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The Pion Nucleon Sigma Term

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Declaration

This thesis contains no material that has been presented for any degree other than as indicated on the title page. To the best of my knowledge, this thesis contains no material presently published, except where due reference is made in the text.

Charles J. Brasted November 1995

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Abstract

The pion-nucleon sigma term is a measure of symmetry breaking in QCD. Despite the general acceptance that its experimental value is 45 ± 12 MeV, there is still some division regarding the role of certain processes contributing to its theoretical value. In this thesis we calculate and compare the pion nucleon sigma term with and without the contributions arising from processes that involve decuplet baryons. Furthermore, we examine the wider problem of the validity of SU(3) phenomenology, and its accuracy in treating processes involving strangeness.

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Chapter 1

Introduction

1.1 Introduction

Quantum Chromo-dynamics (QCD) is a theory of strong interactions. In the light of it's successes, it is probably **the** theory of strong interactions. However, one cannot be certain, as it is yet to be proven that it models strongly interacting particles in every way. Clearly, it is worthwhile to analyse QCD as critically as possible.

The origins of QCD can be traced back to 1961, when Gell-Mann developed a model of hadrons. Given the nature of the baryon spectrum, he proposed that hadrons were made up of more fundamental particles with half-integer spin and fractional charge, quarks. The quarks possessed an extra quantum number, flavour. Gell-Mann proposed that the baryons were made up of three quarks, and the mesons composed of a quark/anti-quark pair. We can see how this conclusion was arrived at if we consider an irreducible representation of 3 triplets,

$$\mathbf{3} \times \mathbf{3} \times \mathbf{3} = \mathbf{10} + \mathbf{8} + \mathbf{8} + \mathbf{1}. \tag{1.1}$$

In other words, the coupling of three triplets will manifest itself physically as an octet, decuplet or singlet. The singlet baryon has not been observed.

The main problem with Gell-Mann's model of the hadrons was that certain hadron states, such as the Δ^{++} resonance possessed bad symmetry. The Δ^{++} particle has a spin-flavour wave function, $|u \uparrow u \uparrow u \uparrow\ >$ which is clearly symmetric under an exchange of quarks. Real baryons are fermions, and so the wave function should not be symmetric under such a transformation. As a result a new quantum number was introduced, colour, which is carried by quarks and gluons, hence the name Quantum Chromo-dynamics. The quarks came in three colours and the hadrons were colourless.

Quantum electrodynamics(QED) has been very successful in describing electromagnetic interactions. QED is a theory based on a locally gauge invariant Lagrangian, where the gauge field is the photon. Similiarly in QCD the gauge field is associated with the messenger particles between quarks, the gluons. The development of a locally gauge invariant theory with corresponding non-Abelian Lie algebra had been done by Yang and Mills in 1954 [1]. It was not until 1971 that Gross and Wilczek [2] and Politzer [3] showed that a non-abelian gauge theory exhibited asymptotic freedom, that is, the quarks behaved as free particles at high energies.

1.2 Origin of Mass

In the gauge theory of weak interactions, the quark masses are not fundamental constants of nature, but are generated by the particle's motion through a scalar condensate resulting from the spontaneous breaking of the symmetry of the Lagrangian. Various theories describing the spontaneous breakdown of gauge symmetries have been developed, however it is not possible to predict the quark masses using these theories. Our interest is in the treatment of these quark masses, rather than their origins, so we will choose the QCD Lagrangian as the starting point for our investigations.

The fact that QCD exhibits asymptotic freedom at high energies (i.e. coupling becomes small) means that we can apply perturbation theory in the usual way. At low energies the coupling between quarks is large, so conventional perturbative techniques (as used successfully in QED) do not work. The alternatives to a perturbative treatment are to use lattice theories, or phenomenological models to describe QCD at low energies.

1.3 Chiral Symmetry

We can define left and right handed Dirac fields as

$$\psi_L = \frac{1-\gamma_5}{2}\psi \text{ and } \psi_R = \frac{1+\gamma_5}{2}\psi, \tag{1.2}$$

where $\frac{1\pm\gamma_5}{2}$ is the helicity projection operator ($\Pi^{\pm}$). Consider

$$\bar{\psi}_L\psi_L = \bar{\psi}\Pi^+\Pi^-\psi = 0. \tag{1.3}$$

Clearly then, a particle with definite handedness will be massless. The Lagrangian

$$\mathcal{L} = i\bar{\psi}_L \overleftrightarrow{\partial\!\!\!/} \psi_L + i\bar{\psi}_R \overleftrightarrow{\partial\!\!\!/} \psi_R, \tag{1.4}$$

