	26.8	-0.2	0.8	49	27.3	-0.2	1.0
		L	18	26.3		1.1	22	27.2		1.2	6	26.6		1.2	49	27.1		1.3
MANDIBLE	Incisor		18	32.7		2.0	22	32.9		1.5	6	31.7		2.2	49	30.7		1.8
	Canine	R	18	29.7	0.1	1.6	22	29.9	0.2	1.2	6	29.5	-0.9	2.4	49	28.8	-0.2	1.6
		L	18	29.8		1.6	22	30.1		1.4	6	28.6		1.5	49	28.6		1.6
	Premolar	R	18	23.9	0.0	1.2	22	24.2	0.2	1.0	6	23.7	0.3	0.7	49	23.7	-0.3	1.2
		L	18	23.9		1.1	22	24.4		0.9	6	24.0		1.2	49	23.4		1.0
	Molar	R	18	23.6	0.0	0.9	22	24.0	0.0	0.9	6	23.7	0.5	0.6	49	24.2	0.0	1.0
		L	18	23.6		1.0	22	24.0		0.8	6	24.2		1.0	49	24.2		1.1

* $p < 0.05$

vectors for males and females respectively. The differences between right side and left side values were small and only two were statistically significant.

2.2 Polar vectors and diagrammatic representation

Figures 28 and 29 show heptagonal shapes representing dental arches for males and females respectively. Values of the incisor vector and the means of the right and left polar vectors presented in Tables 12 and 13 were used to draw the shapes.

2.3 Sex differences

Tables 14(a and b) and Figure 30 show sex differences in average values of right and left polar vectors by dental groups. Polar vectors for males were greater than those for females with only one exception. Polar vector differences were statistically significant in the canine, premolar and molar regions for the maxilla, and in the premolar and molar regions for the mandible.

2.4 Differences between dental groups

Table 15 compares polar vectors between dental groups for males and females. The same data are shown diagrammatically in Figures 31 and 32. For males, polar vectors in group B were greater than their corresponding polar vectors in group A. Polar vectors in group C were less than those in group B except for the

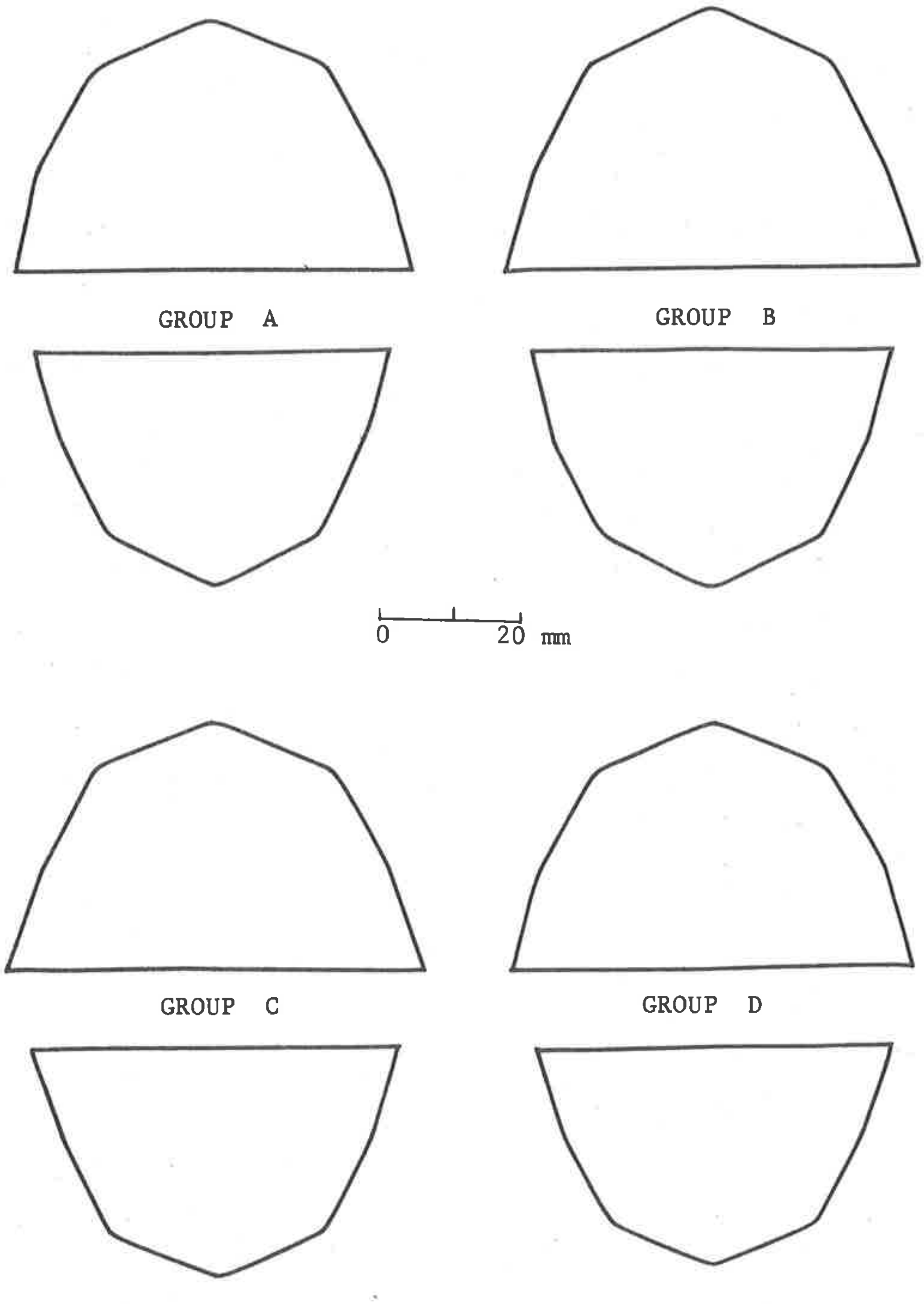

Figure 28. Dental-arch heptagons representing dental arches for males by dental groups

