Optimal Water Allocation and Scheduling for Irrigation Using Ant Colony Algorithms

by

Duc Cong Hiep Nguyen

MEng (Civil and Environmental)

Thesis submitted to The University of Adelaide,
Faculty of Engineering, Computer and Mathematical Sciences,
School of Civil, Environmental and Mining Engineering
in fulfilment of the requirements
for the degree of Doctor of Philosophy

Contents

Abstract

In most regions of the world, irrigation is vital for food production. However, under increased water demands due to population growth, economic development, and climate change in recent decades, there is likely to be a significant reduction in the amount of water available for irrigation. Therefore, it is imperative to make the best use of water that is available for irrigation. This applies to: 1) the optimal allocation of land and water resources for irrigation management to achieve maximum net return, subject to constraints on area and water allocations at the district or regional scale; and 2) the optimal irrigation scheduling of available water, as well as fertilizer, in order to maximise net return at the farm scale. In order to rigorously address these problems, metaheuristic optimization algorithms have been used extensively due to their abilities in terms of finding globally optimal or near-globally optimal solutions and relative ease of linkage with complex simulation models. However, the application of these algorithms to real-world problems has been challenging due to the generally large size of the search space and the computational effort associated with realistic long-term simulation of crop growth and associated soil-water processes.

In this thesis, general simulation-optimization frameworks for optimal irrigation management (including optimal crop and water allocation, and optimal irrigation water and fertilizer application scheduling) have been developed in order to make the application of metaheuristic optimization methods to the above problems more computationally efficient. As part of this approach the problems are represented in the form of decision graphs which are solved using ant colony optimization (ACO) as the optimization engine. The frameworks enable dynamic reduction of the size of the search space by using dynamic decision variable option (DDVO) adjustment during solution construction. This also ensures only feasible solutions are obtained as part of the stepwise solution generation process. In addition, the computational efficiency of the ACO algorithms within the framework for optimal crop and water allocation has been increased by biasing the options at each node in the decision-tree graph based on domain knowledge

(represented by a visibility factor, VF). Furthermore, the framework for optimal irrigation scheduling was linked with a process-based crop growth model to enable optimal or near-optimal irrigation water and fertilizer application schedules to be identified.

This thesis is arranged as a series of three publications that present the main research contributions. The first paper introduces a generic simulation-optimization framework for optimal crop and water allocation at the regional or district scale using decision-tree graphs, ACO and the search space reduction technique based on dynamically adjusting decision variable options during stepwise solution construction. The performance of this technique in terms of finding feasible solutions, solution quality, computational efficiency and convergence speed was compared with that of linear programming (LP) and a “standard” ACO approach using static decision variable options (SDVO) on a benchmark case study from the literature. The second paper extends the ACO formulation for optimal crop selection and irrigation water allocation in the first paper by incorporating domain knowledge through VFs to bias the search towards selecting crops that maximize net returns and water allocations that result in the largest net return for the selected crop, given a fixed total volume of water. This improvement enables locally optimal solutions related to the factors (i.e., crops and water) affecting net return to be identified, and enables promising regions of the search space to be explored. The benefits of this improved formulation were tested on the benchmark case study used in the first paper and a real-world case study based on an irrigation district located in Loxton, South Australia near the River Murray. In the final paper, the formulation for detailed optimal irrigation water and fertilizer application scheduling at the farm scale is introduced and applied to a case study considering corn production under center pivot irrigation in Colorado, USA. The Root Zone Water Quality Model 2 (RZWQM2) was used for this case study to simulate the detailed impacts of irrigation water and fertilizer application scheduling on crop growth at a fixed time step. The utility of the proposed framework was demonstrated in terms of finding better net returns while using less fertilizer and similar amounts of water, or similar net returns while using less water and fertilizer, compared with the Microsoft Excel spreadsheet-based Colorado Irrigation Scheduler (CIS) tool for annual crops.

Statement of Originality

I certify that this work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide and where applicable, any partner institution responsible for the joint-award of this degree.

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Acknowledgements

First of all, I would like to thank my supervisors, Prof. Holger Maier and Prof. Graeme Dandy, for their supervision, support, and dedication during my PhD research. I am particularly grateful to Prof. Holger Maier for his guidance, determination and enthusiasm in helping me to finish the thesis and improve my research skills (including presentation, writing, and publication). I would also like to thank Prof. Graeme Dandy for his encouragement, constant motivation and inspiration to keep my morale positive for the four years of my PhD candidature.

I would like to thank Dr James C. Ascough II (Research Hydrologic Engineer, USDA-ARS-PA, Agricultural Systems Research Unit, Fort Collins, Colorado, USA) and Associate Prof. Allan A. Andales (Department of Soil and Crop Sciences, Colorado State University, Fort Collins, Colorado, USA) for their support and supervision involving the provision of data, crop growth models and technical assistance for the success of my PhD research.

I am grateful to Darran King (CSIRO Ecosystem Sciences) for providing the data of the River Murray case study. I am also grateful to the staff of eResearch South Australia for the kind assistance to run the models using supercomputers.

I acknowledge my PhD peer, Joanna Szemis, for her support in developing the initial model. I also acknowledge all the staff and other postgraduate students in the School of Civil, Environmental and Mining Engineering who have helped me enjoy my PhD life.

I am grateful to my friends for their encouragement.

I would like to thank my parents (Nguyen Duc Ngoc and Nguyen Thi Huong), my sister (Hong Chau), my brother (Cong Song), and my family for their encouragement, love and trust. Finally, I would like to thank my wife, Hai Duong, for her love and support. She has accompanied me over the journey of my PhD candidature.

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CHAPTER 1

Introduction

In most regions of the world, irrigation is vital for food production. Water shortages for irrigation can potentially constrain agricultural production, while efficient crop water allocation can possibly increase crop yields, contributing to food security and sustainable socio-economic development (Young *et al.*, 1994). Irrigation generally accounts for the largest percent of available total water, for example, about 70% in the world (Fry, 2005), about 80% in the basins of southern Spain (Reca *et al.*, 2001) and up to 94% in the Murray-Darling Basin, Australia (Murray Darling Basin Authority, 2011). While the importance of irrigation should increase in the near future as a result of population growth (Dyson, 1999; Rosegrant and Ringler, 2000), economic development (Malla and Gopalakrishnan, 1999), environmental needs (Burke *et al.*, 2004), and the impact of climate change (Arnell, 1999; Liu *et al.*, 2010), there is likely to be a significant reduction in the amount of water available for irrigation. Therefore, it is imperative to make the best use of water that is available for irrigation. This applies to: 1) the optimal allocation of land and water resources for irrigation management to achieve the maximum net return, subject to constraints on area and water allocations at the district or regional scale; and 2) the optimal irrigation scheduling of available water, as well as fertilizer, in order to maximise net return at the farm scale.

In order to address the above problems, optimization techniques have generally been applied in previous studies (Singh, 2012, 2014), including dynamic programming (Rao *et al.*, 1988; Naadimuthu *et al.*, 1999), nonlinear programming (Ghahraman and Sepaskhah, 2004), multi-objective programming (Lalehzari *et al.*, 2015), and simulated annealing (Brown *et al.*, 2010). Although these “conventional algorithms” (CAs) for optimization have the advantage of being simple and efficient to apply, they are somewhat limited in terms of handling nonlinear problems and by the “curse of dimensionality" (i.e., the search space size grows exponentially with the number of state variables), as is the case in irrigation management (Singh, 2014).

In recent years, metaheuristic optimization algorithms have been used extensively to identify the optimal solutions for irrigation problems (Nicklow *et al.*, 2010; Maier *et al.*, 2014; Maier *et al.*, 2015). This is because these algorithms are able to find globally optimal or near-globally optimal solutions. However, the application of these algorithms to real-world problems is complicated by a number of factors (Maier *et al.*, 2014), one of which is the generally large size of the search space, which may limit the ability to find globally optimal or near-globally optimal solutions in an acceptable time period. In addition, metaheuristic algorithms are also able to be linked with simulation models (e.g., crop growth models) that are commonly applied to evaluate objective functions in irrigation management. These models may take on simple forms, such as water production functions (Jensen (1968); Doorenbos and Kassam (1979)), to calculate crop yield response to water (Reca *et al.*, 2001; Evans *et al.*, 2003; Azamathulla *et al.*, 2008; Georgiou and Papamichail, 2008; Brown *et al.*, 2010; Prasad *et al.*, 2011) or the FAO Penman-Monteith method crop evapotranspiration (ET) and the crop growth coefficient approach of Doorenbos and Pruitt (1977) to estimate crop water requirements (Shyam *et al.*, 1994; Sethi *et al.*, 2006; Khare *et al.*, 2007). Although these quasi-empirical modelling approaches have been widely used in optimization studies due to their computational efficiency (Singh, 2012), they are unable to represent the underlying physical processes affecting crop water requirements, crop growth and agricultural management strategies (e.g., fertilizer application) in a realistic manner. These shortcomings have been addressed by more complex simulation models, such as ORYZA2000 (Bouman *et al.*, 2001), RZQWM2 (Bartling *et al.*, 2012), AquaCrop (Vanuytrecht *et al.*, 2014), EPIC (Zhang *et al.*, 2015) and STICS (Coucheney *et al.*, 2015). However, due to their relatively long runtimes (e.g., several minutes per evaluation), these models are normally used to simulate a small number of management strategy combinations (Camp *et al.*, 1997; Rinaldi, 2001; Arora, 2006; DeJonge *et* *al.*, 2012; Ma *et al.*, 2012b), rather than being used in combination with metaheuristic algorithms to identify (near) globally optimal solutions.

A potential solution to the above problems (i.e., large size of search space and long simulation-optimization model run times) is to reduce the search space size, as this has been successfully applied in many application areas of water resources, such as the optimal design of water distribution systems (Gupta *et al.*, 1999; Wu and Simpson, 2001; Kadu *et al.*, 2008; Zheng *et al.*, 2011, 2014), the optimal design of stormwater networks (Afshar, 2006, 2007), the optimal design of sewer networks (Afshar, 2012), the calibration of hydrologic models (Ndiritu and Daniell, 2001), the optimization of maintenance scheduling for hydropower stations (Foong *et al.*, 2008a; Foong *et al.*, 2008b) and the optimal scheduling of environmental flow releases in rivers (Szemis *et al.*, 2012, 2014). Reduction in the size of the search space has the potential to enable near-globally optimal solutions to be found within a reasonable timeframe or the best possible solution to be found for a given computational budget. Of the metaheuristic algorithms, ant colony optimization (ACO) is particularly suited to problems where there is dependence between decision variables, such that the selected values of particular decision variables restrict the available options for other decision variables, as is often the case in scheduling and allocation problems (e.g., Afshar, 2007; Afshar and Moeini, 2008; Foong *et al.*, 2008a; Foong *et al.*, 2008b; Szemis *et al.*, 2012, 2014; Szemis *et al.*, 2013).

Another potential solution for addressing the difficulties of applying metaheuristics to solving real-world irrigation optimization problems is to improve computational efficiency by incorporating domain knowledge during the optimization process. This is demonstrated in a number of other water resources planning and management problem domains (Kadu *et al.*, 2008; Zheng *et al.*, 2011; Creaco and Franchini, 2012; Kang and Lansey, 2012; Afshar *et al.*, 2015; Bi *et al.*, 2015). As the trial solutions of the metaheuristics, such as genetic algorithms (GAs), are modified from previous ones as the search progresses from one iteration to the next, the incorporation of domain knowledge can generally only be achieved by seeding the initial population. As a result, these algorithms are unable to take advantage of the potential benefits of incorporating domain knowledge into the construction of trial solutions. The use of ACO can overcome this limitation as trial solutions are constructed using past experience contained in the search space, which is presented in the form of a decision-tree graph. Therefore, ACO offers great potential for increasing the computational efficiency in relation to solving irrigation-related optimization problems via the incorporation of domain knowledge.

1.1 Research objectives

In order to address the problems outlined above, this thesis develops general simulation-optimization frameworks for irrigation management, including optimal crop and water allocation and optimal irrigation water and fertilizer application scheduling. As part of the frameworks, the problems are represented in the form of decision graphs and ACO is used as the optimization engine. Furthermore, reduction in the search space size and incorporation of domain knowledge during the optimization process are utilized to increase computational efficiency. Overall, this study has the following three main objectives:

Objective 1: To develop a generic simulation-optimization framework for optimal crop and water allocation (Papers 1 and 2).

Objective 1.1: To formulate a generic simulation-optimization framework that reduces search space size by incorporating dynamic decision variable options (Paper 1).

Objective 1.2: To improve the computational efficiency of the generic simulation-optimization framework in Objective 1.1 by incorporating domain knowledge through the addition of visibility factors (VFs) (Paper 2).

Objective 2: To develop a generic simulation-optimization framework for irrigation water and fertilizer application scheduling that reduces search space size by adjusting dynamic decision variable options (Paper 3).

Objective 3: To evaluate the utility of the frameworks in Objectives 1 and 2 (Papers 1, 2 and 3).

Objective 3.1: To apply the framework in Objective 1 to a benchmark case study from the literature, using simple crop production functions (Papers 1 and 2).

Objective 3.2: To apply the framework in Objective 1 to a real-world case study in Loxton, South Australia, near the Murray River, using simple crop production functions (Paper 2).

Objective 3.3: To apply the framework in Objective 2 to a case study in Colorado State, USA, using a detailed, process-based crop-growth model (Paper 3).

1.2 Thesis overview

This thesis is organized into five chapters, where the main contributions are presented in Chapters 2 to 4. Each of these chapters is presented in the form of a technical paper. The first of these (Chapter 2) has been published in Environmental Modelling & Software and the second (Chapter 3) has been accepted for publication in the Journal of Water Resources Planning and Management.

Chapter 2 introduces a simulation-optimisation framework for optimal crop and water allocation at the regional scale (Objective 1), where the problem is represented in the form of a decision-tree graph and ACO is used as the optimization engine. As part of the framework, the search space size is dynamically reduced by using dynamic decision variable option (DDVO) adjustment during stepwise solution construction (Objective 1.1). The options that violate any constraint are not available to be selected as a trial crop and a water allocation plan is constructed. The utility of the framework is then illustrated (Objective 3) by comparing it to linear programming (LP) and a "standard" ACO approach using static decision variable options (SDVO) using a benchmark case study from the literature (Objective 3.1).

Chapter 3 improves the computational efficiency of the ACO formulation for optimal crop selection and irrigation water allocation in the framework introduced in **Chapter 2** (Objective 1). It does this by incorporating domain knowledge through VFs to bias the search towards selecting crops that maximize net returns and water allocations that result in the largest net return for the selected crop, given a fixed total volume of water available (Objective 1.2). The benefits of this improved formulation are tested on the benchmark case study in the first paper (Objective 3.1) and a real-world case study based on an irrigation district located in Loxton, South Australia near the River Murray (Objective 3.2).

Chapter 4 introduces a simulation-optimisation framework for detailed irrigation water and fertilizer application scheduling at the farm scale (Objective 2), which represents the problem in the form of a decision-tree graph, uses ACO as the optimization engine and is linked with a complex, process-based crop growth model. To demonstrate the utility of the framework, it is applied to a realistic case study which uses the Root Zone Water Quality Model 2 (RZWQM2) for corn production under center pivot irrigation in Colorado, USA (Objective 3.3).

The linking of each of the papers to the objectives is shown in Table 1-1. The scale, optimization problem addressed, ACO algorithmic improvements introduced, case studies considered and crop growth models used in each of the papers are summarised in Table 1-2. Although the manuscripts have been reformatted in accordance with University guidelines, and sections renumbered for inclusion within this thesis, the material within these papers is otherwise presented herein as published. A copy of the first paper "as published" is provided in Appendix A.

Conclusions of the research within this thesis are provided in **Chapter 5**, which summarises: 1) the research contributions, 2) limitations and 3) future directions for further research.

Table 1-1. Linking of each of the papers to the objectives

	Objectives	Paper 1	Paper 2	Paper 3
1	To develop a generic simulation-optimization framework for optimal crop and water allocation	X	X	
1.1	To formulate a generic simulation-optimization framework that reduces search space size by incorporating dynamic decision variable options.	X		
1.2	To improve the computational efficiency of the generic simulation-optimization framework in Objective 1.1 by incorporating domain knowledge through the addition of VFs.		X	
2	To develop a generic simulation-optimization framework for irrigation water and fertilizer application scheduling that reduces search space size by adjusting dynamic decision variable options.			X
3	To evaluate the utility of the frameworks in Objectives 1 and 2.	X	X	X
3.1	To apply the framework in Objective 1 to a benchmark case study from the literature, using simple crop production functions.	X	X	
3.2	To apply the framework in Objective 1 to a real-world case study in Loxton, South Australia, near the Murray River, using simple crop production functions.		X	
3.3	To apply the framework in Objective 2 to a case study in Colorado State, USA, using a detailed, process-based crop-growth model.			X

Table 1-2. Classification of the papers by the different features addressed

		Paper 1	Paper 2	Paper 3
Scale	Regional	X	X	
	Farm			X
Optimization Problem	Crop allocation	X	X	
	Water allocation	X	X	
	Irrigation water scheduling			X
	Fertilizer application scheduling			X
ACO improvements	Dynamic decision variable option adjustment	X	X	X
	Visibility factors		X	
Case Study	Benchmark case study introduced by Kumar and Khepar (1980)	X	X	
	Loxton, SA near the Murray River		X	
	Colorado, USA			X
Crop Growth Model	Crop production functions	X	X	
	Process-based model			X

CHAPTER 2

Framework for computationally efficient optimal crop and water allocation using ant colony optimization (Paper 1)

Statement of Authorship

Title of Paper	Framework for computationally efficient optimal crop and water allocation using ant colony optimization
Publication Status	Published
Publication Details	Nguyen, D.C.H., Maier, H.R., Dandy, G.C. & Ascough II, J.C., 2016. Framework for computationally efficient optimal crop and water allocation using ant colony optimization. Environmental Modelling & Software, 76, 37-53.

Principal Author

Name of Principal Author (Candidate)	Duc Cong Hiep Nguyen		
Contribution to the Paper	To review the literature, construct the framework, develop computer programming, run simulations, do analysis and discussion of results, and write the manuscript.		
Overall percentage (%)	70%		
Certification:	This paper reports on original research I conducted during the period of my Higher Degree by Research candidature and is not subject to any obligations or contractual agreements with a third party that would constrain its inclusion in this		
Signature		Date	16 Feb 2016

Co-Author Contributions

By signing the Statement of Authorship, each author certifies that:

i. the candidate's stated contribution to the publication is accurate (as detailed above);

ii. permission is granted for the candidate in include the publication in the thesis; and

iii. the sum of all co-author contributions is equal to 100% less the candidate's stated contribution.

Name of Co-Author	Holger R. Maier		
Contribution to the Paper 10%	Supervision of the research and review of the manuscript		
Signature		Date	16/2/16

Name of Co-Author	Graeme C. Dandy		
Contribution to the Paper 10%	Supervision of the research and review of the manuscript		
Signature		Date	16/2/16

Name of Co-Author	James C. Ascough II		
Contribution to the Paper 10%	Supervision of the research and review of the manuscript		
Signature		Date	11 February, 2016

Abstract

A general optimization framework is introduced with the overall goal of reducing search space size and increasing the computational efficiency of evolutionary algorithm application to optimal crop and water allocation. The framework achieves this goal by representing the problem in the form of a decision tree, including dynamic decision variable option (DDVO) adjustment during the optimization process and using ant colony optimization (ACO) as the optimization engine. A case study from literature is considered to evaluate the utility of the framework. The results indicate that the proposed ACO-DDVO approach is able to find better solutions than those previously identified using linear programming. Furthermore, ACO-DDVO consistently outperforms an ACO algorithm using static decision variable options and penalty functions in terms of solution quality and computational efficiency. The considerable reduction in computational effort achieved by ACO-DDVO should be a major advantage in the optimization of real-world problems using complex crop simulation models.

2.1 Introduction

Evolutionary algorithms (EAs) have been used extensively and have contributed significantly to the optimization of water resources problems in recent decades (Nicklow *et al.*, 2010; Maier *et al.*, 2014). However, the application of EAs to real-world problems presents a number of challenges (Maier *et al.*, 2014). One of these is the generally large size of the search space, which may limit the ability to find globally optimal or near-globally optimal solutions in an acceptable time period (Maier *et al.*, 2014). In order to address this problem, different methods to reduce the size of the search space have been proposed in various application areas to either enable near-globally optimal solutions to be found within a reasonable timeframe or to enable the best possible solution to be found for a given computational budget. Application areas in which search space reduction techniques have been applied in the field of water resources include the optimal design of water distribution systems (WDSs) (Gupta *et al.*, 1999; Wu and Simpson, 2001; Kadu *et al.*, 2008; Zheng *et al.*, 2011, 2014), the optimal design of stormwater networks (Afshar, 2006, 2007), the optimal design of sewer networks (Afshar, 2012), the calibration of hydrologic models (Ndiritu and Daniell, 2001), the optimization of maintenance scheduling for hydropower stations (Foong *et al.*, 2008a; Foong *et al.*, 2008b) and the optimal scheduling of environmental flow releases in rivers (Szemis *et al.*, 2012, 2014). Some of the methods used for achieving reduction in search space size include:

1. *Use of domain knowledge*. Domain knowledge of the problem under consideration has been widely applied for search space size reduction in specific application areas. For example, in the design of water distribution systems, the known physical relationships between pipe diameters, pipe length, pipe flows, and pressure head at nodes has been considered to reduce the number of diameter options available for specific pipes, thereby reducing the size of the search space significantly (Gupta *et al.*, 1999; Kadu *et al.*, 2008; Zheng *et al.*, 2011; Creaco and Franchini, 2012; Zheng *et al.*, 2014; Zheng *et al.*, 2015). This enables the search process to concentrate on promising regions of the feasible search

space. Other examples of this approach include the design of watershed-based stormwater management plans (Chichakly *et al.*, 2013), optimal locations and settings of valves in water distribution networks (Creaco and Pezzinga, 2015), optimization of multi-reservoir systems (Li *et al.*, 2015), and model calibration (Dumedah, 2015).

2. *Level of discretization.* When using discrete EAs, the level of discretization of the search space, which refers to the resolution with which continuous variables are converted into discrete ones, has also been used in order to reduce the size of the search space. As part of this approach, a coarse discretization of the search space is used during the initial stages of the search, followed by use of a finer discretization in promising regions of the search space at later stages of the search. Approaches based on this principle have been used for model calibration (Ndiritu and Daniell, 2001), the design of WDSs (Wu and Simpson, 2001), and the design of sewer networks (Afshar, 2012).

3. *Dynamic decision trees.* When ant colony optimization algorithms (ACOAs) are used as the optimization engine, solutions are generated by moving along a decision tree in a stepwise fashion. These decision trees can be adjusted during the solution generation process by reducing the choices that are available at a particular point in the decision tree as a function of choices made at preceding decision points (with the aid of domain knowledge of the problem under consideration). This approach has been applied successfully to scheduling problems in power plant maintenance (Foong *et al.*, 2008a; Foong *et al.*, 2008b), environmental flow management (Szemis *et al.*, 2012, 2014), the design of stormwater systems (Afshar, 2007) and the optimal operation of single- or multi-reservoir systems (Afshar and Moeini, 2008; Moeini and Afshar, 2011; Moeini and Afshar, 2013).

One application area where search space reduction should be beneficial is optimal crop and water allocation. Here the objective is to allocate land and water resources for irrigation management to achieve maximum economic return, subject to constraints on area and water allocations (Singh, 2012, 2014). One reason for this is that the search spaces of realistic crop and water allocation problems are very large (Loucks and Van Beek, 2005). For example, in a study by Kuo *et al.* (2000) on optimal irrigation planning for seven crops in Utah, USA, the search space size was 5.6 x 10^{14} and in a study by Rubino *et al.* (2013) on the optimal allocation of irrigation water and land for nine crops in Southern Italy, the search space size was 3.2 x 10^{32} and 2.2 x 10^{43} for fixed and variable crop areas, respectively.

Another reason for considering search space size reduction for the optimal crop and water allocation problem is that the computational effort associated with realistic long-term simulation of crop growth can be significant (e.g., on the order of several minutes per evaluation). While simple crop models (e.g., crop production functions or relative yield – water stress relationships) have been widely used in optimization studies due to their computational efficiency (Singh, 2012), these models typically do not provide a realistic representation of soil moisture - climate interactions and the underlying physical processes of crop water requirements, crop growth, and agricultural management strategies (e.g., fertilizer or pesticide application). In order to achieve this, more complex simulation models, such as ORYZA2000 (Bouman *et al.*, 2001), RZQWM2 (Bartling *et al.*, 2012), AquaCrop (Vanuytrecht *et al.*, 2014), EPIC (Zhang *et al.*, 2015) and STICS (Coucheney *et al.*, 2015) are typically employed. However, due to their relatively long runtimes, these models are normally used to simulate a small number of management strategy combinations (Camp *et al.*, 1997; Rinaldi, 2001; Arora, 2006; DeJonge *et al.*, 2012; Ma *et al.*, 2012b), rather than being used in combination with EAs to identify (near) globally optimal solutions. Given the large search spaces of optimal crop and water allocation problems, there is likely to be significant benefit in applying search-space size reduction methods in conjunction with hybrid simulation-optimization approaches to this problem (Lehmann and Finger, 2014).

Despite the potential advantages of search space size reduction, to the authors’ knowledge this issue has not been addressed thus far in previous applications of EAs to optimal crop and water allocation problems. These applications include GAs (Nixon *et al.*, 2001; Ortega Álvarez *et al.*, 2004; Kumar *et al.*, 2006; Azamathulla *et al.*, 2008; Soundharajan and Sudheer, 2009; Han *et al.*, 2011; Fallah-Mehdipour *et al.*, 2013; Fowler *et al.*, 2015), particle swarm optimization (PSO) algorithms (Reddy and Kumar, 2007; Noory *et al.*, 2012; Fallah-Mehdipour *et al.*, 2013), and shuffled frog leaping (SFL) algorithms (Fallah-Mehdipour *et al.*, 2013). In order to address the absence of EA application to search space size reduction for the optimal crop and water allocation problem outlined above, the objectives of this paper are:

1. To develop a general framework for reducing the size of the search space for the optimal crop and water allocation problem. The framework makes use of dynamic decision trees and ant colony optimization (ACO) as the optimization engine, as this approach has been used successfully for search space size reduction in other problem domains (Afshar, 2007; Foong *et al.*, 2008a; Foong *et al.*, 2008b; Szemis *et al.*, 2012, 2014).

2. To evaluate the utility of the framework on a crop and water allocation problem from the literature in order to validate the results against a known benchmark. It should be noted that although the search space of this benchmark problem is not overly large and does not require running a computationally intensive simulation model, it does require the development of a generic formulation that is able to consider multiple growing seasons, constraints on the maximum allowable areas for individual seasons, different areas for individual crops, and dissimilar levels of water availability. Consequently, the results of this case study provide a proof-of-concept for the application of the proposed framework to more complex problems involving larger search spaces and computationally expensive simulation models.

The remainder of this paper is organized as follows. A brief introduction to ACO is given in Section 2.2. The generic framework for optimal crop and water allocation that caters to search space size reduction is introduced in Section 2.3, followed by details of the case study and the methodology for testing the proposed framework on the case study in Section 2.4. The results are presented and discussed in Section 2.5, while conclusions and recommendations are given in Section 2.6.

2.2 Ant Colony Optimization (ACO)

ACO is a metaheuristic optimization approach first proposed by Dorigo *et al.* (1996) to solve discrete combinatorial optimization problems, such as the traveling salesman problem. As ACO has been used in a number of previous studies (Maier *et al.*, 2003; Zecchin *et al.*, 2005; Zecchin *et al.*, 2006; Afshar, 2007; Foong *et al.*, 2008a; Szemis *et al.*, 2012), only a brief outline is given here. For a more extensive treatment of ACO, readers are referred to Dorigo and Di Caro (1999). ACO is inspired by the behavior of ants when searching for food, in that ants can use pheromone trails to identify the shortest path from their nest to a food source. In ACO, a colony (i.e., population) of artificial ants is used to imitate the foraging behavior of real ants for finding the best solution to a range of optimization problems, where the objective function values are analogous to path length. As part of ACO, the decision space is represented by a graph structure that represents the decision variables or decision paths of the optimization problem. This graph includes decision points connected by edges that represent options. Artificial ants are then used to find solutions in a stepwise fashion by moving along the graph from one decision point to the next.

The probability of selecting an edge at a particular decision point depends on the amount of pheromone that is on each edge, with edges containing greater amounts of pheromone having a higher probability of being selected. While the pheromone levels on the edges are generally allocated randomly at the beginning of the optimization process, they are updated from one iteration to the next based on solution quality. An iteration consists of the generation of a complete solution, which is then used to calculate objective function values. Next, larger amounts of pheromone are added to edges that result in better objective function values. Consequently, an edge that results in better overall solutions has a greater chance of being selected in the next iteration. In this way, good solution components receive positive reinforcement. In contrast, edges that result in poor objective function values receive little additional pheromone, thereby decreasing their chances of being selected in subsequent iterations. In fact, the pheromone on these edges is likely to decrease over time as a result of pheromone evaporation. In addition, artificial ants can be given visibility, giving locally optimal solutions a higher probability of being selected at each decision point. This is achieved by weighting these two mechanisms via pheromone and visibility importance factors, respectively. The basic steps of ACO can be summarized as follows:

1. Define the number of ants, number of iterations, initial pheromone (τ_o) on each edge, pheromone importance factor (α), visibility importance factor (β), pheromone persistence (ρ) to enable pheromone evaporation, and pheromone reward factor (q) to calculate how much pheromone to add to each edge after each iteration.

2. Calculate the selection probability p for each edge (path) of the decision tree, as illustrated here for the edge joining decision points A and B:

$$p_{AB} = \frac{[\tau_{AB}(t)]^{\alpha}[\eta_{AB}]^{\beta}}{\sum_{B=1}^{N_A}[\tau_{AB}(t)]^{\alpha}[\eta_{AB}]^{\beta}} \tag{2.1}$$

where t is the index of iteration, $\tau_{AB}(t)$ is the amount of pheromone on edge (A, B) at iteration t, η_{AB} is the visibility of edge (A, B), and N_A is the set of all decision points neighboring decision point A.

3. After all ants have traversed the decision tree and the objective function value corresponding to the solution generated by each ant has been calculated, update pheromone on all edges, as illustrated here for edge (A, B):

$$\tau_{AB}(t + 1) = \rho\tau_{AB}(t) + \Delta\tau_{AB} \tag{2.2}$$

where $\Delta\tau_{AB}$ is the pheromone addition for edge (A, B).

It should be noted that there are different ways in which pheromone can be added to the edges, depending on which ACO algorithm is used. Any of these approaches can be applied to the proposed framework, as the proposed framework is primarily concerned with dynamically adjusting the structure of the decision-tree graph and not the way optimal solutions can be found on this graph, which can be done with a variety of algorithms. The only difference between the ACO algorithms is the way the pheromone update in Equation 2.2 is performed. The pheromone can be updated on: 1) all of the selected paths, as in the ant system (AS) (Dorigo *et al.*, 1996); 2) only the path of the global-best solution from the entire colony after each iteration, as in the elitist ant system (ASelite) (Bullnheimer *et al.*, 1997); 3) the paths from the top ranked solutions, which are weighted according to rank (i.e., higher ranked solutions have a larger influence in the pheromone updating process), as in the elitist-rank ant system (ASrank) (Bullnheimer *et al.*, 1997); or 4) the path of the iteration-best solutions or the global-best solutions after a given number of iterations, as in the Max-Min Ant System (MMAS) (Stützle and Hoos, 2000). In this study, the MMAS algorithm is used as it has been shown to outperform the alternative ACO variants in a number of water resources case studies (e.g., Zecchin *et al.* (2006); Zecchin *et al.* (2007); Zecchin *et al.* (2012)). As part of this algorithm, pheromone addition is performed on each edge, as shown for edge AB for illustration purposes:

$$\Delta\tau_{AB}(t) = \Delta\tau_{AB}^{ib}(t) + \Delta\tau_{AB}^{gb}(t) \tag{2.3}$$

where $\Delta\tau_{AB}^{ib}(t)$ and $\Delta\tau_{AB}^{gb}(t)$ are the pheromone additions for the iteration-best solution (s^{ib}) and the global-best solution (s^{gb}), respectively. While s^{ib} is used to update the pheromone on edge (A, B) after each iteration, s^{gb} is applied with the frequency f_{global} (i.e., $\Delta\tau_{AB}^{gb}(t)$ is calculated after each f_{global} iterations). $\Delta\tau_{AB}^{ib}(t)$ and $\Delta\tau_{AB}^{gb}(t)$ are given by:

$$\Delta\tau_{AB}^{ib}(t) = \begin{cases} \frac{q}{f(s^{ib}(t))} & if\ (A,B) \in s^{ib}(t) \\ 0 & otherwise \end{cases} \tag{2.4}$$

$$\Delta\tau_{AB}^{gb}(t) = \begin{cases} \frac{q}{f(s^{gb}(t))} & if\ (A,B) \in s^{gb}(t)\ and\ \ t\ mod\ f_{global} = 0 \\ 0 & otherwise \end{cases} \tag{2.5}$$

where $f(s^{ib}(t))$ and $f(s^{gb}(t))$ are objective function values of s^{ib} and s^{gb} at iteration t, respectively; and q is the pheromone reward factor.

In MMAS, the pheromone on each edge is limited to lie within a given range to avoid search stagnation, i.e., $\tau_{min}(t) \leq \tau_{AB}(t) \leq \tau_{max}(t)$. The equations for $\tau_{min}(t)$ and $\tau_{max}(t)$ are given as follows:

$$\tau_{max}(t) = \left(\frac{1}{1-\rho}\right)\frac{1}{f(s^{gb}(t-1))} \tag{2.6}$$

$$\tau_{min}(t) = \frac{\tau_{max}(t)(1-\sqrt[n]{p_{best}})}{(avg-1)\sqrt[n]{p_{best}}} \tag{2.7}$$

where n is the number of decision points, avg is the average number of edges at each decision point, and p_{best} is the probability of constructing the global best solution at iteration t, where the edges chosen have pheromone trail values of τ_{max} and the pheromone values of other edges are τ_{min}. Additionally, MMAS also uses a pheromone trail smoothing (PTS) mechanism that reduces the difference between edges in terms of pheromone intensities, and thus, strengthens exploration:

$$\tau_{AB}^{*}(t) = \tau_{AB}(t) + \delta(\tau_{max}(t) - \tau_{AB}(t)) \tag{2.8}$$

where δ is the PTS coefficient ($0 \leq \delta \leq 1$).

As is the case with most metaheuristic optimization algorithms, the parameters controlling algorithm searching behavior are generally determined with the aid of sensitivity analysis (e.g., Simpson *et al.* (2001); Foong *et al.* (2008b); Szemis *et al.* (2012)). Although algorithm performance has been found to be insensitive to certain parameters (e.g., Foong *et al.* (2005)), and for some application areas guidelines have been developed for the selection of appropriate parameters based on problem characteristics and the results of large-scale sensitivity analyses (e.g., Zecchin *et al.* (2005)), parameter sensitivity is likely to be case study dependent.

Over the last decade, ACO has been applied extensively to a range of water resources problems, including reservoir operation and surface water management, water distribution system design and operation, urban drainage system and sewer system design, groundwater management and environmental and catchment management, as detailed in a recent review by Afshar *et al.* (2015). While ACO shares the advantages of other evolutionary algorithms and metaheuristics of being easy to understand, being able to be linked with existing simulation models, being able to solve problems with difficult mathematical properties, being able to be applied to a wide variety of problem contexts and being able to suggest a number of near-optimal solutions for consideration by decision-makers (Maier *et al.*, 2014; Maier *et al.*, 2015), it is particularly suited to problems where there is dependence between decision variables, such that the selected value of particular decision variables restricts the available options for other decision variables, as is often the case in scheduling and allocation problems (e.g., Afshar (2007); Afshar and Moeini (2008); Foong *et al.* (2008a); Foong *et al.* (2008b); Szemis et al., (2012, 2014); Szemis *et al.* (2013)). This is because the problem to be optimized is represented in the form of a decision-tree, as mentioned above, enabling solutions to be generated in a stepwise fashion and decision variable options to be adjusted based on selected values at previous nodes in the decision tree. In other words, as part of the process of generating an entire solution, the available options at nodes in the tree can be constrained based on the values of partial solutions generated at previous nodes.

2.3 Proposed framework for optimal crop and water allocation

2.3.1 Overview

A simulation-optimization framework for optimal crop and water allocation is developed that is based on: 1) a graph structure to formulate the problem, 2) a method that adjusts decision variable options dynamically during solution construction to ensure only feasible solutions are obtained as part of the stepwise solution generation process in order to dynamically reduce the size of the search space, and 3) use of ACO as the optimization engine. The framework is aimed at identifying the seasonal crop and water allocations that maximize economic benefit at district or regional level, given restrictions on the volume of water that is available for irrigation purposes. Use of the framework is expected to result in a significant reduction in the size of the search space for optimal crop and water allocation problems, which is likely to reduce the number of iterations required to identify optimal or near globally optimal solutions.

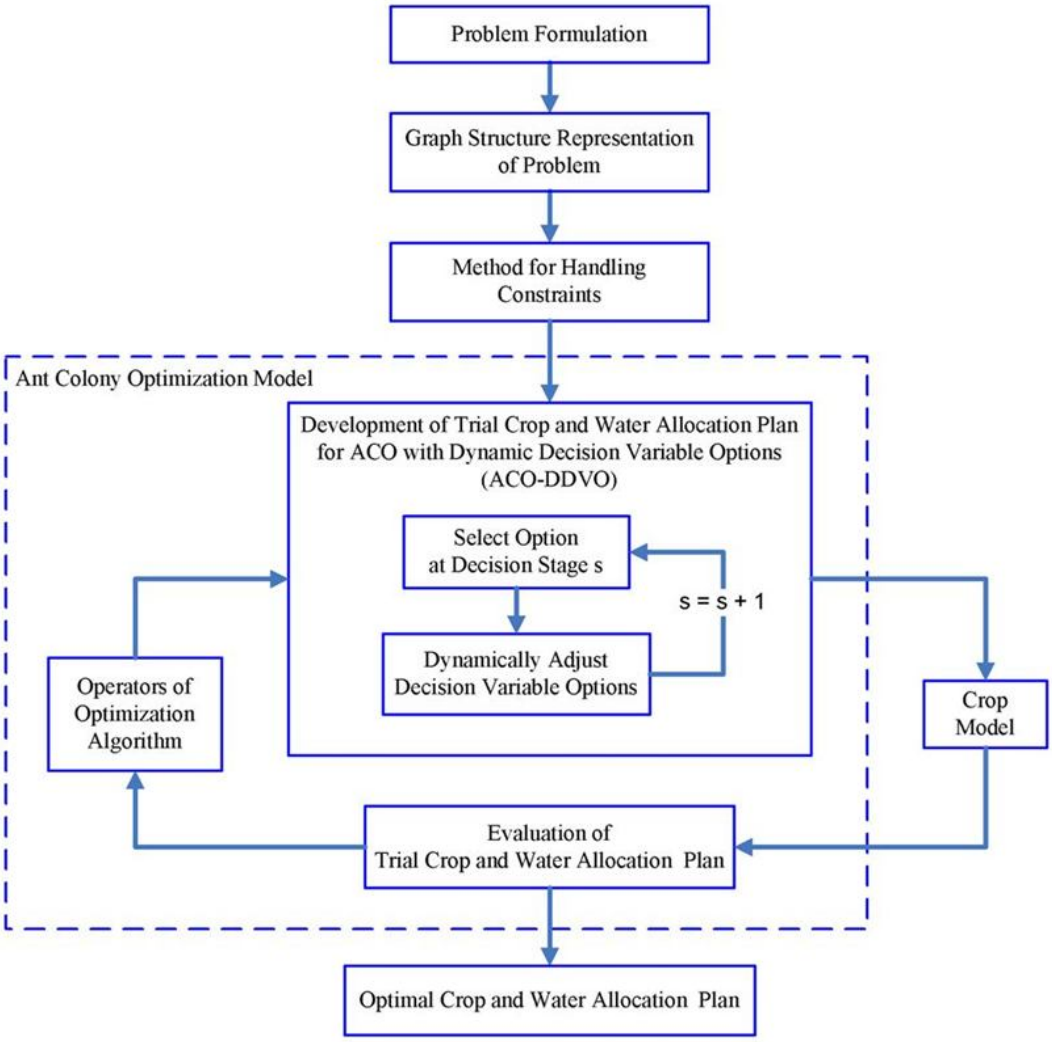


Figure 2-1. Overview of the proposed simulation-optimization framework for optimal crop and water allocation.

An overview of the framework is given in Figure 2-1. As can be seen, the first step is problem formulation, where the objective to be optimized (e.g., economic return) is defined, the constraints (e.g., maximum land area, annual water allocation, etc.) are specified, and the decision variables (e.g., crop type, crop area, magnitude of water application to different crops, etc.)

and decision variable options (e.g., available crops to select, options of watering levels, etc.) are stipulated. Herein, the level of discretization of the total area is also identified, so that the values of the sub-areas are able to reflect the characteristics of the problem considered.

