AN EMPIRICAL STUDY OF PRICE FORMATION ON THE SYDNEY WOOL FUTURES MARKET

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of the requirements for the degree of
Master of Agricultural Science

by

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This thesis contains no material which has been accepted for the award of any other degree or diploma in any University. Further, to the best of my knowledge and belief, it contains no material previously published or written by another person, except where due reference is made in the text of the thesis.

B. F. HUNT.

CHAPTER 1

INTRODUCTION

1.1 Aim of The Study

The aim of this study is to analyse changes in and between futures price series. We have used the random walk theory as our model of price formation in futures markets, which assumes that a particular futures price series behaves as a simple stochastic process.

The random walk model's origins can be traced to the original notions economist Holbrook Working held about price formation in futures markets.[1] Working believed that futures,

> ....prices depend on expectations regarding the future course of events effecting demand and supply of the commodity being traded. Traders base their expectations on market news which in broad and highly organized commodity markets tends to be generally accurate, timely and relevant.
>
> In an ideal market, existing knowledge of market conditions would be reflected in current price, any new information...would produce a price movement. Truly new information emerges randomly, and so price movements would tend to be random. [2]

In this study we examine how closely price changes in the Sydney Greasy Wool Futures Exchange (S.G.W.F.E.) approximate a random walk. In other words, we wish to see whether the Sydney wool futures market conforms to Working's generalized notions about futures markets. We will also investigate the relationships between different wool futures price series and between wool prices and wool futures price series.

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1 Holbrook Working, "New Ideas and Methods for Price Research," *Journal of Farm Economics*, (Dec. 1956) pp.1427-36, and "A Theory of Anticipatory Prices," *American Economic Review*, May 1958, pp.188-99.

2 Arnold B. Larson, "Measurement of a Random Process in Futures Prices," *Food Research Institute Studies*, Vol.1, No.3 (Nov. 1960) p.316.

The remainder of this chapter contains a description of the evolution and mechanics of futures markets. This is necessary to facilitate an understanding of the theory sections of the following chapters. Chapter 2 consists of a detailed exposition of the random walk theory, while Chapter 3 is a short explanation of the data used in our study. Chapter 4 consists of the results of our investigations into the distribution of futures price changes. In this chapter we look for a distribution in both the theoretical and practical sense.

The most important assertion of the random walk is that price changes are independent of one another and Chapter 5 contains the results of testing this independence assertion. In Chapter 6 we have investigated the theoretical and empirical relationships existing between wool prices, near futures prices and more remote futures prices.

There are many esoteric terms associated with commodity futures markets, and although most will be defined in the text and footnotes a glossary is provided for the uninitiated in Appendix I.

1.2 The Development of Futures Markets

Futures markets did not appear overnight as fully developed economic institutions. They were, in fact, the result of modifications to certain precursor institutions and it is thus possible to trace the evolutionary development of futures markets. Bakken contends there are five distinct stages in the process of market evolution.[3] These are

(1) System of gift giving,

(2) Barter,

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[3] Henry H. Bakken, "Historical Evaluation, Theory and Legal Status of Futures Trading in American Agricultural Commodities," *Futures Trading Seminar*, Mimir, Wisconsin (1960); also John Phillips, "The Theory and Practice of Futures Trading," *Review of Marketing and Agricultural Economics*, Vol.34, No.2 (June, 1966).

(3) Cash markets,[4]

(4) Forward delivery markets,

(5) Futures markets.

We will first consider the factors leading to the development of cash markets from barter trade.

Cash Markets

During the sixteenth and seventeenth centuries a great increase in the supply of precious metals to Europe and the invention of a system of coinage heralded the end of the barter system. The use of money and the development of a merchant code of ethics allowed accounts to be settled in cash. The use of money as a medium of exchange increased the efficiency of exchange as a seller no longer had to find a buyer with something to exchange in return, a seller could now settle with any cash buyer.

The next development involved specialization within the existing cash market. Middle-men entered the market to shoulder the burden of future demand and supply prediction. These merchants specialized in price formation, uncertainty bearing and in the provision of large scale storage. The emergence of middle-men enabled the producer and the processor of the commodity to concentrate on their main activities thus reducing industry costs through division of labour and economies of scale. Another modification to the primitive cash market was the localization of the market. The first cash markets were merely random places on a road where two traders per chance met. The establishment of centralized markets at a particular place, e.g. London, enabled buyers, sellers and middle-men to communicate with each other with greater efficiency.

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4 Cash, Spot and Physical markets are synonymous terms describing a simple commodity exchange.

Forward Markets

It was not long before cash market participants realized there existed a time dimension to market organization. Expectations about the future affected the cash market. As the future is not perfectly predictable, production storage and consumption necessarily involve uncertainty. The forward market was the direct result of market participants' attempts to reduce the degree of uncertainty about the future. If a farmer is assured of a price for his crop prior to planting then he can make the most of his resources in producing that crop. In particular he does not have to retain an amount of capital to cover contingencies such as poor harvest time prices. The forward market allowed the function of price formation and accompanying risk bearing to be transferred as in the cash market to those more adept at handling the risks. With a forward contract the risk of an adverse price change is transferred from the producer to the forward buyer or seller. The assurance given by the forward contract allows banks to provide capital over and above what would be normally loaned on a product not sold or bought forward.

The early forward contracts were either verbal or informal memoranda kept by each party specifying the quantity, price and time of delivery. The value of those early contracts were sharply reduced by uncertainty of fulfillment during times of violent price change when protection against risks was most vital. The contracts turned "soft" when the party disadvantaged by the price change attempted to reneg on his contractual obligation.

The development of modern futures markets was not possible until forward markets had undergone further refinement. It was necessary for the contracts to become far more specific as to quantity, grade and time of delivery. The contracts also had to become personal and negotiable.

Emergence of Futures Markets

The early futures markets were first viewed as predominantly delivery markets.[5] The contracts would in the majority of cases be terminated by delivery.[6] They were organized to facilitate the existing merchandising trade by providing uniform rules governing the transactions. The imposition of a Clearing House that coordinated and underwrote impersonal contracts foreshadowed the emergence of the modern futures market geared to hedging[7] needs.

Gray[8] maintains that although futures markets evolved from forward trading markets, they differ in kind rather than degree.[9] In his opinion forward markets facilitate the forward movement of goods, while futures markets provide hedging facilities.

In futures markets the contract is impersonal and thus transferrable. Hence, a trader can satisfy his contractual obligation through undertaking the opposite position. That is, a trader who has previously sold a contract to deliver a specific quantity of goods some time in the future can complete his transaction by buying a contract to accept the same quantity of goods at the same time in the future. Gray insists futures markets did not supplant forward trading but came to complement the existing trade by enabling traders

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5 H.S.Houthakker, "The Scope and Limits of Futures Trading," in Abromovitz et al, *The Allocation of Economic Resources*, Stanford University Press, Stanford (1959).

6 That is, most of the contracts to deliver goods at a future date ended at the specified time with delivery.

7 Hedging is the process of risk reduction, see Appendix I.

8 Roger W. Gray, *Wool Futures Trading in Australia - Further Prospects*, (University of Sydney, Department of Agricultural Economics: Research Bulletin No.5, 1967); also R.W.Gray and David J.S.Rutledge, "The Economics of Commodity Futures Markets: A Survey," *Review of Marketing and Agricultural Economics*, Vol.39, No.4 (Dec., 1971).

9 Gray used the analogy that although the aeroplane evolved from a balloon, the first aeroplane was not an advanced balloon but a distinct new entity.

to conveniently establish prices for a future date in a standardized version of the commodity against which they could buy or sell for immediate or later delivery at negotiable prices for specific lots, specific as to quantity, grade, location and time. Futures markets were better adapted to hedging usage than to direct merchandising[10] usage. This crucial difference between forward and futures markets is reflected in the fact that only a small percentage of futures contracts terminate in delivery of the goods.

One of the earliest futures markets to evolve was that for grains in Chicago.[11] The market arose shortly after the completion of a canal connecting inland rivers to the Great Lakes. This opened the way for cheap transportation of corn and wheat to the Eastern States and Europe. Chicago was a natural centre for merchandising and disposal of the crops. However, any grain harvested after October could not be transported cheaply to Chicago until the thaw in May. There arose a convention of winter dealings in Chicago for corn to be delivered in May,[12] such contracts soon began to change hands. The establishment of a clearing house that certified and backed negotiable contracts heralded the appearance of one of the first futures exchanges.

Historically, a great deal of time has been required for the transition from cash to futures markets.[13] Also, progress towards futures trading has been appreciably slower in some commodities than in others. It would appear that there must exist certain necessary

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10 Merchandising involves the movement of goods between various sections of the market.

11 Harold S. Irwin, *The Evolution of Futures Trading*, Madison, Wisconsin: Mimir (1954).

12 This represented a transition to forward marketing.

13 Bakken, *op.cit.*, p.14.

conditions in the market and its environs to catalyse the transition from cash to futures markets.

Preconditions for Futures Trading

Goss lists six preconditions for futures trading.[14] First, there must exist production rigidities that dictate a commitment to the physical market. Once such a commitment exists it can only be avoided at considerable cost. For example, a sheep farmer with wool on the animal's back is pledged to shearing and selling the wool. Similarly a woollen manufacturer, once he has decided on a production run, is committed to a purchase of raw wool inputs.

Second, the physical[15] price must fluctuate, thus presenting the party with a physical market commitment with a price risk. As a consequence of a combination of the first two conditions, there will be a demand for hedging facilities. Hedging activities are regarded as the *raison d'etre* of futures markets.[16]

The third precondition is that the commodity must lend itself to standardization for the purpose of drawing up the contract of delivery at a specified date of specified quantity and quality. Unless standardization can be assured, traders will not place any faith in the traded contract. Fourth, delivery of a contract must be possible. If this condition is fulfilled, then theoretically the price of a future at maturity will equal the spot price at that date. Thus, futures prices are linked to physical prices. Fifth, storage must be possible to allow arbitrageurs[17] to buy, carry and sell stocks of the commodity. This type of activity again assures

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14 B.A.Goss, *The Theory of Futures Trading*, London, Routledge & Kegan Paul (1972).

15 See Footnote 3 or Appendix I for a definition.

16 Gray & Rutledge, *op.cit.*, pp.60-61.

17 See Appendix I.

a relationship between spot and futures prices.[18] Finally, a pre-requisite for futures trading is that there exists a number of speculators to take up the difference between long and short hedgers.[19] Goss warns us that although all the necessary pre-conditions exist, there may be forces that prevent the establishment of a successful futures market.[20]

1.3 The Sydney Greasy Wool Futures Exchange

The Sydney Greasy Wool Futures Exchange (S.G.W.F.E.) came into being on 11th May 1960. The principal reasons for its emergence were, its nearness to some of the world's largest wool selling centres and the fact that the Reserve Bank refused to allow funds out of Australia for use on overseas futures markets.[21]

Organization

There are two types of exchange members; floor members and associate members. Floor members are the only parties permitted to actually transact futures business. There are thirteen floor members of the S.G.W.F.E. Associate members are allowed to trade on their own account or for clients through any one of the floor members at half the normal brokerage.

The futures contract is at present the greasy equivalent of 1,500 kilograms clean[22] weight 64's quality good topmaking Merino Fleece Wool. The eight types of futures contracts differ by and

18 The relationship between spot and futures prices will be discussed more fully in Chapter 6.

19 A long hedge involves buying a futures contract and a short hedge involves selling a futures contract, see Appendix I.

20 Goss, *op.cit.*, p.6.

21 More detailed discussion of the reasons for the emergence of the Sydney wool futures markets can be found in Gray, *op.cit.*

22 Greasy and clean are defined in Appendix I.

FIGURE 1·1

A SCHEDULE OF QUOTED FUTURES

DURING THE MONTH OF	CURRENT YEAR					FOLLOWING YEAR					SUCCEEDING YEAR	
	MAR.	MAY	JULY	OCT.	DEC.	MAR.	MAY	JULY	OCT.	DEC.	MAR.	MAY
JAN.	★	★	★	★	★	★	★	★				
FEB.	★	★	★	★	★	★	★	★				
MARCH	★	★	★	★	★	★	★	★				
APRIL		★	★	★	★	★	★	★	★			
MAY		★	★	★	★	★	★	★	★			
JUNE			★	★	★	★	★	★	★	★		
JULY			★	★	★	★	★	★	★	★		
AUG.				★	★	★	★	★	★	★		
SEPT.				★	★	★	★	★	★	★	★	
OCT.				★	★	★	★	★	★	★	★	
NOV.					★	★	★	★	★	★	★	★
DEC.					★	★	★	★	★	★	★	★

NOTE:—In each month shown in the left-hand vertical column of this chart, the officially quoted forward months are those starred in the other columns reading horizontally to the right. For example: In January of the current year, the officially quoted forward months will be: current year, March, May, July, October, December; in the year following, March, May and July.

are named after their maturity dates. The contracts mature on the 23rd of their stipulated month. For example, trading in the October future ceases on the 23rd of October.[23] The eight contracts commence trading approximately eighteen months before their maturation. Hence there are usually two or three futures with maturity dates exactly a year apart, say May 1974 and May 1975. The former is called the May future and the latter the New May Future. Figure 1.1 illustrates the particular futures contracts being traded at any particular time during the year.

If a transactor does not liquidate[24] his position in the futures market, then he is obliged to deliver or take delivery of a quantity of wool. To be accepted as deliverable, wool must belong to one of the 73 prescribed types of quality which range from 70's to 60's.[25] The wool must be greasy and contain only 4 per cent vegetable matter. The deliverable wool must have been sold at one of the recognized Australian selling centres. This regulation ensures the futures market against becoming an alternative to the auction system of selling. Deliverable wool can be tendered at any wool selling centre. If wool of a different quality to average 64's is tendered for delivery, it is quoted at a discount or premium to average 64's. The discounts and premiums are revised every month to correct any price movements of other grades, relative to average 64's.

All transactions must take place through a process of open outcry. This ensures prices are the result of the interaction of

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[23] The fact that the 23rd of the month is used as the maturity date is merely a S.G.W.F.E. convention.

[24] The buying back of a sold contract or alternatively the selling of a bought contract.

[25] The index of wool quality is always referred to as a pure number, e.g. 68's, but has its origins in the number of twists that can be put on certain lengths of yarn made from the wool.

unrestricted supply and demand forces. A floor member cannot match two buy and sell orders without first offering them to the rest of the market through open outcry.

The cost of a completed futures market transaction is split up as follows: $40 brokerage fee, $2.60 Clearing House Charges.[26] The charges are symmetrical; i.e. it costs as much to buy as it does to sell a contract. The S.G.W.F.E. also requires $450 per contract opened (either bought or sold) as a deposit to be refunded on liquidation of the contract. The deposits earn interest at 3 per cent per annum. Margins may be called over and above the minimum deposit to cover adverse price changes.[27] There are also appraisal, warehouse and storage charges associated with delivery of a wool futures contract.

A Clearing House is necessarily associated with any futures exchange. The Clearing House coordinates transactions, settles profits and losses and oversees deliveries. No futures contract is valid unless it passes through the Clearing House. It is for all intents and purposes a buyer to every seller and a seller to every buyer as it guarantees all transactions.

The structure and operation of the S.G.W.F.E. is representative of the majority of commodity futures markets the world over. The only significant dissimilarity is the multiplicity of delivery locations.[28]

1.4 Transactor Types

Participants in a futures market can be somewhat arbitrarily divided into three groups; hedgers, arbitrageurs and speculators.

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26 In N.S.W. there is also a $0.15 stamp duty fee.

27 See Appendix I for an explanation of margins. The current margins are $150 per contract per 10 cent movement.

28 Gray, *op.cit.*, p.25.

A hedger is a party with certain commitments independent of any transactions in the futures market who enters the futures market in order to reduce risks arising out of this commitment.[29] In other words, a hedger is one who holds a position in the futures market opposite to that he holds in the physical market. A hedger may make a short or a long hedge: the former involves initially selling futures and purchasing them again at a later date: the latter requires one to buy futures and later sell them. An example of a short hedge is as follows. A farmer with wool on the sheep's back decides to insure against a change in the price of wool, by buying sufficient futures contracts at price F_0 to cover his entire anticipated clip. When at time 1 he sells his wool, price S_1, he also closes out his futures contracts at price F_1. His total profit on the hedged position equals profit on his physical transaction minus the loss on the futures, that is

$$(S_1 - S_0) - (F_1 - F_0) \quad \ldots\ldots\ldots\ldots \quad (1.1)$$

or

$$(S_1 - F_1) - (S_0 - F_0) \quad \ldots\ldots\ldots\ldots \quad (1.2)$$

which may be positive, zero or negative.[30] The price difference between spot and futures is termed the basis which at time zero equals $(S_0 - F_0)$ and at time period one equals $(S_1 - F_1)$. Thus we can see from expression (1.2) that as long as the basis remains constant over time, then the hedge will be perfect, that is

$$(S_1 - F_1) - (S_0 - F_0) = 0 \quad \ldots\ldots\ldots\ldots \quad (1.3)$$

The hedger substitutes a basis risk for a price risk.

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[29] N. Kaldor, *Essays on Economic Stability and Growth.* London, Duckworth (1961).

[30] The notation and approach is taken from Goss, *op.cit.*, pp.2-3.

Arbitrageurs take a position in both spot and futures markets simultaneously, the aim of which is a certain profit. For example, if the futures price exceeds the spot price by more than the cost of carrying the commodity a riskless profit can be made by buying spot, selling futures and later making delivery of the commodity to complete the futures contract. Similarly, if the spot price is greater than the futures price it is possible for a manufacturer, who needs the commodity, at a later date to sell the commodity, buy futures and take delivery later.

A speculator in the futures market is the same animal that speculates in any other area of uncertainty. By virtue of their expertise and willingness to shoulder risks, they aim to make an uncertain profit.

The division of transactions into hedging, speculation and arbitrages is only a half truth. Hedgers do incorporate price expectations into their decisions, and thus they are to some extent speculating.[31]

Sections 1.2 and 1.3 provide only a very brief and superficial description of the operation of futures markets. It is, however, hoped that this will provide sufficient background information to enable the reader to follow the arguments contained in the following chapters.

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31 A fuller discussion of hedging motives is given in H.Working, "Hedging Reconsidered," *Journal of Farm Economics*, (Nov. 1953) and "New Concepts Concerning Futures Markets and Prices," *American Economic Review*, Vol.52 (1962).

CHAPTER 2

THE RANDOM WALK MODEL

2.1 Theory of the Random Walk

The underlying economic foundations for the model of market behaviour known as the random walk is that an "ideal" market is characterized by many highly informed participants who incorporate into their notion of the worth of the traded good all relevant available knowledge. To paraphrase Holbrook Working, the market price is anticipated in that all existing information gleaned from the past and all that can be perceived about the future is discounted in today's price.[1] The price will thus only change in response to truly new information. New information concerning the numerous determinants of future supply and demand for the market good will emerge randomly and be instantaneously translated into a new price. Hence price changes will be random with respect to time.

Symbolically the model is expressed as;

$$P_t = P_{t-1} + U_t \qquad \text{..........} \qquad (2.1)$$

where P_t is the price of the traded good at time t and U_t is a white noise series (i.e. the residuals are independent of each other) with zero mean. That is

$$E[U_t] = 0 \qquad \text{..........} \qquad (2.2)$$

and $$E[U_t U_{t-s}] = 0 \qquad s \neq 0 \qquad \text{..........} \qquad (2.3)$$

Another criterion of the random walk is the series of residuals form a probability distribution. This second, far less important

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1 Holbrook Working, "New Ideas and Methods for Price Research," *Quarterly Journal of Economics*, (May 1927), pp.394-411.

assertion of the random walk is somewhat controversial; Granger and Labys believe the random walk model makes no assumption as to the distribution of U_t.[2]

The random walk model of market action can be illustrated by a simple dice game. Suppose today's price is 20 and tomorrow's price is determined by tossing two dice, one of which has the usual 1 to 6 on the faces and the other with -1 to -6 as its alternative outcomes. Tomorrow's price is 20 plus the sum of the two dice outcomes. Thus tomorrow's price today has a probability distribution e.g.probability $(P_{t+1} = 17) = .083$, probability $(P_{t+1} = 20) = .16$. The distribution is a function of the generating system only, in this case the dice represent the many small independent influences on prices. The number that determines tomorrow's price is in no way dependent upon past results of the dice. In the "ideal" market, there also exists an independence of today's and tomorrow's prices. It is not possible to conclude anything about future price changes using only past price changes.

> In competitive markets there is a buyer for every seller. If one could be sure that a price will rise, it would have already risen.[3]

The theoretical basis for random walks, as a description of ideal markets is sound but what assumptions about ideal markets need be relaxed when describing real world markets? That is, is the random walk the best description of price changes in an efficient real world market such as the S.G.W.F.E.?

That any market reacts instantaneously to new information is obviously a doubtful truth. First the process of information

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2 Walter C. Labys and C.W.J. Granger, *Speculation Hedging and Commodity Price Forecasts*, (Heath Lexington, Massachusetts, 1970), p.63.

3 Paul A. Samuelson, "Proof that Properly Anticipated Prices Fluctuate Randomly," *Industrial Management Review*, 6 (1965), p.42.

generation may be slow or perhaps monopolized by one section of the market. Second, information once generated may be incorporated into a new price over time as some sections of the market will probably be more adept at analysing the new information. Cargill and Rausser[4] suggest although human and technological limitations prevent any market reacting immediately to new information the delayed response may be only a small portion of the total reaction and should not substantially alter the underlying process. Fama[5] notes that real world markets are not frictionless,[6] and information is not freely available to investors, who all agree on its implications. However, he believes markets will be "efficient" in that price reflects all available information, if sufficient numbers of investors have ready access to the available information.

An important deviation of commodity markets from the ideal markets is the possibility of very high transaction costs. The independence assumption of the random walk is a by product of unhampered competitive trading actions. Thus if the cost of market participation is prohibitive, patterns in the series of residuals, $\{U_t\}$ equation (1), may emerge. The independence assumption of equation (3) would no longer be met. The major components of transaction costs are brokerage, interest charges and cost of storage.

Most agricultural commodities show seasonality of supply which would be translated into seasonality of prices, unless goods

4 Thomas F. Cargill and Gordon C. Rausser, "Time and Frequency Domain Representations of Futures Prices as a Stochastic Process," *Journal of the American Statistical Association*, (March, 1972), Volume 67, No. 337, pp.23-31.

5 E. Fama, "Efficient Capital Markets - A Review of Theoretical and Empirical Work," *Journal of Finance*, (1970), No. 25, pp.383-416.

6 Negligible transaction costs.

are withheld from market during the months of surfeit and added to market in periods of scarcity.[7] The cost of storage, for example refrigeration of apples, may prevent the above arbitraging taking place thus leaving the market with a degree of seasonality of prices.

Samuelson[8] and Snape[9] argue that commodity futures prices will fluctuate randomly even if the particular commodity market contains some seasonality or trend. The basis of their belief is as follows. Arbitrage will ensure the equality of futures and commodity prices at the date of maturation of the specific future. Therefore at any time prior to maturation the futures price will be set by competitive bidding to the expected level of the spot price[10] on the expiration date of the future. Thus today's futures price already contains in itself all that is known about the future, for example, seasonality or inflation.

Samuelson, Working, Cargill, Rausser and Stevenson have provided theoretical justification for believing efficient commodity futures markets possess random price changes as summarized in the random walk model. This model implies futures price changes are independent of past price changes and can be assigned a distribution. Equation (2) of the random walk model says, today's price is the best estimate of tomorrow's price.

2.2 Modifications of the Random Walk

One of the possible modifications of the strict random walk allows a constant percentage change in price over time. This model,

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7 Walter C. Labys and C.W.J. Granger, *op.cit.*, p.84.

8 Samuelson, *op.cit.*, pp.43-49.

9 R.H. Snape, "Price Relationships on the Sydney Wool Futures Market," *Economica*, (May, 1968).

10 Commodity price

referred to as the random walk with drift can be expressed symbolically as

$$P_t = \alpha P_{t-1} + U_t \qquad \alpha \neq 1 \quad \ldots\ldots\ldots\ldots \qquad (2.4)$$

where U_t is a white noise series with zero mean, that is $E[U_t] = 0$ and $E[U_t U_{t+s}] = 0$, $s \neq 0$. The strict random walk can be considered a special case of, $\alpha = 1$ of this general model.

Theoretical justification for the random walk with drift has derived mainly from considerations about share markets. Risk averting share market investors will only participate in the market, if there exists evidence, in past prices, of a normal return on capital invested. The New York Stock Exchange has exhibited a price increase of 6.8 per cent per annum over a 34 year period.[11] The notion of expected returns can be expressed as,

$$E[r] = E[P_{t+s}] - P_t \qquad \ldots\ldots\ldots\ldots \qquad (2.5)$$

where E is the expectation operator; r is return; P_t is the price at time t; P_{t+s} is the price s periods into the future.[12] The strict random walk assumes P_t is bid to $E[P_{t+s}]$ ensuring independence of P_t and P_{t+s}.

Fama's[13] concept of a market's equilibrium expected return being greater than zero implies a degree of dependence between P_t and P_{t-1}, that is $\alpha > 1$ (equation (4)). In such a "fair game"[14] market P_t is set by competitive bidding at a point specifying a level

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11 That is $\alpha = .068$. L.Fisher and J.Louie, "Rates of Return on Investments in Common Stock; The Year-by-Year Record, 1926-1965," *Journal of Business*, Vol.XXXVII, (Jan. 1964), p.315.

12 Strictly speaking P_t and P_{t+s} must be log prices if r is to represent returns.

13 Fama, *op.cit.*, pp.384-388.

14 *Loc.cit.*

of $E[r]$ that justifies the use of funds in the market. Fama contends the market is efficient in that all available information is discounted in P_t. However, included in the discounted information is the notion of a normal expected return.

A market described by the random walk with drift remains a "fair game" in that a trader operating on available information cannot expect to reap greater reward than the equilibrium expected profit. For example, a "chartist"[15] cannot rationally expect greater returns than a policy of always buying-and-holding the security during the future period in question.

The random walk with drift model appears more applicable to the share market than to commodity or commodity futures markets, where inflationary increases in prices are dwarfed by supply and demand induced price changes.

The possibility of seasonal fluctuations in the price of a commodity was discussed in the preceding section. Transaction costs such as storage, insurance and brokerage may prevent seasonality of supply and demand being completely traded[16] out. A modification of the random walk to include seasonality can be expressed thus

$$P_t = P_{t-1} + S(t) + U_t \quad .... \quad (2.6)$$

where U_t is a white noise series obeying equations (2) and (3); $S(t)$ is a seasonal function with a mean equal to zero.

Samuelson has argued that properly anticipated futures prices will not contain any seasonality.[17] However, it is possible a

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[15] The name "chartist" describes a person who uses graphs of past prices to predict future prices by extrapolation.

[16] That is buying when prices are high and selling when they are low.

[17] Samuelson, *op.cit.*, pp.41-49.

futures market watched by only a few speculators and where there is little research could retain systematic seasonality of prices.

Cootner postulates markets are described by yet another modification of the random walk, the random walk with reflecting barriers.[18] To some extent his model was foreshadowed by Taussig's work.[19] He argued supply and demand only roughly specified price and prices wandered aimlessly within a penumbra of uncertainty.

Cootner's justification for the random walk with reflecting barriers comes from a belief that the market can be divided into "naive investors" and market professionals. Naive investors will not recognize any divergence of prices from the intrinsic worth of the traded good. The naive investor is responsible for aimless price fluctuations. When the price differs significantly from its true intrinsic value the professional investor finds it profitable to correct this deviation. That is, professionals erect reflecting barriers around the intrinsic value. Cootner believes this model is an adequate explanation not only of negative correlation between price changes but also the empirically observed "fat tailed" leptokurtic distributions of price changes.

Whether price changes in the S.G.W.F.E. are described by the strict random walk, the random walk with drift, the random walk with seasonality, the random walk with reflecting barriers or some other model will be determined in the following chapters.

2.3 The Effect of Market Structure on Price Changes

Section one of this chapter asserted an ideal efficient market would exhibit stochasticity with respect to price change. This

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18 P.H. Cootner, "Stock Prices: Random Versus Systematic Changes," *Industrial Management Review*, Vol.3, No.2, (Spring, 1962), pp.24-45.

19 F.W. Taussig, "Is Market Price Determinate?" *Quarterly Journal of Economics*, May 1921, pp.394-411.

section considers all possible deviations of market structure and performance from the ideal state. In particular the effect of these deviations on price formation are to be studied. It is only after examination of the theoretical effects of efficient and inefficient market structure on price changes that it will be possible to interpret the statistical results of the following chapters.

Randomly fluctuating prices are only consistent with rapid assimilation of market news into a new price. This occurs when information concerning future supply and demand is quickly and accurately analysed by many competitive market operators. If information is only slowly processed, then prices will only slowly adjust. This form of market imperfection will present itself, statistically, as trends in the price change data. Inaccurate processing of new information could be manifest as price over reaction. For example, prices could reach their new levels by a series of damped oscillations. Larson[20] and Brinegar[21] have shown futures price changes are followed by sharp reactions[22] and longer trends. Unless a market has physically efficient channels of information promulgation, to many highly skilled market participants, a random walk model of price changes is impossible.

The random walk model of market pricing has important implications for persons who claim to be able to predict price changes. For example, investment services and wool brokers. Forecasters can be divided into two categories; technical analysts

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[20] Arnold B. Larson, "Measurement of a Random Process in Futures Prices," *Food Research Institute Studies*, Vol.1, No.3, (Nov., 1960), pp.313-324.

[21] C.S.A. Brinegar, "A Statistical Analysis of Speculative Price Behavior," *Food Research Institute Studies*, Vol.IX, Supplement (1970).

[22] Reaction is a price change in the opposite direction to the initial movement.

and fundamentalists.[23] Technical analysts believe the operation of supply and demand for the traded good shows up as patterns in past prices. In the extreme form technical theorists maintain that only patterns of past prices need be studied, since the effect of everything else is reflected "on the tape."[24] Random walk theorists, while acknowledging that certain patterns may arise expost,[25] deny that these patterns have any prediction properties.

Thus any market which has the property of allowing technical theorists to consistently make a profit is necessarily non ideal. Samuelson summarizes the position of the random walk model with respect to the "chartists". He insists randomly fluctuating prices imply,

> ...there is no way of making an expected profit by extrapolating past changes in the futures price by chart or any other esoteric devices of magic or mathematics [26]

Fundamentalists believe it possible to predict future price changes not by extrapolating past prices, but by examining the underlying economic determinants of market supply and demand. The fundamentalists seek early knowledge of the external factors affecting price changes. Fundamentalists would profit in a market where information assessment is inaccurate or slow.

Granger and Morgenstern, proponents of the random walk, state,

> Any 'capable' or well-informed expert is able to perform better than the majority of players in the market 'game' and so is worth consulting.[27]

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23 Harry V. Roberts, "Stock Market 'Patterns' and Financial Analysis," *Journal of Finance*, Vol.14, No.1, (March, 1959), p.1.

24 George E. Pinches, "The Random Walk Hypothesis and Technical Analysis," *Financial Analysts Journal*, (March-April, 1970), pp.104-110.

25 Pinches, *ibid.*, pp.4-6.

26 Samuelson, *op.cit.*, p.44.

27 C.W.J. Granger and O.Morgenstern, *Predictability of Stock Market Prices*, (Heath Lexington, Massachusetts, 1970).

This may be so in some markets, but a little consideration leads to the conclusion, such markets do not follow a random walk. The ability of a research service to obtain information in advance of the rest of the market would necessarily lead to trends in the price series. Trends would be the result of prices gradually adjusting as information, gathered by the research service, diffused through the entire market.

A random walk market cannot contain sections receiving prior information. Conversely, any market where information reaches some sections more quickly than others, either because of superior research resources or techniques, or because of some quirk of market structure allowing information monopolization, must show a non-random series of prices. A random walk market contains many independent market researchers with similar resources and techniques.

The effect of transaction costs on price changes was discussed when the random walk with seasonality model was postulated. Fama,[28] however, believes even large transaction costs that inhibit the flow of transactions, do not necessarily imply, when transactions do take place, that the price will not fully reflect information. That is, price changes will remain random. This opinion is obviously fallacious in a situation where there exists seasonality of supply or demand and large transaction costs. These large transaction costs are responsible for non-random seasonality of prices. There is also a possibility of alternative price cycles such as daily or weekly cycles, resulting from market imperfections.

What are the effects of market structure and organization on the distribution of price changes? The more "volatile" a market, the more likely it is to have a "fat tailed" non-normal distribution.

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28 Fama, *op.cit.*, p.388.

A volatile market has many quiet trading days interspersed with days of very active trading and large price jumps. Such a market because of the changing nature of transactions from day to day will have far more area under the tails of its distribution, than an even even market where activity is constant. An investor in a "fat tailed" market will need a greater amount of capital to cover all possible contingencies. This is especially pertinent in futures markets where margins are required to cover adverse price changes in an open contract.[29]

The discussion has been general so far, and could equally well apply to commodity stock or futures markets. The next section deals briefly with market structure peculiar to futures and the possibility of futures markets deviating from the ideal efficient futures market as presented by Samuelson[30] and Working.[31]

Gray has shown empirically that certain U.S. commodity futures exhibit "characteristic bias".[32] Such a situation arises when a large portion of transactions involve little or no consideration of the level of the transaction price. This is usually associated with "routine hedging". Provided there are enough well informed "speculators", routine hedging does not alter the underlying process of random fluctuations. However, if there is little conscious formation of expected future prices, the futures price will no longer reflect all available knowledge, thus destroying one of the assumptions of the random walk model.

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29 *Futures Trading and Valuing of Greasy Clips*, Sydney Greasy Wool Futures Limited, (1968).

30 Samuelson, *op.cit.*, pp.41-49.

31 Working, *op.cit.*, pp.394-411.

32 R.W. Gray, "The Characteristic Bias in Some Thin Futures Markets," *Food Research Institute Studies*, Vol.I (Nov., 1960), pp.296-312.

Another possible source of non-random price movements is price squeezing before the maturation of a future. Price squeezing arises, due to the cost of delivering or taking delivery of a futures contract. These costs include purchase costs, transport costs as well as the cost of accepting delivery of a commodity of doubtful quality. If prior to maturation, a substantial portion of one side of all open contracts is controlled by one interest, then they can hold the other market side to ransom. The greater the cost of delivery the greater the possible price squeeze in the last days of a maturing future.

It is clear that unrestrained competition between many highly skilled traders in a futures market would result in random price changes. However, it is also clear there may exist many inefficient market structures causing non-random price changes.

2.4 The Random Walk Theory and the Spot-Futures Price Complex

It has been argued in Section 2.1 that the price of an individual futures contract can be simulated by a random walk model. In any futures market, there is, however, more than one price. In the S.G.W.F.E., there are typically seven futures contracts traded, thus there are at any one time seven futures prices. In addition to the futures prices, there is also a going price for wool, the spot price. What, therefore, is the relationship between these prices, and can the random walk model be extended to cover the entire complex of prices in the spot - futures market?

It is possible that, viewed in isolation, each price series generated in the spot - futures market follows a random walk, but viewed as a whole, only one price series follows a random walk and the other prices are generated from this dominant price series. For example, it could be supposed that all new information concerning future supply and demand factors is translated into a new price in, say, the July future and that all other futures price changes are lagged images of the July future price changes.

The above example is not, however, an accurate description of events in an efficient futures market. In such a market, any pertinent new information affecting the expected future price of wool would produce simultaneous price changes in the spot, and all futures prices. If some of the futures prices tended to lag other futures prices in the adjustment of prices to new information, then speculators would be presented with a profitable situation. Any consistent lag would be removed by speculative intervention. Thus, in an efficient futures market, the random walk model is just as applicable to the whole market with all the associated futures prices as it is to the price of an individual futures contract.[33]

2.5 Previous Research on the Random Walk Model

Bachelier published the first specific exposition of the random walk hypothesis.[34] He tested his theory against prices generated in the French Government bond market. Although it is now recognized that some of his assumptions were invalid,[35] his work remains a path breaking study, marking a beginning in the theory of stochastic processes.

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33 The theoretical aspects of the adjustment of spot and futures prices to new information is discussed in greater detail in Section 6.2.

34 Louis Bachelier, "Theory of Speculation," (1900). Translated and reprinted in P.H. Cootner, *The Random Character of Stock Market Prices*, M.I.T. Press, Cambridge Massachusetts (1964).

35 P.H. Cootner, *The Random Character of Stock Market Prices*, Cambridge Massachusetts, M.I.T. Press (1964), p.4.

During the 1930's, attention was focused on the inability of so-called experts to predict future share price changes. Cowles' demonstration of the inability of the majority of financial specialists to do no better than market averages implicitly supported the random walk model.[36]

Working simulated stock prices by generating a series consisting of cumulative sums of random numbers,[37] Roberts also performed such a simulation.[38] He (Roberts) showed that all the classical indicators of the technical analysts, for example "head and shoulders"[39] could be reproduced by a random generator.

Kendall's work was the first attempt to analyse price fluctuations using contemporary time series techniques.[40] The overwhelming proportion of his results supported independence of price changes. He also concluded the distribution of price changes was approximately normal. Kendall had no preconceived theoretical ideas concerning stock price fluctuations. Thus his empirical results supporting the random walk were surprising to him.

Cowles' and Jones' study in 1937 applied the theory of runs to several indices of stock prices.[41] The results did not support

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[36] A. Cowles, "Can Stock Market Forecasters Forecast?" *Econometrica*, (1933), No.1, pp.309-324.

[37] Holbrook Working, "A Random Difference Series for Use in the Analysis of Time Series," *Journal of the American Statistical Association*, Vol.XXIX, No. 85, (Mar. 1934), pp.11-24.

[38] Roberts, *op.cit.*, pp.3-7.

[39] A term used by technical theorists to describe a series of prices where one peak is flanked by two smaller peaks.

[40] M.G. Kendall, "The Analysis of Economic Time-Series - Part 1: Prices," *Journal of the Royal Statistical Society*, Vol. 96, Part I, (1953), pp.11-25.

[41] A. Cowles and H. Jones, "Some *A Posteriori* Probabilities in Stock Market Action," *Econometrica*, No. 5 (1937), pp. 280-294.

the random walk. Working,[42] however, pointed out the averaging process necessary to produce an index automatically transmitted to the index a degree of serial correlation. Cowles later revised their results.[43] His reworked analysis supported the random walk model. Osborne applied to price changes the theory of molecular movement, viz. Brownian motion.[44] Brownian motion is a specific example of random walk where the distribution of changes is normal. Cootner postulated a modified random walk to explain empirically observed negative correlation and leptokurtosis.[45] He suggested that a random walk with reflecting barriers would describe price changes both theoretically and empirically, Alexander[46] pioneered the use of mechanical trading rules, or filters, as a tool for testing independence of price change. Mechanical trading rules were introduced to counter the market professional's claim that common statistical techniques were unable to measure the complex dependencies the chartist sees in past prices. Alexander's initial results, showing profit accruing to filter rules was interpreted as evidence against the random walk model. However, Mandelbrot demonstrated Alexander's computations incorporated bias which led to serious overstatement of profitability.[47] Alexander's reworked results showed a drastic reduction in profit.

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42 Holbrook Working, "Note on the Correlation of First Differences of Averages in a Random Chain," *Econometrica*, Vol.28 No.4 (Oct. 1960), pp.916-918.

43 Alfred Cowles, "A Revision of Previous Conclusions Regarding Stock Price Behavior," *Econometrica*, Vol.28, No.4 (Oct. 1960), pp.909-915.

44 M.F.M. Osborn, "Periodic Structure in the Brownian Motion of Stock Prices," *Operations Research*, Vol.10 (May-June, 1960), pp.345-379.

45 P.H.Cootner, "Stock Prices: Random versus Systematic Changes", *op.cit.*

46 S.Alexander, "Prices Movements in Speculative Markets: Trends or Random Walks," *Industrial Management Review II*, No.2, 7-26 (May,1961).

47 B.Mandelbrot, "The Variation of Certain Speculative Prices," *Journal of Business*, Vol.36, pp.394-419.

Fama in a series of articles thoroughly examined the hypothesis that share market price changes follow a random walk.[48] In his extensive article, he examined price fluctuation of thirty stocks listed on the New York exchange. Fama used three techniques to test the independence of price changes. These were; serial correlation, runs analysis and Alexander's filter technique. He concluded the independence assumption of the random walk seemed to be an adequate description of reality. However, unlike Bachelier, Osborne, Moore and Kendall, he did not agree that the distribution of price changes was Gaussian.

Mandelbrot and Fama both contended past research had neglected certain consistent departures from normality. Mandelbrot proposed an alternative theoretical distribution best known as stable Paretian.[49]

Spectral analysis as a technique for testing a time series for independence, has recently been applied to commodity, share and futures markets. Granger, Morgenstern and Godfrey,[50] Granger and Morgenstern[51] produced some evidence to suggest the existence of long term cycles. Granger and Labys'[52]investigation into many U.S. commodity futures revealed some cyclical behavior. Leuthold's conclusions regarding price cycles in live cattle futures were ambiguous.[53]

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48 E.Fama, "The Behavior of Stock Market Prices," *Journal of Business*, Vol.38, (1965), pp.34-105.

49 The stable Paretian distribution has "fat tails" relative to the normal distribution.

50 M.D. Godfrey, C.W.F. Granger and O. Morgenstern, "The Random Walk Hypothesis of Stock Market Behavior," *Kyklos*, Vol.17, (1964), pp. 1-30.

51 C.W.J. Granger, and O.Morgenstern, "Spectral Analysis of New York Stock Market Prices," *Kyklos*, Vol.16, (1963), pp.1-27 and Granger and Morgenstern, *op.cit.*

52 Granger and Labys, (1970), *op.cit.*, pp.63-85.

53 Raymond M.Leuthold, "Random Walk and Price Trends: The Live Cattle Futures Market," *Journal of Finance*, Vol.XXVII, (Sept.1972), pp.879-887.