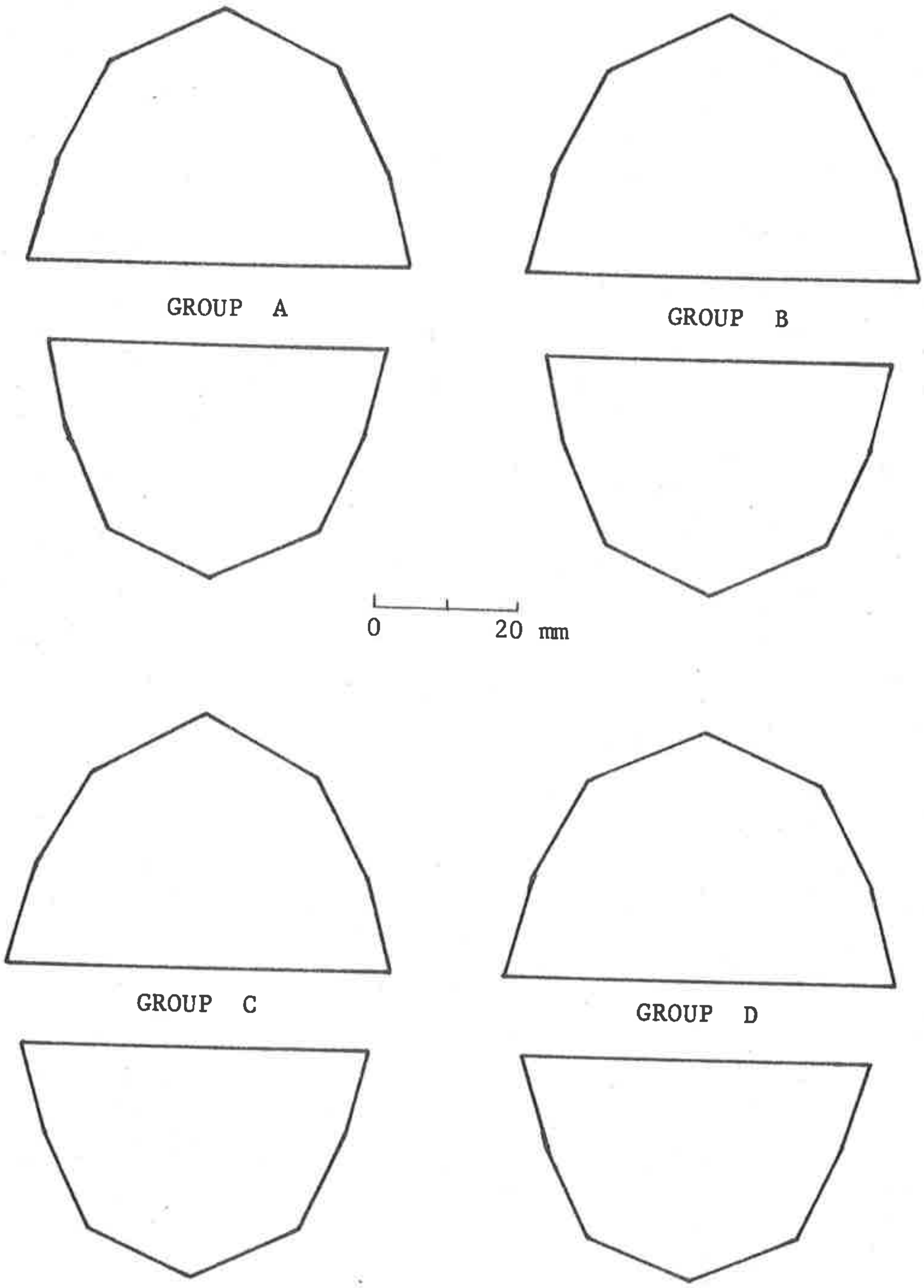

Figure 29. Dental-arch heptagons representing dental arches for females by dental groups

Table 14a. Sex differences in average values of right and left maxillary polar vectors by dental groups - sample numbers, and means, standard deviations and differences in millimetres

MAXILLA

Polar vector	Sex	Group A				Group B			
		N	Mean	Dif.	S.D.	N	Mean	Dif.	S.D.
Incisor	M	32	35.9	0.7	1.9	35	36.7	0.6	2.0
	F	18	35.2		2.2	22	36.1		2.0
Canine	M	32	33.0	0.9*	1.3	35	33.4	1.0†	1.7
	F	18	32.1		1.3	22	32.4		1.2
Premolar	M	32	27.9	1.2†	1.1	35	28.5	1.3†	1.0
	F	18	26.7		1.0	22	27.2		0.9
Molar	M	32	27.8	1.5†	1.2	35	28.6	1.5†	1.1
	F	18	26.3		1.1	22	27.1		1.2
		Group C				Group D			
Incisor	M	12	35.6	0.6	2.7	93	34.6	0.0	2.1
	F	6	35.0		0.8	49	34.6		2.2
Canine	M	12	33.0	1.9*	1.9	93	32.7	1.0†	1.6
	F	6	31.1		0.9	49	31.7		1.2
Premolar	M	12	28.2	2.0†	0.9	93	27.9	1.0†	1.0
	F	6	26.2		0.7	49	26.9		0.8
Molar	M	12	28.8	2.1†	1.7	93	28.3	1.1†	1.1
	F	6	26.7		1.0	49	27.2		1.1

* $p < 0.05$ † $p < 0.01$

Table 14b. Sex differences in average values of right and left mandibular polar vectors by dental groups - sample numbers, and means, standard deviations and differences in millimetres

M A N D I B L E

Polar vector	Sex	Group A				Group B			
		N	Mean	Dif.	S.D.	N	Mean	Dif.	S.D.
Incisor	M	32	33.1	0.4	1.7	35	33.3	0.4	1.8
	F	18	32.7		2.0	22	32.9		1.5
Canine	M	32	30.1	0.3	1.3	35	30.3	0.3	1.6
	F	18	29.8		1.5	22	30.0		1.1
Premolar	M	32	24.8	0.9†	1.0	35	25.1	0.8†	1.1
	F	18	23.9		1.1	22	24.3		0.8
Molar	M	32	24.8	1.2†	1.1	35	25.2	1.2†	1.0
	F	18	23.6		0.9	22	24.0		0.8
		Group C				Group D			
Incisor	M	12	32.3	0.6	1.4	93	30.8	0.1	1.5
	F	6	31.7		2.2	49	30.7		1.8
Canine	M	12	30.1	1.0	1.4	93	28.6	-0.1	1.4
	F	6	29.1		1.9	49	28.7		1.5
Premolar	M	12	25.0	1.1*	0.9	93	24.0	0.4*	1.2
	F	6	23.9		0.9	49	23.6		1.0
Molar	M	12	25.5	1.6†	0.9	93	25.2	1.0†	1.1
	F	6	23.9		0.7	49	24.2		1.1