After problem formulation, the problem is represented in the form of a decision-tree graph. This graph includes a set of nodes (where values are selected for the decision variables) and edges (which represent the decision variable options). A crop and water allocation plan is constructed in a stepwise fashion by moving along the graph from one node to the next. In the next step, the method for handling constraints needs to be specified. As part of the proposed framework, it is suggested to dynamically adjust decision variable options during the construction of a trial crop and water allocation plan in order to ensure constraints are not violated. This is achieved by only making edges available that ensure that all constraints are satisfied at each of the decision points. However, as this is a function of choices made at previous decision points in the graph, the edges that are available have to be updated dynamically each time a solution is constructed. This approach is in contrast to the approach traditionally used for dealing with constraints in ACO and other evolutionary algorithms, which is to allow the full search space to be explored and to penalize infeasible solutions. However, the latter approach is likely to be more computationally expensive, as the size of the search space is much larger. Consequently, it is expected that the proposed approach of dynamically adjusting decision variable options will increase computational efficiency as this approach reduces the size of the search space and ensures that only feasible solutions can be generated during the solution construction process.

As part of the proposed framework, ACO algorithms are used as the optimization engine because they are well-suited to problems that are represented by a graph structure and include sequential decision-making (Szemis *et al.*, 2012), as is the case here. In addition, they have been shown to be able to accommodate the adjustment of dynamic decision variable options by handling constraints in other problem domains (Foong *et al.*, 2008a; Szemis *et al.*, 2012). As part of the optimization process, the evaluation of the objective function is supported by crop models. In this way, improved solutions are generated in an iterative fashion until certain stopping criteria are met, resulting in optimal or near-optimal crop and water allocations. Further details of the problem formulation, graph structure representation, method for handling constraints, crop model options, and ACO process are presented in subsequent sections.

2.3.2 Problem formulation

The process of problem formulation includes the following steps:

1. Identify the number of the seasons (e.g., winter, monsoon, etc.), the seasonal (e.g., wheat) and annual (e.g., sugarcane) crops, the total cultivated area and the volume of available water.

2. Identify economic data in the study region, including crop price, production cost, and water price.

3. Specify decision variables (e.g., crop type, crop area, and irrigated water).

4. Specify decision variable options. For crop type, a list of potential options is given by the crops identified in step 1 (e.g., wheat, sugarcane, cotton, etc.). For continuous variables (i.e., crop area and irrigated water), the specification of the options includes selection of the range and level of discretization for each decision variable. The level of discretization (e.g., sub-area or volume of irrigated water for each crop) can significantly impact on either the quality of solutions found or the search space size (due to the exponential growth of this size). A discretization that is too coarse could exclude the true global optimal solution, while a fine discretization could result in a significant increase in computational time. While the depth of irrigated water can be discretized depending on the type and capacity of irrigation system, the acreage of each sub-area can be set equal to a unit area (e.g., 1 ha) or be the same as that of a standard field in the studied region. The

discretization of area can also be implemented depending on soil type or land-use policy. Each sub-area should reflect different conditions (e.g., soil type, evapotranspiration, and rainfall in season, etc.), and thus the discretization process will support the planning of the cropping patterns more realistically. Consequently, instead of selecting the area and the depth of irrigated water for each crop, as part of the proposed framework, the total area of the studied region is discretized into a number of sub-areas with each sub-area requiring decisions on which crop type should be planted and how much water should be supplied to the selected crop.

5. Specify the objective function and constraints. The objective function is to optimize the economic benefit and has the following form:

$$F = \mathrm{Max}\left\{\sum_{i=1}^{N_{sea}}\sum_{j=1}^{N_{ic}}\sum_{k=1}^{N_{SA}}\left(A_{ijk} \times \left[\begin{matrix} Y_{ijk}\left(W_{ijk}\right) \times P_{ij} - \\ \left(C_{FIXij} + W_{ijk} \times C_{W}\right)\end{matrix}\right]\right)\right\} \tag{2.9}$$

where F is the total net annual return (currency unit, e.g., \$ $year^{-1}$), N_{sea} is the number of seasons in a year (an annual crop is considered as the same crop for all seasons in a year), N_{ic} is the number of crops for season i (i = 1, 2, …, N_{sea}; for annual crop, i = a), N_{SA} is the number of sub-areas, A_{ijk} is the area of crop j in season i in sub-area k (ha), W_{ijk} is the depth of water supplied to crop j in season i in sub-area k (mm), Y_{ijk} is the yield of crop j in season i in sub-area k (depending on W_{ijk}) (kg ha^{-1}), P_{ij} is the price of crop j in season i (\$ kg^{-1}), C_{FIXij} is the fixed annual cost of crop j in season i (\$ ha^{-1} $year^{-1}$), and C_W is the unit cost of irrigated water (\$ mm^{-1} ha^{-1}).

As noted in Section 2.3.1 the objective is to maximize the total net annual return at the district or regional level rather than the net return to individual irrigators. Hence, the framework represents the perspective of an irrigation authority or farmer co-operative.

The objective function is maximized subject to limits on available resources, such as water and area of land. Consequently, the following constraints will be considered in order to provide a flexible and generic formulation:

- Constraints for maximum allowable area of each season A_i:

$$\sum_{j=1}^{N_{ic}} \sum_{k=1}^{N_{SA}} A_{ijk} \leq A_i \quad (2.10)$$

- Constraints for maximum allowable crop area A_{ijMax} for each season:

$$\sum_{k=1}^{N_{SA}} A_{ijk} \leq A_{ijMax} \quad (2.11)$$

- Constraints for minimum allowable crop area A_{ijMin} for each season:

$$\sum_{k=1}^{N_{SA}} A_{ijk} \geq A_{ijMin} \quad (2.12)$$

- Constraints for available volume of irrigation water W:

$$\sum_{i=1}^{N_{sea}} \sum_{j=1}^{N_{ic}} \sum_{k=1}^{N_{SA}} W_{ijk} \times A_{ijk} \leq W \quad (2.13)$$

2.3.3 Graph structure problem representation

As discussed in Section 2.3.2, a crop and water allocation plan can be established by determining the crop type and the depth of irrigated water for the selected crop in each sub-area. Thus, the full decision-tree graph for the optimal crop and water allocation problem is as shown in Figure 2-2.

The decision tree includes a set of decision points corresponding to the number of discrete sub-areas in the irrigated/studied area. At each decision point, a subset of decision points is used to consider each season in turn in order to decide which crop will be chosen to be planted at this sub-area in season i (i.e., C_{i1}, C_{i2}, …, C_{iNic}), and then what depth of water (i.e., W_1, W_2, …, W_{Nw}) will be supplied to the selected crop. If the selected crop at a decision point is an annual crop, then that decision point only considers the depth of irrigated water for that crop and skips the other seasons. A complete crop and water allocation plan is developed once a decision has been made sequentially at each decision point.

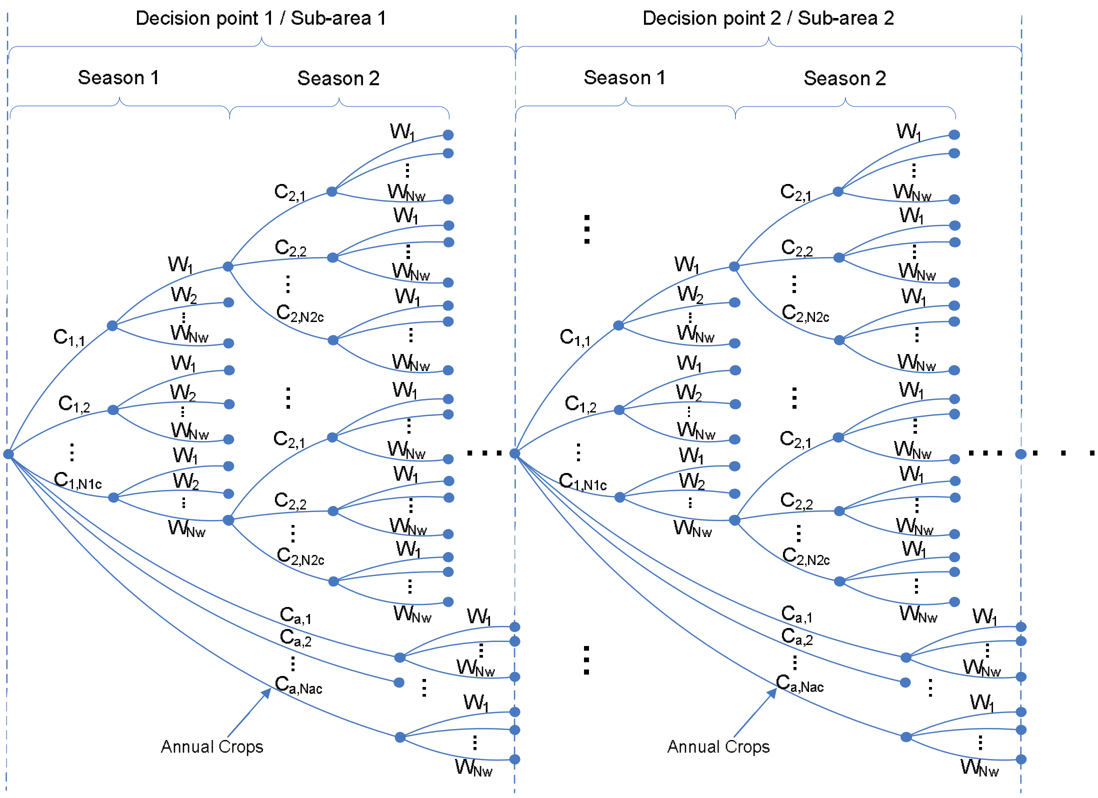


Figure 2-2. Proposed decision-tree graph for the optimal crop and water allocation problem.

It should be noted that the sequential solution generation steps are internal to the ACO process and do not reflect the sequence with which actual decisions are made, as the output of every ACO run is a complete annual crop and water allocation plan. While the order of solutions in the decision tree is likely to have an impact on the solutions obtained in a particular iteration, it would be expected that as the number of iterations increases, this effect would disappear as a result of the identification of globally optimal solutions via pheromone trail adjustment. It should also be noted that while the current formulation is aimed at identifying seasonal crop and water allocations, it could be extended to cater to more frequent (e.g., monthly, weekly, or daily) water allocations for the selected crops by adding the required number of decision points for water allocation. For example, if the frequency of water allocation decisions was changed from seasonally to monthly, there would be six decision points related to water allocation for each crop (one for each month), rather than a single seasonal decision point as shown in Figure 2-2.

2.3.4 Method for handling constraints

The available decision variable options are adjusted *dynamically* by checking all constraints (Equations 2.10-2.13) at each decision point and removing any options (i.e., crops or irrigated water) that result in the violation of a constraint based on paths selected at previous decision points (i.e., the number of available decision variable options is dynamically adjusted *during* the stepwise solution construction process). As mentioned previously, the purpose of this process is to dynamically reduce the size of the search space during the construction of trial solutions by each ant in each iteration, which is designed to make it easier and more computationally feasible to identify optimal or near-optimal solutions.

Details of how the decision variable options that result in constraint violation are identified for each of the constraints are given below. It should be noted that the four constraints in Equations 2.10-2.13 are considered for the choice of crops at the beginning of each season in a sub-area during the construction of a trial solution. However, to select the depth of irrigated water for the crop selected in the previous decision, only the constraint for available volume of irrigated water is checked.

- *Key steps for handling constraints for maximum allowable area for each season (Equation 2.10):*

1. Keep track of the total area allocated to each season as the decision tree is traversed from sub-area to sub-area.
2. Add the area of the next sub-area in the decision tree to the already allocated area for each season.
3. Omit all crops in a particular season and all annual crops from the choice of crops for this and subsequent sub-areas if the resulting area exceeds the maximum allowable area for this season.

- *Key steps for handling constraints for maximum allowable crop area (Equation 2.11):*

1. Keep track of the total area allocated to each crop type as the decision tree is traversed from sub-area to sub-area.

2. Add the area of the next sub-area in the decision tree to the already allocated area for each crop.

3. Omit a particular crop from the choice of crops for this and subsequent sub-areas if the resulting area exceeds the maximum allowable area for this crop.

- *Key steps for handling constraints for minimum allowable crop area (Equation 2.12):*

1. Keep track of the total area allocated to each crop type as the decision tree is traversed from sub-area to sub-area.

2. Sum the sub-areas in the decision tree remaining after this current decision.

3. Restrict the crop choices at this and subsequent decisions (i.e. subsequent sub-areas) to the ones that have minimum area constraints that are yet to be satisfied if the total area remaining after the current decision is less than the area that needs to be allocated in order to satisfy the minimum area constraints.

- *Constraints for maximum available volume of irrigated water (Equation 2.13):*

The key steps for handling this constraint for the *choice of crops* include:

1. Keep track of the total volume of irrigation water allocated to all crops as the decision tree is traversed from sub-area to sub-area.

2. Sum the volume of irrigation water for each crop in the decision tree remaining after this current decision.

3. Restrict the crop choices at this and subsequent decisions (i.e. subsequent sub-areas) to the ones that have minimum area constraints

that are yet to be satisfied if the total volume of water remaining after the current decision is less than the volume of water that needs to be supplied in order to satisfy the minimum area constraints.

The key steps for handling this constraint for the *choice of the depth of irrigated water* include:

1. Keep track of the total volume of irrigation water allocated to crops as the decision tree is traversed from sub-area to sub-area.

2. Calculate the available volume of irrigation water for the crop selected in the previous decision at this current decision.

3. Omit the choices of the depth of irrigated water for the crop selected in the previous decision if the volumes of irrigation water corresponding to these choices exceed the available volume of water in Step 2.

2.3.5 Crop models

In the proposed framework, a crop model coupled with the ACO model is employed as a tool for estimating crop yield to evaluate the utility of trial crop and water allocation plans. Generally, the crop model can be a simplified form (e.g., regression equation of crop production functions or relative yield – water stress relationships) which has the advantage of computational efficiency, or a mechanistic, process-based form which is able to represent the underlying physical processes affecting crop water requirements and crop growth in a more realistic manner.

2.3.6 Ant colony optimization model

The ACO model is used to identify optimal crop and water allocation plans by repeatedly stepping through the dynamic decision tree (see Section 2.3.4). At the beginning of the ACO process, a trial schedule is constructed by each ant in the population in accordance with the process outlined in Section 2.2. Next, the corresponding objective function values are calculated with the aid of the crop model and the pheromone intensities on the decision paths are updated (see Section 2.2). These steps are repeated until the desired stopping criteria have been met.

2.4 Case study

The problem of optimal cropping patterns under irrigation introduced by Kumar and Khepar (1980) is used as the case study for testing the utility of the framework introduced in Section 2.3. As discussed in Section 2.1, various challenges (e.g., a large search space or relatively long runtimes) have restricted the application of complex crop models for solving optimal crop and water allocation problems. Although the search space in the Kumar and Khepar (1980) case study is not overly large and the study uses crop water production functions rather than a complex crop-growth model, the problem has a number of useful features, including:

1. It requires a generic formulation, including multiple seasons, multiple crops, and constraints on available resources (e.g., a minimum and maximum area of each season, each crop, and water availability), as mentioned in Section 2.1.

2. Due to its relative computational efficiency, it enables extensive computational trials to be conducted in order to test the potential benefits of the proposed framework in a rigorous manner, and thus, may play an important role in increasing the efficacy of the optimization of crop and water allocation plans utilizing complex crop model application. Consequently, the use of crop production functions for the proposed framework is important for providing a proof-of-concept prior to its application with complex crop simulation models.

3. As optimization results for this case study have already been published by others, it provides a benchmark against which the quality of the solutions obtained from the proposed approach can be compared.

Details of how the proposed framework was applied to this case study are given in the following sub-sections.

2.4.1 *Problem formulation*

2.4.1.1 *Identification of seasons, crops, cultivated area and available water*

The case study problem considers two seasons (winter and monsoon) with seven crop options: wheat, gram, mustard, clover (referred to as berseem in Kumar and Khepar, 1980), sugarcane, cotton and paddy. While sugarcane is an annual crop, the other crops are planted in winter (e.g., wheat, gram, mustard and clover) or the monsoon season (e.g., cotton and paddy) only. The total cultivated area under consideration is 173 ha and the maximum volume of water available is 111,275 ha-mm. Three different water availability scenarios are considered for various levels of water losses in the main water courses and field channels, corresponding to water availabilities of 100%, 90% and 75%, as stipulated in Kumar and Khepar (1980).

2.4.1.2 *Identification of economic data*

The economic data for the problem, consisting of the price and fixed costs of crops for the region, are given in Table 2-1. The water price is equal to 0.423 Rs mm^{-1} ha^{-1}.

Table 2-1.
Details of crops considered, crop price, fixed costs of crop and the seasons in which crops are planted (from Kumar and Khepar, 1980).

Season	Crop	Price of crop (Rs qt^{-1})	Fixed costs of crop (Rs ha^{-1} $year^{-1}$)
Winter	Wheat	122.5	2669.8
	Gram	147.8	1117.0
	Mustard	341.4	1699.55
	Clover	7.0	2558.6
Annual	Sugarcane	13.5	5090.48
Summer	Cotton	401.7	2362.55
	Paddy	89.0	2439.68

Notes: Rs is a formerly used symbol of the Indian Rupee; qt is a formerly used symbol of weight in India.

2.4.1.3 Specification of decision variables

As was the case in Kumar and Khepar (1980), two separate formulations were considered, corresponding to different decision variables. In the first formulation, the only decision variable was area, i.e., how many hectares should be allocated to each crop in order to achieve the maximum net return. In the second formulation, the decision variables were area and the depth of irrigated water applied to each crop. In this case study, as discussed in Section 2.3.2, the decision variables are which crop to plant in each sub-area, and the depth of irrigated water supplied to the selected crop.

2.4.1.4 Specification of decision variable options

The number of decision points for area is generally equal to the maximum area (i.e., 173 ha in this case) divided by the desired level of discretization, which is selected to be 1 ha here. This would result in 173 decision points, each corresponding to an area of 1 ha to which a particular crop is then allocated (Figure 2-2). However, in order to reduce the size of the search space, a novel discretization scheme was adopted. As part of this scheme, the number of decision points for area was reduced from 173 to 29 with 10, 10, and 9 points corresponding to areas of 5, 6, and 7 ha, respectively. As a choice is made at each of these decision points as to which crop choice to implement, this scheme enables any area between 5 and 173 ha (in increments of 1 ha) to be assigned to any crop, with the exception of areas of 8 and 9 ha. For example:

- An area of 6 ha can be allocated to a crop by selecting this crop at 1 of the 10 areas corresponding to an area of 6 ha and not selecting this crop at any of the decision points corresponding to areas of 5 and 7 ha.

- An area of 27 ha can be allocated to a crop by selecting this crop at 4 of the 10 areas corresponding to an area of 5 ha and at 1 of the 9 areas corresponding to an area of 7 ha and not selecting this crop at any of the decision points corresponding to an area of 6 ha.

- An area of 173 ha can be allocated to a crop by selecting this crop at all of the decision points (i.e., 10x5 + 10x6 + 9x7 = 173).

Based on the above discretization scheme, for each of the 29 decision points for each sub-area, there are six decision variable options for crop choice for Season 1 (i.e. N1c = 6, see Figure 2-2), including dryland, wheat, gram, mustard, clover and sugarcane, and three decision variable options for crop choice for Season 2 (i.e. N2c = 3, see Figure 2-2), including dryland, cotton and paddy.

An obvious limitation of this scheme is that it is not possible to allocate areas of 1-4, 8 and 9 ha to any crop. However, the potential loss of optimality associated with this was considered to be outweighed by the significant reduction in the size of the solution space. Another potential shortcoming of this scheme is that it leads to a bias in the selection of crops during the solution generation process (i.e., intermediate areas have higher possibilities of being selected than extreme values). While this has the potential to slow down overall convergence speed, it would be expected that as the number of iterations increases, this bias would disappear as a result of the identification of globally optimal solutions via pheromone trail adjustment. The potential loss in computational efficiency associated with this effect is likely to be outweighed significantly by the gain in computational efficiency associated with the decrease in the size of solutions space when adopting this coding scheme.

It should be noted that in general terms, a discretization scheme of 1, 2 and 4 can be used for any problem, as the sum of combinations of these variables enable the generation of any integer. However, if there is a lower bound that is greater than one, then alternative, case study dependent optimization schemes can be developed in order to reduce the size of the search space further, as demonstrated for the scheme adopted for the case study considered in this paper. This is because the number of decision points resulting from the selected discretization scheme is a function of the sum of the integer values used in the discretization scheme. For example, if a scheme of 1, 2 and 4 had been used in this study, the required number of decision points for sub-area for each integer value in the scheme would have been 173/(1+2+4)=24.7. In contrast, for the adopted scheme, this was only 173/(5+6+7)=9.6 (resulting in the adopted distribution of 10, 10, 9).

Table 2-2.
Optimization problem details for each of the two problem formulations considered.

Formulation	Water availability	Decision variables	No. of decision points for area	No. of crop options for each sub-area	No. of irrigated water options for each crop	Size of total search space
1	100%	Crop type	29	6 for Season 1, 3 for Season 2	1	2.5×10^{36}
	90%					
	75%					
2	100%	Crop type and depth of irrigated water	29	6 for Season 1, 3 for Season 2	150 for each crop	4.1×10^{162}
	90%					
	75%					

Note: The size of total search space is equal to ($6^{29} \times 3^{29}$) for Formulation 1 and ($6^{29} \times 3^{29} \times 150^{29} \times 150^{29}$) for Formulation 2.

For Formulation 2, decision variable options also have to be provided for the depth of irrigated water for each of the selected crops at each of the sub-areas (see Figure 2-2). Based on the irrigation depth that corresponds to maximum crop yield for the crop production functions (see Section 2.4.4) and an assumed discretization interval of 10 mm ha^{-1}, the number of irrigated water options for each crop was 150 (i.e. NW = 150, see Figure 2-2), corresponding to choices of 0, 10, 20, …, 1490 mm ha^{-1}.

Details of the decision variables, decision variable options, and search space size for three scenarios of both formulations are given in Table 2-2.

2.4.1.5 Objective function and constraints

As there are only two seasons, the objective function for both formulations is as follows in accordance with the general formulation of the objective function given in Equation 2.9:

$$F = \text{Max}\left\{ \begin{array}{l} \sum_{j=1}^{6}\sum_{k=1}^{29}\left(A_{1jk} \times \left[\begin{array}{c} Y_{1jk}(W_{1jk}) \times P_{1j} - \\ (C_{FIX1j} + W_{1jk} \times C_W) \end{array}\right]\right) \\ + \sum_{j=1}^{3}\sum_{k=1}^{29}\left(A_{2jk} \times \left[\begin{array}{c} Y_{2jk}(W_{2jk}) \times P_{2j} - \\ (C_{FIX2j} + W_{2jk} \times C_W) \end{array}\right]\right) \end{array} \right\} \tag{2.14}$$

where the variables were defined in Section 2.3.2.

The objective function is subject to the following constraints, which are in accordance with those stipulated in Kumar and Khepar (1980).

- *Constraints for maximum allowable areas in winter and monsoon seasons:*

The total planted area of crops in each season must be less than or equal to the available area for that season. As stipulated in Kumar and Khepar (1980), the maximum areas A_i in the winter and monsoon seasons are 173 and 139 ha, respectively.

$$\sum_{j=1}^{N_{ic}}\sum_{k=1}^{29} A_{ijk} \leq A_i \tag{2.15}$$

- *Constraints for minimum and maximum allowable crop area:*

The area of a crop must be less than or equal to its maximum area and greater than or equal to its minimum area. At least 10% of the total area in the winter season (approximately 17 ha) has to be planted in clover, and the maximum areas of sugarcane and mustard are equal to 10% and 15% of the total area in the winter season (approximately 17 ha and 26 ha, respectively).

$$\sum_{k=1}^{29} A_{ijk} \leq A_{ijMax} \tag{2.16}$$

$$\sum_{k=1}^{29} A_{ijk} \geq A_{ijMin} \tag{2.17}$$

- *Constraints for available volume of irrigated water*

The total volume of irrigated water applied to the crops is less than or equal to the maximum volume of water available for irrigation in the studied region. As mentioned above, three scenarios are considered with 75%, 90%

and 100% of water entitlement, respectively. The corresponding volumes of available water for these scenarios are 844,570, 1,001,780 and 1,112,750 m^3, respectively.

$$\sum_{j=1}^{6}\sum_{k=1}^{29} W_{1jk} \times A_{1jk} + \sum_{j=1}^{3}\sum_{k=1}^{29} W_{2jk} \times A_{2jk} \leq W \tag{2.18}$$

2.4.2 Graph structure representation of problem

There are separate decision-tree graphs for Formulations 1 and 2. In Formulation 1, the graph includes 29 decision points corresponding to 29 sub-areas, as discussed in Section 2.4.1. At each decision point, there are only two choices of crops corresponding to Seasons 1 and 2 as the depth of irrigated water for each crop is fixed (Figure 2-3). As can be seen, there are six crop options (dryland, wheat, gram, mustard, clover and sugarcane) in Season 1 (five crops in the winter season and one annual crop) and three options (dryland, cotton, and paddy) in Season 2 (i.e., the monsoon season). It should be noted that if sugarcane is selected, there is no crop choice for Season 2, as sugarcane is an annual crop. A complete solution is developed once the crops for all sub-areas are selected.

In similar fashion to Formulation 1, the decision-tree graph for Formulation 2 also includes 29 decision points for area, but each decision point includes two choices of crops (one for each season) and two choices of the depth of irrigated water (one for each crop in each season). This graph has the same structure as the decision-tree graph in Figure 2-2, but includes two seasons, six crop options for Season 1, three crop options for Season 2, and 150 depth of irrigated water options for each crop (see section 2.4.1.4). After a crop is selected for each season at each decision point, the depth of irrigated water for the selected crop is determined (unless the crop is dryland in which case there is no irrigation option). Furthermore, at each decision point, if an annual crop (i.e., sugarcane) is selected in Season 1, there is only the choice of the depth of irrigated water for the annual crop. Although other choices in Season 2 are skipped in this case, the available area and depth of water after that decision point will be reduced by annual crop use. A complete crop and water allocation plan is developed once a decision has been made sequentially at each decision point.

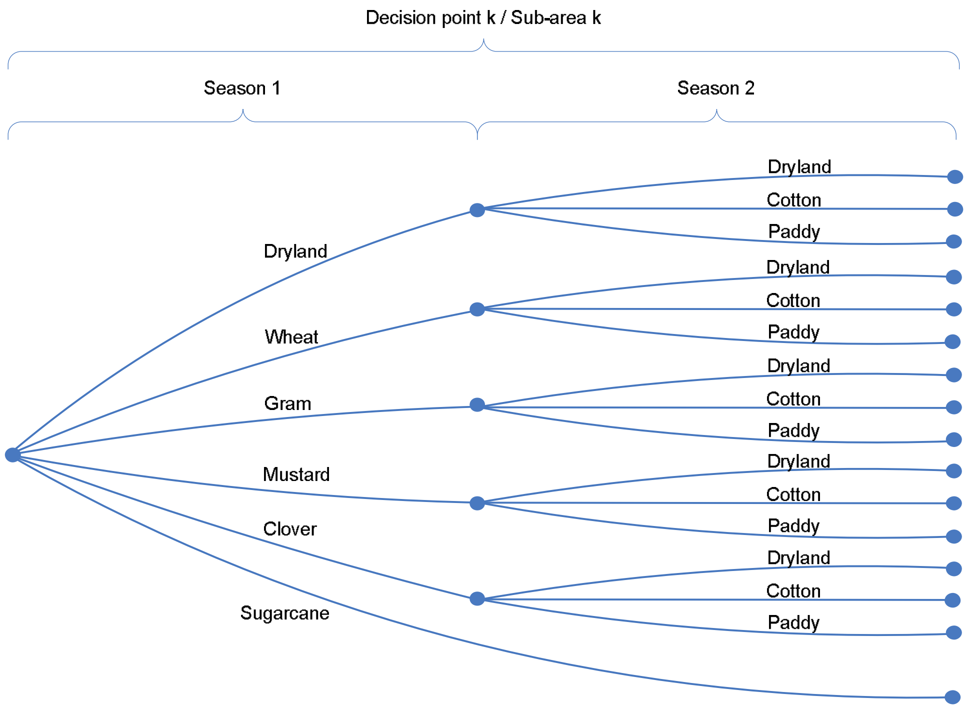


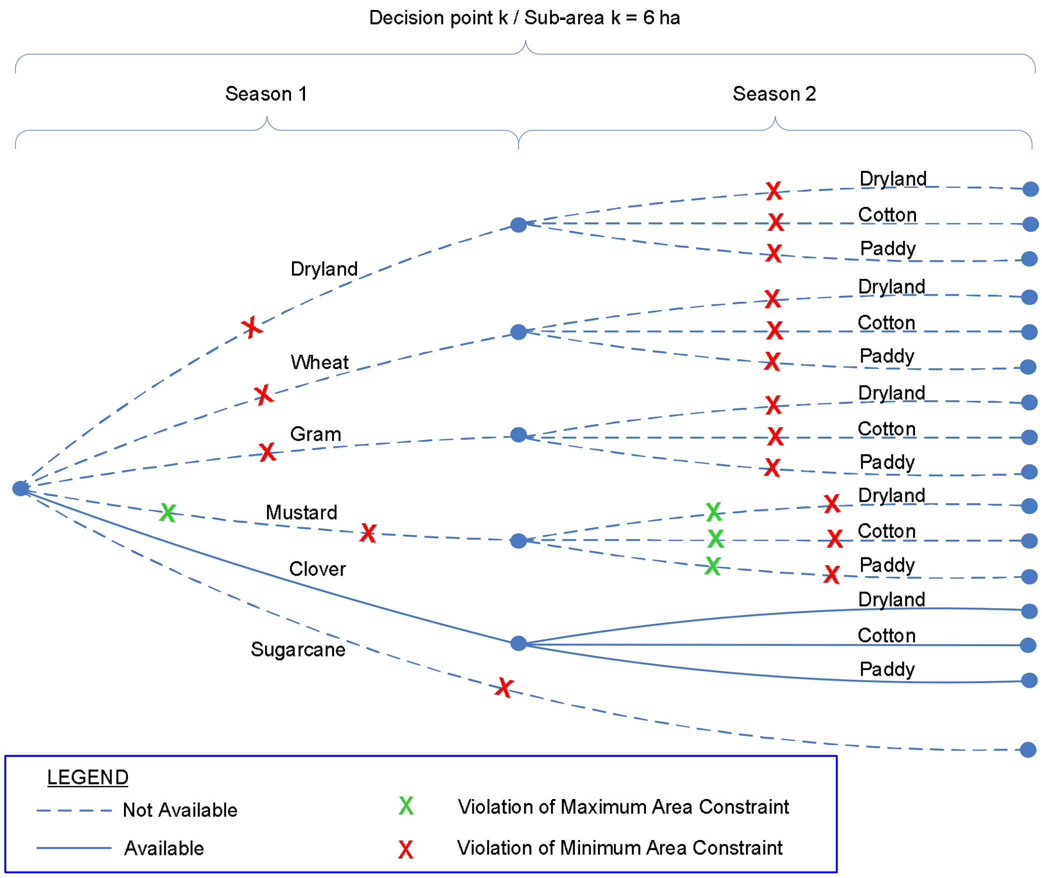
Figure 2-3. A single decision point for area of the decision-tree graph for Formulation 1.

2.4.3 Method for handling constraints

In addition to the proposed dynamic decision variable options (DDVO) adjustment approach for dealing with constraints in ACO, the traditional and most commonly used method via the use of penalty functions was also implemented. This was undertaken in order to assess the impact on search space size reduction of the proposed DDVO approach. Details of both approaches are given below.

2.4.3.1 DDVO adjustment approach

As part of this approach, the decision trees for Formulations 1 and 2 described in Section 2.4.2 were dynamically adjusted based on the procedure outlined in Section 2.3.4. An example of how this works for the case study is shown in Figure 2-4.



Crops in Season 1	Cumulative Area Already Allocated	Minimum Allowable Crop Area	Maximum Allowable Crop Area	Column (1) + Sub-Area k (i.e., 6 ha)	Constraints for Maximum Allowance Crop Area	Constraints for Minimum Allowance Crop Area
(0)	(1)	(2)	(3)	(4)	(5)	(6)
Dryland	23	0	173	29	Available	Not available
Wheat	50	0	173	56	Available	Not available
Gram	48	0	173	54	Available	Not available
Mustard	22	0	26	28	Not available	Not available
Clover	5	17	173	11	Available	Available
Sugarcane	10	0	17	16	Available	Not available
Total	158	17				

Figure 2-4. Example of decision variable option adjustment process for one decision point for Formulation 1.

In this example, two constraints for maximum and minimum allowable crop area were considered to check the available crop options at decision point k (which corresponds to one of the 10 sub-areas with an area of 6 ha - see Section 2.4.1.4 - for the sake of illustration) in Formulation 1. In the figure, the cumulative area that has already been allocated to each crop is shown in column (1) and the resulting total area allocated to each crop if this particular crop is selected at this decision point is shown in column (4). It is clear that when the constraint for maximum allowance crop area was checked, mustard could be removed as an option at this decision point (column (5)) because its total cumulative allocated area in column (4) was larger than the maximum allowable area for this crop in column (3), thereby reducing the size of the search space (Figure 2-4). When checking the minimum allowable area constraint by comparing the areas in columns (2) and (4), and comparing the remaining area after this decision point (i.e. 173 – 158 – 6 = 9 ha) and the remaining minimum area at this decision point (i.e., 17 – 5 = 12 ha), clover provided the only feasible crop choice (column (6)) at this decision point. This enables all other crop choices to be removed, thereby further reducing the size of the search space and ensuring only feasible solutions are generated (Figure 2-4).

2.4.3.2 Penalty function approach

As part of the penalty function approach to constraint handling, there is no dynamic adjustment of decision variable options based on solution feasibility. Consequently, infeasible solutions can be generated and in order to ensure that these solutions are eliminated in subsequent iterations, a penalty value (P) is added to the objective function value (F) for these solutions.

Penalty function values are generally calculated based on the distance of an infeasible solution to the feasible region (Zecchin *et al.*, 2005; Szemis *et al.*, 2012; Zecchin *et al.*, 2012). Therefore, the following penalty functions were used for the constraints in Equations 2.10-2.13:

- Penalty for maximum allowable area of each season A_i (corresponding to Equation 2.10):

$$P(1) = \begin{cases} 0 & \text{if } \sum_{j=1}^{N_{ic}} \sum_{k=1}^{N_{SA}} A_{ijk} \leq A_i \\ \left(\sum_{j=1}^{N_{ic}} \sum_{k=1}^{N_{SA}} A_{ijk} - A_i\right) \times 1{,}000{,}000 & \text{if } \sum_{j=1}^{N_{ic}} \sum_{k=1}^{N_{SA}} A_{ijk} > A_i \end{cases} \quad (2.19)$$

- Penalty for maximum allowable crop area A_{ijMax} (corresponding to Equation 2.11):

$$P(2) = \begin{cases} 0 & \text{if } \sum_{k=1}^{N_{SA}} A_{ijk} \leq A_{ijMax} \\ \left(\sum_{k=1}^{N_{SA}} A_{ijk} - A_{ijMax}\right) \times 1{,}000{,}000 & \text{if } \sum_{k=1}^{N_{SA}} A_{ijk} > A_{ijMax} \end{cases} \quad (2.20)$$

- Penalty for minimum allowable crop area A_{ijMin} (corresponding to Equation 2.12):

$$P(3) = \begin{cases} 0 & \text{if } \sum_{k=1}^{N_{SA}} A_{ijk} \geq A_{ijMin} \\ \left(A_{ijMin} - \sum_{k=1}^{N_{SA}} A_{ijk}\right) \times 1{,}000{,}000 & \text{if } \sum_{k=1}^{N_{SA}} A_{ijk} < A_{ijMin} \end{cases} \quad (2.21)$$

- Penalty for available volume of irrigated water W (corresponding to Equation 2.13):

$$P(4) = \begin{cases} 0 & \text{if } \sum_{i=1}^{N_{sea}} \sum_{j=1}^{N_{ic}} \sum_{k=1}^{N_{SA}} W_{ijk} \times A_{ijk} \leq W \\ \left(\sum_{i=1}^{N_{sea}} \sum_{j=1}^{N_{ic}} \sum_{k=1}^{N_{SA}} W_{ijk} \times A_{ijk} - W\right) \times 1{,}000{,}000 & \text{vice versa} \end{cases} \quad (2.22)$$

where the variables in Equations 2.19-2.22 are defined in Section 2.3.2.

The following equation was used as the overall fitness function to be minimized during the optimization process:

$$\text{Min } f(.) = \frac{1{,}000{,}000}{1{,}000{,}000 + F} + \text{Penalty} \quad (2.23)$$

where F is given in Equation 2.14; and Penalty is the sum of four penalties in Equations 2.19-2.22. The form of this function, including the multiplier of 1,000,000, was found to perform best in a number of preliminary trials.

2.4.4 Crop models

As mentioned previously, this case study utilizes simple crop production functions, rather than complex mechanistic crop models. Details of these functions are given in Table 2-3. The area without crops, referred to as Dryland, was not irrigated and has a yield equal to zero.

Table 2-3.
Crop water production functions (from Kumar and Khepar, 1980).

Crop type	Formulation 1		Formulation 2
	Irrigation water W (mm)	Crop yield Y (qt ha^{-1})	
Wheat	307	36.60	$Y = 26.5235 - 0.03274\ W + 1.14767\ W^{0.5}$
Gram	120	18.21	$Y = 15.4759 + 0.04561\ W - 0.00019\ W^{2}$
Mustard	320	18.44	$Y = 14.743 - 0.011537\ W + 0.41322\ W^{0.5}$
Clover	716	791.20	$Y = 25.5379 - 1.0692\ W + 57.2238\ W^{0.5}$
Sugarcane	542	782.50	$Y = -11.5441 + 2.92837\ W - 0.0027\ W^{2}$
Cotton	526	13.76	$Y = 6.6038 - 0.013607\ W + 0.62418\ W^{0.5}$
Paddy	1173	47.25	$Y = 5.9384 - 0.035206\ W + 2.412043\ W^{0.5}$

2.4.5 Computational experiments

Two computational experiments were implemented to test the utility of the proposed approach to search-space size reduction. The first experiment used static decision variable options (SDVO) in conjunction with the penalty function method for handling constraints (referred to as ACO-SDVO henceforth), and the second used the proposed ACO-DDVO approach for handling constraints. Each computational experiment was conducted for the two formulations and three water availability scenarios in Table 2-2, and for eight different numbers of evaluations ranging from 1,000 to 1,000,000. A maximum number of evaluations of 1,000,000 was selected as this is commensurate with the values used by Wang *et al.* (2015) for problems with search spaces of similar size. The pheromone on edges for both ACO-SDVO and ACO-DDVO were updated using MMAS.

In order to select the most appropriate values of the parameters that control ACO searching behavior, including the number of ants, alpha, beta, initial pheromone, pheromone persistence and pheromone reward (see Section 2.2), a sensitivity analysis was carried out. Details of the parameter values included in the sensitivity analysis, as well as the values selected based on the outcomes of the sensitivity analysis, are given in Table 2-4. It should be noted that visibility factor β was set to 0 (i.e., ignoring the influence of visibility on searching the locally optimal solutions), as was the case in other applications of MMAS to scheduling problems (Szemis *et al.*, 2012). Due to the probabilistic nature of the searching behavior of the ACO algorithms, the positions of starting points are able to influence the optimization results (Szemis *et al.*, 2012). Thus, each optimization run was implemented with 10 replicates, i.e., 10 randomly generated values for starting points in the solution space.

Table 2-4.
Details of the ACO parameter values considered as part of the sensitivity analysis and the optimal values identified and ultimately used in the generation of optimization results presented.

Parameter	Values for sensitivity analysis	Values selected
Number of ants	50; 100; 200; 500; 1,000; 2,000; 5,000; 10,000	100; 1,000; 10,000
Alpha (α)	0.1, 0.5, 1.0, 1.2, 1.5	1.2
Beta (β)	0	0
Initial pheromone (τ_o)	0.5, 1.0, 2.0, 5.0, 10.0, 20.0	10.0
Pheromone persistence (ρ)	0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9	0.6
Pheromone reward (q)	0.5, 1.0, 2.0, 5.0, 10.0, 20.0, 50.0	20.0

In addition, the best final solutions of the computational experiments from the ACOAs were compared with those obtained by Kumar and Khepar (1980) using linear programming (LP).