Stevenson and Bear employed serial correlation analysis and filter techniques to examine prices of July futures of corn and soybeans.[54] They concluded the random walk hypothesis did not offer a satisfactory explanation of the movement of speculative price series. Contrary evidence has been put forward by Cargill and Rausser.[55] These researchers, using serial correlation and spectral techniques on a range of futures, argued the random walk was the simplest viable model for commodity futures markets.

Praetz's work is the latest in a series of investigations exploiting the majority of statistical techniques applicable to time series.[56] The techniques employed by Praetz to investigate price changes on the Sydney Stock Exchange were serial correlation, runs analysis, spectral analysis and cross spectral analysis. He summarized his results thus:

> ...Australian share price changes form at best a crude first approximation to a random walk.[57]

He also suggested the empirical observed non-normal distribution could be best described by a rescaled t distribution.[58]

Some of the more recent investigations into various markets have produced results contrary to the random walk model of market action. However, no viable alternative to the random walk has emerged. The random walk remains the most strongly supported description of price changes in share, commodity and futures markets.

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54 Richard A.Stevenson and Robert M.Bear, "Commodity Futures: Trends or Random Walks?" *Journal of Finance*, Vol.25, (March, 1970), pp.65-81.

55 Cargill and Rausser, *op.cit.*

56 P.D. Praetz, *A Statistical Study of Fluctuations of Australian Share Prices*, (University of Adelaide: unpublished Ph.D. thesis, July, 1971).

57 *Ibid*, p.179.

58 *Ibid*, pp.42-51.

CHAPTER 3

DATA

3.1 Construction of Continuous Series

A future has a finite life resulting from a futures contract necessarily having a delivery date. Thus there can be no trading in a particular future after the specified delivery data. The life of a Sydney greasy wool futures contract is approximately eighteen months. To obtain a series of futures prices over a period greater than eighteen months, it is necessary to join together two or more price series.

The price series generated by joining together several series of futures prices are described by their average distance from maturity. For example, a *near* futures price series consists of prices of the future nearest maturity. In March, the *near* futures series consists of prices of the March future until its maturation on the 23rd of March. After the expiration of the March future the *near* futures prices are taken from the price of the May future. The *four month* futures series contains prices of the futures approximately four months from maturation. In March, the *four month* series consists of prices of the July future. Similarly, the *twelve month* and *distant* futures series are made up of futures prices approximately a year and those furtherest from maturity. For example, in March, the new March and new July make up the *twelve month* and *distant* price series respectively.

It is usual in futures markets for the prices of the more distant futures to be at a discount or premium to the nearer months. Table 3.1, a matrix of futures prices for the week beginning 19/3/73 for all seven futures contracts, demonstrates clearly the discounting of prices of the more distant futures *vis-a-vis* the nearer futures.

TABLE 3.1

Daily Closing Prices of All Contracts
(Cents/Kilo)

Month of Delivery	Date				
	19/3/73	20/3/73	21/3/73	22/3/73	23/3/73
March	560	565	575	560	Deleted
May	548	555	565	572	562
July	530	525	542	547	528
October	583	485	498	516	501
December	452	454	467	483	473
New March	411	406	420	440	438
New May	385	393	400	422	419

Source: General Report of the Sydney Greasy Wool Futures Exchange, 23/3/73.

The price differential between adjacent futures is an obstacle to the creation of a continuous homogeneous price series. The price changes within a futures series are of a different nature to the price changes between series. For example, the *near* futures series for the week commencing 19/3/73 (see Figure 3.1) is ...560, 565, 575, 560, 562,.... The price change 560 to 562 represents the change from the March future to the May future and thus, is of a different nature to any other within future price changes. In all price series formed from more than one future the price change between futures has been discarded. The exception to this rule is the monthly price series, to have discarded all between futures price changes would have resulted in removal of five out of twelve observations per year.

Another problem arising during the construction of continuous price series was the occurrence of non-trading days such as public holidays. Again, the price changes either side of the non-trading days (excluding weekends) have been ignored.

3.2 Data Series Used in the Study

The various series are summarized in Table 3.2. Greater detail concerning the collection and formation of the data series is given below.

Monthly Prices

Four monthly future price series were taken from various issues of *The Wool Record and Textile World*. These were *near*, *four months*, *twelve months* and *distant* futures prices. The series were formed by tabulating the closing price (in cents per kilo) on the first day of the month. A series of monthly *physical* prices was also constructed. The series consisted of prices on the first day of the month of average 64's quality clean wool as listed in *Australian Wool Board; Weekly Report*. All five monthly price series consisted of 153 observations covering the period July 1960 to March 1970.

Weekly Prices

Near, *four month*, *twelve month* and *distant* weekly futures price series were constructed from Friday's closing prices as listed in *Wool Record and Textile World*. The four series covering the period 6/1/67 to 7/4/72 contained 271 observations.

Daily Prices

A series of daily *near* futures prices was similarly constructed

TABLE 3.2

Data Series

Observation Interval	No. of Series	Names of Series	Period of Observation	No. of Observations
Monthly	5	Physical, Near, 4 month, 12 month, Distant	1/7/60-1/3/73	153
Weekly	4	Near, 4 month, 12 Month, Distant	6/1/67-7/4/72	271
Daily	1	Near	5/1/64-31/12/71	1,305
Daily	4	Near, 4 month, 12 month, Distant	4/1/72-31/5/73	240
Daily	3	Open, Close Volume	11/12/71-16/7/73	655
Seven Times Per Day	4	July, Oct., Mar., New July	1/5/72-31/5/72	161
Transaction	1	July	1/5/72-31/5/72	1,141
	1]	May	1/5/72-22/5/72	363

from daily closing prices listed in *Wool Record and Textile World*. The 1305 observations covered the period 5/1/64 to 31/12/71.

Four series of daily closing buyer quotes were constructed from prices listed in a weekly communication from the Sydney Greasy Wool Futures Market. The daily *near, four month, twelve month* and *distant* price series covered the period 4/1/72 to 31/5/73.

Using material supplied from G.H. Michell and Sons Pty. Ltd., we constructed two series of daily *near* futures prices. The first series consisted of prices recorded at the opening of each trading day. The second series of a list of prices taken at the close of each trading day. These two series called the daily *open* and *close* futures prices covered the period 11/12/71 to 16/7/73 (655 observations).

A *volume* series consisting of the number of transactions per day was supplied by the S.G.W.F.E. for the *near* future. Combination of daily, *open* and *close* prices and *volume* per day allowed a comparison of price and volume changes.

Seven Times Per Day Prices

This set of series was constructed from information supplied by the S.G.W.F.E. Seven observations on price per day were recorded. The times of the observations (chosen somewhat arbitrarily) were 11 a.m. call, 12.00 Noon, 12.30 p.m., 3.00 p.m. call, 4.00 p.m., 4.20 p.m. and 4.30 p.m. call. Four series resulted from, and were named after, 161 observations on the *July, October, March* and *new July* futures.

Transaction Prices

Two series consisting of the price of all transactions in the July contract during May 1972 were devised from information supplied by S.G.W.F.E. The series were called the *May* and *July transaction* price series.

3.3 Data Transformation

The price series have been transformed by taking logarithms to the base 10 and then first differences. Symbolically, this transformation is represented by

$$y_t = \log.P_{t+1} - \log.P_t \quad \ldots\ldots\ldots\ldots \qquad (3.1)$$

where P_t is price of the traded good at time t.

Rationale for the transformation is forwarded by Moore,[1] Fama[2] and Granger and Morgenstern.[3] The theoretical justifications for the above transformation are summarized below. First, the distribution of prices is bounded from below at zero, but it has no upper limit, while the log. transformed distribution is symmetrically unbounded. Thus the transformation may reduce skewness about the mean. Second, a change in log. prices represents the yield from holding the security. To most market participants the percentage price change is all they are interested in, to them the absolute price change is irrelevant and for small changes, e.g. less than 15 per cent, the log. price change is very close to the percentage change.

Moore's work demonstrated another reason for the use of the log. transformation. He showed the variability of simple price changes, for a given stock, was an increasing function of the price level of the stock. The log. transformation neutralized this price level effect.[4]

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1 Arnold Moore, "A Statistical Analysis of Common-Stock Prices," unpublished Ph.D. dissertation, Graduate School of Business, University of Chicago (1962), pp.13-15.

2 E. Fama, "Behavior of Stock Market Prices," *Journal of Business*, 38, pp.45-46.

3 C.W.J. Granger and O. Morgenstern, *Predictability of Stock Market Prices*, D.C. Heath and Co. Lexington, Massachusetts, (1970), p.74.

4 Moore, *op.cit.*, pp.13-15; also the empirical results of Section 4.5 demonstrate the stability of the log. transformed series.

CHAPTER 4

DISTRIBUTION OF FUTURES PRICE CHANGES

4.1 Preamble

The random walk model of price changes makes two basic assertions. First, price changes should be independent of one another and second, price changes conform to some probability distribution. The independence assumption is generally considered the more important of the two assertions, and historically, has commanded greater attention from econometricians. Nevertheless, investigation of the distribution of price changes remains a valid academic pursuit for both theoretical and practical reasons, some of these reasons are presented below.

A knowledge of the distribution of price changes assists in understanding the underlying process generating the price changes.[1] For example, frequent large scale price changes indicate that the underlying determinants of market supply and demand are themselves subject to violent change. Evidence of the usefulness of price change distributions is presented by Cootner, who has utilized distributional facts to gain insight into market mechanisms.[2] His model of market segmentation is partly based on the shape of his empirical distributions.

Fama[3] and Praetz[4] agree that the distribution of price changes is helpful to the erstwhile investor. It is the distribution of

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1 E.Fama, "The Behavior of Stock Market Prices," *Journal of Business*, Vol.38, (1965), pp.34-105.

2 P.H. Cootner, "Stock Prices: Random vs. Systematic Changes," *Industrial Management Review*, Vol.3 (1962), pp. 26-45.

3 E.Fama, "Mandelbrot and the Stable Paretian Hypothesis," *Journal of Business*, Vol.36, No.4, (Oct., 1963), pp.420-429.

4 P.D. Praetz, *A Statistical Study of Fluctuations of Australian Share Prices*, unpublished Ph.D. thesis, Adelaide, (July, 1971), p.66.

price changes that determines the riskiness of an investment. Thus a market with a distribution function represented by A(y), (Figure 4.1) is less likely to have large price changes than a market that has distribution function B(y), (Figure 4.1). Fama considers a market with a price change distribution B(y) has important practical implications for the investor who attempts to minimize his risk of a loss by placing "stop-loss"[5] orders. In his opinion prices in the market may change so rapidly, as to render a stop-loss order inoperable.

Finally, Fama[6] and Mandelbrot[7] believe the shape of the price change distribution is important to anyone wishing to undertake empirical work upon speculative markets. Most statistical techniques are dependent upon certain assumptions about the mean and variance of the data under consideration. In particular, acceptance of Mandelbrot's infinite variance hypothesis[8] invalidates most statistical tools such as correlation, regression and spectral analysis.

4.2 The Theory of Normal Price Changes

Bachelier[9] initially and later Osborne [10] outlined theoretical reasons for believing price changes in speculative markets to be normally distributed.

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5 A "Stop loss" strategy involves placing a sell order with a broker at a price determined by the maximum loss an individual is prepared to take.

6 E. Fama, "The Behavior of Stock Market Prices," *op.cit.*, p.41.

7 Benoit Mandelbrot, "Variation of Certain Speculative Prices," *Journal of Business*, Vol. 36, No.4, (Oct., 1963), pp.394-419.

8 *Loc.cit.*

9 Louis Bachelier, "Theory of Speculation" translated and reprinted in P.H. Cootner, *The Random Character of Stock Market Prices*, M.I.T.Press, Cambridge, Massachusetts (1964).

10 M.Osborne, "Periodic Structure in the Brownian Motion of Stock Prices," *Operations Research*, 10, pp.345-379.

FIGURE 4.1

ALTERNATIVE DISTRIBUTIONS OF PRICE CHANGE

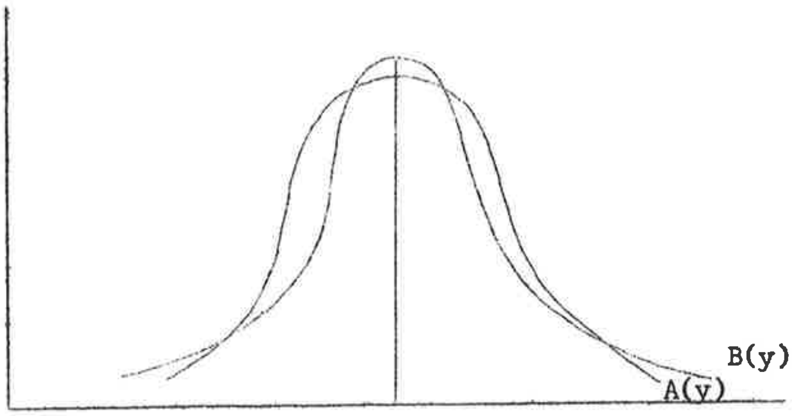

TABLE 4.1

Values of the Mean, $\bar{y}$ (x 10^4)

Interval Series	*Daily*	*Weekly*	*Monthly*
Physical Prices			25.92
Near future	10.68	-13.71	26.09
Four month future	13.35	-10.52	23.16
Twelve month future	10.79	- 6.73	18.40
Distant future	9.29	- 2.94	17.09

Let $y_t = \log.P_t - \log.P_{t-1}$ .... (4.1)

where P_t represents the price in period t and similarly P_{t-1} is the price in period t-1. Then if $\{y_t\}$ is normally distributed it can be represented by the probability function

$$f(y) = \frac{1}{(2\pi\sigma^2)^{\frac{1}{2}}} e^{-(y^2/2\sigma^2)} \quad .... \quad (4.2)$$

where σ^2 represents the variance of $\{y_t\}$. The rationale behind the normal distribution of price changes is a simple appeal to the central limit theorem. The central limit theorem states that the sum of many independent random variables (irrespective of their individual distribution functions) is asymptotically normal.[11]

The Bachelier-Osborne model assumes price changes between transactions are independent random variables. As long as the number of transactions per time period is large and approximately constant then the central limit theorem should hold true. This is, price change per period, being the sum of many independent random variables and thus should be normally distributed. Moreover the variances of the distributions should be proportional to the differencing interval. That is, the variance of weekly price changes should be approximately five times the variance of daily price changes.[12]

The market model incorporating normality of price changes is intuitively appealing. There are many different determinants of price in any market. A small change in any of these numerous variables will result in a flow of transactions. For example, the price of wool futures can be affected by changes in the weather,

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11 The distribution of the sum of n random variables approaches a normal distribution as the number n becomes very large.

12 E. Fama, "Behavior of Stock Market Prices," *op.cit.*, p.41.

tastes, production, income, interest rates, exchange rates, liquidity, etc.

Moore[13] and Kendall[14] are two investigators who have provided empirical evidence supporting normal distribution of price changes in stock markets. Both are studies on share prices movements and while there existed within their results some evidence of leptokurtosis they concluded that their empirical distributions were Gaussian. Given that wool futures price changes derived from many varied sources and that some researchers had concluded that price changes in some share markets were random, it could be argued that there were *a priori* reasons for believing Sydney wool futures price changes would be normally distributed.

4.3 Empirical Distribution of Wool Futures Price Changes

The various price series were transformed to log. first differences for distributional analysis. The philosophy behind such a transformation was outlined in Section 3.3. In addition to twelve series of futures prices, a monthly series of physical prices was studied.

Mean

The variable under study was y_t as given in equation (4.1), that is,

$$y_t = \log.P_t - \log.P_{t-1} \qquad \ldots\ldots\ldots\ldots \qquad (4.3)$$

The mean of a series $\{y_t\}$, $\bar{y}$ is defined as

$$\bar{y} = \frac{1}{n}\sum_{t=1}^{n} y_t \qquad \ldots\ldots\ldots\ldots \qquad (4.4)$$

where n is the number of observations in the series $\{y_t\}$. The

13 Arnold B. Moore, "Some Characteristics of Changes in Common Stock Prices," in P.H. Cootner, *op.cit.*, pp.139-161.

14 M.G. Kendall, "The Analysis of Economic Time Series: Part 1: Prices," *Journal of the Royal Statistical Society*, Vol. 96, (1953), pp.11-25.

empirically determined values for the mean are given in Table 4.1.

It would seem that the futures furthermost from maturity have means closest to zero. The monthly series stretch across the entire period of operation of the S.G.W.F.E. and thus the inference from Table 4.1 is that wool prices and wool futures prices have risen over the period of operation of the Sydney wool futures market.

Variance

The variance of a series $\{y_t\}, s^2$ is defined thus

$$s^2 = \sum_{t=1}^{n} (y_t - \bar{y})^2/n \quad \ldots\ldots\ldots\ldots \quad (4.5)$$

where n is the number of observations and $\bar{y}$ is the mean of the series. The variance measures the spread of observations about the mean. The calculated variances for thirteen price series are given in Table 4.2.

The variance of the *physical* price series is greater than any of the *monthly futures* prices. This is important to any person with an option on either a wool market or wool futures market transaction. For example, if a market observer reckons that wool prices will rise he has the choice of either buying wool or buying wool futures. Given risk is correlated with price variability it follows that a futures market transaction is less risky.

If price changes are independent, the variance of the distributions should be proportional to the differencing interval.[15] Hence, the variance of the weekly series should be approximately one quarter the variance of the monthly price series. The weekly empirical variances listed in Table 4.2 are indeed approximately

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15 E.Fama, "The Behavior of Stock Market Prices," *op.cit.*, p.42; C.W.J. Granger and O.Morgenstern, *Predictability of Stock Market Prices*, Lexington, Massachusetts (1970), p.171; also see section 4.2.

TABLE 4.2

Estimated Variances, s^2

Interval / Series	*Daily*	*Weekly*	*Monthly*
Physical Prices			53.4
Near future	25.1	12.7	49.2
Four month future	22.0	9.4	42.1
Twelve month future	20.3	6.3	26.9
Distant future	18.2	7.5	24.7

one quarter the monthly variances. However, the variance of the daily series certainty is not one fifth of the variance of the weekly series. The daily series spans a period of very high "activity".[16] We believe the variance of price changes is itself a variable and thus the daily variance relative to the weekly or monthly variance will not simply be proportional to the differencing interval. This is discussed in much greater detail in Section 4.5. An alternative hypothesis that the anomalous daily variances are the result of dependencies within the series is not supported by the evidence of Chapter 5.

The results of Table 4.2 agree with the oft stated but little tested notion of price variability as a function of the distance to maturity.

16 A period of high activity is accompanied by a high volume of transactions stemming from traders widely fluctuating expectations.

> It is a well known rule of thumb that nearness to expiration date involves greater variability of riskiness per hour per day or per month than does farness.[17]

The consequence of increasing variance (as the future approaches maturity) is increasing risk of transaction as the time elapsing between closing out a transaction and maturation becomes smaller. It is this increasing risk that prompts wool futures brokers to suggest that hedging should be undertaken in a future that matures a couple of months after the hedger has sold his wool, rather than in a future that expires at the same time he sells his wool.

Skewness

The second and third moments about the mean, u_2 and u_3 respectively are defined as,

$$u_2 = \sum_{t=1}^{n} (y_t - \bar{y})^2/n \qquad \text{..........} \qquad (4.6)$$

and

$$u_3 = \sum_{t=1}^{n} (y_t - \bar{y})^3/n \qquad \text{..........} \qquad (4.7)$$

A standardized measure of skewness, β_3, can be expressed symbolically as

$$\beta_3 = \frac{u_3{}^2}{u_2{}^3} \qquad \text{..........} \qquad (4.8)$$

If the distribution is symmetric $\beta_3 = 0$, if the distribution is skewed to the right $\beta_3 > 0$ and if the distribution is skewed to the left $\beta_3 < 0$.

Pearson has shown three standard deviation (3s) limits for β_3 are given by $\pm 9(6/n)$.[18] We decided to accept the distributions as being symmetrical if the observed values of β_3 lie within the 3s limits.

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17 P.A.Samuelson, "A Proof that Properly Anticipated Futures Prices fluctuate randomly," *Industrial Management Review*, Vol.6 (1965), p.44.

18 E.S.Pearson, "A Further Development of Tests for Normality," *Biometrika*, Vol.11 (1930), pp.239-249.

Table 4.3 contains observed values of the standardized measure of skewness.

TABLE 4.3

Values of the Standardized Measure of Skewness β_3

Interval Series	*Daily*	*Weekly*	*Monthly*
Physical			11.8*
Near future	.14	1.76*	1.4*
Four month future	.25*	1.51*	3.1*
Twelve month future	.00	.86*	1.9*
Distant future	.00	.04	1.8*

* indicates significance at the ±3s level.

The majority (9 out of 13) of the distributions were skewed. These were all positively skewed, that is, the body of observations lie to the left of the mean as shown diagrammatically in Figure 4.2.

Of the four daily futures price series only one is skewed. It would seem that short run, e.g. daily distributions are less skewed than the longer run, e.g. monthly distributions. Skewed empirical distributions have been encountered by other researchers, Mandelbrot[19] and Press[20] implied their distributions were occasionally skewed but presented no actual evidence to support their statements. Praetz[21]

[19] Mandelbrot, *op.cit.*

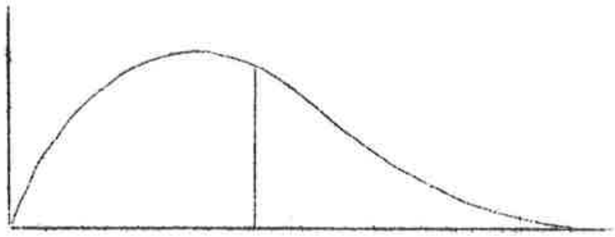
[20] S.J. Press, "A Compound Events Model for Security Prices," *Journal of Business*, Vol.40 (1967), pp.317-335.

[21] Praetz, *op.cit.*, pp.29-30.

FIGURE 4.2

Diagrammatic Representation

of a Positively Skewed Distribution.



demonstrated that over a quarter of his distributions were skewed, but he made no further comment. There is no *a priori* reason to expect skewed distributions. Perhaps the empirical results derive from violation of the stationarity assumptions of equations (2.2) and (2.3).[22] For example, one could easily simulate a similar skewed distribution to Figure 4.2, by permitting the mean of a symmetrical distribution to increase over time.

Kurtosis

The second and fourth moments about the mean, u_2 and u_4 respectively, are defined thus:

$$u_2 = \sum_{t=1}^{n} (y_t - \bar{y})^2/n \qquad \text{..........} \qquad (4.9)$$

---

22 The stationarity assumptions are that mean $\{y_t\} = 0$ and variance of $\{y_t\}$is constant. A shift in the mean coupled with changing variance could result in skewness.

and $$u_4 = \sum_{t=1}^{n} (y_t - \bar{y})^4/n \qquad \text{..........} \qquad (4.10)$$

Let $$\beta_4 = \frac{u_4}{u_2^2} \qquad \text{..........} \qquad (4.11)$$

β_4 is a prime number and represents a measure of kurtosis (or peakedness). β_4 can be any positive number, $\beta_4 = 3$ for the normal distribution. Pearson gives the 3 standard deviations limits for kurtosis of a normal distribution as $3 \pm 3(24/n)^{\frac{1}{2}}$.[23] The empirically observed values of β_4 are tabulated in Table 4.4.

TABLE 4.4

Values of the Measure of Kurtosis, β_4

Internal Series	*Daily*	*Weekly*	*Monthly*
Physical price			28.0*
Near future	6.0*	9.5*	8.5*
Four month future	6.4*	10.6*	12.2*
Twelve month future	4.8*	8.04*	9.4*
Distant future	6.4*	9.01*	9.2*

* means significant at the 3s limit.

All series were significantly leptokurtic, $\beta_4 > 3$,[24] that is, the observed distributions are somewhat more peaked relative to the normal distribution. Too many of the observations fall near the

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23 Pearson, *op.cit.*

24 If A(y) of Figure 3.1 represents the standardized normal distribution, then B(y) is leptokurtic.

TABLE 4.5

Comparison of the Empirical Frequency Distribution with the Expected Normal Distribution for *Weekly* Futures Price Changes

Interval (in standard deviation units)	*Near* (1)	*4 Month* (2)	*12 Month* (3)	*Distant* (4)	Average (5)	Normal Expected Frequency (6)	Normal Minus Average (7)
s ≤ -2.0	1	8	6	6	7.25	5.45	-1.80
-2.0 ≤ s ≤ -1.8	0	1	2	1	1.00	3.13	2.13
-1.8 " -1.6	4	1	3	7	3.75	4.52	0.77
-1.6 " -1.4	3	0	3	1	1.75	6.21	4.46
-1.4 " -1.2	3	2	5	1	2.75	8.20	5.45
-1.2 " -1.0	3	4	8	3	4.5	10.42	5.92
-1.0 " -0.8	2	7	8	12	7.25	12.72	5.47
-0.8 " -0.6	10	12	14	17	13.25	14.91	1.66
-0.6 " -0.4	23	18	14	16	17.75	16.80	-0.95
-0.4 " -0.2	29	26	15	23	23.25	18.19	-5.06
-0.2 " 0.	32	32	28	28	30.00	18.95	-11.05
0. " 0.2	26	31	34	32	30.75	18.95	-11.80
0.2 " 0.4	24	36	34	33	31.75	18.19	-13.56
0.4 " 0.6	24	17	11	14	16.50	16.80	0.30
0.6 " 0.8	15	19	18	13	16.25	14.91	-1.34
0.8 " 1.0	10	6	12	7	8.75	12.72	3.97
1.0 " 1.2	4	5	4	6	4.75	10.42	5.67
1.2 " 1.4	6	4	7	6	5.75	8.20	2.45
1.4 " 1.6	3	3	3	5	3.5	6.21	2.71
1.6 " 1.8	2	2	2	3	2.25	4.52	2.27
1.8 " 2.0	0	0	4	1	1.25	3.13	1.88
2.0 ≤ s	7	5	3	5	5.0	5.45	0.45
	239	239	239	239	239.0	239.0	0.0

mean and in the extremities of the distribution. The results of Table 4.3 are hardly unique as leptokurtic distributions of speculative price changes have been observed by Cootner,[25] Granger and

[25] Cootner, *op.cit.*

Morgenstern,[26] Mandelbrot,[27] Fama[28] and Praetz.[29]

4.4 Normality of Empirical Distributions

The kurtosis results suggested the empirical distributions were non-normal. All series were leptokurtic with values of β_4 ranging from 6 to 28. The next three sections contain the results of three additional techniques designed to test for departure from normality. The three techniques are: exposition of frequency distributions, normal probability graphs and chi squared goodness of fit test.

Frequency Distributions

A simple way of analysing the distribution of price changes is to construct frequency distributions for individual series. The observed frequencies are grouped into standard deviation (s) intervals and can thus be compared to the normal distribution. Given the total number of observations, the expected frequencies (for each interval) of the normal distribution can be computed and compared to the actual observed frequencies.

Table 4.5 contains the observed frequencies for four weekly futures price series. Columns (1) to (4) contain the observed frequency and column (5) the result of averaging (1) to (4). Column (6) consists of the expected normal frequencies and finally, column (7) is the result of subtracting (5) from (6).

It is apparent from the results tabulated in column (7) that the observed distributions are more peaked in the centre and have longer tails than the Gaussian distribution.[30] Figure 4.3 diagrammatically

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26 Granger and Morgenstern, *Op. cit.*, p.184.

27 Mandelbrot, *op.cit.*

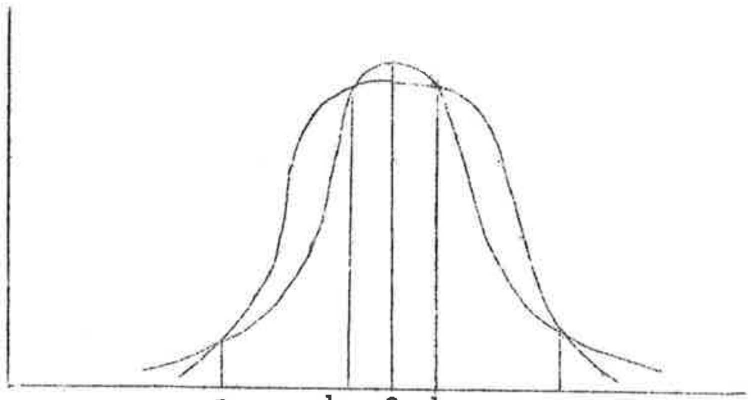
28 Fama, "Behavior of Stock Market Prices," *op.cit.*pp.47-61.

29 Praetz, *op.cit.*, p.30.

30 This result reinforces the conclusions drawn from kurtosis values.

FIGURE 4.3

Diagrammatic Comparison of Observed and Normal Distributions



superimposes the empirical and normal distributions. The broken line represents the empirical distribution and the solid curve the normal distribution. There are four points where the two distributions cross over, -a, -b, b and a. We estimated from Table 4.5 that a $\simeq$ 2.0 and b $\simeq$ 0.6.

Normal Probability Plots.

This technique consists of graphing the relative cumulative frequency (R.C.F.) of a particular observation against the relative cumulative frequency of the unit normal distribution. If in fact the empirical distribution was normally distributed the resulting plot would be a straight line passing through coordinates (0,0) and (1,1). Normal probability graphing is a simple visual test of the Gaussian hypothesis. The normal hypothesis is rejected if the plot deviates from a 45 degree straight line. Figures 4.4, 4.5, 4.6 and 4.7 are normal probability plots of daily log price changes in the *near four month, twelve month* and *distant* futures series.

FIGURE 4·4

NORMAL PROBABILITY PLOT ;

DAILY NEAR FUTURE PRICES

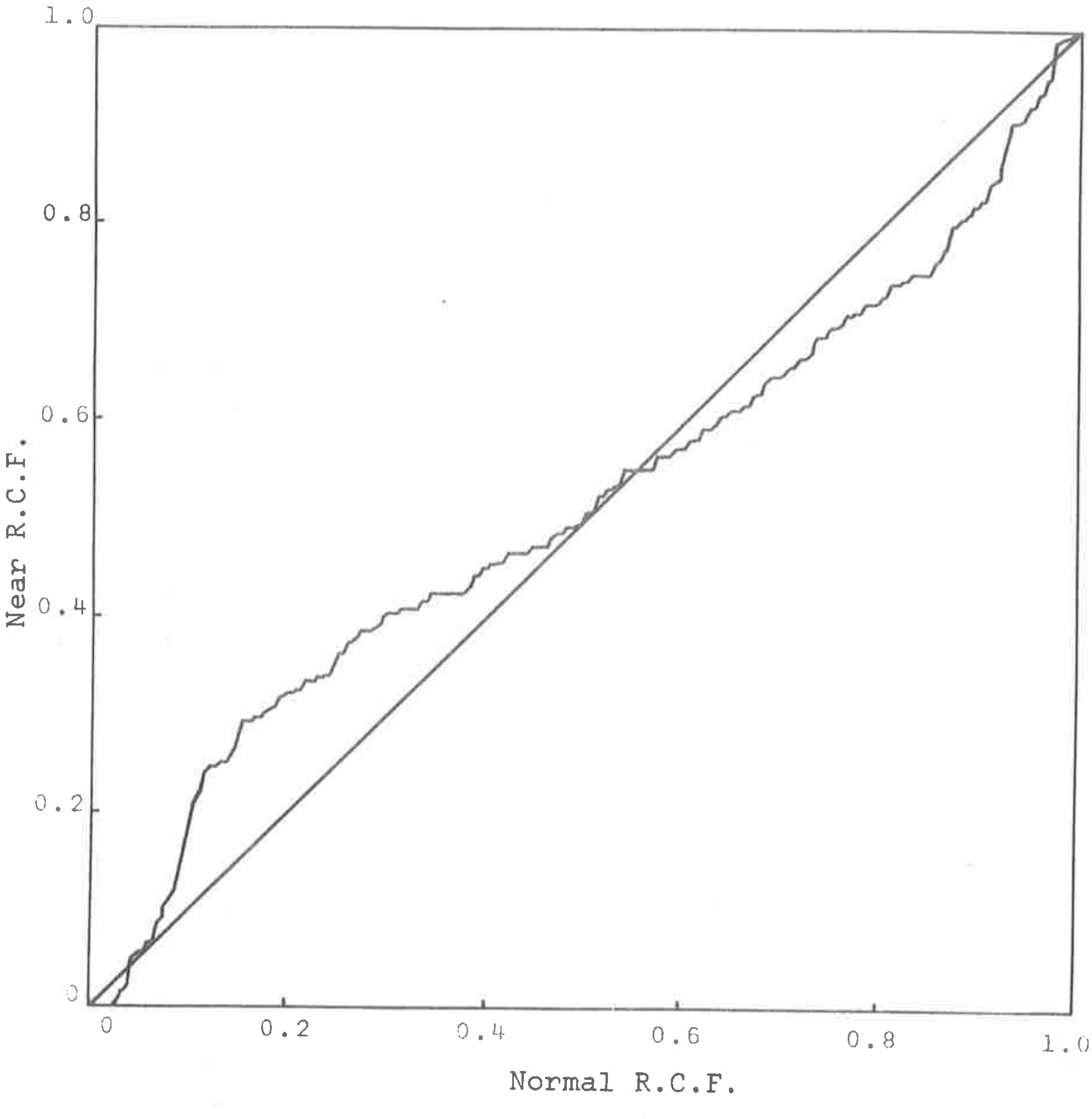


FIGURE 4·5

NORMAL PROBABILITY PLOT;

DAILY FOUR MONTH FUTURE PRICES

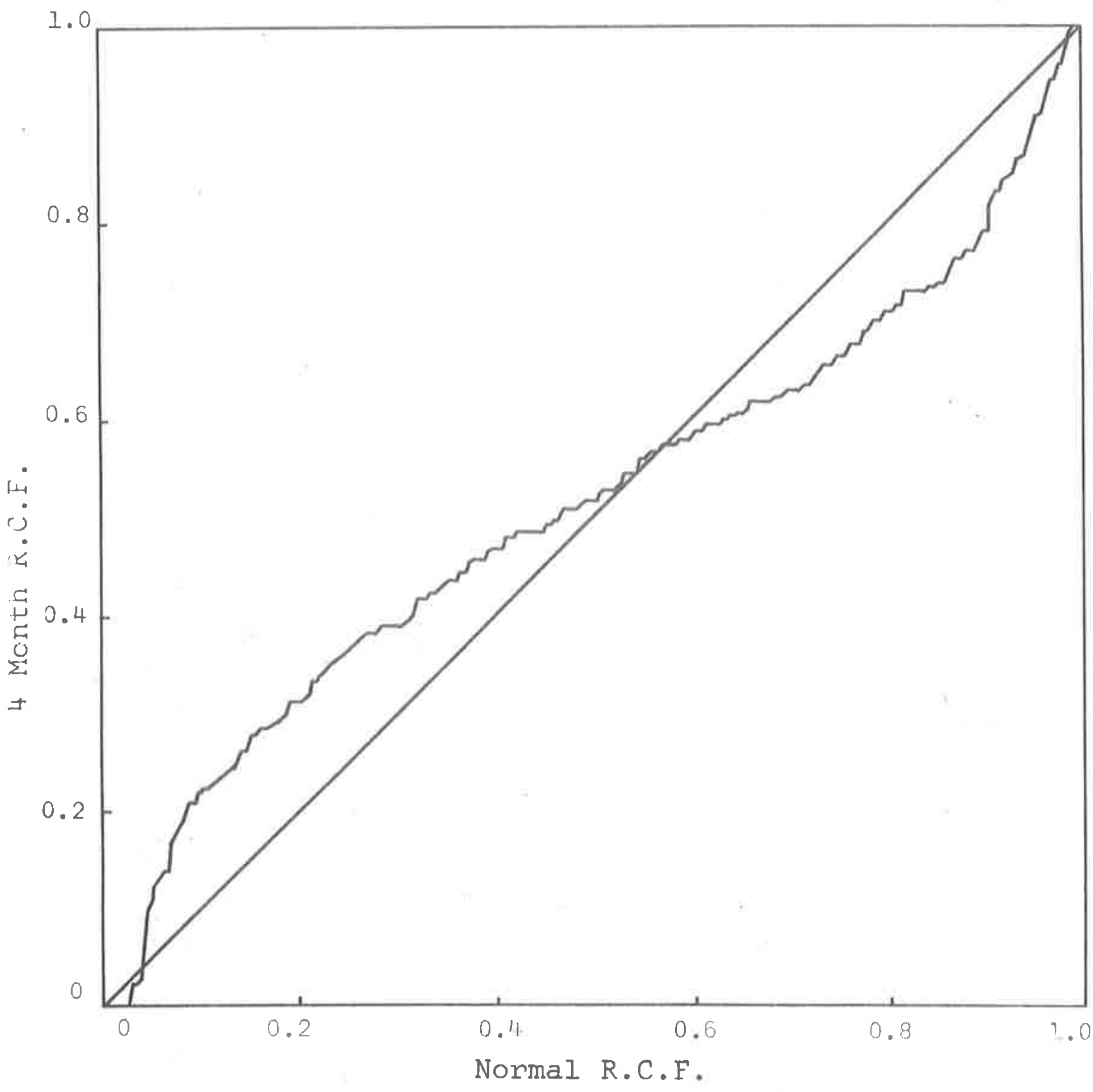


FIGURE 4·6

NORMAL PROBABILITY PLOT;

DAILY TWELVE MONTH FUTURE PRICES

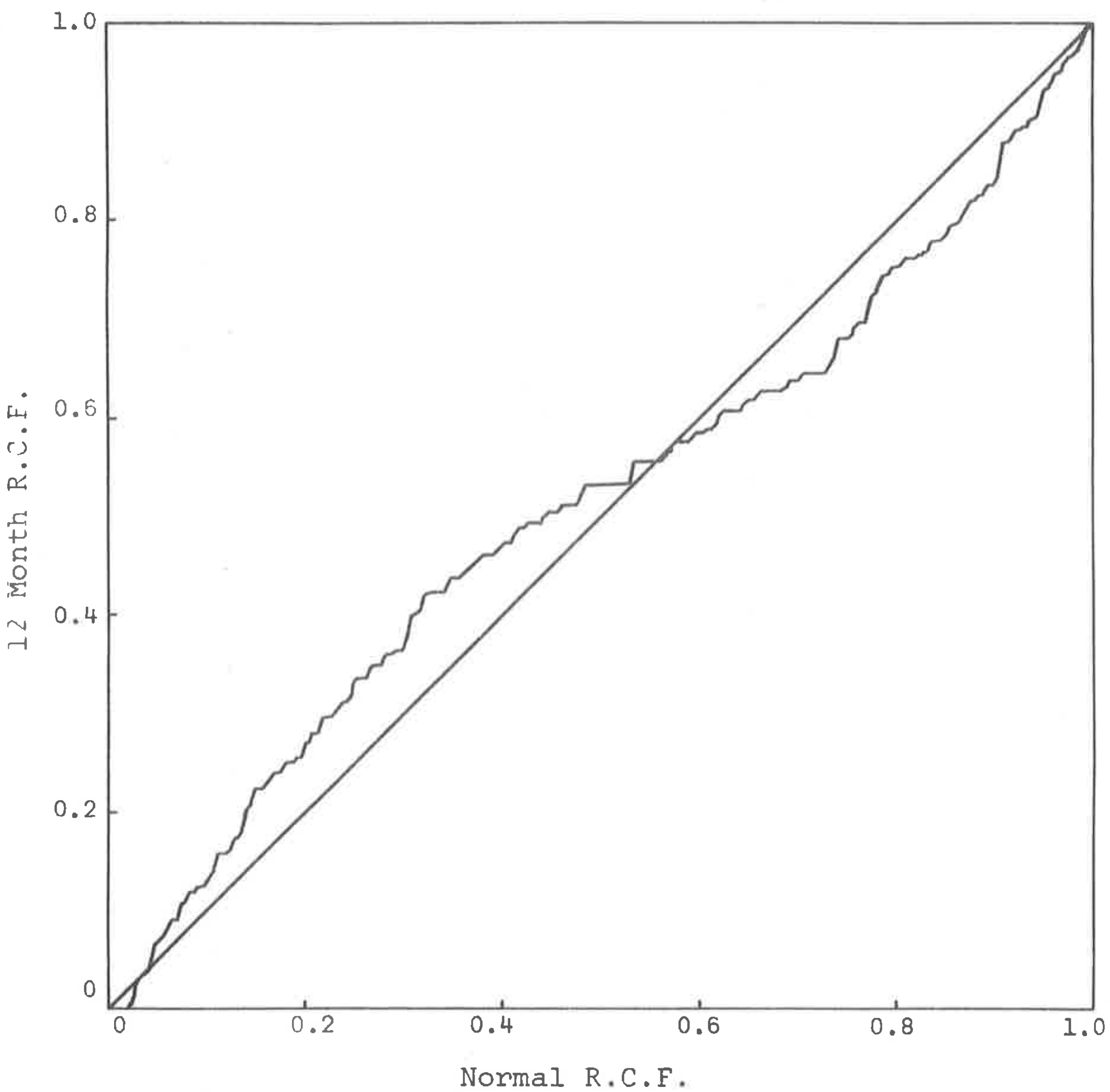


FIGURE 4·7

NORMAL PROBABILITY PLOT ;

