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# On the coarse-scale residual opening of hydraulic fractures created using the Channel Fracturing technique

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**Abstract.** Channel fracturing is a novel technique utilised to achieve discontinuous placement of proppant within a hydraulic fracture and create a network of open channels or voids between the proppant-filled regions (proppant columns), which can significantly increase the conductivity of the fracture. The problem of deformation and fluid flow in a partially-filled fracture involves two length scales: a large scale comparable to the length of the fracture ~O($10^2$) m and a fine scale comparable to the length of the proppant filled regions or 'columns' ~ O(1) m. In this paper, a homogenisation procedure is developed to obtain the residual opening profile and effective fracture conductivity at the large scale from the solution of a 'unit-cell' problem at the fine scale. The application of the model in a practical scenario is demonstrated by performing a mock numerical simulation.

## Introduction

The open channels created by channel fracturing technique are very conductive pathways for fluid flow from oil/gas reservoir to the wellbore, so the effective fracture conductivity can be increased up to several folds higher than that using the conventional hydraulic fracturing techniques [1,2]. The effective fracture conductivity can be maximised by selecting the optimal width of the open channels, i.e. the optimal spacing between the proppant-filled regions or 'columns'.

The optimisation requires solution to the problem of rock deformation and fluid flow in a partially-filled fracture at two length-scales: a coarse length-scale, $\hat{X}$ comparable to the half-length of the fracture, $L \sim O(10^2)$ m, and a fine length-scale, $\hat{x}$ comparable to the half-length of the proppant-filled regions or 'columns', $b \sim O(1)$ m. The initial opening of the fracture, $\delta_o$, the fluid pressure within the fracture, $p_f$, as well as the compressive overburden stress normal to the fracture plane, $\sigma_{yy}^{\infty}$, vary at the coarse scale. However, these variations are expected to be negligible at the fine scale, except close to the wellbore ($X = 0$) or the fracture tips ($|X| = \pm L$). Hence, the problem geometry can be treated as periodic at the length-scale of the proppant columns, and the fine scale can be formulated as a "unit cell" problem (see Fig. 1) [2].

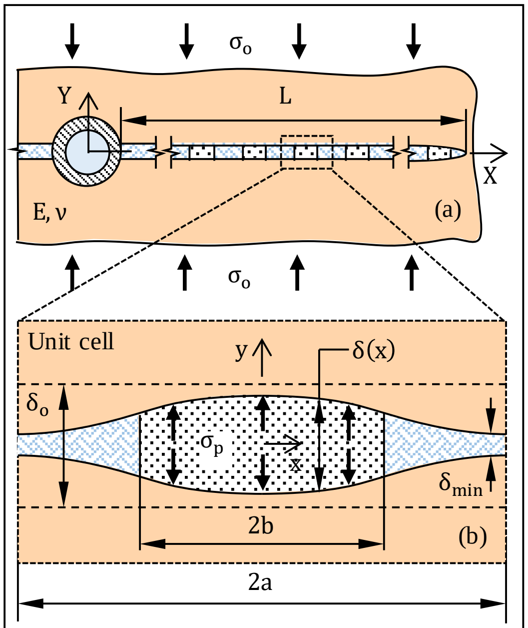


Fig. 1: (a) 2D model of a partially-filled hydraulic fracture, (b) detailed view of unit-cell (not-to-scale)

From a practical viewpoint, it is of greater interest to consider the effective conductivity of the entire fracture, rather than a unit-cell. The solution at the coarse scale can be obtained in a computationally efficient manner by adopting a homogenisation procedure. The homogenisation or averaging procedure replaces the system of discrete proppant columns along the fracture length by a continuously-distributed 'fictitious' porous medium. The purpose of this paper is to develop the displacement-dependent traction for the fictitious medium, which is a necessary first step towards the solution of homogenised problem.

## Mathematical model for the unit-cell problem

The present study adopts a simple one-dimensional model for proppant consolidation, i.e. the lateral expansion of the proppant columns is ignored. The compressive stress at a given location in the proppant column, $\sigma_p(x)$ is related to the change in height of the proppant column, $\delta_o - \delta(x)$ using the following power-law relationship [2]:

$$\sigma_p(x) = \alpha \left( \frac{\delta_o - \delta(x)}{\delta(x)} \right)^{\beta}, \tag{1}$$

where the constants $\alpha$ and $\beta$ are the fitting parameters determined from experimental data. The height of the proppant column $\delta(x)$ lies in the interval $(0, \delta_0]$ and Eq. (6) implies that $\sigma_p = 0$ at $\delta(x) = \delta_0$ and $\sigma_p \to \infty$ as $\delta(x) \to 0$.