is invariant under the transformations

$$U(1)_L : \psi_L \rightarrow e^{i\alpha_L}\psi_L \tag{1.5}$$

$$U(1)_R : \psi_R \rightarrow e^{i\alpha_R}\psi_R \tag{1.6}$$

where α_L and α_R are global phases. These transformations are simply phase transformations. The important feature of this is that the left handed and the right handed fields transform independently. The Lagrangian (1.4) is invariant under these chiral transformations, and as a result the Noether currents for transformations (1.5) and (1.6)

$$J^\mu_L = \bar{\psi}_L\gamma^\mu\psi_L \text{ and } J^\mu_R = \bar{\psi}_R\gamma^\mu\psi_R \tag{1.7}$$

are conserved.

The QCD lagrangian is written

$$\mathcal{L} = -i\sum_f \bar{q}_f \overleftrightarrow{D\!\!\!\!/}\, q_f - m_f\bar{q}_f q_f - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} \;+\; \text{gauge fixing and ghost terms} \tag{1.8}$$

where q_f is the quark field for a quark with flavour $f = u, d, s$. This Lagrangian is invariant under the $SU(3)_L \otimes SU(3)_R$ group of transformations ($q_- = q_L$ and $q_+ = q_R$)

$$q_\pm \to \exp(-\frac{i}{2}\theta \cdot \lambda) q_\pm \tag{1.9}$$

The corresponding conserved Noether currents are[1]

$$J_\pm^{a\mu} = \sum_{ll'} \bar{q}_l(x) \gamma^\mu \lambda^a_{ll'} \frac{1 \pm \gamma_5}{2} q_{l'}(x) \tag{1.10}$$

which can be rewritten as $J_\pm^\mu = V^\mu \pm A^\mu$,

$$\text{with } V^{a\mu} = \sum_{ll'} \bar{q}_l \lambda^a_{ll'} \gamma^\mu q_{l'}(x), \tag{1.11}$$

$$\text{and } A^{a\mu}(x) = \sum_{ll'} \bar{q}_l \lambda^a_{ll'} \gamma^\mu \gamma_5 q_{l'}. \tag{1.12}$$

The Lagrangian is still invariant under the $U(1)_L \otimes U(1)_R$ group of transformations, so we still have currents

$$V^\mu_{qq'} = \bar{q}\gamma^\mu q' \tag{1.13}$$

$$A^\mu_{qq'} = \bar{q}\gamma^\mu \gamma_5 q' \tag{1.14}$$

The naive divergences of these currents are

$$\partial_\mu V^\mu_{qq'} = i(m_q - m_{q'})\bar{q}q' \tag{1.15}$$

$$\partial_\mu A^\mu_{qq'} = i(m_q + m_{q'})\bar{q}\gamma_5 q' \tag{1.16}$$

Thus we can see that the vector current is conserved in the limit that all quarks have equal

[1] $\lambda^a_{ll'}$ is the relevant component of the particular Gell-Mann matrix.

mass. The axial current is conserved in the limit that all quarks are massless.

Massless QCD is symmetric under the $SU(3)_A \times SU(3)_V \times U(1)_V$ group of transformations.

1.4 The Sigma Term

We can rewrite (1.8) as

$$\mathcal{L}_0 - \sum m_q \bar{q} q, \tag{1.17}$$

where $\mathcal{L}_0$ is invariant under the chiral transformations. As well as can be established, the mass term in the Lagrangian is the only term that explicitly breaks chiral symmetry. There is no theoretical reason why this should be so. Given this, the motivation for studying a measure of symmetry breaking is twofold. Firstly, we can predict a theoretical value for this measure, and draw a comparison with an experimental value, based on the assumption that $-\sum m_q \bar{q} q$ is the only symmetry breaking term in $\mathcal{L}_{\mathrm{QCD}}$. The result will tell us whether this assumption is correct (or not), and also provide information as to whether the treatment is correct. If there is agreement between theory and experiment, we will conclude that it is likely our initial assumption was correct, as was our treatment.

The measure of symmetry breaking that we will consider is the pion-nucleon sigma term, formally defined as

$$\sigma_{\pi N}(t) = \sum_{i=1}^{3} \frac{1}{3} < N(p')|[Q_i^5, [Q_i^5, H]]|N(p) >, \tag{1.18}$$

where $t = -(p-p')^2$ and H the strong interaction Hamiltonian. Early calculations of $\sigma_{\pi N}$ gave deviations from the experimental value (45 MeV) by $\sim 50\%$. Many solutions to this problem have been proposed, the most common a large strange quark content of the nucleon. More recently it has been proposed that the current quark mass of the light quark is larger than 6 MeV [4]. It is our contention that one important source of the discrepancy between the experimental and theoretical values is the failure to include the decuplet processes in the one loop corrections to $\sigma_{\pi N}$.