DAILY DISTANT FUTURE PRICES

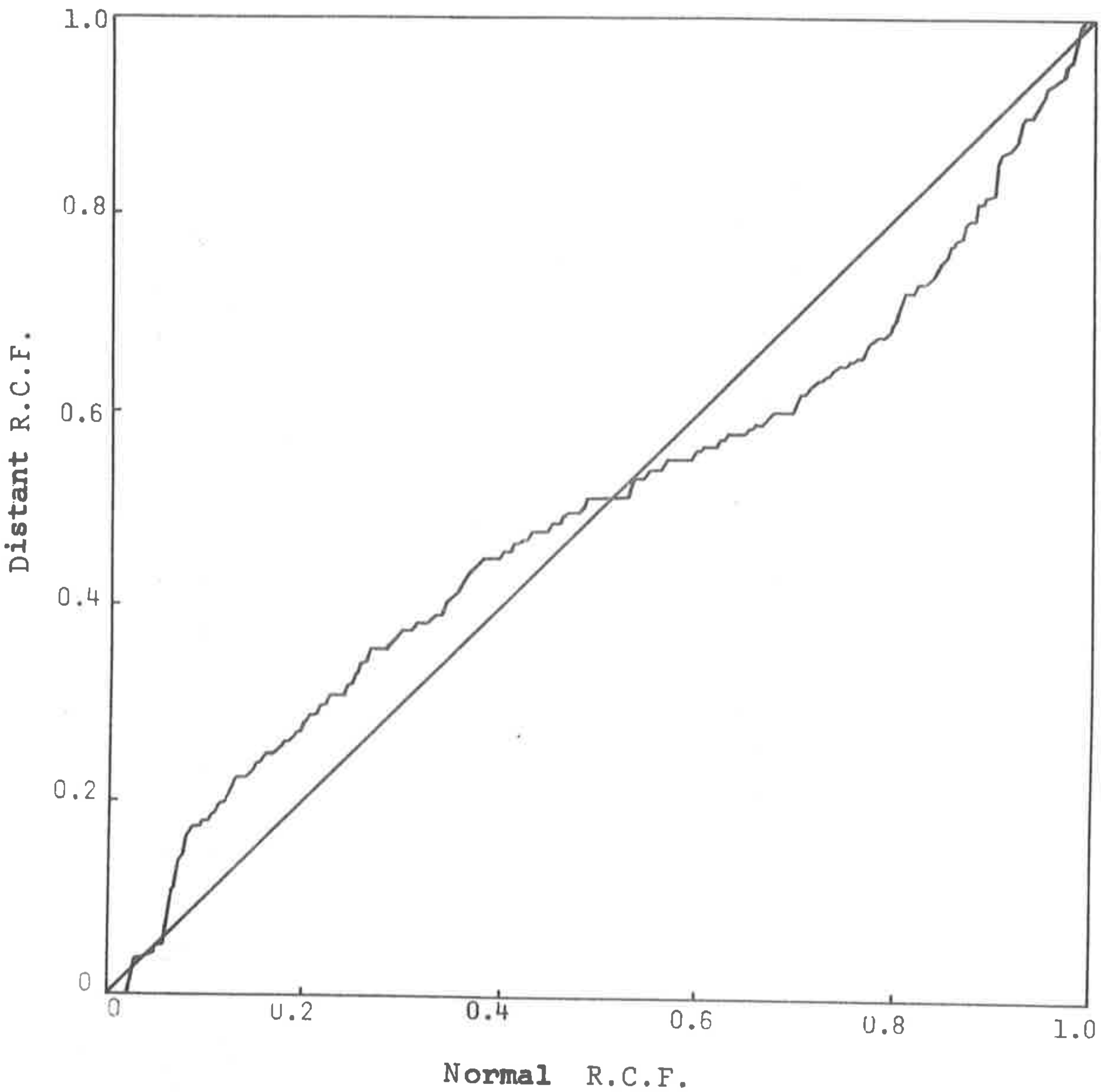
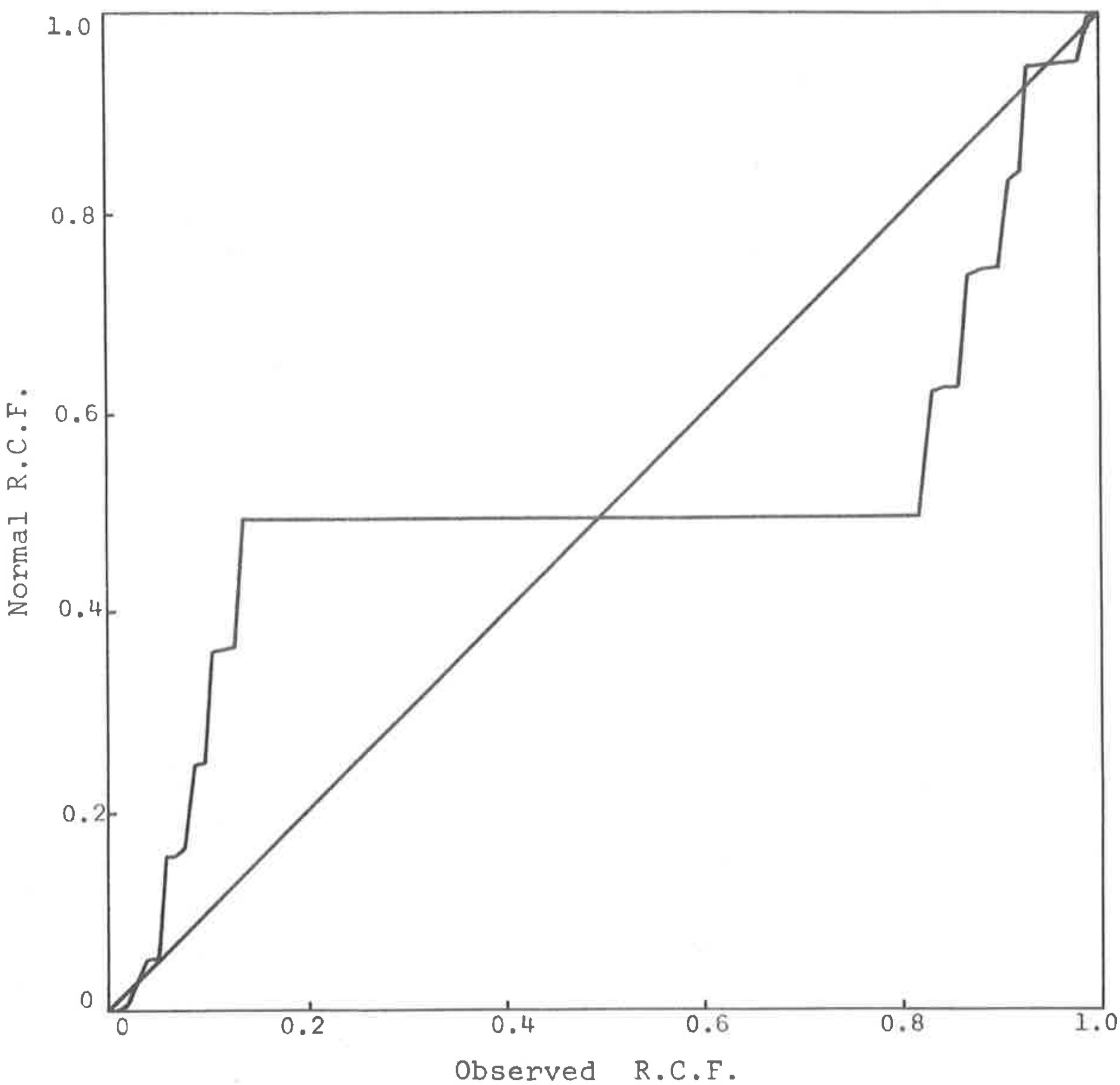


FIGURE 4·8

NORMAL PLOT OF TRANSACTION PRICE CHANGES IN THE MAY 1974 FUTURE



Departure from normality of the four *daily* series is manifest in the characteristic S shaped curve. In point of fact, all thirteen log price series listed in Tables 4.1, 4.2, 4.3 and 4.4 were typically S shaped. The normal probability plots corroborated the evidence of non-normality of price change presented in previous sections.

Figure 4.8 is a normal plot of log transaction price changes of the May future. The kinks in the plot high-light the discontinuous nature of the series.[31] The discontinuity is a natural consequence of prices being negotiated in multiples of 1/10 of a cent per kilo. As expected, the essentially discrete nature of futures price series is more apparent when the differencing interval is small.[32] The evidence of discontinuity in daily price changes is not as great as that presented in transaction price changes. Granger and Morgenstern[33] make the valid point that the necessarily discontinuous nature of futures price changes means that the empirical distributions can never be perfectly fitted by a continuous theoretical distribution, such as the normal curve.

Chi Squared Test for Normality

Daily, weekly and *monthly* futures price series were subjected to a chi squared goodness of fit test for normality. The results are presented in Table 4.6. The degrees of freedom are in parentheses. With one exception, the null hypothesis of normal price changes was rejected at the 1 per cent level.

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31 Figures 4.5 and 4.6 also show some evidence of discontinuity about zero price change.

32 The differencing interval in daily series is small, (24 hours).

33 Granger and Morgenstern, *op.cit.*, p.183.

The evidence of this section is in agreement with that presented in Sections 4.3 and 4.4. Log. price changes in the Sydney wool futures market are definitely non-Gaussian. The empirical distributions are peaked in the centre and have fat tails (Figure 4.3).

TABLE 4.6

Chi Squared Values, Degrees of Freedom in Parentheses

Log first difference price series	Interval		
	Daily	*Weekly*	*Monthly*
Physical			38.72** (11)
Near future	51.10** (18)	63.14** (19)	14.52 (11)
Four month future	58.27** (17)	76.77** (19)	39.73** (11)
Twelve month future	70.23** (19)	49.04** (19)	37.93** (11)
Distant future	61.63** (19)	61.50** (19)	38.76** (11)

** significant at the 1% level.

The results indicate that distributions of price changes in the Sydney wool futures market is similar to the share and commodity markets described by Fama,[34] Praetz[35] and Mandelbrot.[36]

4.5 Alternative Hypotheses to Explain the Empirical Distributions

The evidence of preceding sections[37] denies strongly, the hypothesis that price changes are distributed normally. The

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34 Fama, "Behavior of Stock Market Prices," *op.cit.*

35 Praetz, *op.cit.*

36 Mandelbrot, *op.cit.*

37 Sections 4.3 and 4.4.

proceeding sections of this chapter are concerned with plausible alternative explanations of the empirical distributions.

The Stable Paretian Hypothesis

The stable Paretian hypothesis had its beginnings in the work of Benoit Mandelbrot.[38] Mandelbrot contended that past research had neglected certain regularly observed departures from normality. He was of the opinion that variances of empirical distributions behaved as if they were infinite and that the empirical distributions conformed best to a family of stable Paretian curves.

The logarithm of the characteristic function for the stable Paretian family of distributions is

$$\log f(t) = \log \int_{-\infty}^{\infty} \exp(iut)dP(\tilde{u} < u) \quad \ldots\ldots\ldots\ldots \quad (4.12)$$

$$= i\delta t - \gamma|t|^{a}[1 + i\beta(t/|t|)\tan(a\pi/2)] \quad \ldots \quad (4.13)$$

where a is the parameter determining the area beneath the tails of the distribution. If $a = 2$ the distribution is normal,[39] but if $a < 2$ then the distribution has fat tails relative to the Gaussian distribution.[40]

Fama suggests price changes from transaction to transaction have infinite variance. He points out that summation of variables possessing infinite variance produces a limiting stable Paretian distribution, i.e. $0 < a < 2$.[41] Thus if price changes between

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[38] Mandelbrot, "The Variation of Certain Speculative Prices," *op.cit.*; Mandelbrot, "New Methods in Statistical Economics," *Journal of Political Economy* (Oct. 1963).

[39] The Gaussian distribution is a highly specified stable Paretian curve.

[40] Fama, "Mandelbrot and the Stable Paretian Hypothesis," *op.cit.*

[41] *Loc. cit.*, p.425.

transactions is a stable Paretian variable then daily, weekly or monthly price changes will be stable Paretian with the same value of a. The stable Paretian theory's ability to explain deviation from normality without resorting to arguments concerning instability in underlying parameters (e.g. mean and/or variance) is its greatest attribute.

Praetz in his criticism of Mandelbrot's hypothesis, lists three short-comings of the stable Paretian distribution. First, the distribution functions are specified in only a few cases. Second, acceptance of the notion of infinite variance invalidates most statistical techniques and finally the estimation methods used to specify distribution parameters are not satisfactory.[42]

A method of testing for infinite variance involves plotting the sequential sample variance estimate s_N^2 based on the first N observations against N, where,

$$s_N^2 = \sum_{t=1}^{N} (y_t - \bar{y})^2/N \qquad \ldots\ldots\ldots\ldots \qquad (4.14)$$

$\{y_t\}$ is a log. first difference

y_t.

If the underlying process generating $\{y_t\}$ is truly stable Paretian, such that $\{y_t\}$ has infinite variance, then the sequential variance s_N^2 will diverge as N increases, that is as: $N \to \infty$, $s_N^2 \to \infty$ Fama[43] and Praetz[44] have applied the above test to share price data. Praetz considered his empirical results did not support the stable Paretian hypothesis. Similarly Fama's calculated variances showed little evidence of divergence.[45]

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42 Praetz, *op.cit.*, p.35.

43 Fama, "Behavior of Stock Market Prices," *op.cit.*, pp.65-66.

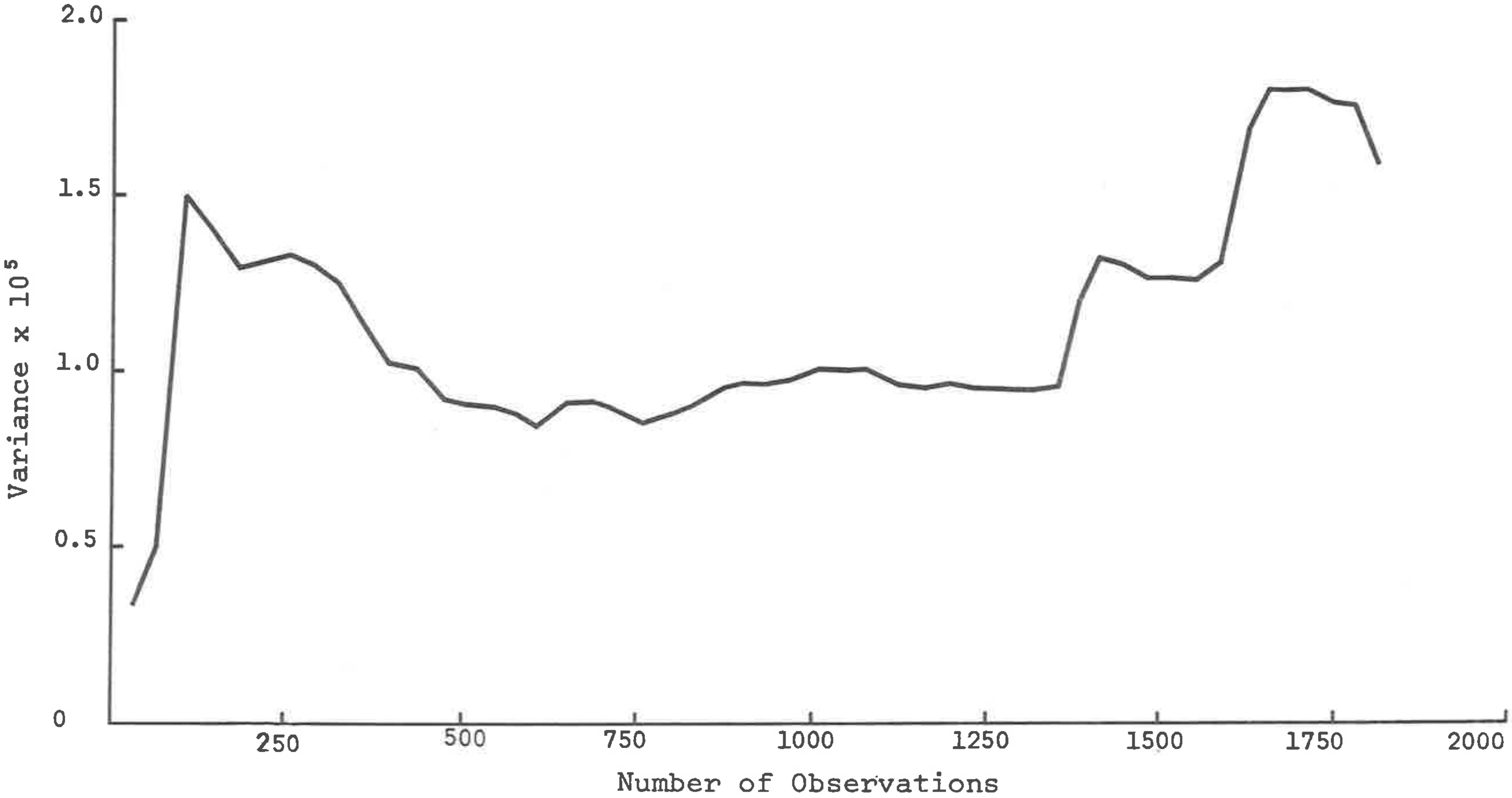
44 Praetz, *op.cit.*, p.35.

45 Strictly speaking, Fama did not consider his results to be a denial of infinite variance.

FIGURE 4·9

SEQUENTIAL VARIANCE:

LOG. PRICE CHANGES OF NEAR FUTURE



A sequential variance function s_N^2 was determined for the extended (2,088 observations) *daily near* future price series.[46] Each increment in N was 36 observations giving 58 values of s_N^2. The plot of s_N^2 against N is presented in Figure 4.9.

Sequential variance of wool futures prices show little, if any, evidence of divergence. The evidence of figure 4.9 coupled with Praetz's criticisms of the stable Paretian hypothesis led us to reject that theory as an efficacious explanation of Sydney wool futures price changes.

Stability of the Distributions

Any statement concerning the stationarity of a time series $\{u_t\}$ implies the mean (μ) and the variance σ^2 are independent of time t. To test for stability over time of mean and variance the twelve futures price series were somewhat arbitrarily divided into n subseries of equal length. For the *monthly* series n = 12, that is, the *monthly* series of 144 observations were split into twelve, yearly series. This negated any seasonal effects. For both the *weekly* and *daily* series n = 30.

Stability of means was studied using one way analysis of variance, i.e. an F statistic described by the mathematical definition

$$F_{(k-1),k(n-1)} = \frac{k(n-1).SSB}{(k-1).SSE} \quad \ldots\ldots\ldots\ldots \quad (4.15)$$

where:
- k = no. of sub-series
- n = no. of observations per sub-series
- SSB = sums of squares between the sub-series
- SSE = total sums of squares

and F is distributed with k - 1 and k(n - 1) degrees of freedom.[47]

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[46] Unfortunately this was the only price series of sufficient length to perform sequential variance analysis.

[47] A fuller account of this technique is given by Roger L.Burford, *Statistics. A Computer Approach*, Merrill Publishing Co., Columbus, Ohio (1968), pp.257-65.

The results of one way analysis of variance on log. first differences of futures prices are given in Table 4.7. The null hypothesis of stability of means was not rejected for any of the twelve series at either the 1 or 5 per cent level. Praetz[48] and Scheffé[49] suggest that non-normality is not a serious problem for inferences concerning stability of means.

These results do not lend support to explanations of non-normality of price changes involving the hypothesis of shifting means. One way analysis of variance tests were also performed on ordinary price data (as opposed to log. first difference). The results of F tests on untransformed data revealed gross instability of means. This result would indicate the log. first difference transformation (equation 3.1) imparts stability of means to time series of speculative prices.

Stability of variance was checked using Bartlett's test of homogeneity of variances. Scheffe warns that inferences on variances are sensitive to departures from normality. However, Gartside's work demonstrates Bartlett's statistic is only marginally affected by non-normality.[50] Also Praetz's adjustments to allow for non-normality did not appreciably affect his conclusions about share price variances.

Bartlett's statistic is computed thus

$$\chi^2_{(n-1)} = \frac{n(k-1)[\log_e(\frac{1}{n}\sum s_i^2) - \sum \log_e s_i^2]}{1 + (3(n-1)^{-1}(n/k-1) - 1/n(k-1)} \quad \ldots\ldots\ldots\ldots \quad (4.16)$$

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48 Praetz, *op.cit.*, p.38.

49 H. Scheffé, *The Analysis of Variance*, New York, (1959), Wiley, Chapter 10.

50 Peter S.Gartside, "A Study of Methods for Comparing Several Variances," *Journal of the American Statistical Association*, (June, 1972), Vol.67, No.338, pp.342-346.

TABLE 4.7

Values of F(Stable Mean) and χ^2(Stable Variance); Degrees of Freedom in Parentheses

Interval	Series	F	χ^2
Monthly	*Near future*	.98 (11,120)	19.86* (11)
	Four month	1.03 (11,120)	27.95** (11)
	Twelve month	1.12 (11,120)	52.69** (11)
	Distant	1.18* (11,120)	49.48** (11)
Weekly	*Near*	0.34 (15,150)	120.3** (7)
	Four month	0.36 (14,150)	185.2** (7)
	Twelve month	0.80 (14,150)	160.5** (7)
	Distant	0.76 (14,150)	160.3** (7)
Daily	*Near*	0.46 (9,120)	91.7** (7)
	Four month	0.53 (9,120)	112.5** (7)
	Twelve month	1.47 (9,120)	45.4** (7)
	Distant	0.80 (9,120)	116.12** (7)

* *indicates significance at 5 per cent level.*

** *indicates significance at 1 per cent level.*

where n = no. of sub-series

k = no. in each sub-series

s_i^2 = variance of the ith sub-series

The statistic has an approximate χ^2 distribution with (n-1) degrees of freedom. The results of the determination of Bartlett's statistic for twelve series are tabulated in 4.7.

An hypothesis of changing variances is suggested by the results. Such a result is not inconsistent with common beliefs about futures markets. It is widely acknowledged that activity in a particular future changes from time to time. It is for this reason that a price quote about a particular future on the sydney exchange is usually qualified by statements about the nature of trading; for example, quiet, slow, steady, active etc.

Stochastic Variance as an Explanation of Non-Normal Empirical Distributions

The previous section demonstrated the distribution of futures price changes did not possess constant variance. Praetz[51] and Granger[52] have provided plausible accounts of changing variance. Praetz accepts that the variance is itself a function, but does not relate the function to any specific economic variables. On the other hand Granger believes the variance is a direct function of the number of transactions undertaken. For instance, Granger believes the variance of daily price changes is proportional to the day's turnover. Praetz argues the variance of daily price changes is the result of the interaction of many economic variables, such as expected

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51 Praetz, *op.cit.*, pp.33-52.

52 Granger and Morgenstern, *op.cit.*, p.180, and Granger and Orr, "Infinite Variance" and"Research Strategy in Time Series Analysis," *Journal of the American Statistical Association*, Vol.67, No.338, (June, 1972), pp.277-279.

supply and demand for wool and its substitutes, exchange rates, interest rates, etc. He does not think these variables can be related to variance in a simple way and thus at best the variance of daily price changes can be assigned a distribution function. We will first consider Granger's approach to stochastic variance.

4.6 Granger's Transformation

The hypothesis, central to this transformation is that individual transaction prices follow a simple random walk.

$$y_{j,t} = \log.(P_{j,t}) - \log.(P_{j-1,t}) \quad \ldots\ldots\ldots\ldots \quad (4.17)$$

where $P_{j,t}$ is the price of the jth transaction on day t.[53] If the random walk model holds $\{y_{j,t}\}$ is a white noise series with zero mean and variance, σ^2. Daily, weekly or monthly price changes are created by the summation of $y_{j,t}$ over the period t.[54] Thus, according to Granger, daily price changes will be normally distributed with the variance proportional to N_t the number of transactions per day. In other words, if $\{N_t\}$ is not a constant series, daily price changes will be composed of a mixture of normal distributions. The price change on day t may be expressed as

$$x_t = \sum_{j=1}^{N_t} y_{j,t} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4.18)$$

as there are N transactions on day t

$$\therefore \quad \sigma_t^2 = N_t\sigma^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4.19)$$

where σ_t^2 is the daily variance and σ^2 is the transaction variance.

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53 t can represent any time period over which prices are quoted e.g. day, week, month.

54 This is expressed in equation (4.18).

The probability function of daily price changes is therefore dependent on the number of transactions

$$f(x|N_t) = \frac{1}{(2\pi N_t\sigma^2)^{\frac{1}{2}}} e^{-(x^2/2N_t\sigma^2)} \quad .... \quad (4.20)$$

Thus if we allow for the stochastic nature of N_t, the distribution of daily price changes is given by the joint probability function

$$f(x,N) = \frac{1}{(2\pi N\sigma^2)^{\frac{1}{2}}} e^{-(x^2/2N\sigma^2)} \cdot f(N) \quad .... \quad (4.21)$$

where f(N) is the function describing the distribution of the number of transactions per day N_t. The distribution function of daily price changes is found by integrating

$$F(x) = \int_0^{\infty} f(x, N) \cdot d\,N \quad .... \quad (4.22)$$

Granger's transformation (equation 4.23) involves dividing the daily price change by the square root of the number of transactions on that day. The transformation can be expressed symbolically as

$$z = \frac{x}{N^{\frac{1}{2}}} \quad .... \quad (4.23)$$

$$\therefore \frac{dx}{dz} = N^{\frac{1}{2}} \quad .... \quad (4.24)$$

using the transformation formula

$$f(z) = f[z(x)] \mid dx/dz \mid \quad .... \quad (4.25)$$

and substituting equation (4.25) in (4.21)

$$f(z,N) = \frac{1}{(2\pi N\sigma^2)^{\frac{1}{2}}} e^{-(z^2N/2N\sigma^2)} \cdot f(N) \cdot N^{\frac{1}{2}} \quad ... \quad (4.26)$$

$$= \frac{1}{(2\pi\sigma^2)^{\frac{1}{2}}} e^{-(z^2/2\sigma^2)} \cdot f(N) \quad \ldots\ldots\ldots\ldots \quad (4.27)$$

$$= \phi(z) \cdot f(N) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4.28)$$

where $$\phi(z) = \frac{1}{(2\pi\sigma^2)^{\frac{1}{2}}} e^{-(z^2/2\sigma^2)} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4.29)$$

$\phi(z)$ is independent of N, thus a distribution function of z can be obtained by integrating over the range of N.

$$F(z) = \int_0^\infty \phi(z) \cdot f(N) \cdot dN \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4.30)$$

$$= \phi(z) \int_0^\infty f(N)dN \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4.31)$$

$$= \phi(z) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4.32)$$

The transform z should thus be normally distributed as given in equation (4.29).

An empirical test of Granger's transform was performed using daily price when trading starts in the morning (*open* price) and the price when trading finishes at the end of trading (*close* price) and the daily *volume* of transactions.[55] If in fact Granger's ideas about the nature of price changes are correct then the variable of equation (4.23), that is, log. *open* price - log. *close* price divided by the square root of daily *volume* should be normally distributed.

Some inadequacies of the transform were foreseen before the results of the empirical tests were available. Consider the transform as described in equation (4.23).

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55 The source and method of preparation of these three series is described in Section 3.2.

$$z_t = x_t/N_t^{\frac{1}{2}} \qquad \text{..........} \qquad (4.33)$$

If $N_t = 0$, that is there was no trading on day t in the near future, then z is undefined. This problem was overcome by removing all observations where $N_t = 0$. However, there still remained the related problem of anomalous values of z_t on observations where N_t was very small. It was the possibility of very large values of z_t when N_t was small that prompted us to prepare a slightly transformed series. This series consisted of all observations containing greater than twenty-five transactions per day.

Another predictable inadequacy of Granger's transform was its inability to take account of turnover in other futures. Price changes in one future are necessarily related to price changes in other futures.[56] The situation could arise where the day's trading on the Sydney market was predominantly in one particular future. Trading in the other futures would be minimal, just sufficient for prices to adjust relative to the dominating future. Thus the variance of price changes in the lightly traded futures would not be related to its turnover but to the turnover of the dominant future. Unfortunately, there is no easy way of relating price changes in one particular future to some weighted average of turnover in all futures. However, in the end it was practical rather than theoretical considerations that stopped us attempting to modify Granger's transform. Data on the daily turnover of all individual futures was not available.

Two series using Granger's transform were constructed, the first containing all daily observations where at least one contract was traded. The second, included only observations where turnover was greater than 25. Both were divided into three approximately

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[56] See Section 6.3.

TABLE 4.8

Chi Squared Values for Granger's Transformed Series: Degrees of Freedom in Parentheses

Sub-Series No.	≥ 1 Transaction per day	≥25 Transactions per day
1	106.48** (17)	59.12** (17)
2	79.53** (17)	49.25** (15)
3	150.80** (17)	13.61 (15)

** Significant at 1% level.

equal sub-series and fitted to the normal distribution.

The results of a chi squared goodness of fit to the normal distribution for the two sets of sub-series are presented in Table 4.8.

The results of Table 4.8 do not support Granger's thesis that a series can be returned to normality by dividing the price change by the square root of the number of transactions.

Yet another test of the efficacy of Granger's transform involved plotting the variance of daily price change against average daily turnover. The variance was established for 50 observations and a corresponding value for turnover was determined by averaging over the same 50 observations. Equation (4.19) requires the function to be a straight line passing through zero. The slope of the function should be equal to σ^2, the variance of the price change between transactions. The result of graphing variance against average turnover is displayed in Figure 4.10. Even the most generous interpretation of Figure 4.10 could not describe it as a straight line passing through the origin.

The empirical evidence of Table 4.8 and Figure 4.10 would seem to deny the validity of Granger's theory of price change. However, even if the empirical evidence had demonstrated a return to normality of the transformed variable Granger's theory has limited areas of validity. It could never, for example, be applicable to overnight price changes, which, because the market is closed, are not accompanied by a flow of transactions. The evidence of the following section will demonstrate that, although differences in the daily turnover explains to some extent the shifting variance of the empirical distributions, there exist other important influences or variances.

The final test of Granger's transform consisted of ranking the *daily open* minus *close* price series by increasing *daily turnover*. The series was then split into 11 sub-series (each with approximately 50 observations). A chi squared goodness of fit test to the normal distribution was performed on each sub-series. Two control groups of eleven sub-series were prepared for comparison against the volume ranked sub-series. The first control consisted of creating eleven sub-series by random sampling. The second control group of eleven sub-series was prepared by simply dividing the original time series into eleven segments, that is, the observations were ranked chronologically. Table 4.9 lists the chi squared values for the three sets of sub-series.

Granger's contention is that the empirical distribution is made up of a mixture of normal distributions of differing variances depending on the number of transactions per time interval. Thus, if we restrict the variation in daily volume, as we have done in

FIGURE 4·10

PLOT OF VARIANCE

DAILY (LOG. OPEN PRICE – LOG. CLOSE PRICE)

AGAINST AVERAGE TURNOVER FOR THE NEAR FUTURE

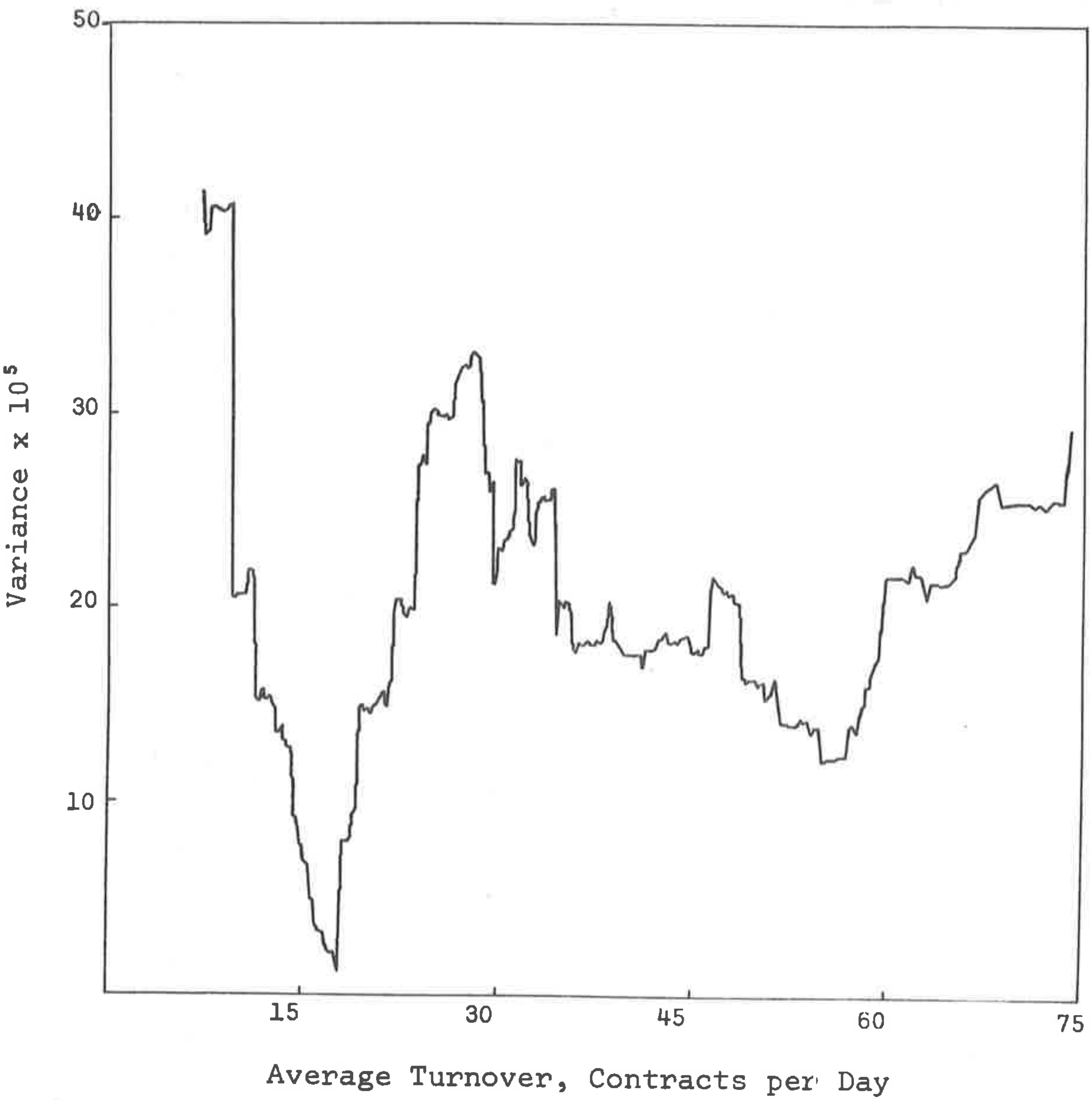

TABLE 4.9

Chi Squared Values for *Daily* Log.*Open* Minus *Close* Price Sub-Series.

Sub-Series Number	Randomly Selected Observations	Volume Ranked Observations	Time Ranked Observations
1	26.5**	59.5**	26.4**
2	18.6*	24.1**	23.4**
3	14.5	14.3	34.2**
4	70.2**	35.1**	10.6
5	24.3**	26.7**	6.1
6	21.1**	43.0**	6.0
7	33.2**	13.6	13.1
8	12.8	15.6	15.5
9	14.2	16.8	11.9
10	16.6*	14.9	10.1
11	18.2*	6.9	7.0

* significant at 1% level

** significant at 5% level

the set of volume ranked sub-series, the sub-series should approach normality. We would expect that the randomly selected sub-series would have greater chi squared values than the transaction ranked sub-series. The results presented in Table 4.9 support the prediction based on Granger's theory of price changes. Of the eleven randomly selected sub-series eight were non-normal at the 5 per cent significance level, while only five of the volume ranked sub-series were non-normal. Certainly, some of the movements in the variance can be attributed to changes in the daily turnover.

The chi squared values for the time ranked observations are much lower than those for the other two sets of sub-series. Only three of the time ordered sub-series displayed non-normality. This result convinced us that Praetz's approach to stochastic variance was superior to the one offered by Granger. Praetz believes the distribution of price change is normal, but with variance changing over time. Thus, if variance changes slowly and the period over which the observations are collected is made small, the distribution should approximate normality. Each sub-series of the time ranked set represented approximately a ten week period.[57] The chi squared results of Table 4.9 indicated that the variance was constant over the ten week period in eight of the eleven sub-series.

4.7 Praetz's Rescaled t_n Distribution

Praetz[58] argues that the distribution of share price changes is Gaussian conditional on a fixed value of the variance σ^2. That is, the normal probability density function, as used by Oxborne,[59]

$$f(y) = \frac{1}{(2\pi\sigma^2)^{\frac{1}{2}}} e^{-(y^2/2\sigma^2)} \qquad \ldots\ldots\ldots\ldots \qquad (4.34)$$

now becomes conditional on a fixed value of the variance σ^2, the conditional density function can be expressed as

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[57] It was impossible to reduce this ten week period and retain enough observations to perform a meaningfull chi squared test.

[58] Praetz, *op.cit.*, pp.42-52.

[59] Osborne, *op.cit.*

$$f(y|\sigma) = \frac{1}{(2\pi\sigma^2)^{\frac{1}{2}}} e^{-(y^2/2\sigma^2)} \quad \text{..........} \quad (4.35)$$

A marginal distribution function is obtained by integration

$$F(y) = \int_o^\infty f(y|\sigma^2) \;.\; g(\sigma^2) \;.\; d\sigma^2 \quad \text{..........} \quad (4.36)$$

$$\text{with } o < \sigma^2 < \infty$$

$g(\sigma^2)$ is the probability density function for the variance. Praetz suggests the probability density function of the variance is best represented by

$$g(\sigma^2) = \sigma_o^{2m}(m-1)^m \; \sigma^{-2(m+1)} e^{-(m-1)\;\sigma_o^2/\sigma^2} \;/\; \Gamma(m) \quad \text{.....} \quad (4.37)$$

When $g(\sigma^2)$ is substituted into equation (4.38) and F(y) is produced by integration

$$F(y) = [1+y^2/\sigma_o^2(2m-2)]^{-(m+\frac{1}{2})} \;.\; \Gamma(m) \;.\; [(2m-2)\pi]^{\frac{1}{2}} \colon \sigma_o \quad \text{...} \quad (4.38)$$

F(y) is a t_n distribution (with 2m = n degrees of freedom) scaled by a factor $[n/(n-2)]^{\frac{1}{2}}$.

Praetz used an eclectic approach to determine the particular form of $g(\sigma^2)$ given in equation (4.36). The function $g(\sigma^2)$ represents a prior distribution for an unknown parameter σ^2. Goodness of fit was Praetz's major criterion for selecting the particular prior distribution $g(\sigma^2)$.

> It is theoretically possible that other prior distributions exist to describe σ^2, but it would seem unlikely that they would provide a better fit to the data studied.[60]

The distribution function $g(\sigma^2)$ represents the "activity" or "energy" of the market. $g(\sigma^2)$ is a mathematical presentation of the long recognized phenomenon of changing market volatility. That is,

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60 Praetz, *op.cit.*, p.47.

TABLE 4.10

Chi Squared Values of Rescaled t_n Distribution;

Values of n in Parentheses

Price Series	Interval		
	Daily	*Weekly*	*Monthly*
Physical			12.2 (3)
Near Future	33.2** (3)	22.1 (3)	3.2 (4)
Four month Future	36.7** (3)	17.3 (3)	17.2 (3)
Twelve month Future	20.1 (3)	15.1 (3)	10.2 (3)
Distant Future	19.7 (3)	10.8 (3)	16.2 (3)

** significant at 1% level

price changes are more violent in some periods than in others. An example of the acceptance of the phenomenon of changing variance is given in the general report of the S.G.W.F.E. on 16/2/73.

> The market closed on Friday with an upsurge in activity volume and prices....this trend overcame a lull in trading on Tuesday and Wednesday with the foreign exchange crisis, and the market appears poised for the reopening of auction sales next week.[61]

Words like upsurge, activity and lull[62] are descriptions of points on the x axis of $g(\sigma^2)$. The general report offers some explanation of the determinants of $g(\sigma^2)$ such as currency exchange crises and events of the wool auction sales. The evidence of Table 4.9 suggests turnover

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61 S.G.W.F.E., "Weekly Report,", 16/2/73.

62 Other adjectives frequently used to describe market energy are, slow, quiet, steady, brisk, active, etc.

is one of the major influences on $g(\sigma^2)$, but there exist many others, such as the abovementioned currency fluctuations and changing expectations of future physical wool prices. The problem of relating the many determining variables to $g(\sigma^2)$ appears to be intractable.

Results of Fitting Empirical Distributions to Rescaled t_n Distribution

The empirical distributions were subjected to a chi squared goodness of fit test. The expected frequencies were derived from the probabilities of the rescaled t_n distribution, where n is the degrees of freedom, as specified by equation (4.39). n was varied from 3 to 8 to produce a family of t_n distributions. The particular distribution chosen to represent the empirical distribution was selected on the basis of minimization of the chi squared value.

It is clear from Table 4.10 that the rescaled t_n distribution provided a very good description of the empirical distributions. Eleven of the thirteen empirical distributions did not differ significantly (at 1 per cent level) from a t_n distribution. The fits to the *daily* series were not as good as the fits to the *weekly* and *monthly* series. A preponderance of zero price changes over the short period gave rise to lumpy empirical daily distributions and consequent poor fits to the t_n distribution. The lumpy short period distributions are the result of market inertia, that is, prices tend to be sticky in the short run.[63] Numerous zero price changes were the major contributions to the significant chi squared value for the *near daily* futures price series. The poor fit of the *twelve month* future was the result of excessive skewness in the price series.[64] Despite the two anomalous series we believed the rescaled t_n distribution is a

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63 The necessarily discrete nature of price changes in the short run also contributed to the poor daily price change fit. This is discussed in detail in Section 4.4.

64 See Table 4.3.

justifiable description, on both theoretical and practical grounds, of price changes in the S.G.W.F.E.

4.8 Conclusion

The results of our investigation have shown price changes in the S.G.W.F.E. to be similar to most documented studies of price changes in share and futures markets, in that our empirical price changes were highly non-normal. Our empirical results suggested that price changes in the S.G.W.F.E. were stationary with respect to the mean but had shifting variance. The fluctuating variance was somewhat explained by movements in daily turnover. We believe the failure of Granger's transform to describe the empirical distributions stemmed from its inability to incorporate any other determinants of variance other than daily turnover. We believe that many other economic variables influence variance. In particular we think turnover in other futures is a major influence. The price change series were best simulated by a rescaled t_n distribution which resulted from the assumption that price changes are normal distributed with a stochastic variance.

What is the relevance of assuming price changes on the S.G.W.F.E. are described by a t_n distribution? First, compared to the normal distribution the t_n distribution has far greater area under the tails, thus the chance of a very large price change is much higher in a rescaled t_n market. Second, if we accept the rescaled t_n distribution as a valid description of price change, it also means we accept that the variance is finite. This is contrary to the infinite variance, implied in Mandelbrot's stable Paretian theory which invalidates a host of statistical tools. Hence the use of statistical techniques, such as regression and spectral analysis are perfectly compatible with the rescaled t_n distribution philosophy.