* $p < 0.05$ † $p < 0.01$

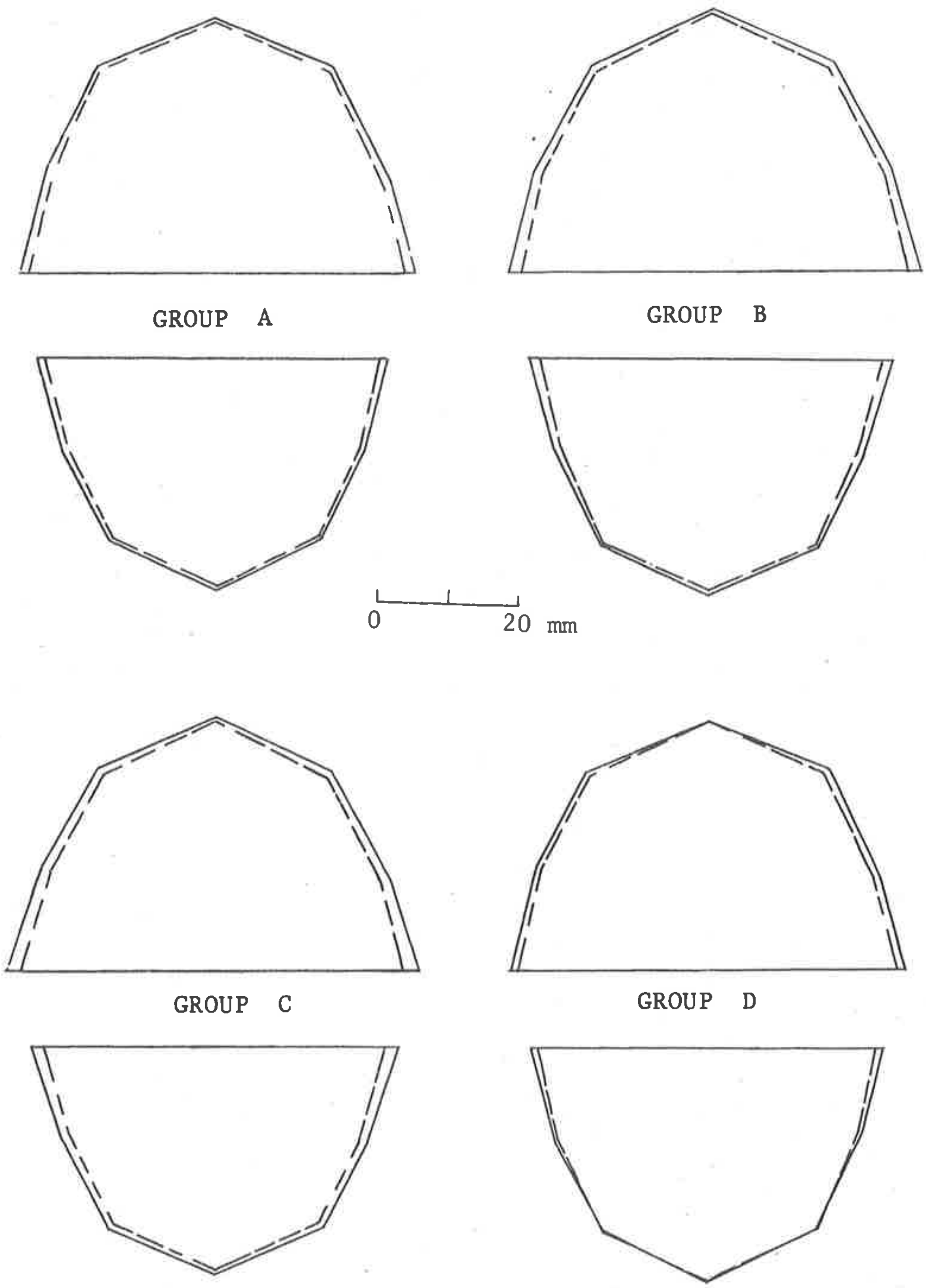

Figure 30. Dental-arch heptagons showing sex differences by dental groups - male groups shown by continuous lines

Table 15. Dental group differences in average values of right and left polar vectors in millimetres for males and females

Polar vector	Arch	A & B	B & C	C & D	A & D
M A L E S					
Incisor	Max	0.8	-1.1	-1.0	-1.3†
	Man	0.2	-1.0	-1.5†	-2.3†
Canine	Max	0.4	-0.4	-0.3	-0.3
	Man	0.2	-0.2	-1.5†	-1.5†
Premolar	Max	0.6*	-0.3	-0.3	0.0
	Man	0.3	-0.1	-1.0†	-0.8†
Molar	Max	0.8†	0.2	-0.5	0.5
	Man	0.4	0.3	-0.3	0.4*
F E M A L E S					
Incisor	Max	0.9	-1.1	-0.4	-0.6
	Man	0.2	-1.2	-1.0	-2.0†
Canine	Max	0.3	-1.3*	0.6	-0.4
	Man	0.2	-0.9	-0.4	-1.1*
Premolar	Max	0.5	-1.0*	0.7	0.2
	Man	0.4	-0.4	-0.3	-0.3
Molar	Max	0.8*	-0.4	0.5	0.9†
	Man	0.4	-0.1	0.3	0.6*

Groups compared: A & B, B & C, C & D, A & D

* $p < 0.05$ † $p < 0.01$

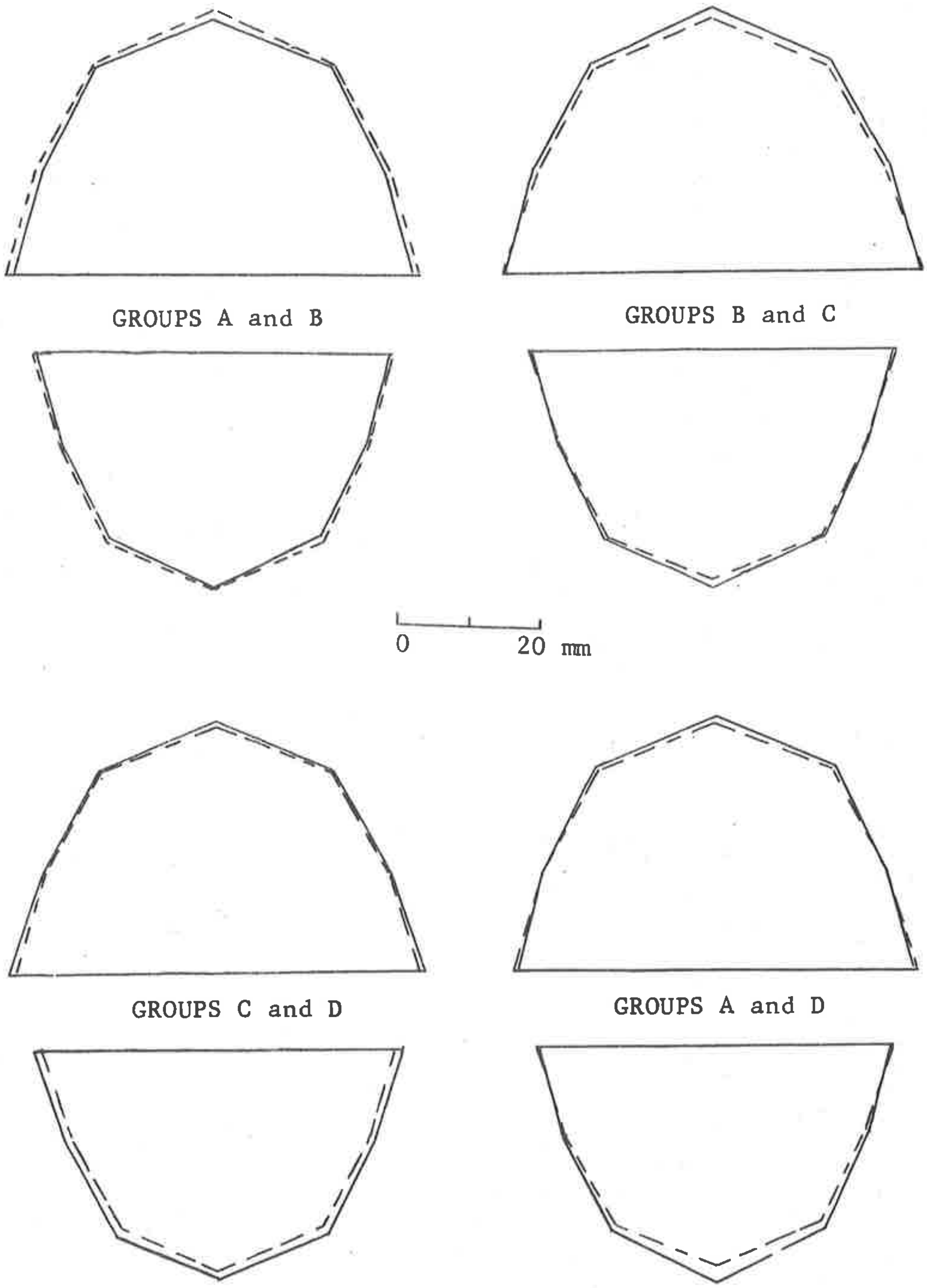

Figure 31. Dental-arch heptagons showing dental group differences for males - the first-mentioned group is shown by a continuous lines