2.5 Results and discussion

2.5.1 Objective function value

The best solutions from the ACO models over the 10 runs with different random starting positions (i.e., ACO-SDVO and ACO-DDVO) and those obtained by Kumar and Khepar (1980) using LP are given in Table 2-5. As can be seen, ACO outperformed LP for five out of the six experiments in terms of net returns. For Formulation 1, there was very little difference between the results from the ACO models and LP, in which the percentage deviations for three scenarios (i.e., 100%, 90% and 75% of water availability) were 0.48%, -0.06%, and 0.74%, respectively. This was as expected since the problem formulation is linear. For 90% water availability, the net return using LP was slightly better than that of the ACO models (741,157.3 Rs vs. 740,731.4 Rs, respectively). However, this is because the optimal solution obtained using LP could not be found using ACO because of the discretization interval used. For Formulation 2, which is a nonlinear problem, ACO outperformed LP by between 5.58% and 11.23% for ACO-DDVO and by between 4.46% and 11.21% for ACO-SDVO. This demonstrates that the use of EAs is beneficial when solving realistic problems, which are likely to be non-linear. This difference is likely to be exacerbated for more complex problems.

Table 2-5.
Comparison between best-found solutions using the two ACO formulations (ACO-SDVO and ACO_DDVO) and those obtained by Kumar and Khepar (1980) which are used as a benchmark.

Formulation	Water Availability	Net Return (Rs)		
		Benchmark	ACO-SDVO	ACO-DDVO
1	100%	785,061.1	**788,851.4** (0.48%)	**788,851.4 (**0.48%)
	90%	**741,157.3**	740,731.4 (-0.06%)	740,731.4 (-0.06%)
	75%	647,627.2	**652,438.3** (0.74%)	**652,438.3** (0.74%)
2	100%	800,652.6	890,404.0(11.21%)	**890,600.7** (11.23%)
	90%	799,725.6	865,441.8 (8.22%)	**873,190.2** (9.19%)
	75%	792,611.2	827,998.0 (4.46%)	**836,839.2** (5.58%)

Note: The bold numbers are the best-found solution. The numbers in parentheses are the percentage deviations of the optimal solutions obtained using the ACO algorithms relative to the benchmark results obtained by Kumar and Khepar (1980). Positive percentages imply that the ACO models performed better than the Benchmark, and vice versa.

ACO-SDVO and ACO-DDVO performed very similarly in terms of the best solution found. For Formulation 1, as the search space size was relatively small (2.5 x 10^{36}), identical solutions were found. However, with the larger search space size in Formulation 2 (2.1 x 10^{160}), ACO-DDVO obtained slightly better solutions (about 1% better for the two scenarios with tighter constraints) and the difference between these solutions increased for decreases in water availability. This is most likely because it is easier to find better solutions in the smaller search spaces obtained by implementing the proposed ACO-DDVO approach.

2.5.2 Ability to find feasible solutions

The ability of each algorithm to find feasible solutions after a given number of nominal evaluations was represented by the number of times feasible solutions were found from different starting positions in solution space (as represented by the 10 repeat trials with different random number seeds) (see Table 2-6).

Table 2-6.
The number of times feasible solutions were identified out of ten trials with different random number seeds for different numbers of function evaluations, constraint handling techniques and water availability scenarios for the two different formulations of the optimization problem considered.

Formulation	Water Availability	Constraint Handling	Number of Nominal Evaluations (x 1,000)							
			1	2	5	10	50	100	500	1,000
1	100%	SDVO	5	5	5	5	10	10	10	10
		DDVO	10	10	10	10	10	10	10	10
	90%	SDVO	2	2	2	2	4	4	4	4
		DDVO	10	10	10	10	10	10	10	10
	75%	SDVO	0	0	1	1	4	4	4	4
		DDVO	10	10	10	10	10	10	10	10
2	100%	SDVO	9	9	9	9	10	10	10	10
		DDVO	10	10	10	10	10	10	10	10
	90%	SDVO	9	9	9	9	10	10	10	10
		DDVO	10	10	10	10	10	10	10	10
	75%	SDVO	7	8	8	8	9	9	9	9
		DDVO	10	10	10	10	10	10	10	10

As expected, ACO-DDVO always found feasible solutions from all 10 random starting positions for all trials (Table 2-6), as the DDVO adjustment guarantees that none of the constraints are violated. However, this was not the case for ACO-SDVO. As shown in Table 2-6, the ability of ACO-SDVO to identify feasible solutions was a function of the starting positions in solution space. It can also be seen that the ability to identify feasible solutions from different starting position decreased with a reduction in the size of the feasible region, as is the case for Formulation 1 compared with Formulation 2 and when water availability is more highly constrained. The ability to identify feasible regions increased with the number of function evaluations, but this comes at the expense of computational efficiency.

2.5.3 Convergence of solutions

The convergence of the feasible solutions (i.e., how quickly near-optimal solutions were found) was evaluated against the best-found solution (Figure 2-5). It should be noted that the average and maximum net return values for ACO-SDVO were only calculated for the trials that yielded feasible solutions from among the 10 different starting positions in solution space. In general, Figure 2-5 shows that convergence speed for the ACO-DDVO solutions is clearly greater than convergence speed for the ACO-SDVO solutions.

In Formulation 1, the difference between the average and maximum results from ACO-SDVO was fairly large at 50,000 and 1,000 – 2,000 nominal evaluations for 75% and 100% water availability, respectively. As only one solution was found at 5,000 – 10,000 nominal evaluations for 75% water availability, the average and maximum results are identical. On the contrary, there was no large difference between these solutions for all three scenarios for ACO-DDVO. This demonstrates that the quality of the solutions from the various random seeds for ACO-DDVO was more consistent than that obtained from ACO-SDVO. In addition, the speed of convergence of the results from ACO-SDVO at the best-found solution had an increasing trend when the available level of water increased. The number of nominal evaluations to obtain this convergence were 500,000, 50,000 and 50,000 for 75%, 90%, and 100% water availability, respectively. The corresponding solutions from ACO-DDVO always converged to the best-found solution after 10,000 nominal evaluations.

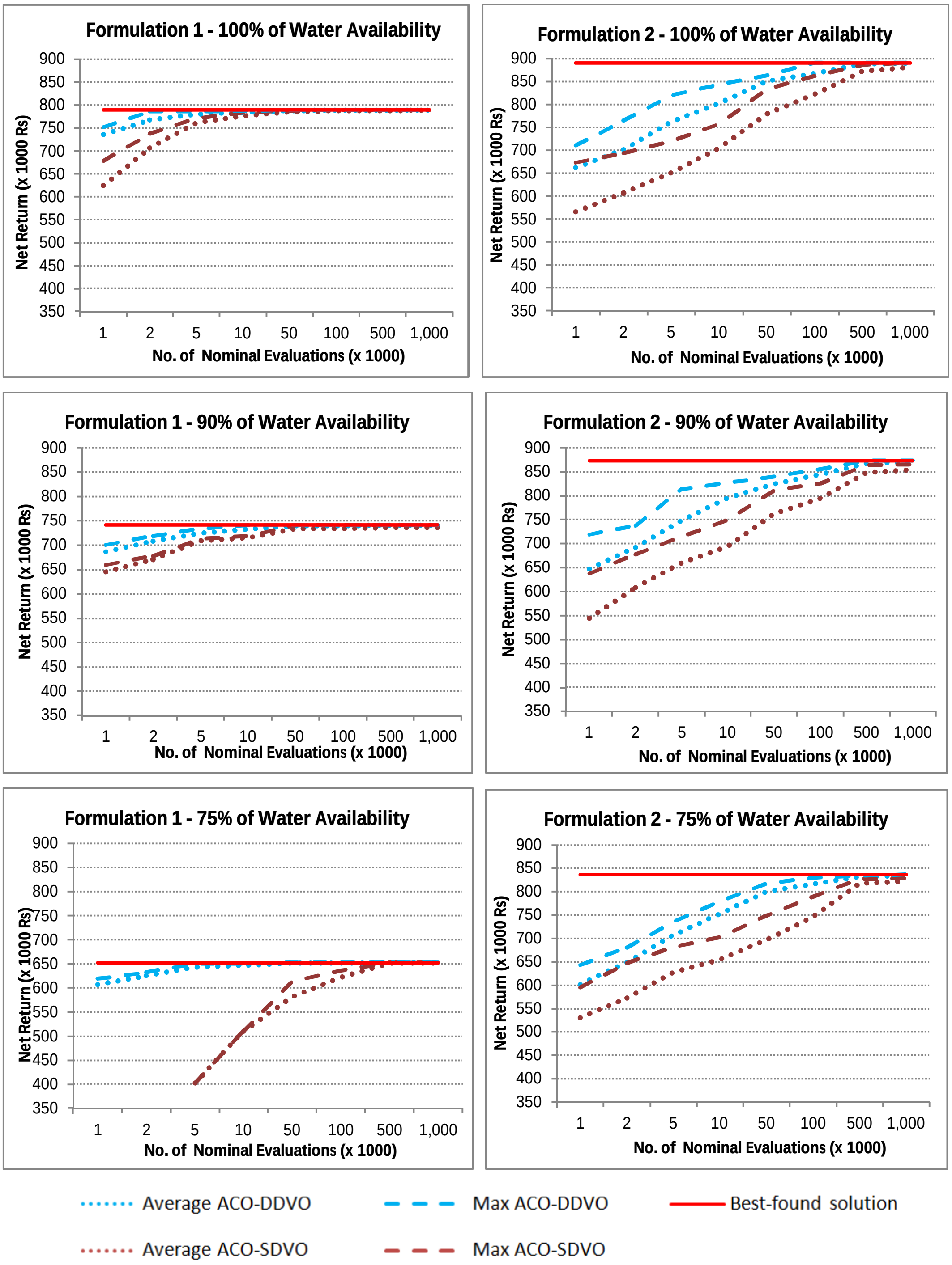


Figure 2-5. Convergence of average and maximum optimal solutions obtaining from ACO.

In Formulation 2, the search space and the number of feasible solutions were larger because of the increase in the number of decision variables (Table 2-2). As a result, there was a clear difference between the average and maximum solutions for ACO-SDVO. Furthermore, these solutions did not converge to the best-found solution, even with the maximum number of evaluations of 1,000,000. In contrast, although the difference between the average and maximum results from ACO-DDVO increased compared to those from ACO-DDVO in Formulation 1, it was still markedly smaller than those from ACO-SDVO. The solutions obtained from ACO-DDVO always converged at 500,000 nominal evaluations for all three scenarios. Consequently, the results demonstrate that the method of handling constraints in ACO-DDVO resulted in much better convergence towards the best-found solution compared to that of ACO-SDVO, which is most likely due to the reduced size of the search space and the fact that the search is restricted to the feasible region when ACO-DDVO is used.

2.5.4 Tradeoff between computational effort and solution quality

The increased computational efficiency of ACO-DDVO compared with that of ACO-SDVO is demonstrated by the relationship between computational effort and solution quality (Figure 2-6). It should be noted that the best results over the 10 runs were used to calculate the deviation from the best-found solution and the % computational effort was calculated from the number of actual evaluations. As shown in Section 2.5.1, ACO-DDVO and ACO-SDVO attained identical solutions for Formulation 1 and ACO-DDVO was able to find slightly better solutions than ACO-SDVO for Formulation 2. However, Figure 2-6 shows that these better solutions were obtained at a much reduced computational effort, ranging from 74.4 to 92.7% reduction in computational effort for Formulation 1 and from 63.1 to 90.9% reduction for Formulation 2 (for the same percentage deviation from the best found solution). In addition, near-optimal solutions could be found more quickly. For example, for Formulation 1 ACO-DDVO only needed a very small computational effort to reach a solution with 5% deviation from the best-found solution (about 1.5%, 5.3%, and 1.0% of total computational effort for 100%, 90% and 75% of water availability, respectively). The corresponding values for ACO-SDVO were 7.1%, 12.1%, and 39.9% of total computational effort, respectively. Similar results were found for Formulation 2, in which ACO-DDVO needed less than 5% of the total computational effort and ACO-SDVO required over 40% of the total computational effort for the two scenarios with tighter constraints.

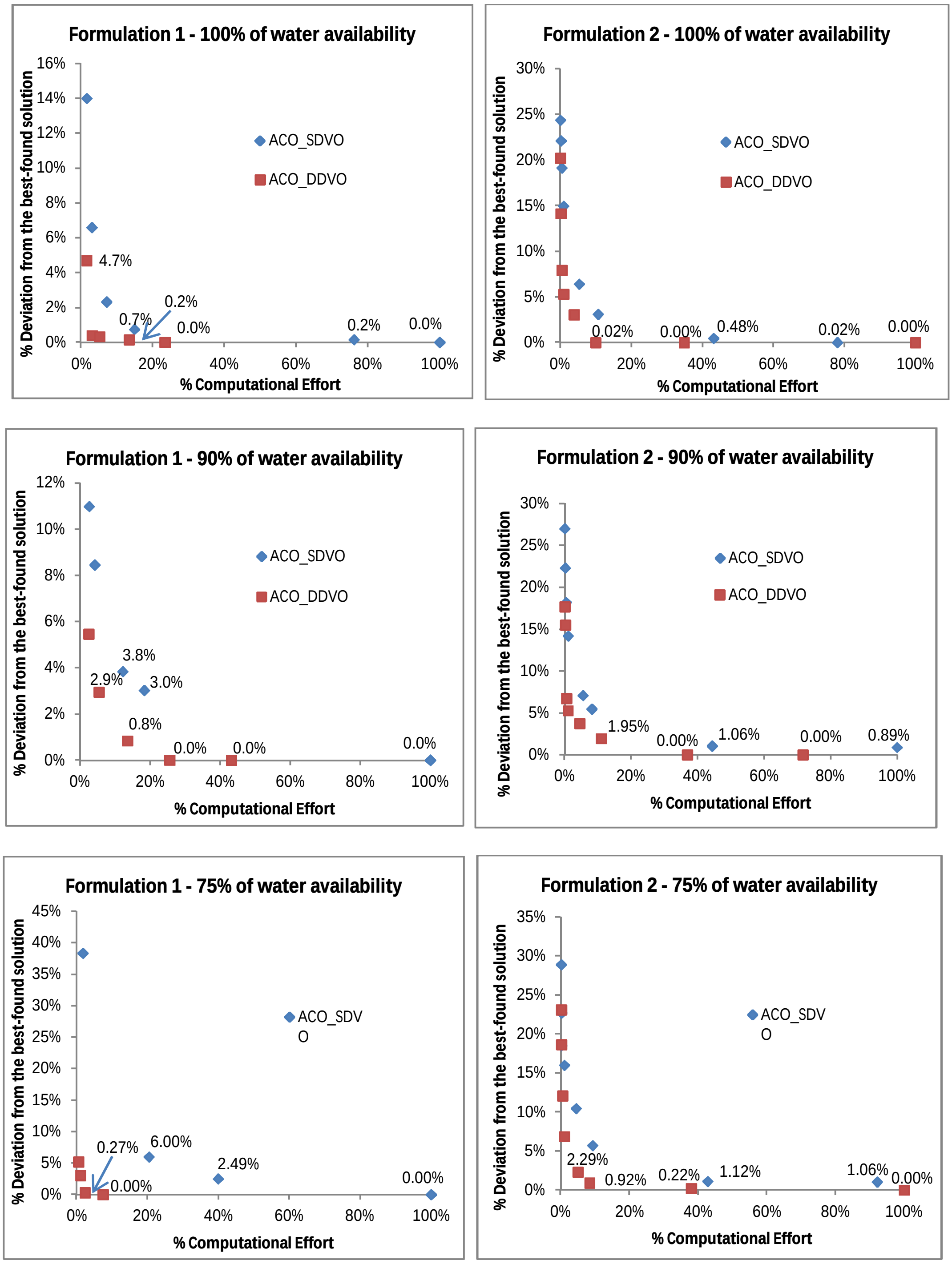


Figure 2-6. Computational effort vs. solution quality for the different ACO variants, formulations and water availability scenarios.

For this case study, the actual savings in CPU time are not that significant (~ 1.5 CPU hours was saved by using ACO-DDVO for Formulation 2 with 1,000,000 evaluations). However, if complex simulation models were used for objective function evaluation (where a single evaluation could take several minutes), a 63.1% reduction in computational effort would result in significant time savings. For example, if the number of evaluations corresponding to this computational saving was reduced from 872,204 to 322,181, the actual CPU time would be reduced by 5,500,230 seconds (over 2 months) for a 10-sec. simulation model evaluation. This demonstrates that the proposed ACO-DDVO approach has the potential to significantly reduce the computational effort associated with the simulation-optimization of crop and water allocation, while increasing the likelihood of finding better solutions.

2.6 Summary and conclusions

A general framework has been developed to reduce search space size for the optimal crop and water allocation problem when using a simulation-optimization approach. The framework represents the constrained optimization problem in the form of a decision tree, uses dynamic decision variable option (DDVO) adjustment during the optimization process to reduce the size of the search space and ensures that the search is confined to the feasible region and uses ant colony optimization (ACO) as the optimization engine. Application of the framework to a benchmark crop and water allocation problem with crop production functions showed that ACO-DDVO clearly outperformed linear programming (LP). While LP worked well for linear problems (i.e., Formulation 1 where the only decision variable was area), ACO-DDVO was able to find better solutions for the nonlinear problem (i.e., Formulation 2 with decision variable options for depth of irrigated water for each of the selected crops at each of the sub-areas) and for more highly constrained search spaces when different levels of water availability were considered. The ACO-DDVO approach was also able to outperform a “standard” ACO approach using static decision variable options (SDVO) and penalty functions for dealing with infeasible solutions in terms of the ability to find feasible solutions, solution quality, computational efficiency and convergence speed. This is because of ACO-DDVO’s ability to reduce the size of the search space and exclude infeasible solutions during the solution generation process.

It is important to note that while the results presented here clearly illustrate the potential of the proposed framework as a proof-of-concept, there is a need to apply it to more complex problems with larger search spaces, as well as in conjunction with more realistic irrigation demands (e.g., Foster *et al.* (2014)) and mechanistic crop growth simulation models (see Section 2.1). However, based on the demonstrated benefits for the simple case study considered in this paper, the proposed ACO-DDVO simulation-optimization framework is likely to have even more significant advantages when applied to real-world problems using complex crop models with long simulation times.

CHAPTER 3

Improved ant colony optimization for optimal crop and irrigation water allocation by incorporating domain knowledge (Paper 2)

Statement of Authorship

Title of Paper	Improved Ant Colony Optimization for Optimal Crop and Irrigation Water Allocation by Incorporating Domain Knowledge
Publication Status	Accepted f or Publication
Publication Details	Nguyen, D.C.H., Dandy, G.C., Maier, H.R. & Ascough II, J.C., 2016. Improved ant colony optimization for optimal crop and irrigation water allocation by incorporating domain knowledge. ASCE J. Water Resources Planning and Management.

Principal Author

Name of Principal Author (Candidate)	Duc Cong Hiep Nguyen		
Contribution to the Paper	To review the literature, construct the framework, develop computer programming, make case studies, run simulations, do analysis and discussion of results, and write the manuscript.		
Overall percentage (%)	70%		
Certification:	This paper reports on original research I conducted during the period of my Higher Degree by Research candidature and is not subject to any obligations or contractual agreements with a third party that would constrain its inclusion in this		
Signature		Date	16 Feb 2016

Co-Author Contributions

By signing the Statement of Authorship, each author certifies that:

i. the candidate's stated contribution to the publication is accurate (as detailed above);

ii. permission is granted for the candidate in include the publication in the thesis; and

iii. the sum of all co-author contributions is equal to 100% less the candidate's stated contribution.

Name of Co-Author	Graeme C. Dandy		
Contribution to the Paper 10%	Supervision of the research and review of the manuscript		
Signature		Date	16/2/16

Name of Co-Author	Holger R. Maier		
Contribution to the Paper 10%	Supervision of the research and review of the manuscript		
Signature		Date	16/2/16

Name of Co-Author	James C. Ascough II		
Contribution to the Paper 10%	Supervision of the research and review of the manuscript		
Signature		Date	11 February, 2016

Abstract

An improved ant colony optimization (ACO) formulation for the allocation of crops and water to different irrigation areas is developed. The formulation enables dynamic decision variable option (DDVO) adjustment and makes use of domain knowledge through visibility factors (VFs) to bias the search towards selecting crops that maximize net returns and water allocations that result in the largest net return for the selected crop, given a fixed total volume of water. The performance of this formulation is compared with that of other ACO algorithm variants (without and with domain knowledge) for two case studies, including one from the literature and one introduced in this paper for different water availability scenarios within an irrigation district located in Loxton, South Australia near the River Murray. The results for both case studies indicate that the use of VFs: 1) increases the ability to identify better solutions at all stages of the search; and 2) reduces the computational time to identify near-optimal solutions. Furthermore, the savings in computational time obtained by using VFs and DDVO adjustment should be considerable for ACO application to problems such as detailed irrigation scheduling that rely on more complex crop models than those used in the case studies presented herein.

3.1 Introduction

In recent years, ant colony optimization (ACO) has been used extensively to identify optimal solutions to a range of water resources problems (Afshar *et al.*, 2015). One of the reasons for the popularity of ACO is that, like other metaheuristic optimization methods such as the more commonly used genetic algorithms (GAs), it is able to be linked with complex simulation models in order to identify globally optimal or near-globally optimal solutions to complex water resource problems (Nicklow *et al.*, 2010; Maier *et al.*, 2014; Maier *et al.*, 2015). However, unlike other metaheuristics, ACO is able to more easily facilitate the incorporation of system domain knowledge *within the construction of trial solutions* rather than by *seeding initial populations*, as has been done for Gas (Keedwell and Khu, 2006; Kadu *et al.*, 2008; Zheng *et al.*, 2011; Creaco and Franchini, 2012; Kang and Lansey, 2012; Bi *et al.*, 2015). This is because the trial solutions of ACO are constructed using past experience contained in the search space instead of modifying previous solutions, as is done when GAs are used (Maier *et al.*, 2003). Consequently, ACO offers great potential in terms of increasing computational efficiency of the optimization process, which is particularly important in cases where complex simulation models are used for objective function or constraint evaluation.

In ACO, problems are represented in the form of a decision-tree graph which artificial ants have to traverse in a stepwise fashion in order to generate trial solutions. As a result, the number of options available at each node in the decision tree can be adjusted based on choices made at previous nodes with the aid of domain knowledge about the system to be optimized. In addition, the edges of the graph at each node can be biased in accordance with an understanding of which of the available options are likely to result in locally optimal solutions (Dorigo and Di Caro, 1999). This provides a mechanism for combining the power of global optimization algorithms with system understanding and experience and is likely to enable better solutions to be identified more quickly, as less computational effort is likely to be expended on the exploration of poor regions of the solution space. The benefits of incorporating domain knowledge into ACO has previously been demonstrated in a number of water resources planning and management problem domains (Afshar *et al.*, 2015). Examples include optimal design of stormwater networks (Afshar, 2007, 2010), optimal maintenance scheduling for hydropower stations (Foong *et al.*, 2008a; Foong *et al.*, 2008b), optimal design of water distribution systems (Maier *et al.*, 2003; Zecchin *et al.*, 2005; López-Ibáñez *et al.*, 2008; Zecchin *et al.*, 2012), optimal operation of reservoir systems (Afshar and Moeini, 2008; Moeini and Afshar, 2011; Moeini and Afshar, 2013), and scheduling of environmental flow releases in rivers (Szemis *et al.*, 2012, 2014).

The potential increase in computational efficiency via the incorporation of domain knowledge into ACO is particularly important in the application to real-world problems where search spaces are typically large and system simulation models are generally computationally demanding (Bonissone *et al.*, 2006; Maier *et al.*, 2014; Maier *et al.*, 2015). An important application area where this is the case is optimal crop and water allocation (Singh, 2012, 2014). However, to date, the application of EAs to this problem (e.g., Nixon *et al.* (2001), Ortega Álvarez *et al.* (2004), Kumar *et al.* (2006), Reddy and Kumar (2007), Azamathulla *et al.* (2008), Soundharajan and Sudheer (2009), Han *et al.* (2011), Noory *et al.* (2012), and Fallah-Mehdipour *et al.* (2013)) has ignored the potential of increasing computational efficiency by incorporating domain knowledge into the optimization process. The likely reason for this is that the incorporation of such knowledge is difficult to achieve when using the evolutionary algorithms that have been applied to this problem thus far (e.g., GAs).

In order to address this shortcoming, Nguyen *et al.* (2016b) introduced a formulation that enables ACO to be used to identify crop and irrigation water allocations that maximize net return. As part of the formulation, a generic decision-tree graph for the problem was presented, enabling decisions to be made about seasonal allocation of crops to available plots of land and how much water to irrigate for each plot in order to maximize the net return. Computational efficiency was increased by dynamically reducing the size of the search space during the construction of each trial solution, based on decisions made at previous nodes in the graph. The benefits of this formulation were demonstrated for a simple benchmark problem from the literature; however, the potential of increasing the computational efficiency of the algorithm by biasing the edges at each node based on domain knowledge (heuristic information) was not investigated. Therefore, the objectives of this paper are:

1. To extend the ACO formulation for optimal crop selection and irrigation water allocation developed by Nguyen et al. (2016) to include the use of domain knowledge to bias the selection of crops and water allocations at each node in the decision-tree graph in order to increase computational efficiency.

2. To test the benefits of the improved formulation on the benchmark case study used by Nguyen et al. (2016) and a real-world case study based on an irrigation district located in Loxton, South Australia near the River Murray.

The remainder of this paper is organized as follows. The Proposed ACO Formulation Incorporating Domain Knowledge section provides a brief overview of ACO and describes a novel approach for incorporating domain knowledge into ACO. This is followed by the Methodology, including details of the two cases studies used to evaluate the performance of the proposed formulation. Next, the four ACO algorithm variants evaluated in this study are summarized in the Computational Experiments section. Finally, the results of the two case studies are presented and discussed in the Results and Discussion section, followed by the Summary and Conclusions.

3.2 Proposed ACO Formulation Incorporating Domain Knowledge

ACO is a metaheuristic algorithm based on the foraging behavior of ants that enables them to find the shortest path connecting a food source and their nest (Dorigo *et al.*, 1996). This is achieved by ants in a colony communicating with each other indirectly via pheromone. Ants deposit pheromone on the paths they traverse and are also more likely to follow paths with higher pheromone concentrations. As shorter paths take less time to traverse, more ants traverse them per unit time. This increases the pheromone concentration on these paths, making them more likely to be selected in future (see Maier *et al.* (2003)).

In ACO, the decision space of the optimization problem is represented in terms of a graph that includes a set of nodes (or decision variables) and edges (or decision variable options). A solution to the problem is a set of edges that is selected in a stepwise fashion as ants move along the graph from one node (decision point) to the next, depositing pheromone as they go. In a particular iteration, a number of ants will traverse the graph, each constructing independent trial solutions. At the end of an iteration, paths that lead to better overall solutions are rewarded with more pheromone, making them more likely to be selected in subsequent iterations. In this way, better solutions evolve as the number of iterations increases.

The ACO decision-tree graph for the optimal crop and irrigation water allocation problem considered in this paper as proposed by Nguyen et al. (2016) is shown in Figure 3-1. As can be seen, the decision points of the graph correspond to which crop type should be planted in season i (C_{i1}, C_{i2}, …, C_{iNic}) in a particular sub-area and how much water should be supplied to the selected crop (W_1, W_2, …, W_{Nw}). The above decisions are made for each season and sub-area. If an annual crop is chosen for a particular sub-area, the other seasons corresponding to this sub-area are ignored. A trial solution (i.e., crop and irrigation water allocation plan for each sub-area and each season) is constructed by an ant traversing the graph and selecting an edge at each node. As detailed in Nguyen et al. (2016), the objective of the optimization process is to determine the crop and irrigation water allocation plan that maximizes net return, subject to constraints on maximum allowable area for each season, minimum and maximum allowable area for each crop, and the available volume of irrigation water.

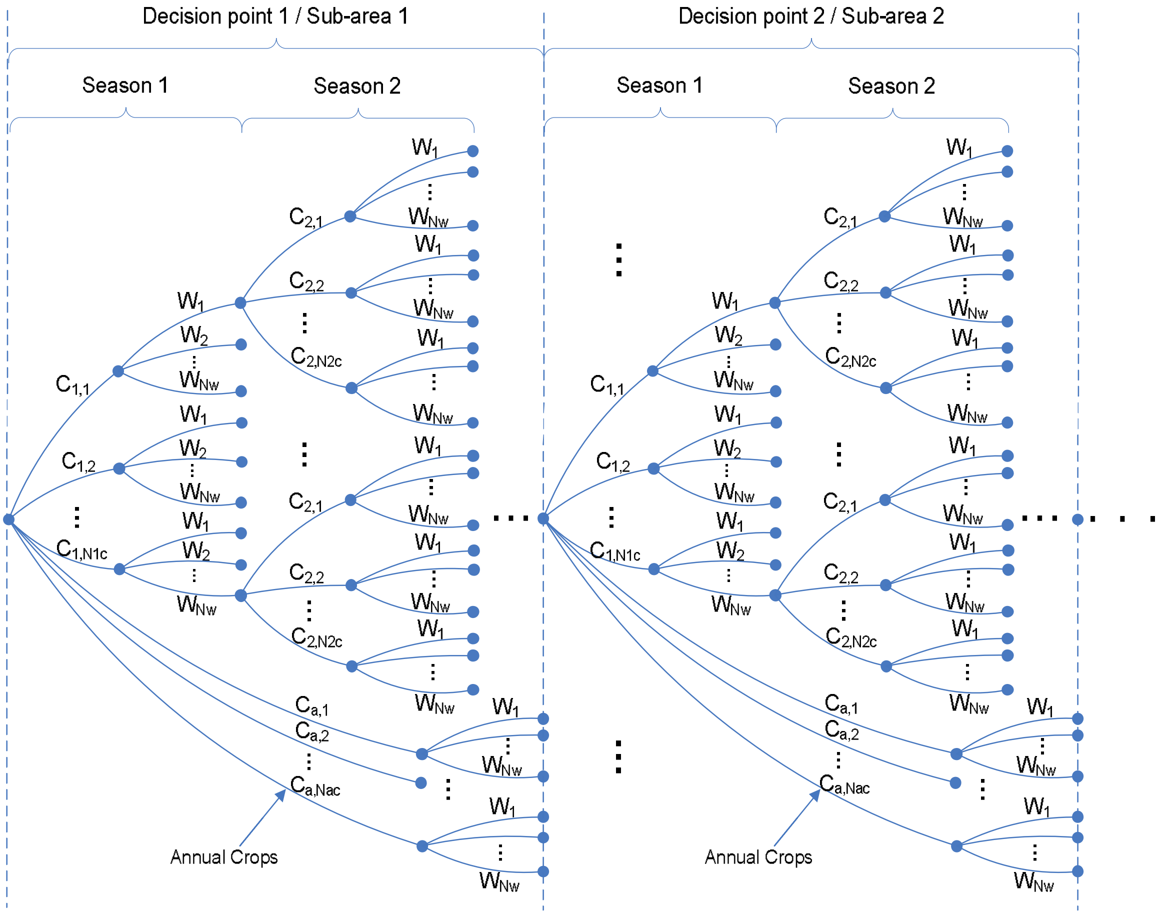


Figure 3-1. Decision-tree graph for the crop and water allocation problem. N_{ic} is the number of crops for season i (i =1, 2, …; for annual crop, i = a); N_w is the number of options for the depth of irrigated water supplied to a crop

The size of the search space can be reduced, and hence computational efficiency increased, by dynamically adjusting the number of edges at each decision point during the construction of each trial solution (Nguyen et al. 2016). In order to achieve this, all constraints in relation to available water and allowable areas for particular crops in particular seasons are checked at each decision point and any options that violate any of the constraints are removed. The above process occurs dynamically every time an ant generates a trial solution in a particular iteration, so that different adjustments to the size of the search space are likely to be made each time an ant traverses the decision-tree graph. This is made possible because individual trial solutions in ACO are constructed in a step-wise manner by ants traversing graphs, thus enabling alteration of the decision-variable options available at decision points based on selections made at previous decision points by a particular ant during a particular iteration, rather than having to define decision variable options statically at the beginning of the optimization process.

At each node, the decision about which edge to select by an ant is made in accordance with the following decision policy (Dorigo et al. 1996):

$$p_{AB} = \frac{[\tau_{AB}(t)]^{\alpha}[\eta_{AB}]^{\beta}}{\sum_{B=1}^{N_A}[\tau_{AB}(t)]^{\alpha}[\eta_{AB}]^{\beta}} \tag{3.1}$$

where p_{AB} is the probability of selecting edge (A, B), t is the index of iteration, $\tau_{AB}(t)$ is the amount of pheromone on edge (A, B) at iteration t, η_{AB} is the visibility of edge (A, B), α is the pheromone importance factor, β is the visibility importance factor, and N_A is the set of all edges that connect node A to the next node.

As given in Equation 3.1, the probability of selecting each edge depends on two factors, pheromone concentration and visibility. As explained previously, pheromone deposition is the means by which ants communicate with each other in order to explore the search space. As the number of iterations increases, pheromone concentrations that favor globally optimal solutions are identified. While real ants are almost completely blind and rely on pheromone trails for the identification of optimal solutions, the addition of visibility to the artificial ants used in ACO can increase computational efficiency, as it enables locally optimal solutions to be favored (Dorigo et al. 1996; Maier et al. 2003; Foong et al. 2008a; Foong et al. 2008b; López-Ibáñez et al. 2008; Zecchin et al. 2012). Consequently, visibility provides a means of including domain knowledge of the problem under consideration in the ACO process, as it enables a biased selection of edges at each node.

For the optimal crop and water allocation problem considered in this paper, the domain knowledge (represented by a visibility factor, VF) that can be used to identify locally optimal solutions relates to factors that increase net return (i.e., the objective function), including which crop is selected and how much water is applied to the selected crop. Consequently, separate VFs are required for the decision points for crop selection and water allocation, as detailed below.

3.2.1 Visibility Factor for Crop Choice

The VF for crop choice introduces bias so that crops that are likely to result in greater net return have a greater chance of being selected for each of the sub-areas, as follows:

$$\eta_{cropj} = 1 - \frac{1}{NR_j\left(Min\left(W_{Avai}, W_{NRMaxj}\right)\right)} \tag{3.2}$$

where η_{cropj} is the VF for crop j, NR_j is the net return of crop j, W_{Avai} is the volume of water available at a decision point, and W_{NRMaxj} is the volume of water that results in the maximum net return of crop j. Note that as the actual volume of water allocated to a particular crop is not known at the crop selection node (see Figure 3-1), the proposed formulation assumes that the volume of water allocated to a crop is either the volume that maximizes net return (W_{NRMaxj}, as given in Equation 3.2) or the volume that is still available for allocation at the decision point under consideration (if this is less than W_{NRMaxj}). W_{NRMaxj} is obtained with the aid of a crop growth model used in this study as part of the optimization process (see the Methodology-Overview section). The functional form of the VF shown in Equation 3.2 is suggested as it performed best among a number of candidate functional forms considered as part of preliminary trials. Note that if different sub-areas have different soil types or are exposed to different meteorological conditions (e.g., rainfall, evapotranspiration), different VFs can be used for individual sub-areas to reflect any differences in optimal water requirements and net returns resulting from these differences.

3.2.2 Visibility Factor for Water Allocation Choice

The VF for water allocation choice introduces bias so that water allocations likely to result in greater net returns have a greater chance of being chosen for the selected crop in each of the sub-areas, as follows:

$$\eta_{jw} = \begin{cases} 0 & \text{if } (NR_{jw} = NR_{jw1}) \text{ and } (w > w1) \\ 1 - \frac{1}{NR_{jw}} & \text{if (other cases)} \end{cases} \quad (3.3)$$

where η_{jw} is the VF for the level of water allocation w; NR_{jw} and NR_{jw1} are the net returns of crop j for water allocations w and w1, respectively. Similar to W_{NRMaxj} in Equation 3.2, the net return for a given water allocation is obtained with the aid of a crop growth model used in this study as part of the optimization process. The functional form of the VF shown in Equation 3.3 is suggested as it performed best among a number of candidate functional forms considered as part of preliminary trials.

3.2.3 ACO Pheromone Updating Process

After all ants in the colony have traversed the decision-tree graph and the objective function value corresponding to the solution generated by each ant has been calculated, pheromone values are updated so as to increase the chances that edges that contributed to better solutions receive more pheromone and are therefore more likely to be selected in subsequent iterations. The pheromone update for edge (A, B) is therefore given by Dorigo et al. (1996):

$$\tau_{AB}(t+1) = \rho\tau_{AB}(t) + \Delta\tau_{AB} \quad (3.4)$$

where ρ is the pheromone persistence factor, which accounts for the evaporation of pheromone from one iteration to the next and $\Delta\tau_{AB}$ is the pheromone addition for edge (A, B).

In this paper, the Max-Min Ant System (MMAS) algorithm (Stützle and Hoos 2000) is used to calculate $\Delta\tau_{AB}$, as it has been applied successfully to a number of water resources studies (Zecchin *et al.*, 2006; Zecchin *et al.*, 2007; Afshar and Moeini, 2008; Szemis *et al.*, 2012; Zecchin *et al.*, 2012), including the crop and irrigation water allocation problem (Nguyen et al. 2016). As part of this algorithm, pheromone addition on each edge is performed as follows (illustrated here for edge (A, B)):

$$\Delta\tau_{AB}(t) = \Delta\tau_{AB}^{ib}(t) + \Delta\tau_{AB}^{gb}(t) \quad (3.5)$$

where $\Delta\tau_{AB}^{ib}(t)$ and $\Delta\tau_{AB}^{gb}(t)$ are the pheromone additions for the iteration-best solution (s^{ib}) and the global-best solution (s^{gb}), respectively. While s^{ib} is used to update the pheromone on edge (A, B) after each iteration, s^{gb} is applied with the frequency f_{global} (i.e., $\Delta\tau_{AB}^{gb}(t)$ is calculated after each f_{global} iterations). $\Delta\tau_{AB}^{ib}(t)$ and $\Delta\tau_{AB}^{gb}(t)$ are given by:

$$\Delta\tau_{AB}^{ib}(t) = \begin{cases} \frac{q}{f(s^{ib}(t))} & if\ (A,B) \in s^{ib}(t) \\ 0 & otherwise \end{cases} \quad (3.6)$$

$$\Delta\tau_{AB}^{gb}(t) = \begin{cases} \frac{q}{f(s^{gb}(t))} & if\ (A,B) \in s^{gb}(t)\ and\ \ t \bmod f_{global} = 0 \\ 0 & otherwise \end{cases} \quad (3.7)$$

where $f(s^{ib}(t))$ and $f(s^{gb}(t))$ are the objective function values of s^{ib} and s^{gb} at iteration t, respectively; and q is the pheromone reward factor.

In MMAS, the pheromone on each edge is limited to lie within a given range to avoid search stagnation, i.e., $\tau_{min}(t) \le \tau_{AB}(t) \le \tau_{max}(t)$. The equations for $\tau_{min}(t)$ and $\tau_{max}(t)$ are given as follows:

$$\tau_{max}(t) = \left(\frac{1}{1-\rho}\right)\frac{1}{f(s^{gb}(t-1))} \quad (3.8)$$

$$\tau_{min}(t) = \frac{\tau_{max}(t)(1-\sqrt[n]{p_{best}})}{(avg-1)\sqrt[n]{p_{best}}} \quad (3.9)$$

where n is the number of decision points, avg is the average number of edges at each decision point, and p_{best} is the probability of constructing the global best solution at iteration t, where the edges chosen have pheromone trail values of τ_{max} and the pheromone values of other edges are τ_{min}. Additionally, MMAS uses a pheromone trail smoothing (PTS) mechanism that reduces the difference between edges in terms of pheromone intensities, thus strengthening exploration.

$$\tau_{AB}^{*}(t) = \tau_{AB}(t) + \delta(\tau_{max}(t) - \tau_{AB}(t)) \quad (3.10)$$

where δ is the PTS coefficient ($0 \le \delta \le 1$).

The pheromone updating process is repeated until certain stopping criteria have been met, such as the completion of a fixed number of iterations or until there is no further improvement in the objective function.

3.3 Methodology

3.3.1 Overview

This section provides details of how the approach outlined in the Proposed ACO Formulation Incorporating Domain Knowledge section is applied to the two case studies, including the benchmark case study used in Nguyen et al. (2016) and a real-world case study introduced in this paper based on an irrigation district located in Loxton, South Australia near the River Murray. A summary of the main steps in the methodology is given in Figure 3-2. In the first step the problem is formulated to: 1) identify the seasons (e.g., winter, monsoon), the seasonal crops (e.g., wheat), the annual crops (e.g., sugarcane), the total cultivated area and the volume of water available for irrigation purposes; 2) identify economic data (e.g., crop price, production cost, and water price) in the study region; 3) specify decision variables (e.g., crop type and the depth of irrigated water allocated to each sub-area); 4) specify decision variable options; and 5) specify the objective function (e.g., maximum economic return) and constraints on the availability of land and water resources.

Next, trial crop and water allocations are generated in a stepwise fashion by moving along the decision-tree graph (Figure 3-1) from one node to the next. As outlined in the Proposed ACO Formulation Incorporating Domain Knowledge section, dynamic adjustment of decision variable options and the use of VFs to favor solutions that result in a local increase in net return are considered during the generation of trial solutions. In the next step, a crop growth model is used to calculate the objective function values for the developed trial crop and water allocation plans. In this study, crop production functions were used for this purpose as they are sufficiently computationally efficient to enable different optimization algorithm formulations to be compared in a more rigorous manner. Consequently, the results of this study provide a proof-of-concept for the application of the proposed approach to problems using more computationally intensive and physically-based crop growth simulation models.

