Beam Space Signal Processing for Directional Transmission Phased Arrays

Ruiting Yang

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School of Electrical & Electronic Engineering
Faculty of Engineering, Computer & Mathematical Sciences
The University of Adelaide
Adelaide, South Australia

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Contents

Abstract

Beam space (BS) processing is a spatial signal processing technique using beam output data. For example, the BS beamformer applies weights to a set of beam outputs, which are then summed to form a new output. In this way, advanced optimum spatial signal processing algorithms can be applied when the element outputs are not accessible. However, existing BS processing algorithms are based on a model that assumes a passive receiving system or for active systems that the transmission is omni-directional and can be ignored. When the transmission is directional as is typical for phased arrays that electronically scan over a given sector, such methods are mismatched and result in significant performance degradation.

The first part of this thesis presents a new formulation of BS processing for the scenario where relatively narrow beams are directionally transmitted and received and then scanned over a given sector of interest. New formulae are developed for this case and the performance of the new formulae is analysed.

The second part of this thesis is focused on the properties of directional transmission BS processing. When beams are formed in a sector of interest, problems related to the region outside the sector of interest are investigated, including analysing the output in the direction-of-arrival (DOA) of an interference lying outside the sector of interest, removing the high response in the region outside the sector of interest and mitigating a spurious output peak caused by the interference. Additionally, phased array errors cause the array response to be different from that being assumed and can seriously degrade the performance of the BS beamformer, a robust BS beamformer is developed to improve the tolerance to errors. Cramér–Rao Bounds (CRB) for DOA estimation for the directional transmission BS are derived and compared with the omni-directional element space (ES) and BS cases. The performance of the optimum BS beamformer for a non-stationary scatterer is evaluated.

The third part of this thesis deals with BS processing for coherent signals. The commonly used subarray algorithms for removing coherence in the ES processing cannot be applied to the BS problem directly. A method of reconstructing the ES signal subspace is developed for the omni-directional transmission BS case, and then existing methods, such as MUSIC, in ES processing can be applied. For the directional transmission BS case, a method is proposed to reconstruct a matrix which is a summation of weighted self-outer products of ES signal steering vectors, and this matrix allows the DOAs of coherent signals to be estimated regardless of coherence.

Finally, the developed algorithms are investigated by carrying out spatial processing on real experimental data containing stationary targets.

Declaration

I certify that this work contains no material which has been accepted for the award of any other degree or diploma in my name, in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission in my name, for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide and where applicable, any partner institution responsible for the joint-award of this degree.

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Signed　　　　　　　　　　Date

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Functions and Operators

$(\cdot)^*$	complex conjugate
$(\cdot)^{\mathrm{H}}$	Hermitian transpose
$(\cdot)^{\mathrm{T}}$	transpose
$()^{+}$	Moore-Penrose pseudoinverse
$\lfloor\cdot\rfloor$	integer part of the expression
$\circ$	Hadamard product
$\otimes$	Kronecker product
$\frac{d\boldsymbol{v}(\theta)}{d\theta}$	the first order derivative of $\boldsymbol{v}(\theta)$ respect to θ
$\frac{\partial\boldsymbol{v}(\theta_{sl})}{\partial\theta_{sl}}$	the first order partial derivative of $\boldsymbol{v}(\theta_{sl})$ regards to θ_{sl}
$\det(\cdot)$	determinant of a matrix
E{ }	expectation or ensemble averaging
$\exp(\cdot)$	exponential function
$ln(\cdot)$	natural logarithm, logarithm to the base *e*
$\mathrm{rank}(\cdot)$	rank of a matrix
$Re[\cdot]$	real part of a complex expression
$span\{\cdot\}$	form a space spanned by vectors
$\mathrm{tr}[\cdot]$	trace of a matrix
$\mathrm{unif}(\cdot)$	uniform distribution
$\mathrm{vec}\,[\cdot]$	stack the columns of a matrix on top of each other