The sigma term has been calculated using QCD sum rules [5, 6], however to date, it is difficult to determine the accuracy of such calculations, as the final result depends upon quantities such as the accuracy of kaon PCAC, and the value of $< \bar{s}s >_0 / < \bar{u}u >_0$, for which there is some uncertainty. Most importantly, to date no one has been able to provide an estimate of the uncertainties associated with sum rules.

1.5 An alternative to QCD

If one is willing to forgo the ideological 'elegance' of gauge field theory, phenomenological models can be used with great success. A phenomenological model is based on the idea that physical systems can be decribed in terms of their observable properties[1] without recourse to the

[1] Husserl's phenomenalism

fundamental basis (in this case non-Abelian gauge invariance) of the system. These models should respect the underlying symmetries and degrees of freedom of QCD, however they are less fundamental in their formulation. One such model is the Cloudy Bag Model (CBM).

A persistent problem with all models for strongly interacting particles with quark degrees of freedom is that they do not predict confinement. No free quarks have been observed, so it is reasonable to assume, up to energy levels of TeV, quarks only exist in 3 quark states (baryons) and quark-antiquark pairs (mesons). The main idea behind all bag models is that confinement is provided by virtue of the quarks being massless inside some region in space (the bag) and possessing infinite mass outside the bag. The early bag models have been developed and improved. The most successful to date is the CBM. The CBM is an improvement of one of the original bag models, the MIT bag model, which described massless quarks moving freely inside a region of space (the bag). The problem with the MIT bag model was that, although it was confining, it violated chiral symmetry. The CBM corrected this by introducing a pseudo-scalar field, which coupled to the quarks at the surface of the bag. This tied in nicely with the idea that the nucleon was surrounded by a pion cloud. The CBM has been very successful in predicting many properties of baryons [7] , and so is a reasonable starting point for the analysis of the sigma term.

This remainder of this report will be presented as follows. Firstly, we will discuss chiral symmetry, chiral symmetry breaking and the link between the quark masses and symmetry breaking. Secondly we will discuss a measure of chiral symmetry breaking, the pion-nucleon sigma term. Following this, we will examine the model used to carry out all of our calculations, the cloudy bag model. Finally, we devote a chapter to general observations regarding previous studies of the pion-nucleon sigma term.

Chapter 2

Chiral Symmetry

2.1 Chiral symmetry

Chiral symmetry is the invariance of the Lagrangian under the chiral transformation. As massless QCD is chirally symmetric, we choose this symmetry as a starting point for our investigations. Nature is nearly chirally symmetric, however the only exact symmetry of QCD is the $U(1)_V$ group, that is, conservation of Baryon number[1]. Chiral symmetry in QCD is broken in 2 ways. The quark masses explicitly break the symmetry of the Lagrangian, and irrespective of the quark mass values, chiral symmetry is spontaneously broken by virtue of the symmetry of the system not matching that of the vacuum.

When considering chiral symmetry, it is important to understand the largest symmetry group involved, and the subgroups that are revealed as the symmetry is broken. The largest symmetry group that we will consider is the $SU(3)_V \otimes SU(3)_A$[2]. The Lagrangian is invariant

[1]In the entire body of this report, colour is not a consideration

[2]We will confine ourselves to the study of the study of the three lightest quarks, as the masses of the heavier quarks makes treatment of the theory as a perturbation about the exact symmetry limit unproductive.

under $q \to Uq$ where

$$U_V(x) = \exp(i\underline{\lambda} \cdot \underline{\alpha}(x))$$

$$U_A(x) = \exp(i\gamma_5\underline{\lambda} \cdot \underline{\beta}(x)) \tag{2.1}$$

which leads to the conserved currents

$$V_\mu^a = \bar{q}\gamma_\mu \frac{\lambda^a}{2} q \tag{2.2}$$

and eight conserved axial currents

$$A_\mu^a = \bar{q}\gamma_\mu\gamma_5 \frac{\lambda^a}{2} q \tag{2.3}$$

for $a = 1, 2..8$. QCD is invariant under the $U(1)_V$ group of transformations (conservation of Baryon number). There are (of course) axial charges associated with the conserved currents,

$$Q^a = \int d^3x V_0^a(x)$$

$$Q_5^a = \int d^3x A_0^a(x) \tag{2.4}$$

which satisfy their own Lie Algebra.