A knowledge of the distribution of price changes in any market has obvious consequences for potential investors. It is only from a knowledge of the exact distribution, such as the rescaled t_n distribution, that the risk of a particular investment can be evaluated. The results of Section 4.3 suggest that the risk accompanying a transaction in the S.G.W.F.E. increases as the future nears maturity. This section also demonstrated that price changes in the physical wool market showed greater variability than price changes of any of the futures series. In other words, there is greater risk attached to a physical market transaction than a futures market transaction.

CHAPTER 5

INDEPENDENCE OF PRICE CHANGES

5.1 Preamble

The random walk model, as described in Chapter 2, makes two basic assertions. The first is that a distribution can be assigned to price changes. The second and far more important assertion of the random walk is that price changes are independent of one another. In this chapter we are concerned with determining how closely price changes in the Sydney Greasy Wool Futures Exchange (S.G.W.F.E.) follow the second assertion of the random walk model.

The random walk model can be expressed symbolically as

$$u_t = P_t - P_{t-1} \qquad \text{..........} \qquad (5.1)$$

where P_t is the price in period t. The independence criterion of the random walk demands that the series of residuals u_t be independent of one another. That is

$$E[U_t U_{t-s}] = 0 , \qquad s \neq 0 \qquad \text{..........} \qquad (5.2)$$

If equation (5.2) holds true, it implies past prices contain no information about future prices. In other words, it is impossible to utilize past prices to predict the next move of prices. This can be expressed mathematically after the fashion of Fama[1] as

$$P_r(P_t = P | P_{t-1}, P_{t-2} \ldots .) = Pr(P_t = P) \quad \text{......} \qquad (5.3)$$

where Pr represents probability and P_t price in period t.

We have used a range of statistical and mechanical techniques to test for and measure any dependencies in the series of residuals

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1 E.Fama, "Efficient Capital Markets - A Review of Theoretical and Empirical Work," *Journal of Finance*, 25 (1970), pp.383-416.

u_t. Each technique enabled us to gain some insight into the underlying process generating price changes in the S.G.W.F.E. The need for more than one test of independence is admirably expressed by Leuthold,

> ...conclusions drawn by any investigator who uses only one basic approach must be looked at with suspicion.[2]

We have employed autocorrelation analysis, runs analysis, spectral analysis, seasonality indices and mechanical filters in the search for dependencies in u_t.

5.2 Autocorrelation Function

The variable tested was the log. first difference of price changes y_t, where

$$y_t = \log.P_t - \log.P_{t-1} \qquad \text{..........} \qquad (5.4)$$

The estimate of the autocovariance function is

$$C_y(g) = \frac{1}{n-g}\sum_{t=1}^{n-g}(y_t - \bar{y})(y_{t+g} - \bar{y}) \qquad \text{..........} \qquad (5.5)$$

where $\qquad g = 0, 1, \ldots a \qquad$ .... (5.6)

and $\bar{y}$ is the mean of $\{y_t\}$.

The estimate of the autocorrelation function is

$$R_y(g) = C_y(g)/C_y(0) \qquad \text{..........} \qquad (5.7)$$

Under the random walk model

$$E[R_y(g)] = 0 \qquad \text{..........} \qquad (5.8)$$

it has been shown by Kendall and Stuart[3] that,

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[2] Raymond M.Leuthold, "Random Walk and Price Trends: The Live Cattle Futures Market," *Journal of Finance*, Vol.XXVII, (Sept.1972), pp.879-889.

[3] M.Kendall and A.S.Stuart, *The Advanced Theory of Statistics*, Vol.3, Griffin, London, (1966).

$$\text{Variance } R_y(g) = \frac{1}{n - a} \qquad \ldots\ldots\ldots\ldots \qquad (5.9)$$

Confidence intervals for estimates of the correlation coefficients were calculated from the Var.$R_y(g)$, equation (5.9). Previous researchers have found the autocorrelation function to be a somewhat inadequate technique for testing for dependencies in economic series.[4] Larson suggested that extreme values associated with the observed leptokurtic distributions combine to produce odd empirical correlation coefficients.[5] On the other hand, Praetz believes the correlation function is applicable to non-normal data so long as the sample is large.[6]

Previous Research

One of the assumptions of the random walk is that prices react instantly to new information.[7] Obviously in real world markets, there must exist some delay between information arrival and its incorporation in a new price. In an efficient market, the delayed response should only be a small portion of total reaction. Nevertheless price changes in a real world market will be a moving average of the price change which would have occurred if the market had been "perfect". Larson demonstrated that the estimation of the parameters of the moving average price adjustment process in corn futures futures from the serial correlation function was extremely difficult.[8]

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4 Arnold B. Larson, "Measurement of a Random Process in Futures Prices," *Food and Research Institute Studies*, Vol.1, No.3, (Nov. 1960), pp.222-225; and Thomas F. Cargill and Gordon C.Rausser, "Time and Frequency Domain Representation of Futures Prices as a Stochastic Process," *Journal of the American Statistical Association*, March 1972, Vol.67, No.337, pp.25-26.

5 Larson, *op.cit.*, p.223.

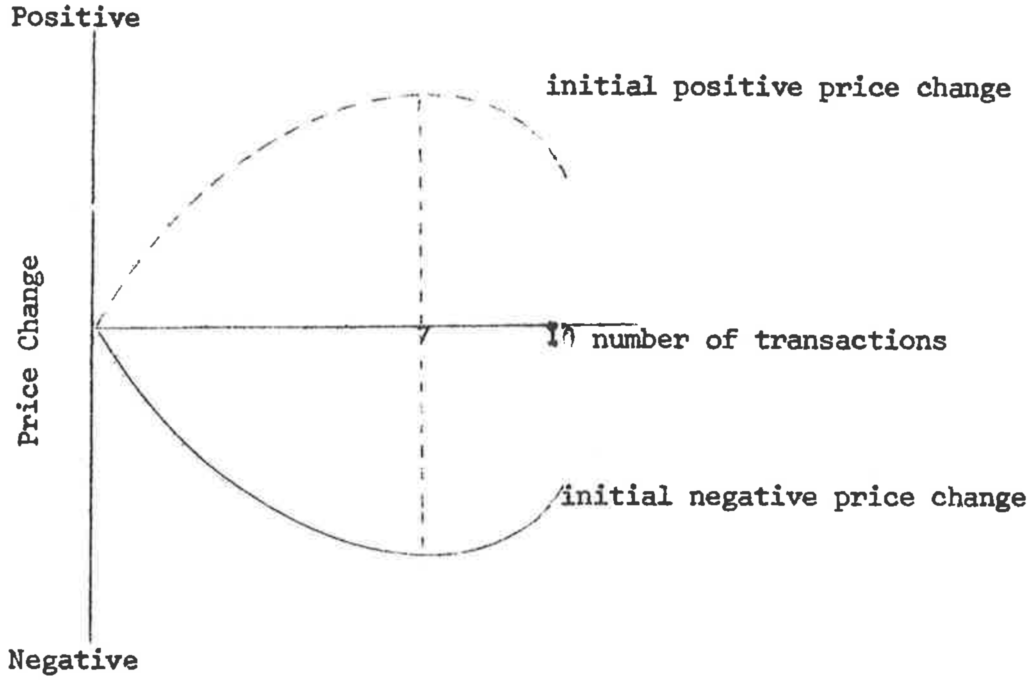
6 P.D.Praetz, *A Statistical Study of Fluctuations of Australian Share Prices*, Unpublished Ph.D. thesis, Adelaide, (July, 1971), p.53.

7 See Chapter 2 for more detail.

8 Larson, *op.cit.*, p.223.

FIGURE 5.1

Diagrammatic Representation of Transaction Price Changes



He considered that the interpretation of his empirically obtained autocorrelation function was next to impossible. Several of the coefficients were significant but no clear price pattern emerged. Stevenson and Bear[9] and Cargill and Rausser[10] used the serial correlation technique on various U.S. commodity futures markets. Like Larson they found the interpretation of significant departures from the independence hypothesis well nigh impossible. Kendall[11]

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9 Richard A. Stevenson and Robert M. Bear, "Commodity Futures: Trends or Random Walks," *Journal of Finance*, Vol. 25, (March, 1970), pp.65-81.

10 Cargill and Rausser, *op.cit.*, pp.23-30.

11 M.G. Kendall, "The Analysis of Economic Time Series, Part 1. Prices," *Journal of the Royal Statistical Society*, Series A, No. 116, (1953), pp.11-25.

and Cootner[12] have published results showing small negative autocorrelation amongst their share price data.

Empirical Results of Autocorrelation Analysis

The values of the autocorrelation function for the first ten lags is given in Table 5.1. The serial correlation value at lag one for the *July transaction* price series is positive and significant. This tends to lend support to the Working, Larson contention that price adjustments follow a moving average mechanism.[13] The *July transaction* series' autocorrelation function is diminishingly positive for lags 1 to 6 and then becomes negative for lags 7 to 10. A typical price change sequence would consist of dampened price movement in one direction for the first seven transactions. After that there would be some reaction against the initial price change as prices changed direction. A simple illustration of price changes at the level of transactions is presented in Figure 5.1. The transaction price change mechanism is not clearly defined for the *May transaction* series. There is, however, a nearly significant positive autocorrelation value at lag 1, which could be interpreted as evidence for a moving average model of price adjustment.

The first lag autocorrelation values for the *seven times/day* series are predominantly negative (three negative and one zero value). These series are made up of prices recorded seven times during a trading day.[14] The results for the *seven times/day* series should be analysed in the light of the results for the Transaction series.

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12 P.H. Cootner, "Stock Prices: Random vs. Systematic Changes," *Industrial Management Review*, Vol.3, No.2, (Spring, 1962), pp.24-25.

13 It was Working's theory of anticipatory prices that led Larson to formulate a moving average model of price adjustment; Larson, *op.cit.*, p.222.

14 The prices are recorded at seven arbitrarily chosen times within the trading day. Refer to Section 3.2 for more detail on the construction of *seven times/day* series.

TABLE 5.1

Values of Serial Correlation Coefficients

at Ten Lags for Sixteen Price Series.

Interval	Series	Lags									
		1	2	3	4	5	6	7	8	9	10
Trans-action	*July*	.20*	.03	.08	.07	.04	.04	-.04	-.01	-.13	-.11
	May	.10	-.01	.01	0.0	-.01	-.01	-.09	-.04	.02	.01
Seven Times Per Day	*July*	-.08	-.07	-.06	.04	-.02	.04	.01	-.10	.10	-.04
	October	-.29*	.05	.10	-.08	-.10	.07	.09	-.12	.14	-.02
	March	-.05	.15	0.0	.09	-.13	.02	.03	.02	-.07	-.02
	New July	0.0	.03	0.0	-.09	.04	.01	.09	-.02	0.0	.01
Daily	*Near*	.12	-.07	-.06	-.08	.10	0.0	-.20*	.03	.03	.09
	Four Month	.06	0.0	-.10	.03	.07	-.02	-.10	-.10	.05	.08
	Twelve Month	-.03	.09	.03	-.06	.09	.05	-.05	0.0	.09	0.0
	Distant	.10	-.02	0.0	-.06	.12	.02	-.11	.03	.12	.05
Weekly	*Near*	-.04	-.10	-.06	-.12*	.04	-.03	.03	-.01	-.03	.11
	Four Month	-.10	-.03	-.10	-.12*	.11	-.06	0.0	0.0	0.0	.10
	Twelve Month	-.08	-.05	-.06	-.09	.09	-.04	0.0	-.01	.07	.09
	Distant	-.10	-.08	-.07	.09	.05	-.03	0.0	.05	.11	.04
Monthly	*Near*	.01	.24	.30*	.05	.08	.05	.07	.04	.12	.12
	Four Month	-.02	.12	.29*	.09	.03	.15	.04	.04	.12	.09
	Twelve Month	.08	.07	.27*	.08	.05	.19*	.09	0.0	.11	.10
	Distant	.04	.08	.23*	.06	.04	.21*	.05	.01	.14	.06

* indicates significance at the 5% level.

It is feasible that the time elapsing between each *seven times/day* series represents seven or so individual transactions. If this is the case then there would exist negative correlation between adjacent observations in the *seven times/day* series. We believe that the interval between *seven times/day* observations is too long to register the trend aspect of Figure 5.1. However, the reactionary element of Figure 5.1 is transferred to the *seven times/day* series and is subsequently registered in the autocorrelation function of those series.

There was very little information available from the serial correlation functions of the *daily* and *weekly* series. Of the 80 values only three were significant. Neither positive nor negative values dominated the serial correlation functions.

The autocorrelation function of the four monthly series contained six significant values. Four of these six values occurred at lag three. The significant values at lag three may foreshadow the existence of a quarterly price cycle.[15]

Summary of Autocorrelation Results

Serial correlation analysis revealed some deviations from the pure random walk model in the very short run. The transaction price data analysis produced evidence of positive autocorrelation for the first seven price changes. This is in complete agreement with the Working-Larson[16] suggestion that price changes can be described by a moving average model. We agree with Working and Larson and believe that it would be a naive economist that expected to find complete independence of price changes at the transaction level. Human and

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15 The existence of price cycles will be investigated using spectral analysis in Section 5.4.

16 See Footnote 13.

machine limitations necessarily dictate that new information is incorporated into a new price over a period of time, this means that price changes are autocorrelated. The results of serial correlation analysis led us to believe that a typical price response to new information involves price changes in the one direction for approximately seven transactions followed by a reaction against the initial price movement.

The reactionary aspect of price adjustment has been observed in other speculative markets.[17] Negative correlation between price changes in the S.G.W.F.E. was observed in both the longer lags of the Transaction series and the first lag of the *seven times/day* series. The *daily* and *weekly* series price changes were random, as far as the empirical autocorrelation function was concerned. The *monthly* series contained some confusing dependencies. The importance of the monthly significant values was determined by further investigation with alternative statistical techniques.[18]

Autocorrelation analysis of the transaction data demonstrated that it is theoretically possible for an astute trader to profit by the short run deviations from the random walk. Figure 5.1 shows that if a trader could consistently buy (or sell) at transaction one and sell (or buy) at transaction seven he would be rewarded with a potential profit. However, it may be that transaction costs would be larger than the potential profit. It would also be difficult for a trader seeking the profit to be had from short term price trends to recognize the beginning and end of such trends. With transactions occurring rapidly in the S.G.W.F.E. it would be

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17 Larson, *op.cit.*, pp.225-230; Claude S. Brinegar, "A Statistical Analysis of Speculative Price Behavior," *Food Research Institute Studies*, Supplement to Vol.IX, (1970), pp.1-58; Kendall, *op.cit.*, pp.11-25.

18 The results of spectral analysis of the monthly price series revealed some dependencies in the data, see Table 5.8.

an almost impossible task for a trader to ensure his entry and departure from the market occurred at transaction one and seven respectively. We believe the short term price trends occur too rapidly and the transaction costs are too high for an average participant in the S.G.W.F.E. to make a profit. We do, however, recognize that a trader closely associated with the market with small transaction costs, e.g. a wool futures broker, may be able to extract a consistent profit from short run price trends.

5.3 Analysis of Runs

Introduction

A run is defined as a sequence of price changes of the same sign. For the purpose of our analysis price changes were categorized into one of three possible signs, that is, empirical price changes were classified as being either positive, negative or zero. The hypothetical series of price changes,

+, +, -, +, 0, 0, 0, 0

where + represents a price increase

0 represents no change in price

and - represents a price decrease

contains four runs. There are two positive runs, one of length one and the other of length two. The other two sequences are a negative run of length one and a zero run of length four.

The underlying theory of runs analysis evolved from the controversies surrounding games of chance.[19] Runs analysis has definite advantages when compared with more conventional tests of

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[19] The possibility of developing a systematic rule for winning at roulette was central to the development of runs theory. A concise summary of the evolution and current theory of runs distributions is given by: A.M. Mood, "The Distribution Theory of Runs," *Annals of Mathematical Statistics*, No.11, (1940), pp.367-392.

independence of price changes, such as serial correlation or spectral analysis. These last two techniques require that the error terms are normally distributed with constant variance. Analysis of the distribution of price changes revealed that the underlying assumptions of serial correlation analysis were violated with respect to price changes in the S.G.W.F.E. That is price changes were non-normal with stochastic variance. Runs testing is non parametric and hence is not affected by changing variance or non-normality. It is for these reasons that the results of runs analysis are to be preferred to the results of serial correlation.[20]

The application of runs theory to speculative prices was pioneered by Fama.[21] Praetz[22] and Stevenson and Bear[23] have adopted similar approaches to runs tests of the independence of price changes. Our experimental approach to runs testing was also similar to Fama's method. We analysed the difference between expected and actual empirical number of runs in three different ways. First by totals, then by sign and finally by length.

Total Actual and Expected Number of Runs

Mood[24] has shown that if the sample proportions of positive, negative and zero price changes are good estimates of the population proportions, then, under a hypothesis of independence the total

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20 In all fairness to the serial correlation technique, it must be pointed out that many researchers feel that the technique is reasonably robust with respect to non normality, see C.W.J. Granger and D. Orr. "Infinite Variance and Research Strategy in Time Series Analysis," *Journal of the American Statistical Association,* (June, 1972), Vol.67, No.338, p.280.

21 E. Fama, "Behavior of Stock Market Prices," *Journal of Business*, (1965), 38, pp.34-105.

22 P. Praetz, *op.cit.*

23 Stevenson and Bear, *op.cit.,* pp.73-75.

24 Mood, *op.cit.*, pp.367-377.

expected number of runs for all signs can be computed as

$$m = \frac{1}{N} [N(N + 1) - \sum_{i=1}^{3} n_i^2] \quad .... \quad (5.10)$$

where N equals the total number of observations and $\{n_i\}$ are the numbers of price changes of each sign.[25] The variance of the total expected number of runs m is given by the expression

$$\sigma_m^2 = \frac{\{\sum_{i=1}^{3} n_i^2[\sum_{i=1}^{3} n_i^2 + N(N+1)] - 2N\sum_{i=1}^{3} n_i^3 - N^3\}}{N^2(N - 1)} \quad ..... \quad (5.11)$$

and for large N the sampling distribution of m is approximately normal. This is an extremely robust test of independence as the asymptotic properties of the sampling distribution of m do not depend on normality of the distribution of price changes.[26] As the variable m is approximately normal with standard error σm, the difference between the expected number of runs m, the empirically observed number of runs R, can be expressed as a standardized random variable

$$z = \frac{(R + \frac{1}{2}) - m}{\sigma_m} \quad .... \quad (5.12)$$

where the ½ in the numerator is an adjustment for continuity. Table 5.2 sets out the expected number of runs m, the actual number of runs R and the standard normal deviate z for seventeen series.

Five of the seventeen calculated values of the standard normal deviate z were significant at the 5 per cent level. Thus runs analysis, like serial correlation analysis presented results contrary to the random walk model of price changes. Although the results of

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25 i = negative, zero, positive or i = 1, 2, 3,

26 W.A.Wallis and H.V.Roberts, *Statistics, a New Approach*, Glencoe Illinois: Free Press (1956).

TABLE 5.2

Values for the Expected (m) and Actual (R) Numbers of Runs, and the Standardized Variable (z)

Interval	Series	m	R	z
Transaction	*July*	433.4	348	-4.96**
Seven Times per day	*July*	106.2	96	-1.64
	October	102.5	88	-2.44**
	March	103.9	102	-0.24
	New July	100.1	106	1.13
Daily	*Near*	113.1	111	-0.23
	Four month	108.9	95	-1.96*
	Twelve month	112.5	95	-2.57**
	Distant	111.1	93	-2.66**
Weekly	*Near*	126.1	136	1.38
	Four month	130.5	144	1.87
	Twelve month	133.0	139	.86
	Distant	124.8	137	1.67
Monthly	*Near*	80.9	82	.26
	Four month	80.8	91	1.79
	Twelve month	84.6	84	-0.18
	Distant	81.6	91	1.67

* Indicates significance at 5% level.

** Indicates significance at 1% level.

total number of runs tests and autocorrelation tests were not in complete agreement, there were some areas where the results of the two techniques indicated the same departure from the random walk model. The number of runs for the *July transaction* series was significantly less than was expected under an hypothesis of independence of price change. This supports the notion that prices tend to move in the one direction in the very short run. The finite rate of information incorporation into a new price is revealed in these short term trends. Thus prices tend to move in the one direction until the new price is reached.[27]

The results of runs analysis on the *seven times per day* series also show that price changes tend to persist. This is in conflict with the serial correlation results that showed reactionary tendencies in the *seven times per day series*. Perhaps price reaction was outweighed by price trends. Of the daily series, three showed significantly less runs than we would have expected under the random walk model. This evidence of price persistence is again in conflict with the results of serial correlation analysis.

There were no significant values of z amongst the results for the *weekly* and *monthly* series. In all series bar one, the actual number of runs exceeded the expected number.

Comparison of total actual and expected numbers of runs presented in Table 5.2 provides some evidence against the hypothesis that price changes in the S.G.W.F.E. are random. The evidence was strongest for the short run series. Runs analysis revealed a degree of price persistence in the series consisting of price observation at intervals less than or equal to a day. These series contain a greater number of price trends than would be expected under the random walk model.

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[27] See Section 5.2.

Where the evidence of price dependencies tested by serial correlation and runs analysis is in conflict, the results of runs testing should be given more weight as runs analysis, unlike serial correlation analysis, is not dependent upon doubtful assumptions about the nature of the distribution of price changes.

Actual and Expected Numbers of Runs of Each Sign

Suppose the signs of each run are generated by an independent Bernoulli process, where P(+), P(-) and P(0) represent the probabilities for the three possible types of price change. Thus for large samples the expected number of plus runs of length i in a sample of N price changes is given by the formula

$$R(+)_i = NP(+)^i \, [1 - P(+)]^2 \quad \ldots\ldots\ldots\ldots \quad (5.13)$$

where $R(+)_i$ equals the expected number of positive runs of length i. The expected number of runs of all length is determined by the summation of equation (5.13)

$$R(+) = \sum_{i=1}^{\infty} N \, P(+)^i \, [1 - P(+)]^2 \quad \ldots\ldots\ldots \quad (5.14)$$

$$= N \, P(+) \, [1 - P(+)] \quad \ldots\ldots\ldots\ldots \quad (5.15)$$

Similarly the expected numbers of minus and zero runs are

$$R(-) = N \, P(-) \, [1 - P(-)] \quad \ldots\ldots\ldots\ldots \quad (5.16)$$

and

$$R(0) = N \, P(0) \, [1 - P(0)] \quad \ldots\ldots\ldots\ldots \quad (5.17)$$

The probability of a plus run can be expressed as the ratio of the expected number of plus runs in a sample of size N, to the total expected number of runs of all signs.

$$P(+\ \text{run}) = \frac{N\left\{P(+)\left[1 - P(+)\right]\right\}}{m} \quad \ldots\ldots\ldots\ldots \quad (5.18)$$

m is the variable defined in equation (5.10). The expected number of plus runs as a fraction of the total actual number of runs is as follows:

$$R\ (+\ \text{runs}) = R\ [P\ (+\ \text{run})\ ] \quad \ldots\ldots\ldots\ldots \quad (5.19)$$

where R is the actual total runs of all signs. Similarly,

$$R\ (-\ \text{runs}) = R\ [P\ (-\ \text{run})] \quad \ldots\ldots\ldots\ldots \quad (5.20)$$

and

$$R\ (0\ \text{runs}) = R\ [P\ (0\ \text{run})] \quad \ldots\ldots\ldots\ldots \quad (5.21)$$

Using R to compute the breakdown of runs by sign means that the summation of the expected number of runs of all signs equals the total actual number of runs. This avoids any confusion resulting from the fact that the number of runs of all signs was often very different from the expected number of runs of all signs.[28] Table 5.3 lists the breakdown of runs by signs.

The percentage difference between the actual and the expected number of runs for each sign was small for the majority of series. We concluded from the small difference between actual and expected numbers that the sign of the runs were generated by an independent Bernoulli process. That is, runs of one sign did not dominate the total number of runs at the expense of runs of the other signs, given the proportion of positive, negative and zero price changes.

Distribution of Runs by Length

In this the last section of runs analysis, the distribution of runs by length as a portion of total actual numbers of runs is

28 See Table 5.2.

TABLE 5.3

RUNS ANALYSIS BY SIGNS

Interval	Series	Positive Runs			Zero Runs			Negative Runs		
		Actual	Expected	% Difference	Actual	Expected	% Difference	Actual	Expected	% Difference
Transaction	*July*	122	120.6	1	175	178.8	-2	87	83.8	4
Seven Times/Day	*July*	35	33.4	5	33	34.1	-3	28	27.6	1
	October	33	28.5	16	27	34.2	-21	28	24.1	16
	March	35	32.6	7	39	38.0	3	28	29.4	-5
	New July	29	34.3	-15	45	42.3	6	32	28.4	13
Daily	*Near*	51	50.5	1	11	10.2	8	49	49.4	-1
	Four Month	45	44.5	1	7	5.9	19	43	43.7	-2
	Twelve Month	44	42.9	3	12	10.3	17	39	41.0	-5
	Distant	43	42.6	1	10	8.7	15	40	40.8	-2
Weekly	*Near*	64	64.0	0	6	6.3	-5	66	64.6	2
	Four Month	68	66.6	2	9	9.7	-7	67	66.7	0
	Twelve Month	66	64.4	2	9	9.1	-1	64	64.5	-1
	Distant	66	65.6	1	4	4.3	-7	67	66.0	2
Monthly	*Near*	39	38.7	1	4	4.0	0	39	38.4	2
	Four Month	43	43.0	0	3	4.4	-32	45	42.5	6
	Twelve Month	40	38.0	5	8	7.5	7	36	37.5	-4
	Distant	44	42.5	4	5	5.4	-7	42	42.0	0

examined. As before - equation (5.13) - the expected number of plus runs of length i in a sample of N price changes is given by,

$$R(+)_i = N\,P(+)^i\,[1 - P(+)]^2 \qquad \text{..........} \qquad (5.22)$$

and the total expected number of plus runs,

$$R(+) = N\,P(+)\,[1 - P(+)]$$

Therefore out of the total expected number of runs the expected number of plus runs of length i is

$$\bar{R}(+)_i = \frac{N\,P(+)^i[1 - P(+)]^2}{N\,P(+)\,[1 - P(+)]} \qquad \text{..........} \qquad (5.23)$$

$$= P(+)^{i-1}\,[1 + P(+)] \qquad \text{..........} \qquad (5.24)$$

Thus the expected distribution by length of the total actual number of runs can be computed

$$\bar{R}(+)_i = \hat{R}(+)\,P(+)^{i-1}[1 + P(+)\,] \qquad \text{..........} \qquad (5.25)$$

$$\bar{R}(-)_i = \hat{R}(-)\,P(-)^{i-1}\,[1 + P(-)] \qquad \text{..........} \qquad (5.26)$$

$$\bar{R}(0)_i = \hat{R}(0)\,P(0)^{i-1}\,[1 + P(0)] \qquad \text{..........} \qquad (5.27)$$

where $\bar{R}(+)_i$ is the expected number of plus runs of length i and $\hat{R}(+)$ is the actual number of plus runs . The number of runs of any sign of length i can be computed by summing equations (5.25), (5.26) and (5.27).

$$\bar{R}(+-0)_i = \bar{R}(+)_i + \bar{R}(0)_i + \bar{R}(-)_i \qquad \text{..........} \qquad (5.28)$$

Table 5.4 contains the expected and actual numbers of runs of length one to nine, for four typical series.

TABLE 5.4

Expected and Actual Distributions of Runs of Length (i)

Length	*Seven Times/Day July Series*		*Daily Near Series*		*Weekly Near Series*		*Monthly Near Series*	
	Expected	*Actual*	*Expected*	*Actual*	*Expected*	*Actual*	*Expected*	*Actual*
1	63.6	52	63.0	62	72.4	81	44.0	48
2	21.2	31	25.2	26	32.7	28	19.5	17
3	7.3	8	11.8	11	16.2	16	9.4	7
4	2.5	3	5.6	6	7.8	7	3.3	6
5	.9	2	2.7	3	3.9	0	1.6	2
6	.3	0	1.3	1	1.9	2	.8	1
7	.1	0	.7	2	1.0	1	.4	0
8	.0	0	.3	0	.3	0	.2	0
9	.0	0	.2	0	.2	1	.1	1

There are no consistent discrepancies between the expected number and the actual number of runs of length i. It is possible that there is a slight tendency to an excess of longer runs. For example, there were five runs equal to or greater than length seven, compared to the expected number of 3.5 under the hypothesis of independence.

We applied a chi squared goodness of fit test to seventeen series decomposed into their frequency distributions of length of runs. We consider this test of the empirical distribution of length of runs to be complimentary to the test of total actual and expected number of runs of Section 5.3. For instance, it is possible for a series to contain non random elements such that the number of short runs is increased and the number of long runs decreased. Thus it is possible the overall number of runs of a non random series could equal the overall number of runs of a random series. A series containing the above described dependencies would not be revealed as non random by the z test of Section 5.3, but would be picked up by the chi squared test of distribution of runs length. Hence a series may be presumed to contain non random associations if z or the chi squared values are significantly different from zero.

In Table 5.5, the series are numbered rather than named. For the *daily*, *weekly* and *monthly* series numbers 1, 2, 3 and 4 represent respectively the *near*, *four months*, *twelve months* and *distant* series. However, for the *Seven Times/Day* series, numbers 1, 2, 3 and 4 represent the *July*, *October*, *March* and *New July* series; e.g. Series 1 for the transaction price series is the *July* series.

The significant values for the goodness of fit of expected versus actual runs of each length are listed in Table 5.5. The significant chi squared result for the *July Transaction* is due to a preponderance of very long zero runs. The other three significant chi squared values are the result of slight positive correlation between futures prices. The positive correlation resulted in a paucity of short runs and surfeit of longer runs.

Summary of Runs Analysis Results

The runs tests provided more evidence that prices changes in the S.G.W.F.E. do not follow a random walk in the short run. Tests of the total actual number of runs[29] showed that for intervals of one day or less there was a significant tendency for prices to move in one direction. Runs analysis did not, however, corroborate the evidence of serial correlation analysis that demonstrated reaction in the price series.

Analysis of runs of each sign failed to show any anomalous behaviour. The signs of price runs are as if generated by a Bernoulli

[29] Table 5.2.

TABLE 5.5

Chi Square Values for Distribution of Runs

Series No.	Interval				
	Transaction	*Seven Times/ Day*	*Daily*	*Weekly*	*Monthly*
1	28.4**	6.98*	.22	3.2	3.5
2		5.53	6.3	4.6	4.5
3		1.78	7.0	5.9	.5
4		1.56	10.0*	9.6*	4.3

* indicates significance at 5% level

** indicates significance at 1% level.

process.

Tests of the distribution of runs showed small departures from the random walk hypothesis. There was some evidence of an excess of longer runs in some series.

It is our belief that the deviations from the random walk model, as demonstrated by runs analysis are too small to be profitably exploited, the results of the filter technique certainly reinforce this opinion.[30]

5.4 Spectral Analysis

Spectral analysis is a recently developed technique to test for dependencies in economic time series.[31] Spectral analysis is concerned with decomposing time series into a number of components,

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30 See Section 5.7.

31 Probably the most comprehensive text concerning spectral and cross spectral analysis is by C.W.J. Granger and M. Hatanaka, *Spectral Analysis of Economic Time Series*, (1964), Princeton University Press, Princeton, New Jersey.

each associated with a frequency or period.[32] This spectral decomposition of a time series yields a spectral density function and measures the relative importance of each frequency band in terms of its contribution to the overall variance of a time series. Essentially, spectral analysis is an examination of the variance of a time series with respect to frequency components.

Theory and Estimation of the Spectrum

If a time series is stationary in the wide sense[33] then it is possible to estimate the mean, variance and covariance function by averages over time. That is, if we have a time series

$x_1\ x_2\ \ldots\ x_t \ldots x_n$ then

$$\bar{x} = \frac{1}{n}\sum_{t=1}^{n} x_t \qquad \ldots\ldots\ldots\ldots \qquad (5.29)$$

$$s^2 = \frac{1}{n}\sum_{t=1}^{n} (x_t - \bar{x})^2 \qquad \ldots\ldots\ldots\ldots \qquad (5.30)$$

$$C_g = \frac{1}{n-g}\sum_{t=1}^{n-g} (x_t - \bar{x})\ (x_{t+g} - \bar{x}) \qquad \ldots\ldots\ldots\ldots \qquad (5.31)$$

The next step in understanding spectral analysis is to consider the time series x_t not as a set of real numbers, but as a set of complex numbers. Thus x_t becomes a set of a complex stationary process with

$$E\ [X_t] = 0 \qquad \ldots\ldots\ldots\ldots \qquad (5.32)$$

$$E\ [X_t \bar{X}_t] = \sigma^2 \qquad \ldots\ldots\ldots\ldots \qquad (5.33)$$

$$E\ [X_t\ \bar{X}_{t+g}] = u_g \qquad \ldots\ldots\ldots\ldots \qquad (5.34)$$

where $\bar{X}$ is the complex conjugate.

32 Frequency indicates the number of cycles per unit of time and the period describes the length of time required for one complete cycle.

33 A series is stationary in the wide sense if the mean, variance and autocorrelation function are independent of t the time of observation.

The two formulae which provide the basis for the entire spectral method are the spectral representation of the covariance sequence

$$u(g) = E(S_t X_{t-g}) = \int_{-\pi}^{\pi} e^{igw} dF(w) \quad \ldots\ldots\ldots\ldots \quad (5.36)$$

and the spectral representation of the series

$$X_t = \int_{-\pi}^{\pi} e^{itw} dZ(w) \quad \ldots\ldots\ldots\ldots \quad (5.36)$$

where $Z(w)$ is a complex random function having the properties

$$E\,[dZ(w)\,\overline{dZ}\,(\lambda)\,] = 0 \qquad w \neq \lambda \quad \ldots\ldots\ldots\ldots \quad (5.37)$$

$$= dF(w) \qquad w = \lambda \quad \ldots\ldots\ldots\ldots \quad (5.38)$$

and $F(w)$ is an increasing step function.

The above presentation of spectral theory can be represented much more simply by considering an elementary model. Consider a time series $\{x_t\}$ made up of a number of purely cyclical components with amplitudes a_j, frequency w_j and phase θ_j. The time series can be represented by

$$x_t = \sum_{j=1}^{m} a_j \cos(tw_j + \theta_j) \quad \ldots\ldots\ldots\ldots \quad (5.39$$

x_t is the finite sum of independent components each of which has associated with it a different frequency. By direct evaluation it follows that[34]

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[34] The following exposition of spectral analysis was taken from; W.C. Labys and C.W.J. Granger, *Speculation Hedging and Commodity Price Forecasts*, (1970), Heath Lexington, Lexington, Massachusetts, pp.42-44.

$$\text{var}(x_t) = \frac{1}{2}\sum_{j=1}^{m}\sigma_j^2 \qquad \text{..........} \qquad (5.40)$$

and

$$u(g) = \text{cov}(x_t x_{t-g}) = \frac{1}{2}\sum_{j=1}^{m}\sigma_j^2\cos gw_j \qquad \text{.......} \qquad (5.41)$$

so the importance of each component can be measured in terms of the contribution it makes $(\frac{1}{2}\sigma_j^2)$ to the total variance of x_t. Similarly the covariance sequence $u(g)$, $g = 0, 1, 2, \ldots$ is given by

$$u(g) = \int_{-\pi}^{\pi}\cos gw\,dF(w) \qquad \text{...........} \qquad (5.42)$$

where $F(w)$ is a step function with steps of size $\frac{1}{2}\sigma_j^2$ (the individual contribution to variance) at w_j and flat elsewhere. Equation (5.42) is a real version of equation (5.35).

A simpler version of equation (5.42) is

$$u(g) = \int_{-\pi}^{\pi}\cos gwf(w)\,dw$$

where $f(w)$ is the derivative of $F(w)$ and is called the power spectral density function. It arises when the number of components becomes extremely large, and thus no one component makes a finite contribution to $\text{var}(x_t)$ but the sum of components with frequencies in any small band do make a finite contribution.

The idea of decomposing a time series into various components is not a new one. One of the classical tools for analysis of time series concerns the separation of time series into trend, cyclical, seasonal and irregular. Spectral analysis is merely a more sophisticated technique for the decomposition of an economic variable into its frequency components.

The first step in estimating the spectrum of the time series $\{x_t\}$, $t = 1, 2, \ldots n$, involves the estimation of the covariance function,

$$\hat{C}_j = \frac{1}{n-j} \sum_{t=1}^{n-j} (x_t - \bar{x})(x_{t+j} - \bar{x}) \quad \ldots\ldots\ldots\ldots \quad (5.43)$$

where $j = 0, 1, 2 \ldots m$.

The spectral estimates are calculated thus

$$\hat{f}(w_j) = \frac{1}{2\pi} [\ \lambda_0 \hat{C}_0 + 2 \sum_{j=1}^{m-1} \lambda_j \hat{C}_j \cos w_j j\ ] \quad \ldots\ldots \quad (5.44)$$

where

$$w_j = \frac{\pi j}{m}, \quad j = 0, 1, 2 \ldots m \quad \ldots\ldots\ldots\ldots \quad (5.45)$$

λ_0 = system of weights

and $\hat{f}(w_j)$ = spectral estimate at frequency w_j.

There are many possible systems of weights. We chose the Parzen system of weights to provide our empirical spectral functions.[35] The Parzen estimate allows very small leakage from one frequency band to another and it also has the useful property of never giving negative estimates. The Parzen weights are defined as,

$$\lambda_j = 1 - \frac{6j^2}{m^2}\left(1 - \frac{j}{m}\right) \quad 0 \leq j \leq m/2 \quad \ldots\ldots\ldots \quad (5.46)$$

$$= 2\left(1 - \frac{j}{m}\right)^3 \quad m/2 \leq j \leq m \quad \ldots\ldots\ldots\ldots \quad (5.47)$$

The spectrum is estimated at m + 1 equidistant points. The choice of m is entirely up to the researcher. However, the larger is m, the greater is the variance of the estimate of the spectrum at each point. The smaller is m the better is the estimate. However, bias may creep in if m becomes too small. Granger and Hatanaka recommend that m be less than n/3 and greater than n/6 where n is the number of observations of the time series.[36]

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35 A full discussion of Parzen weights is given in G.M.Jenkins, "General Considerations in the Analysis of Spectra," *Technometrics*, Vol.3, (1961) pp.133-166.

36 Granger and Hatanaka, *op.cit.*, p.61.

If the spectrum is estimated at m + 1 points, that is m is the maximum number of lags, then it is a simple process to calculate the frequency and period associated with each point of estimation. Let j equal the specific point of estimation, then the period is

$$P_j = \frac{2m}{j} \qquad j = 0, 1, 2, \ldots m \qquad \text{..........} \qquad (5.48)$$

and the frequency is

$$w_j = \frac{j}{2m} \qquad \text{..........} \qquad (5.49)$$

Five and ten per cent confidence intervals for the theoretical spectrum were calculated by Jenkins and reprinted in Granger and Hatanaka.[37] The confidence limits are

$$T_\alpha\ (m, n)\ \sigma^2/2\pi \qquad \text{and} \qquad T'_\alpha\ (m, n)\ \sigma^2/2\pi$$

These are (100 - 2α) per cent confidence limits

$T_\alpha(m, n)$ is $\chi^2_{100-\alpha}(k) / k$

and $T'_\alpha(m, n)$ is $\chi^2_\alpha\ (k) / k$

where $\chi^2_\beta(S)$ is the β per cent value of the χ^2 distribution with S degrees of freedom. k the equivalent degrees of freedom is defined

$$k = 2n/m \qquad \text{..........} \qquad (5.50)$$

where n is the number of observations and m is the number of lags used in the estimate.

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37 Granger and Hatanaka, *op.cit.*, p.61.

Previous Research

Granger and Morgenstern applied spectral analysis to stock price series and indices. They concluded that their spectra of log. price differences were flat for all series and thus strongly supported the random walk model.[38] Praetz also applied spectral techniques to share price data.[39] He showed that Sydney share price data contained yearly cycles. In his opinion, the seasonality was a function of the Australian tax system. He also implied business cycle frequencies were prominent in his empirical spectra.

Labys and Granger[40] tested many cash and future price series for U.S. commodities for random behaviour using spectral techniques. Quite a number of their series contained significant seasonal components. They concluded that their series were basically random walks, with an occasional seasonal cycle. The hypothesis of random behavior in U.S. commodity futures markets was also investigated by Cargill and Rausser.[41] They found approximately one fifth of their 196 empirical spectra exhibited non random behavior.

Live beef cattle futures prices were subjected to spectral analysis by Leuthold.[42] His results suggested that although prices follow a simple random walk some of the time, there were occasions when this simple stochastic process did not describe beef futures prices.

In summary, it can be said that most spectral studies on speculative price series have revealed some deviations from the

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38 C.W.J. Granger and O. Morgenstern, *Predictability of Stock Market Prices*, Lexington, Massachusetts: Heath Lexington, (1970).

39 Praetz, *op.cit.*

40 Labys and Granger, *op.cit.*

41 Cargill and Rausser, *op.cit.*

42 Leuthold, *op.cit.*

characteristic flat spectra of the random walk model. These deviations from the simple stochastic process have been more obvious in commodity and commodity futures prices than in share prices.

Results of Spectral Analysis

The random walk model of price formation dictates that the terms of a price time series $\{x_t\}$; t = 1, 2, ... n, are independent of each other. The spectrum of an independent sequence of terms is a horizontal line parallel to the x-axis over the complete frequency range.