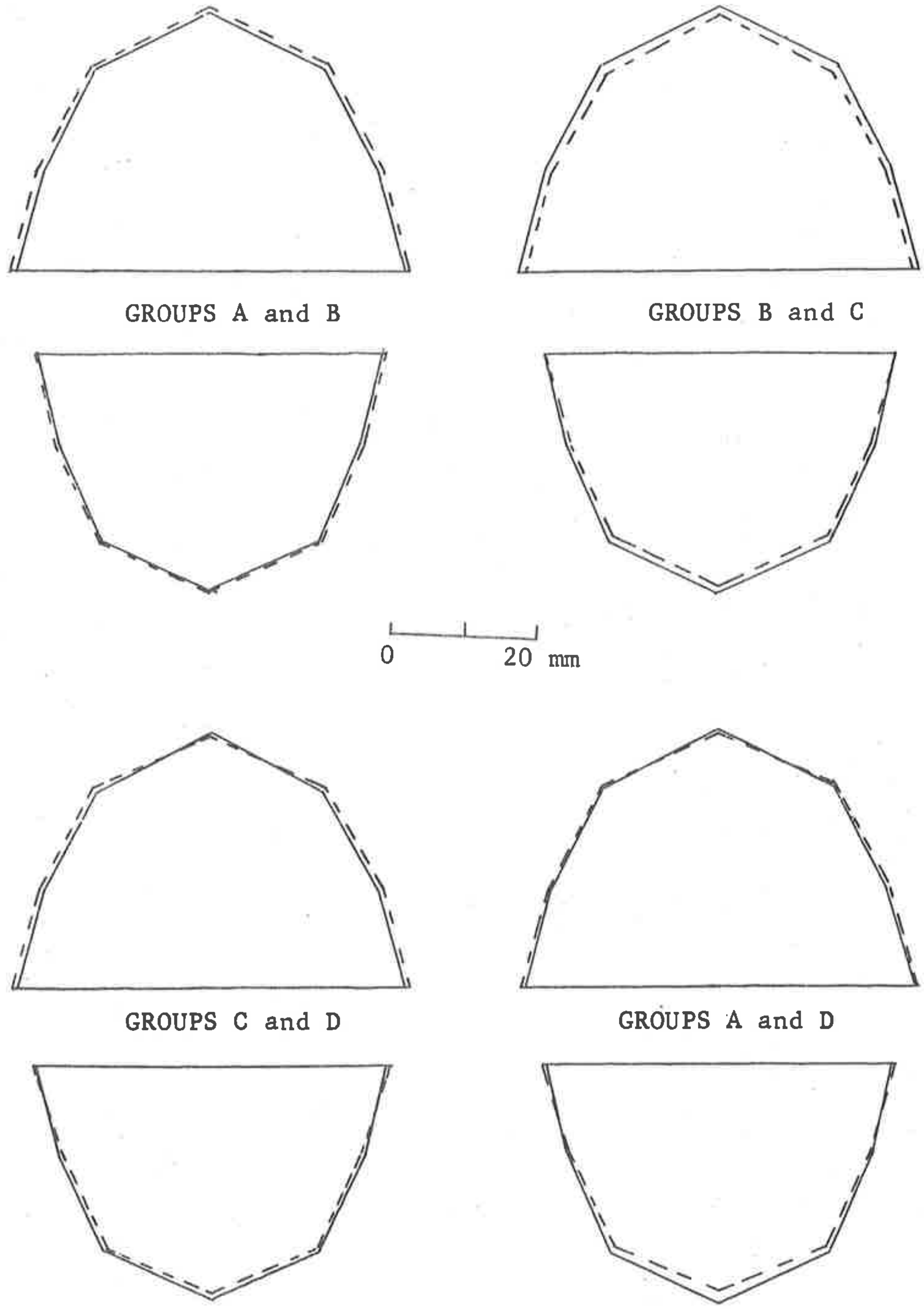

Figure 32. Dental-arch heptagons showing dental group differences for females - the first-mentioned group is shown by a continuous line

molar vectors. Polar vectors in group D were less than those in group C or in group A except for the molar vectors in group A.

For females, a similar trend of increase or decrease in polar vector values was found with only a few exceptions when group C was compared.

Polar vector differences between dental groups were statistically significant, mainly for the molar vector when group A was compared; and for most of the mandibular polar vectors when group D was compared in males, and when groups A and D were compared in females.

3. POLAR OVERJETS

Polar overjets were derived from differences in values of the polar vectors between the maxillary and the corresponding mandibular polar vectors with the arches related in the position of maximum intercuspation. They were analysed by sex and by dental group to provide statistics describing the vestibular projection of the maxillary arch curve outside the mandibular arch curve.

3.1 Right side compared with left side values

Tables 16 and 17 show the sample numbers, means, standard deviations and differences between right and left side polar

Table 16. Polar overjet values for males - sample numbers, and means, standard deviations and differences between right and left side values in millimetres

M A L E S

Polar overjet	Side	Group A				Group B			
		N	Mean	Dif.	S.D.	N	Mean	Dif.	S.D.
Incisor		32	3.2		1.3	35	3.8		1.3
Canine	R	32	3.3	-0.4	1.1	35	3.7	-0.7*	1.3
	L	32	2.9		1.3	35	3.0		1.4
Premolar	R	32	3.2	0.0	0.8	35	3.6	0.1	0.9
	L	32	3.2		0.8	35	3.7		1.6
Molar	R	32	3.4	0.1	0.9	35	3.5	0.4	1.0
	L	32	3.5		1.1	35	3.9		1.3
		Group C				Group D			
Incisor		12	4.3		1.4	93	3.5		1.1
Canine	R	12	4.0	-0.7	1.4	93	4.0	-0.3	1.4
	L	12	3.3		1.6	93	3.7		1.4
Premolar	R	12	3.4	0.3	1.1	93	4.0	-0.1	1.9
	L	12	3.7		1.1	93	3.9		0.8
Molar	R	12	3.9	0.0	1.8	93	3.2	0.3	1.1
	L	12	3.9		0.9	93	3.5		1.0

* $p < 0.05$

Table 17. Polar overjet values for females - sample numbers, and means, standard deviations and differences between right and left side values in millimetres

FEMALES

Polar overjet	Side	Group A N	Group A Mean (R)	Group A Mean (L)	Group A Dif.	Group A S.D.	Group B N	Group B Mean (R)	Group B Mean (L)	Group B Dif.	Group B S.D.
Incisor		18	3.4			1.9	22	3.6			1.6
Canine	R	18	2.8		0.0	1.4	22	2.7		0.8	1.2
	L	18		2.8		1.4	22		3.5		1.6
Premolar	R	18	3.0		0.3	0.5	22	3.0		0.1	1.0
	L	18		3.3		0.4	22		3.1		1.0
Molar	R	18	3.1		0.3	0.6	22	3.4		0.2	0.9
	L	18		3.4		0.9	22		3.6		1.1

Polar overjet	Side	Group C N	Group C Mean (R)	Group C Mean (L)	Group C Dif.	Group C S.D.	Group D N	Group D Mean (R)	Group D Mean (L)	Group D Dif.	Group D S.D.
Incisor		6	3.6			1.2	49	3.9			1.8
Canine	R	6	2.5		-0.7	1.9	49	3.4		-0.4	1.4
	L	6		1.8		2.7	49		3.0		1.3
Premolar	R	6	2.7		0.0	1.2	49	3.3		-0.1	0.8
	L	6		2.7		1.0	49		3.2		1.0
Molar	R	6	2.8		0.4	1.5	49	3.2		0.4	1.2
	L	6		3.2		1.2	49		3.6		0.7

overjets for males and females respectively. Differences between right side and left side values were small and only one was statistically significant.