When the mass term, $-\sum m_q \bar{q}q$, is included in the QCD Lagrangian the symmetry is explicitly broken. The current treatment of this is chiral perturbation theory (χPT), in which the behaviour of the system is studied as the masses are turned on.

Historically, the main focus of investigations into chiral symmetry breaking has been the path taken by the system as the various symmetry breaking processes are 'turned on'. Initially there was no consideration given to the origin of chiral symmetry breaking (χSB), simply the properties of the symmetry breaking components in the Hamiltonian. Following the development of the quark model, a more mechanical procedure was developed, whereby a perturbation scheme was developed with the quark masses as the source of symmetry breaking, and therefore the perturbation parameter. These various schemes had to take into account the divergences that occurred within the theory as a result of the way chiral symmetry is realised. The resultant techniques developed were known as chiral perturbation theories. We will examine chiral symmetry and its treatment in more detail now.

2.1.1 Realisations of Chiral Symmetry

Consider Coleman's theorem, that is

$$Q^a|0\rangle = 0 \Rightarrow [Q^a, H] = 0, \tag{2.5}$$

which means that the physical states of H can be classified according to the irreducible representations of the group generated by Q^a. Goldstone's theorem considers the possibility that a symmetry of the system is not a symmetry of the vacuum. That is, we can have either

$$[Q^a, H] = 0 \Rightarrow Q^a|0\rangle = 0, \tag{2.6}$$

which is the converse of Coleman's theorem, or

$$[Q^a, H] = 0 \quad \text{but} \quad Q^a|0\rangle \neq 0. \tag{2.7}$$

If the second case is true, then the symmetry associated with the charge that breaks the vacuum symmetry has no bearing on the baryon spectrum.

If the charges satisfy the following algebra,

$$[Q^a, Q^b] = i\epsilon^{abc}Q^c \qquad [Q^a, Q_5^b] = i\epsilon^{abc}Q_5^c \qquad [Q_5^a, Q_5^b] = i\epsilon^{abc}Q_5^c, \tag{2.8}$$

then we can consider $Q_\pm^a = Q^a \pm Q_5^a$ to illustrate the point. If $[Q_\pm^a, H] = 0$ and $Q_\pm^a|0>= 0$ then we have what is known as the Wigner-Weyl realisation of the symmetry group. If the group was the $SU(2)_V$ group then we would have a spectrum of iso-multiplets. However, if $Q_5^a|0>= 0$ then we would have iso-multiplets with parity partners (degenerate in mass, but with opposite parity). The observed baryon spectrum supports the notion that $Q^a|0>= 0$ but not $Q_5^a|0>= 0$. Therefore, we assume that $Q_5^a|0>\neq 0$, which is the Nambu-Goldstone realisation of the symmetry associated with Q_5^a. Goldstone's theorem tells us that if the latter is true, then the vacuum symmetry is broken by the transformation and there exists massless Goldstone bosons. As $Q_5^a|0>\neq 0$, there must exist a state $Q_5^a|0>$ which has the same energy eigenvalue as $|0>$. Clearly this state will be a pseudoscalar, and we identify pions as pseudo-scalar Goldstone bosons.

2.1.2 Partially Conserved Axial Currents

We know that the pion decays via the axial current, i.e.

$$< 0|A_\mu^a|\pi^b(q) >= if_\pi q_\mu \delta^{ab}, \tag{2.9}$$

where a, b are the iso-spin indices and f_π is the pion decay constant[3]. In the chiral limit, $\partial^\mu A_\mu = 0$ and therefore $< 0|\partial^\mu A_\mu^a(0)|\pi^b(q) >= f_\pi m_\pi^2 \delta^{ab} = 0$. Thus, in the chiral limit, we require either $m_\pi^2 = 0$ or $f_\pi = 0$. Since $Q_5^a|0 >\neq 0$, f_π cannot be equal to zero, so in the chiral limit $m_\pi^2 = 0$.

There is a link between the breaking of chiral symmetry and the conservation of the axial current. If we consider the pion-nucleon-nucleon coupling

$$< N(p')|A_\mu^a|N(p) >= \bar{u}(p')(\frac{\tau^a}{2})[\gamma_\mu\gamma_5 g_A(q^2) + q_\mu\gamma_5 h(q^2)]u(p), \tag{2.10}$$

where $q = p' - p$, $g_A(q^2)$ and $h(q^2)$ are the axial and pseudoscalar form factors respectively, and $g_A(0) = 1.25$ [8] If k is small, then, in the non-relativistic limit, we find

$$\gamma_\mu\gamma_5 \to \vec{\sigma}, \text{ and } q_\mu\gamma_5 \to \frac{\vec{\sigma}\cdot\vec{q}}{2m_N}\vec{q}, \tag{2.11}$$

and hence (dropping isospin labels)

$$< n|\vec{A}^-|p >= \frac{1}{\sqrt{2}}\chi_{p'}^\dagger \left[g_A(q^2)\vec{\sigma} - \frac{h(q^2)}{2m_N}\vec{\sigma}\cdot\vec{q}\vec{q}\right]\chi_p. \tag{2.12}$$

[3]Equation (2.9) is the definition of f_π.