To obtain each empirical spectrum it was necessary to specify m, the maximum number of lags. We followed Granger and Hatanaka's advice specifying m so that,

$$n/6 \leq m \leq n/3$$

The length of the specific cycle under investigation provides another criterion for choosing m. If j equals the particular point of estimation, j = 1, 2, ... m. The period associated with the estimation point j is given by

$$p_j = 2m/j \qquad \ldots\ldots\ldots\ldots \qquad (5.51)$$

Thus, if we are interested in a particular cycle m must be specified to allow j to be an integer at that frequency. For example, suppose we are using monthly data to test for a yearly cycle, that is, p = 12. If m equals 24, the power of the yearly cycle will be estimated at the fourth lag (j = 4).[43] After some experimentation, we decided upon the maximum number of lags m, as given below.

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[43] For further clarification see Granger and Hatanaka, *op.cit.*, pp.61-63.

TABLE 5.6

Maximum Number of Lags (m) Used in Spectral Estimation

Series	(m)
Transaction	40
Seven Times/Day	35
Daily	35
Weekly	36
Monthly	24 and 48

Some of the empirical spectra are presented in Figures 5.2 - 5.14. Each consists of log. spectrum $f(w_j)$ plotted against the number of estimation lags j. Log. $f(w_j)$ is used rather than $f(w_j)$ for two reasons. First, it is an historical fact that for economic series, the lower frequencies are generally more important than the higher frequencies. Thus plotting on a log. scale allows all points to be shown conveniently. Second, log. plotting means that confidence bands can easily be added as horizontal straight lines.[44]

The x-axis of Figures 5.2 - 5.14 consists of the number of lags j. The lags j may be quickly transferred into the associated period or frequency. The period p_j is given by

$$p_j = 2m/j \qquad \text{..........} \qquad (5.52)$$

where m is the maximum number of lags given in Table 5.6. The frequency w_j is the inverse of the period and is therefore given by

$$w_j = j/2m \qquad \text{..........} \qquad (5.53)$$

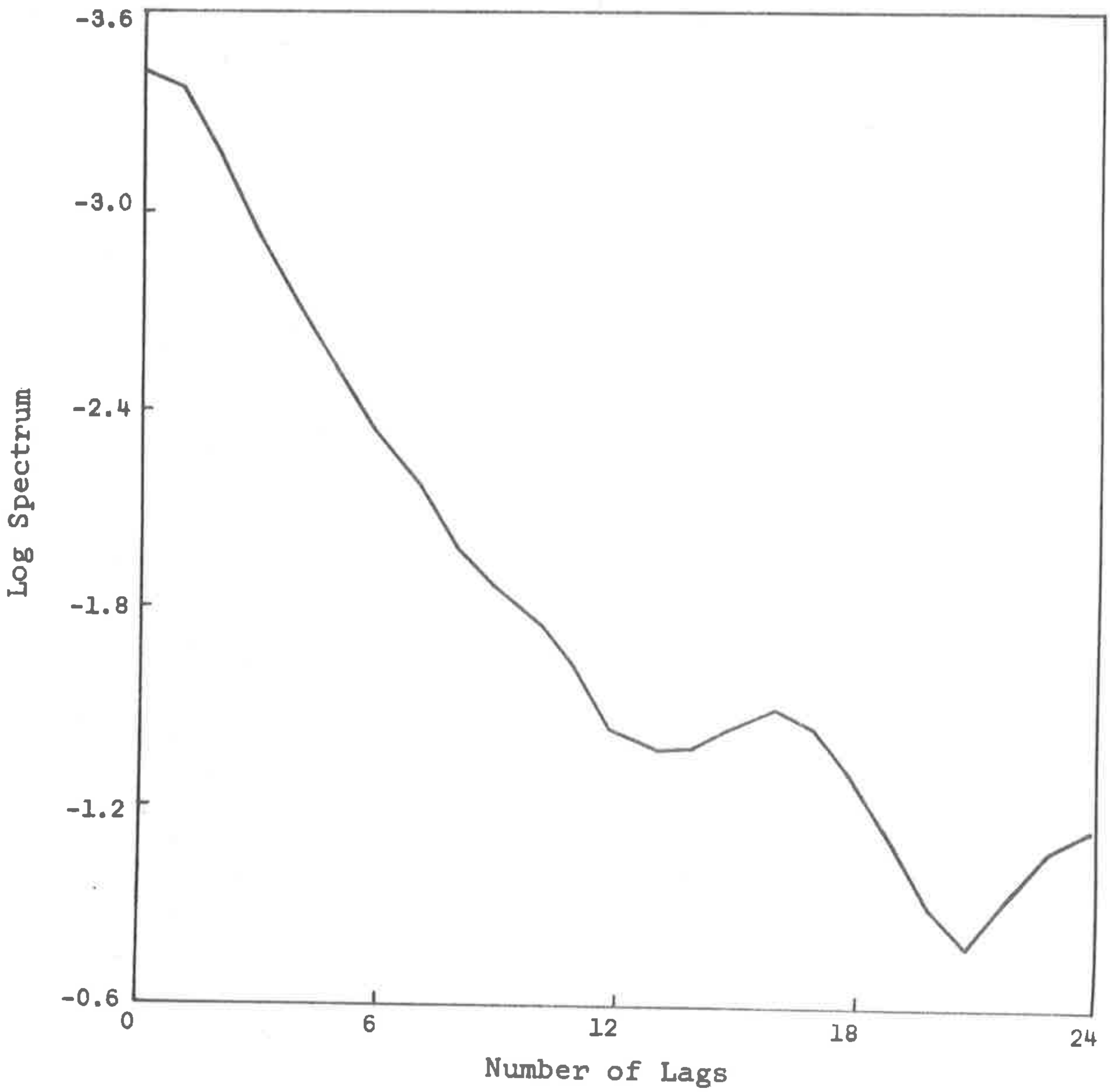
It follows from equations(5.52) and (5.53) that as the number of estimation lags j, increases, the associated frequency w_j increases

[44] Granger and Hatanaka, *op.cit.*, p.62.

FIGURE 6·2

SPECTRUM OF MONTHLY FUTURES PRICE CHANGES

UNTRANSFORMED NEAR MONTHLY SERIES



and the period p_j decreases.

The spectral output, Figures 5.2 - 5.14 have a figure in parenthesis, above any significant spectral peak. This figure is the period associated with the spectral peak and unless stipulated otherwise, is in the same units as the observations. For example, Figure 5.4 has a period of 12 months associated with a spectral peak.

Figure 5.2 is the estimated spectrum for the untransformed n *near monthly* futures price series. The spectrum is typical of most untransformed economic time series. Power is concentrated at the lower frequencies and declines almost exponentially as the frequency increases. The high power at the low frequencies is the result of trend in the mean of the time series. Any trend will give the zero frequency a large value and will be inclined to raise the value of neighbouring bands.[45] Thus before we could use spectral analysis to test for cycles, it was necessary to remove trend.

We first used simple regression on time in an attempt to remove trend in the series. However, it was quickly apparent that this method failed to remove the inordinate amount of power at the low frequencies. The log. first difference transformation was found to be the most efficient method of removing the trend component of the empirical spectra. Hence it was the familiar variable y_t that was examined for cyclical behavior using the spectral technique.

$$y_t = \log. P_t - \log. P_{t-1} \qquad \ldots\ldots\ldots\ldots \qquad (5.54)$$

. 5.7 and 5.8 represent in summary form, the results of spectral analysis of 18 price series. A more detailed discussion of the results is left until the next section. Table 5.7 lists the number of spectral points outside the five and ten per cent confidence limits in both absolute and percentage terms. Table 5.8 classifies

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[45] Granger and Hatanaka, *op.cit.*, pp.132-134.

TABLE 5.7

Spectral Points Outside the 5% and 10% Confidence Intervals

Interval	Series	5%		10%	
		Number	Percentage	Number	Percentage
Transaction	*May*	0	0	3	7.5
Seven Times/Day	*July*	0	0	0	0
	October	3	8.3	10	27.8
	March	0	0	2	5.6
	New July	0	0	0	0
Daily	*Near*	0	0	0	0
	Four Months	3	8.3	5	13.9
	Twelve Months	3	8.3	8	22.2
	Distant	1	2.8	3	8.3
Weekly	*Near*	1	2.8	7	19.4
	Four Months	2	5.6	6	6.7
	Twelve Months	4	11.1	4	11.1
	Distant	1	2.8	2	5.6
Monthly	*Physical*	4	16.7	6	2.4
	Near	10	33.3	14	45.8
	Four Months	5	20.8	10	41.7
	Twelve Months	4	8.3	7	16.7
	Distant	4	8.3	5	20.8

each series into one of three categories. The series are listed as being either, random (R), almost random (A.R), or non random (N.R.). The classification was performed subjectively and is only partly determined by the number of points outside the confidence limits. If, for example, there is evidence of a spectral peak at a logical frequency, such as the seasonal frequency, then we would not consider the series to be random. In our opinion this is so even if the spectral peak does not traverse the particular confidence interval.

TABLE 5.8

Classification and Major Period Component

Interval	Series	Classification	Major Period Component
Transaction	*May*	R.	13.3, 4.7, 3.2
Seven Times/Day	*July*	R.	4.4, 3.0, 2.1
	October	A.R.	3.2, 2.2
	March	R.	35, 4.4, 2.3
	New July	A.R.	35, 7, 3.5, 2.3
Daily	*Near*	R.	10, 5.4, 2.9
	Four Months	N.R.	5, 2.6
	Twelve Months	N.R.	5, 2.6
	Distant	A.R.	5
Weekly	*Near*	A.R.	10, 5.4, 2.4
	Four Months	A.R.	7, 4.4, 3.3, 2.1
	Twelve Months	N.R.	4.7, 3.4
	Distant	A.R.	4.7, 2.4
Monthly	*Physical*	A.R.	12.0,
	Near	N.R.	12.0, 2.8, 2.4
	Four Month	N.R.	10.7, 3.1
	Twelve Month	A.R.	12.0, 3.2
	Distant	A.R.	12.0, 3.1, 2.2

It is obvious from Tables 5.7 and 5.8 that the simple random walk model is an over-simplification of price formation in the S.G.W.F.E. There was definite evidence of the existence of a weekly cycle. We were very surprized that our initial spectral analysis did not reveal the much touted seasonality of wool prices.[46]

Spectral Results for the Monthly Series

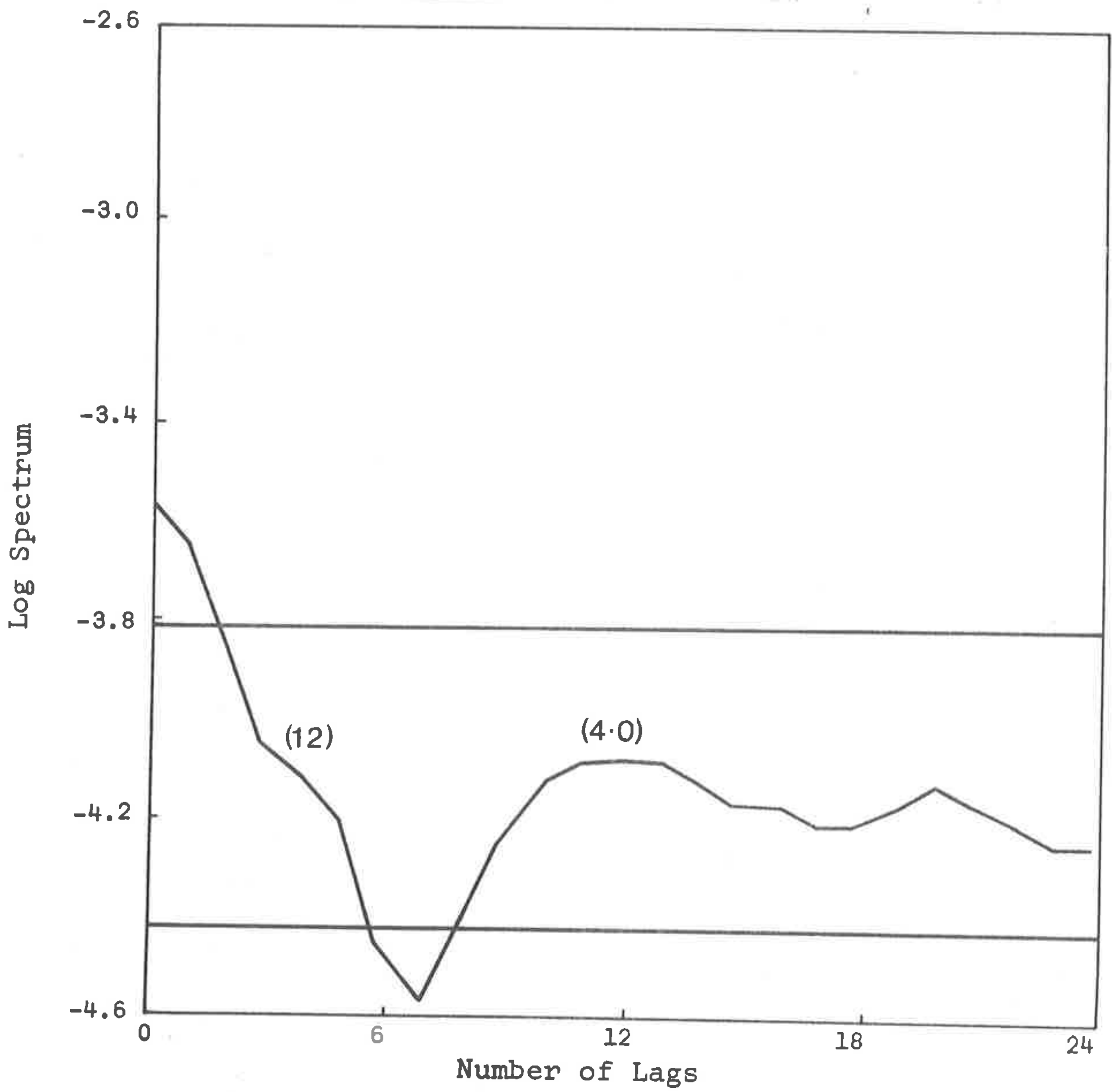
The spectrum of *physical* wool prices (Figure 5.3) showed no evidence of a seasonal cycle. The absence of seasonality is contrary

46 R.H. Snape, "Price Relationships on the Sydney Wool Futures Market," *Econometrica*, (May, 1968).

FIGURE 5·3

SPECTRUM OF MONTHLY PHYSICAL PRICE CHANGES,

24 LAGS



to the evidence of other researchers. R. H. Snape writes,

> It is well known that wool prices exhibit a seasonal movement, being relatively low early in the selling season (late August, September and October) and high towards the end (May, June and July).[47]

On *a priori* grounds, we expected to discover some seasonality. Wool certainly has seasonality of supply which would *ceteris paribus* be converted into seasonality of price. Offsetting this potential yearly price movement is the fact that wool is a durable good and thus can be carried over from a period of excess supply to a period of short supply. Nevertheless, we believed the cost of storage would have ensured that some seasonality remained in wool prices.

The spectrum of physical wool prices (Figure 5.3) contains a great deal of power at the lower frequencies. This is not surprising as wool prices were quite high during the early 1960's, then low during the late sixties and finally recovered in the early 1970's. Such price movements would show up as a very long cycle, perhaps 10 years in length. This is the reason for large spectral estimates at the very low frequencies.

It is plausible that the excess power at lags 0, 1 and 2 have leaked into and possibly masked the seasonal cycle effect, estimated at lag 4. This hypothesis is given some weight by the fact that there is a spectral peak at the four month harmonic of the 12 month seasonal cycle.[48] Also the index of monthly seasonal variation (Table 5.8) would seem to indicate a quite strong seasonal pattern in physical prices. The values of the seasonal index lends support to Snape's hypothesis about wool prices, namely, that they are high in May, June and July and low in August, September and October.

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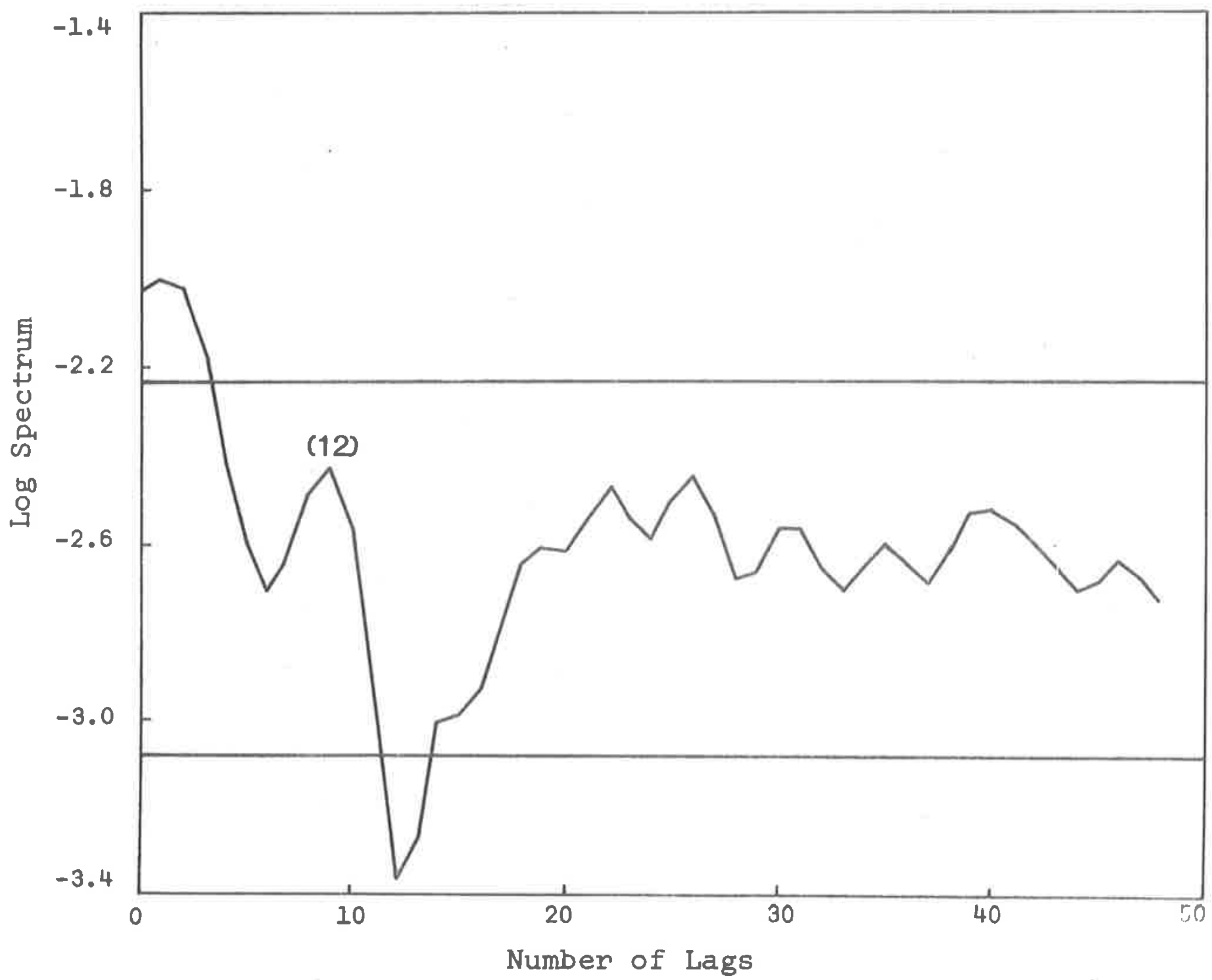
[47] R.H. Snape, *op.cit.*, p.170.

[48] The relationship of harmonics to their fundamental frequency is outlined in Granger and Hatanaka, *op.cit.*, pp.46-47.

FIGURE 5·4

SPECTRUM OF MONTHLY PHYSICAL PRICE CHANGES, 48 LAGS



For the above reasons we believed there existed seasonality of physical wool prices, but it was being masked by the high powered low frequency cycles. To separate the cycles we increased the number of estimations points by setting m = 48.[49] The resulting spectrum (Figure 5.4) displayed a non-significant yet discernible peak at the seasonal frequency.

What is the effect on futures prices of the commodity having a seasonal cycle? Samuelson argues that an individual futures price will follow a random walk irrespective of any seasonality in the commodity market.[50] Briefly the rationale is as follows: a futures price represents an estimation by the market traders of the commodity price on the day the future matures. This is so because arbitrage ensures the equality of the futures price and the physical price on the day the futures contract expires. As the futures price represents an estimation of the physical price on a particular day, it is not affected by what the expected physical price will be on the next day or the next month. The price of the *March* future is an estimation of the price of wool on the 23rd of March; it is irrespective of the fact that seasonality of prices dictates that the price on the 23rd of March will be lower than the price on the 1st of June. Thus prices of the March future should follow a random walk even if wool prices do not.

Samuelson's conclusions hold true for an individual future, however our monthly futures price series consist of a combination of futures prices. For example, in a year the monthly *near* futures price

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49 The reason why we were initially reluctant to set m = 48 was that with 153 monthly observations such a level of m is at the outer limit of our rule of thumb that $m < n/3$.

50 The argument is presented in Section 2.2; P.A.Samuelson, "Proof that Properly Anticipated Prices Fluctuate Randomly," *Industrial Management Review*, Vol.6, p.41, (1965).