3.2 Polar overjets and diagrammatic representation

Figure 33 shows heptagonal shapes representing the maxillary and the mandibular arches superimposed in functional relation to illustrate polar overjets of the four dental groups for males and females. Values of the incisor polar overjet and the means of the right and left polar vectors presented in Tables 16 and 17 were used to draw the shapes.

3.3 Sex differences

Table 18 shows sex differences in average values of right and left polar overjets by dental groups. Polar overjets were generally greater for males than females with only three exceptions, mainly in the incisor regions. Polar overjet differences were statistically significant, mostly in the canine and premolar regions.

3.4 Differences between dental groups

Table 19 compares polar overjets between dental groups for males and females. In general, the polar overjet values for

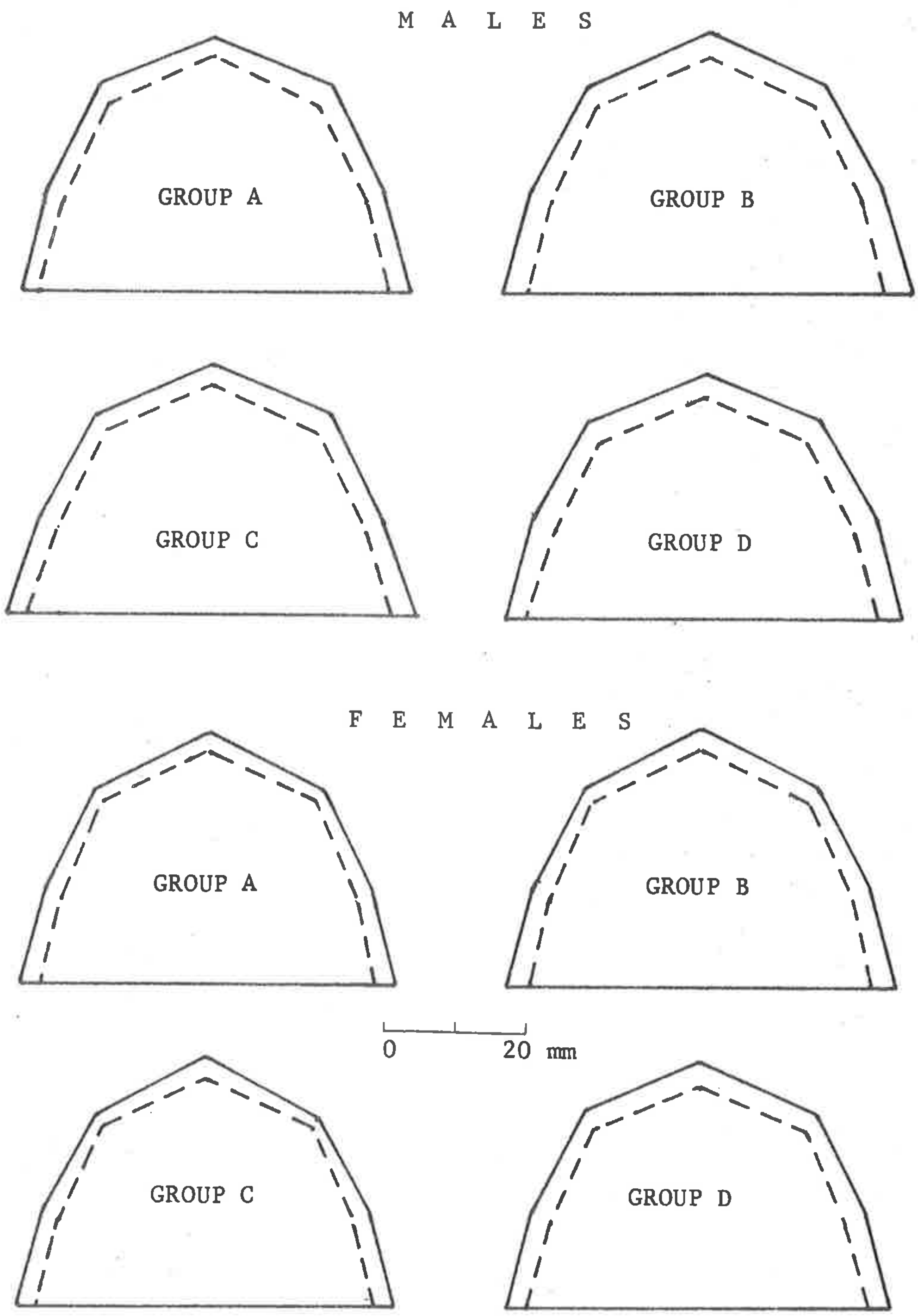

Figure 33. Superimposed maxillary (continuous line) and mandibular (broken line) dental heptagons showing polar overjets for males and females by dental groups

Table 18. Sex differences in average values of right and left polar overjets by dental groups - sample numbers, and means, standard deviations and differences in millimetres

Polar overjet	Sex	Group A				Group B			
		N	Mean	Dif.	S.D.	N	Mean	Dif.	S.D.
Incisor	M	32	3.2	-0.2	1.3	35	3.8	0.2	1.3
	F	18	3.4		1.9	22	3.6		1.6
Canine	M	32	3.1	0.3	1.1	35	3.3	0.7*	1.1
	F	18	2.8		1.3	22	2.6		1.1
Premolar	M	32	3.2	0.1	0.7	35	3.6	0.6*	1.0
	F	18	3.1		0.4	22	3.0		0.9
Molar	M	32	3.5	0.3	0.9	35	3.7	0.2	0.9
	F	18	3.2		0.7	22	3.5		0.8
		Group C				Group D			
Incisor	M	12	4.3	0.7	1.4	93	3.5	-0.4	1.1
	F	6	3.6		1.2	49	3.9		1.8
Canine	M	12	3.7	1.6	1.3	93	3.8	0.6†	1.2
	F	6	2.1		1.9	49	3.2		1.2
Premolar	M	12	3.5	0.8*	1.0	93	3.9	0.6†	1.1
	F	6	2.7		0.6	49	3.3		0.7
Molar	M	12	3.9	0.9	1.3	93	3.3	-0.1	0.9
	F	6	3.0		0.9	49	3.4		0.9

* $p < 0.05$ † $p < 0.01$

Table 19. Dental group differences in average values of right and left polar overjets in millimetres for males and females

MALES

Polar overjet	Groups compared			
	A & B	B & C	C & D	A & D
Incisor	0.6	0.5	-0.8	0.3
Canine	0.2	0.4	0.1	0.7†
Premolar	0.4	-0.1	0.4	0.7†
Molar	0.2	0.2	-0.6	-0.2

FEMALES

Polar overjet	A & B	B & C	C & D	A & D
Incisor	0.2	0.0	0.3	0.5
Canine	-0.2	-0.5	1.1	0.4
Premolar	-0.1	-0.3	0.6	0.2
Molar	0.3	-0.5	0.4	0.2

† $p < 0.01$

males increased between successive groups - with exceptions in the incisor and molar regions when group D was compared.

