Local Duality Predictions for $x \sim 1$ Structure Functions

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Recent data on the proton F_2 structure function in the resonance region suggest that local quark-hadron duality works remarkably well for each of the low-lying resonances, including the elastic, to rather low values of Q^2. We derive model-independent relations between structure functions at $x \sim 1$ and elastic electromagnetic form factors, and predict the $x \to 1$ behavior of nucleon polarization asymmetries and the neutron to proton structure function ratios from available data on nucleon electric and magnetic form factors.

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The nucleon's deep-inelastic structure functions and elastic form factors parametrize fundamental information about its quark substructure. Both reflect dynamics of the internal quark wave functions describing the same physical ground state, albeit in different kinematic regions. Recent work on generalized parton distributions [1] has provided a unifying framework within which both form factors and structure functions can be simultaneously embedded.

Exploration of the structure function/form factor interface is actually as old as the first deep-inelastic scattering experiments themselves. In the early 1970s the inclusive/exclusive connection was studied in the context of deep-inelastic scattering in the resonance region and the onset of scaling behavior. In their pioneering work, Bloom and Gilman [2] observed that the inclusive F_2 structure function at low W generally follows a global scaling curve which describes high W data, to which the resonance structure function averages. Furthermore, the equivalence of the averaged resonance and scaling structure functions appears to hold for each resonance, over restricted regions in W, so that the resonance-scaling duality also exists locally.

More recently, high precision data on the F_2 structure function from Jefferson Lab [3] have confirmed the original observations of Bloom and Gilman, demonstrating that local duality works remarkably well for each of the low-lying resonances, including surprisingly the elastic, to rather low values of Q^2. In the context of the operator product expansion in QCD, the existence of Bloom-Gilman duality can be attributed to the small size of higher twist ($1/Q^2$ suppressed) contributions to the structure function. In this Letter, we examine local duality for the elastic case more closely, and derive *model-independent* relations between structure functions at $x \sim 1$ and elastic electromagnetic form factors. Using the most recent data on the nucleon electric and magnetic form factors, we apply local duality to make quantitative predictions for the asymptotic behavior of unpolarized and polarized structure function ratios.

To illustrate the interplay between resonances and scaling functions, one can observe [2,4,5] that (in the narrow resonance approximation) if the contribution of a resonance of mass M_R to the F_2 structure function at large Q^2 is given by $F_2^{(R)} = 2M\nu[G_R(Q^2)]^2\delta(W^2 - M_R^2)$, then a form factor behavior $G_R(Q^2) \sim (1/Q^2)^n$ translates into a scaling function $F_2^{(R)} \sim (1 - x_R)^{2n-1}$, where $x_R = Q^2/(M_R^2 - M^2 + Q^2)$. On purely kinematical grounds, therefore, the resonance peak at x_R does not disappear with increasing Q^2, but rather moves towards $x = 1$.

For elastic scattering, the connection between the $1/Q^2$ power of the elastic form factors at large Q^2 and the $x \to 1$ behavior of structure functions was first established by Drell and Yan [6] and West [7]. More recently, interest in large-x structure functions has arisen in connection with the polarization asymmetry $A_1 = g_1/F_1$, and the F_2^n/F_2^p ratio, whose $x \to 1$ limits reflect mechanisms for the breaking of spin-flavor SU(6) symmetry in the nucleon [8].

If the inclusive/exclusive connection via local duality is taken seriously, one can use measured structure functions in the resonance region at large x to directly extract elastic form factors [9]. Conversely, empirical electromagnetic form factors at large Q^2 can be used to predict the $x \to 1$ behavior of deep-inelastic structure functions [2]. To quantify this connection, we begin by noting that the elastic contributions to the inclusive spin-averaged structure functions can be expressed through electric and magnetic form factors as [4]

$$F_1^{\mathrm{el}} = M\tau G_M^2 \delta\left(\nu - \frac{Q^2}{2M}\right), \tag{1a}$$

$$F_2^{\mathrm{el}} = \frac{2M\tau}{1+\tau}(G_E^2 + \tau G_M^2)\delta\left(\nu - \frac{Q^2}{2M}\right), \tag{1b}$$

where $\tau = Q^2/4M^2$, while for spin-dependent structure functions [4,10]:

$$g_1^{\mathrm{el}} = \frac{M\tau}{1+\tau} G_M(G_E + \tau G_M)\delta\left(\nu - \frac{Q^2}{2M}\right), \tag{1c}$$

$$g_2^{\mathrm{el}} = \frac{M\tau^2}{1+\tau} G_M(G_E - G_M)\delta\left(\nu - \frac{Q^2}{2M}\right). \tag{1d}$$