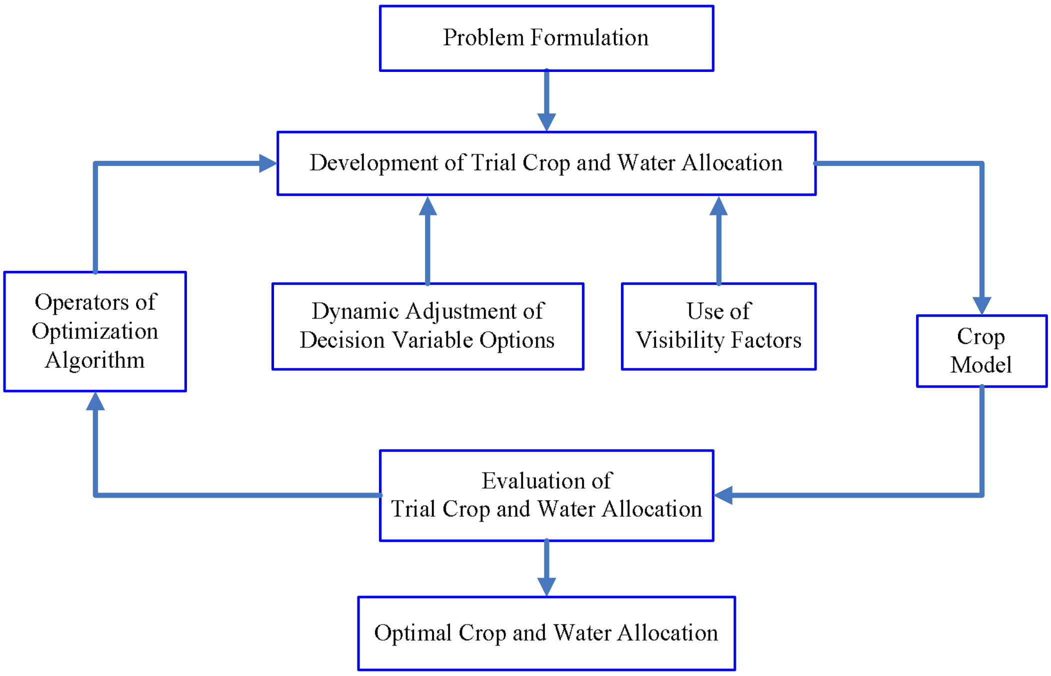


Figure 3-2. Summary of the main steps in the methodology of optimal crop and water allocation using ACO formulation incorporating domain knowledge

Based on the results of the evaluation of the trial solutions, the pheromone levels on the edges of the decision-tree graph are updated so as to favor crop choices and water allocations that lead to better solutions, as outlined in the Proposed ACO Formulation Incorporating Domain Knowledge section. This iterative loop of developing trial solutions using the ACO algorithm and evaluating their utility with the aid of the crop production functions is repeated until the specified stopping criteria have been met. In general, the stopping criterion is the completion of a certain number of evaluations. Details of the two case studies and how the above methodology was applied to them are given in the subsequent sections.

3.3.2 *Case Study 1*

3.3.2.1 *General Description*

This case study was introduced by Kumar and Khepar (1980) and considers seven crops (i.e., wheat, gram, mustard, clover (referred to as berseem in Kumar and Khepar 1980), sugarcane, cotton and paddy). Except for sugarcane, which is planted annually, all other crops are either planted in the winter (i.e., wheat, gram, mustard and clover) or monsoon season (i.e., cotton and paddy). The total cultivated area under consideration is 173 ha. The prices and total fixed costs of the different crops are shown in Table 3-1. The water price is equal to 0.423 Rs mm^{-1} ha^{-1}.

Table 3-1.
Economic Data for Crops and Crop Production Functions in Case Study 1 (from Kumar and Khepar, 1980)

Crop	Crop price (Rs qt^{-1})	Crop total fixed cost (Rs ha^{-1} $year^{-1}$)	Crop production function
Wheat	122.5	2669.8	$Y = 26.5235 - 0.03274\ W + 1.14767\ W^{0.5}$
Gram	147.8	1117.0	$Y = 15.4759 + 0.04561\ W - 0.00019\ W^{2}$
Mustard	341.4	1699.55	$Y = 14.743 - 0.011537\ W + 0.41322\ W^{0.5}$
Clover	7.0	2558.6	$Y = 25.5379 - 1.0692\ W + 57.2238\ W^{0.5}$
Sugarcane	13.5	5090.48	$Y = -11.5441 + 2.92837\ W - 0.0027\ W^{2}$
Cotton	401.7	2362.55	$Y = 6.6038 - 0.013607\ W + 0.62418\ W^{0.5}$
Paddy	89.0	2439.68	$Y = 5.9384 - 0.035206\ W + 2.412043\ W^{0.5}$

Note: Rs = Indian Rupee; qt = quintal; Y = crop yield (qt ha^{-1}); W = depth of irrigated water (mm).

3.3.2.2 *Problem Formulation*

The two decision variables are which crop to plant on each sub-area, and the depth of irrigated water supplied to the selected crop (Figure 3-1). For each sub-area, there are six crop options for Season 1 (consisting of dryland, wheat, gram, mustard, clover and sugarcane) and three crop options for Season 2 (consisting of dryland, cotton and paddy). Here dryland refers to an area without crops, so it is not irrigated and has no yield. As suggested in

Nguyen et al. (2016), 150 options of the depth of irrigated water were considered for each crop, corresponding to 0, 10, 20, …, 1490 mm ha^{-1}. Furthermore, the novel discretization scheme introduced by Nguyen et al. (2016) was applied to reduce the size of the search space. As part of this scheme, the total area was discretized into 29 sub-areas, 10 corresponding to an area of 5 ha, 10 to an area of 6 ha and 9 to an area of 7 ha. The corresponding search space size for this case study is equal to (6^{29} x 3^{29} x 150^{29} x 150^{29}) or approximately 4.1 x 10^{162}.

The objective function is given as:

$$F = \mathrm{Max}\left\{\begin{array}{l} \sum_{j=1}^{6}\sum_{k=1}^{29}\left(A_{1jk}\left[Y_{1jk}P_{1j} - \left(C_{FIX1j} + W_{1jk}C_W\right)\right]\right) \\ + \sum_{j=1}^{3}\sum_{k=1}^{29}\left(A_{2jk}\left[Y_{2jk}P_{2j} - \left(C_{FIX2j} + W_{2jk}C_W\right)\right]\right) \end{array}\right\} \quad (3.11)$$

where F is the total net annual return (Rs $year^{-1}$), A_{1jk} and A_{2jk} are the area of crop j at sub-area k in seasons 1 and 2 (ha), Y_{1jk} and Y_{2jk} are the yields of crop j at sub-area k in seasons 1 and 2 (kg ha^{-1}), P_{1j} and P_{2j} are the prices of crop j in seasons 1 and 2 (Rs kg^{-1}), C_{FIX1j} and C_{FIX2j} are the fixed annual cost of crop j in seasons 1 and 2 (Rs ha^{-1} $year^{-1}$), W_{1jk} and W_{2jk} are the depth of irrigated water supplied to crop j at sub-area k in seasons 1 and 2 (mm), and C_W is the unit cost of irrigated water (Rs mm^{-1} ha^{-1}).

The objective function is subject to the following constraints (as stipulated in Kumar and Khepar 1980):

- *Constraints for maximum allowable areas in each season*

For Season 1 (winter): $$\sum_{j=1}^{6}\sum_{k=1}^{29} A_{1jk} \leq 173 \quad (3.12)$$

For Season 2 (monsoon): $$\sum_{j=1}^{4}\sum_{k=1}^{29} A_{2jk} \leq 139 \quad (3.13)$$

It is important to realize that the area allocated to sugarcane needs to be added to the accumulated area of Season 2 because sugarcane is an annual crop. Consequently, although there are only three crops in this season, Equation 3.13 is calculated for four crops, consisting of dryland, cotton, paddy and sugarcane.

- *Constraints for minimum and maximum allowable crop areas*

$$\begin{cases} A_{1jMin} \leq \sum_{k=1}^{29} A_{1jk} \leq A_{1jMax} & j = 1, \ldots, 6 \\ A_{2jMin} \leq \sum_{k=1}^{29} A_{2jk} \leq A_{2jMax} & j = 1, \ldots, 3 \end{cases} \tag{3.14}$$

where A_{1jMin} and A_{1jMax} are the minimum and maximum areas of crop j allowed in Season 1 and A_{2jMin} and A_{2jMax} are the minimum and maximum areas of crop j allowed in Season 2. As stipulated by Kumar and Khepar (1980), the minimum area of clover is 17 ha, and the maximum areas of mustard and sugarcane are equal to 26 ha and 17 ha, respectively. The other minimum areas are 0, while the other maximum areas are 173 ha for each of the winter crops and 139 ha for each of the monsoon crops.

- *Constraints for available volume of irrigated water W*

$$\sum_{j=1}^{6} \sum_{k=1}^{29} W_{1jk} A_{1jk} + \sum_{j=1}^{3} \sum_{k=1}^{29} W_{2jk} A_{2jk} \leq W \tag{3.15}$$

As was the case in Kumar and Khepar (1980), three different values of W were considered as part of three scenarios corresponding to 75%, 90% and 100% of water availability (corresponding to limits on the available volume of irrigated water of 84,457, 100,178, and 111,275 m^3, respectively). Crop production functions from Kumar and Khepar (1980) were used to estimate crop yield, as detailed in Table 3-1.

3.3.3 Case Study 2

3.3.3.1 General Description

This case study is based on an irrigation district of 130 ha, including 50 sub-areas of various sizes (see Table 3-2), located in Loxton, South Australia near the River Murray (Figure 3-3). In this case study, six typical crops, including wine grapes, apricots, almonds, oranges, irrigated wheat and potatoes, were selected to represent the major crop groups in Loxton (consisting of grapes, stone fruit, nuts, citrus, field crops and vegetable) as mentioned in King *et al.* (2012). These crops are harvested once per year.

The economic data of the crops for the studied region are summarized in Table 3-3. The water price is equal to AU$0.100 per m^3 (King et al. 2012).

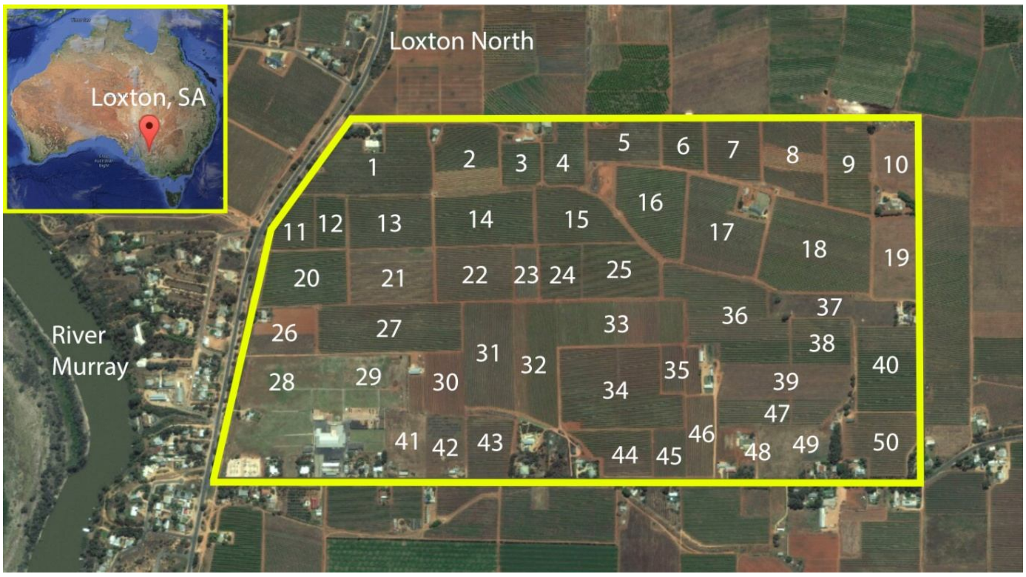


Figure 3-3. Schematic of the sub-areas for Case Study 2 (inset satellite photo: Imagery © 2016 TerraMetrics, Map data © 2016 GBRMPA, Google; subarea photo: Imagery © 2016 CNES/Astrium, Cnes/Spot Image, DigitalGlobe, Map data © 2016 Google)

Table 3-2.
Sub-area Details for Case Study 2

No.	Area (ha)	No.	Area (ha)	No.	Area (ha)	No.	Area (ha)	No.	Area (ha)
1	4.7	11	1.3	21	3.0	31	3.5	41	1.7
2	2.8	12	1.0	22	2.7	32	3.3	42	1.6
3	1.6	13	2.9	23	1.0	33	3.8	43	2.1
4	1.7	14	3.4	24	1.7	34	5.4	44	1.9
5	2.1	15	2.5	25	2.5	35	1.0	45	1.0
6	1.3	16	3.2	26	2.2	36	5.3	46	1.4
7	1.9	17	4.5	27	4.7	37	2.1	47	1.7
8	3.0	18	6.5	28	1.6	38	1.9	48	2.4
9	2.8	19	2.3	29	1.8	39	2.7	49	1.8
10	3.0	20	3.1	30	1.8	40	3.6	50	3.2

Table 3-3.
Economic Data for Crops in Loxton, South Australia [a]

Crop	Price of crop (AU$ t^{-1})	Irrigation costs (AU$ ha^{-1} $year^{-1}$)	Operating costs (AU$ ha^{-1} $year^{-1}$)	Fixed capital costs of irrigation infrastructure [b] (AU$ ha^{-1} $year^{-1}$)
Wine grapes	1,400.0	286.0	3,650.0	2,623.9
Apricots	1,100.0	3,389.0	6,285.0	2,476.9
Almonds	7,000.0	486.0	4,413.0	2,473.3
Oranges	300.0	586.0	5,284.0	2,598.6
Irrigated wheat	290.0	180.0	345.0	566.0
Potatoes	370.0	236.0	4,062.0	1,326.8

[a] Source: King et al. (2012).

[b] Based on applying pivot irrigation to wine grapes, apricots, almonds, and oranges; flood irrigation to wheat; and drip irrigation to potatoes.

3.3.3.2 Problem Formulation

The two decision variables are crop type and the volume of irrigated water applied to each sub-area (Figure 3-1). For each sub-area there are seven crop options consisting of dryland, wine grapes, apricots, almonds, oranges, irrigated wheat, and potatoes. The dryland option indicates that there is no irrigation and therefore a yield of zero. In addition, as the maximum water application rate for each crop is 9,000 m^3 ha^{-1} (Table 3-4) and a discretization interval of 500 m^3 ha^{-1} was assumed, there are 19 options of irrigated water for each sub-area, corresponding to choices of 0, 500, 1,000, …, 9,000 m^3 ha^{-1}. Consequently, the size of the total search space for case study 2 is (6^{50} x 19^{50}), which is approximately 1.6 x 10^{106}. The objective function of this problem is given as:

$$F = \mathrm{Max}\left\{\sum_{j=1}^{7}\sum_{k=1}^{50}\left(A_{jk}\left[Y_{jk}\,P_j - \left(C_{FIXj} + W_{jk}\,C_W\right)\right]\right)\right\} \tag{3.16}$$

where A_{jk} is the area of crop j at sub-area k (ha), Y_{jk} is the yield of crop j at sub-area k (depending on W_{jk}) (kg ha^{-1}), P_j is the price of crop j (AU$ kg^{-1}), C_{FIXj} is the fixed annual cost of crop j (AU$ ha^{-1} $year^{-1}$), and W_{jk} is the depth of water supplied to crop j at sub-area k (m^3 ha^{-1}). The remaining variables have been defined previously in Equation 3.11.

Table 3-4.
Crop Production Functions for Crops in Loxton, South Australia [a]

Crop type	Crop production function	
Wine grapes	$Y = 0.1093\ W^3 - 2.3108\ W^2 + 15.489\ W - 8.3295$	$0 \le W \le 9{,}000$
Apricots	$Y = 0.1438\ W^3 - 3.035\ W^2 + 21.324\ W - 32.318$	$0 \le W \le 9{,}000$
Almonds	$Y = 0.0155\ W^3 - 0.3584\ W^2 + 2.792\ W - 4.357$	$0 \le W \le 9{,}000$
Oranges	$Y = 0.1938\ W^3 - 5.2001\ W^2 + 45.865\ W - 82.849$	$0 \le W \le 9{,}000$
Irrigated wheat	$Y = 0.0008\ W^3 - 0.1492\ W^2 + 2.0857\ W + 1.3937$	$0 \le W \le 9{,}000$
Potatoes	$Y = -\ 0.6915\ W^2 + 13.038\ W - 9.1178$	$0 \le W \le 9{,}000$

Note: Y = crop yield (kg ha^{-1}); W = volume of irrigated water (1,000 $m^3\ ha^{-1}$).
[a] Source: King et al. (2012).

This objective function is subject to the following constraints:

- *Constraints for maximum allowable areas*

$$\sum_{j=1}^{7} \sum_{k=1}^{50} A_{jk} \le 130 \tag{3.17}$$

- *Constraints for minimum (A_{jMin}) and maximum (A_{jMax}) allowable crop area*

$$A_{jMin} \le \sum_{k=1}^{50} A_{jk} \le A_{jMax} \tag{3.18}$$

where the values of A_{jMin} and A_{jMax} are given in Table 3-5.

Table 3-5.
Maximum and Minimum Areas for Different Crop Types for Case Study 2

Crop j	Crop type	Minimum area (ha)	Maximum area (ha)
1	Wine grapes	0	100
2	Apricots	0	130
3	Almonds	0	130
4	Oranges	0	130
5	Irrigated wheat	0	50
6	Potatoes	5	15
7	Dryland	0	130

- *Constraints for available volume of irrigated water W*

$$\sum_{j=1}^{7} \sum_{k=1}^{50} W_{jk} \times A_{jk} \leq W \quad (3.19)$$

According to King et al. (2012), there are five levels of annual water allocations in the study region that are provided to irrigators at the start of each season based on current reservoir storage and forecasts of inflows. In normal precipitation years, there are two types of water allocations including wet (95-100% allocation) and dry (80-95% allocation). During drought years, there are three types of water allocations including dry (60-80% allocation), very dry (25-60% allocation), and extremely dry (less than 25% allocation). In order to account for the different water allocation levels, six scenarios, corresponding to six different levels of W, were considered. These included allocations of 100%, 85%, 70%, 50%, 35% and 10% which cover each of the five water allocation levels, with two allocations (50% and 35%) corresponding to the very dry allocation category as this spans a wide range of allocations (i.e., 25-60%). In the case of 100% water allocation, crops can be supplied at up to the 9,000 m^3 ha^{-1} maximum water application rate and thus the total water availability of this allocation for the studied area (130 ha) is 1,170,000 m^3. Consequently, the various percentage water allocations considered correspond to limits on available volume of irrigated water of 1,170,000 (100%), 994,500 (85%), 819,000 (70%), 585,000 (50%), 409,500 (35%) and 117,000 m^3 (10%), respectively. As mentioned above, crop production functions (Table 3-4) were used for the calculation of net returns and the dryland yield was considered to be zero.

3.4 Computational Experiments

Four ACO algorithm variants (Table 3-6) were applied to each case study, including: 1) a "standard" algorithm which uses static decision variable options (SDVOs) but does not incorporate domain knowledge through the use of VFs (henceforth referred to as ACO-SDVO); 2) an algorithm which uses SDVOs and incorporates domain knowledge through the use of VFs (henceforth referred to as ACO-SDVO-VF); 3) an algorithm that uses the dynamic decision variable option (DDVO) adjustment introduced by Nguyen et al. (2016) but does not incorporate domain knowledge through the use of VFs (henceforth referred to as ACO-DDVO); and 4) an algorithm that uses the Nguyen et al. (2016) DDVO adjustment and incorporates domain knowledge through the use of VFs (henceforth referred to as ACO-DDVO-VF). The FORTRAN code developed for implementing the different ACO variants can be downloaded from Nguyen (2015). As was carried out by Nguyen et al. (2016), each algorithm was run for 1,000,000 function evaluations, with results compared after 1,000, 2,000, 5,000, 10,000, 50,000, 100,000, 500,000 and 1,000,000 function evaluations. The MMAS algorithm was used for pheromone updating, as mentioned previously. Based on the results of a sensitivity analysis, the best values of the ACO parameters were selected for each algorithm, case study and percent water availability, as summarized in Tables 3-7 and 3-8.

In order to assess the relative merits of incorporating the VFs introduced in this paper, the results obtained using ACO-SDVO-VF and ACO-DDVO-VF were compared with those obtained using the ACO-SDVO and ACO-DDVO algorithms that do not use VFs (Table 3-6). Each experiment was repeated 30 times with different starting positions in solution space (i.e., different random number seeds). This enabled the null hypothesis that the mean objective function values of the algorithms using the VFs are not significantly better than those of the corresponding algorithms without the VFs to be tested using a Student's t-test. This is similar to the process used by Zecchin et al. (2007) when assessing the relative performance of different ACO algorithms. In addition, for Case Study 1, the best results obtained using ACO were also compared with those obtained using linear programming by Kumar and Khepar (1980).

Table 3-6.
Summary of Computational Experiments Conducted for each Case Study

ACO algorithm variant	Decision variable options		Visibility factor	Replicates
	Static	Dynamic		
ACO-SDVO	X			30
ACO-DDVO		X		30
ACO-SDVO-VF	X		X	30
ACO-DDVO-VF		X	X	30

Table 3-7.
ACO Parameter Values Selected Based on a Sensitivity Analysis

Parameter	Values considered	Values selected
Number of ants	50; 100; 200; 500; 1,000; 2,000; 5,000; 10,000	100[a] and 1,000[b]
Pheromone importance factor (α)	0.5, 1.0, 1.2, 1.5, 2.0	See Table 3.8
Visibility importance factor (β)	0, 0.5, 0.7, 1.0, 1.2, 1.5, 2.0	
Initial pheromone (τ_o)	0.5, 1.0, 2.0, 5.0, 10.0, 20.0	10.0
Pheromone persistence (ρ)	0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9	0.6
Pheromone reward (q)	0.5, 1.0, 2.0, 5.0, 10.0, 20.0, 50.0	20.0

Note: a = if the number of evaluations ≤ 10,000; b = if the number of evaluations ≤ 1,000,000.

Table 3-8.
Pheromone Importance and Visibility Importance Factors (α, β) Selected (for each ACO Formulation, Percent Water Availability, and Case Study) Based on a Sensitivity Analysis

	Water availability (%)	ACO-SDVO	ACO-DDVO	ACO-SDVO-VF	ACO-DDVO-VF
Case Study 1	100	(1.2, 0)	(1.2, 0)	(1.2, 0.7)	(1.2, 1.2)
	90	(1.2, 0)	(1.2, 0)	(1.2, 0.5)	(1.2, 2.0)
	75	(1.2, 0)	(1.2, 0)	(1.2, 0.7)	(1.2, 1.2)
Case Study 2	100	(1.5, 0)	(1.2, 0)	(1.2, 1.0)	(1.0, 0.5)
	85	(1.2, 0)	(1.5, 0)	(1.2, 1.0)	(1.5, 1.2)
	70	(1.2, 0)	(1.5, 0)	(1.2, 0.5)	(1.2, 1.0)
	50	(1.2, 0)	(1.5, 0)	(1.2, 1.0)	(1.0, 0.5)
	35	(1.2, 0)	(1.5, 0)	(1.2, 0.5)	(1.5, 0.5)
	10	(1.2, 0)	(1.2, 0)	(1.2, 1.2)	(1.2, 1.0)

3.5 Results and Discussion

3.5.1 Impact of the Visibility Factors

The average objective function (OF) values over the 30 trials from different random starting positions in solution space, as well as the p-values for the t-test which indicate the level of significance (i.e., a p-value of 0.05 indicates significance at the 5% level) are shown in Table 3-9 for Case Study 1 and Tables 3-10 and 3-11 for Case Study 2. In these tables, the percentage deviation of the mean value of the 30 runs from the best-found solution over all runs at a particular number of function evaluations for a particular water availability scenario is also given.

As shown in Table 3-9 for Case Study 1, the addition of the VFs resulted in improved mean objective function values at all stages of the search (i.e., for all evaluation numbers considered) and for all water availability scenarios. When comparing ACO-SDVO-VF with ACD-SDVO, the improvements in mean objective function values resulting from the addition of the VFs were significant at $p < 0.0001$. The same was true when comparing ACO-DDVO-VF with ACO-DDVO, except for three cases where $p = 0.0003$ (90% water availability, 1,000,000 function evaluations), $p = 0.0016$ (100% water availability, 1,000,000 function evaluations) and $p = 0.0001$ (100% water availability, 500,000 function evaluations).

Based on the results from Tables 3-10 and 3-11, the addition of the VFs also resulted in improved mean objective function values at all stages of the search (i.e., for all evaluation numbers considered) and for all water availability scenarios for Case Study 2. However, the p-values were much more variable for Case Study 2 compared to Case Study 1. When comparing ACO-SDVO-VF with ACO-SDVO, the former performed significantly better at the $p < 0.05$ level (i.e., the 95% confidence level) and above for all evaluation numbers for the 10%, 35%, 50% and 100% water availability scenarios, while for the other two scenarios (70% and 85% water availability) this was not the case for evaluation numbers of 50,000 and above. When comparing ACO-DDVO-VF with ACO-DDVO, the former only performed significantly better at the $p < 0.05$ level and above for all evaluation numbers for the 50% water availability scenario. For the 10% and 100% scenarios, the $p < 0.05$ significance level was not satisfied for evaluation numbers of 100,000 and above, while for the 35% and 75% scenarios the $p < 0.05$ level occurred at 50,000 evaluations and above. Finally, for the 85% water availability scenario, the $p < 0.05$ significance threshold was 10,000 evaluations.

Overall, the results indicate that the addition of the VFs introduced in this paper led to significant improvements in the ability to find solutions with better objective function values. This benefit is more pronounced for smaller numbers of evaluations, suggesting that the use of VFs can have significant benefits for problems for which the computational requirements are great and computational budgets are limited, such as when detailed, process-based crop growth models are used. The results in Tables 3-9 to 3-11 confirm the benefits of using the dynamic decision variable constraints (DDVO) introduced by Nguyen et al. (2016) and also suggest that the use of DDVO, in conjunction with VFs, results in the best overall ACO performance.

Table 3-9.

Comparison of Average Objective Function Values (Total Net Annual Return in Rs Year^{-1}) Obtained over 30 Trials, the Deviation of these Values from the Best-Found Solution (in Parentheses Below the Average Values) and the p-Values for the t-Tests comparing ACO Formulations With and Without VF for Different Evaluation Numbers for Case Study 1.

Water availability	No. of evaluations	ACO-SDVO vs ACO-SDVO-VF			ACO-DDVO vs ACO-DDVO-VF		
		ACO-SDVO	ACO-SDVO-VF	p-value for t-test	ACO-DDVO	ACO-DDVO-VF	p-value for t-test
100%	1,000	565,932.3 (36.45%)	707,730.4 (20.52%)	< 0.0001	661,634.6 (25.70%)	796,684.2 (10.54%)	< 0.0001
	2,000	606,467.1 (31.89%)	753,582.1 (15.37%)	< 0.0001	701,541.2 (21.22%)	830,964.0 (6.69%)	< 0.0001
	5,000	651,481.0 (26.84%)	826,891.0 (7.14%)	< 0.0001	762,838.2 (14.34%)	867,150.2 (2.63%)	< 0.0001
	10,000	704,423.8 (20.89%)	867,469.8 (2.58%)	< 0.0001	802,504.5 (9.88%)	878,966.9 (1.30%)	< 0.0001
	50,000	770,186.0 (13.51%)	879,862.0 (1.19%)	< 0.0001	855,207.5 (3.97%)	887,956.7 (0.29%)	< 0.0001
	100,000	819,262.3 (8.00%)	888,153.2 (0.26%)	< 0.0001	874,009.5 (1.86%)	889,928.1 (0.07%)	< 0.0001
	500,000	872,905.9 (1.97%)	890,426.3 (0.00%)	< 0.0001	889,173.5 (0.15%)	890,499.9 (0.00%)	0.0001
	1,000,000	881,838.7 (0.97%)	890,465.4 (0.00%)	< 0.0001	890,046.4 (0.05%)	890,529.5 (0.00%)	0.0016
90%	1,000	545,105.4 (37.57%)	708,527.0 (18.85%)	< 0.0001	647,114.1 (25.90%)	784,343.3 (10.18%)	< 0.0001
	2,000	608,342.3 (30.33%)	765,390.2 (12.34%)	< 0.0001	692,580.5 (20.69%)	821,282.5 (5.95%)	< 0.0001
	5,000	659,347.1 (24.49%)	835,879.7 (4.27%)	< 0.0001	747,986.9 (14.35%)	849,461.1 (2.73%)	< 0.0001
	10,000	695,005.5	852,861.7	< 0.0001	795,470.0	859,270.5	< 0.0001

Water availability	No. of evaluations	ACO-SDVO vs ACO-SDVO-VF			ACO-DDVO vs ACO-DDVO-VF		
		ACO-SDVO	ACO-SDVO-VF	p-value for t-test	ACO-DDVO	ACO-DDVO-VF	p-value for t-test
		(20.40%)	(2.32%)		(8.91%)	(1.60%)	
	50,000	752,121.2	861,724.7	< 0.0001	835,106.5	866,212.5	< 0.0001
		(13.86%)	(1.31%)		(4.37%)	(0.81%)	
	100,000	791,330.5	868,887.5	< 0.0001	852,020.7	869,569.0	< 0.0001
		(9.37%)	(0.49%)		(2.43%)	(0.42%)	
	500,000	846,912.6	872,902.7	< 0.0001	868,906.6	872,909.2	< 0.0001
		(3.00%)	(0.03%)		(0.50%)	(0.04%)	
	1,000,000	856,367.6	873,149.7	< 0.0001	871,667.0	873,273.0	0.0003
		(1.92%)	(0.00%)		(0.18%)	(0.00%)	
75%	1,000	516,907.6	703,384.8	< 0.0001	589,000.4	764,290.6	< 0.0001
		(38.38%)	(16.15%)		(29.78%)	(8.89%)	
	2,000	560,790.0	747,464.5	< 0.0001	645,478.4	793,265.6	< 0.0001
		(33.15%)	(10.89%)		(23.05%)	(5.43%)	
	5,000	616,682.8	798,435.3	< 0.0001	714,719.6	817,347.3	< 0.0001
		(26.48%)	(4.82%)		(14.80%)	(2.56%)	
	10,000	640,954.3	815,255.6	< 0.0001	758,497.2	824,559.1	< 0.0001
		(23.59%)	(2.81%)		(9.58%)	(1.70%)	
	50,000	709,464.2	827,565.1	< 0.0001	805,907.7	833,097.1	< 0.0001
		(15.42%)	(1.34%)		(3.93%)	(0.68%)	
	100,000	752,978.7	832,567.0	< 0.0001	817,627.6	836,382.8	< 0.0001
		(10.24%)	(0.75%)		(2.53%)	(0.29%)	
	500,000	814,688.6	837,100.3	< 0.0001	832,292.4	837,959.7	< 0.0001
		(2.88%)	(0.21%)		(0.78%)	(0.11%)	
	1,000,000	822,062.0	837,647.8	< 0.0001	835,136.3	838,293.5	< 0.0001
		(2.00%)	(0.14%)		(0.44%)	(0.07%)	

Table 3-10.

Comparison of Average Objective Function Values (Total Net Annual Return in AU$ year^{-1}) Obtained over 30 Trials, the Deviation of these Values from the Best-Found Solution (in Parentheses Below the Average Values) and the p-Values for the t-Tests comparing ACO Formulations With and Without VF for Different Evaluation Numbers for Case Study 2 (River Murray – Normal Precipitation Years)

Water availability	No. of evaluations	ACO-SDVO vs ACO-SDVO-VF			ACO-DDVO vs ACO-DDVO-VF		
		ACO-SDVO	ACO-SDVO-VF	p-value for t-test	ACO-DDVO	ACO-DDVO-VF	p-value for t-test
100%	1,000	1,998,171	2,485,737	< 0.0001	2,477,420	2,588,004	0.0047
		(37.52%)	(22.28%)		(22.54%)	(19.08%)	
	2,000	2,529,891	2,948,301	< 0.0001	2,905,378	2,951,132	0.0517
		(20.90%)	(7.81%)		(9.16%)	(7.72%)	
	5,000	3,148,770	3,183,345	< 0.0001	3,168,960	3,186,494	< 0.0001
		(1.55%)	(0.46%)		(0.91%)	(0.37%)	
	10,000	3,186,217	3,195,066	0.0046	3,194,176	3,195,113	< 0.0001
		(0.37%)	(0.10%)		(0.13%)	(0.10%)	
	50,000	3,197,077	3,197,512	0.00913	3,197,344	3,197,357	0.0008
		(0.03%)	(0.02%)		(0.03%)	(0.03%)	
	100,000	3,197,577	3,197,908	0.01817	3,197,581	3,197,624	0.0588
		(0.02%)	(0.01%)		(0.02%)	(0.02%)	
	500,000	3,197,684	3,198,007	0.0434	3,197,900	3,198,118	0.1467
		(0.02%)	(0.01%)		(0.01%)	(0.00%)	
	1,000,000	3,197,729	3,198,055	0.0257	3,198,048	3,198,150	0.1559
		(0.01%)	(0.00%)		(0.00%)	(0.00%)	

Water availability	No. of evaluations	ACO-SDVO vs ACO-SDVO-VF			ACO-DDVO vs ACO-DDVO-VF		
		ACO-SDVO	ACO-SDVO-VF	p-value for t-test	ACO-DDVO	ACO-DDVO-VF	p-value for t-test
85%	1,000	2,026,555 (36.63%)	2,485,737 (22.28%)	< 0.0001	2,411,376 (24.60%)	2,522,697 (21.12%)	0.0013
	2,000	2,549,290 (20.29%)	2,951,448 (7.72%)	< 0.0001	2,875,900 (10.08%)	2,972,386 (7.06%)	< 0.0001
	5,000	3,112,901 (2.67%)	3,182,822 (0.48%)	< 0.0001	3,189,343 (0.28%)	3,193,058 (0.16%)	0.0346
	10,000	3,176,135 (0.69%)	3,195,297 (0.09%)	< 0.0001	3,197,197 (0.03%)	3,197,312 (0.03%)	0.07873
	50,000	3,196,938 (0.04%)	3,197,436 (0.02%)	0.0819	3,197,577 (0.02%)	3,197,618 (0.02%)	0.7956
	100,000	3,197,778 (0.01%)	3,197,820 (0.01%)	0.9069	3,197,670 (0.02%)	3,197,917 (0.01%)	0.288
	500,000	3,198,018 (0.01%)	3,198,056 (0.00%)	0.6549	3,197,922 (0.01%)	3,198,155 (0.00%)	0.0815
	1,000,000	3,198,092 (0.00%)	3,198,111 (0.00%)	0.8203	3,198,053 (0.00%)	3,198,163 (0.00%)	0.1295

Table 3-11.

Comparison of Average Objective Function Values (Total Net Annual Return in AU$ year-1) Obtained over 30 Trials, the Deviation of these Values from the Best-Found Solution (in Parentheses Below the Average Values) and the p-Values for the t-Tests comparing ACO Formulations With and Without VF for Different Evaluation Numbers for Case Study 2 (River Murray – Drought Years)

Water availability	No. of evaluations	ACO-SDVO vs ACO-SDVO-VF			ACO-DDVO vs ACO-DDVO-VF		
		ACO-SDVO	ACO-SDVO-VF	p-value for t-test	ACO-DDVO	ACO-DDVO-VF	p-value for t-test
70%	1,000	1,941,279 (39.29%)	2,468,732 (22.79%)	< 0.0001	2,399,936 (24.94%)	2,598,329 (18.74%)	< 0.0001
	2,000	2,474,036 (22.62%)	2,940,475 (8.04%)	< 0.0001	2,839,908 (11.18%)	3,017,716 (5.62%)	< 0.0001
	5,000	3,092,723 (3.27%)	3,177,186 (0.63%)	< 0.0001	3,182,876 (0.46%)	3,185,093 (0.39%)	0.02167
	10,000	3,174,491 (0.72%)	3,191,884 (0.17%)	< 0.0001	3,192,746 (0.15%)	3,193,968 (0.11%)	0.02337
	50,000	3,192,713 (0.15%)	3,195,293 (0.07%)	0.0821	3,196,095 (0.04%)	3,196,180 (0.04%)	0.4208
	100,000	3,195,924 (0.05%)	3,196,152 (0.04%)	0.738	3,196,391 (0.03%)	3,196,414 (0.03%)	0.3551
	500,000	3,197,233 (0.01%)	3,197,247 (0.01%)	0.3574	3,197,378 (0.00%)	3,197,382 (0.00%)	0.2442
	1,000,000	3,197,396 (0.00%)	3,197,423 (0.00%)	0.7885	3,197,459 (0.00%)	3,197,479 (0.00%)	0.8064
50%	1,000	1,597,131 (46.72%)	2,267,891 (24.35%)	< 0.0001	2,112,120 (29.57%)	2,338,744 (22.01%)	< 0.0001

Water availability	No. of evaluations	ACO-SDVO vs ACO-SDVO-VF			ACO-DDVO vs ACO-DDVO-VF		
		ACO-SDVO	ACO-SDVO-VF	p-value for t-test	ACO-DDVO	ACO-DDVO-VF	p-value for t-test
	2,000	2,101,179 (29.91%)	2,710,367 (9.58%)	< 0.0001	2,553,502 (14.85%)	2,766,652 (7.75%)	< 0.0001
	5,000	2,812,515 (6.18%)	2,943,016 (1.82%)	< 0.0001	2,930,579 (2.28%)	2,935,671 (2.11%)	0.02634
	10,000	2,935,203 (2.08%)	2,965,683 (1.07%)	0.0005	2,953,652 (1.51%)	2,955,463 (1.45%)	0.0239
	50,000	2,956,886 (1.36%)	2,979,432 (0.61%)	0.0233	2,967,134 (1.06%)	2,981,524 (0.58%)	0.0044
	100,000	2,980,347 (0.58%)	2,989,604 (0.27%)	0.02755	2,970,068 (0.96%)	2,991,094 (0.26%)	0.0006
	500,000	2,989,721 (0.27%)	2,995,927 (0.06%)	0.03249	2,972,670 (0.88%)	2,998,187 (0.02%)	< 0.0001
	1,000,000	2,991,437 (0.21%)	2,997,675 (0.00%)	0.0318	2,979,074 (0.66%)	2,998,910 (0.00%)	< 0.0001
35%	1,000	1,170,280 (54.99%)	1,823,726 (29.86%)	0.0081	1,738,890 (33.12%)	1,805,180 (30.57%)	0.0209
	2,000	1,474,953 (43.27%)	2,182,345 (16.06%)	0.0285	2,079,631 (20.01%)	2,143,743 (17.55%)	0.0414
	5,000	1,528,140 (41.23%)	2,518,042 (3.15%)	0.0251	2,428,474 (6.60%)	2,462,407 (5.29%)	0.02634
	10,000	2,354,982 (9.42%)	2,562,864 (1.43%)	0.0014	2,475,551 (4.79%)	2,532,633 (2.59%)	0.0239
	50,000	2,470,212 (4.99%)	2,564,565 (1.36%)	0.0248	2,546,467 (2.06%)	2,572,819 (1.04%)	0.0725

Water availability	No. of evaluations	ACO-SDVO vs ACO-SDVO-VF			ACO-DDVO vs ACO-DDVO-VF		
		ACO-SDVO	ACO-SDVO-VF	p-value for t-test	ACO-DDVO	ACO-DDVO-VF	p-value for t-test
	100,000	2,537,560 (2.40%)	2,570,122 (1.15%)	0.0061	2,555,555 (1.71%)	2,574,947 (0.96%)	0.1715
	500,000	2,546,848 (2.04%)	2,576,867 (0.89%)	0.0171	2,571,720 (1.09%)	2,579,036 (0.81%)	0.4932
	1,000,000	2,552,653 (1.82%)	2,579,696 (0.78%)	0.0129	2,578,593 (0.82%)	2,586,769 (0.51%)	0.3818
10%	1,000	No feasible solution	616,966 (18.38%)	No solution for ACO-SDVO	571,089 (24.45%)	657,147 (13.07%)	< 0.0001
	2,000		693,270 (8.29%)		658,859 (12.84%)	710,134 (6.06%)	< 0.0001
	5,000		723,830 (4.25%)		710,601 (6.00%)	732,665 (3.08%)	< 0.0001
	10,000		735,897 (2.65%)		727,866 (3.71%)	742,242 (1.81%)	0.0002
	50,000		749,416 (0.86%)		736,500 (2.57%)	749,400 (0.86%)	0.0022
	100,000		749,829 (0.81%)		746,117 (1.30%)	749,722 (0.82%)	0.1592
	500,000		750,991 (0.65%)		750,783 (0.68%)	752,124 (0.50%)	0.6258
	1,000,000		753,847 (0.28%)		752,630 (0.44%)	753,184 (0.36%)	0.5277

3.5.2 Optimal Solutions

The optimal solutions obtained using ACO-DDVO-VF (the best performing ACO algorithm) and the corresponding net returns for Case Study 1 are given in Table 3-12. As can be seen, the same four crops (mustard, clover, sugarcane, cotton) were included in the optimal solutions for all three water availability scenarios. In addition, the area allocated to each of these crops was also the same, and the total area is fully planted in winter and summer. However, the volume of water applied to these crops was increased as the volume of available water increased from 75% to 90%, and then to 100%, thereby increasing the net return. The major difference between the optimal solutions identified using ACO and LP was the inclusion of wheat in the LP solution, at the expense of including larger areas of clover. The whole area of wheat in the LP solutions (i.e., 113 ha) was transferred to clover in the ACO-DDVO-VF solutions due to the higher net return of clover. As shown in Table 3-12, this resulted in a marked decrease in net return for all water availability scenarios.