Variables

α	Hermitian product of $\boldsymbol{w}(\theta_{s1})$ and $\boldsymbol{h}(\theta_{s2})$
$\beta(\theta_{s1},\theta_{s2})$	Hermitian product of $\boldsymbol{h}(\theta_{s1})$ and $\boldsymbol{h}(\theta_{s2})$
$\beta_d(\theta_{s1},\theta_{s2})$	Hermitian product of $\boldsymbol{v_y}(\theta_{s1})$ and $\boldsymbol{v_y}(\theta_{s2})$
γ	amplitude of correlation coefficient between two signals
δ_k	position error at the k-th element
ϵ	Maximum squared Euclidean distance from the assumed steering vector in an uncertainty set
θ	azimuthal angle
$\varDelta\theta$	azimuthal angle difference of a non-stationary scatter between two adjacent pulses
$\varTheta_s$	DOAs of source signals
$\hat{\varTheta}_s$	estimated DOAs of signals
$\varTheta_B$	azimuthal angles of transmit beam centres
$\varTheta_{sc}$	DOAs of coherent signals
$\varTheta_{su}$	DOAs of uncorrelated signals
λ	wavelength of transmit signal
λ_i	the i-th eigenvalue of ES covariance matrix
λ_{yi}	the i-th eigenvalue of BS covariance matrix
$\boldsymbol{\varLambda}$	A diagonal matrix with eigenvalue of ES covariance matrix on the diagonal
$\boldsymbol{\varLambda_y}$	A diagonal matrix with eigenvalue of BS covariance matrix on the diagonal
μ	Langrage multiplier
μ_k	complex gain error on the k-th element
ρ_{ij}	correlation coefficient between two signals $s_i(t)$ and $s_j(t)$

ρ'_l	weighted correlation coefficient in (6-26)
σ_I^2	power of interference
σ_n^2	power of noise
σ_s^2	combined power of a group of coherent signals
σ_{si}^2	power of the i-th signal
$\boldsymbol{\Sigma}$	Singular values
$\boldsymbol{\varphi}$	a vector, whose l-th element is $\pi sin\theta_{sl}$
$e^{j\varphi}$	random phase shift
ϕ_m	difference between the actual and assumed MRA of the m-th beam
$\omega(\theta)$	constrained optimisation function for MVDR
$B(\theta_1, \theta_2)$	Hermitian product of directional transmission BS steering vectors $\boldsymbol{v_y}(\theta_1)$ and $\boldsymbol{v_y}(\theta_2)$
$B_H\left(\theta_{1,}\ \theta_2\right)$	squared Hermitian product of $\boldsymbol{h}(\theta_1)$ and $\boldsymbol{h}(\theta_2)$
$bp(\theta', \theta)$	beam pattern at θ' when the MRA is at a look angle θ
$bp_l(\theta_m)$	beam pattern of the m-th transmitted beam at the DOA of the l-th signal
BW	beam width
c	number of constraints for LCMV
$\boldsymbol{C}$	matrix of c LCMV constraints
$\boldsymbol{C_e}(\Theta_s)$	covariance matrix of estimation errors
$\boldsymbol{C}_{CR}(\Theta_s)$	ES CRB for estimating Θ_s
$\boldsymbol{C}_{CR-\boldsymbol{h}}(\Theta_s)$	omni-directional transmission BS CRB for estimating Θ_s
$\boldsymbol{C}_{CR-\boldsymbol{v_y}}(\Theta_s)$	directional transmission BS CRB for estimating Θ_s
d	separation space between receivers
$\boldsymbol{d}_l$	the l-th column of $\boldsymbol{D}$, the first order partial derivative of ES steering vector, $\boldsymbol{v}(\theta_{sl})$, regards to θ_{sl}
$\boldsymbol{D}$	the first order derivative of $\boldsymbol{V_s}$