Unlike the males, polar overjet values for females decreased between successive groups A, B and C - except in the incisor and molar regions in group B. However, polar overjets for group D were greater than those for group C or group A.

Only two of the polar overjet differences were statistically significant - the canine and premolar regions between groups A and D for males.

4. DENTAL-ARCH AREAS

Areas of the dental arches were computed from the various polynomial equations. Table 20 shows the sample numbers, means and standard deviations of the dental-arch areas for males and females.

4.1 Sex differences

Table 21 shows the sex differences in dental-arch areas by dental groups. For both the maxillary and the mandibular arches dental-arch areas were greater for males than females. Differences in areas were statistically significant for all the groups in the maxillary arch and for group B in the mandibular arch.

Table 20. Dental-arch areas in square millimetres for males and females - sample numbers, means and standard deviations

Sex	Arch	Group A			Group B			Group C			Group D		
		N	Mean	S.D.	N	Mean	S.D.	N	Mean	S.D.	N	Mean	S.D.
Males	Max	32	1515	116	35	1573	122	12	1526	132	93	1485	120
	Man	32	1234	103	35	1260	107	12	1229	91	93	1125	85
Females	Max	18	1413	103	22	1469	84	6	1373	63	49	1408	81
	Man	18	1175	101	22	1203	49	6	1141	117	49	1107	86

Table 21. Sex differences in dental-arch areas in square millimetres - by dental groups

Arch	Group A	Group B	Group C	Group D
Max	102†	104†	153†	77†
Man	59	57†	88	18

† $p < 0.01$

Table 22. Dental group differences in dental-arch areas in square millimetres - for males and females

Sex	Arch	Groups compared			
		A & B	B & C	C & D	A & D
Males	Max	58*	-47	-41	-30
	Man	26	-31	-104†	-109†
Females	Max	56	-96*	35	- 5
	Man	28	-62	- 34	- 68*

* $p < 0.05$ † $p < 0.01$

4.2 Differences between dental groups

Table 22 shows dental group differences in dental-arch areas for males and females. For males, dental-arch areas of both the maxillary and the mandibular arches were greater in group B than in group A; less in group C than group B, and in group D than group C or group A.

For females, similar results were found except that the maxillary arch areas in group D were greater than those in group C.

Dental-arch area differences were statistically significant mainly for the mandibular arch when group D was compared.

5. DENTAL-ARCH PERIMETERS

Perimeters of the dental arches were computed from the various polynomial equations. Table 23 shows the sample numbers, means and standard deviations of the dental-arch perimeters for males and females.

5.1 Sex differences

Table 24 shows the sex differences in dental-arch perimeters by dental groups. For both the maxillary and the mandibular arches dental-arch perimeters were greater for males

Table 23. Dental-arch perimeters in millimetres for males and females - sample numbers, means and standard deviations

Sex	Arch	Group A			Group B			Group C			Group D		
		N	Mean	S.D.	N	Mean	S.D.	N	Mean	S.D.	N	Mean	S.D.
Males	Max	32	99.5	4.8	35	102.1	5.1	12	99.9	5.2	93	98.1	4.6
	Man	32	96.2	5.2	35	95.1	4.3	12	93.4	4.3	93	90.4	5.0
Females	Max	18	94.9	4.8	22	98.4	3.2	6	94.6	3.5	49	96.8	4.2
	Man	18	93.7	4.6	22	93.3	3.5	6	91.6	3.9	49	89.5	5.7

Table 24. Sex differences in dental-arch perimeters in millimeters - by dental groups

Arch	Group A	Group B	Group C	Group D
Max	4.6†	3.7†	5.3*	1.3
Man	2.5	1.8	1.8	0.9

* $p < 0.05$ † $p < 0.01$

Table 25. Dental group differences in dental-arch perimeters in millimetres - for males and females

Sex	Arch	Groups compared			
		A & B	B & C	C & D	A & D
Males	Max	2.6*	-2.2	-1.8	-1.4
	Man	-1.1	-1.7	-3.0*	-5.8†
Females	Max	3.5*	-3.8*	2.2	1.9
	Man	-0.4	-1.7	-2.1	-4.2†

* $p < 0.05$ † $p < 0.01$

than females. Differences in maxillary arch perimeters were statistically significant in groups A, B and C.

5.2 Differences between dental groups

Table 25 shows dental group differences in dental-arch perimeters for males and females. For males, dental-arch perimeters were smaller in the latter group of any two groups compared - except for the maxillary arch between groups A and B.

For females, a similar trend of increase or decrease in the values of dental-arch perimeters was found - except for the maxillary arch perimeter in group D which was greater than the perimeter in group C or group A.

Dental-arch perimeter differences were statistically significant, mainly for the maxillary arch when group B was compared, and for the mandibular arch when group D was compared.

SUMMARY OF RESULTS

The findings detailed in previous sections are summarized below. The numbers in parentheses denote the sections.

1. Differences between corresponding right side and left side Cartesian coordinates were small. The left side abscissa values tended to be greater than the right side in the

maxilla in the early mixed dentition stage. However, the right side abscissa values were usually greater in the mandible for all the stages and for the maxilla in the late mixed dentition and early permanent dentition. Significant differences were mainly in the anterior tooth regions. (1.1)

2. Although degree five polynomial equations described the dental arches better than degree four polynomials there were deficiencies in representing the shapes of a few arches. However, using polynomial with the same number of terms simplified presentation. (1.2)

3. The abscissa and ordinate values were generally greater for males than females except for the abscissa values of the mandibular anterior tooth regions in group D. Significant differences were mainly for the maxillary abscissa values of teeth distal to the incisors and for the mandibular abscissa values of the premolar and permanent first molar regions. (1.3)

4. For both males and females the maxillary and mandibular abscissa values between successive groups showed a gradual decrease in values in the incisor tooth regions and a gradual increase in values in the posterior tooth regions with exceptions when group C females were compared. However, the ordinate values for both arches showed a

gradual decrease in values between successive groups except for the anterior tooth regions when groups A and B were compared. Significant differences were mainly for the ordinate values in the mandibular premolar tooth regions and for most of the abscissa and ordinate values when groups A and D were compared. (1.4)

5. The differences between corresponding right side and left side polar coordinates were small and only two were statistically significant. (2.1)

6. Polar vectors for males were greater than those for females. Significant differences were found for all the maxillary polar vectors except the incisor vectors, and for the mandibular premolar and molar vectors. (2.3)

7. For males, polar vectors of both arches showed an increase in values between groups A and B, and a decrease in values between subsequent successive groups except for the molar vectors between groups B and C. Similar results were found for females with some exceptions when group C was compared. Significant differences were mainly for the molar vectors when group A was compared, and for most of the mandibular polar vectors when group D was compared. (2.4)

8. Differences between corresponding right side and left side polar overjets were small and only one was statistically significant. (3.1)

9. Polar overjets were generally greater for males than females. Significant differences were mainly in the canine and premolar regions. (3.3)

10. Polar overjets generally increased between successive groups for males with exceptions in the incisor and molar regions of group D. For females, however, most of the polar overjet values decreased between successive groups A, B and C but increased between groups C and D. Significant differences were found in the canine and premolar regions when groups A and D males were compared. (3.4)

11. Dental-arch areas were greater for males than females. Significant differences were mainly for the maxillary arches. (4.1)

12. For both males and females dental-arch areas generally increased between groups A and B, and subsequently decreased between the remaining successive groups. Significant differences were mainly for the mandibular arches when group D was compared. (4.2)

13. Dental-arch perimeters were greater for males than females. Significant differences were mainly for the maxillary arches in groups A, B and C. (5.1)

14. For males, dental-arch perimeter values decreased between successive groups except for the maxillary dental-arch perimeter between groups A and B. For females, the results were similar except that the group D maxillary dental-arch perimeter was greater. Significant differences were found mainly for the maxillary arch when group B was compared and for the mandibular arch when group D was compared. (5.2)

DISCUSSION

In the introduction to this work it was pointed out that there was no simple method of describing the size and shape of dental arches. The review of literature has emphasized the problem. Few results of the many investigations referred to could be directly compared because of the variety of methods used.