FIGURE 5·5

SPECTRUM OF MONTHLY FUTURES PRICE CHANGES,

NEAR SERIES

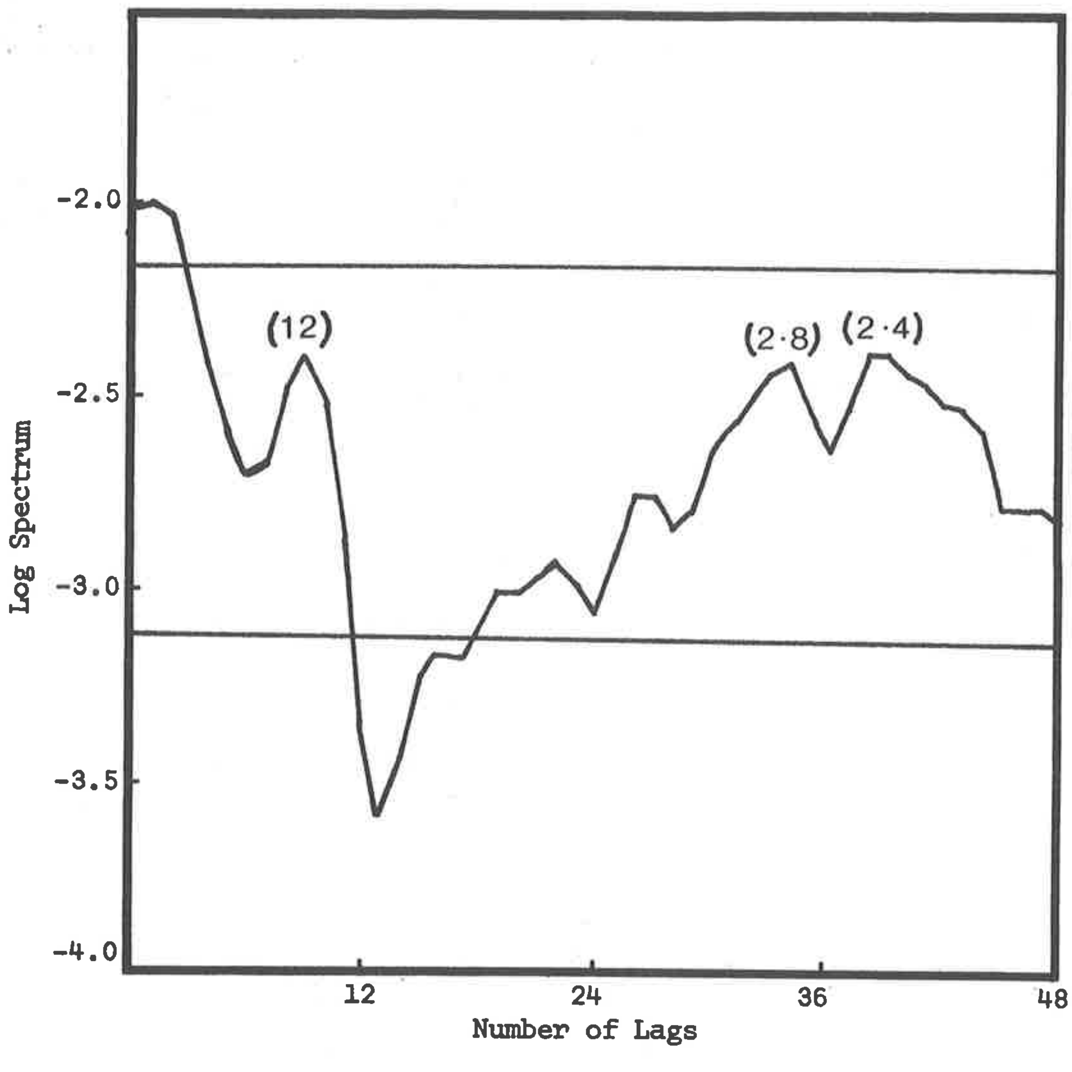


FIGURE 5·6

SPECTRUM OF MONTHLY FUTURES PRICE CHANGES,

FOUR MONTH SERIES

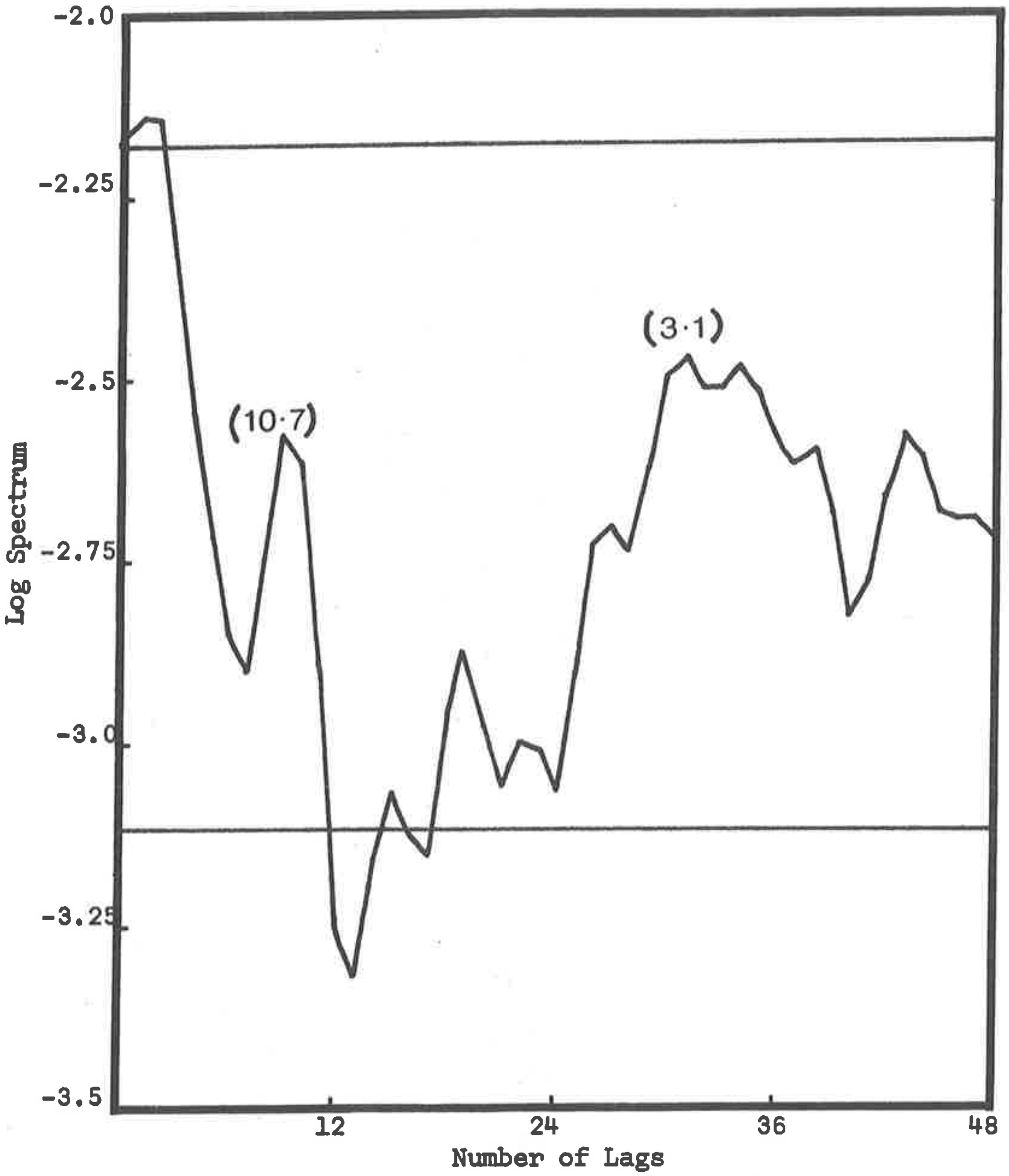


FIGURE 5·7

SPECTRUM OF MONTHLY FUTURES PRICE CHANGES,

TWELVE MONTH SERIES

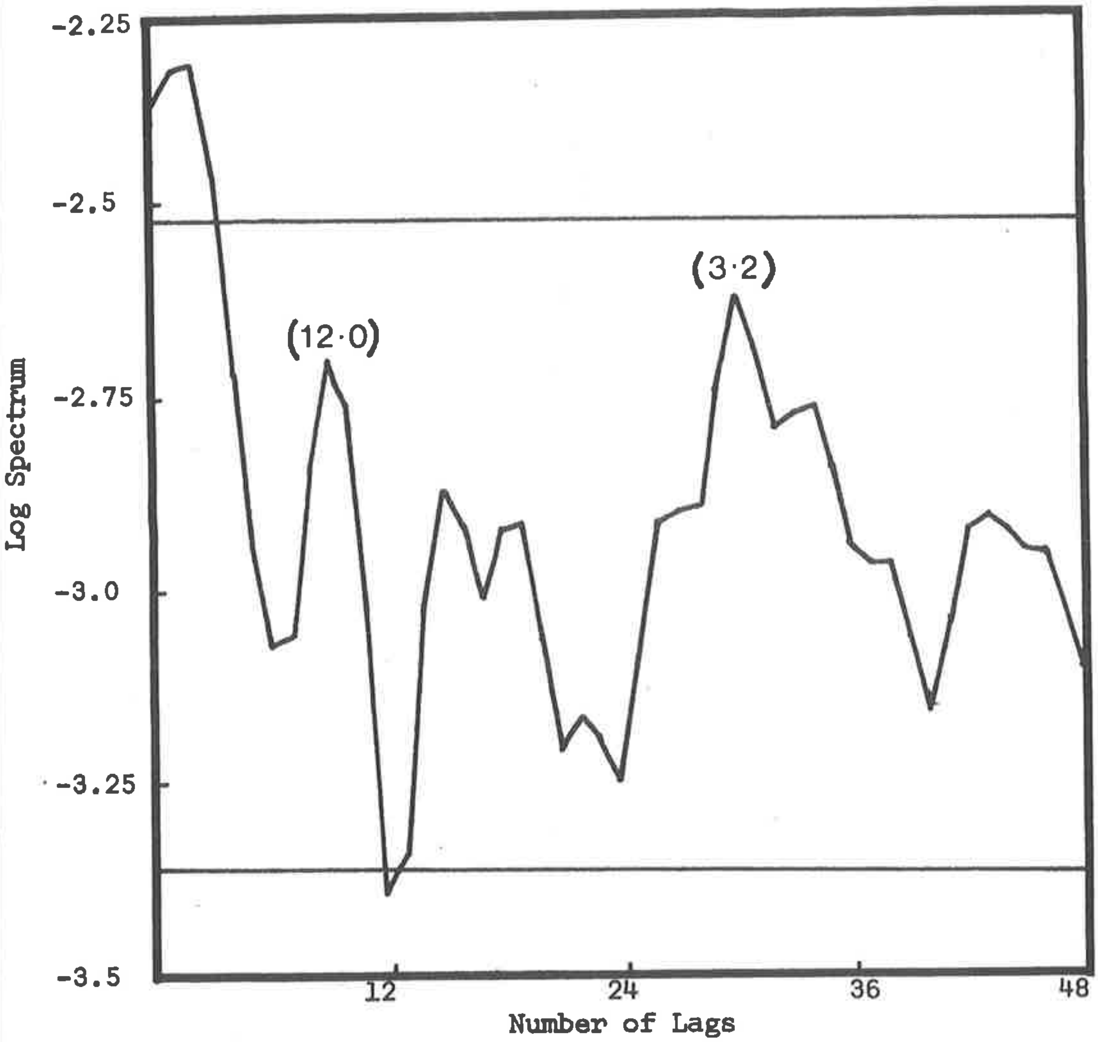


FIGURE 5·8

SPECTRUM OF MONTHLY FUTURES PRICE CHANGES,

DISTANT SERIES

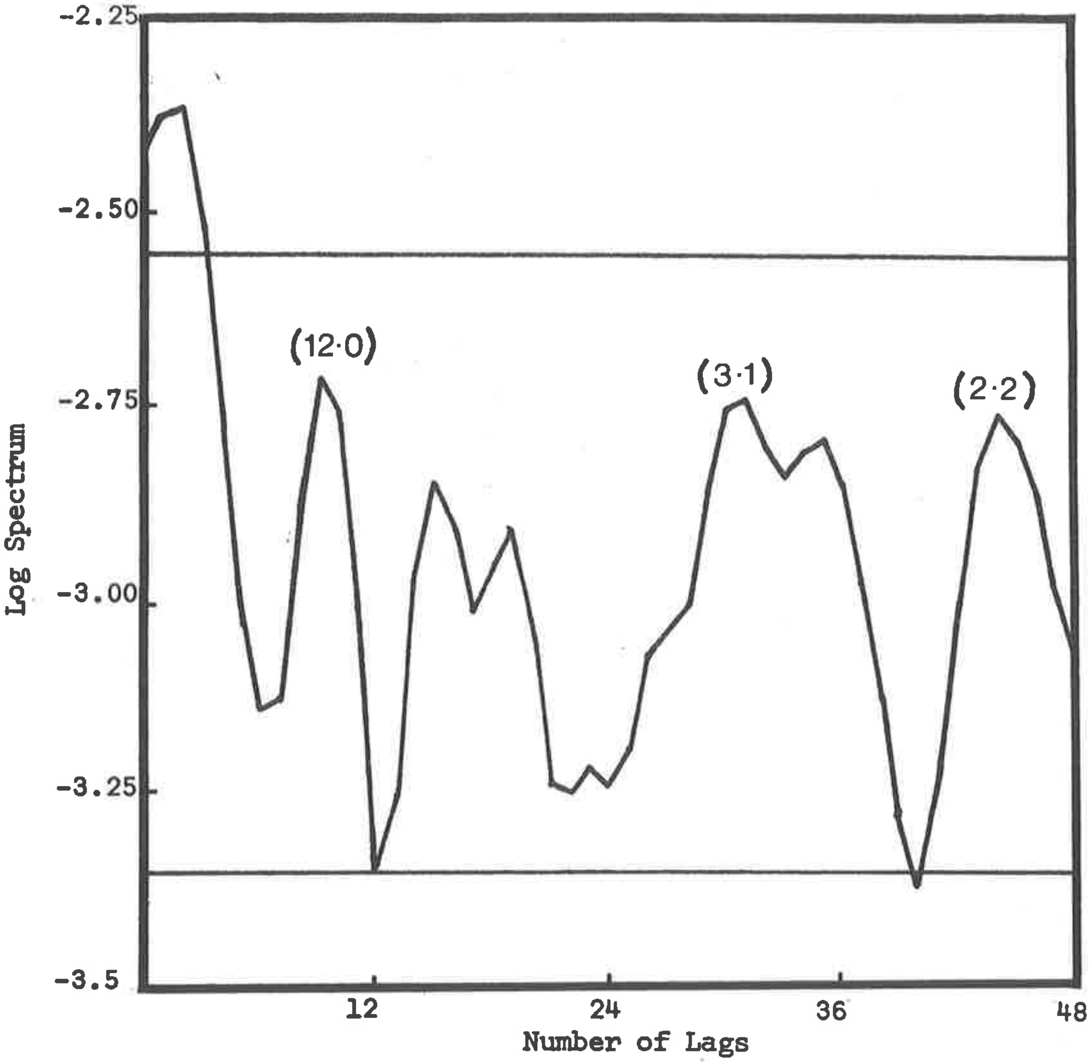


FIGURE 5·9

SPECTRUM OF DAILY FUTURES PRICE CHANGES, NEAR SERIES

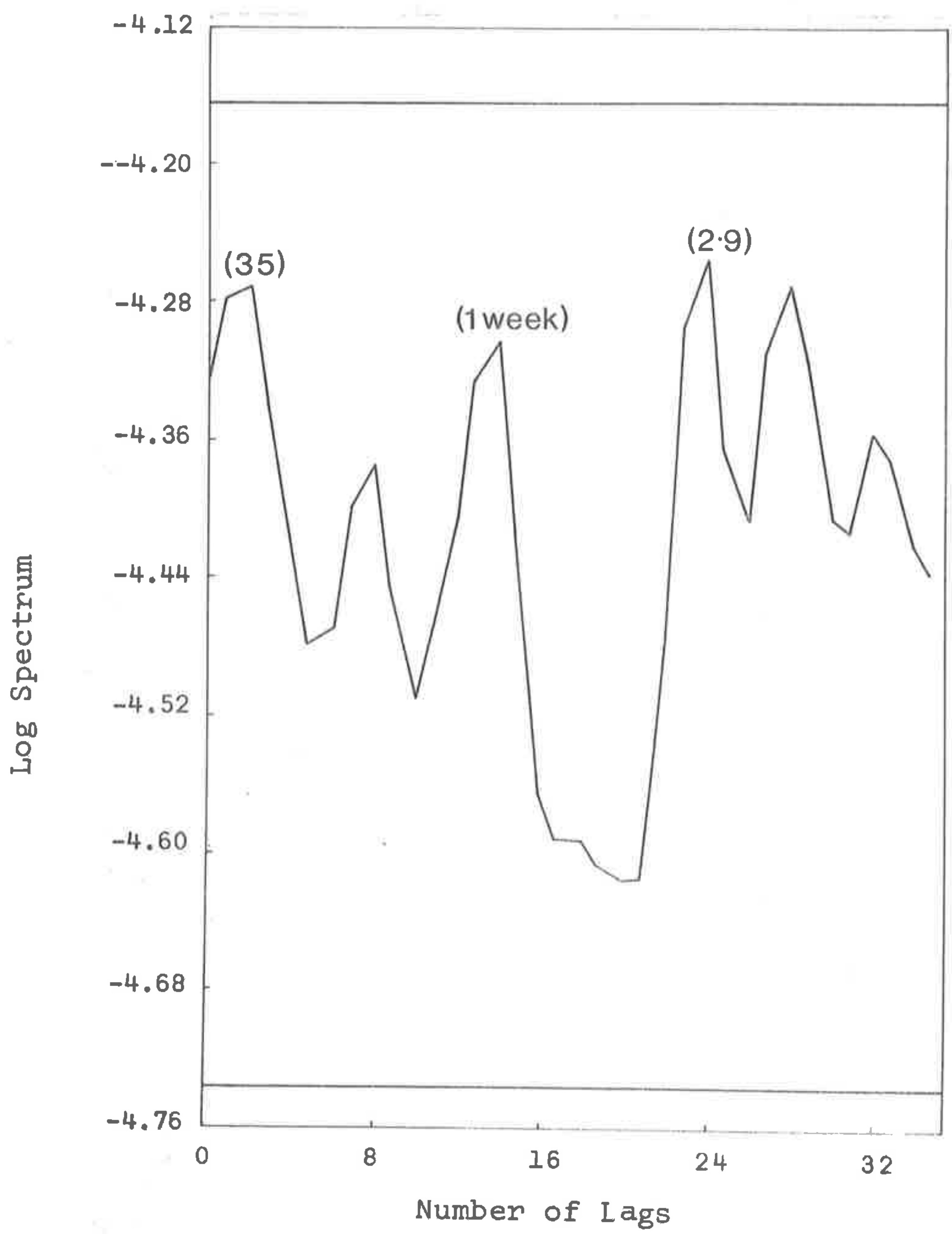


FIGURE 5·10

SPECTRUM OF DAILY FUTURES PRICE CHANGES,

TWELVE MONTH SERIES

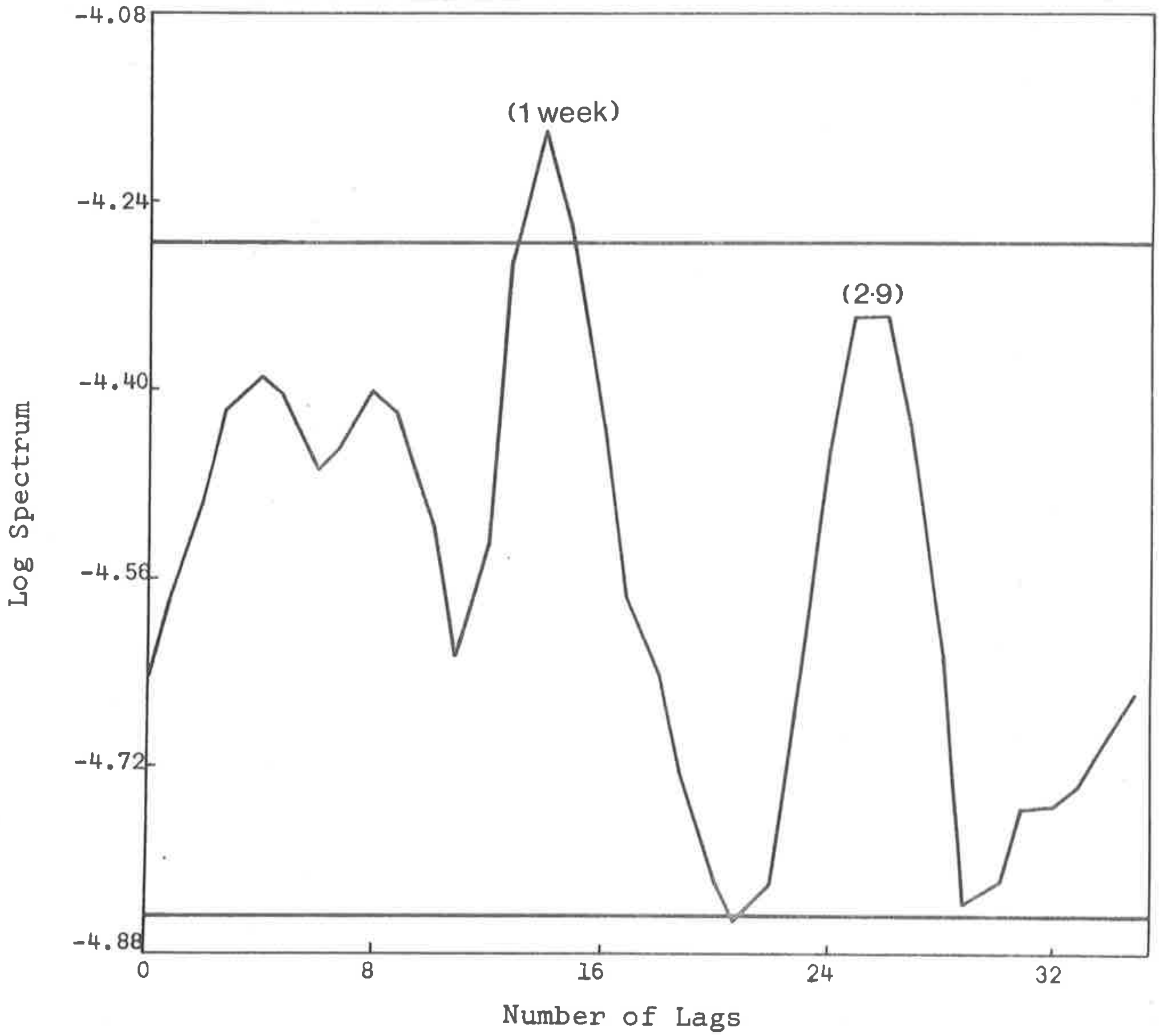


FIGURE 5·11

SPECTRUM OF SEVEN TIMES/DAY FUTURES PRICE CHANGES, OCTOBER SERIES

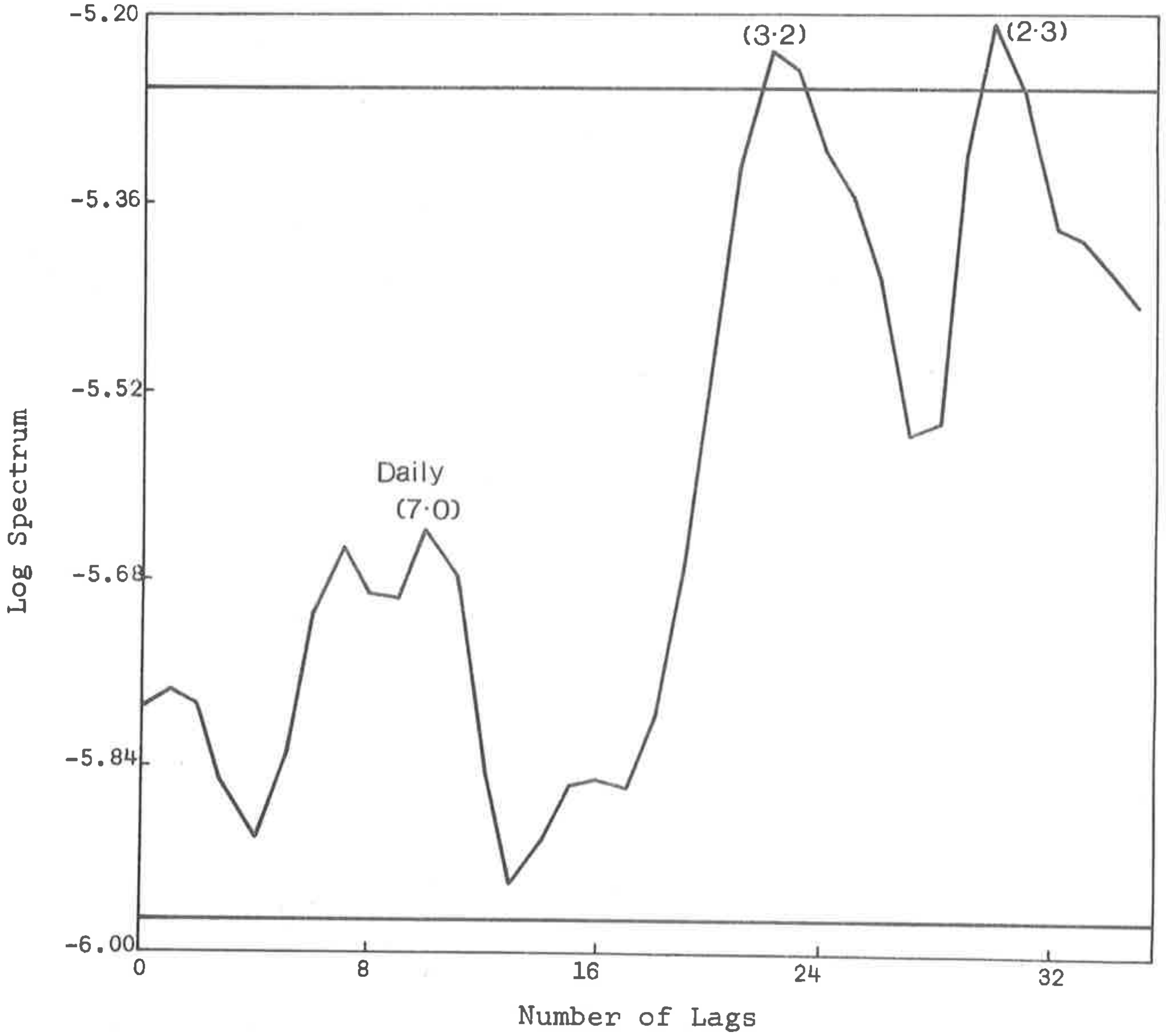


FIGURE 5·12

SPECTRUM OF SEVEN TIMES/DAY FUTURES PRICE CHANGES, MARCH SERIES

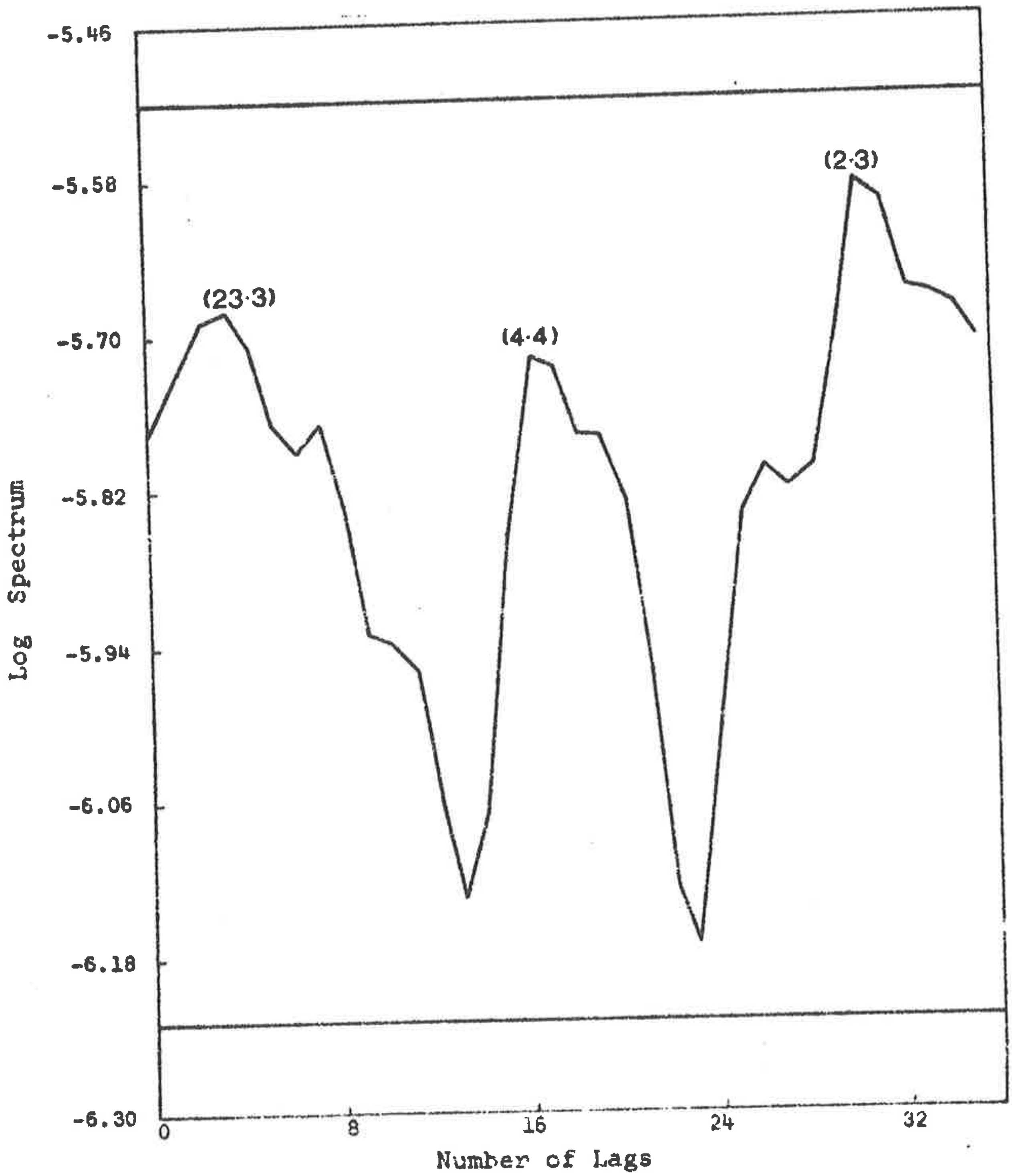


FIGURE 5·13

SPECTRUM OF SEVEN TIMES/DAY FUTURES PRICE CHANGES, NEW JULY SERIES

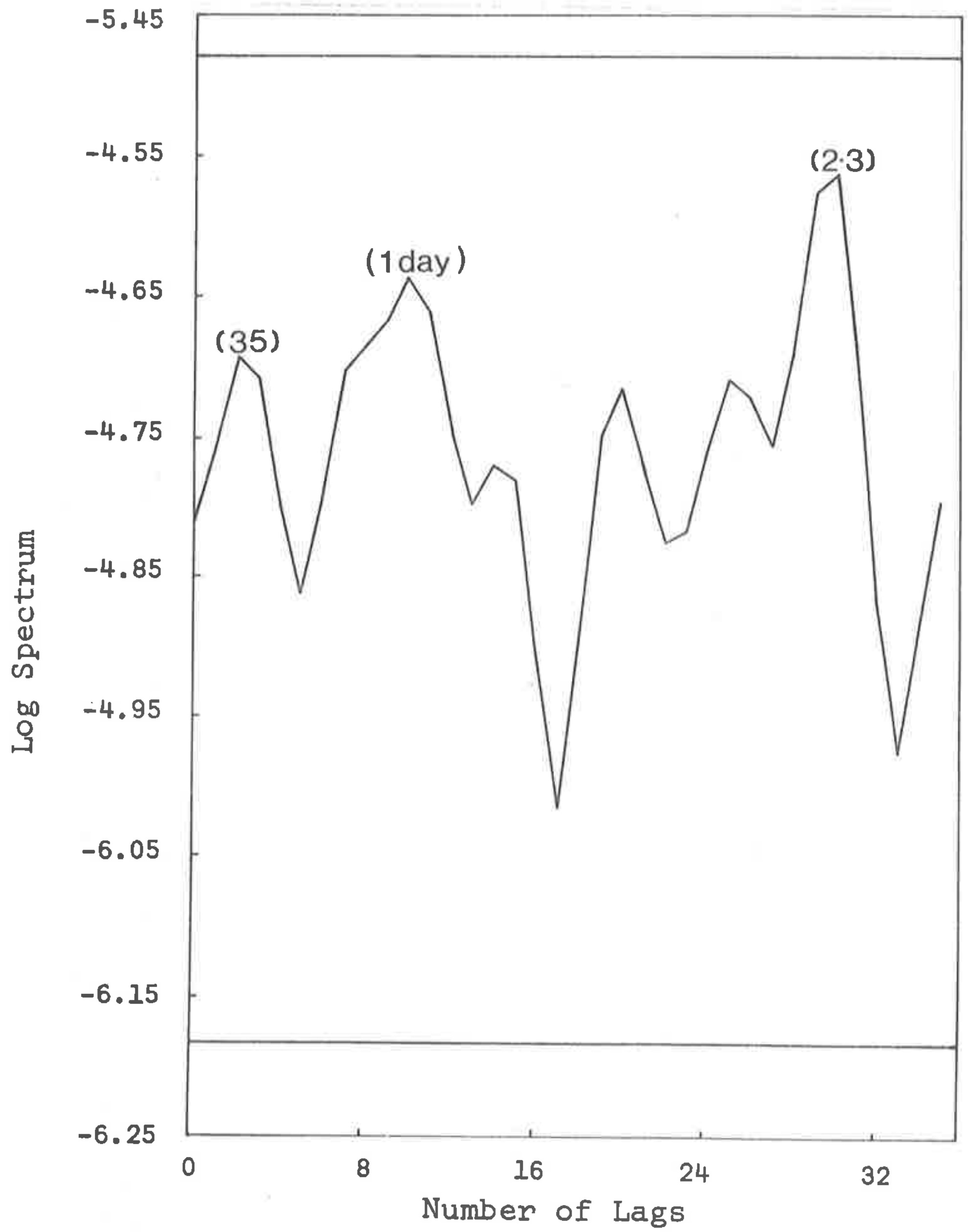
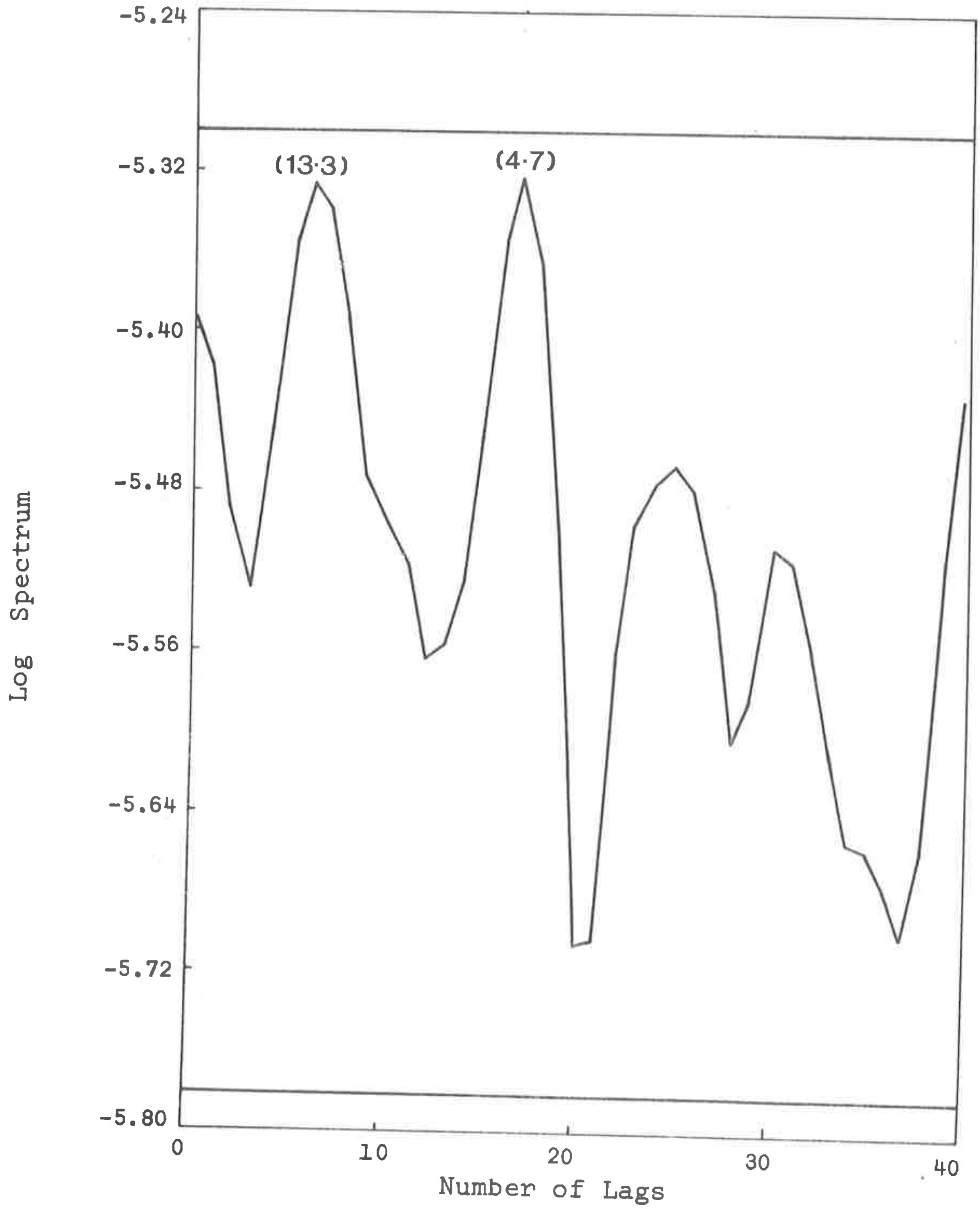


FIGURE 5·14

SPECTRUM OF TRANSACTION FUTURES PRICE CHANGES, MAY SERIES



series consists of prices of the March, May, July, October and December contracts. Thus, if the price of wool contained some seasonality we would expect that the monthly futures prices would also contain some residual seasonality. Figures 5.5 to 5.8 show that this was indeed the case.

Spectral Results for the Weekly Series

The spectra of the *weekly* series were very hard to interpret. All had peaks at 4.7 and 2.4 week cycles. These peaks are perhaps the harmonics of an approximate nine week period. Every nine weeks or so in each series a new future replaces the old one. Quite often prices will move quite dramatically in the period just before the old future matures.[51] Such price movements could form the basis of a nine week cycle. The possibility of such a cycle is given credence by the fact that most of the monthly spectra have peaks at 2.2 months, or approximately nine weeks.

Spectral Results of the Daily Series

Three of the four daily spectra displayed peaks at points equivalent to a five trading days cycle. The evidence of such a cycle is very strong in the *four month, twelve month* and *distant* futures price series. Figures 5.9 and 5.10 are the spectra of the *near* and *twelve month* daily series. The indices of variation presented in Table 5.10 show that prices are typically high on Monday and Friday but low during the week. The cycle can be interpreted in terms of market supply and demand, that is, buying orders dominate the market at the beginning and end of the week, while selling orders are a majority during the middle of the week. Possible explanations to account for the weekly cycle will be further discussed in Section 5.5.

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51 See Section 2.3.

TABLE 5.9

Indices of Seasonal Variation(Minus 1 Multiplied by 1000)

Month	Monthly Series				
	Physical	*Near Future*	*Four Month Future*	*Twelve Month Future*	*Distant Future*
January	3	-2	2	-2	2
February	0	-2	1	-2	5
March	12	-4	5	-3	-1
April	28	2	3	6	1
May	31	5	8	12	6
June	26	1	4	12	2
July	6	2	-3	1	3
August	-10	6	2	5	7
September	-33	-5	-8	-13	-11
October	29	3	-6	-15	-11
November	-14	-1	5	2	1
December	-15	-4	-7	-11	-6

TABLE 5.10

Indices of Weekly Variation(-1 x 1000)

Day	Daily Futures Price Series			
	Near	Four Month	Twelve Month	Distant
Monday	4	7	8	4
Tuesday	0	0	1	0
Wednesday	-8	-10	-10	-7
Thursday	-2	-3	-3	-3
Friday	5	4	3	5

Labys and Granger have also recorded weekly cycles in commodity prices.[52] However, they did not think that weekly cycles were ever really important and they doubted whether they could be used profitably.

Spectral Results for the Seven Times/Day Series

Spectral analysis of the four *seven times per day* series yielded unspectacular results. The *March* spectrum displayed evidence of a cycle with a period of 35 units. Thirty-five units of *seven times per day* is equivalent to a working week.[53] Thus, the weekly cycle discovered in the daily data is supported by the results of the March *seven times per day* spectrum, Figure 5.13.

The daily cycle peaks are shown on the *October* and *New July* (Figures 5.11 and 5.13) empirical spectra. Both peaks were very insignificant. This manifestation combined with the indices of daily variation, Table 5.11, form a very strong argument against the proposition that there exists a daily price cycle.

Spectral Results for the Transaction Series

The spectrum of the *May transaction* prices (Figure 5.14) had two significant peaks (at the 10 per cent level). These peaks corresponded to cycles whose periods were 13.3 and 4.7 transactions. The results of autocorrelation analysis (Section 5.2) suggested that transaction prices respond to new information in a cyclical way. That is, prices gradually adjust to new information and then react against the initial price change.[54] Spectral analysis results indicate this process takes approximately 13 transactions.

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52 Labys and Granger, *op.cit.*, p.81.

53 Seven times/day multiplied by the five working days in a week equals 35 Seven times/Day units in a working week.

54 See Figure 5.1.

Granger and Morgenstern[55] used spectral techniques to analyse transaction prices of I.T.T. stock. Their empirical spectrum sloped upward to the right. They suggested this was a result of negative correlation between adjacent transaction prices. Following their logic, it can be deduced that there is no simple negative correlation in transactions' price changes in the S.G.W.F.E. as our empirical spectrum slopes downward to the right.

Summary of Spectral Analysis

If price changes in the S.G.W.F.E. followed a random walk, then our estimated spectra would be flat. That is, the estimated spectral points would be independent of the frequency.

Our empirical spectra showed considerable deviations from the theoretical random walk spectrum. In particular, our investigations revealed a significant weekly cycle of prices.

The spectrum of monthly physical prices (48 lags) showed evidence of a small seasonal cycle. However, we believe the seasonal peaks of the monthly futures price spectra to be the result of the method of formation of the series and thus do not necessarily signify any seasonal cycle in futures prices.

5.5 Indices of Cyclical Variation

Spectral analysis suggested that futures price changes may contain some cyclical elements. The strongest evidence was that for a weekly price cycle, there was also indications of seasonality in both physical and futures price series. One could suppose there also existed some very weak evidence for a daily cycle. Once the possibility of cyclical variation was detected by spectral analysis, the logical next step was to quantify the regular price movements through the construction of indices of cyclical variation.

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[55] Granger and Morgenstern, *op.cit.*, pp.154-156.

The method of construction of the indices is identical to that described by Karmel.[56] A matrix of ratios of empirical observations to the moving average of length equal to the cycle period was formed for each series. The ratios were then averaged and adjusted to produce indices of periodic variation. The indices were then transformed by subtracting one so that they were given a mean of zero. Finally the transformed indices were multiplied by 1,000 to emphasize the periodic variation.

The monthly index for wool prices (Table 5.9) reveals some seasonal variation. For instance, the variation from September to April is of the order of 6 per cent.[57] The underlying factor causing a yearly cycle in wool prices is seasonality of supply. The bulk of shearing in Australia is carried out in the spring and early summer months. This increase in supply forces prices down in the months August to December. The relative shortage of supply in the Autumn and Winter results in higher prices during these months. It is possible that there also exists seasonality of demand. Obviously, there is a greater demand for woollen goods during winter.[58] However, the production process is long and smooth enough to absorb such changes in demand. It is doubtful whether demand seasonality has any effect on auction prices for wool. The seasonal cycle of wool prices is not traded out because of the carrying costs associated with arbitrage, that is, buying wool when the price is low, say September and holding until the price has risen, in say May. Table 5.9 also suggests some seasonality of wool futures prices. However, we believe

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[56] P.H. Karmel, *Applied Statistics for Economists*, Melbourne, I. Pitman and Sons, 1963, pp.233-238.

[57] The difference between the May and the September physical wool price index is 64. This is a variation of 6.4% about the average price.

[58] Northern Hemisphere winter.

this is a function of the method of preparation of the futures price series and does not reflect an innate seasonality in individual futures contract.

The weekly cycle which showed out very clearly under spectral analysis, is revealed in more detail by the construction of an index of daily prices, Table 5.10. Typically, prices are high on Monday, then fall away during the week to bottom on Wednesday, and finally rise again on Friday. An explanation of this strong weekly cycle is to be found upon examination of the rational reactions of the different market transactor groups to uncertainty. To explain such a cycle it is necessary to examine the mechanics of trading and the nature of the traders. If a trader has not yet completed a futures market transaction, that is, he has bought but not sold a contract or *vice versa*, he is said to be maintaining an open position in the market. There is associated with every futures transaction a buyer and a seller. It thus follows that for every long open position, there is necessarily a short open position, where long and short refer to the different approaches to completed futures transactions. A long transaction involves initially buying a contract, while a short transaction requires that the trader initially sell a contract. The equivalence of open long and short contracts can be symbolically expressed as

$$C_L = C_S \qquad \text{..........} \qquad (5.55)$$

where C_L represents the long contracts and C_S the short contracts. As mentioned before there are broadly two types of transactions, the risk reducing hedge and the profit seeking speculation. The open positions can thus be divided into hedging and speculative transactions. Hence, equation (5.55) becomes

$$H_L + S_L = H_S + S_S \qquad \text{..........} \qquad (5.56)$$

where the subscripts L and S refer to long and short and H and S designate the number of open transactions concerned with hedging or speculation respectively.

The hedger, to reduce risk, takes the opposite position in the futures market to his commitment in the commodity market. The producers of wool, the farmers, are long in the commodity market and hence to reduce price risk they take up a short position in the futures market. The users of wool, top makers, spinners, weavers, etc., having perhaps sold their products forward are short in the commodity market, therefore they go long in the futures market. H_S of equation (5.56) is identified with the producers of wool and H_L with the users of raw wool.

It is an acknowledged historical fact that the producers of wool have been very slow to use the futures market.[59] The reasons for this phenomenon range from liquidity problems to sheer ignorance of the mechanics of hedging. The great demand for hedging facilities originates from the processors of raw wool.

> ...the majority of hedgers at Sydney may be long rather than short. Certainly there appears to have been little hedging by growers; and it is significant that the move for the establishment of the Sydney market came mainly from those who would be more likely to sell wool forward (and be long hedgers when hedging) than to hold stocks of actual wool.[60]

This opinion was reinforced during private communication between the author and the Secretary of the S.G.W.F.E.

Long hedging transactions exceed short hedging transactions, that is

$$H_L > H_S \qquad \ldots\ldots\ldots\ldots \qquad (5.57)$$

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59 R.W. Gray, "Wool Futures Trading in Australia - Further Prospects," *University of Sydney, Department of Agricultural Economics, Research Bulletin*, No.5, (1967), pp.18-22; also Phillips, *op.cit.*, pp. 61-63.

60 R.H. Snape, *op.cit.*, p.178.

referring to equation (5.56) it is possible to deduce that

$$S_S > S_L$$

and $\therefore \quad S_S - S_L > 0$ .... (5.58)

that is, speculation is net short. The cyclical behavior of futures is a direct result of speculators being on average short transactors.

The speculator whose aim is an uncertain profit is far more sensitive to the nuances of the market, than is the hedger who is merely reducing risk by maintaining a position in the futures market. It is apparent that the maintenance of an open futures market position during the weekend involves a considerable amount of risk. Price changing events do take place over the holiday period. The potential consequences of a major price changing event to speculators are much greater during the weekend because the speculator cannot alter his position until Monday. It is our proposition that many speculators regard the risk of holding a position over the weekend as intolerable. For this reason they close out their market position on Friday. As speculators are net short, they must buy back futures to liquidate their position in the market. This increased buying pressure means prices rise on Friday.

Such an hypothesis explains why futures prices are high on Friday, but it does not explain why prices remain relatively high on Monday after the weekend period has passed. Monday's high prices are again the result of risk averse speculators leaving the market prior to a period of uncertainty. The major price changing influence upon wool future prices is the auction price of wool. Auctions in Australia generally commence on a Tuesday. This means Monday becomes a period of gross uncertainty for a speculator in wool futures. Thus some speculators prefer to vacate the futures market on Monday forcing prices to a weekly high.

TABLE 5.11

Indices of Daily Variation (-1 x 1000)

Time	Seven Times per Day Futures Price Series			
	July	*October*	*March*	*New July*
11.00 A.M.	2	2	1	1
12.00 NOON	0	1	1	1
12.30 P.M.	1	0	1	1
3.00 P.M.	0	1	0	1
4.00 P.M.	-1	-1	0	-1
4.20 P.M.	-1	0	-1	-1
4.30 P.M.	-1	-2	-1	-2

The low prices on Tuesday, Wednesday and Thursday are a manifestation of speculators resuming their net short position. Selling pressure drops the futures price as speculators come back into the futures market.

The indices presented in Table 5.11 quantify the daily cycle. It is obvious from the indices that the daily cycle is an insignificant perturbation representing only .2 to .4 per cent of the market value of the futures contract.

5.6 Implications of Futures Price Cycles

The existence of a significant weekly cycle does have some ramifications for futures market traders and the random walk model. The cycle represents about 1.5 per cent of futures price. The value of the cycle in dollars is dependent upon the price of wool futures. The average price over the period was approximately 350 cents/kilo. Thus, the cycle represents a fluctuation of about $78 per contract. The cost of a completed futures market transaction is made up of $40 brokerage and $2.50 Clearing House charge, a total of $42.50.

It is thus theoretically possible to make a sure profit solely from the knowledge that prices are high on Monday and low on Wednesday. However, it is our belief that this profit margin is insufficient to cover the indirect costs of a futures market transaction, such as the $350 per contract deposit, and the margins that may be called in to cover adverse price changes. It is these indirect costs that prevent the value of the weekly cycle being reduced to the cost of a futures transaction.

The weekly price cycle has implications for those traders committed to hedging transactions in the futures markets. It obviously makes sense for these traders to buy on Wednesday afternoon and sell on Monday morning. This way they can retrieve all of their direct transaction costs.

The random walk model and price cycles are by definition incompatible. Thus, although the weekly cycle is numerically not very significant, it does constitute a rebuff to the random walk model. The model must be modified from

$$P_t = P_{t-1} + U_t \qquad \text{..........} \qquad (5.59)$$

where P_t represents the price at time t and U_t is a white noise series to

$$P_t = P_{t-1} + W(a) + U_t \qquad \text{..........} \qquad (5.60)$$

where W(a) is a weekly function with zero mean. In the weekly cycle we have a price phenomenon that is not the result of some inefficiency in what is basically a random walk market. The weekly cycle is the result of market transactors rational response to risk. The random walk proponent's assumption that well informed competitive traders are sufficient to ensure random price changes has been shown to be unjustified.

5.7 Filter Analysis

As the random walk model of share prices gathered more and more support it provoked a strong reaction from parties with vested interests in share price prediction. Typical of such reaction was R.A. Rotnam who wrote that random walk proponents were

> like the scientist who proved conclusively that a fly couldn't fly.[61]

Market professionals claimed that common statistical tools were unable to measure the complex dependence they claimed existed in speculative markets. For example, they believed the linear relationships that underlie the serial correlation model were much too unsophisticated to identify the complex patterns that the "chartist" sees in stock prices. It was to test such claims that filter techniques[62] were evolved.

Preamble

The random walk theory hypothesizes that successive price changes are independent. The implication is that past price changes contain no information about the next price change. This is important to a market participant as he wants to know if a history of past prices can be used to increase expected gains. In a random walk market with negative zero or positive drift, no mechanical trading rule applied to futures prices would consistently out perform a policy of buying and holding for the entire period. Thus the random walk model should be accepted if a trading policy incorporating past prices is unable to produce profit greater than those of a buy and hold policy.

Alexander[63] formulated the filter most widely used as a test of

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61 Granger and Morgenstern, *op.cit.*, p.83.

62 A filter is simply a market trading rule.

63 S.S. Alexander, "Price Movements in Speculative Markets: Trends on Random Walks," *Industrial Management Review*, Vol.2, No.2, (May, 1961), pp.7-26.

price dependence. His mechanical trading rule (filter) was as follows: f the price of a security (future) moves up x per cent, he bought and maintained his position until the price fell x per cent from a subsequent peak. At this point in time Alexander would sell and sell again, thus putting himself in a short position. He would remain short until prices rose x per cent from a local low, when he would buy twice thus taking up a long position again. Alexander's filter is schematically represented in Figure 5.15.

Alexander formulated the technique to test the belief widely held amongst market professionals that prices gradually adjust to new information. Market professionals refused to believe that prices reacted to new information immediately in a single jump.[64] They believed that prices trend to their new levels and that it was possible to recognize and profit from these trends. If a price after moving up (down) x per cent has a greater probability of moving up(down) more than x per cent than the probability of any other moves, then Alexander's filter will consistently show a profit relative to a policy of buy and hold. Thus Alexander's filter technique essentially tests for trends in the price data.

Alexander applied his filter technique to two share price indices. His published results showing significant profit accruing to most filters ostensibly refuted the random walk model. However, Mandelbrot was quick to point out that Alexander's computations incorporated bias which led to serious overstatement of profitability.[65] Of Mandelbrot's three objections to Alexander's method, only one is

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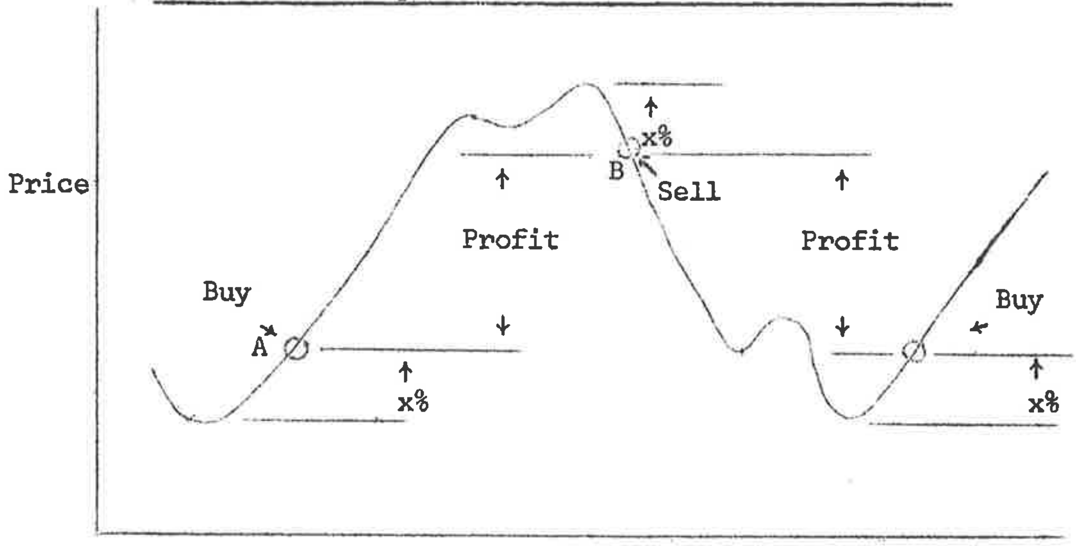
[64] The empirical autocorrelation results for the transaction price data support the proposition that futures prices do not react instantly to new information. See Section 5.2.

[65] B.Mandelbrot, "The Variation of Certain Speculative Prices," *Journal of Business*, XXXVI (Oct., 1963), pp.394-419.

relevant to the application of Alexander's filter to futures prices.[66] The relevant objection concerns the difference between the theoretical transaction price of the filter and the actual transaction price

FIGURE 5.15

Diagrammatic Representation of Alexander's Filter



that could be obtained in the market. Alexander assumed it was always possible to buy at a low plus x per cent (point A of Figure 5.15) and sell at a high minus x per cent (point B of Figure 5.15). It was Mandelbrot's valid claim that this was impossible in a real world market. The very presence of a buyer or seller in the market will necessarily have a price changing effect.

We believe we have overcome this objection to filter analysis by using buyer and seller daily closing quotes. Any decision to purchase is enacted at the end of each day at the price quoted by the sellers. We think this is the buying price that would be obtained in reality. Conversely, the selling price is generated from the buyer's quote.

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66 The two other objections were that he used share price indices and not share prices, and that he ignored the effect of dividends.

Data and Method

The data used covers two distinct time periods. Series A is comprised of 750 daily observations of buyer and seller closing quotations over the period 4/1/1965 - 20/11/1967. Series B consists of 655 buyer and seller daily close quotes during period 11/1/71 - 17/7/73. The gap in the two series was not intentional and it was only circumstances beyond our control that prevented us using the entire period 1965 to 1973. To test the significance of using buyer and seller quotes, as opposed to one series of prices, we also prepared two more series of average prices by taking the mean of the buyer and seller prices. It should also be pointed out that as the data is from the near futures series a compulsory transaction must occur about every two months as the current future matures. Thus a policy of buy and hold will therefore incur some transaction costs.[67]

We used Alexander's filter as our mechanical trading rule. If the selling price rose x per cent above a local minimum we purchased one clean kilo at the selling price. We held on to the hypothetical kilo until the buying price dropped x per cent from a local high. At this point we sold two clean kilos at the price determined by the buying quote. Again we maintained our position until the selling price rose x per cent, when we purchased two kilos and so on. On the day before the future matured we liquidated our position and took up the same position in the new future. The variable x ranged from .05 per cent to 100 per cent.

The programme used to determine filter profits generated values for gross profit, net profit (gross profit minus transaction costs), net profit for a buy and hold policy (the profit from buying at the start of the series and selling on the last day) and the relative profit of the filter rule. The relative profit was determined by

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67 Series A had 21 and series B 14 compulsory transactions.

subtracting the net profit of the buy and hold technique from the net filter profit and then dividing by modulus of the buy and hold profit.

Results of Filter Analysis

The results of hypothetical trading using Alexander's filter rules are summarized in Table 5.12 and Figures 5.16, 5.17 and 5.18. They provide no joy for a person seeking an easily won fortune from the Sydney futures market. Gross filter profits for series A transactions peak at values associated with the 2 per cent and 18 per cent filter (see Figure 5.15). For series B the 3 to 5 per cent and the 14 - 20 per cent filters provide the greatest gross profit. For both series A and B the gross filter profits are somewhat bimodal, with modes at 2-5 per cent and at 18-20 per cent.

The results would tend to suggest that there are two types of trends in the data. The success of the 2-5 per cent filter is the result of short term trends, while the 15-20 per cent filter profits result from longer term trends.

The gross filter profits of series A surely supports the random walk model as only one filter produced a positive gross profit. However, the gross filter profits for series B revealed the presence of some profitable trends in the price data. Eleven filter gross profits were positive. The inference from such a result is that series B data is non random. Evidence of dependence between price changes by itself is of little use to the potential investor. The investor wishes to know whether the degree of dependence is sufficient to cover transaction costs and thus allow him a profit. The results in Table 5.12 and Figure 5.17 show that seven series B filters produce positive net profits. These positive albeit small net profits indicate that persons using only past prices could profit by trading in the near future during the period.

TABLE 5.12

Filter Profits (cents per clean kilo)

Filter Size %	A Series (Buyer/Seller Quotes)				B Series (Buyer/Seller Quotes			
	No.of Trans-Actions	Gross Profit	Net Profit	Relative Profit %	No. of Trans-actions	Gross Profit	Net Profit	Relative Profit %
.05	296	-1.83	-9.18	-560	247	-2.80	-8.17	-385
.5	143	-1.27	-5.04	-262	208	-3.27	-7.80	-372
1.0	103	-0.98	-3.38	-143	156	-1.21	-4.60	-260
2.0	54	-0.31	-1.66	-19	102	-0.63	-2.85	-199
3.0	42	-1.14	-2.29	-65	69	2.02	0.52	-82
4.0	38	-1.13	-2.22	-60	58	1.07	-0.19	-107
5.0	34	-1.08	-1.84	-33	54	2.53	1.35	-53
6.0	29	-0.87	-1.50	-8	52	-1.95	-3.08	-208
7.0	29	-0.82	-1.43	-2	46	-2.61	-3.61	-226
8.0	28	-1.05	-1.66	-19	40	-2.78	-3.65	-227
9.0	26	-0.85	-1.42	-2	34	-1.31	-2.05	-172
10.0	25	-0.82	-1.39	0	34	-2.70	-3.44	-220
12.0	25	-1.32	-1.91	-37	24	-0.10	-0.62	-122
14.0	24	-0.31	-0.79	43	18	1.56	1.17	-59
16.0	22	-0.31	-0.79	43	18	0.96	.57	-80
18.0	22	0.03	-0.45	67	16	2.90	2.55	-11
20.0	22	-0.08	-0.56	60	16	2.82	2.47	-14
25.0	22	-0.85	-1.33	5	16	1.63	1.28	-55
30.0	21	-0.93	-1.39	0	14	2.23	1.92	-33
35.0	21	-0.93	-1.39	0	14	1.03	0.72	-75
40.0	21	-0.93	-1.39	0	14	.43	0.12	-96
45.0	21	-0.93	-1.39	0	14	3.13	2.87	0
50.0	21	-0.93	-1.39	0	14	3.13	2.87	0
100.0	21	-0.93	-1.39	0	14	3.13	2.87	0

Surprisingly enough, the series B filter net profits at no stage exceeded the net profit from the very simple policy of buying at the start and holding until the end of the particular period. For Series A the picture was slightly different with four of the longer filters having associated with them positive relative profits. However, the net profit of these four filters was negative and is thus discouraging to the potential investor.

The results displayed in Figures 5.17 and 5.18 vindicate Mandelbrot's objection that the impossibility of obtaining the desired transaction price leads to erroneous profit figures. In Figures 5.17 and 5.18 we have plotted the filter net profits obtained from both the buyer/seller price series and an average price series. For the majority of filter sizes the net profit figures for the average price series was greater than the corresponding net profit for the buyer/seller series. The difference in filter net profits for the two series is a result of the unfavourable price movement that a trader produces when he enters the market. That is, if a trader wants to sell a futures contract immediately he must take the going buyer's quotation, which in all probability is below the last transaction price.

