



AN APPLICATION OF FUNCTIONAL ANALYSIS TO A PROBLEM IN GEOPHYSICS

by

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Thesis submitted for the degree of
Master of Science
in the Department of Pure Mathematics,
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Revised Version January, 1992 .

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SUMMARY

The aim of this thesis was to provide a rigorous justification for certain aspects of a perturbation method used to calculate an approximation of transient electromagnetic field. This method used the quasi-static approximation to the solution of the vector wave equations ' in a region of two half-spaces of differing conductivities.

In the case in which neither of the half-spaces was insulating, it was shown, via a variational approach, that a unique solution of the time-domain problem exists. If one of the spaces was insulating it was only possible to establish existence in the scalar case. The solution to the scalar diffusion equation was shown to exist in a weighted Sobolev space.

It was shown that the elements of the fundamental matrix of the Laplace transformed (with respect to t) vector wave equation, tended spatially pointwise to their value at $\epsilon = 0$, as $\epsilon \rightarrow 0$. Formulae for the fundamental matrix obtained previously were verified. It was shown that the perturbation method gave a solution to the problem in the half-space of non-zero conductivity if the current source was considered to reside in this half-space. Further restrictions on the source were shown to be necessary if it was considered to reside in the insulating half-space. The spatial asymptotic behaviour of the field was determined.

SIGNED STATEMENT

This thesis contains no material which has been accepted for the award of any other degree or diploma in any University and that, to the best of my knowledge and belief, the thesis contains no material previously published or written by another person other than where due reference is made in the text of the thesis.

I consent to the thesis being made available for photocopying and loan if applicable if accepted for the award of the degree.

Ken W. McNamara.

ACKNOWLEDGEMENTS

The author wishes to thank Dr. Alan Carey for his help and advice. During this research the author was supported by an ABSTUDY grant.