ANALYSIS OF A COMBINATORIAL APPROACH TO THE TRAVELLING SALESMAN PROBLEM

by

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SUMMARY

The subject of this thesis is the study of the recently introduced concept of p-optimality of tours in a network and the application of this concept to the Travelling Salesman Problem.

This study consists of two parts. The first is the theoretical derivation of the number of trials required to test whether a given tour is p-optimal, subject to various assumptions on the nature of an algorithm for carrying out such a test. The main part of this derivation consists of the solution of a number of combinatorial problems concerning the permutations and combinations of points and arcs on the circumference of a circle. These combinatorial results are proved using the notation of tours in a network, although the results themselves can be stated quite generally. These general statements and some physical interpretations are included in an appendix.

The second part consists of a study of the practical application of p-optimality to the Travelling Salesman Problem. A general algorithm for generating p-optimal tours is described and its applicability to the Travelling

Salesman Problem is discussed. A practical version of this algorithm and its application to finding suboptimal solutions to Travelling Salesman Problems is then described. This algorithm is based on the generation of 3-optimal tours, and is a modification of an existing method due to S. Lin. It differs from Lin's method both in the manner in which 3-optimal tours are generated and in the way in which the probability of having obtained an optimal tour at a given stage is estimated. A summary of extensive computational results enables some empirical conclusions to be drawn on the applicability of this algorithm to different sized networks. Finally, an accelerated algorithm is described. This algorithm differs from the above systematic algorithm in that instead of generating 3-optimal tours, it generates "almost 3-optimal" tours by selectively omitting parts of the systematic algorithm. The effect of these omissions in practice is a great increase in computational efficiency, accompanied by only a slight increase in the number of trial tours requiring to be generated. Further computational results demonstrate the efficacy of the accelerated algorithm.

SIGNED STATEMENT

This thesis contains no material which has been accepted for the award of any other degree or diploma in any University. To the best of my knowledge and belief, the thesis contains no material previously published or written by any other person, except where due reference is made in the text of the thesis.

(Glen R. Thompson)

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