Following de Rújula *et al.* [9], one can integrate Eq. (1) over the targer-mass corrected scaling variable $\xi =$

$2x/(1+\sqrt{1+x^2/\tau})$ between the pion threshold and 1, allowing the "localized" moments of scaling functions to be expressed in terms of elastic form factors. The assumption of local elastic duality is that the area under the elastic peak [given by integrating the right-hand side of Eq. (1) at a given Q^2] is the same as the area under the scaling function (at much larger Q^2) when integrated from the pion threshold to the elastic point [2]. Using the local duality hypothesis, de Rújula *et al.* [9], and more recently Niculescu *et al.* [3], extracted the proton's G_M form factor (assuming that the ratio G_E/G_M is sufficiently constrained) from resonance data on the F_2 structure function at large ξ, finding better than ~30% agreement over a large range of Q^2.

Applying the argument in reverse, one can formally differentiate the local elastic duality relation [2] with respect to Q^2 to express the scaling functions, evaluated at threshold, in terms of Q^2 derivatives of elastic form factors:

$$F_1(x = x_{\rm th}) = \beta \frac{dG_M^2}{dQ^2}, \tag{2a}$$

$$F_2(x = x_{\rm th}) = \beta\left\{\frac{G_M^2 - G_E^2}{2M^2(1+\tau)^2} + \frac{2}{1+\tau}\left(\frac{dG_E^2}{dQ^2} + \tau\frac{dG_M^2}{dQ^2}\right)\right\} \rightarrow 2\beta\frac{dG_M^2}{dQ^2} \quad \text{as } \tau\rightarrow\infty, \tag{2b}$$

$$g_1(x = x_{\rm th}) = \beta\left\{\frac{G_M(G_M - G_E)}{4M^2(1+\tau)^2} + \frac{1}{1+\tau}\left(\frac{d(G_E G_M)}{dQ^2} + \tau\frac{dG_M^2}{dQ^2}\right)\right\} \rightarrow \beta\frac{dG_M^2}{dQ^2} \quad \text{as } \tau\rightarrow\infty, \tag{2c}$$

$$g_2(x = x_{\rm th}) = \beta\left\{\frac{G_M(G_M - G_E)}{4M^2(1+\tau)^2} + \frac{\tau}{1+\tau}\left(\frac{d(G_E G_M)}{dQ^2} + \frac{dG_M^2}{dQ^2}\right)\right\} \rightarrow \beta\frac{d}{dQ^2}(G_M^2 + G_E G_M) \quad \text{as } \tau\rightarrow\infty, \tag{2d}$$

where $x_{\rm th} = Q^2/(W_{\rm th}^2 - M^2 + Q^2)$, with $W_{\rm th} = M + m_\pi$, corresponds to the pion production threshold, and the kinematic factor $\beta = (Q^4/M^2)(\xi_0^2/\xi^3)(2x - \xi)/(4 - 2\xi_0)$. It is interesting to observe that asymptotically in the $Q^2 \rightarrow \infty$ limit each of the structure functions F_1, F_2, and g_1 is determined by the slope of the square of the magnetic form factor, while g_2 which in deep-inelastic scattering is associated with higher twists is determined by a combination of G_E and G_M.

The interpretation of the relations in Eq. (2) follows that given by Bloom and Gilman in the context of finite-energy sum rules [2]. Formulated originally by Dolen, Horn, and Schmid [11] for hadron scattering, finite-energy sum rules relate resonance structure functions at finite Q^2, averaged over appropriate intervals in W (or ν), to smooth scaling functions, such as those measured in the deep-inelastic region at much larger Q^2, which (modulo perturbative $\log Q^2$ corrections) depend on x only. For local elastic duality, the relevant interval over which the structure functions are averaged is between the pion production threshold at $x = x_{\rm th}$ and the elastic point, $x = 1$. Clearly, in the subthreshold region the only contribution is from elastic scattering, which is given entirely by the elastic form factors on the right-hand side of Eq. (1).