The optimal solutions and the corresponding net returns for Case Study 2 are given in Table 3-13. As can be seen, wine grapes and potatoes feature in the optimum solutions for all water availability scenarios. Potatoes were included because there was a minimum area constraint of 5 ha for this crop. In contrast, wine grapes were included because they resulted in the highest net return. As the volume of available water increased from 10% to 35%, and then to 50%, the area allocated to wine grapes was increased until the maximum allowable area of 100 ha was reached, at which point almonds were added to the mix of crops and the amount of water applied to potatoes doubled from 2,500 m^3 to 5,000 m^3. As available water was increased further to 70%, the constraint for maximum total area of 130 ha was reached by increasing the area allocated to almonds. Net return also increased by increasing the amount of water allocated to all three crops. When water availability was increased to 85%, there was only a slight increase in the amount of water allocated to potatoes, but no further changes occurred when water availability reached 100%. This is because the maximum allowable area has been allocated, wine grapes have their optimal water allocation and the amounts of water that can be applied to the other two crops have reached their upper limit at an allocation of 85%; thus, there is no additional benefit by increasing the amount of water that is available.

Table 3-12.

Details of Best Solutions Obtained Using Linear Programming (LP) and ACO-DDVO-VF for Case Study 1

Water % (volume, m^3)	Algorithm (net return, Rs)	Decision variable	Crop							Total	
			Wheat	Gram	Mustard	Clover	Sugarcane	Cotton	Paddy	Winter	Summer
100% (1,112,750)	LP (800,652.6)	Area (ha)	113	0	26.0[a]	17.0[b]	17.0[a]	122	0	173[a]	139[a]
		Water (mm ha^{-1})	200	0	190	608.1	542.3	526.1	0	1,112,750[a]	
	ACO (890,600.7)	Area (ha)	0	0	26.0[a]	130	17.0[a]	122	0	173[a]	139[a]
		Water (mm ha^{-1})	0	0	140	470	510	310	0	1,112,300	
90% (1,001,780)	LP (799,725.6)	Area (ha)	113	0	26.0[a]	17.0[b]	17.0[a]	122	0	173[a]	139[a]
		Water (mm ha^{-1})	200	0	190	550	542.3	443.2	0	1,001,780[a]	
	ACO (873,457.6)	Area (ha)	0	0	26.0[a]	130	17.0[a]	122	0	173[a]	139[a]
		Water (mm ha^{-1})	0	0	140	430	520	260	0	1,001,000	
75% (844,570)	LP (792,611.2)	Area (ha)	113	0	26.0[a]	17.0[b]	17.0[a]	122	0	173[a]	139[a]
		Water (mm ha^{-1})	200	0	190	550	542.3	306.1	0	834,533	
	ACO (838,840.8)	Area (ha)	0	0	26.0[a]	130	17.0[a]	122	0	173[a]	139[a]
		Water (mm ha^{-1})	0	0	100	360	480	220	0	844,000	

[a] At maximum value

[b] At minimum value

Table 3-13.
Details of Best Solutions Obtained Using ACO-DDVO-VF for Case Study 2

Water % (volume, m^3)	Net return (AU$)	Decision variable	Crop						Total
			Wine Grapes	Apricots	Almonds	Oranges	Wheat	Potatoes	
100%	3,198,173	Area (ha)	100[a]	0	25	0	0	5.0[b]	130.0[a]
(1,170,000)		Water (m^3 ha^{-1})	5,500	0	9,000[a]	0	0	9,000[a]	820,000
85%	3,198,173	Area (ha)	100[a]	0	25	0	0	5.0[b]	130.0[a]
(994,500)		Water (m^3 ha^{-1})	5,500	0	9,000[a]	0	0	9,000[a]	820,000
70%	3,197,556	Area (ha)	100[a]	0	25	0	0	5.0[b]	130.0[a]
(819,000)		Water (m^3 ha^{-1})	5,500	0	9,000[a]	0	0	8,500	817,500
50%	2,999,943	Area (ha)	100[a]	0	20	0	0	5.0[b]	125
(585,000)		Water (m^3 ha^{-1})	4,500	0	5,500	0	0	5,000	585,000[a]
35%	2,599,976	Area (ha)	99.2	0	0	0	0	5.0[b]	104.2
(409,500)		Water (m^3 ha^{-1})	4,000	0	0	0	0	2,500	409,300
10%	755,929	Area (ha)	34.8	0	0	0	0	5.0[b]	39.8
(117,000)		Water (m^3 ha^{-1})	3,000	0	0	0	0	2,500	116,900

[a] At maximum value
[b] At minimum value

3.6 Summary and Conclusions

In this paper, an improved ACO formulation for the allocation of crops and water to different irrigation areas was introduced. The proposed formulation incorporates domain knowledge related to the factors affecting net return into the optimization process via the use of visibility factors (VFs). In addition, the formulation enables the number of decision variable options to be adjusted dynamically, as proposed by Nguyen et al. (2016). These novel improvements to the standard ACO approach enable the size of the search space to be reduced and enable better regions of the search space to be explored.

In order to comprehensively evaluate overall efficacy, the proposed approach was applied to the case study introduced by Kumar and Khepar (1980), used by Nguyen et al. (2016) to evaluate the benefits of dynamic decision variable option adjustment, and a real-world case study based on an irrigation district located in Loxton, South Australia near the River Murray. Various water allocation scenarios were considered for the two case studies, and four ACO variants were tested including: 1) a standard algorithm which uses SDVOs but does not incorporate domain knowledge through the use of VFs (ACO-SDVO); 2) an algorithm which uses SDVOs and incorporates domain knowledge through the use of VFs (ACO-SDVO-VF); 3) an algorithm that uses the DDVO adjustment introduced by Nguyen et al. (2016) but does not incorporate domain knowledge through the use of VFs (ACO-DDVO); and 4) an algorithm that uses the Nguyen et al. (2016) DDVO adjustment and also incorporates domain knowledge through the use of VFs (ACO-DDVO-VF).

The results obtained for both case studies indicate that the use of VFs increased the ability to identify better solutions at all stages of the search, especially at smaller numbers of function evaluations. This highlights the potential of using VFs to identify near-optimal solutions for problems such as detailed irrigation scheduling for individual crops where the use of more computationally expensive, mechanistic crop growth models may be required.

CHAPTER 4

Optimization of Irrigation Scheduling Using Ant Colony Algorithms and an Advanced Cropping System Model (Paper 3)

Statement of Authorship

Title of Paper	Optimization of Irrigation Scheduling Using Ant Colony Algorithms and an Advanced Cropping System Model
Publication Status	Unpublished and Unsubmitted work written in manuscript style
Publication Details	Nguyen, D.C.H., Ascough II, J.C., Maier, H.R., Dandy, G.C. & Andales, A. A., 2016. Optimization of Irrigation Scheduling Using Ant Colony Algorithms and an Advanced Cropping System Model.

Principal Author

Name of Principal Author (Candidate)	Duc Cong Hiep Nguyen		
Contribution to the Paper	To construct the framework, develop computer programming, run simulations, do analysis and discussion of results, and write the manuscript.		
Overall percentage (%)	60%		
Certification:	This paper reports on original research I conducted during the period of my Higher Degree by Research candidature and is not subject to any obligations or contractual agreements with a third party that would constrain its inclusion in this		
Signature		Date	16 Feb 2016

Co-Author Contributions

By signing the Statement of Authorship, each author certifies that:

i. the candidate's stated contribution to the publication is accurate (as detailed above);

ii. permission is granted for the candidate in include the publication in the thesis; and

iii. the sum of all co-author contributions is equal to 100% less the candidate's stated contribution.

Name of Co-Author	James C. Ascough II		
Contribution to the Paper 10%	Provision of data and simulation model, supervision of the research and review of the manuscript		
Signature		Date	11 February, 2016

Name of Co-Author	Holger R. Maier		
Contribution to the Paper 10%	Supervision of the research and review of the manuscript		
Signature		Date	15/2/16

Name of Co-Author	Graeme C. Dandy		
Contribution to the Paper 10%	Supervision of the research and review of the manuscript		
Signature		Date	16/2/16

Name of Co-Author	Allan A. Andales		
Contribution to the Paper 10%	Provision of data, supervision of the research and review of the manuscript		
Signature		Date	11 February, 2016

Abstract

A generic simulation-optimization framework for optimal irrigation and fertilizer scheduling is developed, where the problem is represented in the form of a decision-tree graph, ant colony optimization (ACO) is used as the optimization engine and a process-based crop growth model is applied to evaluate the objective function. By using dynamic decision variable option (DDVO) adjustment, the framework is able to reduce the size of the search space during the process of trial solution construction, thereby increasing computational efficiency. A real-world case study for irrigation and fertilizer scheduling of corn production in eastern Colorado, USA is implemented to test the utility of the proposed framework, where various fixed irrigation time steps (i.e., 3-day, 5-day and 7-day), levels of water availability (i.e., 100%, 60%, 40% and 35%), and rates of fertilizer application (i.e., 50, 100, 150 and 200 kg N ha-1) are considered. The results from the case study indicate that ACO-DDVO is able to identify irrigation and fertilizer schedules that result in better net returns while using less fertilizer and similar amounts of water, or similar net returns while using less water and fertilizer, than those obtained using the Microsoft Excel spreadsheet-based Colorado Irrigation Scheduler (CIS) tool for annual crops. Another advantage of ACO-DDVO compared to CIS is the identification of both optimal irrigation and fertilizer schedules.

4.1 Introduction

In many regions of the world, irrigation is vital for food production. While the importance of irrigation should increase in the near future as a result of population growth (Dyson, 1999), economic development (Schneider *et al.*, 2011) and climate change (Döll, 2002), there will most likely be a reduction in the amount of water available for irrigation due to increased domestic (Rosegrant and Ringler, 2000), industrial, commercial (Malla and Gopalakrishnan, 1999) and environmental (Burke *et al.*, 2004; Szemis *et al.*, 2013) demands, as well as over-allocation of existing resources (Jury and Vaux, 2005) and the impact of climate change (Arnell, 1999; Liu *et al.*, 2010). Consequently, there is a need to identify irrigation management strategies (e.g., sequential irrigation scheduling) that maximize economic return for a given water allocation. However, this is not a trivial task due to the typically large search space for this type of problem (Nguyen *et al.*, 2016b). This is because each irrigation management strategy involves a number of associated choices to be made in relation to various components, including crops (type, rotation, area planted), irrigation method and scheduling (magnitude, duration, and timing), as well as fertilizer application method and scheduling (magnitude and timing).

In order to address the irrigation management strategy problem as described above, optimization, simulation, and combined simulation-optimization approaches have typically been employed. For the optimization approach (Singh, 2012, 2014), irrigation has been scheduled using dynamic programming (Rao *et al.*, 1988; Naadimuthu *et al.*, 1999), nonlinear programming (Ghahraman and Sepaskhah, 2004) and multi-objective programming (Lalehzari *et al.*, 2015) to maximize crop yield or economic profit. Although these “conventional algorithms” (CAs) for optimization have the advantage of being simple and efficient to apply, they are somewhat limited in terms of handling nonlinear and “curse of dimensionality” (i.e., the search space size grows exponentially) problems, such as those that occur in irrigation management (Singh, 2014). In the past decade, metaheuristic algorithms, such as evolutionary algorithms (EAs), have been used extensively to overcome the shortcomings of CAs for solving computationally demanding (i.e., NP-hard) sequential irrigation scheduling problems. For example, development and evaluation of genetic algorithms (GAs) for the irrigation scheduling problem have been presented by Wardlaw and Bhaktikul (2004), Haq *et al.* (2008), Haq and Anwar (2010), Anwar and Haq (2013), and Sadati *et al.* (2014).

The simulation approach for solving irrigation management problems varies widely in the level of model complexity and soil-water-plant process representation. Simplistic crop models used for irrigation management include: (a) those based on crop-water production functions (Jensen (1968); Doorenbos and Kassam (1979)) to calculate crop yield response to irrigation water (Reca *et al.*, 2001; Evans *et al.*, 2003; Azamathulla *et al.*, 2008; Georgiou and Papamichail, 2008; Brown *et al.*, 2010; Prasad *et al.*, 2011; Nguyen *et al.*, 2016a; Nguyen *et al.*, 2016b); and (b) the FAO Penman-Monteith method crop evapotranspiration (ET) and the crop growth coefficient approach of Doorenbos and Pruitt (1977) to estimate crop water requirements (Shyam *et al.*, 1994; Sethi *et al.*, 2006; Khare *et al.*, 2007). While these quasi-empirical modeling approaches are computationally efficient, they are unable to represent the underlying physical processes affecting crop water requirements and crop growth in a realistic manner. This limits the usefulness of the results obtained and prevents investigation of certain management strategies (i.e., fertilizer application timing and rate) on the optimal trade-offs between water allocation and net return. To assess the impact of different irrigation management strategies in a more realistic manner (i.e., through a detailed description of ET and/or crop growth), a number of process-based soil water balance/dynamics (George *et al.*, 2000; Shang *et al.*, 2004; Shang and Mao, 2006) and crop growth (Ma *et al.*, 2012b; Sun and Ren, 2014; Seidel *et al.*, 2015; Linker *et al.*, 2016) modeling studies have been conducted. These have utilized well-known cropping and agroecosystem models, including CERES-Maize (Jones *et al.*, 1986), CROPGRO (Boote *et al.*, 1998), RZQWM2 (Ma *et al.*, 2012a), AquaCrop (Vanuytrecht *et al.*, 2014), EPIC (Zhang *et al.*, 2015), STICS (Coucheney *et al.*, 2015), and SWAT (Arnold *et al.*, 2012).

The above modeling studies have generally focused on a small number of irrigation management strategy combinations (e.g., Camp *et al.* (1997); Rinaldi (2001); Arora (2006); Ma *et al.* (2012b)). Consequently, there is a need to combine detailed process-based crop growth simulation models with optimization approaches so that better irrigation management solutions resulting in maximum net return can be identified more efficiently. The majority of simulation-optimization studies in the literature have employed conventional optimization algorithms (Cai *et al.*, 2010; Karamouz *et al.*, 2012; Hejazi *et al.*, 2013). An exception to this is the work of Kloss *et al.* (2012), who developed a stochastic simulation framework combining the CropWat (Smith, 1992), PILOTE (Mailhol *et al.*, 1997), Daisy (Abrahamsen and Hansen, 2000), and APSIM (Keating *et al.*, 2003) cropping system models with an evolutionary algorithm to optimize irrigation management and water productivity. In general, simulation-optimization approaches utilizing CAs have been somewhat restricted due to the generally large size of the search space, which may limit the ability to find globally optimal or near-globally optimal solutions in an acceptable time frame.

In addition to evolutionary algorithms such as GAs, other metaheuristic search algorithms, such as ant colony optimization (ACO) algorithms, have contributed significantly to a range of water resources problems (Afshar *et al.*, 2015), including irrigation management problems (Nguyen *et al.*, 2016a; Nguyen *et al.*, 2016b). In ACO, the problems are represented in the form of a decision-tree graph which artificial ants have to traverse in a stepwise fashion in order to generate trial solutions. Therefore, use of ACO can increase the probability of finding globally optimal or near-globally optimal solutions and improve computational efficiency through reduction in search space size and incorporation of domain knowledge during the optimization process. Similar to other metaheuristic algorithms, another advantage of ACO for irrigation management problems is the ability to easily connect to simulation models (Maier *et al.*, 2014; Maier *et al.*, 2015).

Nguyen *et al.* (2016b) developed a general optimization framework for the crop and water allocation problem that utilized a dynamic decision tree graph and ACO as the optimization engine. The framework was subsequently extended (Nguyen *et al.*, 2016a) to include the use of domain knowledge to bias the selection of crops and water allocations at each node in the decision-tree graph in order to increase computational efficiency. However, these studies only focused on the annual optimal crop and water allocation problem (i.e., each sub-area of the total area in the studied region required decisions on which crop should be planted and how much water should be supplied to the selected crop), but did not consider irrigation water scheduling throughout the year (i.e., timing and magnitude of water allocation) for each crop in a sub-area. In addition, both studies used crop water production functions to calculate yield (instead of a physically-based and more computationally expensive crop growth simulation model) and did not consider the application of fertilizer, which can have a significant influence on achieving maximum net return.

As evidenced from the above discussion, many, if not most, existing simulation-optimization approaches for solving the irrigation management problem have either used simplified representations of crop growth processes (which has a number of disadvantages for irrigation scheduling problems) or mathematical optimization algorithms that are not especially amenable to linkage with process-based crop growth models. Furthermore, while metaheuristic algorithms can be linked to detailed crop growth models, there are often inherent issues with simulation run-times and size of search space (Loucks and Van Beek, 2005; Nguyen *et al.*, 2016b). Despite the potential advantages of ACO with respect to search space size reduction, to the authors' knowledge, ACO has not been combined with process-based crop simulation models to identify realistic irrigation and fertilization schedules that maximize net return for a given water allocation. This type of approach is needed to rigorously assess the large number of combinations associated with the different components of the irrigation scheduling problem. Consequently, the specific objectives of this paper are:

1. To develop an innovative metaheuristic simulation-optimization framework that links ant colony optimization (ACO) with a process-based crop growth model, enabling optimal or near-optimal irrigation water and fertilizer application schedules to be identified.

2. To demonstrate the proposed optimization framework for an irrigation management case study in eastern Colorado, USA.

The remainder of this paper is organized as follows. A brief introduction to ACO is given in Section 4.2. The generic simulation-optimization framework for irrigation and fertilizer scheduling is introduced in Section 4.3, followed by a case study description and methodology for evaluating the proposed framework with the case study in Section 4.4. The results and discussion are presented in Section 4.5 before a summary and conclusions are given in Section 4.6.

4.2 Ant Colony Optimization (ACO)

ACO is a metaheuristic optimization algorithm inspired by the foraging behavior of ants to identify the shortest path from their nest to a food source using pheromone trails (Dorigo *et al.*, 1996). In ACO, the decision space of the optimization problem is represented by a graph, the nodes and edges of which represent decision variables and decision variable options, respectively. A solution is constructed by an ant traversing the graph and selecting an edge at each node. As ants travel along a path, they deposit pheromone. The paths that are traversed more often have higher pheromone concentrations and are more likely to be selected by other ants in the future (Maier *et al.*, 2003). An example of the pheromone distribution for the Travelling Salesman Problem (TSP) with 7 cities is shown in Figure 4-1, where the thicker edges (6-7-1-2-3-4-5) of the right-hand graph are proportional to the higher pheromone level. During each iteration of the ACO process, all members of a colony traverse the graph, each generating a solution. After each iteration, paths that led to better overall solutions are rewarded with more pheromone, making them more likely to be selected in subsequent iterations. In this way, better solutions evolve as the number of iterations increases.

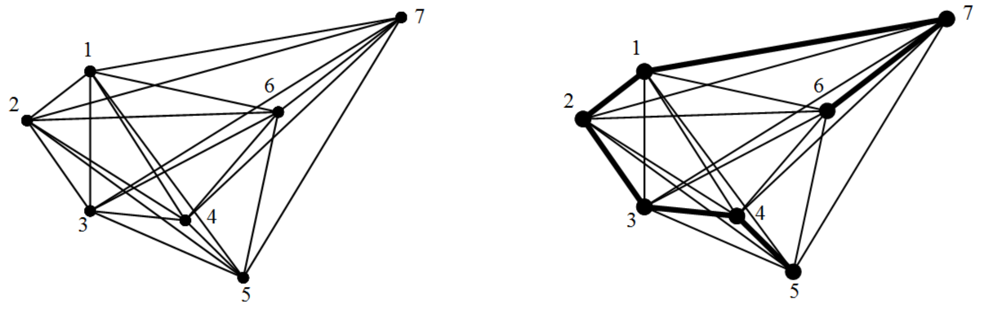


Figure 4-1. An example of pheromone distribution for of a 7-city TSP

At each decision point, the probability that an ant selects a particular edge (e.g., edge A,B) is given by the following equation (Dorigo *et al.*, 1996):

$$p_{AB} = \frac{[\tau_{AB}(t)]^{\alpha}[\eta_{AB}]^{\beta}}{\sum_{B=1}^{N_A}[\tau_{AB}(t)]^{\alpha}[\eta_{AB}]^{\beta}} \quad (4.1)$$

where t is the index of iteration, $\tau_{AB}(t)$ is the amount of pheromone on edge (A,B) at iteration t, η_{AB} is the visibility of edge (A,B), which provides a user-defined bias towards locally optimal solutions at the decision point under consideration, N_A is the set of all decision options at decision point A, α is the pheromone importance factor, and β is the visibility importance factor. The pheromone update on each edge (e.g. edge A,B) after each iteration is given by the following equation (Dorigo *et al.*, 1996):

$$\tau_{AB}(t+1) = \rho\tau_{AB}(t) + \Delta\tau_{AB}(t) \quad (4.2)$$

where $\Delta\tau_{AB}(t)$ is the pheromone addition for edge (A,B) during iteration t, which can be achieved using a range of approaches, such as the ant colony system, elitist ant system, elitist-rank ant system and min-max ant system (Zecchin et al., 2007). The ACO iterations continue until specific stopping criteria have been met, such as the completion of a specified number of iterations or until there is no further improvement in the objective function.

4.3 Proposed Simulation – Optimization Framework

4.3.1 Overview

An overview of the proposed ACO optimization – simulation framework for the irrigation and fertilizer scheduling problem is given in Figure 4-2. The first step in the framework is problem formulation, including specification of decision variables (e.g., irrigation method, magnitude and timing of irrigation water application, rate and timing of fertilizer application, etc.), stipulation of decision variable options (e.g., irrigation water and fertilizer application time step), identification of economic data (e.g., crop price, production cost, water cost, and fertilizer cost), specification of constraints (e.g., annual water allocation), and definition of the objective function (e.g., economic return).

Next, optimal irrigation water and fertilizer application schedules are identified using ACO. As introduced in Section 4.2, ACO algorithms work in an iterative fashion, in that they generally start with a number (population) of randomly selected solutions (i.e., irrigation water and fertilizer application schedules). As part of the proposed framework, the utility of these solutions is assessed with the aid of a crop growth simulation model, before the ants use this information to select a new (and generally better) population of solutions. This loop is repeated until the stopping criteria are met. In this way, ACO algorithms can be linked with the crop growth simulation model so that the solutions developed are run through the model and the corresponding model outputs are used to calculate the objectives and constraints. The objectives and constraints are then passed back to the ACO algorithm to assist with the selection of trial solutions in the next iteration. It should be noted that in order to assess the utility of a proposed irrigation or fertilizer application schedule with the aid of a crop growth simulation model, the model must be sufficiently detailed to enable all of the decision variables to be represented as model inputs.

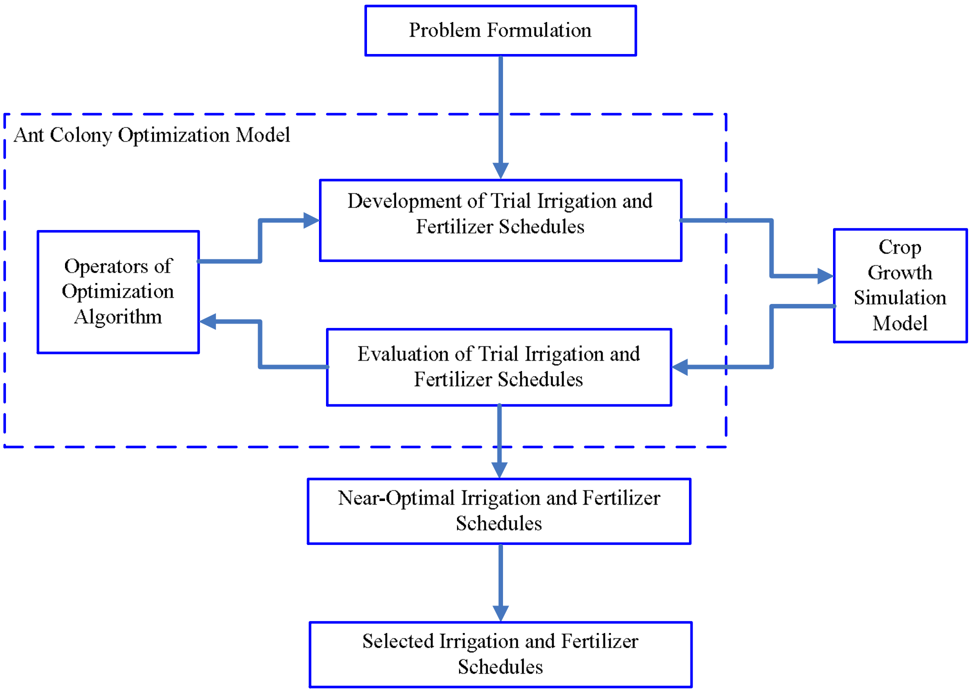


Figure 4-2. Overview of proposed optimization – simulation framework for irrigation and fertilizer scheduling

As ACO algorithms work with populations of solutions, it is generally possible to identify a number of near-optimal solutions. By using a global search technique in combination with a process-driven crop growth simulation model, the shortcomings of existing simulation-optimization approaches can be overcome, thus enabling the identification of realistic management strategies that maximize net return for a given water allocation. Details of the Problem Formulation, Optimization, and Simulation steps are given in the following sections.

4.3.2 Problem Formulation

The process of the problem formulation for irrigation and fertilizer scheduling includes the following main steps:

- *Specify decision variables and decision variable options*

In irrigation water and fertilizer application scheduling problems, the timing and magnitude of irrigation events and fertilizer applications are decision variables. The corresponding decision variable options are irrigation time steps (e.g., daily, 3-day, weekly, etc.), the depth of irrigation water (e.g., 0.5 cm, 1.0 cm, etc.), when to apply fertilizer (e.g., pre-planting, pre-emergence, etc.) and the fertilizer application rate (e.g., 50 kg/ha, 100 kg/ha, etc.). The water volume of the irrigation event will be selected depending on the capacity of the irrigation system.

- *Specify the objective function and constraints*

The proposed objective function for maximizing the net return has the following form:

$$F = \sum_{i=1}^{ny} XY_i P_i - \sum_{i=1}^{ny} X \begin{pmatrix} C_i + \sum_{j=1}^{n_{irr(i)}} W_{ij} C_{wi} \\ + \sum_{k=1}^{n\_fer(i)} FER_{ik} C_{feri} \end{pmatrix} \qquad (4.3)$$

where F is the total net return ($\$$ year^{-1}), ny is the number of years planted, n_irr(i) is the number of irrigation days in year i, n_fer(i) is the number of fertilizer applications in year i, X is the area of the crop planted (ha), Y_i is the crop yield in year i (kg ha^{-1}), P_i is the crop price in year i ($\$$ kg^{-1}), C_i is the fixed crop production cost in year i ($\$$ ha^{-1}), W_{ij} is the depth of water supplied on irrigated day j of year i (cm), C_{wi} is the unit cost of irrigated water in year i ($\$$ ML^{-1}), FER_{ik} is the rate of fertilizer applied in application k of year i (kg ha^{-1}), C_{feri} is the unit cost of fertilizer in year i ($\$$ kg^{-1}).

The objective function is optimized subject to the following constraints:

- Constraint for annual water availability:

The total volume of irrigation water applied in a year must not exceed the corresponding available supply (including both surface water and ground water) in that year.

$$\sum_{j=1}^{n\_irr(i)} W_{ij} X \leq W_{maxi} \qquad (4.4)$$

where W_{maxi} is the total volume of water available in year i.

- Constraint for capacity of irrigation system:

The volume of irrigation water supplied in a day must not exceed the capacity of the irrigation system.

$$W_{ij}X \leq Irr \tag{4.5}$$

where Irr is the maximum capacity (cm day^{-1}) of the irrigation system.

4.3.3 Optimization Model

4.3.3.1 Graph structure problem representation

As discussed in Section 4.3.2, an irrigation and fertilizer schedule can be established by identifying the timing and magnitude of irrigation events and fertilizer applications. Therefore, there are two separate decision-tree graphs for irrigation and fertilizer scheduling as shown in Figures 4-3 and 4-4.

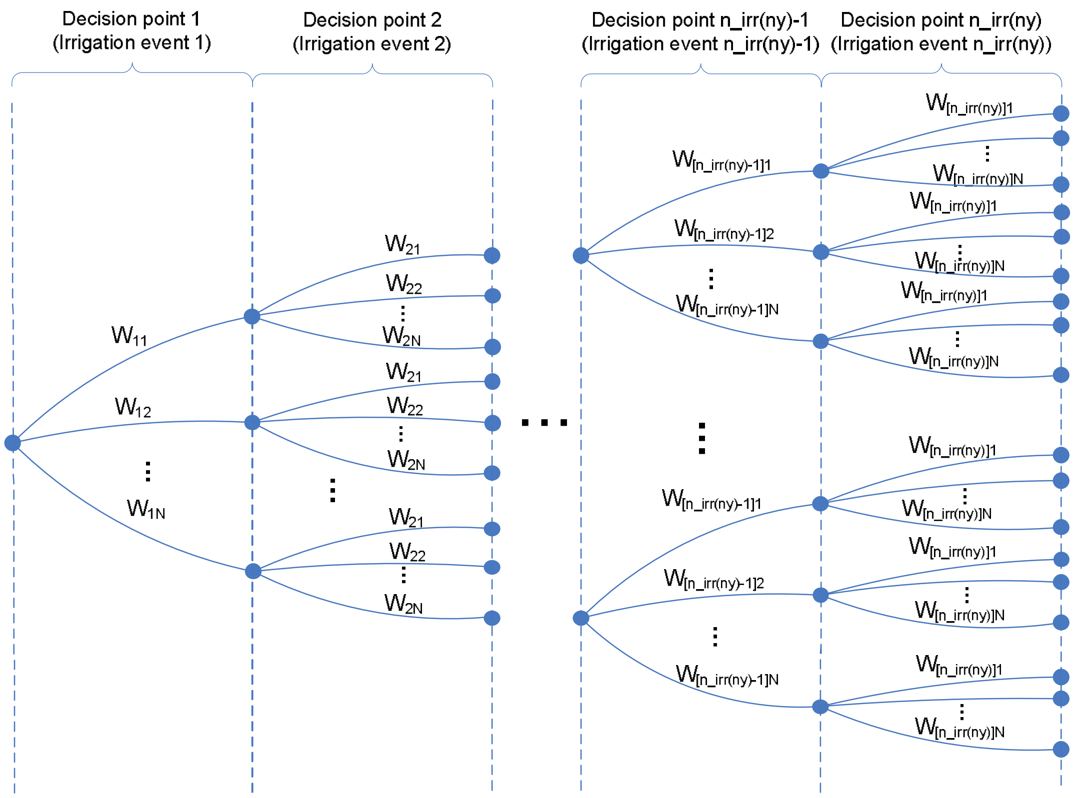


Figure 4-3. Decision tree graph for irrigation scheduling for fixed irrigation time steps

Note: W_{1N}, W_{2N}, etc = the depths of irrigation; other variables are defined in Equation 4.3.

For the irrigation water scheduling problem, the decision tree includes a set of decision points corresponding to the number of irrigation events (days) for the scheduling period (Figure 4-3). It should be noted that the irrigation time step is fixed and irrigation events only occur during the growth period of the crop (i.e., from the planting date to the harvesting date). As a planning horizon of more than one year is considered, the maximum period of irrigation is the sum of the crop growth periods. At the beginning of each decision point, a depth of irrigation water (i.e., W_{11}, W_{12}, …, W_{1N}, W_{21}, …, $W_{[n\_irr(ny)]N}$) is selected for each irrigation event. If the depth selected is zero, there is no irrigation event. A complete irrigation schedule is constructed once all decisions have been made from the start to the finish of the maximum irrigation period.

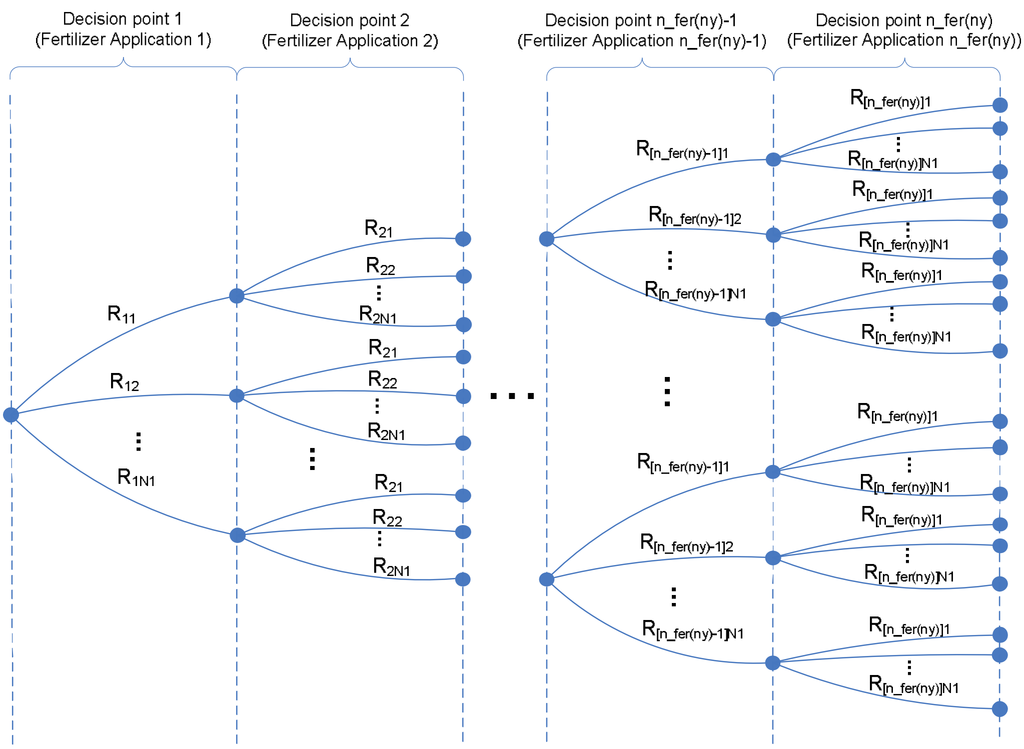


Figure 4-4. Decision tree graph for fertilizer scheduling

Note: R_{1N1}, R_{2N1}, etc = the rate of fertilizer application; other variables are defined in Equation 4.3.

In a similar way to irrigation water scheduling, the fertilizer application scheduling decision tree includes a set of decision points corresponding to the number of fertilizer applications for the studied period (Figure 4-4). As can be seen, a rate (i.e., R_{11}, R_{12}, …, R_{1N1}, R_{21}, …, $R_{[n\_fer(ny)]N1}$) for each fertilizer application is selected at the beginning of each decision point. In fact, the timings for these applications may be pre-plant or pre-emergence. A complete fertilizer schedule is constructed once a decision has been made sequentially at each decision point.

4.3.3.2 Ant colony optimization model

As mentioned in Section 4.3.1, the ACO model is used to identify optimal irrigation water and fertilizer application scheduling based on the decision trees in Figures 4-3 and 4-4. The ACO process is outlined in Section 4.2. In the framework, the constraint for the capacity of the irrigation system (Equation 4.5) can be included into the options for the irrigation depth. For example, if 2.5 cm is the daily capacity of the irrigation system, the options should be less than or equal to 2.5 cm. For the constraint of annual water availability, the method of dynamically adjusting decision variable options as introduced in the optimal crop and water allocation problem of Nguyen *et al.* (2016b) is applied to reduce search space size. In this method, all constraints are checked at each decision point to remove any infeasible options, thereby adjusting the decision variable options dynamically during the optimization process. The key steps for handling the constraint for annual water availability include:

1. Keep track of the total volume of irrigation water used after each irrigation event as the solution is constructed along the decision tree.

2. Calculate the remaining volume of irrigation water that is available for the irrigation event at this current decision.

3. Omit the choices of the depth of irrigated water for the irrigation event if the volumes of irrigation water corresponding to these choices exceed the available volume of water in Step 2.

4.3.4 Simulation Model

In the proposed framework for irrigation management, a crop model that is integrated with the ACO model is employed as a tool for estimating crop yield to evaluate the trial irrigation management strategies. As discussed in Section 4.1, a mechanistic, process-based crop model (e.g., CROPGRO, RZWQM2) was selected, as it can provide a realistic representation of soil moisture-climate interactions and the underlying physical processes of crop water requirements, crop growth, and agricultural management strategies (e.g., irrigation water and fertilizer application). When linked with the ACO model, a mechanistic crop growth model is able to calculate the value of the objective function (e.g., the net return) that does not contain the decision variables directly, as is the case here (Soundharajan and Sudheer, 2009).

4.4 Case Study

In order to test the utility of the proposed framework, the case study of irrigation water and fertilizer application scheduling of corn production introduced by Gleason (2013) is applied. The case study utilizes data from a field experiment initiated in 2010 near Greeley, Colorado (latitude N40.46°, longitude W104.58°, altitude 1429 m above mean sea level). The field was planted with corn in early May of each year of the study from 2010-2012. The soil at the site is an Olney fine sandy loam (fine-loamy, mixed, superactive, mesic Ustollic Haplargids) and is fairly uniform throughout the 200 cm soil profile. Soil bulk density in the corn field was determined using a Madera probe (Precision Machine Company Inc., Lincoln, NE) and methods described by Evett (2008). Two samples were taken from the field at the end of the 2011 growing season at depths of 0-15, 15-30, 30-45, 45-60, 60-90, and 90-105 cm and again at the beginning of the 2012 growing season at the same depths. The permanent wilting point (PWP) for each soil layer was determined using a WP4-T Dewpoint PotentiaMeter (Decagon Devices, Inc., Pullman WA). A soil water retention curve (SWRC) was created to obtain gravimetric water content at 1.5 MPa tension. Volumetric water content at PWP was then calculated using methods described by Evett (2008). Field capacity (FC) for each soil layer was determined *in situ* by soil core sampling 24 hours after several deep irrigation/precipitation events in 2011. Gravimetric water content at FC determined in 2011 was used to calculate volumetric water content at FC for all years. Total plant available water content (TAW) was determined using the equation TAW = FC - PWP, and TAW was calculated for each soil layer sampled. Additional soil core samples were taken from the corn field on a weekly basis throughout each growing season using a JMC Backsaver handle and a “dry” sampling tube with a core diameter of 1.905 centimeters (Clements Associates Inc., Newton, IA). The soil samples were taken at the same depth increments as described above, weighed to obtain fresh mass, and then oven dried at 105^{o}C until a constant mass was obtained. Gravimetric (g g^{-1}) and volumetric (cm^3 cm^{-3}) water content in the corn field was then determined using methods described by Evett (2008).

Weather data were recorded on site with a standard Colorado Agricultural Meteorological Network (http://ccc.atmos.colostate.edu/~coagmet) weather station (GLY04). Missing data at the beginning of the study were estimated with data from a nearby station 800 m to the east (GLY03). Average daily temperature during the growing season was 18.2^{o}C in 2010, 17.9^{o}C in 2011, and 17.3^{o}C in 2012. Corresponding growing season precipitation in each year was 24.5 cm, 23.7 cm, and 21.1 cm, respectively. The 26.3 ha field was divided into 9 m by 44 m small plots. Maize (‘Dekalb 52-59’) was planted at an average rate of 85,000 seeds per hectare with 0.76 m row spacing on May 12 in 2010 and May 11 in 2011 and 2012, and harvested on November 6 in 2010, November 12 in 2011, and October 29 in 2012. Fertilizer as urea-ammonium-nitrate (UAN) was applied at planting. The application rates were 134 kg N ha^{-1} in 2010, 160 kg N ha^{-1} in 2011, and 146 kg N ha^{-1} in 2012. The corn field was irrigated via a center pivot irrigation system. The irrigation system and irrigation water application was maintained and managed by a cooperative grower who participated in the study. Irrigation application efficiency of the system was estimated to be 90%, based on guidelines outlined by Martin et al. (2007). Irrigation was applied every 3-7 days, total irrigation amounts were 46.9 cm in 2010, 41.7 cm in 2011, and 39.5 cm in 2012. The amount of crop water used (actual ET) for the field was estimated on a daily basis based on reference ET demand, a crop coefficient, rainfall, and soil water deficit (FAO 56, Allen et al., 1998).