The most common method used in studies of arch size was measurement of various dimensions - breadths, depths and perimeters. However, the particular dental-arch dimensions selected for measurement were often differently defined. The methods used in studies of shape varied. As mentioned previously the subjective classification of dental-arch shapes following Hrdlička (1916) and the use of dental-arch indices following De Terra (1905) are unreliable. The introduction of the use of polar coordinates and Cartesian coordinates by Numata (1947) set the pattern for a number of dental-arch studies using mathematical equations. Most mathematical methods aimed at the description of basic dental-arch shapes. However, they were inadequate for shape comparisons. The catenary curve suggested by MacConaill and Scher (1949) provided a means for arch shape comparison. However, all dental arches cannot be adequately described by catenary curves, and the single measurement used to represent the shape of each dental arch does not convey to the reader the shape it describes.

The same problem of methodology was encountered in the present study. An early decision was taken to use a coordinate system of measurement and analysis. Having made the measurements on 276 sets of casts representing 44 subjects, several different analyses were carried out. The following paragraphs discuss the various analyses performed, each of which contributed to a new method developed in this study - namely, the use of areas to measure size and polar vectors to measure shape.

The refined data obtained from measurements of the dental casts were in the form of Cartesian coordinates. The location of each of the dental-arch reference points was specified in two dimensions by the abscissa and ordinate values. The coordinate values of the reference points were also used to compare the positional differences between teeth and their antimeres of the same dental arch, or between two corresponding teeth in dental arches of different groups of subjects. Although some general conclusions could be drawn from the results the analysis of Cartesian coordinate values was a complicated and tedious procedure. The results did not give a clear indication of changes in dental-arch size and shape. However, after obtaining the Cartesian coordinates of the dental-arch curve reference points polynomial equations were fitted to the points and a selection of the equations made to determine the curves best describing the dental arches.

The polynomial equations as such proved to be of little use for size and shape description. Only the first terms of the equations were directly comparable because they were equivalent to dental-arch depth values. However, when the equations were plotted their curves satisfactorily described the shape of the arches diagrammatically. Table 8 shows the complexity of the polynomial equations, and it would be a difficult task for anyone to plot the equations without the use of a computer. Even when the curves were plotted, the problem of describing the shapes in meaningful words still remained. It was possible to demonstrate differences in size and shape of dental arches by superimposing plotted curves upon each other, but when the differences between the curves were small it was difficult to distinguish visually any differences between the curves. Although the polynomial equations achieved the objective of expressing dental-arch shape in mathematical terms, yet it was impossible to compare dental-arch size and shape by the equations except when the curves were plotted. However, having computed the polynomial equations a number of dental-arch parameters - namely, breadth, depth, perimeter, area and the polar vectors - could be derived.

Of the dental-arch parameters mentioned only the polar vectors - a newly introduced term for the present investigation - and the dental-arch areas were not found in the available literature. A possible reason why area measurements have not been used previously for dental arch comparison might be the lack of a simple method whereby dental-arch areas can be readily determined. The area measurement is a good indication of the size of the dental arch, although it does not describe the shape. Tables 21 and 22 illustrate how dental-arch areas can be used for comparison. From the findings in Table 21 it is obvious that the male dental arches were larger than female arches, especially the maxillary arches. However, information was lacking as to how their shapes differed from one group to another. It was not possible to come to any conclusion from the findings when dental-arch perimeters were compared, nor when dental-arch breadths or dental-arch depths were compared. The perimeter measurements did not show either the size or the shape of the dental arches, but they provided valuable supplementary information for the interpretation of the findings. Comparing dental-arch shapes using breadth and depth parameters alone is no better than the use of dental-arch indices as suggested by De Terra (1905). Therefore, the polar vector parameters were introduced for use with area measurements so that dental-arch size and shape could be described and compared satisfactorily.

Seven polar vestors were derived from each of the polynomial equations. The incisor vector measurement represented dental-arch depth, whilst the sum of the right and left molar vector measurements indicated dental-arch breadth at the permanent first molar region. In addition to these more common depth and breadth measurements the canine vector and premolar vector measurements showed simultaneously the arch depth and breadth at the canine and premolar regions. Thus, the polar vectors defined dental-arch shape. It was possible to connect the seven polar coordinate measurements by straight lines forming the dental-arch heptagon. Furthermore, direct analyses of the polar vector measurements could be made to study symmetry of dental arches, differences in shape of dental arches from one group to another, and the degree of buccal and labial protrusion or overjet of the maxillary dental arch in relation to the mandibular arch when occluded in maximum intercuspation. In addition, it would be possible to fit polynomial curves to the seven polar coordinates to approximate the arch shape.

The sex differences and dental group differences in size and shape of the dental arches studied are summarized and discussed in the following paragraphs. The findings refer only to the arch regions from the permanent first molars forwards.

Differences in size and shape of dental arches between males and females showed that the male arches were larger in area and perimeter than the female arches at corresponding stages of dental development. They were also broader and more pronounced in overjet. The finding that the male arches were larger is consistent with the report by Barrett, Brown and Macdonald (1965) from a study of the dental arches of Australian Aborigines. An explanation of the sex differences in size of the dental arches could be the larger mesiodistal crown diameters of the teeth of males compared with females. This explanation is supported by the finding of the present study that dental-arch perimeters were greater for males than females, and by the finding of Barrett, Brown and Macdonald (1963a) that the mean values of the mesiodistal crown diameters of deciduous molars, the maxillary deciduous canine, and all the permanent teeth except the mandibular first premolars were significantly greater in males than in females.

Developmental changes in the size and shape of the dental arches from the early mixed dentition stage to the early permanent dentition stage were deduced from the findings of dental group differences. Changes during the early mixed dentition, both in males and females, involved increases in areas and perimeters and broadening of the arches. These changes suggested a general expansion of the dental arches as the permanent incisors emerged to their occlusal positions.

However, arch changes between the late mixed dentition stage and the early permanent dentition stage involved decreases in areas, perimeters and arch depths. For the maxilla the male arches showed a continuous decrease in size. In contrast, the females arches over a similar period of dental development showed an initial decrease in size followed by an increase. No explanation is offered for this apparent sex difference. For the mandible the male arches showed a greater decrease in size than females. Changes in shape were mainly due to the arch depth reductions. These changes in size and shape of the arches were almost certainly the result of positional adjustments of the teeth after exfoliation of the deciduous molars and their replacement with permanent successors having tooth crowns narrower mesiodistally. The considerably greater size reduction in mandibular arches for males compared with females could be the result of relatively greater differences between the diameters of deciduous molars and their successors. This explanation is supported by the measurements of the mesiodistal crown diameters of the teeth of Australian Aborigines reported by Barrett, Brown and Macdonald (1963a) and by Barrett, Brown and Luke (1963).