Differentiating the finite-energy sum rule relations for the elastic case [2], local duality then allows one to equate the right-hand side of Eq. (2), which represents the elastic contribution to the structure functions at finite Q^2, with the left-hand side, which corresponds to structure functions in the scaling region. Aside from perturbative QCD corrections, in the scaling region the latter are functions only of x.

The scaling functions on the left-hand side of Eq. (2) are evaluated at $x = x_{\rm th}$, with $x_{\rm th}$ corresponding to the particular value of Q^2 on the right-hand side of (2) [2]. However, since the results in the scaling limit are Q^2 independent, the predictions should also valid for $x > x_{\rm th}$. Note that in the limit $Q^2 \rightarrow \infty$ the location of the pion threshold $x_{\rm th} \rightarrow 1$, and the kinematic factor $\beta \rightarrow Q^4/(2\xi_0 M^2)$. In this limit one can explicitly verify that the right-hand side of (2) gives the correct asymptotic behavior of the structure functions as $x \rightarrow 1$. If $G_M(Q^2) \sim (1/Q^2)^n$ at large Q^2, then the right-hand sides of (2) must scale like $(1/Q^2)^{2n-1}$. At fixed W, since $(1 - x)$ behaves like $1/Q^2$, the x dependence of the scaling functions at large x is $(1 - x)^{2n-1}$, as required by the asymptotic scaling laws [6,7].

Equation (2) allows the large-x behavior of structure functions to be predicted from empirical electromagnetic form factors. Of particular interest is the $x \rightarrow 1$ behavior of the polarization asymmetry, A_1, which at large Q^2 is given by the ratio of spin-dependent to spin-averaged structure functions, $A_1 = g_1/F_1$. From spin-flavor SU(6) symmetry one expects, at leading twist, $A_1 = 5/9$ for the proton, and $A_1 = 0$ for the neutron. A number of models which incorporate SU(6) breaking, through either perturbative or nonperturbative mechanisms [8], suggest that $A_1 \rightarrow 1$ as $x \rightarrow 1$, in dramatic contrast to the SU(6) predictions, especially for the neutron.

Using the parametrization of global form factor data from Ref. [12], the proton and neutron asymmetries arising from the local quark-hadron duality relations (2) are shown in Fig. 1 as a function of x, with x corresponding to $x_{\rm th}$. One sees that while for $x \lesssim 0.9$ (which for the pion threshold corresponds to $Q^2 \approx 2.5$ GeV^2) the asymmetries are qualitatively consistent with the SU(6) expectations, the trend as $x \rightarrow 1$ is for both asymmetries to approach unity. Since $x_{\rm th} \rightarrow 1$ as $Q^2 \rightarrow \infty$, this is consistent with the operator product expansion interpretation of de Rújula *et al.* [9] in which duality should be a better approximation with increasing Q^2.

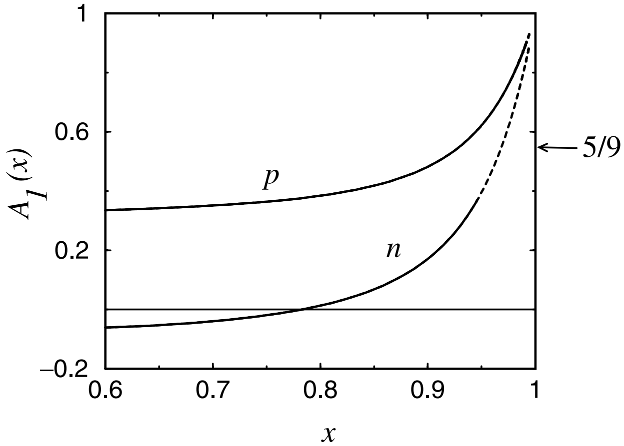

FIG. 1. Polarization asymmetries A_1 for the proton and neutron at large x. The SU(6) predictions are 5/9 for p and 0 for n. The dashed extensions represent asymmetries calculated from extrapolations of form factors beyond the currently measured regions of Q^2.