4.4.1 Problem Formulation

In this case study, the decision variables are the timing and magnitude of irrigation events and fertilizer applications. Fixed time steps between irrigation events of either 3-, 5- and 7-days were used. The annual irrigation period started on 15 May and ended on 30 September, i.e., the maximum number of decision points was 144 (48 per year). The maximum daily depth of irrigation was 2.54 cm and five options for irrigation depth were considered (0, 0.635, 1.27, 1.905 and 2.54 cm). The two possible fertilizer applications for each year were pre-planting (i.e., before 2 May) and a mid-summer application on 1 July. The fertilizer (NO_3-N and NH_4-N) rate options were 0, 50, 100, 150, and 200 kg N ha^{-1}. In this case study, the unit cost of water for center pivot irrigation was considered to be negligible. Other economic data for the problem are given in Table 4-1.

Table 4-1.
Economic data from 2010-2012 for agricultural production in Colorado, US

Items	2010	2011	2012
Crop price (\$ kg^{-1})[a]	0.237	0.271	0.299
Fixed crop production cost (\$ ha^{-1})[b]	1,171.6	1,299.1	1,483.0
Fertilizer cost (\$ kg^{-1})[c]	0.418	0.509	0.529

Source: [a] Agricultural prices, the National Agricultural Statistics Service (NASS), Agricultural Statistics Board, United States Department of Agriculture (USDA); [b] Estimated Production Costs and Returns, Colorado State University; [c] Fertilizer Prices, United States Department of Agriculture (USDA).

The objective function of this problem is given as:

$$F = \sum_{i=1}^{3} XY_i P_i - \sum_{i=1}^{3} X\left(C_i + \sum_{k=1}^{2} FER_{ik} C_{feri}\right) \qquad (4.6)$$

where the variables were defined in Section 4.3.2.

There were no constraints in this case study, as the constraint for the capacity of the irrigation system was included into the options for irrigation depth. Although annual water availability was not limited, different levels of available water were tested to evaluate their impacts on net return.

4.4.2 Optimization model

The decision space of the irrigation water scheduling problem considered is represented by the graph in Figure 4-3. The decision tree of fertilizer application included six decision points (two per year), each of which is the choice of fertilizer application rate. The Max-Min Ant System (MMAS) algorithm was used as the ACO optimization engine, as MMAS has been applied successfully in a number of water resources case studies (e.g., Zecchin *et al.* (2006); Zecchin *et al.* (2007); Afshar and Moeini (2008); Szemis *et al.* (2012); Zecchin *et al.* (2012); Nguyen *et al.* (2016b); Nguyen *et al.* (2016a)). As part of this algorithm, pheromone addition for edge (A,B) (Equation 4.2) is given by:

$$\Delta\tau_{AB}(t) = \Delta\tau_{AB}^{ib}(t) + \Delta\tau_{AB}^{gb}(t) \tag{4.7}$$

where $\Delta\tau_{AB}^{ib}(t)$ and $\Delta\tau_{AB}^{gb}(t)$ are the pheromone additions for the iteration-best solution (s^{ib}) and the global-best solution (s^{gb}), respectively. While s^{ib} is used to update the pheromone on edge (A, B) after each iteration, s^{gb} is applied with the frequency f_{global} (i.e., $\Delta\tau_{AB}^{gb}(t)$ is calculated after each f_{global} iteration). $\Delta\tau_{AB}^{ib}(t)$ and $\Delta\tau_{AB}^{gb}(t)$ are given by:

$$\Delta\tau_{AB}^{ib}(t) = \begin{cases} \frac{q}{f(s^{ib}(t))} & if\ (A,B) \in s^{ib}(t) \\ 0 & otherwise \end{cases} \tag{4.8}$$

$$\Delta\tau_{AB}^{gb}(t) = \begin{cases} \frac{q}{f(s^{gb}(t))} & if\ (A,B) \in s^{gb}(t)\ and\ \ t\ mod\ f_{global} = 0 \\ 0 & otherwise \end{cases} \tag{4.9}$$

where $f(s^{ib}(t))$ and $f(s^{gb}(t))$ are the objective function values of s^{ib} and s^{gb} at iteration t, respectively; and q is pheromone reward factor.

In MMAS, the pheromone on each edge is limited to lie within a given range in order to avoid search stagnation, i.e., $\tau_{min}(t) \le \tau_{AB}(t) \le \tau_{max}(t)$. The equations for $\tau_{min}(t)$ and $\tau_{max}(t)$ are given as follows:

$$\tau_{max}(t) = \left(\frac{1}{1-\rho}\right)\frac{1}{f(s^{gb}(t-1))} \tag{4.10}$$

$$\tau_{min}(t) = \frac{\tau_{max}(t)(1-\sqrt[n]{p_{best}})}{(avg-1)\sqrt[n]{p_{best}}} \tag{4.11}$$

where n is the number of decision points, avg is the average number of edges at each decision point, and p_{best} is the probability of constructing the global best solution at iteration t, where the edges chosen have pheromone trail values of τ_{max} and the pheromone values of other edges are τ_{min}. Additionally, MMAS also uses a pheromone trail smoothing (PTS) mechanism that reduces the difference between edges in terms of pheromone intensities, and thus, strengthens exploration.

$$\tau^*_{AB}(t) = \tau_{AB}(t) + \delta\big(\tau_{max}(t) - \tau_{AB}(t)\big) \tag{4.12}$$

where δ is the PTS coefficient ($0 \leq \delta \leq 1$).

4.4.3 Simulation Model

The Root Zone Water Quality Model (RZWQM2, version 3.0) with the Decision Support Systems for Agrotechnology Transfer (DSSAT, version 4.5) CERES-Maize crop module was used to evaluate the objective function. RZWQM2 was selected because it has been successfully applied in numerous studies for crop yield estimation, including Saseendran *et al.* (2010), Islam *et al.* (2012), Ma *et al.* (2012b), Qi *et al.* (2012), and Saseendran *et al.* (2014).

4.4.3.1 RZWQM2 description

RZWQM2 is a process-oriented agricultural systems model that integrates various physical, chemical and biological processes and simulates the impacts of soil-crop-nutrient management practices on soil water, crop production, and water quality under different climate conditions (Ahuja *et al.*, 2000). The crop simulation modules (CSM) in the DSSAT 4.5 package facilitate detailed growth and development simulations of 16 different crops (Jones *et al.*, 2003). The soil and water routines of RZWQM2 are linked with the CSM-DSSAT 4.5 crop modules in the current version of RZWQM2 (Ma *et al.*, 2009). RZWQM2 uses the Green-Ampt equation for infiltration and Richards' equation for redistribution of water in the soil profile (Ahuja *et al.*, 2000). Potential evapotranspiration is calculated using the extended Shuttleworth-Wallace equation, which is the Penman-Monteith equation modified to include partial crop canopy and the surface crop residue dynamics on aerodynamics and energy fluxes (Farahani and DeCoursey, 2000). The soil carbon/nitrogen dynamic module contains two surface residue pools, three soil humus pools and three soil microbial pools. N mineralization, nitrification, denitrification, ammonia volatilization, urea hydrolysis, and microbial population processes are simulated in detail (Shaffer *et al.*, 2000). Management practices simulated in the model include tillage, applications of irrigation, application of manure and fertilizer at different rates and times by different methods, planting and harvesting operation, and surface crop residue dynamics (Rojas and Ahuja, 2000). The DSSAT 4.5-CERES (Crop Environment Resource Synthesis) crop plant growth module for corn was used in this study. The DSSAT 4.5-CERES-Maize plant growth module in RZWQM2 simulates phenological stage, vegetative and reproductive growth, and crop yield and its components. This module calculates net biomass production using the radiation use efficiency (RUE) approach. Biomass production per day is a product of photosynthetic active radiation intercepted by the canopy and the RUE. Water stress effects on photosynthesis are simulated by CERES using empirically calculated stress factors, with respect to potential transpiration and crop water uptake (Ritchie and Otter-Nacke, 1985).

4.4.3.2 RZWQM2 parameterization

RZWQM2 was calibrated for yield, biomass, leaf area index (LAI), and soil water content for the 2010 experimental year. The model was first manually calibrated with a set of RZWQM2-DSSAT plant cultivar parameters (P1, P2, P5, G2, G3, and PHINT) from Ma *et al.* (2012b) and a laboratory derived soil water retention curve (SWRC). The manually calibrated set of plant parameters simulated plant biomass, yield, and LAI reasonably well for 2010; however, the soil water content was not simulated well with a Root Mean Square Deviation (RMSD) of 23% for total profile soil water and 27% for soil water content. Automated calibration was then performed with the laboratory-measured SWRC to optimize the above RZWQM2-DSSAT plant cultivar parameters to improve soil water content simulation. The autocalibration was performed using a single-objective calibration tool based on the Shuffled Complex Evolution (SCE) algorithm (Duan et al., 1992). The SCE algorithm has been widely used in multiple agroecosystem model calibration arenas (i.e., hydrology, crop growth, soil erosion, land surface modeling, etc.) and has generally been found to be robust, effective, and efficient (Duan, 2003). In this study, the sum of squares of residuals was selected as the objective function to be minimized. After autocalibration, simulated yield and biomass for 2010 were both within 10% of the observed values. In addition, simulated anthesis day was 88 days after planting (DAP) and simulated physiological maturity date was 146 DAP, which were within 5% of observed dates in the field. Simulated maximum LAI was 4.74 versus the observed 4.54 and simulated RMSD was 0.653 for LAI, 0.44 cm^3 cm^{-3} for soil water content, and 4.35 cm for soil profile water. RZWQM2 was used with the calibrated plant cultivar parameters to simulate crop yield for 2011 and 2012.

4.4.4 Computational experiments

The irrigation water schedule obtained from the Microsoft Excel spreadsheet-based Colorado Irrigation Scheduler (CIS) tool for annual crops (Gleason, 2013) was used as a benchmark against which to compare the results from the proposed ACO framework. The CIS was applied to calculate daily total soil water deficit, which can be compared to a management allowed depletion value for scheduling irrigation events for crops in Colorado. A potential advantage of the proposed ACO approach is that it utilizes knowledge of future rainfall events, whereas the CIS does not (i.e., actual rainfall data, corresponding to perfect knowledge of future rainfall, are used when the RZWQM2 model is run for a particular irrigation and fertilizer schedule developed using ACO). The irrigation schedules from the application of CIS for corn production during 2010-2012 were presented in Gleason (2013). The fertilizer (NO_3-N and NH_4-N) rates of two applications each year for pre-planting (i.e., before 2 May) and mid-summer on 1 July were assumed to be 100 kg N ha^{-1} in this paper.

Six computational experiments were conducted to test the utility of the proposed framework in terms of the impact of using different levels of water availability and different irrigation intervals (Table 4-2). As mentioned previously, three potential fixed irrigation intervals were considered, namely 3-, 5- and 7-days. As the maximum amount of water that can be delivered on a given day is restricted by the capacity of the available irrigation infrastructure (2.54 cm day^{-1}), the maximum total amount of water that can be applied is reduced as the irrigation interval is increased. This is shown in Table 4-2, where the maximum total amount of water that can be delivered when a 3-day irrigation interval is used is designated as 100%, which is reduced to 60% and 44% when the irrigation interval is increased to 5- and 7-days, respectively. In order to be able to separate the impact of irrigation interval and total water availability, 60% and 44% water availability levels were also applied with an irrigation interval of 3 days. In order to be able to compare the results between the proposed ACO framework and Gleason (2013) in terms of water use, a 3-day irrigation interval was also combined with a water availability level of 35% (about 42.7 cm $year^{-1}$), as this level was just below the smallest annual amount of irrigation water used in Gleason (2013) (i.e., 46.1 cm in 2010), whereas the other three irrigation levels considered (i.e. 100%, 60% and 44%) all correspond to irrigation depths that are greater than this.

The ACO algorithm described in Section 4.3.3.2 that uses the method of dynamically adjusting decision variable options (henceforth referred to as ACO-DDVO) was used in all experiments. Each experiment was implemented for 10 different numbers of objective function evaluations, ranging from 250 to 2,500, to enable algorithm convergence to be examined. A sensitivity analysis was carried out to select the most appropriate values of the ACO parameters (see Table 4-3). Due to the probabilistic nature of the searching behavior of ACO, each experiment was repeated 30 times from different starting points in the solution space using the selected values of the ACO parameters given in Table 4-3. All computational experiments were implemented on e-Research South Australia's Tizard supercomputer, which includes 48 SGI computer nodes connected by a high-speed QDR Infiniband network, 48 cores (4 AMD 6238 12-core 2.6Ghz CPUs) and 128GB memory (2.7GB per core) for each node, as well as a total of 2304 cores with a peak performance of 24 TFLOPS.

Table 4-2.
Details of computational experiments.

Experiment	No. of decision points for irrigation	Choices for irrigation days	Choices for irrigation water	Choices for fertilizer	Water Availability	Size of total search space
1	144	3-day	0, 0.635, 1.27, 1.905 and 2.54 cm	0, 50, 100, 150, and 200 kg N ha^{-1}	100%	7.0 x 10^{104}
2	144	3-day	0, 0.635, 1.27, 1.905 and 2.54 cm	0, 50, 100, 150, and 200 kg N ha^{-1}	60%	7.0 x 10^{104}
3	144	3-day	0, 0.635, 1.27, 1.905 and 2.54 cm	0, 50, 100, 150, and 200 kg N ha^{-1}	44%	7.0 x 10^{104}
4	144	3-day	0, 0.635, 1.27, 1.905 and 2.54 cm	0, 50, 100, 150, and 200 kg N ha^{-1}	35%	7.0 x 10^{104}
5	87	5-day	0, 0.635, 1.27, 1.905 and 2.54 cm	0, 50, 100, 150, and 200 kg N ha^{-1}	60%	1.0 x 10^{65}
6	63	7-day	0, 0.635, 1.27, 1.905 and 2.54 cm	0, 50, 100, 150, and 200 kg N ha^{-1}	44%	1.7 x 10^{48}

Table 4-3.
The ACO parameter values considered as part of the sensitivity analysis and their values selected.

Parameter	Values for sensitivity analysis	Values selected
Number of ants	10; 20; 50; 100	50
Pheromone importance factor (α)	0.1, 0.5, 1.0, 1.2, 1.5	1.0
Initial pheromone (τ_o)	0.5, 1.0, 2.0, 5.0, 10.0, 20.0	10.0
Pheromone persistence (ρ)	0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9	0.6
Pheromone reward (q)	0.5, 1.0, 2.0, 5.0, 10.0, 20.0, 50.0	20.0

4.5 Results and discussion

4.5.1 Impact of using different water availability and irrigation intervals

The best total net return obtained over the 3-year period investigated (2010-2012) from the 30 ACO runs for each experiment, as well as the corresponding total irrigation water used, the total fertilizer used and the number of function evaluations required to achieve this result, are shown in Table 4-4. The values of net return, irrigation water used and fertilizer used obtained by Gleason (2013) are also shown, as these values were used as a benchmark against which to compare the ACO results. Table 4-4 shows that the best net return was found in Experiments 1-3, even though the total amount of water that was available in these experiments was significantly different (i.e. 100%, 60% and 44%). This suggests that the total amount of water that can be applied is much greater than that needed to produce optimal net returns, highlighting the potential benefits of using a formal optimization approach (such as the one presented in this paper) to not only maximize net return, but to reduce the amount of water that is used to achieve this. Table 4-4 also shows that once the total amount of available water was reduced to 35% (Experiment 4), there was a slight reduction in net return from \$7760.7 to \$7611.9 (1.9% reduction) with the full usage of the available water (i.e., 42.7 cm per year, as shown in Figure 4-8), suggesting that the minimum amount of total irrigation water required to maximize net return is likely to be between 44 and 35%.

Table 4-4.
Comparison between best-found solutions for 2010-2012 (over the 30 trials from different starting positions) of the various ACO experiments and those obtained by Gleason (2013), which are used as a benchmark

Experiment	Net Return ($)	Irrigation water (cm)	Fertilizer (kg)	No. of Evaluations
1	**7,760.7 (1.80%)**	169.6 (-13.44%)	**1,000 (16.67%)**	980
2	**7,760.7 (1.80%)**	169.6 (-13.44%)	**1,000 (16.67%)**	980
3	**7,760.7 (1.80%)**	150.5 (-0.67%)	**1,000 (16.67%)**	2,201
4	7,611.9 (-0.15%)	**127.5 (14.72%)**	**1,000 (16.67%)**	2,258
5	**7,760.7 (1.80%)**	158.8 (-6.22%)	**1,000 (16.67%)**	987
6	6,309.8 (-17.23%)	130.2 (12.91%)	**1,000 (16.67%)**	2,474
Gleason (2013)	7,623.4	149.5	1,200	N/A

Note: The numbers in bold are the best overall solution. The numbers in parentheses are the percentage deviations of the best-found solutions from the experiments using ACO relative to the solutions from Gleason (2013). Positive percentages imply that the experiments performed better than Gleason (2013) and vice versa.

Comparison of the results from Experiments 2 and 5 indicate that increasing the irrigation interval from 3 to 5 days did not have an impact on net return or fertilizer usage when the same amount of total irrigation water was available, although the number of evaluations required to identify the optimal solution when an irrigation interval of 5 days was used was greater (Table 4-4) despite the reduced size of the search space (Table 4-2). This suggests that it is slightly more difficult to find a solution that results in the optimal net return when a greater irrigation interval is used, which is not unexpected. When the irrigation interval was increased to 7 days (Experiment 6), the optimum net return could no longer be obtained, even though the reduced total amount of water available was not a restricting factor, as shown by the results of Experiment 3. This indicates that the available irrigation water needs to be applied at intervals of less than 7 days for the case study considered in order maximize net returns.

The convergence behavior of the ACO runs for the different experiments, as well as the differences in the best and average results from the 30 runs with different random number seeds, are shown in Figures 4-5 and 4-6. As can be seen for Experiments 1-5, the ACO algorithms converged very quickly (Figure 4-5). In addition, the variation in results between the 30 runs from different starting positions was quite small. This highlights the potential of using the proposed ACO formulation in conjunction with computationally expensive process-based crop growth models, as optimal solutions can be identified within a relatively small number of function evaluations and the impact of the starting position does not play a large role, which enables the number of required replicates to be reduced. This is an important consideration, as a single run of 2,500 evaluations took approximately 60 hours of computer time.

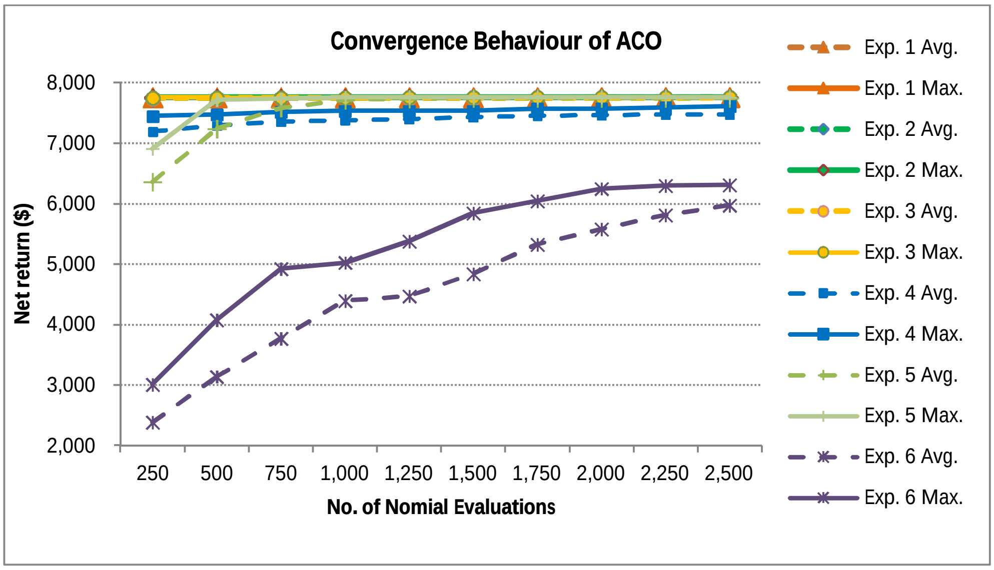


Figure 4-5. Maximum and average solutions for all computational experiments

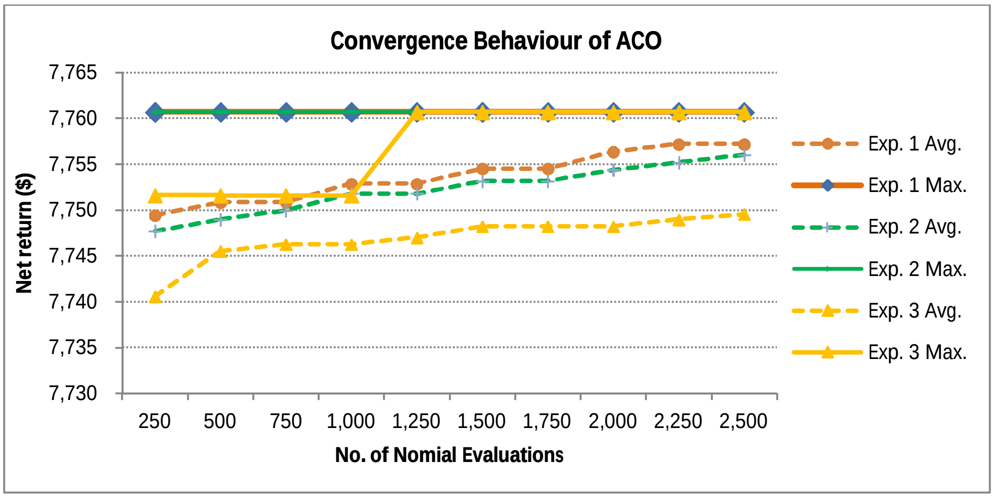


Figure 4-6. Maximum and average solutions for Experiments 1, 2 and 3

In general, convergence rate decreased and the spread in solutions increased with the degree of problem difficulty, such as reducing the amount of water available (Experiments 1-4, Figures 4-5 and 4-6) and increasing the irrigation interval (Experiments 5 and 6, Figure 4-5). These effects were particularly noticeable for Experiment 6, where the best net return that could be identified was 18.7% less than the best found value. This was due to the inability of the 7-day minimum irrigation interval to provide sufficient water at the times it is needed for optimal crop growth, as mentioned above. Consequently, computational efficiency issues could be a potential problem as the degree of difficulty of the problem considered increases.

4.5.2 Comparison with benchmark results of Gleason (2013)

As shown in Table 4-4, the maximum net return identified with the aid of the proposed ACO approach was 1.8% higher than that identified by Gleason (2013). This improved net return was achieved with a 16.67% reduction in fertilizer use and a small 0.67% increase in irrigation water use (Experiment 3). However, as the amount of available irrigation water was set as a constraint during the ACO runs, it is likely that the maximum net return could be identified with the aid of ACO with the same water usage as Gleason (2013), given that a slightly reduced net return was identified in Experiment 4 (0.15% less than that identified by Gleason (2013), but with a

14.72% reduction in the amount of water used and 16.67% reduction in the amount of fertilizer used). These results highlight the efficacy of the proposed approach, although it should be noted that the results obtained using the ACO algorithm made use of perfect knowledge of actual rainfall, which was not the case in the approach used by Gleason (2013).

The distributions of net return, water applied and fertilizer applied over the three years considered for Experiments 3 and 4 compared with those of Gleason (2013) are shown in Figures 4-7 and 4-8, respectively. As can be seen, the net returns obtained using ACO were higher in 2012, whereas the opposite applies in 2010. In Experiment 3, in which the ACO approach resulted in an overall greater net return, the net return obtained using ACO was greater in 2011. In contrast, in Experiment 4, in which the ACO approach resulted in a slightly lower overall net return, the net return obtained by Gleason (2013) was slightly higher in 2011. For both Experiments 3 and 4, ACO water usage was relatively constant across the three years considered, whereas the amount of irrigation water applied by Gleason (2013) varied noticeably from year to year. Unlike water usage, fertilizer usage across years was constant for Gleason (2013), as this was one of the modelling assumptions, as discussed previously. However, the reverse was true when the ACO approach was employed. For Experiment 3, the greatest amount of fertilizer was applied in 2010 (significantly greater than that applied by Gleason, 2013), followed by much smaller amounts (significantly smaller than those applied by Gleason, 2013) in 2011 and 2012. In contrast, for Experiment 4, the greatest amount of fertilizer was applied in 2011 (noticeably greater than that applied by Gleason, 2013), with much smaller amounts (significantly smaller than those applied by Gleason, 2013) applied in 2010, and particularly in 2012. This highlights the potential benefits of using the formal optimization approach presented in this paper in being able to identify optimal irrigation and fertilizer schedules that are customized to the specific circumstances under consideration, leading to either increased net returns with similar water usage and reduced fertilizer usage (as in Experiment 3), or similar net returns with reduced water and fertilizer usage (as in Experiment 4).

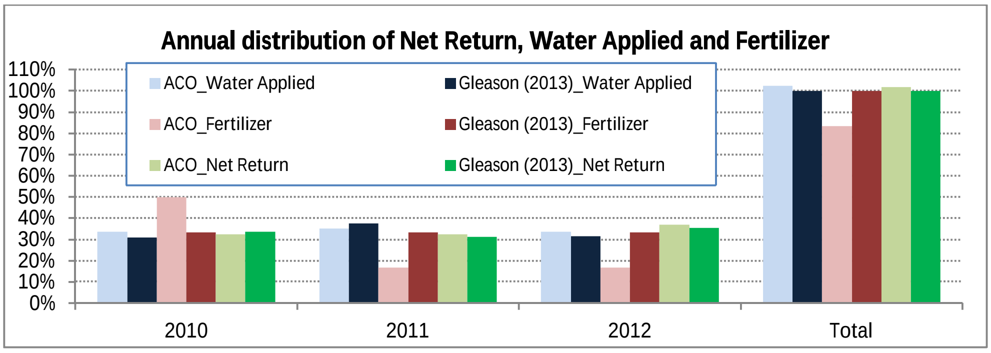


Figure 4-7. Comparison of water applied, fertilizer, and net return between Experiment 3 using ACO and Gleason (2013). The water applied, fertilizer and total net return used in Gleason (2013) are used as a benchmark and assigned values of 100%.

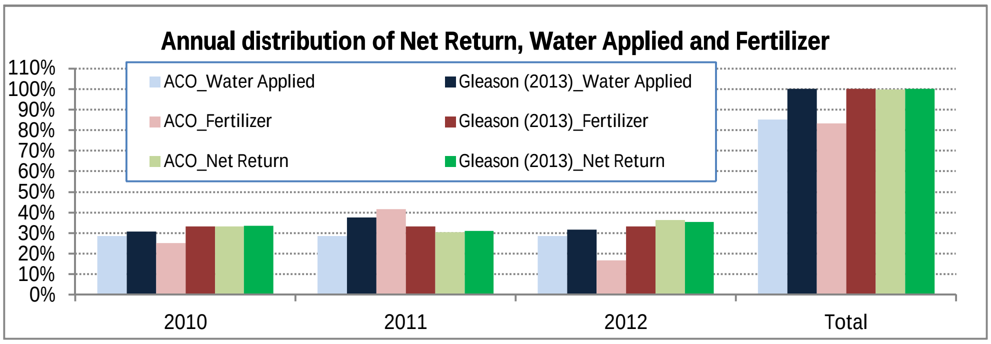


Figure 4-8. Comparison of water applied, fertilizer, and net return between Experiment 4 using ACO and Gleason (2013). The water applied, fertilizer and total net return used in Gleason (2013) are used as a benchmark and assigned values of 100%.

The detailed irrigation schedules for Experiment 3 and those obtained by Gleason (2013) are given in Figure 4-9. As can be seen, even though the total amount of irrigation water applied by Gleason (2013) from one year to the next was more variable than that applied using ACO (Figure 4-7), the pattern (amount and timing) of application was more regular and the vast majority of irrigation events were confined to July and August. In contrast, even though the total amount of water applied by using the ACO approach was reasonably constant from year to year, the pattern of application was rather irregular, with significant variation in the amount of water applied and in the application timing. In addition, the application of irrigation water was less confined to the July/August window. This again highlights the potential advantages of using a formal optimization approach to customize irrigation schedules, although some of this customization is likely to be in response to rainfall patterns, which were assumed to be known as part of the ACO approach, but not by Gleason (2013), as discussed previously.

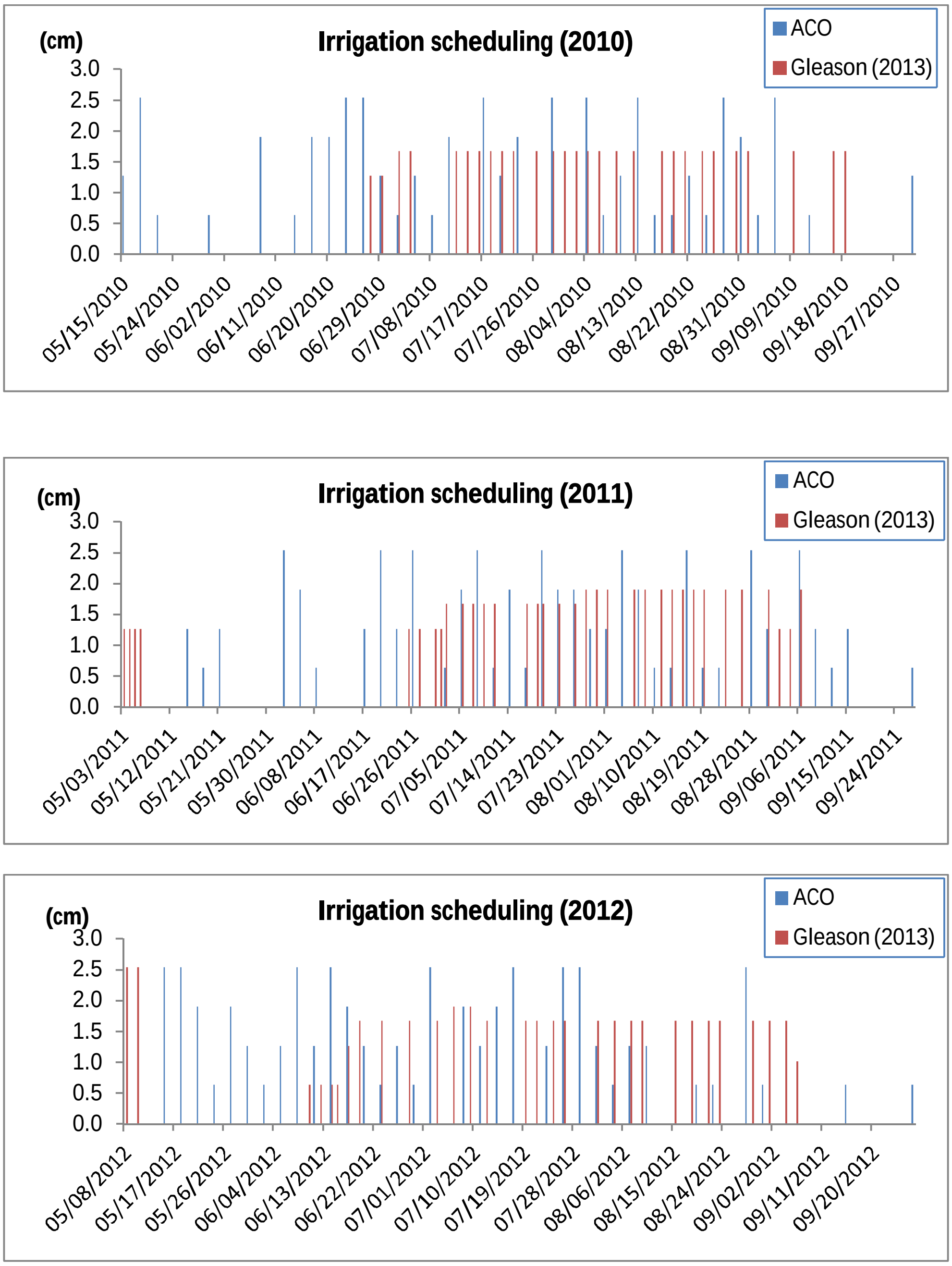


Figure 4-9. Irrigation scheduling for the best-found solution from ACO Experiment 3 and those obtained from Gleason (2013).

4.6 Summary and conclusions

A generic simulation-optimization framework for optimal irrigation and fertilizer scheduling was developed and tested, where the problem is represented in the form of a decision-tree graph, ant colony optimization (ACO) is used as the optimization engine, and a mechanistic, process-based crop growth model is used to estimate crop yield. By dynamically adjusting the number of decision variable options (DDVO) during the solution generation process, the framework is able to reduce the size of the search space during the process of trial solution construction, thereby increasing computational efficiency.

In order to test the utility of the proposed framework, it was applied to a case study introduced by Gleason (2013) to determine optimal irrigation and fertilizer schedules for corn production in eastern Colorado, USA. The Root Zone Water Quality Model (RZWQM2) was used to simulate crop growth at a daily time step. Six experiments were conducted investigating the impact of different fixed irrigation time steps (3-, 5- and 7- days) and different levels of water availability (100%, 60%, 44% and 35%). Each experiment was repeated 30 times from different starting positions in the solution space. The results of the experiments were compared with those obtained by Gleason (2013).

The results from the case study highlight the efficacy of the proposed ACO-DDVO approach, as it was able to identify irrigation water and fertilizer application schedules that resulted in almost an identical net return (0.15% less) to those obtained by Gleason (2013) while using significantly less water (14.72%) and fertilizer (16.67%). In addition, irrigation and fertilizer schedules were identified that resulted in an increased net return (1.8%) compared with those developed by Gleason (2013) while using significantly less fertilizer (16.67%) and only slightly more water (0.67%). However, the schedules developed using the proposed ACO-DDVO approach made use of perfect knowledge of rainfall, the impact of which should be investigated as part of future studies (see e.g., Szemis *et al.*, 2014 and Delgoda *et al.*, 2016). The optimal ACO results were obtained using a relatively small number of function evaluations (< 2,500, corresponding to approximately 60 hours of computing time for each replicate) and were reasonably robust to different starting positions in solution space, highlighting the potential of the proposed approach for linking global optimization methods with advanced crop growth models in order to identify optimal or near-optimal irrigation and fertilizer schedules within feasible computational budgets.

CHAPTER 5

Conclusion

Irrigation management, which consists of crop and water allocation and irrigation water and fertilizer application scheduling, plays an important role for food production, especially in the context of the increasing constraints that are placed on the amount of irrigation water available. Consequently, the identification of optimal crop and water allocation plans and optimal irrigation water and fertilizer application schedules designed to maximise economic benefit is vitally important. While the use of metaheuristic optimisation algorithms is well suited to this task, as these algorithms can be linked with existing crop-growth models, their application also presents a number of challenges. One such challenge is being able to identify globally optimal or near-optimal solutions for large search spaces within a reasonable computational effort. This problem is exacerbated when process-based models are used for objective function evaluation, due to their relatively high computational expense. In order to address this issue, two simulation-optimization frameworks using ant colony optimisation were introduced in this thesis, which are able to: 1) improve computational efficiency by dynamically adjusting decision variable options during the optimisation run, and 2) guiding the search to more promising regions of the search space through incorporation of domain knowledge via visibility factors (VFs). The first framework is suited to the allocation of crop and water at the district or regional scale, while the second framework is designed to cater to irrigation water and fertilizer application scheduling at the farm scale.

5.1 Research Contribution

The overall contribution of this research is the development and testing of two simulation-optimization frameworks for optimal irrigation management using ant colony optimization (ACO) algorithms. In the first framework, optimal crop and water allocation schedules at the regional or district scale are identified to maximize net return, subject to limits on the available land and water resources. The benefits of this framework are demonstrated using a benchmark case study from the literature and a real-world case study based on an irrigation district located at Loxton, South Australia, near the River Murray. The second framework produces optimal irrigation water and fertilizer application schedules for obtaining the maximum net return at the farm scale and is applied to a real case study in eastern Colorado, USA.

The specific research contributions to address the objectives stated in the Introduction are as follows:

1. A generic simulation-optimization framework for optimal crop and water allocation at the regional or district scale using decision-tree graphs and ACO is developed in Paper 1. This framework is able to dynamically reduce the size of the search space, as only feasible solutions are obtained by using dynamic decision variable option (DDVO) adjustment during the process of solution construction. A benchmark crop and water allocation problem from the literature with crop production functions was used to test the utility of the framework. The results showed that the proposed framework is able to find better solutions for nonlinear problems and for more highly constrained search spaces (i.e., different levels of water availability) than linear programming. Due to the ability to reduce the search space size and exclude infeasible solutions during the solution generation process, the proposed ACO-DDVO approach also outperformed a "standard" ACO approach using static decision variable options (SDVO) and penalty functions for dealing with infeasible solutions in terms of the ability to find feasible solutions, solution quality, computational efficiency and convergence speed. The results also demonstrated the potential of the proposed framework for application to real-world problems using complex crop models with long simulation times.

2. The framework for optimal crop and water allocation is improved by incorporating domain knowledge via the use of VFs to bias the selection of crops (i.e., crops likely to result in greater net return are assigned a greater chance of being selected for each of the sub-areas) and water allocations (i.e., water allocations likely to result in greater net returns are given a greater chance of being chosen for the selected crop in each of the sub-areas) at each node in the decision-tree graph (Paper 2). This improvement enables locally optimal solutions related to the factors (i.e., crops and water) affecting net return to be identified, and enables better regions of the search space to be explored. The overall effectiveness of the framework was validated using the benchmark case study and a real-world case study based on an irrigation district located in Loxton, South Australia, next to the River Murray. The results obtained for both case studies demonstrated that the use of VFs increases the ability to identify better solutions, especially at smaller numbers of function evaluations, and thus reduces the computational time to identify near-optimal solutions.

3. A generic simulation-optimization framework for optimal irrigation water and fertilizer application scheduling at the farm scale is developed, where the problem is represented in the form of a decision-tree graph and ACO is used as the optimization engine (Paper 3). In this framework, the ACO model is linked with a process-based crop growth model, which can provide a realistic representation of soil moisture-climate interactions and the underlying physical processes of crop water requirements, crop growth, and agricultural management strategies (e.g., irrigation water and fertilizer application) in order to evaluate trial irrigation management strategies. Furthermore, this framework allows for dynamically adjusting decision variable options during the solution generation process in order to eliminate infeasible solutions, thereby dynamically reducing the search space size of the problem. The utility of the framework was demonstrated using a case study with

the Root Zone Water Quality Model (RZWQM2) for corn production under center pivot irrigation in eastern Colorado, USA. The results from the case study indicate that the proposed framework is able to identify irrigation water and fertilizer application schedules that result in better net returns, while using less fertilizer and similar amounts of water, or similar net returns, while using less water and fertilizer, than those obtained using the Microsoft Excel spreadsheet-based Colorado Irrigation Scheduler (CIS) tool for annual crops. Furthermore, the framework also takes advantage of identifying both optimal irrigation water and fertilizer application schedules, which is not the case for CIS.

5.2 Limitations

The limitations of this research are discussed below.

1. In Papers 1 and 2, a discretization scheme was suggested to reduce the search space size of the benchmark problem. However, this scheme is limited in terms of obtaining all the values of decision variables and biasing the selection of the decision variables during the solution generation process, as intermediate values have higher possibilities of being selected than extreme values. Although this limitation is handled in the case study, it needs to be considered for each specific case.

2. In Papers 1 and 2, the search space of the benchmark problem is not overly large, and the simulation model (i.e., crop production functions) used is simplistic and has a low computational overhead. In order to demonstrate the generality of the method, there is a need to apply the proposed framework to more complex problems with larger search spaces and more computationally expensive, mechanistic crop growth simulation models.

3. The two proposed frameworks only consider a single, economic objective (i.e., maximize net return). However, there would be value in expanding the frameworks to consider multiple objectives, such as

social objectives (e.g., to minimize the total water shortage of the whole system) and environmental objectives (e.g., to minimize the excess leaching of fertilizer into groundwater).

4. The generic simulation-optimisation framework for optimal crop and water allocation at the regional scale developed in Chapter 2 and applied in Papers 1 and 2 is aimed at achieving the maximum total net benefits of crop production of the system as a whole. In many irrigation systems, individual irrigators make decisions regarding which crops to plant and how much water to use. This will limit the ability to attain the optimum for the system as a whole.

5. The utility of the proposed frameworks has been demonstrated via the three case studies (from simple to complex), as their application enabled optimal solutions to be identified within a given computational budget. However, application of the frameworks has not necessarily identified solutions to support real-world decision makers, particularly in places where future hydrometeorological conditions are unknown.

6. Although economic factors (e.g., input and output prices) have been included in the proposed frameworks, there is no consideration of the sensitivity of the optimal irrigation schedules obtained to different pricing assumptions.

5.3 Future Work

From the above limitations, some future studies are recommended below.

1. As the performance of ACO algorithms for a particular problem depends on the trade-off between the exploitation and exploration mechanisms, which are represented by the pheromone and visibility operators, the different algorithms are expected to generate a variety of different final levels of performance (Zecchin *et al.*, 2012). Therefore, research into the searching behaviour of ant colony optimization algorithms for irrigation management should be

considered. In addition, comparisons of the performance of ACO and other metaheuristic algorithms should also be carried out.

2. Other objectives (e.g., social, environmental or ecological objectives) need to be considered to evaluate the impact of agricultural production on related problems, such as shortage of irrigation water or pollution of groundwater. This is an opportunity to develop new frameworks to address these problems and methods to quantify them.