$\boldsymbol{D_H}$	the first order derivative of $\boldsymbol{H}$
$\boldsymbol{D_{V_y}}(\Theta_s)$	the first order derivative of $\boldsymbol{V_y}(\Theta_s)$
$\boldsymbol{e}$	a vector contains errors occur at different elements
$\boldsymbol{E_{IS}}$	oblique projection matrix
$\boldsymbol{E_n}$	noise subspace
$\boldsymbol{E_s}$	signal subspace
$\boldsymbol{\mathcal{F}}$	Fisher's information matrix
$\boldsymbol{g}$	vector of constraint values
$G(\theta)$	gain of conventional BS beamformer at θ
$\boldsymbol{h}(\theta)$	omni-directional transmission beam space steering vector at θ
$\boldsymbol{h}'(\theta)$	SFS BS steering vector
$\dot{\boldsymbol{h}}(\theta_{sl})$	the l-th column of $\boldsymbol{D_H}$
$\boldsymbol{h}(\Theta_{su})$	the uncorrelated signal vectors
$\boldsymbol{h_s}$	omni-directional transmission generalised coherent BS steering vector
$\boldsymbol{h_{sc}}$	omni-directional transmission generalised coherent BS steering vector(s)
$\boldsymbol{H}$	A matrix whose columns are the omni-directional BS steering vectors of signals
$H(\theta)$	squared Euclidean norm of BS steering vector $\boldsymbol{h}(\theta)$
H_c	the approximated constant value of $H(\theta)$
$H_y(\theta_s)$	squared Hermitian product of $\boldsymbol{h}(\theta_s)$ and $\boldsymbol{v_y}(\theta_s)$
$\boldsymbol{i}$	interference(s)
$\boldsymbol{I}$	identity matrix
$\Delta\boldsymbol{I}$	diagonal loading in a manner of identity matrix
$\boldsymbol{J}$	exchange matrix
$k_y(\theta)$	squared Euclidean norm of BS steering vector $\boldsymbol{v_y}(\theta)$

K	number of receivers
K_y	an approximated constant value of $k_y(\theta)$ in the sector of interest
L	number of signals
$L(\Theta_s)$	likelihood function
Lc	number of groups of coherent signals
$\boldsymbol{m_s}$	mean of signal component in receiver outputs
M	number of beams
$\boldsymbol{n}$	receiver noise
N	number of pulses (samples), number of averaging in spatial smoothing
N_e	number of extra virtual beams
$p_{\boldsymbol{x}\|\Theta_s}$	probability density for a single snapshot vector
$p_{CB-ES}(\theta)$	output power of conventional ES beamformer at θ
$p_{MUSIC-BS}(\theta)$	BS MUSIC output at θ
$p_{MUSIC-ES}(\theta)$	ES MUSIC output at θ
$p_{MUSIC-Q_s}(\theta)$	MUSIC output at θ using singular vectors of $\boldsymbol{Q_s}$
$p_{MVDR-BS}(\theta)$	output power of optimum BS beamformer at θ
$p_{MVDR-ES}(\theta)$	output power of optimum ES beamformer at θ
$\boldsymbol{P}_{\boldsymbol{V_I}}^{\perp}$	orthogonal projection to $\boldsymbol{V_I}$
$\boldsymbol{q}_1(\theta_m)$	the principal eigenvector of $\boldsymbol{R_x}(\theta_m)$
$\boldsymbol{q}_{1s}$	the principal eigenvector the stacked ES covariance matrix
$\boldsymbol{q}_i$	eigenvector associated with the i-th eigenvalue of ES covariance matrix
$\boldsymbol{q}_{yi}$	eigenvector associated with the i-th eigenvalue of BS covariance matrix
$\boldsymbol{q}_{yc1}$	the principal eigenvector of BS coherent signal covariance matrix
$\boldsymbol{Q_s}$	summation of weighted self-outer products of ES signal steering vectors