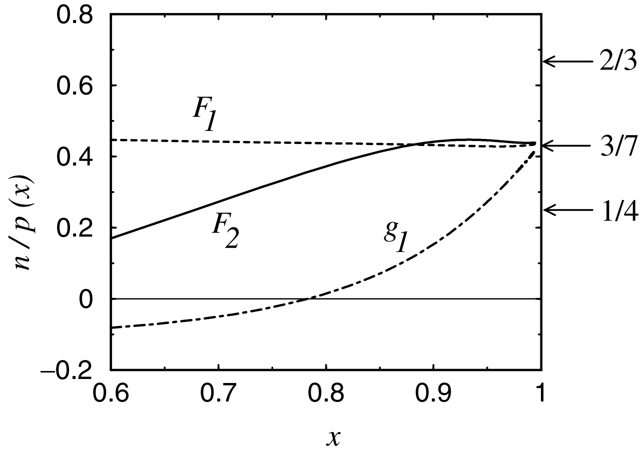

FIG. 2. Neutron to proton ratio for F_1 (dashed), F_2 (solid), and g_1 (dot-dashed) structure functions at large x. Several leading twist model predictions for F_2 in the $x \to 1$ limit are indicated by the arrows: 2/3 from SU(6), 3/7 from SU(6) breaking via helicity retention, and 1/4 from SU(6) breaking through d quark suppression.

Although the curves in Fig. 1 are shown for $x > 0.6$, one should note that the region below $x = x_{\rm th} \approx 0.8$ corresponds to $Q^2 \lesssim 1$ GeV^2, where duality is not expected to be as good an approximation, so the reliability of the local duality predictions there would be more questionable. Unfortunately the current data on A_1 extend only out to an average $\langle x \rangle \sim 0.5$, and are inconclusive about the $x \to 1$ behavior. While the proton A_1 data do indicate a steep rise at large x, the neutron asymmetry is, within errors, consistent with zero over the measured range [13]. It will be of great interest in the future to observe whether, and at which x and Q^2, the A_1 asymmetries start to approach unity.

Another quantity of current interest is the ratio of the neutron to proton F_2 structure functions at large x [8]. There are a number of leading twist predictions for this ratio, ranging from 2/3 in the SU(6) symmetric quark model, to 1/4 in broken SU(6) through d quark suppression [14], to 3/7 in broken SU(6) via helicity flip suppression [15]. Although it is well established that the large-x F_2^n/F_2^p data deviate from the SU(6) prediction, they are at present inconclusive about the precise $x \to 1$ limit because of large nuclear corrections in the extraction of F_2^n from deuterium cross sections beyond $x \sim 0.6$ [16].

The ratios of the neutron to proton F_1, F_2, and g_1 structure functions are shown in Fig. 2 as a function of x, with x again evaluated at $x_{\rm th}$. While the F_2 ratio varies somewhat with x at lower x, beyond $x \sim 0.85$ it remains almost x independent, approaching the asymptotic value $(dG_M^{n2}/dQ^2)/(dG_M^{p2}/dQ^2)$. Because the F_1 n/p ratio depends only on G_M, it remains flat over nearly the entire range of x (and Q^2). At asymptotic Q^2 the model predictions for $F_1(x \to 1)$ coincide with those for F_2; at finite Q^2 the difference between F_1 and F_2 can be used to predict the $x \to 1$ behavior of the longitudinal structure function, or the $R = \sigma_L/\sigma_T$ ratio.

The spin dependence of the proton vs neutron duality predictions is also rather interesting. Since A_1^n is zero for all x according to SU(6), the ratio of the neutron to proton g_1 structure functions is also zero in the spin-flavor symmetric limit. The pattern of SU(6) breaking for g_1^n/g_1^p essentially follows that for F_2^n/F_2^p, namely 1/4 in the d quark suppression [14] and 3/7 in the helicity flip suppression [15] scenarios. According to local duality, the g_1 structure function ratio in Fig. 2 approaches the asymptotic limit in Eq. (2c), albeit somewhat slowly, reflecting the relatively slow approach towards unity of the polarization asymmetry in Fig. 1. This may indicate a larger role played by higher twists in g_1 compared with F_2, a result consistent with analyses of higher twist corrections to moments of the g_1 [10] and F_2 structure functions [17].

It appears to be an interesting coincidence that the helicity retention model [15] prediction of 3/7 is very close to the empirical ratio of the squares of the neutron and proton magnetic form factors, $\mu_n^2/\mu_p^2 \approx 4/9$. Indeed, if one approximates the Q^2 dependence of the proton and neutron form factors by dipoles, and takes $G_E^n \approx 0$, then the structure function ratios are all given by simple analytic expressions, $F_2^n/F_2^p \approx F_1^n/F_1^p \approx g_1^n/g_1^p \to \mu_n^2/\mu_p^2$ as $Q^2 \to \infty$. On the other hand, for the g_2 structure function, which depends on both G_E and G_M at large Q^2, one has a different asymptotic behavior, $g_2^n/g_2^p \to \mu_n^2/[\mu_p(1 + \mu_p)] \approx 0.345$.