3. The trade-off between the various objectives of irrigation management problems or between irrigation water and other water uses (e.g., ecological flow) is very important. This presents opportunities to extend the proposed framework to multi-objective frameworks for irrigation management.

4. Research is needed into the effects of decisions made by individual irrigators on the performance of the overall system. Agent based modelling could be used to study this issue in conjunction with the frameworks developed in this research.

5. In the proposed framework for optimal irrigation water and fertilizer application scheduling, the irrigation water and fertilizer application schedules are developed using perfect knowledge of rainfall. Therefore, future studies on the impact of perfect knowledge of rainfall should be investigated.

6. As economic sensitivities are important for real-world irrigation scheduling, there is a need to take into account this factor in further studies. Furthermore, risk management should be also addressed to evaluate the impact of price sensitivities.

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Appendix A

Copy of Paper from Chapter 2

Nguyen, D.C.H., Maier, H.R., Dandy, G.C. & Ascough II, J.C., 2016. Framework for computationally efficient optimal crop and water allocation using ant colony optimization. Environmental Modelling & Software, 76, 37-53.

Environmental Modelling & Software 76 (2016) 37–53

Contents lists available at ScienceDirect

Environmental Modelling & Software

journal homepage: www.elsevier.com/locate/envsoft

Framework for computationally efficient optimal crop and water allocation using ant colony optimization

Duc Cong Hiep Nguyen [a,*], Holger R. Maier [a], Graeme C. Dandy [a], James C. Ascough II [b]

[a] School of Civil, Environmental and Mining Engineering, University of Adelaide, Adelaide, South Australia, Australia
[b] USDA-ARS-PA, Agricultural Systems Research Unit, Fort Collins, CO, USA

A R T I C L E I N F O

Article history:
Received 21 April 2015
Received in revised form
31 August 2015
Accepted 9 November 2015
Available online 9 December 2015

Keywords:
Optimization
Irrigation
Water allocation
Cropping patterns
Ant colony optimization
Search space

A B S T R A C T

A general optimization framework is introduced with the overall goal of reducing search space size and increasing the computational efficiency of evolutionary algorithm application to optimal crop and water allocation. The framework achieves this goal by representing the problem in the form of a decision tree, including dynamic decision variable option (DDVO) adjustment during the optimization process and using ant colony optimization (ACO) as the optimization engine. A case study from literature is considered to evaluate the utility of the framework. The results indicate that the proposed ACO-DDVO approach is able to find better solutions than those previously identified using linear programming. Furthermore, ACO-DDVO consistently outperforms an ACO algorithm using static decision variable options and penalty functions in terms of solution quality and computational efficiency. The considerable reduction in computational effort achieved by ACO-DDVO should be a major advantage in the optimization of real-world problems using complex crop simulation models.

Software availability

Name of Software: ACO-SDVO, ACO-DDVO
Description: ACO-DDVO and ACO-SDVO are two applications of ant colony optimization (ACO) for optimal crop and water allocation. While ACO-DDVO can reduce the search space size by dynamically adjusting decision variable options during the optimization process, ACO-SDVO uses static decision variable options and penalty functions.
Developers: Duc Cong Hiep Nguyen, Holger R. Maier, Graeme C. Dandy, James C. Ascough II
Available Since: 2015
Hardware Required: PC or MAC
Programming Language: FORTRAN
Program size: 31.5 MB
Contact Address: School of Civil, Environmental and Mining Engineering, University of Adelaide, Adelaide, South Australia.
Contact E-mail: duc.nguyen01@student.adelaide.edu.au
Source Code: https://github.com/hiepnguyendc/ACO-SDVO_and_ACO-DDVO
Cost: Free for non-commercial use.

1. Introduction

Evolutionary algorithms (EAs) have been used extensively and have contributed significantly to the optimization of water resources problems in recent decades (Nicklow et al., 2010; Maier et al., 2014). However, the application of EAs to real-world problems presents a number of challenges (Maier et al., 2014). One of these is the generally large size of the search space, which may limit the ability to find globally optimal or near-globally optimal solutions in an acceptable time period (Maier et al., 2014). In order to address this problem, different methods to reduce the size of the search space have been proposed in various application areas to either enable near-globally optimal solutions to be found within a reasonable timeframe or to enable the best possible solution to be found for a given computational budget. Application areas in which search space reduction techniques have been applied in the field of water resources include the optimal design of water distribution systems (WDSs) (Gupta et al., 1999; Wu and Simpson, 2001; Kadu et al., 2008; Zheng et al., 2011, 2014), the optimal design of stormwater networks (Afshar, 2006, 2007), the optimal design of sewer networks (Afshar, 2012), the calibration of hydrologic models (Ndiritu and Daniell, 2001), the optimization of maintenance

* Corresponding author.
E-mail address: hiepndc@yahoo.com (D.C.H. Nguyen).

http://dx.doi.org/10.1016/j.envsoft.2015.11.003
1364-8152/

scheduling for hydropower stations (Foong et al., 2008a, 2008b) and the optimal scheduling of environmental flow releases in rivers (Szemis et al., 2012, 2014). Some of the methods used for achieving reduction in search space size include:

1. *Use of domain knowledge.* Domain knowledge of the problem under consideration has been widely applied for search space size reduction in specific application areas. For example, in the design of water distribution systems, the known physical relationships between pipe diameters, pipe length, pipe flows, and pressure head at nodes has been considered to reduce the number of diameter options available for specific pipes, thereby reducing the size of the search space significantly (Gupta et al., 1999; Kadu et al., 2008; Zheng et al., 2011; Creaco and Franchini, 2012; Zheng et al., 2014, 2015). This enables the search process to concentrate on promising regions of the feasible search space. Other examples of this approach include the design of watershed-based stormwater management plans (Chichakly et al., 2013), optimal locations and settings of valves in water distribution networks (Creaco and Pezzinga, 2015), optimization of multi-reservoir systems (Li et al., 2015), and model calibration (Dumedah, 2015).
2. *Level of discretization*. When using discrete EAs, the level of discretization of the search space, which refers to the resolution with which continuous variables are converted into discrete ones, has also been used in order to reduce the size of the search space. As part of this approach, a coarse discretization of the search space is used during the initial stages of the search, followed by use of a finer discretization in promising regions of the search space at later stages of the search. Approaches based on this principle have been used for model calibration (Ndiritu and Daniell, 2001), the design of WDSs (Wu and Simpson, 2001), and the design of sewer networks (Afshar, 2012).
3. *Dynamic decision trees*. When ant colony optimization algorithms (ACOAs) are used as the optimization engine, solutions are generated by moving along a decision tree in a stepwise fashion. These decision trees can be adjusted during the solution generation process by reducing the choices that are available at a particular point in the decision tree as a function of choices made at preceding decision points (with the aid of domain knowledge of the problem under consideration). This approach has been applied successfully to scheduling problems in power plant maintenance (Foong et al., 2008a, 2008b), environmental flow management (Szemis et al., 2012, 2014), the design of stormwater systems (Afshar, 2007) and the optimal operation of single- or multi-reservoir systems (Afshar and Moeini, 2008; Moeini and Afshar, 2011, 2013).

One application area where search space reduction should be beneficial is optimal crop and water allocation. Here the objective is to allocate land and water resources for irrigation management to achieve maximum economic return, subject to constraints on area and water allocations (Singh, 2012, 2014). One reason for this is that the search spaces of realistic crop and water allocation problems are very large (Loucks and Van Beek, 2005). For example, in a study by Kuo et al. (2000) on optimal irrigation planning for seven crops in Utah, USA, the search space size was 5.6×10^{14} and in a study by Rubino et al. (2013) on the optimal allocation of irrigation water and land for nine crops in Southern Italy, the search space size was 3.2×10^{32} and 2.2×10^{43} for fixed and variable crop areas, respectively.

Another reason for considering search space size reduction for the optimal crop and water allocation problem is that the computational effort associated with realistic long-term simulation of crop growth can be significant (e.g., on the order of several minutes per evaluation). While simple crop models (e.g., crop production functions or relative yield – water stress relationships) have been widely used in optimization studies due to their computational efficiency (Singh, 2012), these models typically do not provide a realistic representation of soil moisture – climate interactions and the underlying physical processes of crop water requirements, crop growth, and agricultural management strategies (e.g., fertilizer or pesticide application). In order to achieve this, more complex simulation models, such as ORYZA2000 (Bouman et al., 2001), RZQWM2 (Bartling et al., 2012), AquaCrop (Vanuytrecht et al., 2014), EPIC (Zhang et al., 2015) and STICS (Coucheney et al., 2015) are typically employed. However, due to their relatively long runtimes, these models are normally used to simulate a small number of management strategy combinations (Camp et al., 1997; Rinaldi, 2001; Arora, 2006; DeJonge et al., 2012; Ma et al., 2012), rather than being used in combination with EAs to identify (near) globally optimal solutions. Given the large search spaces of optimal crop and water allocation problems, there is likely to be significant benefit in applying search-space size reduction methods in conjunction with hybrid simulation–optimization approaches to this problem (Lehmann and Finger, 2014).

Despite the potential advantages of search space size reduction, to the authors' knowledge this issue has not been addressed thus far in previous applications of EAs to optimal crop and water allocation problems. These applications include GAs (Nixon et al., 2001; Ortega Álvarez et al., 2004; Kumar et al., 2006; Azamathulla et al., 2008; Soundharajan and Sudheer, 2009; Han et al., 2011; Fallah-Mehdipour et al., 2013; Fowler et al., 2015), particle swarm optimization (PSO) algorithms (Reddy and Kumar, 2007; Noory et al., 2012; Fallah-Mehdipour et al., 2013), and shuffled frog leaping (SFL) algorithms (Fallah-Mehdipour et al., 2013). In order to address the absence of EA application to search space size reduction for the optimal crop and water allocation problem outlined above, the objectives of this paper are:

1. To develop a general framework for reducing the size of the search space for the optimal crop and water allocation problem. The framework makes use of dynamic decision trees and ant colony optimization (ACO) as the optimization engine, as this approach has been used successfully for search space size reduction in other problem domains (Afshar, 2007; Foong et al., 2008a, 2008b; Szemis et al., 2012, 2014).
2. To evaluate the utility of the framework on a crop and water allocation problem from the literature in order to validate the results against a known benchmark. It should be noted that although the search space of this benchmark problem is not overly large and does not require running a computationally intensive simulation model, it does require the development of a generic formulation that is able to consider multiple growing seasons, constraints on the maximum allowable areas for individual seasons, different areas for individual crops, and dissimilar levels of water availability. Consequently, the results of this case study provide a proof-of-concept for the application of the proposed framework to more complex problems involving larger search spaces and computationally expensive simulation models.

The remainder of this paper is organized as follows. A brief introduction to ACO is given in Section 2. The generic framework for optimal crop and water allocation that caters to search space size reduction is introduced in Section 3, followed by details of the case study and the methodology for testing the proposed framework on the case study in Section 4. The results are presented and discussed in Section 5, while conclusions and recommendations are given in Section 6.

2. Ant colony optimization (ACO)

ACO is a metaheuristic optimization approach first proposed by Dorigo et al. (1996) to solve discrete combinatorial optimization problems, such as the traveling salesman problem. As ACO has been used in a number of previous studies (Maier et al., 2003; Zecchin et al., 2005, 2006; Afshar, 2007; Foong et al., 2008a; Szemis et al., 2012), only a brief outline is given here. For a more extensive treatment of ACO, readers are referred to Dorigo and Di Caro (1999). ACO is inspired by the behavior of ants when searching for food, in that ants can use pheromone trails to identify the shortest path from their nest to a food source. In ACO, a colony (i.e., population) of artificial ants is used to imitate the foraging behavior of real ants for finding the best solution to a range of optimization problems, where the objective function values are analogous to path length. As part of ACO, the decision space is represented by a graph structure that represents the decision variables or decision paths of the optimization problem. This graph includes decision points connected by edges that represent options. Artificial ants are then used to find solutions in a stepwise fashion by moving along the graph from one decision point to the next.

The probability of selecting an edge at a particular decision point depends on the amount of pheromone that is on each edge, with edges containing greater amounts of pheromone having a higher probability of being selected. While the pheromone levels on the edges are generally allocated randomly at the beginning of the optimization process, they are updated from one iteration to the next based on solution quality. An iteration consists of the generation of a complete solution, which is then used to calculate objective function values. Next, larger amounts of pheromone are added to edges that result in better objective function values. Consequently, an edge that results in better overall solutions has a greater chance of being selected in the next iteration. In this way, good solution components receive positive reinforcement. In contrast, edges that result in poor objective function values receive little additional pheromone, thereby decreasing their chances of being selected in subsequent iterations. In fact, the pheromone on these edges is likely to decrease over time as a result of pheromone evaporation. In addition, artificial ants can be given visibility, giving locally optimal solutions a higher probability of being selected at each decision point. This is achieved by weighting these two mechanisms via pheromone and visibility importance factors, respectively. The basic steps of ACO can be summarized as follows:

1. Define the number of ants, number of iterations, initial pheromone (τ_o) on each edge, pheromone importance factor (α), visibility importance factor (β), pheromone persistence (ρ) to enable pheromone evaporation, and pheromone reward factor (q) to calculate how much pheromone to add to each edge after each iteration.
2. Calculate the selection probability p for each edge (path) of the decision tree, as illustrated here for the edge joining decision points A and B:

$$p_{AB} = \frac{[\tau_{AB}(t)]^{\alpha}[\eta_{AB}]^{\beta}}{\sum_{B=1}^{N_A}[\tau_{AB}(t)]^{\alpha}[\eta_{AB}]^{\beta}} \tag{1}$$

where t is the index of iteration, $\tau_{AB}(t)$ is the amount of pheromone on edge (A, B) at iteration t, η_{AB} is the visibility of edge (A, B), and N_A is the set of all decision points neighboring decision point A.

3. After all ants have traversed the decision tree and the objective function value corresponding to the solution generated by each ant has been calculated, update pheromone on all edges, as illustrated here for edge (A, B):

$$\tau_{AB}(t+1) = \rho\tau_{AB}(t) + \Delta\tau_{AB} \tag{2}$$

where $\Delta\tau_{AB}$ is the pheromone addition for edge (A, B).

It should be noted that there are different ways in which pheromone can be added to the edges, depending on which ACO algorithm is used. Any of these approaches can be applied to the proposed framework, as the proposed framework is primarily concerned with dynamically adjusting the structure of the decision-tree graph and not the way optimal solutions can be found on this graph, which can be done with a variety of algorithms. The only difference between the ACO algorithms is the way the pheromone update in Equation (2) is performed. The pheromone can be updated on: 1) all of the selected paths, as in the ant system (AS) (Dorigo et al., 1996); 2) only the path of the global-best solution from the entire colony after each iteration, as in the elitist ant system (ASelite) (Bullnheimer et al., 1997); 3) the paths from the top ranked solutions, which are weighted according to rank (i.e., higher ranked solutions have a larger influence in the pheromone updating process), as in the elitist-rank ant system (ASrank) (Bullnheimer et al., 1997); or 4) the path of the iteration-best solutions or the global-best solutions after a given number of iterations, as in the Max–Min Ant System (MMAS) (Stützle and Hoos, 2000). In this study, the MMAS algorithm is used as it has been shown to outperform the alternative ACO variants in a number of water resources case studies (e.g., Zecchin et al., 2006, 2007, 2012). As part of this algorithm, pheromone addition is performed on each edge, as shown for edge AB for illustration purposes:

$$\Delta\tau_{AB}(t) = \Delta\tau_{AB}^{ib}(t) + \Delta\tau_{AB}^{gb}(t) \tag{3}$$

where $\Delta\tau_{AB}^{ib}(t)$ and $\Delta\tau_{AB}^{gb}(t)$ are the pheromone additions for the iteration-best solution (s^{ib}) and the global-best solution (s^{gb}), respectively. While s^{ib} is used to update the pheromone on edge (A, B) after each iteration, s^{gb} is applied with the frequency f_{global} (i.e., $\Delta\tau_{AB}^{gb}(t)$ is calculated after each f_{global} iterations). $\Delta\tau_{AB}^{ib}(t)$ and $\Delta\tau_{AB}^{gb}(t)$ are given by:

$$\Delta\tau_{AB}^{ib}(t) = \begin{cases} \dfrac{q}{f\left(s^{ib}(t)\right)} & if\,(A,B)\in s^{ib}(t) \\ 0 & otherwise \end{cases} \tag{4}$$

$$\Delta\tau_{AB}^{gb}(t) = \begin{cases} \dfrac{q}{f\left(s^{gb}(t)\right)} & if\ (A,B)\in s^{gb}(t) and\ t\ mod\ f_{global} = 0 \\ 0 & otherwise \end{cases} \tag{5}$$

where $f(s^{ib}(t))$ and $f(s^{gb}(t))$ are objective function values of s^{ib} and s^{gb} at iteration t, respectively; and q is the pheromone reward factor.

In MMAS, the pheromone on each edge is limited to lie within a given range to avoid search stagnation, i.e., $\tau_{min}(t) \le \tau_{AB}(t) \le \tau_{max}(t)$. The equations for $\tau_{min}(t)$ and $\tau_{max}(t)$ are given as follows:

$$\tau_{max}(t) = \left(\frac{1}{1-\rho}\right)\frac{1}{f\left(s^{gb}(t-1)\right)} \tag{6}$$

$$\tau_{min}(t) = \frac{\tau_{max}(t)\left(1 - \sqrt[n]{p_{best}}\right)}{(avg - 1)\sqrt[n]{p_{best}}} \quad (7)$$

where n is the number of decision points, avg is the average number of edges at each decision point, and p_{best} is the probability of constructing the global best solution at iteration t, where the edges chosen have pheromone trail values of τ_{max} and the pheromone values of other edges are τ_{min}. Additionally, MMAS also uses a pheromone trail smoothing (PTS) mechanism that reduces the difference between edges in terms of pheromone intensities, and thus, strengthens exploration:

$$\tau^*_{AB}(t) = \tau_{AB}(t) + \delta(\tau_{max}(t) - \tau_{AB}(t)) \quad (8)$$

where δ is the PTS coefficient ($0 \leq \delta \leq 1$).

As is the case with most metaheuristic optimization algorithms, the parameters controlling algorithm searching behavior are generally determined with the aid of sensitivity analysis (e.g., Simpson et al., 2001; Foong et al., 2008b; Szemis et al., 2012). Although algorithm performance has been found to be insensitive to certain parameters (e.g., Foong et al., 2005), and for some application areas guidelines have been developed for the selection of appropriate parameters based on problem characteristics and the results of large-scale sensitivity analyses (e.g., Zecchin et al., 2005), parameter sensitivity is likely to be case study dependent.

Over the last decade, ACO has been applied extensively to a range of water resources problems, including reservoir operation and surface water management, water distribution system design and operation, urban drainage system and sewer system design, groundwater management and environmental and catchment management, as detailed in a recent review by Afshar et al. (2015). While ACO shares the advantages of other evolutionary algorithms and metaheuristics of being easy to understand, being able to be linked with existing simulation models, being able to solve problems with difficult mathematical properties, being able to be applied to a wide variety of problem contexts and being able to suggest a number of near-optimal solutions for consideration by decision-makers (Maier et al., 2014, 2015), it is particularly suited to problems where there is dependence between decision variables, such that the selected value of particular decision variables restricts the available options for other decision variables, as is often the case in scheduling and allocation problems (e.g., Afshar, 2007; Afshar and Moeini, 2008; Foong et al., 2008a, 2008b; Szemis et al., 2012, 2013, 2014). This is because the problem to be optimized is represented in the form of a decision-tree, as mentioned above, enabling solutions to be generated in a stepwise fashion and decision variable options to be adjusted based on selected values at previous nodes in the decision tree. In other words, as part of the process of generating an entire solution, the available options at nodes in the tree can be constrained based on the values of partial solutions generated at previous nodes.

3. Proposed framework for optimal crop and water allocation

3.1. Overview

A simulation–optimization framework for optimal crop and water allocation is developed that is based on: 1) a graph structure to formulate the problem, 2) a method that adjusts decision variable options dynamically during solution construction to ensure only feasible solutions are obtained as part of the stepwise solution generation process in order to dynamically reduce the size of the search space, and 3) use of ACO as the optimization engine. The framework is aimed at identifying the seasonal crop and water allocations that maximize economic benefit at district or regional level, given restrictions on the volume of water that is available for irrigation purposes. Use of the framework is expected to result in a significant reduction in the size of the search space for optimal crop and water allocation problems, which is likely to reduce the number of iterations required to identify optimal or near globally optimal solutions.

An overview of the framework is given in Fig. 1. As can be seen, the first step is problem formulation, where the objective to be optimized (e.g., economic return) is defined, the constraints (e.g., maximum land area, annual water allocation, etc.) are specified, and the decision variables (e.g., crop type, crop area, magnitude of water application to different crops, etc.) and decision variable options (e.g., available crops to select, options of watering levels, etc.) are stipulated. Herein, the level of discretization of the total area is also identified, so that the values of the sub-areas are able to reflect the characteristics of the problem considered.

After problem formulation, the problem is represented in the form of a decision-tree graph. This graph includes a set of nodes (where values are selected for the decision variables) and edges (which represent the decision variable options). A crop and water allocation plan is constructed in a stepwise fashion by moving along the graph from one node to the next.

In the next step, the method for handling constraints needs to be specified. As part of the proposed framework, it is suggested to dynamically adjust decision variable options during the construction of a trial crop and water allocation plan in order to ensure constraints are not violated. This is achieved by only making edges available that ensure that all constraints are satisfied at each of the decision points. However, as this is a function of choices made at previous decision points in the graph, the edges that are available have to be updated dynamically each time a solution is constructed. This approach is in contrast to the approach traditionally used for dealing with constraints in ACO and other evolutionary algorithms, which is to allow the full search space to be explored and to penalize infeasible solutions. However, the latter approach is likely to be more computationally expensive, as the size of the search space is much larger. Consequently, it is expected that the proposed approach of dynamically adjusting decision variable options will increase computational efficiency as this approach reduces the size of the search space and ensures that only feasible solutions can be generated during the solution construction process.

As part of the proposed framework, ACO algorithms are used as the optimization engine because they are well-suited to problems that are represented by a graph structure and include sequential decision-making (Szemis et al., 2012), as is the case here. In addition, they have been shown to be able to accommodate the adjustment of dynamic decision variable options by handling constraints in other problem domains (Foong et al., 2008a; Szemis et al., 2012). As part of the optimization process, the evaluation of the objective function is supported by crop models. In this way, improved solutions are generated in an iterative fashion until certain stopping criteria are met, resulting in optimal or near-optimal crop and water allocations. Further details of the problem formulation, graph structure representation, method for handling constraints, crop model options, and ACO process are presented in subsequent sections.

3.2. Problem formulation

The process of problem formulation includes the following steps:

1. Identify the number of the seasons (e.g., winter, monsoon, etc.), the seasonal (e.g., wheat) and annual (e.g., sugarcane) crops, the total cultivated area and the volume of available water.

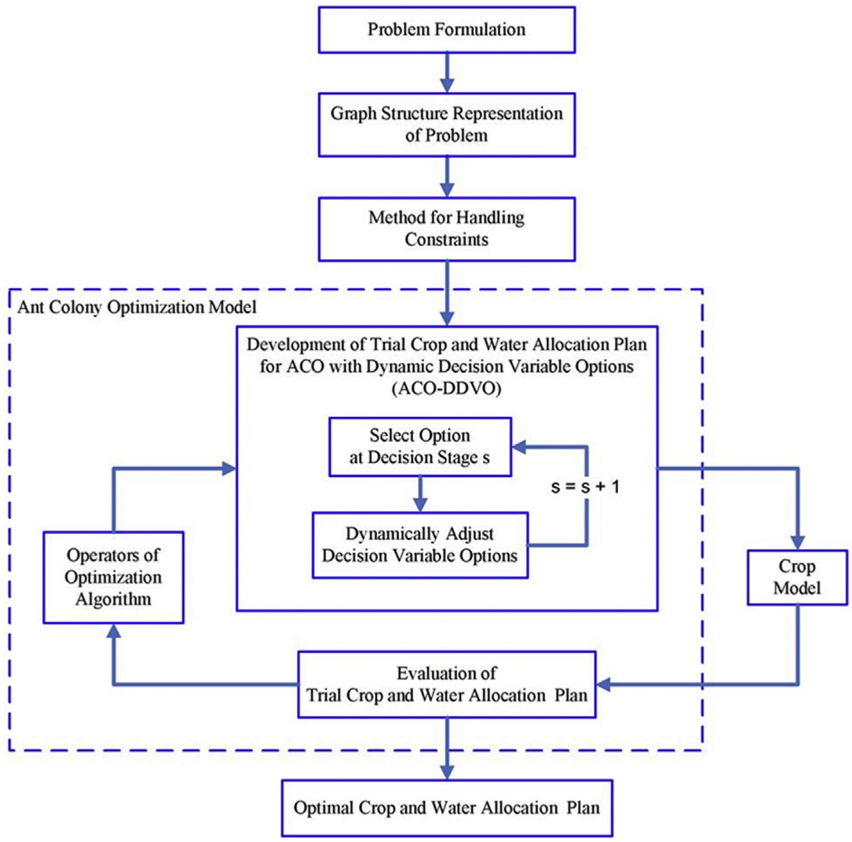


Fig. 1. Overview of the proposed simulation–optimization framework for optimal crop and water allocation.

2. Identify economic data in the study region, including crop price, production cost, and water price.
3. Specify decision variables (e.g., crop type, crop area, and irrigated water).
4. Specify decision variable options. For crop type, a list of potential options is given by the crops identified in step 1 (e.g., wheat, sugarcane, cotton, etc.). For continuous variables (i.e., crop area and irrigated water), the specification of the options includes selection of the range and level of discretization for each decision variable. The level of discretization (e.g., sub-area or volume of irrigated water for each crop) can significantly impact on either the quality of solutions found or the search space size (due to the exponential growth of this size). A discretization that is too coarse could exclude the true global optimal solution, while a fine discretization could result in a significant increase in computational time. While the depth of irrigated water can be discretized depending on the type and capacity of irrigation system, the acreage of each sub-area can be set equal to a unit area (e.g., 1 ha) or be the same as that of a standard field in the studied region. The discretization of area can also be implemented depending on soil type or land-use policy. Each sub-area should reflect different conditions (e.g., soil type, evapotranspiration, and rainfall in season, etc.), and thus the discretization process will support the planning of the cropping patterns more realistically. Consequently, instead of selecting the area and the depth of irrigated water for each crop, as part of the proposed framework, the total area of the studied region is discretized into a number of sub-areas with each sub-area requiring decisions on which crop type should be planted and how much water should be supplied to the selected crop.
5. Specify the objective function and constraints. The objective function is to optimize the economic benefit and has the following form:

$$F = \mathrm{Max}\left\{\sum_{i=1}^{N_{sea}}\sum_{j=1}^{N_{ic}}\sum_{k=1}^{N_{SA}}\left(A_{ijk}\times\left[Y_{ijk}\left(W_{ijk}\right)\times P_{ij} - \left(C_{FIXij}+W_{ijk}\times C_W\right)\right]\right)\right\} \tag{9}$$

where F is the total net annual return (currency unit, e.g., \$ year^{-1}), N_{sea} is the number of seasons in a year (an annual crop is considered as the same crop for all seasons in a year), N_{ic} is the number of crops for season i (i = 1, 2, …, N_{sea}; for annual crop, i = a), N_{SA} is the number of sub-areas, A_{ijk} is the area of crop j in season i in sub-area k (ha), W_{ijk} is the depth of water supplied to crop j in season i in sub-area k (mm), Y_{ijk} is the yield of crop j in season i in sub-area k (depending on W_{ijk}) (kg ha^{-1}), P_{ij} is the price of crop j in season i (\$ kg^{-1}), C_{FIXij} is the fixed annual cost of crop j in season i (\$ ha^{-1} year^{-1}), and C_W is the unit cost of irrigated water (\$ mm^{-1} ha^{-1}).

As noted in Section 3.1 the objective is to maximize the total net annual return at the district or regional level rather than the net return to individual irrigators. Hence, the framework represents the perspective of an irrigation authority or farmer co-operative.

The objective function is maximized subject to limits on available resources, such as water and area of land. Consequently, the following constraints will be considered in order to provide a flexible and generic formulation:

• Constraints for maximum allowable area of each season A_i:

$$\sum_{j=1}^{N_{ic}} \sum_{k=1}^{N_{SA}} A_{ijk} \le A_i \tag{10}$$

• Constraints for maximum allowable crop area A_{ijMax} for each season:

$$\sum_{k=1}^{N_{SA}} A_{ijk} \le A_{ijMax} \tag{11}$$

• Constraints for minimum allowable crop area A_{ijMin} for each season:

$$\sum_{k=1}^{N_{SA}} A_{ijk} \ge A_{ijMin} \tag{12}$$

• Constraints for available volume of irrigation water W:

$$\sum_{i=1}^{N_{sea}} \sum_{j=1}^{N_{ic}} \sum_{k=1}^{N_{SA}} W_{ijk} \times A_{ijk} \le W \tag{13}$$

3.3. Graph structure problem representation

As discussed in Section 3.2, a crop and water allocation plan can be established by determining the crop type and the depth of irrigated water for the selected crop in each sub-area. Thus, the full decision-tree graph for the optimal crop and water allocation problem is as shown in Fig. 2.

The decision tree includes a set of decision points corresponding to the number of discrete sub-areas in the irrigated/studied area. At each decision point, a subset of decision points is used to consider each season in turn in order to decide which crop will be chosen to be planted at this sub-area in season i (i.e., C_{i1}, C_{i2}, …, C_{iNic}), and then what depth of water (i.e., W_1, W_2, …, W_{Nw}) will be supplied to the selected crop. If the selected crop at a decision point is an annual crop, then that decision point only considers the depth of irrigated water for that crop and skips the other seasons. A complete crop and water allocation plan is developed once a decision has been made sequentially at each decision point.

It should be noted that the sequential solution generation steps are internal to the ACO process and do not reflect the sequence with which actual decisions are made, as the output of every ACO run is a complete annual crop and water allocation plan. While the order of solutions in the decision tree is likely to have an impact on the solutions obtained in a particular iteration, it would be expected that as the number of iterations increases, this effect would disappear as a result of the identification of globally optimal solutions via pheromone trail adjustment. It should also be noted that while the current formulation is aimed at identifying seasonal crop and water allocations, it could be extended to cater to more frequent (e.g., monthly, weekly, or daily) water allocations for the selected crops by adding the required number of decision points for water allocation. For example, if the frequency of water allocation decisions was changed from seasonally to monthly, there would be six decision points related to water allocation for each crop (one for each month), rather than a single seasonal decision point as shown in Fig. 2.

3.4. Method for handling constraints

The available decision variable options are adjusted *dynamically* by checking all constraints (Equations (10)–(13)) at each decision point and removing any options (i.e., crops or irrigated water) that result in the violation of a constraint based on paths selected at previous decision points (i.e., the number of available decision variable options is dynamically adjusted *during* the stepwise solution construction process). As mentioned previously, the purpose of this process is to dynamically reduce the size of the search space during the construction of trial solutions by each ant in each iteration, which is designed to make it easier and more computationally feasible to identify optimal or near-optimal solutions.

Details of how the decision variable options that result in constraint violation are identified for each of the constraints are given below. It should be noted that the four constraints in Equations (10)–(13) are considered for the choice of crops at the beginning of each season in a sub-area during the construction of a trial solution. However, to select the depth of irrigated water for the crop selected in the previous decision, only the constraint for available volume of irrigated water is checked.

• *Key steps for handling constraints for maximum allowable area for each season (Equation (10)):*

1. Keep track of the total area allocated to each season as the decision tree is traversed from sub-area to sub-area.
2. Add the area of the next sub-area in the decision tree to the already allocated area for each season.
3. Omit all crops in a particular season and all annual crops from the choice of crops for this and subsequent sub-areas if the resulting area exceeds the maximum allowable area for this season.

• *Key steps for handling constraints for maximum allowable crop area* (Equation (11)):

1. Keep track of the total area allocated to each crop type as the decision tree is traversed from sub-area to sub-area.
2. Add the area of the next sub-area in the decision tree to the already allocated area for each crop.
3. Omit a particular crop from the choice of crops for this and subsequent sub-areas if the resulting area exceeds the maximum allowable area for this crop.

• *Key steps for handling constraints for minimum allowable crop area* (Equation (12)):

1. Keep track of the total area allocated to each crop type as the decision tree is traversed from sub-area to sub-area.
2. Sum the sub-areas in the decision tree remaining after this current decision.
3. Restrict the crop choices at this and subsequent decisions (i.e. subsequent sub-areas) to the ones that have minimum area constraints that are yet to be satisfied if the total area remaining after the current decision is less than the area that needs to be allocated in order to satisfy the minimum area constraints.

• *Constraints for maximum available volume of irrigated water* (Equation (13)):

The key steps for handling this constraint for the *choice of crops* include:

1. Keep track of the total volume of irrigation water allocated to all crops as the decision tree is traversed from sub-area to sub-area.
2. Sum the volume of irrigation water for each crop in the decision tree remaining after this current decision.

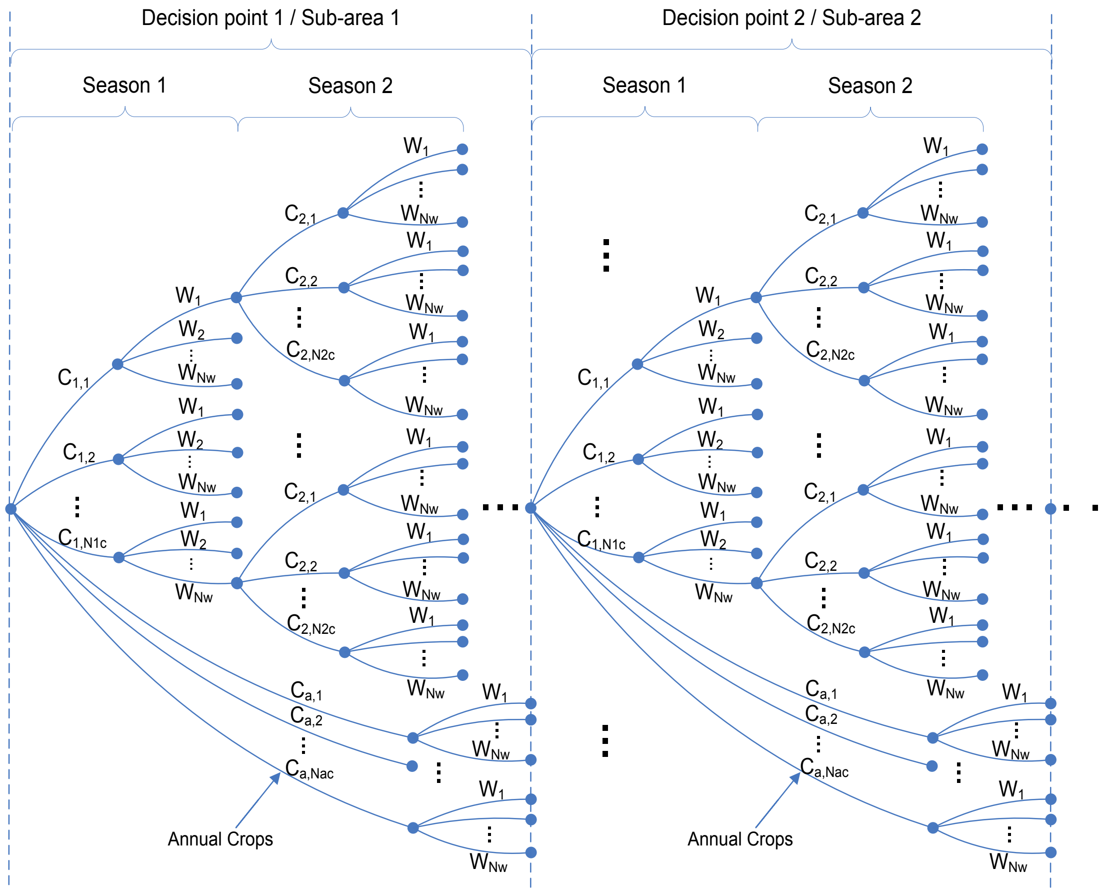


Fig. 2. Proposed decision-tree graph for the optimal crop and water allocation problem.

3. Restrict the crop choices at this and subsequent decisions (i.e. subsequent sub-areas) to the ones that have minimum area constraints that are yet to be satisfied if the total volume of water remaining after the current decision is less than the volume of water that needs to be supplied in order to satisfy the minimum area constraints.

The key steps for handling this constraint for the *choice of the depth of irrigated water* include:

1. Keep track of the total volume of irrigation water allocated to crops as the decision tree is traversed from sub-area to sub-area.
2. Calculate the available volume of irrigation water for the crop selected in the previous decision at this current decision.
3. Omit the choices of the depth of irrigated water for the crop selected in the previous decision if the volumes of irrigation water corresponding to these choices exceed the available volume of water in Step 2.

3.5. Crop models

In the proposed framework, a crop model coupled with the ACO model is employed as a tool for estimating crop yield to evaluate the utility of trial crop and water allocation plans. Generally, the crop model can be a simplified form (e.g., regression equation of crop production functions or relative yield – water stress relationships) which has the advantage of computational efficiency, or a mechanistic, process-based form which is able to represent the underlying physical processes affecting crop water requirements and crop growth in a more realistic manner.

3.6. Ant colony optimization model

The ACO model is used to identify optimal crop and water allocation plans by repeatedly stepping through the dynamic decision tree (see Section 3.4). At the beginning of the ACO process, a trial schedule is constructed by each ant in the population in accordance with the process outlined in Section 2. Next, the corresponding objective function values are calculated with the aid of the crop model and the pheromone intensities on the decision paths are updated (see Section 2). These steps are repeated until the desired stopping criteria have been met.

4. Case study

The problem of optimal cropping patterns under irrigation introduced by Kumar and Khepar (1980) is used as the case study for testing the utility of the framework introduced in Section 3. As discussed in Section 1, various challenges (e.g., a large search space or relatively long runtimes) have restricted the application of complex crop models for solving optimal crop and water allocation problems. Although the search space in the Kumar and Khepar (1980) case study is not overly large and the study uses crop water production functions rather than a complex crop-growth model, the problem has a number of useful features, including:

1. It requires a generic formulation, including multiple seasons, multiple crops, and constraints on available resources (e.g., a minimum and maximum area of each season, each crop, and water availability), as mentioned in Section 1.
2. Due to its relative computational efficiency, it enables extensive computational trials to be conducted in order to test the potential benefits of the proposed framework in a rigorous manner, and thus, may play an important role in increasing the efficacy of the optimization of crop and water allocation plans utilizing complex crop model application. Consequently, the use of crop production functions for the proposed framework is important for providing a proof-of-concept prior to its application with complex crop simulation models.
3. As optimization results for this case study have already been published by others, it provides a benchmark against which the quality of the solutions obtained from the proposed approach can be compared.

Details of how the proposed framework was applied to this case study are given in the following sub-sections.

4.1. Problem formulation

4.1.1. Identification of seasons, crops, cultivated area and available water

The case study problem considers two seasons (winter and monsoon) with seven crop options: wheat, gram, mustard, clover (referred to as berseem in Kumar and Khepar, 1980), sugarcane, cotton and paddy. While sugarcane is an annual crop, the other crops are planted in winter (e.g., wheat, gram, mustard and clover) or the monsoon season (e.g., cotton and paddy) only. The total cultivated area under consideration is 173 ha and the maximum volume of water available is 111,275 ha-mm. Three different water availability scenarios are considered for various levels of water losses in the main water courses and field channels, corresponding to water availabilities of 100%, 90% and 75%, as stipulated in Kumar and Khepar (1980).

4.1.2. Identification of economic data

The economic data for the problem, consisting of the price and fixed costs of crops for the region, are given in Table 1. The water price is equal to 0.423 R mm^{-1} ha^{-1}.

4.1.3. Specification of decision variables

As was the case in Kumar and Khepar (1980), two separate formulations were considered, corresponding to different decision variables. In the first formulation, the only decision variable was area, i.e., how many hectares should be allocated to each crop in order to achieve the maximum net return. In the second formulation, the decision variables were area and the depth of irrigated water applied to each crop. In this case study, as discussed in Section 3.2, the decision variables are which crop to plant in each sub-area, and the depth of irrigated water supplied to the selected crop.

4.1.4. Specification of decision variable options

The number of decision points for area is generally equal to the maximum area (i.e., 173 ha in this case) divided by the desired level of discretization, which is selected to be 1 ha here. This would result in 173 decision points, each corresponding to an area of 1 ha to which a particular crop is then allocated (Fig. 2). However, in order to reduce the size of the search space, a novel discretization scheme was adopted. As part of this scheme, the number of decision points for area was reduced from 173 to 29 with 10, 10, and 9 points corresponding to areas of 5, 6, and 7 ha, respectively. As a choice is made at each of these decision points as to which crop choice to implement, this scheme enables any area between 5 and 173 ha (in increments of 1 ha) to be assigned to any crop, with the exception of areas of 8 and 9 ha. For example:

- An area of 6 ha can be allocated to a crop by selecting this crop at 1 of the 10 areas corresponding to an area of 6 ha and not selecting this crop at any of the decision points corresponding to areas of 5 and 7 ha.
- An area of 27 ha can be allocated to a crop by selecting this crop at 4 of the 10 areas corresponding to an area of 5 ha and at 1 of the 9 areas corresponding to an area of 7 ha and not selecting this crop at any of the decision points corresponding to an area of 6 ha.
- An area of 173 ha can be allocated to a crop by selecting this crop at all of the decision points (i.e., $10 \times 5 + 10 \times 6 + 9 \times 7 = 173$).