$\widetilde{\boldsymbol{Q}}_{\boldsymbol{s}}$	estimated $\boldsymbol{Q}_{\boldsymbol{s}}$
$\boldsymbol{Q}_{\boldsymbol{y}}$	unitary matrix contains eigenvectors of BS covariance matrix
r_b	ratio between the target's location change and beam width
$\boldsymbol{R}_{FB}$	forward/backward spatially smoothed covariance matrix
$\widetilde{\boldsymbol{R}}_{\boldsymbol{n}}$	reconstructed ES noise covariance matrix
$\boldsymbol{R}_{\boldsymbol{s}}$	source signal covariance matrix
$\widehat{\boldsymbol{R}}_{\boldsymbol{s}}$	estimate of the source covariance matrix
$\boldsymbol{R}'_{\boldsymbol{s}}(\theta_m)$	source signal covariance matrix weighted by the beam patterns of the *m*-th transmission beam
$\boldsymbol{R}_{\boldsymbol{sc}}$	coherent source signal covariance matrix
$\boldsymbol{R}_{\boldsymbol{su}}$	uncorrelated source signal covariance matrix
$\boldsymbol{R}_{SS}$	forward spatially smoothed covariance matrix
$\boldsymbol{R}_u^f$	full rank $L \times L$ matrix generated by spatial smoothing
$\boldsymbol{R}_{\boldsymbol{x}}$	ES covariance matrix
$\boldsymbol{R}_{\boldsymbol{x}}(\theta_m)$	ES covariance matrix at the *m*-th directionally transmitted beam
$\boldsymbol{R}_{\boldsymbol{x}}^{(i)}$	ES covariance matrix of the *i*-th subarray
$\widetilde{\boldsymbol{R}}_{\boldsymbol{x}}$	reconstructed ES covariance matrix using matrix inverse
$\widetilde{\boldsymbol{R}}'_{\boldsymbol{x}}$	reconstructed ES covariance matrix from the BS principal eigenvector
$\overline{\boldsymbol{R}}_{\boldsymbol{x}}$	average over ES covariance matrices at different directional transmit beams
$\boldsymbol{R}_{xb}^{(i)}$	covariance matrix of the i-th backward subarray
$\widetilde{\boldsymbol{R}}_{\boldsymbol{xc}}$	reconstructed ES coherent signal covariance matrix from the BS principal eigenvector
$\boldsymbol{R}_{\boldsymbol{xs}}$	stacked ES covariance matrix
$\boldsymbol{R}_{\boldsymbol{y}}$	BS covariance matrix
$\boldsymbol{R}_{\boldsymbol{y}(1)}$	BS covariance matrix for a single signal

$\boldsymbol{R}_{\boldsymbol{y}(1,2)}$	BS covariance matrix for two signals
$\boldsymbol{R}_{\boldsymbol{y}(2)}$	BS covariance matrix when only the second signal is incident
$\boldsymbol{R}_{\boldsymbol{y}\_1}$	BS covariance matrix for a non-stationary scatterer only
$\boldsymbol{R}_{\boldsymbol{y}\_2}$	BS covariance matrix for a stationary scatterer only
$\boldsymbol{R}_{\boldsymbol{y}-\mathrm{aug}}$	BS covariance matrix with virtual beams
$\boldsymbol{R}_{\boldsymbol{y}(\bar{l})}$	BS covariance matrix without the returns from the *l*-th scatterer
$\boldsymbol{R}_{\boldsymbol{yI}}$	Beam space covariance for a model with interference(s)
$\boldsymbol{R}_{\boldsymbol{y}(I)}$	BS covariance matrix for a single interference only
$\boldsymbol{R}_{\boldsymbol{y}}^{\perp \boldsymbol{I}}$	interference orthogonal projected covariance matrix
$\boldsymbol{R}_{\boldsymbol{yn}}$	directional transmission BS noise covariance matrix
$\boldsymbol{R}_{\boldsymbol{y}-ob}$	BS covariance matrix after oblique projection
$\boldsymbol{R}_{\boldsymbol{y}sfs}$	SFS BS covariance matrix
$\boldsymbol{s}$	source signals
$s_{cl}(t)$	part of a mixed signal and is correlated to the transmitted signal
$s_{ul}(t)$	part of a mixed signal and is uncorrelated to the transmitted signal
$s_T(\theta_m, \theta_{sl})$	signal transmitted to θ_{sl} through the sidelobe leakage of a beam whose MRA is at θ_m
$\boldsymbol{s}_{TR}(\theta_m, \theta_{sl})$	array outputs due to a reflected signal at θ_{sl}, which is transmitted through a sidelobe leakage of a beam whose MRA is at θ_m
t	time moment
$\mathbf{U}$	linear transformation between stacked ES and BS covariance matrices
$\boldsymbol{U}_{\boldsymbol{sv}}$	matrix contains the left singular vectors
$\boldsymbol{v}(\theta)$	ES ULA steering vector at angle of θ
$\acute{\boldsymbol{v}}(\theta_{sl})$	ES steering vector at θ_{sl} but with errors
$\boldsymbol{v}_{\boldsymbol{p}}(\theta)$	stacked ES steering vector