Of course the reliability of the duality predictions is only as good as the quality of the empirical data on the electromagnetic form factors. While the duality relations are expected to be progressively more accurate with increasing Q^2 [9], the difficulty in measuring form factors at large Q^2 also increases. Experimentally, the proton magnetic form factor G_M^p is relatively well constrained to $Q^2 \sim 30$ GeV^2, and the proton electric G_E^p to $Q^2 \sim 10$ GeV^2. The neutron magnetic form factor G_M^n has been measured to $Q^2 \sim 5$ GeV^2, although the neutron G_E^n is not very well determined at large Q^2 [fortunately, however, this plays only a minor role in the duality relations, with the exception of the neutron to proton g_2 ratio, Eq. (2d)].

In Fig. 1 the solid curves represent the A_1 asymmetry calculated from actual form factor data, while the dashed extensions illustrate the extrapolation of the form factors in Ref. [12] beyond the currently measured regions of Q^2. The fit in Ref. [12] uses all the available form factor data at lower Q^2, together with perturbative QCD constraints beyond the measured region at large Q^2. To test the sensitivity of the results in Figs. 1 and 2 to form factor shapes we have used several different parametrizations from Refs. [18,19], as well as a pure dipole form. Compared with the latter, the proton polarization asymmetries in Fig. 1 vary by $\sim$18%, 7%, and 2% at $Q^2 = 1$, 10, and 50 GeV^2, respectively, corresponding to values of x at the pion threshold of $x_{\text{th}} = 0.78$, 0.97, and 0.99, respectively. The neutron asymmetries vary by $\sim$100%, 14%, and 3% at the same values (the large relative variation at $Q^2 = 1$ GeV^2 simply reflects the fact that $A_1^n \approx 0$ at $x_{\text{th}} \sim 0.8$). As would be expected, the uncertainties decrease with increasing Q^2, since both the dipole fit and the fit from Ref. [12] incorporate the correct $Q^2 \to \infty$ limits from perturbative QCD.

The differences between the neutron to proton structure function ratios in Fig. 2 and those calculated from dipole parametrizations of form factors are qualitatively similar to those for the polarization asymmetry, namely $\sim$6% and 25% for F_2 and g_1, respectively, at $Q^2 = 5$ GeV^2, decreasing to $\sim$4% and 10% at 20 GeV^2. We have also tested the sensitivity of the ratios to the new data from Jefferson Lab on the G_E^p/G_M^p ratio [20], which show deviations from dipole behavior for $Q^2 \lesssim 3.5$ GeV^2. The differences induced in the ratios in Figs. 1 and 2 are, however, within the quoted range of uncertainty.

Obviously more data at larger Q^2 would allow more accurate predictions for the $x \to 1$ structure functions, and new experiments at Jefferson Lab [20] and elsewhere will provide valuable constraints. However, the most challenging aspect of testing the validity of the local duality hypothesis is measuring the inclusive structure functions at high enough x. Rapidly falling cross sections as $x \to 1$ mean that only very high luminosity facilities will be able to extract these with sufficiently small errors. The most promising possibility at present is the energy-upgraded CEBAF accelerator at Jefferson Lab. Once data on the longitudinal and spin-dependent structure functions at large x become available, a more complete test of local duality between elastic form factors and $x \sim 1$ structure functions can be made. In particular, with data on both the F_1 and F_2 (or g_1 and F_2) structure functions at large x one will be able to extract the G_E and G_M form factors separately, without having to assume the G_E/G_M ratio in extracting G_M from the currently available F_2 [3,9].

Along with the spin dependence, unraveling the flavor dependence of duality is also of fundamental importance. Although the local duality relations discussed here are empirical, a more elementary description of the quark-hadron transition requires understanding the transition from coherent to incoherent dynamics and the role of higher twists for individual quark flavors. This is as relevant for all the $N \to N^*$ transition form factors as for the elastic. The flavor dependence can be determined by either scattering from different hadrons, or tagging mesons in the final state of semi-inclusive scattering in the resonance region. Unraveling the flavor and spin dependence of duality, and more generally the relationship between incoherent (single quark) and coherent (multiquark) processes, will shed considerable light on the nature of the quark $\to$ hadron transition in QCD.

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