The relative opening between the crack faces, $\delta(x)$, is modelled by a continuous distribution of 'edge dislocations'. The singular integral equation which governs the distribution of the dislocations is derived in [2] and can be written as

$$\frac{\bar{E}}{4\pi} \int_0^a B_y(\xi) \left[ \frac{2\xi}{x^2 - \xi^2} + K(x, \xi) \right] d\xi = \sigma_o - \sigma_p(x) H(b - |x|), \quad 0 < x < a, \tag{2}$$

where $B_y(\xi)$ is the unknown dislocation density function which represents the continuous distribution of dislocations, $H(\quad)$ is the Heaviside step function and the kernel $K(x, \xi)$ is given by

$$K(x, \xi) = \sum_{n=1}^{\infty} \frac{4\xi(x^2 - \xi^2 + 4a^2n^2)}{\{(x - \xi)^2 - 4a^2n^2\}\{(x + \xi)^2 - 4a^2n^2\}}. \tag{3}$$

The dislocation density is related to the residual opening profile according to

$$\delta(x) = \delta_{min} + \int_x^a B_y(\xi) d\xi, \quad B_y(\xi) = \frac{d\delta(\xi)}{d\xi}, \quad 0 < x < a. \tag{4}$$

The method of solution of Eq. (2) and the residual opening profile is described in [2].

## Homogenisation procedure

The aim of the homogenisation procedure is to replace the proppant column, which partially occupies the unit-cell, by an effective medium which fills the entire unit-cell. The nonlinear response of the effective medium is also described by Eq. (6), except for a multiplicative constant $C$, which varies with the geometrical parameters a, b and $\delta_o$ and the remote stress, $\sigma_o$. The constant $C$ must be found in such a manner that the potential energy of the unit-cell, defined below, remains conserved [3].

$$\Pi = U_1 + U_2 + W. \tag{5}$$

In Eq. (5), $U_1$ is the strain energy of the rock in the deformed configuration over the region $x \in [-a, a]$, $y \in (-\infty, \infty)$ and can be written as:

$$U_1 = 4 \int_0^{\infty} \int_0^{+a} \left( \frac{\sigma_{xx}^2 + \sigma_{yy}^2 - 2\nu\sigma_{xx}\sigma_{yy}}{2\overline{E}} + \frac{\sigma_{xy}^2}{2G} \right) dx\, dy, \tag{6}$$

where $\sigma_{xx}, \sigma_{yy}, \sigma_{xy}$ are elastic stress components in the rock formation, $\overline{E}$ is the generalised Young's modulus, and G is the shear modulus.

The term $U_2$ in Eq. (5) corresponds to the strain energy stored in the deformed proppant column and can be obtained as:

$$U_2 = 2 \int_0^{+b} \int_0^{\delta_0 - \delta(x)} \alpha \left( \frac{u}{\delta_o - u} \right)^{\beta} du\, dx, \tag{7}$$

where $u = \delta_o - \delta$ denotes the change in height of the proppant column.

Finally, the term W in Eq. (5) represents the work done due to the displacement of the remote boundary upon which the compressive traction $\sigma_{yy}(x, y \to \pm\infty) = -\sigma_o$ is applied. Since the displacement field due to the dislocation density tends to zero at the remote boundary $y \to \infty$, the work done can be written as

$$W = \lim_{y \to \infty} \left( -4a \frac{\sigma_o^2}{\overline{E}} y \right) - 2a(\delta_0 - \delta_{min})\sigma_o. \tag{8}$$

A unit-cell filled entirely with the effective medium undergoes uniaxial compression and the strain energy stored in the rock over the region $x \in [-a, a]$, $y \in (-\infty, \infty)$ is simply given by

$$U_1^* = 4 \int_0^{\infty} \int_0^{+a} \left( \frac{\sigma_o^2}{2\overline{E}} \right) dx\, dy. \tag{9}$$

The strain energy stored in the effective medium can be written as

$$U_2^* = 2 \int_0^{+a} \int_0^{\delta_0 - \delta^*} C\alpha \left( \frac{u}{\delta_o - u} \right)^{\beta} du\, dx, \tag{10}$$

where $\delta^*$ is the constant opening of the fracture filled with the effective medium, and can be obtained as:

$$\delta^* = \delta_o \left( 1 + \left( \frac{\sigma_o}{C\alpha} \right)^{\frac{1}{\beta}} \right)^{-1}. \tag{11}$$