$\boldsymbol{v}_{\boldsymbol{s}}$	generalised coherent steering vector in ES
$\widetilde{\boldsymbol{v}}_{\boldsymbol{s}}$	reconstructed generalised coherent steering vector in ES
$\widetilde{\boldsymbol{v}}_{\boldsymbol{ss}}$	reconstructed stacked generalised coherent steering vector in ES
$\boldsymbol{v}_{\boldsymbol{x}}(\theta_{si}, \theta_m)$	ES steering vector at θ_{si} when a signal is directionally transmitted by a beam whose MRA is at θ_m
$\boldsymbol{v}_{\boldsymbol{y}}(\theta)$	directional transmission BS steering vector at θ
$\acute{\boldsymbol{v}}_{\boldsymbol{y}}(\theta_{sl})$	directional transmission BS steering vector at directional θ_{sl} but with errors
$\boldsymbol{v}_{\boldsymbol{y-aug}}(\theta)$	directional transmission BS steering vector with virtual beams
$\boldsymbol{v}_{\boldsymbol{ys}}$	generalised coherent signal steering vector in directional transmission BS
$\boldsymbol{V}$	matrix containing the ES steering vectors at directions of formed beams
$\boldsymbol{V}'$	SFS BS transformation matrix
$\boldsymbol{V}_{\boldsymbol{B}}(\Theta_B)$	same as $\boldsymbol{V}$, formed beams centres are at Θ_B
$\boldsymbol{V}_{\boldsymbol{I}}$	matrix contains the BS steering vector(s) for interference(s)
$\boldsymbol{V}_{\boldsymbol{s}}(\Theta_s)$	matrix contains the ES steering vectors at DOAs
$\boldsymbol{V}_{\boldsymbol{sv}}$	matrix contains the right singular vectors
$\acute{\boldsymbol{V}}_{\boldsymbol{s}}(\Theta_s)$	matrix contains the ES steering vectors at DOAs but with errors
$\boldsymbol{V}_{\boldsymbol{s}}^{(N)}(\Theta_s)$	matrix containing all the source signal steering vectors in a subarray with *N* elements
$\boldsymbol{V}_{\widehat{\boldsymbol{S}}}$	subspace to keep in the oblique projection
$\boldsymbol{V}_{\boldsymbol{x}}(\Theta_s, \theta_m)$	matrix contains steering vectors of DOAs with a directional transmission beam whose centre is at θ_m
$\boldsymbol{V}_{\boldsymbol{y}}(\Theta_s)$	matrix contains directional transmission BS steering vectors at different DOAs
$\boldsymbol{w}(\theta)$	beamformer weight vector at θ
$\boldsymbol{w}_{MVDR-BS}(\theta)$	optimum BS beamformer weights vector at θ
$\boldsymbol{w}_{MVDR-ES}(\theta)$	optimum ES beamformer weights vector at θ

$\boldsymbol{x}(t)$	element output vector at t
x_k	element output at the k-th element
$\boldsymbol{x}^{(i)}$	element output vector of the i-th subarray
$\boldsymbol{x}_b^{(i)}$	element output vector of the i-th backward subarray
$\boldsymbol{x}_s$	stacked element outputs over different transmit beam
$\boldsymbol{y}$	output vector of M conventional beams
$y(\theta)$	beamformer output in the direction at θ

Glossary

AIC	Akaike Information Criterion
BS	Beam Space
CRB	Cramér–Rao Bound
CRLB	Cramér–Rao Lower Bound
DOA	Direction-of-Arrival
DSTG	Defence Science and Technology Group
ES	Element Space
FBSS	Forward/Backward Spatial Smoothing
INR	Interference to Noise Ratio
IQ	In-phase and Quadrature
LCMV	Linear Constrained Minimum Variance
MDL	Minimum Description Length
MIMO	Multiple-Input and Multiple-Output
ML	Maximum Likelihood
MRA	Maximum Response Axis
MVDR	Minimum Variance Distortionless Response
PRF	Pulse Repetition Frequency
PTWR	Phase Tilt Weather Radar
RCS	Radar Cross Section
SFS	Sector Focused Stability
SNR	Signal to Noise Ratio

STF	System Test Facility
SVD	Singular Value Decomposition
Tx/Rx	Transmit/Receive
ULA	Uniform Linear Array
UCA	Uniform Circular Array