Summary of Filter Results

We feel the results of filter analysis are not completely compatible with a strict random walk. This is especially true for series B. Series B gave positive gross profits for the 3, 4, 5, 14, 16, 18, 20, 25, 30, 35 and 40 per cent filters. The profits remained positive after transaction costs were deducted for the 3, 14, 16, 18, 20, 25, 30, 35 and 40 per cent filters. We believe that 3 per cent net profit was the result of short term trends in the price data. The positive net profit for the filters greater than 14 per cent and for the buy and hold policy resulted from long term price trends. The fact that the buy and hold policy gave the greatest net profit for series B argues that prices during this period follow a random walk with drift.

FIGURE 5·16

FILTER GROSS PROFITS

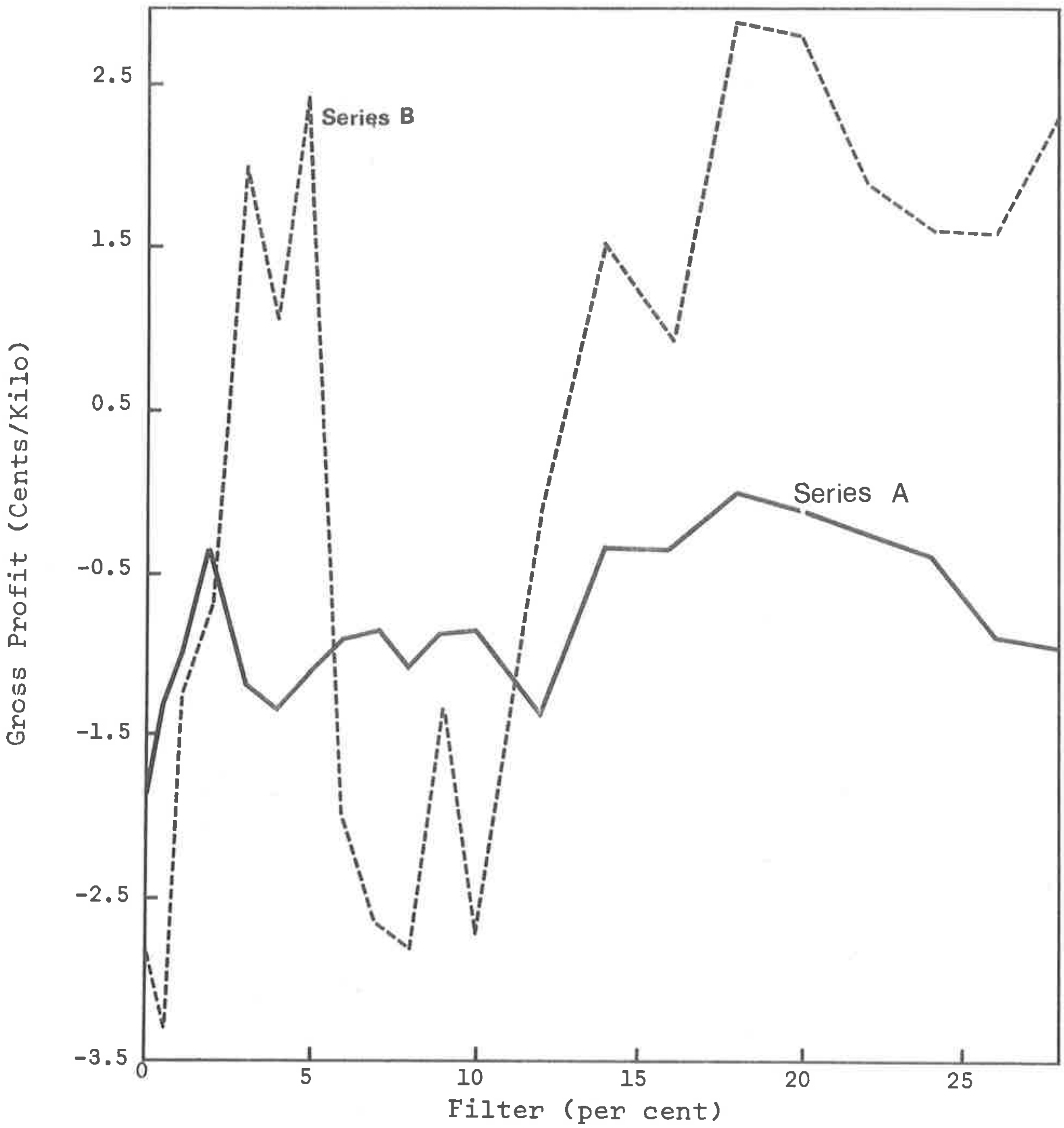


FIGURE 5·17

FILTER NET PROFIT FOR SERIES A,

USING BUYER/SELLER AND AVERAGE PRICES

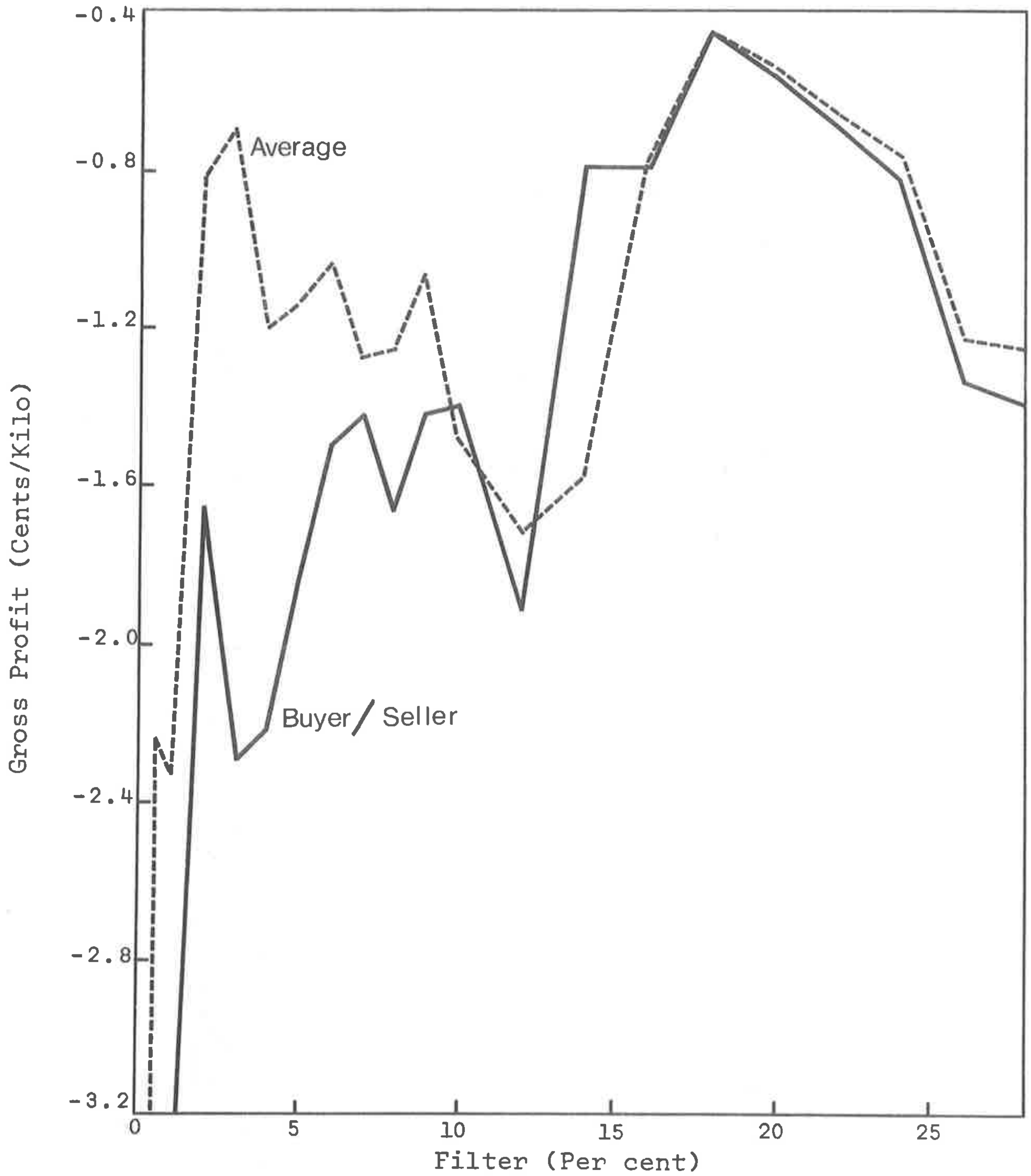
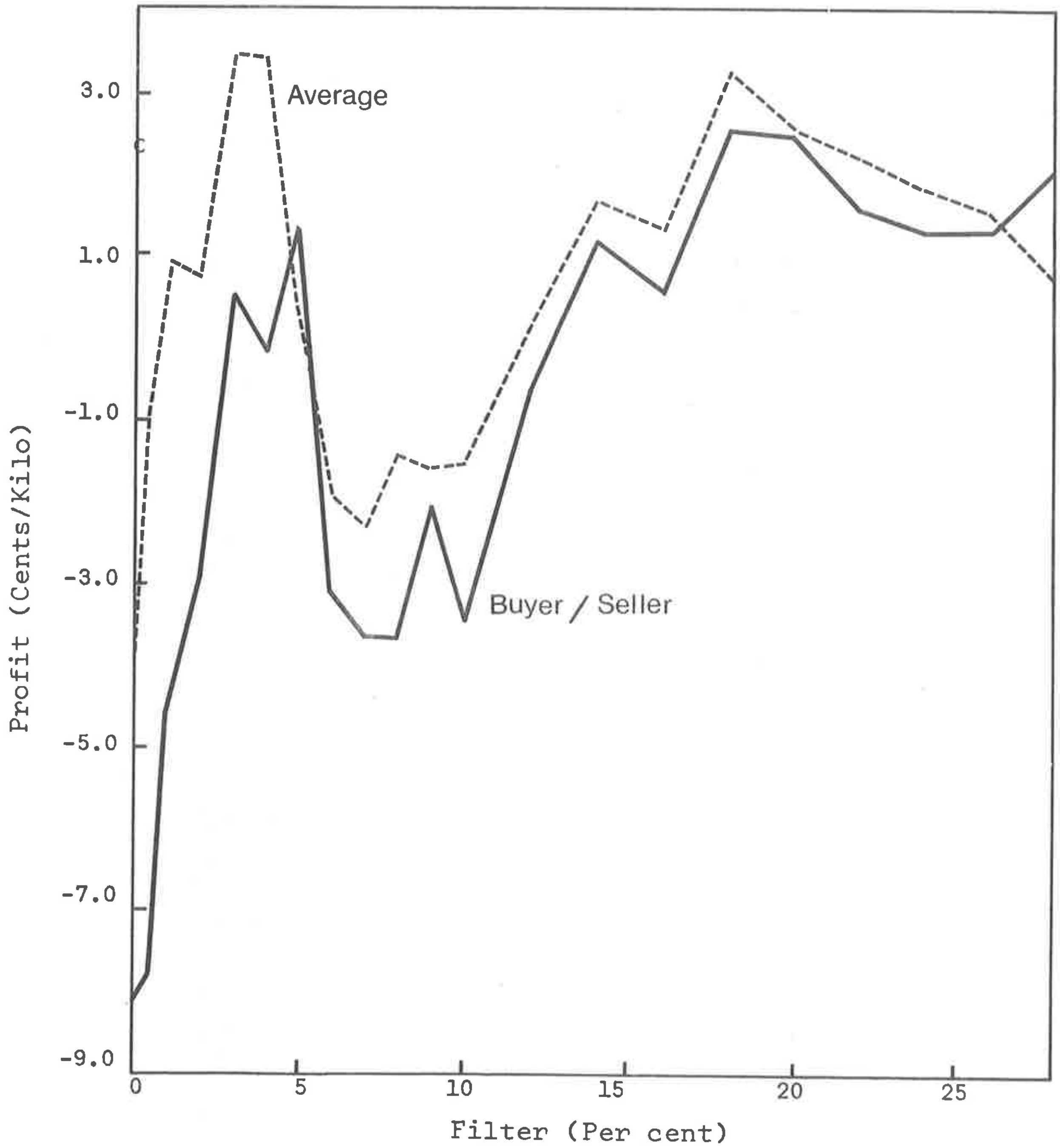


FIGURE 5·18

FILTER NET PROFIT FOR SERIES B, USING BUYER/SELLER AND AVERAGE PRICES



Our results have some implications for the potential investor. The relative profit figures for both series A and series B suggest that a potential systematic investor will be better off if he uses a large filter (greater than 14 per cent) or indeed if he buys and holds the future contract for the entire time period.

Our results also showed that filter analysis on last price or average price data will give erroneous profit figures. Filter profits associated with simple price data will be upward biased compared to the more realistic profit figures associated with buyer/seller price quotation data.

Concluding Remarks

In the light of our empirical results, we find it impossible to recommend that prices in the S.G.W.F.E. follow a random walk. Autocorrelation analysis, runs analysis, spectral analysis and the filter technique all showed up some non random aspect to price changes. It is relevant to ask how great is the departure from the random walk model. The results of filter analysis and the construction of cyclical indices demonstrate that the departure is not great enough to enable an uninformed person to make "easy" money. However, the fact that the non random patterns in prices are small should not be allowed to mask the fact that they exist and are significant statistically, if not financially.

CHAPTER 6

RELATIONSHIPS BETWEEN WOOL PRICES AND WOOL FUTURES PRICES

6.1 Introduction

In any speculative market there exists a great number of firmly entrenched and largely unchallenged beliefs about price relationships. In futures markets, one of these notions is that within futures prices there is contained some information about future spot price movements. For example, some people believe that if futures prices are presently below spot prices, it portends a drop in future spot prices.[1]

We used cross spectral and regression analyses to objectively test the idea that wool futures prices predict wool prices. The results of our investigation are presented in this chapter.

6.2 Theory Governing Spot-Futures Price Relations

Holbrook Working[2] provided a classic theory to explain why spot (or near futures prices) are sometimes at a discount to distant futures and sometimes at a premium. Working suggested the difference between the spot price and the price of a futures contract to mature in T time units represented the price (cost) of storaging a unit of the commodity for T time units.

In order to give a cogent explanation of Working's price of storage theory, let us consider a simplified commodity - futures market complex, peopled only by producers and users of the commodity. Assume that both the producers and the users have some stock on hand and that the users are committed to use these stocks in the productive process at a latter date. Let $F_{0,T}$ represent the price now of a future to mature at time T, S_o represents the spot price now and C_T is cost of storing

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1 This opinion has been expressed to the author by numerous futures market participants.

2 Holbrook Working, "The Theory of Inverse Carrying Charges in Futures Markets," *Journal of Farm Economics*, Vol.XXX, No.1 (Feb.1948), and "The Theory of the Price of Storage," *American Economic Review*, Vol.39, pp.1254-1262, (Dec., 1949).

an additional unit of the commodity from time period 0 to time period T.

Consider, first, the reactions of the producers and users of the commodity to a market price situation where

$$F_{0,T} - S_0 > C_T \quad \text{..........} \quad (6.1)$$

Processors of the commodity who need future inputs in period T would find it cheaper to buy stocks now and carry them forward, rather than pursue the alternative policy,purchasing them in period T. On the other side of the market, producers would recognize that such a price structure - expression (6.1) - creates for them a riskless profit. They would thus withhold their stocks from the spot market, sell futures and deliver the commodity as specified by the futures contract. The combined reaction of market participants would result in a lowering of futures prices and an increase in spot prices to the point where,

$$F_{0,T} - S_0 \leq C_T \quad \text{..........} \quad (6.2)$$

Consider now the reverse situation to that described by expression (6.1), that is

$$F_{0,T} - S_0 < C_T \quad \text{..........} \quad (6.3)$$

The rational reaction of processors of the commodity would be to sell their stocks and buy them back forward by purchasing futures and permitting delivery of the contracts. Such a strategy would prove to be less costly than carrying the stocks over from period 0 to period T. Producers of the commodity would react to the price structure by selling their produce now. The rational strategies of two different sides of the spot - futures market would force futures prices up and spot prices down until,

$$F_{0,T} - S_0 \geq C_T \qquad \text{..........} \qquad (6.4)$$

A comparison of expressions (6.2) and (6.4) leads to the conclusion that spot futures price relations can be described by the equation,[3]

$$F_{0,T} - S_0 = C_T \qquad \text{..........} \qquad (6.5)$$

Working viewed the marginal cost of storage as having two distinct components, the marginal physical cost and the marginal convenience cost, both of which were a function of the level of commodity stocks in the spot-futures market system. The marginal physical cost of storage is made up of rent, handling, interest, insurance and depreciation charges, and is considered to be a constant function of the level of stocks to the point where storage facilities become overloaded.

The marginal convenience yield is a negative cost that arises because manufacturers (and to some extent, producers) find it convenient to hold some stocks. A certain minimum level of stocks means that they are able to meet variation in demand without costly halts in production or loss of customer goodwill through refusing an order. The convenience yield decreases as stocks increase. The marginal net storage cost is derived by the addition of the marginal physical storage cost and the marginal convenience yield. See Figure 6.1.

The marginal storage cost C_T is shown in Figure 6.1 to be a function of the amount of commodity stocks in the spot - futures market system. If stocks are low then carrying costs will be

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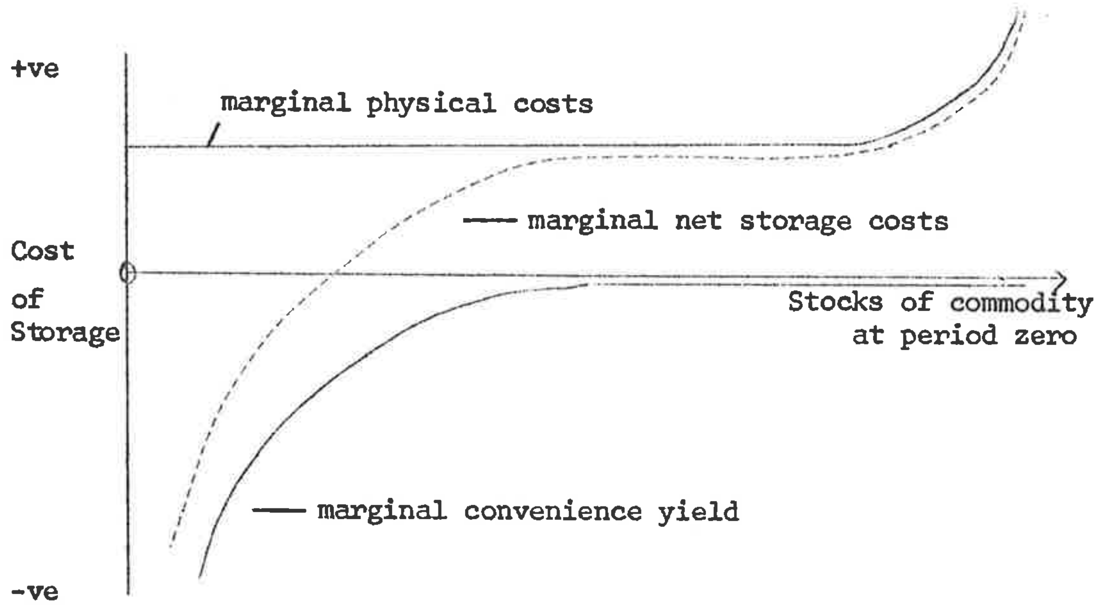
3 A similar result is obtained by Goss, who includes in his analysis the expected spot price as well as the spot and the futures price: B.A. Goss, *The Theory of Futures Trading*, Routledge and Kegan Paul, London, (1972), pp.16-25.

FIGURE 6.1

Marginal Storage Costs



negative and thus from equation (6.5) it follows that futures prices are at a discount to spot prices. If, however, there is an excess of stocks at time zero, carrying costs will be positive and thus $F_{0,T}$ will be greater than S_0, so futures prices will be at premium to the commodity prices.

The point of this rather long-winded explanation of the relationship between spot and futures prices is to ensure that situations where the spot price is at a discount or premium to the futures price should not be misconstrued as a forecast of future rises or falls in the price of the commodity. For example, if the price of clean wool is 350 cents/kilo, while the near future sells for 345 and a distant future sells for 310, this should not be taken as a forecast that prices will fall 40 cents/kilo in the ensuing months. Such a phenomenon should be treated as a market determined carrying (storage) charge of 40 cents/kilo.

This carrying charge is indicative of a shortage of current stocks. Working's price of storage theory minimizes the forecast content in a spot -futures price differential.

Nevertheless expectations of the future do have a very definite effect on spot and futures prices. However, with commodities that can be stored indefinitely expectations are reflected just as much in spot and nearby futures as in distant futures prices.

> The element of expectations is imparted to the whole temporal constellation of price quotations and futures prices reflect essentially no prophesy that is not reflected in the cash price and in that sense is already fulfilled.[4]

The whole spot-futures price complex reacts simultaneously and equally to new information about future demand and supply factors. For example, consider the effect of a Government announcement that in six months time there was to be a total embargo on all wool exports. It is naive to assume that only futures contracts with a delivery date of six months or more would be affected by the decision. Wool prices and all wool futures prices would drop simultaneously as producers attempted to dispose of the clip through auction or by selling futures contracts. A market situation where the physical and futures prices react simultaneously obviously excludes any lead-lag relationships between the price series.

If the contrary were true and futures prices were an accurate forecast of the future commodity price, it would be plausible to hypothesize lead-lag relationships between more and less distant futures price series. If, for example, the near futures price was in fact an estimate of the spot price and the four month futures price was an accurate forecast of wool prices in four months time and similarly the twelve month futures price represented an accurate estimate of

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4 W.G.Tomek and R.W. Gray, "Temporal Relationships Among Prices on Commodity Futures Markets: Their Allocative and Stabilizing Roles," *American Journal of Agricultural Economics*, Vol.52,(1970), p.372.

prices in twelve months, then it follows that the twelve month futures series would lead the four months futures prices which in turn would lead the near futures prices. However, as explained above, this is not, in theory, how a futures market behaves. In fact, the spot price and all futures prices have discounted in themselves all available information about the expected spot price at all future points in time. This means that any new information affects all prices equally and thus should rule out any lead-lag relationships.

Theory would have it that futures price changes are thus derived from two sources. First, changes in the level of stocks will change the price relativity of different futures. Second, new information changes the price level of all futures. Price changes originating from the former influence are much smaller and less frequent than price changes originating from the latter source.[5]

6.3 Cross Spectral Analysis

Theory

The cross spectrum is a statistical device to produce a measure of the degree to which two series are related.[6] Let $f_x(w)$ and $f_y(w)$ represent the spectra of two stationary series $\{X_t\}$ and $\{Y_t\}$. The cross lagged covariance function is thus given by

$$U_{xy}(\tau) = \text{cov}\ (X_t Y_{t-\tau}) \qquad \text{..........} \qquad (6.6)$$

where τ represents the number of lags, that is $\tau = 0, 1, 2 \ldots$ and w is the frequency. The covariances have a spectral representation

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5 W.G.Tomek and R.W. Gray, *op.cit.*, p.373.

6 The theoretical exposition of the cross spectrum is largely taken from: W.C.Labys and C.W.J. Granger, *Speculation, Hedging and Commodity Price Forecasts*, Heath Lexington Books, Lexington, Massachusetts (1972).

$$u_{xy}(\tau) = \int_{-\pi}^{\pi} e^{i\tau w} d\, cr(w) \qquad \ldots\ldots\ldots\ldots \qquad (6.7)$$

cr(w) is called the cross spectrum and is in general a complex function of w. It is usual to consider two other functions derived from cr(w) as they are easier to interpret. These functions are the coherence c(w) defined by

$$c(w) = \frac{|cr(w)|^2}{f_x(w)\, f_y(w)} \qquad \ldots\ldots\ldots\ldots \qquad (6.8)$$

and the phase $\phi(w)$ given by

$$\phi(w) = \tan^{-1}\left[\frac{\text{imaginary part of } cr(w)}{\text{real part of } cr(w)}\right] \ldots \qquad (6.9)$$

The coherence c(w) represents the square of the correlation between the amplitudes of the w frequency components of the two series. Thus, the spectral approach does not give one a single measure of the degree to which two series are related but measures the degree of relatedness in terms of the coherence for each frequency component in the decomposed series. c(w) lies in the range 0 to 1. A value near one suggests the two series are highly related at that frequency.

Another aspect of the investigation of the way series are related is to look for a lag structure. In cross spectral analysis this is done using the phase diagram. In the decomposition of a time series each variance component is associated with a frequency. To characterize fully each component, one needs not only the amplitude but also the phase $\phi(w)$. If there is no lag between the series then $\phi(w)$ will be zero for all w. If there is a simple time lag, that is

$$Y_t = aS_{t-k} + Z_t \qquad \ldots\ldots\ldots\ldots \qquad (6.10)$$

where Z_t is some series uncorrelated with X_t, then $\phi(w) = kw$, so by examining the slope of $\phi(w)$ the lag, k, can be estimated.

It is important to remember when interpreting the phase diagrams to first look at the coherence function before checking for lags as it is nonsensical to attribute to two unrelated series a lag structure. One must look at the coherence diagram to see if the series are related before suggesting that one leads or lags the other.

Previous Cross Spectral Analysis of Futures Prices

There exists only two documented applications of cross spectral technique to futures price series. In one of these studies, Labys and Granger[7] studied cash and futures prices of wheat, oats, soybeans, corn, cocoa and lard. Of their 42 empirical cross spectra only 7 per cent showed any leads or lags. Teir and Kidman[8] examined price movements between wool and twelve months wool futures prices using cross spectral analysis. Their results showed strong cash-futures price coherence over long cycles, however, they failed to find any lead-lag relationships between their empirical price series. These results are predictable from Working's notions about futures markets. He was of the opinion that the whole futures-cash price complex adjusted simultaneously to new information rather than futures prices leading cash prices or *vice versa*. The Labys and Granger study also demonstrated that the coherence between futures contracts with close delivery dates was greater than the coherence between futures contracts with delivery dates spaced wide apart.

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7 W.C.Labys and C.W.J. Granger, *op.cit.*, pp.89-109.

8 T.J. Teir and P.R. Kidman, "Price Movements in and between the Wool, Wool Tops and Worsted Yarn Spot and Futures Markets," *Quarterly Review of Agricultural Economics*, Vol.XXIV, No.2, (April, 1971), pp.63-81.

Results of Cross Spectral Analysis of Sydney Wool Futures Prices

Figures 6.2 to 6.9 show the coherence and phase diagrams for monthly physical wool prices with near futures, four month futures, twelve month futures and distant futures prices.[9] The coherence diagram consists of a plot of c(w) (analagous to the r^2 of correlation analysis) against the number of estimation lags j.[10] The phase diagram consists of a plot of $\phi(w) + 2\pi$ radians against j.[11]

In all cases c(w) is high for the longer run cycles (low value of the lag j) but falls away as the cycles become shorter. In every example the phase for all frequencies was very near to zero or 2π radians, indicative of a structure with little or zero lead-lag relationships.

In order to summarize the cross spectral results the empirical coherence diagrams, for the monthly price series, have been divided into three sets of frequency ranges corresponding to short run, medium run and long run cycles. The short run cycles had periods of two to three months, the medium run cycle periods ranged from 3.2 to 12 months and the long run cycles had periods of 16 months or longer. The coherence c(w) has been averaged for each of these three frequency ranges.

Five price series were subjected to cross spectral analysis resulting in $C_2^5 = 10$, empirical cross spectra. The average coherence values are displayed in Table 6.1, short run cycles, Table 6.2, medium run cycles, and Table 6.3, long run cycles.

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9 Space did not permit us to publish every empirical cross spectral output generated during our investigations. Monthly cross spectra were chosen for listing in the original form because of their relevance to the issue, whether or not physical prices lag futures prices.

10 The relationship between the number of estimation lags j, the frequency w and the period p, is given by

$$p = 1/w \quad \text{and}$$
$$w = j/2m \quad \text{where m is the maximum number of estimation lags.}$$

See equation 5.52 and 5.53.

11 There is, of course, no necessity to add 2π to the phase value $\phi(w)$, however, the output of the program used to estimate the cross spectra presented the phase diagram in this way.