Analogous to (8), the work done at the remote boundary is

$$W^* = \lim_{y \to \infty} \left( -4a \frac{\sigma_o^2}{\overline{E}} y \right) - 2a(\delta_0 - \delta^*)\sigma_o. \tag{12}$$

The equivalence of potential energy requires that $\Pi = \Pi^*$, i.e. $U_1 + U_2 + W = U_1^* + U_2^* + W^*$. Utilising Eqs. (5)-(12), the potential energy equivalence requirement can be stated as

$$4\int_0^{\infty}\int_0^{+a}\left(\frac{\sigma_{xx}^2+\sigma_{yy}^2-2\nu\sigma_{xx}\sigma_{yy}}{2\bar{E}}+\frac{\sigma_{xy}^2}{2G}-\frac{\sigma_o^2}{2\bar{E}}\right)dx\,dy+2\int_0^{+b}\int_0^{\delta_0-\delta(x)}\alpha\left(\frac{u}{\delta_o-u}\right)^{\beta}du\,dx$$
$$-2\int_0^{+a}\int_0^{\delta_0-\delta^*}C\alpha\left(\frac{u}{\delta_o-u}\right)^{\beta}du\,dx-2a(\delta^*-\delta_{min})\sigma_o=0. \tag{13}$$

Eq. (13) is satisfied by a unique value of the constant C which can be obtained using a suitable root finding algorithm.

**Numerical results**

In this section, some numerical results are presented for the effective properties of the homogenised medium. In these numerical calculations, the initial opening $\delta_o$ is fixed at 5 mm and the width of the proppant filled region, 2b is fixed at 1 m. The Young's modulus and Poisson's ratio of the rock are selected to be $E = 10$ GPa and $\nu = 0.3$ and the fitting parameters in Eq. (6) are selected to be $\alpha = 5.543$ MPa and $\beta = 3.873$.

The first step of the analysis is to determine the critical spacing between the proppant columns, 2a, at which the minimum residual opening of the unit-cell, $\delta_{min} = \delta(|x| = a)$ equals to zero, i.e. the fracture walls come in contact (see Fig. 1b). This critical value of proppant column spacing, $2a_{cr}$, corresponds to a drastic reduction in the fluid conductivity of the open channels. The selection of proppant column spacing greater than this critical value will result in sub-optimal increase in the effective fracture conductivity, hence represents a case of little practical interest. The dependence of $a_{cr}$ on the remotely applied compressive stress $\sigma_o$ was obtained through an extensive parametric study and the results are presented in Fig. 2. The best fit equation recovers the limiting cases, i.e. $2a_{cr} \to \infty$ as $\sigma_o \dashrightarrow 0$ and $2a_{cr} \to 2b = 1$ m as $\sigma_o \dashrightarrow \infty$.

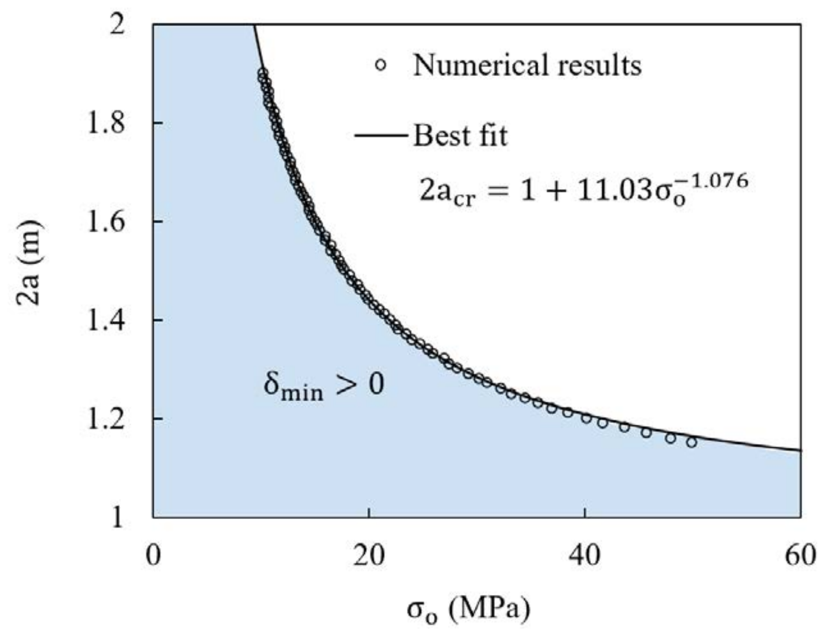


Fig. 2: Envelope showing combinations of proppant column spacing, 2a and remotely applied compressive stress, $\sigma_o$ which ensure that the fracture faces do not come in contact.