Approximal tooth wear could possibly contribute to the reduction in dental-arch size. Both Campbell (1925) and Begg (1954) in their investigations of skull material of Australian Aborigines found that there was a considerable reduction in dental-arch perimeter due to tooth wear. They pointed out that the marked tooth wear of Australian Aborigines living under natural conditions arose from vigorous mastication of coarse, fibrous, gritty food. In the present study, however, any reduction in arch perimeter caused by approximal tooth wear is considered to be minimal for two reasons: first, the developmental stages selected for investigation mainly involved newly erupted teeth and their predecessors; second, under the present settlement living conditions of the Australian Aboriginal subjects studied there was relatively little tooth wear because of the European type foods eaten by these people. The relatively insignificant tooth wear in the Aborigines of the present study compared with

Aborigines living under native conditions might be the reason for the persistence of incisor overbite and overjet in the early permanent dentition. Campbell (1925) reported that an edge-to-edge incisor relation was generally found in the adult dentition and he suggested that the change from the juvenile overbite incisor relation was associated with tooth wear.

The present study has been limited to observations of dental-arch size and shape changes in terms of group statistics. From the early mixed dentition to the early permanent dentition stages, both for males and females, the changes involved decreases in area, perimeter and arch depth. However, the permanent first molar regions showed an increase in arch breadth. There were also reductions in breadth and depth dimensions at the canine and premolar regions as measured by polar vectors. The reduction in size was relatively greater in the mandible than the maxilla, which generally led to a more pronounced overjet for the permanent dentition than the early mixed dentition. These changes are considered to result from positional adjustments of the permanent teeth when replacing their deciduous predecessors. Forces which may have been involved in the size and shape changes - from bone growth, from lips, cheeks and tongue, from the occlusion and from other sources - were not studied. It would be of great interest to study these forces and to continue the investigation of the Yuendumu subjects as they grow older. Changes in size and shape in the adult dentition as well as changes in occlusal relations could be studied in individuals to supplement the findings of the present investigation.

The method developed for the present investigation has proved to be useful. However, it would not have a wide practical application because of the specialised instrumentation required. Simpler methods for obtaining polar vector measurements are being planned. One of the proposed methods obtains direct measurements from casts or from photographs of casts by means of a special protractor-like instrument with graduated scales along the polar vector lines. An alternative method involves the use of a special optical device similar to a bellows camera with ground glass screen. The screen would allow a projected image of the casts at standard magnification to be measured directly in terms of polar vectors or indirectly via a tracing of the arch shape. Further study is required to see if there are any significant differences between results obtained by the present research method and those obtained by simpler methods. It may be possible to compare arch size differences by using dental-arch heptagon areas determined from polar vector measurements and arch shape differences by polar vector values.

In addition to dental-arch studies, the shape-representing polynomial equations may prove to be applicable to clinical orthodontics. The plotted curves would be useful for communicating orthodontic information concerning dental-arch size and shape. For record purposes, ready reference to arch shape would also be provided by the curves. It is likely that polynomial equations could be used to aid diagnosis and treatment planning. With supplementary information from other sources used in orthodontic diagnosis it may be possible to create polynomial curves representing the 'ideal' shape of the arches for each subject. By comparing the perimeters of the created 'ideal' curves and those of the corresponding curves obtained from the pre-treatment dental casts the need for any reduction in tooth substances may be estimated prior to the commencement of orthodontic treatment. Furthermore, the created curves would provide a suitable reference to arch form when shaping arch wires during treatment. However, further research is indicated to develop the practical application of shape-representing polynomial curves in clinical orthodontics.

Appendix 1

THE UNIVERSITY OF ADELAIDE
DEPARTMENT OF RESTORATIVE DENTISTRY

DENTAL CRITERIA FOR GROUPING SUBJECTS IN DENTGRO PROJECT

The grouping method is based on stages of dental development as indicated by the number and type of emerged teeth and the state of exfoliation of the deciduous teeth. A tooth is considered to have emerged when any portion of the crown, however small, has penetrated the gingiva and is visible in the oral cavity or its presence can be detected by a blunt probe through a breach in the gingiva.

Code	Dental Group	Stage of Dental Development
10	Infant 1	The period prior to the emergence of the first deciduous tooth
11	Infant 2	Begins with the emergence of the first deciduous tooth
12	Infant 3	Begins with the emergence of the first of the deciduous second molars
21	Infant 4	Begins with the emergence of the last deciduous tooth
31	Early Juvenile 1	Begins with the emergence of the first permanent tooth
32	Early Juvenile 2	Begins with the emergence of the first of the permanent maxillary central incisors
33	Early Juvenile 3	Begins with the emergence of the first of the maxillary lateral incisors
41	Late Juvenile 1	Begins with the exfoliation of a deciduous canine or molar
42	Late Juvenile 2	Begins with the emergence of the first of the maxillary second premolars
51	Adolescent	Begins with the exfoliation of the last of the deciduous teeth
61	Young Adult 1	Begins with the emergence of the first of the third molars
71	Young Adult 2	Begins with the emergence of the last of the third molars
81	Mature Adult	Begins with attainment of approximately 30 years of age
91	Aged	Begins with attainment of approximately 50 years of age

Appendix 2 The use of the permanent first molars reference points for the construction of abscissa and ordinate

The permanent first molars were selected to define the abscissa because they erupt and attain stable positions in the arch at an early age. They are large teeth, solidly implanted, and subjected to less variation in morphology and less irregularity in position than other teeth. They are common to the early mixed dentition and the adult dentition and can therefore provide a basis for comparison of tooth positions within the arch over a long period of time. The ordinate, derived from the positions of the permanent first molar teeth, is a valid reference line for studies of dental-arch symmetry.

Appendix 3

The tolerance value

A tolerance value of ± 0.5 mm on the abscissa and ordinate was given for accuracy of points read on the screen of the record reader. This was equivalent to a difference in the linear distance of ± $\sqrt{2}$ mm or ± 1.4 mm between the first and second determinations of the same reference point. With enlargement of the distance between the scale points on the screen in the ratio of 136.5:50 the tolerance value for the linear discrepancy in repeated determinations was ± 0.5 mm in actual measurement.

In order to prove the adequacy of the given tolerance value, linear discrepancies between the first and second determinations of the same reference points for the first fifty sets of casts were obtained. After discarding those sets of samples having results of gross disharmony with the general range of figures, the linear discrepancies from thirty sample sets of data were chosen for statistical analysis. The mean linear discrepancy obtained was 0.069 mm with a standard deviation of ± 0.044 mm and the maximum value of the distribution was 0.41 mm. This verified that the suggested tolerance value for this investigation was well chosen.

Appendix 4 An evaluation of the reliability of the register points used for the alignment of the maxillary and mandibular dental arches in their intercuspal or functional relation.

In order to evaluate the differences in abscissa and ordinate values between the register points of the maxillary arch and those of the mandibular arch in their maximum intercuspal relation, Cartesian coordinates of the register points for the first fifteen sets of casts in the series were chosen for the analysis.

Differences in values for the ordinate were zero except in two instances; and for the abscissa were small with a mean of 0.3 mm and standard deviations of ± 0.1 mm. These findings show that errors which could be introduced in the process of transforming coordinate points of the mandibular arch into their maximum intercuspal relation with the maxillary arch were small and the errors were within the tolerance values for linear discrepancies given to the present study.

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