Based on the above discretization scheme, for each of the 29 decision points for each sub-area, there are six decision variable options for crop choice for Season 1 (i.e. $N1c = 6$, see Fig. 2), including dryland, wheat, gram, mustard, clover and sugarcane, and three decision variable options for crop choice for Season 2 (i.e. $N2c = 3$, see Fig. 2), including dryland, cotton and paddy.

An obvious limitation of this scheme is that is not possible to allocate areas of 1–4, 8 and 9 ha to any crop. However, the potential loss of optimality associated with this was considered to be outweighed by the significant reduction in the size of the solution space. Another potential shortcoming of this scheme is that it leads to a bias in the selection of crops during the solution generation process (i.e., intermediate areas have higher possibilities of being selected than extreme values). While this has the potential to slow down overall convergence speed, it would be expected that as the number of iterations increases, this bias would disappear as a result of the identification of globally optimal solutions via pheromone trail adjustment. The potential loss in computational efficiency associated with this effect is likely to be outweighed significantly by the gain in computational efficiency associated with the decrease in the size of solutions space when adopting this coding scheme.

It should be noted that in general terms, a discretization scheme

Table 1
Details of crops considered, crop price, fixed costs of crop and the seasons in which crops are planted (from Kumar and Khepar, 1980).

Season	Crop	Price of crop (Rs qt^{-1})	Fixed costs of crop (Rs ha^{-1} $year^{-1}$)
Winter	Wheat	122.5	2669.8
	Gram	147.8	1117.0
	Mustard	341.4	1699.55
	Clover	7.0	2558.6
Annual	Sugarcane	13.5	5090.48
Summer	Cotton	401.7	2362.55
	Paddy	89.0	2439.68

Notes: Rs is a formerly used symbol of the Indian Rupee; qt is a formerly used symbol of weight in India.

of 1, 2 and 4 can be used for any problem, as the sum of combinations of these variables enable the generation of any integer. However, if there is a lower bound that is greater than one, then alternative, case study dependent optimization schemes can be developed in order to reduce the size of the search space further, as demonstrated for the scheme adopted for the case study considered in this paper. This is because the number of decision points resulting from the selected discretization scheme is a function of the sum of the integer values used in the discretization scheme. For example, if a scheme of 1, 2 and 4 had been used in this study, the required number of decision points for sub-area for each integer value in the scheme would have been $173/(1+2+4) = 24.7$. In contrast, for the adopted scheme, this was only $173/(5+6+7) = 9.6$ (resulting in the adopted distribution of 10, 10, 9).

For Formulation 2, decision variable options also have to be provided for the depth of irrigated water for each of the selected crops at each of the sub-areas (see Fig. 2). Based on the irrigation depth that corresponds to maximum crop yield for the crop production functions (see Section 4.4) and an assumed discretization interval of 10 mm ha^{-1}, the number of irrigated water options for each crop was 150 (i.e. NW = 150, see Fig. 2), corresponding to choices of 0, 10, 20, …, 1490 mm ha^{-1}.

Details of the decision variables, decision variable options, and search space size for three scenarios of both formulations are given in Table 2.

4.1.5. Objective function and constraints

As there are only two seasons, the objective function for both formulations is as follows in accordance with the general formulation of the objective function given in Equation (9):

$$F = \text{Max}\left\{ \sum_{j=1}^{6}\sum_{k=1}^{29}\left(A_{1jk} \times \left[Y_{1jk}\left(W_{1jk}\right) \times P_{1j} - \left(C_{FIX1j} + W_{1jk} \times C_W\right)\right]\right) + \sum_{j=1}^{3}\sum_{k=1}^{29}\left(A_{2jk} \times \left[Y_{2jk}\left(W_{2jk}\right) \times P_{2j} - \left(C_{FIX2j} + W_{2jk} \times C_W\right)\right]\right) \right\} \quad (14)$$

where the variables were defined in Section 3.2.

The objective function is subject to the following constraints, which are in accordance with those stipulated in Kumar and Khepar (1980).

• *Constraints for maximum allowable areas in winter and monsoon seasons:*

The total planted area of crops in each season must be less than or equal to the available area for that season. As stipulated in Kumar and Khepar (1980), the maximum areas A_i in the winter and monsoon seasons are 173 and 139 ha, respectively.

$$\sum_{j=1}^{N_{ic}}\sum_{k=1}^{29} A_{ijk} \le A_i \quad (15)$$

• *Constraints for minimum and maximum allowable crop area*:

The area of a crop must be less than or equal to its maximum area and greater than or equal to its minimum area. At least 10% of the total area in the winter season (approximately 17 ha) has to be planted in clover, and the maximum areas of mustard and sugarcane are equal to 10% and 15% of the total area in the winter season (approximately 17 ha and 26 ha, respectively).

$$\sum_{k=1}^{29} A_{ijk} \le A_{ijMax} \quad (16)$$

$$\sum_{k=1}^{29} A_{ijk} \ge A_{ijMin} \quad (17)$$

• *Constraints for available volume of irrigated water*

The total volume of irrigated water applied to the crops is less than or equal to the maximum volume of water available for irrigation in the studied region. As mentioned above, three scenarios are considered with 75%, 90% and 100% of water entitlement, respectively. The corresponding volumes of available water for these scenarios are 844,570, 1,001,780 and 1,112,750 m^3, respectively.

$$\sum_{j=1}^{6}\sum_{k=1}^{29} W_{1jk} \times A_{1jk} + \sum_{j=1}^{3}\sum_{k=1}^{29} W_{2jk} \times A_{2jk} \le W \quad (18)$$

4.2. Graph structure representation of problem

There are separate decision-tree graphs for Formulations 1 and 2. In Formulation 1, the graph includes 29 decision points corresponding to 29 sub-areas, as discussed in Section 4.1. At each decision point, there are only two choices of crops corresponding to Seasons 1 and 2 as the depth of irrigated water for each crop is fixed (Fig. 3). As can be seen, there are six crop options (dryland, wheat, gram, mustard, clover and sugarcane) in Season 1 (five crops in the winter season and one annual crop) and three options (dryland, cotton, and paddy) in Season 2 (i.e., the monsoon season). It should be noted that if sugarcane is selected, there is no crop choice for Season 2, as sugarcane is an annual crop. A complete solution is developed once the crops for all sub-areas are selected.

In similar fashion to Formulation 1, the decision-tree graph for Formulation 2 also includes 29 decision points for area, but each decision point includes two choices of crops (one for each season) and two choices of the depth of irrigated water (one for each crop in

Table 2
Optimization problem details for each of the two problem formulations considered.

Formulation	Water availability	Decision variables	No. of decision points for area	No. of crop options for each sub-area	No. of irrigated water options for each crop	Size of total search space
1	100% 90% 75%	Crop type	29	6 for Season 1, 3 for Season 2	1	2.5×10^{36}
2	100% 90% 75%	Crop type and depth of irrigated water	29	6 for Season 1, 3 for Season 2	150 for each crop	4.1×10^{162}

Note: The size of total search space is equal to $(6^{29} \times 3^{29})$ for Formulation 1 and $(6^{29} \times 3^{29} \times 150^{29} \times 150^{29})$ for Formulation 2.

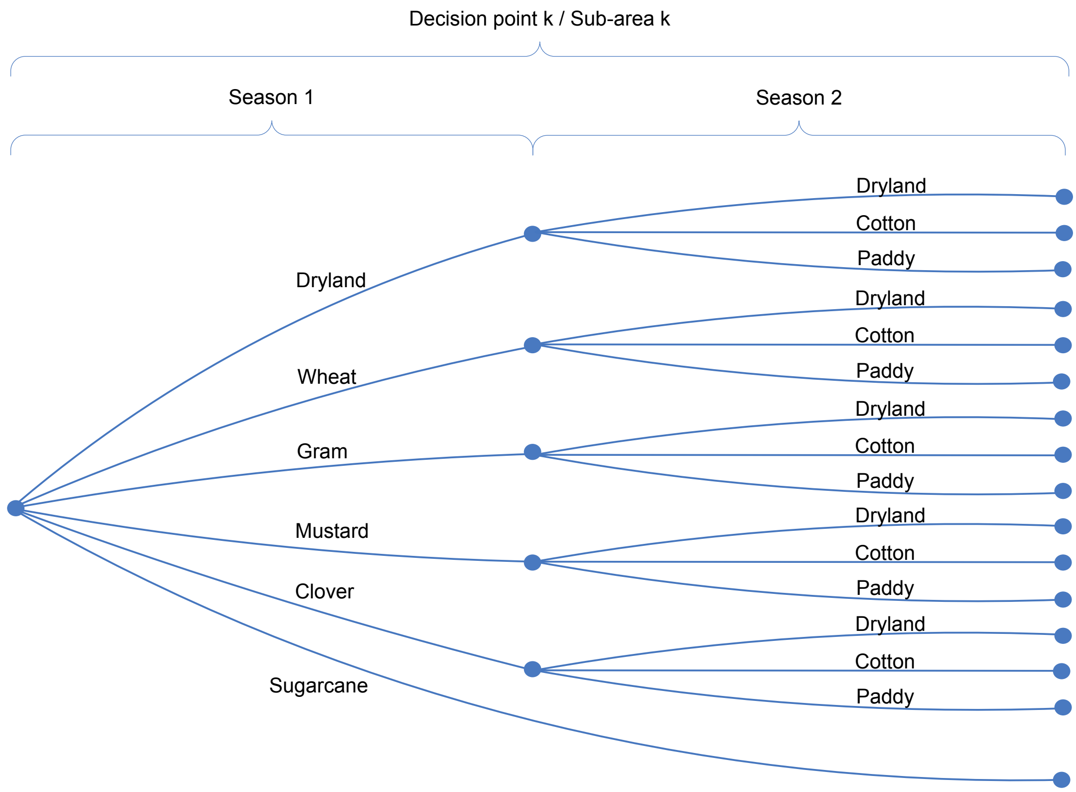


Fig. 3. A single decision point for area of the decision-tree graph for Formulation 1.

each season). This graph has the same structure as the decision-tree graph in Fig. 2, but includes two seasons, six crop options for Season 1, three crop options for Season 2, and 150 depth of irrigated water options for each crop (see Section 4.1.4). After a crop is selected for each season at each decision point, the depth of irrigated water for the selected crop is determined (unless the crop is dryland in which case there is no irrigation option). Furthermore, at each decision point, if an annual crop (i.e., sugarcane) is selected in Season 1, there is only the choice of the depth of irrigated water for the annual crop. Although other choices in Season 2 are skipped in this case, the available area and depth of water after that decision point will be reduced by annual crop use. A complete crop and water allocation plan is developed once a decision has been made sequentially at each decision point.

4.3. Method for handling constraints

In addition to the proposed dynamic decision variable options (DDVO) adjustment approach for dealing with constraints in ACO, the traditional and most commonly used method via the use of penalty functions was also implemented. This was undertaken in order to assess the impact on search space size reduction of the proposed DDVO approach. Details of both approaches are given below.

4.3.1. DDVO adjustment approach

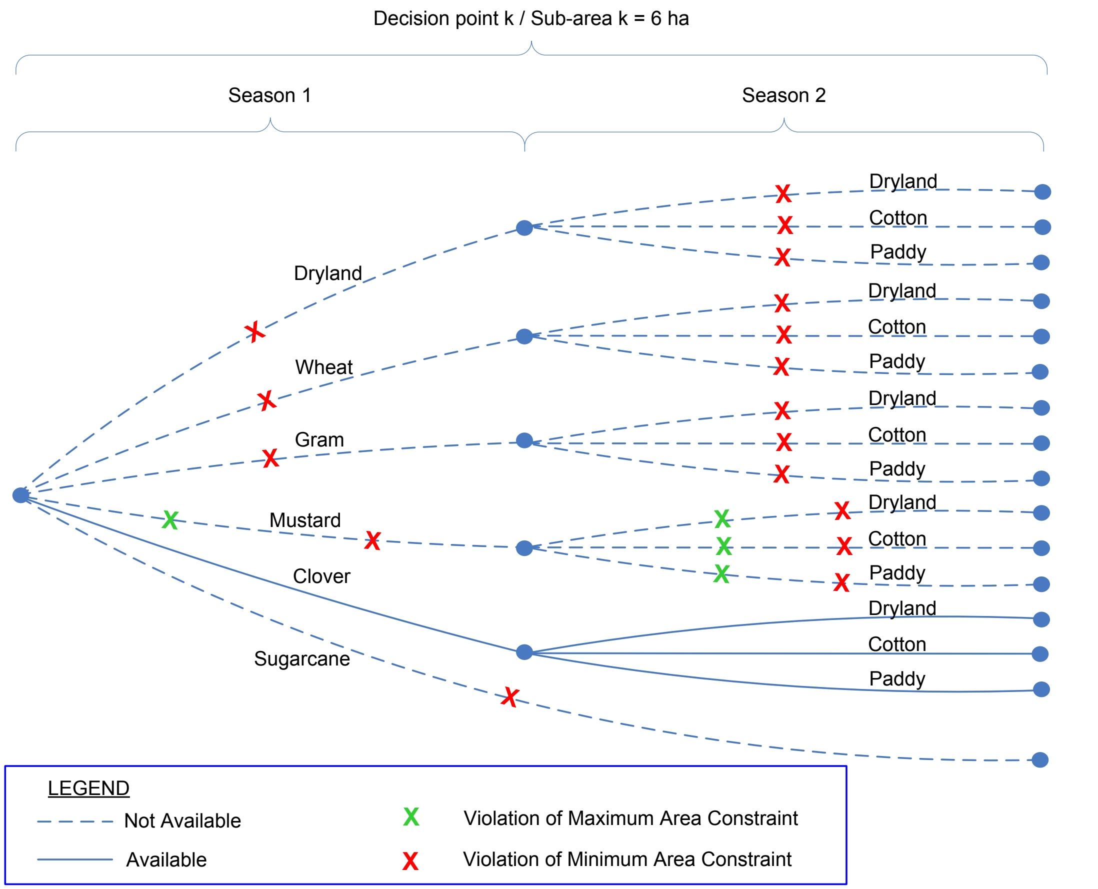
As part of this approach, the decision trees for Formulations 1 and 2 described in Section 4.2 were dynamically adjusted based on the procedure outlined in Section 3.4. An example of how this works for the case study is shown in Fig. 4. In this example, two constraints for maximum and minimum allowable crop area were considered to check the available crop options at decision point k (which corresponds to one of the 10 sub-areas with an area of 6 ha – see Section 4.1.4 – for the sake of illustration) in Formulation 1. In the figure, the cumulative area that has already been allocated to each crop is shown in column (1) and the resulting total area allocated to each crop if this particular crop is selected at this decision point is shown in column (4). It is clear that when the constraint for maximum allowance crop area was checked, mustard could be removed as an option at this decision point (column (5)) because its total cumulative allocated area in column (4) was larger than the maximum allowable area for this crop in column (3), thereby reducing the size of the search space (Fig. 4). When checking the minimum allowable area constraint by comparing the areas in columns (2) and (4), and comparing the remaining area after this decision point (i.e. 173 − 158 − 6 = 9 ha) and the remaining minimum area at this decision point (i.e., 17 − 5 = 12 ha), clover provided the only feasible crop choice (column (6)) at this decision point. This enables all other crop choices to be removed, thereby further reducing the size of the search space and ensuring only feasible solutions are generated (Fig. 4).

4.3.2. Penalty function approach

As part of the penalty function approach to constraint handling, there is no dynamic adjustment of decision variable options based on solution feasibility. Consequently, infeasible solutions can be generated and in order to ensure that these solutions are eliminated in subsequent iterations, a penalty value (P) is added to the objective function value (F) for these solutions.

Penalty function values are generally calculated based on the distance of an infeasible solution to the feasible region (Zecchin et al., 2005; Szemis et al., 2012; Zecchin et al., 2012). Therefore, the following penalty functions were used for the constraints in Equations (10)–(13):

- Penalty for maximum allowable area of each season A_i (corresponding to Equation (10)):



Crops in Season 1	Cumulative Area Already Allocated	Minimum Allowable Crop Area	Maximum Allowable Crop Area	Column (1) + Sub-Area k (i.e., 6 ha)	Constraints for Maximum Allowance Crop Area	Constraints for Minimum Allowance Crop Area
(0)	(1)	(2)	(3)	(4)	(5)	(6)
Dryland	23	0	173	29	Available	Not available
Wheat	50	0	173	56	Available	Not available
Gram	48	0	173	54	Available	Not available
Mustard	22	0	26	28	Not available	Not available
Clover	5	17	173	11	Available	Available
Sugarcane	10	0	17	16	Available	Not available
Total	158	17				

Fig. 4. Example of decision variable option adjustment process for one decision point for Formulation 1.

$$P(1) = \begin{cases} 0 & \textit{if} \ \sum_{j=1}^{N_{ic}} \sum_{k=1}^{N_{SA}} A_{ijk} \le A_i \\ \left(\sum_{j=1}^{N_{ic}} \sum_{k=1}^{N_{SA}} A_{ijk} - A_i \right) \times 1,000,000 & \textit{if} \ \sum_{j=1}^{N_{ic}} \sum_{k=1}^{N_{SA}} A_{ijk} > A_i \end{cases} \tag{19}$$

• Penalty for maximum allowable crop area A_{ijMax} (corresponding to Equation (11)):

$$P(2) = \begin{cases} 0 & \textit{if} \ \sum_{k=1}^{N_{SA}} A_{ijk} \le A_{ijMax} \\ \left(\sum_{k=1}^{N_{SA}} A_{ijk} - A_{ijMax} \right) \times 1,000,000 & \textit{if} \ \sum_{k=1}^{N_{SA}} A_{ijk} > A_{ijMax} \end{cases} \tag{20}$$

• Penalty for minimum allowable crop area A_{ijMin} (corresponding to Equation (12)):

$$P(3) = \begin{cases} 0 & \text{if } \sum_{k=1}^{N_{SA}} A_{ijk} \geq A_{ijMin} \\ \left(A_{ijMin} - \sum_{k=1}^{N_{SA}} A_{ijk}\right) \times 1,000,000 & \text{if } \sum_{k=1}^{N_{SA}} A_{ijk} < A_{ijMin} \end{cases} \quad (21)$$

- Penalty for available volume of irrigated water W (corresponding to Equation (13)):

$$P(4) = \begin{cases} 0 & \text{if } \sum_{i=1}^{N_{sea}} \sum_{j=1}^{N_{ic}} \sum_{k=1}^{N_{SA}} W_{ijk} \times A_{ijk} \leq W \\ \left(\sum_{i=1}^{N_{sea}} \sum_{j=1}^{N_{ic}} \sum_{k=1}^{N_{SA}} W_{ijk} \times A_{ijk} - W\right) \times 1,000,000 & \text{if } \sum_{i=1}^{N_{sea}} \sum_{j=1}^{N_{ic}} \sum_{k=1}^{N_{SA}} W_{ijk} \times A_{ijk} > W \end{cases} \quad (22)$$

where the variables in Equations (19) and (22) are defined in Section 3.2.

The following equation was used as the overall fitness function to be minimized during the optimization process:

$$\text{Min } f(.) = \frac{1,000,000}{1,000,000 + F} + \text{Penalty} \quad (23)$$

where F is given in Equation (14); and Penalty is the sum of four penalties in Equations (19) and (22). The form of this function, including the multiplier of 1,000,000, was found to perform best in a number of preliminary trials.

4.4. Crop models

As mentioned previously, this case study utilizes simple crop production functions, rather than complex mechanistic crop models. Details of these functions are given in Table 3. The area without crops, referred to as Dryland, was not irrigated and has a yield equal to zero.

4.5. Computational experiments

Two computational experiments were implemented to test the utility of the proposed approach to search-space size reduction. The first experiment used static decision variable options (SDVO) in conjunction with the penalty function method for handling constraints (referred to as ACO-SDVO henceforth), and the second used the proposed ACO-DDVO approach for handling constraints. Each computational experiment was conducted for the two formulations and three water availability scenarios in Table 2, and for eight different numbers of evaluations ranging from 1000 to 1,000,000. A maximum number of evaluations of 1,000,000 was selected as this is commensurate with the values used by Wang et al. (2015) for problems with search spaces of similar size. The pheromone on edges for both ACO-SDVO and ACO-DDVO were updated using MMAS.

In order to select the most appropriate values of the parameters that control ACO searching behavior, including the number of ants, alpha, beta, initial pheromone, pheromone persistence and pheromone reward (see Section 2), a sensitivity analysis was carried out. Details of the parameter values included in the sensitivity analysis, as well as the values selected based on the outcomes of the sensitivity analysis, are given in Table 4. It should be noted that visibility factor β was set to 0 (i.e., ignoring the influence of visibility on searching the locally optimal solutions), as was the case in other applications of MMAS to scheduling problems (Szemis et al., 2012). Due to the probabilistic nature of the searching behavior of the ACO algorithms, the positions of starting points are able to influence the optimization results (Szemis et al., 2012). Thus, each optimization run was implemented with 10 replicates, i.e., 10 randomly generated values for starting points in the solution space.

In addition, the best final solutions of the computational experiments from the ACOAs were compared with those obtained by Kumar and Khepar (1980) using linear programming (LP).

5. Results and discussion

5.1. Objective function value

The best solutions from the ACO models over the 10 runs with different random starting positions (i.e., ACO-SDVO and ACO-DDVO) and those obtained by Kumar and Khepar (1980) using LP are given in Table 5. As can be seen, ACO outperformed LP for five out of the six experiments in terms of net returns. For Formulation 1, there was very little difference between the results from the ACO models and LP, in which the percentage deviations for three scenarios (i.e., 100%, 90% and 75% of water availability) were 0.48%, −0.06%, and 0.74%, respectively. This was as expected since the problem formulation is linear. For 90% water availability, the net return using LP was slightly better than that of the ACO models

Table 3
Crop water production functions (from Kumar and Khepar, 1980).

Crop type	Formulation 1		Formulation 2
	Irrigation water W (mm)	Crop yield Y (qt ha^{-1})	
Wheat	307	36.60	$Y = 26.5235 - 0.03274\,W + 1.14767\,W^{0.5}$
Gram	120	18.21	$Y = 15.4759 + 0.04561\,W - 0.00019\,W^2$
Mustard	320	18.44	$Y = 14.743 - 0.011537\,W + 0.41322\,W^{0.5}$
Clover	716	791.20	$Y = 25.5379 - 1.0692\,W + 57.2238\,W^{0.5}$
Sugarcane	542	782.50	$Y = -11.5441 + 2.92837\,W - 0.0027\,W^2$
Cotton	526	13.76	$Y = 6.6038 - 0.013607\,W + 0.62418\,W^{0.5}$
Paddy	1173	47.25	$Y = 5.9384 - 0.035206\,W + 2.412043\,W^{0.5}$

Table 4
Details of the ACO parameter values considered as part of the sensitivity analysis and the optimal values identified and ultimately used in the generation of optimization results presented.

Parameter	Values for sensitivity analysis	Values selected
Number of ants	50; 100; 200; 500; 1000; 2000; 5000; 10,000	100; 1000; 10,000
Alpha (α)	0.1, 0.5, 1.0, 1.2, 1.5	1.2
Beta (β)	0	0
Initial pheromone (τ_o)	0.5, 1.0, 2.0, 5.0, 10.0, 20.0	10.0
Pheromone persistence (ρ)	0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9	0.6
Pheromone reward (q)	0.5, 1.0, 2.0, 5.0, 10.0, 20.0, 50.0	20.0

Table 5
Comparison between best-found solutions using the two ACO formulations (ACO-SDVO and ACO_DDVO) and those obtained by Kumar and Khepar (1980) which are used as a benchmark.

Formulation	Water availability	Net return (Rs)		
		Benchmark	ACO-SDVO	ACO-DDVO
1	100%	785,061.1	**788,851.4** (0.48%)	**788,851.4** (0.48%)
	90%	**741,157.3**	740,731.4 (−0.06%)	740,731.4 (−0.06%)
	75%	647,627.2	**652,438.3** (0.74%)	**652,438.3** (0.74%)
2	100%	800,652.6	890,404.0 Rs (11.21%)	**890,600.7** (11.23%)
	90%	799,725.6	865,441.8 Rs (8.22%)	**873,190.2** (9.19%)
	75%	792,611.2	827,998.0 Rs (4.46%)	**836,839.2** (5.58%)

Note: The bold numbers are the best-found solution. The numbers in parentheses are the percentage deviations of the optimal solutions obtained using the ACO algorithms relative to the benchmark results obtained by Kumar and Khepar (1980). Positive percentages imply that the ACO models performed better than the Benchmark, and vice versa.

(741,157.3 Rs vs. 740,731.4 Rs, respectively). However, this is because the optimal solution obtained using LP could not be found using ACO because of the discretization interval used. For Formulation 2, which is a nonlinear problem, ACO outperformed LP by between 5.58% and 11.23% for ACO-DDVO and by between 4.46% and 11.21% for ACO-SDVO. This demonstrates that the use of EAs is beneficial when solving realistic problems, which are likely to be non-linear. This difference is likely to be exacerbated for more complex problems.

ACO-SDVO and ACO-DDVO performed very similarly in terms of the best solution found. For Formulation 1, as the search space size was relatively small (2.5×10^{36}), identical solutions were found. However, with the larger search space size in Formulation 2 (2.1×10^{160}), ACO-DDVO obtained slightly better solutions (about 1% better for the two scenarios with tighter constraints) and the difference between these solutions increased for decreases in water availability. This is most likely because it is easier to find better solutions in the smaller search spaces obtained by implementing the proposed ACO-DDVO approach.

5.2. Ability to find feasible solutions

The ability of each algorithm to find feasible solutions after a given number of nominal evaluations was represented by the number of times feasible solutions were found from different starting positions in solution space (as represented by the 10 repeat trials with different random number seeds) (see Table 6). As expected, ACO-DDVO always found feasible solutions from all 10 random starting positions for all trials (Table 6), as the DDVO adjustment guarantees that none of the constraints are violated. However, this was not the case for ACO-SDVO. As shown in Table 6, the ability of ACO-SDVO to identify feasible solutions was a function of the starting position in solution space. It can also be seen that the ability to identify feasible solutions from different starting positions decreased with a reduction in the size of the feasible region, as is the case for Formulation 1 compared with Formulation 2 and when water availability is more highly constrained. The ability to identify feasible regions increased with the number of function evaluations, but this comes at the expense of computational efficiency.

Table 6
The number of times feasible solutions were identified out of ten trials with different random number seeds for different numbers of function evaluations, constraint handling techniques and water availability scenarios for the two different formulations of the optimization problem considered.

Formulation	Water availability	Constraint handling	Number of nominal evaluations ($\times$ 1000)							
			1	2	5	10	50	100	500	1000
1	100%	SDVO	5	5	5	5	10	10	10	10
		DDVO	10	10	10	10	10	10	10	10
	90%	SDVO	2	2	2	2	4	4	4	4
		DDVO	10	10	10	10	10	10	10	10
	75%	SDVO	0	0	1	1	4	4	4	4
		DDVO	10	10	10	10	10	10	10	10
2	100%	SDVO	9	9	9	9	10	10	10	10
		DDVO	10	10	10	10	10	10	10	10
	90%	SDVO	9	9	9	9	10	10	10	10
		DDVO	10	10	10	10	10	10	10	10
	75%	SDVO	7	8	8	8	9	9	9	9
		DDVO	10	10	10	10	10	10	10	10

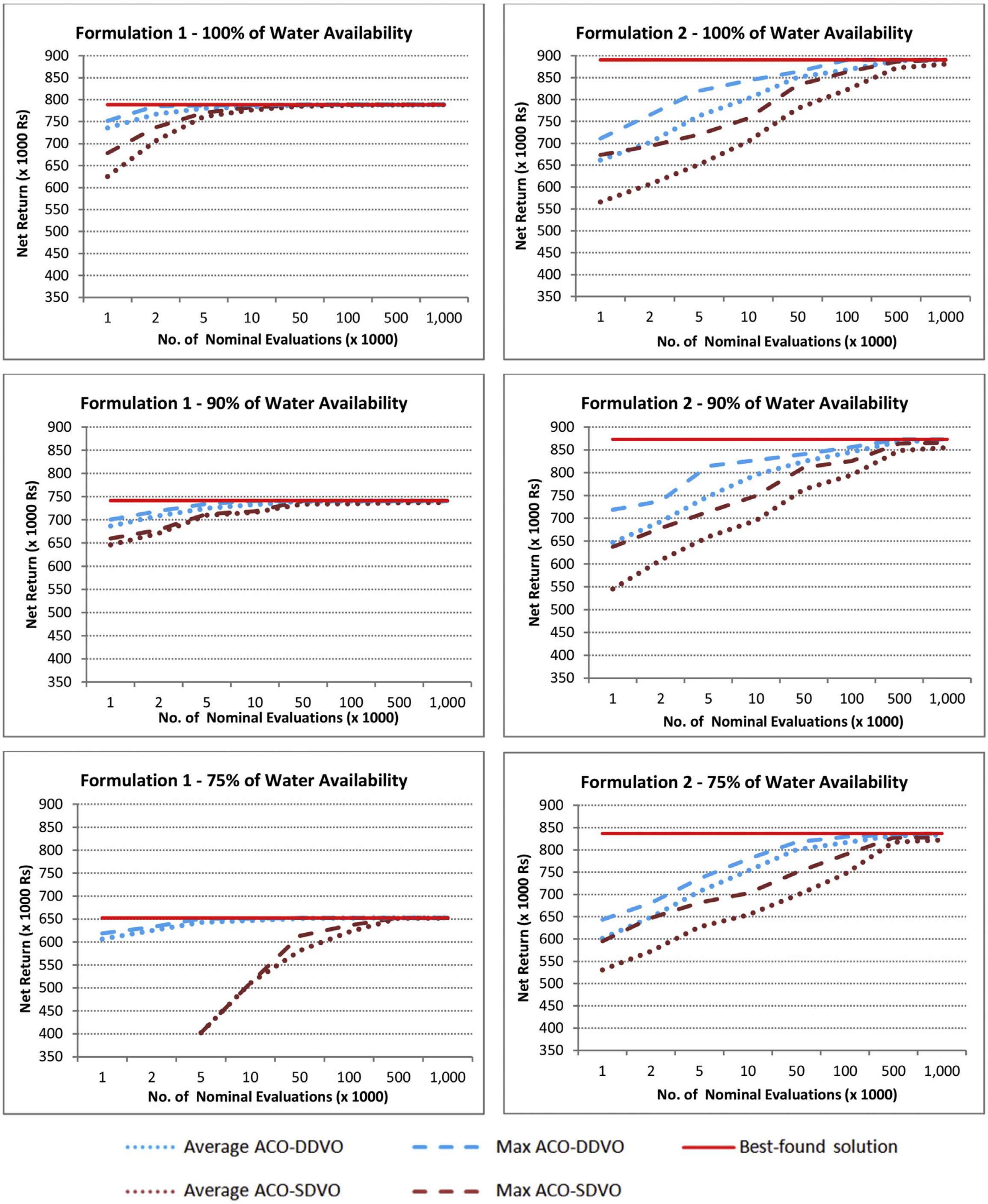


Fig. 5. Convergence of average and maximum optimal solutions obtaining from ACO.

5.3. Convergence of solutions

The convergence of the feasible solutions (i.e., how quickly near-optimal solutions were found) was evaluated against the best-found solution (Fig. 5). It should be noted that the average and maximum net return values for ACO-SDVO were only calculated for the trials that yielded feasible solutions from among the 10 different starting positions in solution space. In general, Fig. 5 shows that convergence speed for the ACO-DDVO solutions is clearly greater than convergence speed for the ACO-SDVO solutions.

In Formulation 1, the difference between the average and maximum results from ACO-SDVO was fairly large at 50,000 and 1000–2000 nominal evaluations for 75% and 100% water availability, respectively. As only one solution was found at 5000–10,000 nominal evaluations for 75% water availability, the average and maximum results are identical. On the contrary, there was no large difference between these solutions for all three scenarios for ACO-DDVO. This demonstrates that the quality of the solutions from the various random seeds for ACO-DDVO was more consistent than that obtained from ACO-SDVO. In addition, the speed of convergence of the results from ACO-SDVO at the best-found solution had an increasing trend when the available level of water increased. The number of nominal evaluations to obtain this convergence were 500,000, 50,000 and 50,000 for 75%, 90%, and 100% water availability, respectively. The corresponding solutions from ACO-DDVO always converged to the best-found solution after 10,000 nominal evaluations.

In Formulation 2, the search space and the number of feasible solutions were larger because of the increase in the number of decision variables (Table 2). As a result, there was a clear difference between the average and maximum solutions for ACO-SDVO. Furthermore, these solutions did not converge to the best-found solution, even with the maximum number of evaluations of 1,000,000. In contrast, although the difference between the average and maximum results from ACO-DDVO increased compared to those from ACO-DDVO in Formulation 1, it was still markedly smaller than those from ACO-SDVO. The solutions obtained from ACO-DDVO always converged at 500,000 nominal evaluations for all three scenarios. Consequently, the results demonstrate that the method of handling constraints in ACO-DDVO resulted in much better convergence towards the best-found solution compared to that of ACO-SDVO, which is most likely due to the reduced size of the search space and the fact that the search is restricted to the feasible region when ACO-DDVO is used.

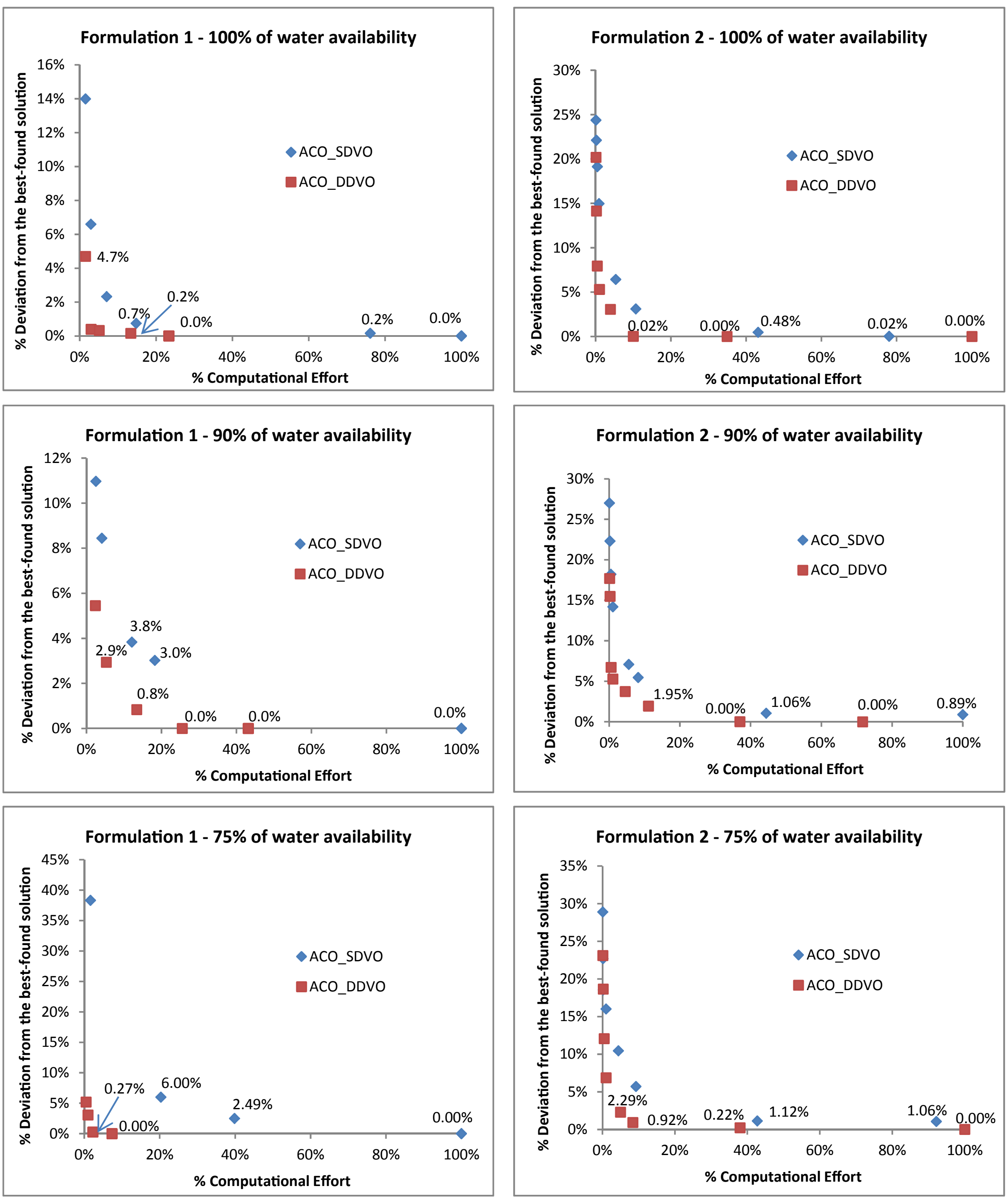


Fig. 6. Computational effort vs. solution quality for the different ACO variants, formulations and water availability scenarios.

5.4. Tradeoff between computational effort and solution quality

The increased computational efficiency of ACO-DDVO compared with that of ACO-SDVO is demonstrated by the relationship between computational effort and solution quality (Fig. 6). It should be noted that the best results over the 10 runs were used to calculate the deviation from the best-found solution and the % computational effort was calculated from the number of actual evaluations. As shown in Section 5.1, ACO-DDVO and ACO-SDVO attained identical solutions for Formulation 1 and ACO-DDVO was able to find slightly better solutions than ACO-SDVO for Formulation 2. However, Fig. 6 shows that these better solutions were obtained at a much reduced computational effort, ranging from 74.4 to 92.7% reduction in computational effort for Formulation 1 and from 63.1 to 90.9% reduction for Formulation 2 (for the same percentage deviation from the best found solution). In addition, near-optimal solutions could be found more quickly. For example, for Formulation 1 ACO-DDVO only needed a very small computational effort to reach a solution with 5% deviation from the best-found solution (about 1.5%, 5.3%, and 1.0% of total computational effort for 100%, 90% and 75% of water availability, respectively). The corresponding values for ACO-SDVO were 7.1%, 12.1%, and 39.9% of total computational effort, respectively. Similar results were found for Formulation 2, in which ACO-DDVO needed less than 5% of the total computational effort and ACO-SDVO required over 40% of the total computational effort for the two scenarios with tighter constraints.

For this case study, the actual savings in CPU time are not that significant (~1.5 CPU hours was saved by using ACO-DDVO for Formulation 2 with 1,000,000 evaluations). However, if complex simulation models were used for objective function evaluation (where a single evaluation could take several minutes), a 63.1% reduction in computational effort would result in significant time savings. For example, if the number of evaluations corresponding to this computational saving was reduced from 872,204 to 322,181, the actual CPU time would be reduced by 5,500,230 s (over 2 months) for a 10-s simulation model evaluation. This demonstrates that the proposed ACO-DDVO approach has the potential to significantly reduce the computational effort associated with the simulation–optimization of crop and water allocation, while increasing the likelihood of finding better solutions.

6. Summary and conclusions

A general framework has been developed to reduce search space size for the optimal crop and water allocation problem when using a simulation–optimization approach. The framework represents the constrained optimization problem in the form of a decision tree, uses dynamic decision variable option (DDVO) adjustment during the optimization process to reduce the size of the search space and ensures that the search is confined to the feasible region and uses ant colony optimization (ACO) as the optimization engine. Application of the framework to a benchmark crop and water allocation problem with crop production functions showed that ACO-DDVO clearly outperformed linear programming (LP). While LP worked well for linear problems (i.e., Formulation 1 where the only decision variable was area), ACO-DDVO was able to find better solutions for the nonlinear problem (i.e., Formulation 2 with decision variable options for depth of irrigated water for each of the selected crops at each of the sub-areas) and for more highly constrained search spaces when different levels of water availability were considered. The ACO-DDVO approach was also able to outperform a "standard" ACO approach using static decision variable options (SDVO) and penalty functions for dealing with infeasible solutions in terms of the ability to find feasible solutions, solution quality, computational efficiency and convergence speed. This is because of ACO-DDVO's ability to reduce the size of the search space and exclude infeasible solutions during the solution generation process.

It is important to note that while the results presented here clearly illustrate the potential of the proposed framework as a proof-of-concept, there is a need to apply it to more complex problems with larger search spaces, as well as in conjunction with more realistic irrigation demands (e.g., Foster et al., 2014) and mechanistic crop growth simulation models (see Section 1). However, based on the demonstrated benefits for the simple case study considered in this paper, the proposed ACO-DDVO simulation–optimization framework is likely to have even more significant advantages when applied to real-world problems using complex crop models with long simulation times.

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