FIGURE 6·2

COHERENCE OF MONTHLY PHYSICAL
AND
NEAR FUTURES PRICES

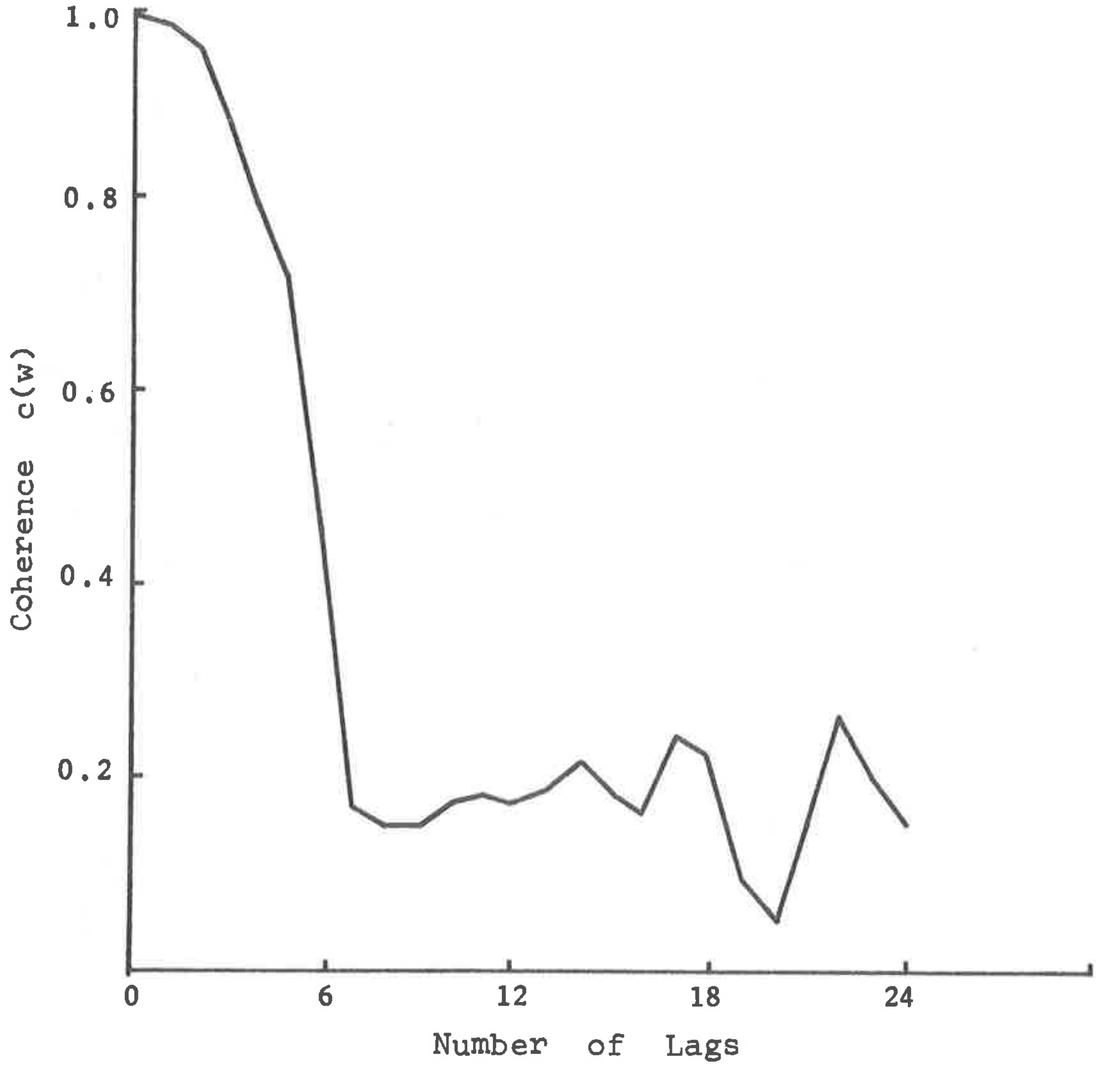


FIGURE 6.3

PHASE OF MONTHLY PHYSICAL
AND
NEAR FUTURES PRICES

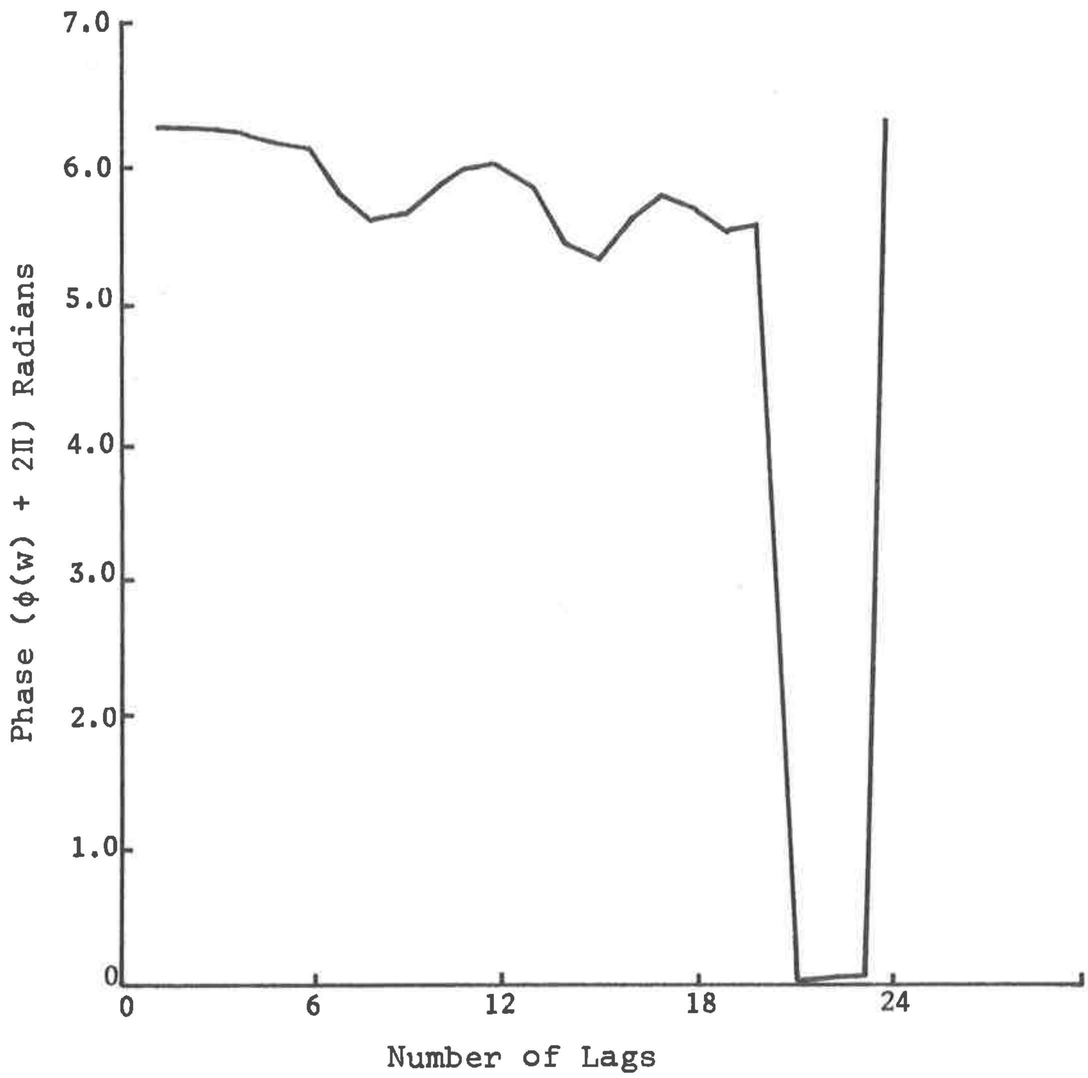


FIGURE 6·4

COHERENCE OF MONTHLY PHYSICAL
AND
FOUR MONTH FUTURES PRICES

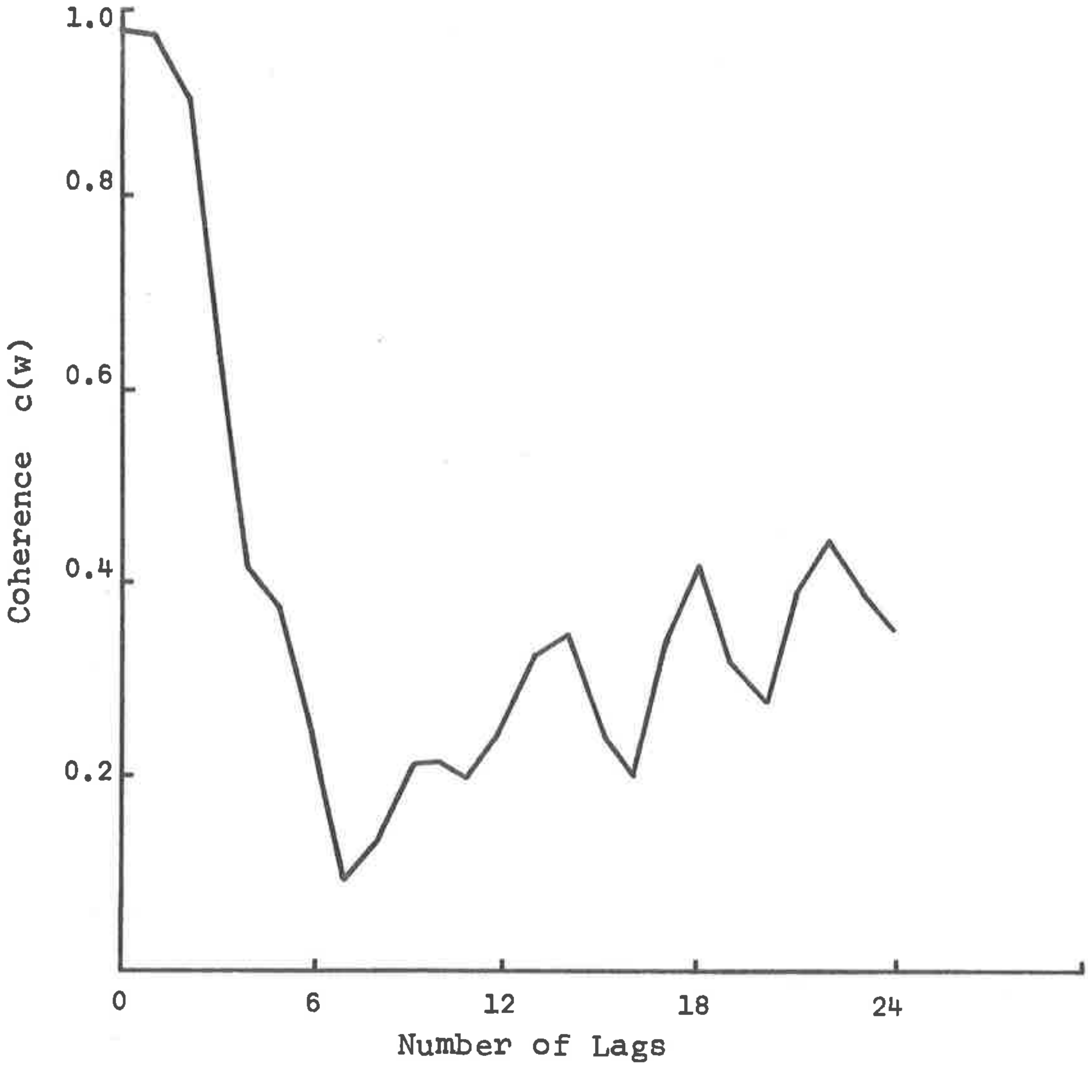


FIGURE 6·5

PHASE OF MONTHLY PHYSICAL
AND
FOUR MONTH FUTURES PRICES

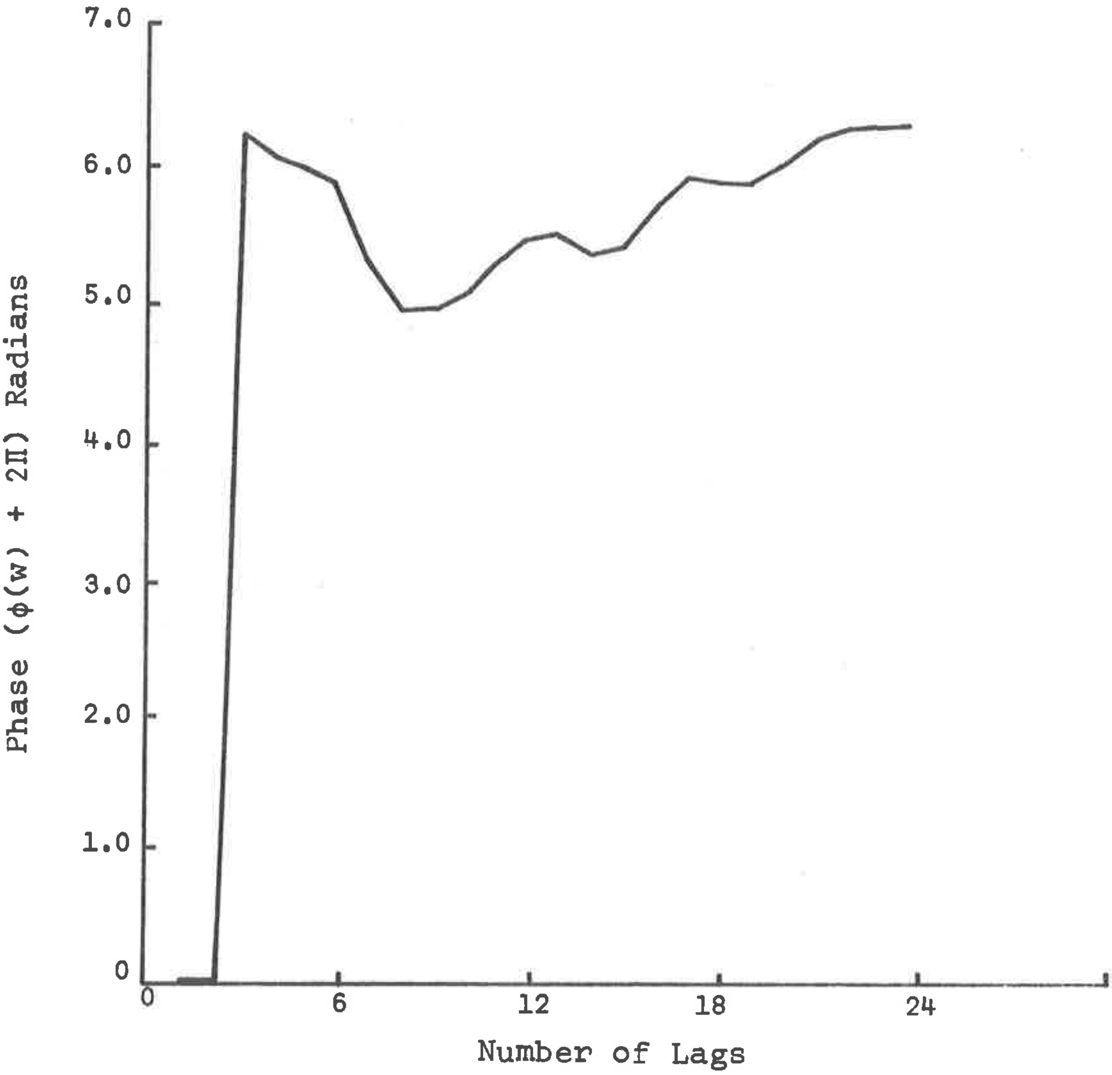


FIGURE 6·6

COHERENCE OF MONTHLY PHYSICAL
AND
TWELVE MONTH FUTURES PRICES

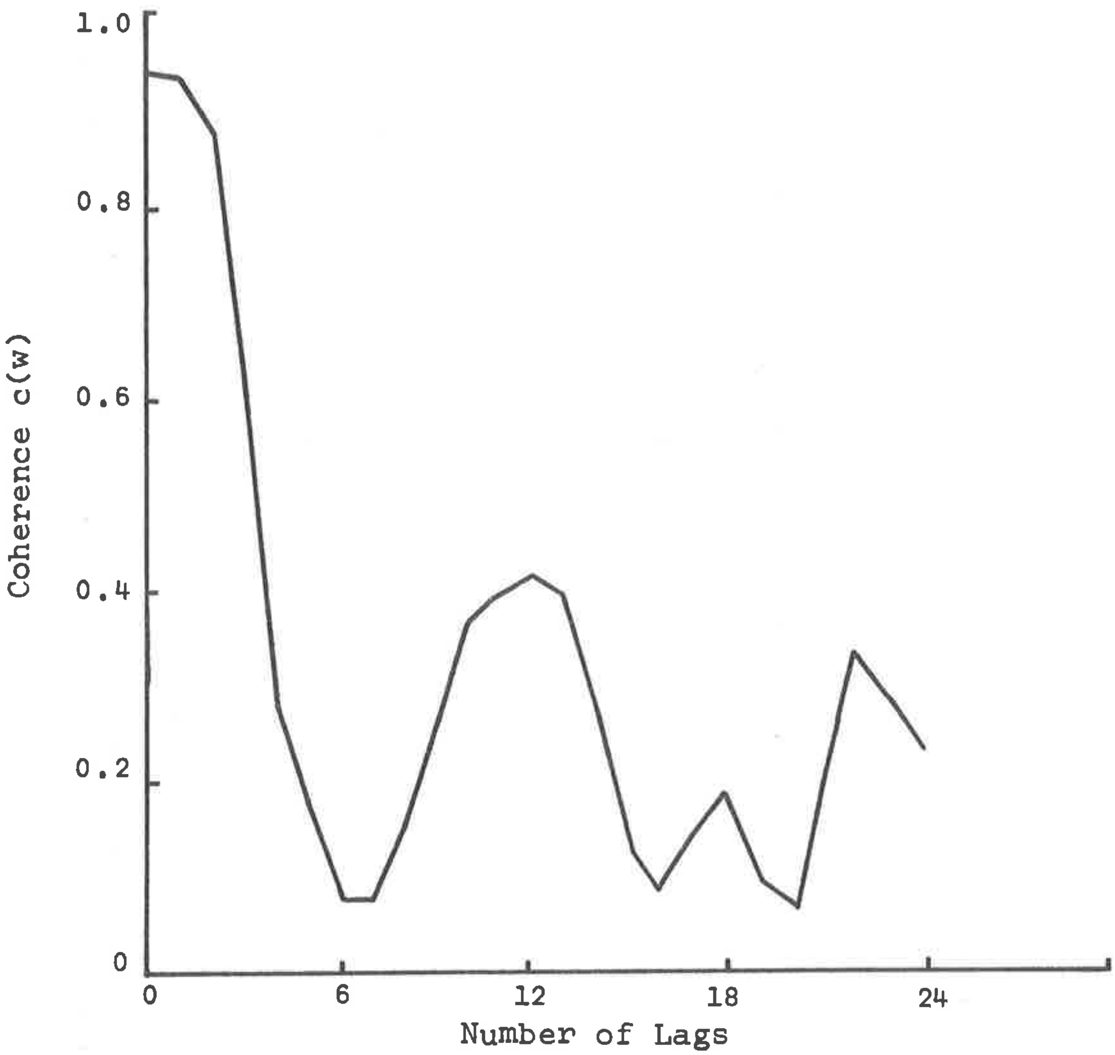


FIGURE 6.7

PHASE OF MONTHLY PHYSICAL
AND
TWELVE MONTH FUTURES PRICES

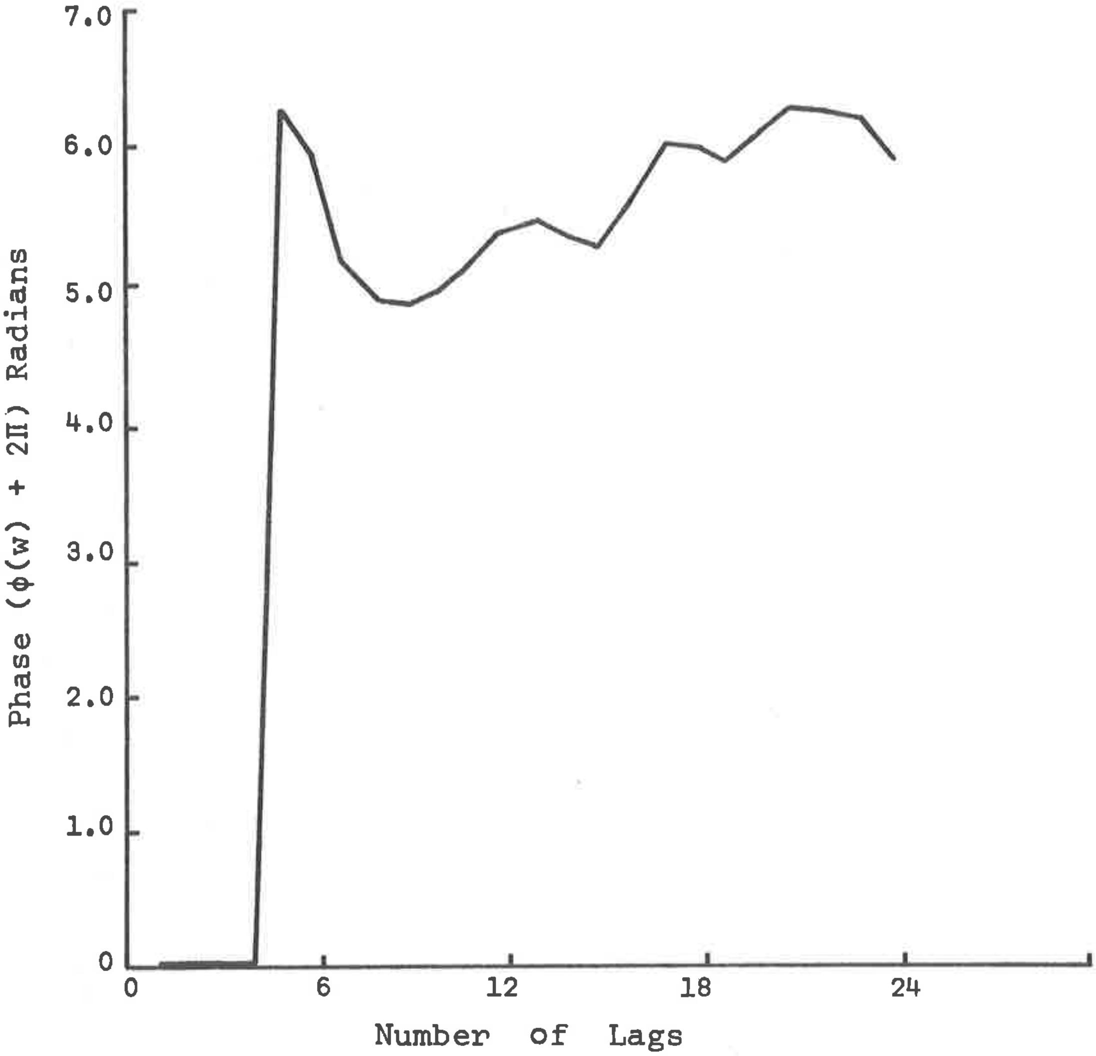


FIGURE 6·8

COHERENCE OF MONTHLY PHYSICAL
AND
DISTANT FUTURES PRICES

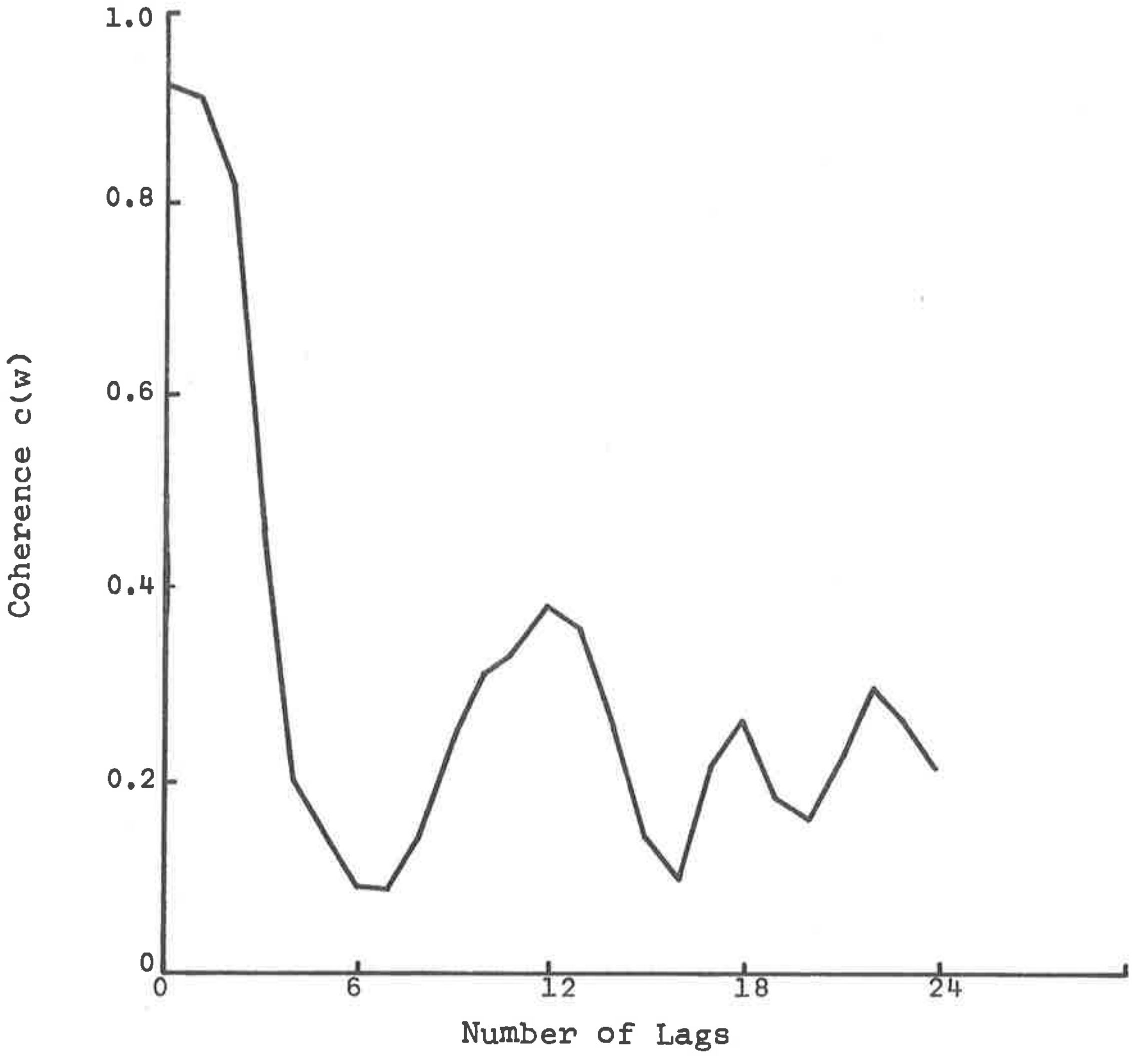
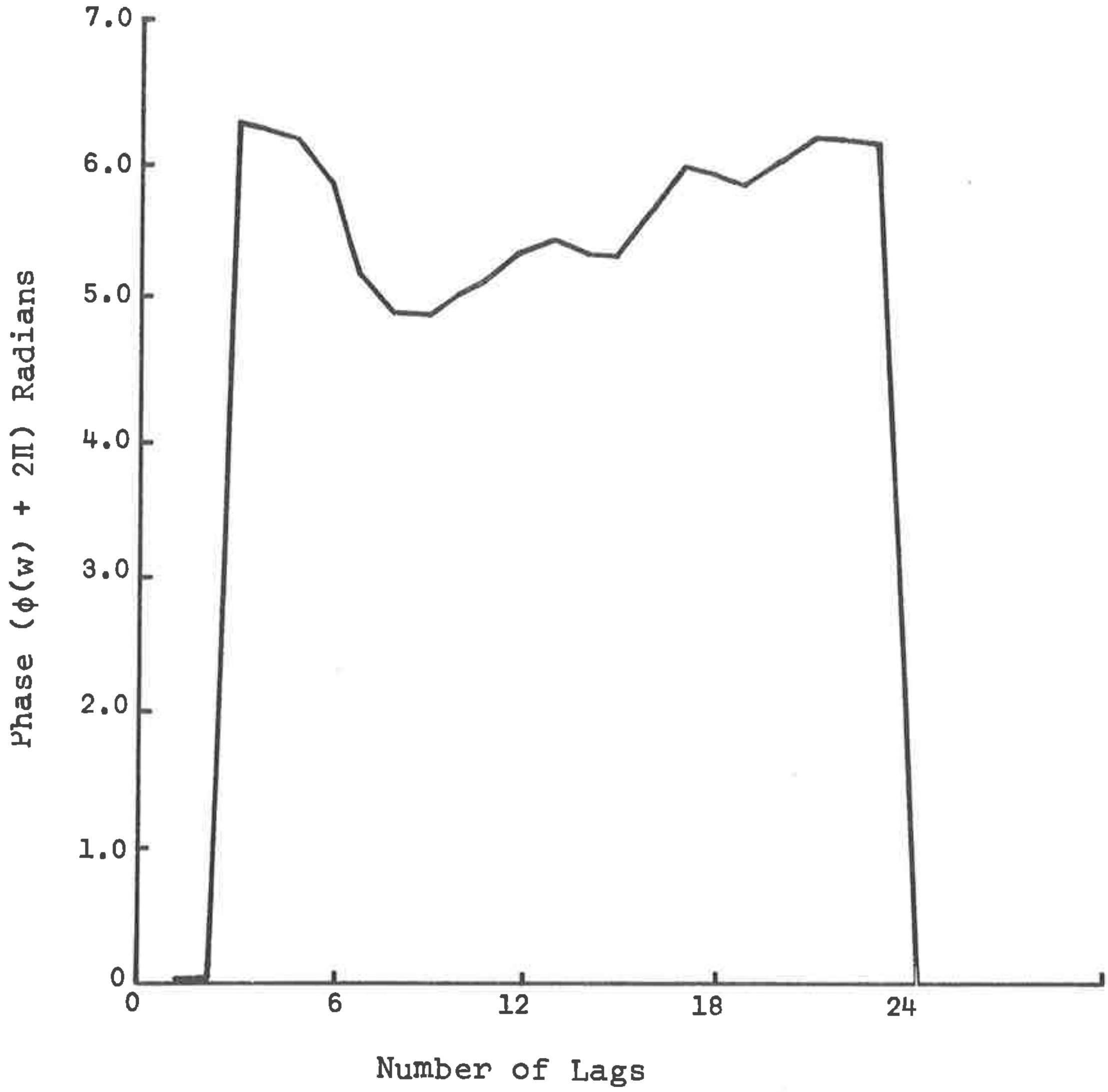


FIGURE 6·9

PHASE OF MONTHLY PHYSICAL

AND

DISTANT FUTURES PRICES



A comparison of the results of Tables 6.1, 6.2 and 6.3 leads to the conclusion that coherence is much higher for the long run fluctuations than it is for the medium and short run fluctuations. Such a result is not particularly unusual.[12] Theoretically all spot-futures prices move together, however, this simultaneity of movement is dependent upon speculators' ability to recognize and correct maladjusted prices. It is plausible to assume that the corrective intervention of speculators is far more efficient over the longer run price swings than for the rapid short run price cycles. Thus coherence of the spot-futures price complex is much greater over long run cycles than it is for the medium or short run fluctuations.

Tables 6.1, 6.2 and 6.3 provide evidence of an inverse relationship between coherence and the distance between futures delivery dates. For example, the near futures series is more closely correlated, at all frequencies, to the four month futures series than to the distant futures series. Similarly the further a future is from maturity the lower its correlation with the spot price. This phenomenon is consistent with the notion that futures price changes originate from two distinct sources. Futures prices are sensitive to either, new information concerning future demand and supply, or changes in the level of stocks which change the price of storage. While new information effects the whole gamut of futures prices simultaneously and equally, changes in the price of storage rearranges the relative prices of futures. As the price (cost) of storage is a direct function of the time over which the storage takes place, any change in the level of stocks effects a greater change in the relative prices of widely separated futures than it does in the relative prices of adjacent futures.

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12 Labys and Granger, *op.cit.*, pp. 93-94.

TABLE 6.1

Average Coherence Values for Short Run Cycles

Price Series	Spot	Near Futures	Four Month Futures	Twelve Month Futures
Near Futures	.40			
Four Month Futures	.58	.73		
Twelve Month Futures	.41	.70	.87	
Distant Futures	.45	.52	.83	.84

TABLE 6.2

Average Coherence Values for Medium Run Cycles

Price Series	Spot	Near Futures	Four Month Futures	Twelve Month Futures
Near Futures	.51			
Four Month Futures	.49	.69		
Twelve Month Futures	.48	.60	.88	
Distant Futures	.45	.51	.86	.94

TABLE 6.3

Average Coherence Values for Long Run Cycles

Price Series	Spot	Near Futures	Four Month Futures	Twelve Month Futures
Near Futures	.97			
Four Month Futures	.93	.95		
Twelve Month Futures	.91	.93	.97	
Distant Futures	.87	.89	.96	.89

As new information is the dominant influence on futures prices it ensures that the matrix of coherence values (Tables 6.1, 6.2 and 6.3) consist largely of high values. However, because of price of storage changes, futures series with close maturation dates are more coherent than futures series widely separated in time.

In examining the lead-lag structure between two series it is necessary that an appreciable cycle exists at frequency w, before any meaningful inferences can be made as to whether one series leads another at frequency w. The results of Section 5.4 provided strong evidence of a weekly cycle. There was also less convincing evidence for a twelve monthly seasonal cycle in physical wool prices and wool futures prices and perhaps an imaginative researcher could argue the seven times/day spectra showed a very small daily cycle. We were thus interested in the coherence and phase figures associated with cycles of periods, twelve months, five days and one day. Tables 6.4, 6.5. and 6.6 show the coherence c(w) and the phase of these particular frequencies. In Tables 6.4, 6.5 and 6.6 the phase value is given in real time.[13] If the value is negative it indicates that the price series on the top of the matrix lags the series on the side of the matrix. A positive phase value indicates the top series leads the side series. The phase figures are shown in parentheses.

The coherence figures for all cycles follow a similar pattern to the generalized coherence results of Tables 6.1, 6.2 and 6.3. That is, the coherence value declines as the distance between the delivery dates of the two series increases.

From the results of Tables 6.4, 6.5 and 6.6, it is possible to draw some conclusions about lead-lag relations in the wool-wool futures market. First, any leads or lags are small, yet despite the diminutive

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[13] Lead/lags are converted from radians to real time using the formula, 2π radians = p time units, where p is the period of the particular cycle. Thus if we are looking at the 12 month cycle, one radian = $12/2\pi$ = .3182 months.

TABLE 6.4

Coherence and Phase Values (in Parentheses) for the Seasonal Cycle. Phase Values in Months

Price Series	Physical	Near Futures	Four Month Futures	Twelve Month Futures
Near Futures	.90 (-.36)			
Four Month Futures	.62 (-.50)	.72 (-.26)		
Twelve Month Futures	.50 (.05)	.68 (.22)	.85 (.31)	
Distant Futures	.40 (-.21)	.54 (.14)	.88 (.31)	.93 (.07)

TABLE 6.5

Coherence and Phase Values (in parentheses) for the Weekly Cycle. Phase Values in Days

Price Series	Near Futures	Four Months Futures	Twelve Months Futures
Four Month Futures	.67 (0.0)		
Twelve Month Futures	.57 (.1)	.86 (.1)	
Distant Futures	.58 (0.0)	.88 (0.0)	.89 (.1)

TABLE 6.6

Coherence and Phase Values (in Parentheses) for the Daily Cycle. Phase Values are in $\frac{1}{7}$ Day Units

Price Series	July Future	October Future	March Future
October Future	.40 (.3)		
March Future	.36 (.8)	.13 (.1)	
New July Future	.14 (.8)	.06 (0.)	.32 (.1)

nature of the empirical leads and lags there does appear to exist a structure somewhat contrary to the notion that spot and all futures prices adjust simultaneously.

Second, for all three price cycles a consistent result was that the futures price series nearer maturity leads the futures price less near maturity. There was only one exception to this result. Third, Table 6.4 would seem to indicate that physical prices lag futures prices at the seasonal frequency.[14]

The results of Tables 6.4, 6.5 and 6.6 would seem to indicate that the transformation of new information into a new price is first enacted in the near future, then in the adjacent future and so on, until the price change has reached the futures contract furtherest from maturity. It appears that it is only after this chain of events has taken place that wool prices finally react.

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14 Too much emphasis should not be placed upon this conclusion as one of the results is rather contradictory with physical prices leading the twelve month futures prices.

It must be remembered that the leads and lags involved in the wool-wool futures market are very small. Nevertheless, there does seem to exist a consistent structure in conflict with Working's idea that all prices react simultaneously. Working's theories assumed that the markets involved were "ideal". It could be that real world markets with transaction costs and finite rates of information promulgation and assimilation necessarily possess a lead-lag structure. It could be that physical wool prices lag wool futures prices simply because the futures market is open every weekday, while the wool market in Australia trades far less regularly. Thus a price change may be reflected in futures prices before wool prices solely because the wool market was not open.

6.4 Futures Prices as Long Range Prophets of Wool Prices

Cross spectral analysis showed that some small yet discernible lead-lag structure existed between the decomposed price series. Cross spectral analysis indicated that futures prices led spot prices. Hence it can be tentatively inferred that a knowledge of today's near futures prices will provide some insight into wool prices in about 10 days time. This is because Table 6.4 reveals that near futures prices lead wool prices by .36 months at the seasonal frequency. In this section of the study we examine the results of using regression analysis to determine whether futures prices retained their predictive power over longer periods of time. Such a test has relevance as many traders continue to subscribe to the theory that futures prices represent an accurate prediction of future spot prices.

Let, $F_{T-t,T}$ represent the price of a futures contract with a maturation date of T,t time units from expiration. Let, P_{T-t} represent the spot price at time T-t and P_T the spot price at time T. If the price today, of a future that matures at time T represents an accurate forecast of spot prices at time T, then the spot price at time T can be visualized as a function of the price of the futures contract, $F_{T-t,T}$. Such a concept can be represented by a single regression equation

$$P_T = \alpha + \beta F_{T-t,T} + u_t \qquad .... \qquad (6.11)$$

where $\alpha = 0$ and $\beta = 1$ and $\{u_t\}$ is a white noise series.

In the theory section of this paper it was contended that the whole futures-spot price complex reacts to new information simultaneously. Thus in the theoretical market system the wool price today would be equally as good as today's futures price in predicting future wool prices. That is, the regression equation

$$P_T = \alpha + \beta\ P_{T-t} + u_t \qquad .... \qquad (6.12)$$

would have associated with it an r^2 comparable to regression equation (6.11).

For the purposes of the empirical investigations t the time to elapse before maturation was varied such that t equalled 3, 6, 12 and 15 months. Of the five futures that mature each year in the Sydney wool futures market, the July, October and December futures were chosen for analysis. As there could only be one observation per future per year and as the S.G.W.F.E. had only been operational from mid 1960, the number of observations for each regression was obviously small. The consequent low number of degrees of freedom was an unavoidable limitation to our investigations.

Table 6.7 contains the empirical estimates for the regression coefficients α and β for equation (6.11). If in fact futures prices were perfect predictors of wool prices on the futures expiration date, then α and β would equal 0 and 1 respectively. For this reason we tested to see if α and β significantly differed from 0 and 1 respectively.[15]

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15 The low number of degrees of freedom meant that the confidence intervals were very wide.

TABLE 6.7

Regression of Future Wool Prices on Wool Futures Prices

Futures Contract	Time to Maturity							
	3 months		6 Months		12 Months		15 Months	
	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\alpha}$	$\hat{\beta}$
July	29.0	.9	138.7**	.3**	147.0*	.28*	186.7*	.1*
October	-13.0	1.1	104.7	.5	257.3**	-.27**	242.0**	-.2*
December	-74.1*	1.4*	60.1	.8	314.8**	-.53**	395.7**	-1.0*

* indicates significance at the 10% level

** indicates significance at the 5% level

Table 6.7 shows that futures prices become less accurate predictions as t increases. Table 6.8 lists the r^2 obtained from regression equations (6.11) and (6.12).

TABLE 6.8

r^2, In Per Cent

Futures Contract	Independent Variable							
	$F_{T-3,T}$	$P_{T-3,T}$	$F_{T-6,T}$	$P_{T-6,T}$	$F_{T-12,T}$	$P_{T-12,T}$	$F_{T-15,T}$	$P_{T-15,T}$
July	74	83	14	56	5	14	0	0
October	45	49	7	12	3	5	11	3
December	82	95	8	16	6	2	18	6

The inability of wool futures prices to predict wool prices over any length of time is illustrated by the results of Table 6.8. The important result in Table 6.8 is that current wool prices proved to be just as good, if not better, than futures prices in predicting future wool prices.[16]

It would, perhaps, be more appropriate to say that today's wool prices and futures prices are similarly inefficient in predicting future wool prices. When t = 6 months, futures prices account for only 10 per cent and wool prices 30 per cent of variation in wool prices six months hence.

This, however, does not mean that the cash-futures price complex was incorrectly priced prior to the futures maturation. Current prices, both spot and futures, are constantly pushed into obsolescence by the arrival of new information. The greater the forecast period the greater the amount of new information and hence the less accurate the prediction.

6.5 Concluding Remarks

The results presented in this chapter have largely supported established ideas about cash-futures price relationships. Although the results did not challenge the validity of the theoretical notion that cash-futures prices react simultaneously, cross spectral analysis did reveal some small leads and lags.

Cross spectral analysis also showed that a futures price series is more highly correlated with a series with a close delivery date than with a series with a distant delivery date. Futures price series were more highly correlated over longer cycles than over the shorter cycles.

It was demonstrated that futures prices contain some predictive power over short periods, however, this quickly disappeared as the forecast period increased. As expected cash prices proved to be just

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16 A *Mann-Whitney* rank test was performed on the r^2 values in Table 6.8 to determine whether there was any significant difference between futures and wool prices as explanators of future wool prices. A standard normal deviate value of .41 indicated there was no significant difference at the 5% level.

as good as, if not better than, futures prices as prophets of future cash prices.

6.6 Thesis Summary

Let us review some of the more important findings of this study. Chapter 4 contains results which show that Sydney Wool futures price changes were non-normal. Also recorded is the fact that Granger's transformation failed to return the empirical distributions to normality. The leptokurtic empirical distributions were finally simulated by Praetz's rescaled t distribution. The search for a theoretical distribution to fit the data may seem a rather trite intellectual pursuit. However, we believe the distribution of price changes to be relevant to many practical economic issues, such as risk assessment and portfolio theory.

The random walk model's assertion that price changes be independent of one another was shown to be of doubtful validity in the S.G.W.F.E. The results presented in Chapter 5 suggest that short run prices, e.g. *transaction* and *daily* do not follow a random walk. In particular, we uncovered a strong weekly cycle. This cycle was not, in our opinion, the result of some market inadequacy but rather was the result of the rational reaction of market traders in the S.G.W.F.E. to uncertainty. The weekly cycle is an example of small short term non-random price movements.

Although there was evidence of certain dependencies in our price series, our results, especially the results of filter analysis, demonstrated that easy profits were not to be had from the wool futures market.

The results shown in Chapter 6 largely supported established ideas about cash-futures price relationships. Although the results did not challenge the validity of the theoretical notion that cash-futures prices react simultaneously, cross spectral analysis did reveal some small leads and lags.

Cross spectral analysis also showed that a futures price series is more highly correlated with a series with a close delivery date than with a series with a distant delivery date. Futures price series were more highly correlated over longer cycles than over the shorter cycles.

It was demonstrated that futures prices contain some predictive power over short periods. However, this quickly disappeared as the forecast period increased. As expected cash prices proved to be just as good as, if not better than, futures prices as prophets of future cash prices.

This thesis has not attempted the provision of explanations as to the causes of wool futures price changes. Ideally, we would like to relate wool futures prices to a number of explanatory variables using the technique of multiple regression. However, such a study is dependent upon advances in model building techniques and data collection.

We hope this thesis affords a foundation for further investigation into wool futures prices. We believe that eventually an econometric model of the entire wool industry, including the wool futures sector, will be produced. It is hoped that this thesis provides some insights to assist in the building of such a model.

APPENDIX I

TERMS USED IN FUTURES TRADING

Arbitrage. The taking of a position in the spot or futures market, or both, the aim of which is a certain profit.

Arbitrageur. One who undertakes arbitrage.

Basis. The difference between the price of the commodity and the price of the commodity future.

Calls. A period in which trading is conducted through a chairman to establish the buyer's and seller's price for each futures month at a particular time, i.e., the opening or closing call.

Cash Market. Commodity market.

Clean. A term used to describe scoured wool.

Clearing House. The central agency which guarantees performance of all contracts and through which all transactions made by Floor Members of the Exchange are cleared and financial settlements effected daily.

Contracts. Futures contracts are legal agreements for the purchase or sale of a standard quantity and quality of wool at a determined price, to be delivered during a specified month in the future. (*Note*: Futures contracts can be bought or sold, and are called "Futures Contracts" because they require delivery of the commodity in a stated month in the future. However, very few contracts ever result in delivery because they can be liquidated before they reach maturity. The main purpose of futures trading therefore is not to give or take delivery, but to reduce the risk of price fluctuations in the future.)

Delivery Month. The calendar month during which a Futures Contract matures.

Deposit. The amount paid by a client or covered under Bank Guarantee to protect the Broker and Clearing House against losses which the client may incur.

Differentials. The premiums and discounts set by the Exchange on the first day of each month for deliverable types and qualities of wool which vary from the standard type.

Greasy. A term used to describe unprocessed wool off the sheep's back.

Hedging. The establishment of an opposite position in the Futures Market from that held in the physical market. In practice therefore it is the operation whereby future production or unsold stock is covered by the sale of futures contracts as protection against a price decline; or conversely whereby anticipated requirements of the commodity, or forward sales, are covered by the purchase of futures as protection against a price rise.

Liquidation. The 'buying back' of a sold contract, or alternatively, the'selling' of a bought contract.

Long. The holder of a 'bought contract' is said to be 'long'.

Margin. The amount of money which may be called to cover an adverse price movement.

Open Positions. The number of contracts on the market which have not been liquidated.

Physical Market. The commodity market.

Short. The holder of a 'sold' contract is said to be 'short'.

Spot Market. The commodity market.

Spot Month. The first quoted month on the market, irrespective of whether it is the current month.

Squeeze. An upward movement of prices in the spot month due to a shortage of certified stock available for prompt delivery.

<u>Stop Loss Order</u>. An order placed with a Broker to operate at best when the market advances or declines to a specified level. Such an order is normally placed to limit losses, or secure profits.

<u>Straddle</u>. The simultaneous buying of one quoted month and the selling of a different quoted month, in anticipation of the price relationship between the two months changing sufficiently for the operator to make a profit.

<u>Switch</u>. The simultaneous liquidation of a futures contract in one month and the opening of a new contract in another month.

BIBLIOGRAPHY

Books:

BURFORD, R.L., *Statistics. A Computer Approach.* Merrill Publishing Co., Columbus, Ohio, (1968).

GOSS, B.A., *The Theory of Futures Trading.* London, Routledge & Kegan Paul, (1972).

GRANGER, C.W.J. and HATANAKA, M., *Spectral Analysis of Economic Time Series.* (1964), Princeton University Press, Princeton, New Jersey.

GRANGER, C.W.J. and MORGENSTERN, O., *Predictability of Stock Market Prices,* Heath Lexington, Massachusetts, (1970).

GRAY, R.W., *Wool Futures Trading in Australia - Further Prospects,* (University of Sydney, Department of Agricultural Economics: Research Bulletin No. 5, (1967).

IRWIN, H.S., *The Evolution of Futures Trading*, Madison, Wisconsin: Mimir (1954).

KALDOR, N., *Essays on Economic Stability and Growth*, London, Duckworth (1961).

KARMEL, P.H., *Applied Statistics for Economists.* Melbourne, I. Pitman and Sons, 1963.

LABYS, W.C. and GRANGER, C.W.J., *Speculation, Hedging and Commodity Price Forecasts*, Heath, Lexington Books, Lexington, Massachusetts, (1970).

MOORE, A., *A Statistical Analysis of Common Stock Prices.* Unpublished Ph.D. dissertation, Graduate School of Business, University of Chicago, (1962).

PRAETZ, P.D. *A Statistical Study of Fluctuations of Australian Share Prices.* (University of Adelaide. Unpublished Ph.D. thesis, July 1971).

SCHEFFE, H., *The Analysis of Variance*, New York: Wiley, (1959).

SYDNEY GREASY WOOL FUTURES LIMITED, *Futures Trading and Valuing of Greasy Clips*. (1968).

WALLIS, W.A. and ROBERTS, H.V., *Statistics. A New Approach.* Glencoe, Illinois: Free Press (1956).

Articles:

ALEXANDER, S., "Prices Movements in Speculative Markets: Trends or Random Walks, *Industrial Management Review II,* No.2, pp. 7-26, (May, 1961).

BAKKEN, H.H., "Historical Evaluation, Theory and Legal Status of Futures Trading in American Agricultural Commodities," *Futures Trading Seminar*, Mimir, Wisconsin (1960).

BACHELIER, L., "Theory of Speculation," (1900), translated and reprinted in P.H. Cootner, *The Random Character of Stock Market Prices*; M.I.T. Press, Cambridge, Massachusetts, (1964).

BRINEGAR, C.S.A., "A Statistical Analysis of Speculative Price Behaviour," *Food Research Institute Studies*. Vol. IX, Supplement, pp.1-58, (1970).

CARGILL, T.F. and RAUSSER, G.C., "Time and Frequency Domain Representation of Futures Prices as a Stochastic Process," *Journal of the American Statistical Association*, Vol.67, No.337, pp.23-31, (March, 1972).

COOTNER, P.H., "Stock Prices: Random vs. Systematic Changes," *Industrial Management Review*, Vol.3, No.2, pp.24-45, (Spring, 1962).

COWLES, A., "Can Stock Market Forecasters Forecast?" *Econometrica*, Vol.1, pp.309-324, (1933).

COWLES, A., "A Revision of Previous Conclusions Regarding Stock Price Behaviour," *Econometrica*, Vol.28, No.4, pp.909-915 (Oct. 1960).

COWLES, A. and JONES, H., "Some *A Posteriori* Probabilities in Stock Market Action," *Econometrica*, No.5, pp.280-294 (1937).

FAMA, E., "Mandelbrot and the Stable Paretian Hypothesis," *Journal of Business*, Vol.36, No.4, pp.420-429, (Oct., 1963).

__________"Behaviour of Stock Market Prices," *Journal of Business*, 38, pp.34-105, (1965).

__________"Efficient Capital Markets - A Review of Theoretical and Empirical Work." *Journal of Finance*, 25, pp.383-416, (1970).

FISHER, L. and LORIE, J., "Rates of Return on Investments in Common Stock; The Year-by-year Record. 1926-1965," *Journal of Business*, Vol.XXXVII, p.315 (Jan. 1964).

GODFREY, M.D., GRANGER, C.W.J. and MORGENSTERN, O., "The Random Walks Hypothesis of Stock Market Behaviour," *Kyklos*, Vol.17, pp.1-30, (1964).

GARTSIDE, P.S., "A Study of Methods for Comparing General Variances," *Journal of the American Statistical Association*, Vol.67, No. 338, pp.342-346, (June, 1972).

GRANGER, C.W.J. and MORGENSTERN, O., "Spectral Analysis of New York Stock Market Prices,",*Kyklos*, Vol.16, pp.1-27, (1963).

GRANGER, C.W.J. and ORR, "'Infinite Variance' and Research Strategy in Time Series Analysis," *Journal of the American Statistical Association*, Vol.67, No.338, pp.277-279, (June, 1972).

GRAY, R.W., "The Characteristic Bias in Some Thin Futures Markets," *Food Research Institute Studies*, Vol.I, pp.296-312, (Nov. 1960).

GRAY, R.W. and RUTLEDGE, D.J.S., "The Economics of Commodity Futures Markets: A Survey," *Review of Marketing and Agricultural Economics*, Vol.39, No.4 (Dec. 1971).

HOUTHAKKER, J.S., "The Scope and Limits of Futures Trading," in Abramovitz et al, *The Allocation of Economic Resources*, Stanford University Press, Stanford, (1959).

JENKINS, G.M., "General Considerations in the Analysis of Spectra," *Technometrics*, Vol.3, pp.133-166 (1961).

KENDALL, M.G., "The Analysis of Economic Time-Series - Part 1: Prices," *Journal of the Royal Statistical Society*, Vol.96, Part I, pp.11-25, (1953).

LARSON, A.B., "Measurement of a Random Process in Futures Prices," *Food Research Institute Studies*, Vol.1, No.3, pp.313-324 (Nov. 1960).

LEUTHOLD, R.M., "Random Walks and Price Trends: The Live Cattle Futures Market," *Journal of Finance*, Vol.XXVII, pp.879-889, (Sept. 1972).

MANDELBROT, B., "New Methods in Statistical Economics, *Journal of Political Economy*, pp.421-440 (Oct. 1963).

______________, "The Variation of Certain Speculative Prices," *Journal of Business*, Vol.36, pp.394-419 (Oct. 1963).

MOOD, A.M., "The Distribution Theory of Runs," *Annals of Mathematical Statistics*, No.11, pp.367-392, (1940).

OSBORN, M.F.M., "Periodic Structure in the Brownian Motion of Stock Prices," *Operations Research*, Vol.10, pp.345-379 (May-June, 1960).

PEARSON, E.S., "A Further Development of Tests for Normality," *Biometrika*, Vol.11, pp.239-249 (1930).

PHILLIPS, J., "The Theory of Practice of Futures Trading," *Review of Marketing and Agricultural Economics*, Vol.34, No.2 (June, 1966).

PINCHES, G.E., "The Random Walk Hypothesis and Technical Analysis," *Financial Analysts Journal*, pp.104-110. (March - April, 1970).

PRESS, S.J., "A Compound Events Model for Security Prices," *Journal of Business*, Vol.40, pp.317-335 (1967).

ROBERTS, H.V., "Stock Market 'Patterns' and Financial Analysis," *Journal of Finance*, Vol.14, No.1, p.1, (March, 1959).

SAMUELSON, P.A., "Proof that Properly Anticipated Prices Fluctuate Randomly," *Industrial Management Review*, Vol.6, p.41, (1965).

SNAPE, R.H., "Price Relationships on the Sydney Wool Futures Market," *Economica*, (May, 1968).

STEVENSON, R.A. and BEAR, R.M., "Commodity Futures: Trends or Random Walks?" *Journal of Finance*, Vol.25, pp.65-81 (March, 1970).

TAUSSIG, F.W., "Is Market Price Determinate?" *Quarterly Journal of Economics*, pp.394-411, May, 1921.

TOMEK, W.G. and GRAY, R.W., "Temporal Relationships Among Prices on Commodity Futures Markets: Their Allocative and Stabilizing Roles," *American Journal of Agricultural Economics*, Vol.52, p.372 (1970).

WORKING, H., "New Ideas and Methods for Price Research," *Quarterly Journal of Economics*, pp.394-411 (May, 1921).

__________, "A Random Difference Series for Use in the Analysis of Time Series," *Journal of the American Statistical Association*, Vol.XXIX, No. 85, pp.11-24 (March, 1934).

__________, "Quotations on Commodity Futures as Price Forecasts," *Econometrica*, Vol.10, p.39 (1942).

__________, "The Theory of Inverse Carrying Charges in Futures Markets," *Journal of Farm Economics*, Vol.XXX, No.1 (Feb. 1948).

WORKING, H., "The Theory of The Price of Storage," *American Economic Review,* Vol.39, pp.1254-1262, (Dec. 1949).

__________, "Hedging Reconsidered," *Journal of Farm Economics,* (Nov. 1953).

__________, "Note on the Correlation of First Differences of Averages in a Random Chain," *Econometrica,* Vol.28, No.4, pp.916-918, (Oct.1960).

__________, "New Concepts Concerning Futures Markets and Prices," *American Economic Review*, Vol.52, (1962).

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