Numerical results for the constant C are obtained for the remotely applied stress in the range $10 < \sigma_o < 50$ MPa with increments of 1 MPa and the proppant column spacing in the range $1.0 < 2a < 2.0$ m, with increments of 0.05 m. A spline function was fitted through the discrete data points to obtain the interpolated value of the constant $C$ for any combination of parameters $2a$ and $\sigma_o$ which yields $\delta_{min} > 0$ as shown in Fig. 3.

The conductivity of the unit cell fully-filled with the effective medium is equivalent with the effective conductivity of the partially-filled unit cell, $K_{eff}$, which is obtained from [4]. Hence, the permeability of the effective medium, $\kappa_p$, can be gained as:

$$\kappa_p = \frac{K_{eff}}{\delta^*} \tag{14}$$

where $\delta^*$ is the constant opening of the unit cell when it is fully-filled with the effective medium, see Eq. (11). $\kappa_p$ is the key to compute the conductivity of each unit cell along the fracture length, and the conductivity of unit cells will be utilised to calculate the conductivity of the entire fracture.

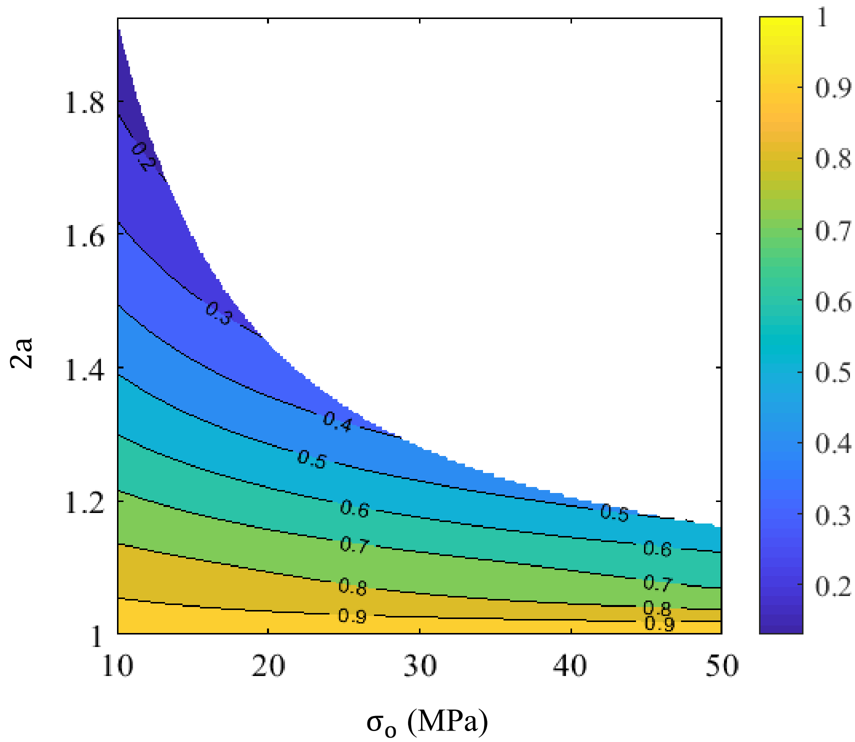


Fig. 3: Contour plot showing the variation of the effective medium stiffness constant C upon proppant column spacing and remotely applied stress.

## Conclusion

In this paper, the periodic system of proppant columns within a hydraulic fracture is replaced by a continuous distribution of springs along the fracture length using a homogeneous procedure. The energy conservation principle and the solution for "unit-cell" developed in [2] are utilised to define the power-law for the nonlinear springs. The numerical results present the effective medium stiffness constant C according to any combination of proppant column spacing and the confining stress. The application of the effective medium stiffness concept allows a significant reduction of the complexity of the problem and an application of well-developed methods of Fracture Mechanics to evaluate the residual opening of a periodically supported fracture. The outcomes of this work provide the first necessary step to analyse the hydraulic channel fracturing technique, which is of great interest to the gas and oil industries.

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