Now, if the axial current is conserved, then

$$\left[g_A(q^2) - \frac{h(q^2)q^2}{2m_N}\right]\vec{\sigma}\cdot\vec{q} = 0. \tag{2.13}$$

Thus

$$h(q^2) = \frac{2m_N g_A(q^2)}{q^2}. \tag{2.14}$$

Since $g_A(0) \approx g_A(q^2)$ for small q^2, $h(q^2)$ must have a pole corresponding to the propagation of a massless, exchanged particle. As A_μ is not conserved, it seems reasonable to rewrite $h(q^2)$ as

$$h(q^2) = \frac{2m_N g_A(q^2)}{q^2 + m_\pi^2}. \tag{2.15}$$

However, when m_π is equal to zero, comparing the explicit calculation of the pion pole diagram we find [9, 10, 7] that

$$m_\pi g_A = 2f_{NN\pi}f_\pi. \tag{2.16}$$

This is the Goldberger-Trieman relation, linking the weak and strong coupling constants. If we do not have massless pions, the divergence of the axial current is

$$\partial^\mu A_\mu = f_\pi m_\pi^2 \phi, \tag{2.17}$$

where ϕ is the pion field. Eq. (2.15), is the definition of the Partially Conserved Axial Current Hypothesis (PCAC). That is, the transition from a massless pion to the massive pion should be smooth. Unless this is the case, the axial current will be broken quite badly. As the pion mass is small, the axial current is almost conserved.

The quantity Δ [9],

$$\Delta = 1 - \frac{g_A m_\pi}{2 f_{NN\pi} f_\pi} = 0.08 \pm 0.02 \tag{2.18}$$

is a measure of the breaking of the $SU(2) \otimes SU(2)$ group. The quantities Δ and $\partial^\mu A^a_\mu$ have the same dependence on m_π, thus the GT relation is a consequence of, and a measure of, chiral symmetry breaking.

2.2 Quark masses as perturbations

Nature is not exactly as described above, the iso-multiplets associated with the vector charge are not degenerate in mass, and the pions are not massless. Both these disrepancies arise from the fact that nature is not exactly chirally symmetric, the symmetry is broken explicitly by the quark masses.

2.2.1 Breakdown of $SU(3) \otimes SU(3)$

The 'pre-quark' theories developed to model the breakdown of chiral symmetry were based on the assumption that the symmetry breaking component of the Hamiltonian transformed under $SU(3)$ in a certain way. It was generally accepted that if we generalise the group associated with the vector charge to include strangeness, that is $SU(3)_V$, where Q^a $(a = 1, 2....8)$ satisfies

$$[Q^a, Q^b] = i f_{abc} Q^c, \tag{2.19}$$

then we could assume that the hadrons would be classified according to the irreducible representations of $SU(3)_V$. There are many possible representations, but we rule out those that predict fractional charge. We are left with the octet and decuplet baryons. The Glashow - Weinberg (GW) model and the Gell-Mann - Oakes - Renner (GMOR) were the most widely used 'pre-quark' models.

It was assumed that the symmetry breaking part of the Hamiltonian has definite transformation properties with respect to $SU(3)_V$ in a representation. Furthermore, it was assumed H_{SB} was a component of the simplest possible representation of $SU(3)_V$, that is, the octet. The Hamiltonian must still commute with iso-spin and hypercharge, as these quantities are conserved. The only component with these transformation properties is the eighth component of the octet, i.e.

$$H = H_0 + \epsilon_8 u_8. \tag{2.20}$$

If one treats $\epsilon_8 u_8$ as the only source of explicit symmetry breaking, then we find

$$2(M_N + M_\Xi) - (3M_\Lambda + M_\Sigma) = O(\epsilon_8^2), \tag{2.21}$$

the Gell-Mann Okubo mass formula, which has been shown to work well. In contrast, mass formulae in other representations, such as **27** are not accurate. This is known as octet enhancement and works surprisingly well. Why this is so is a slightly more complex matter, and various explanations are given in ref. [9].

The Glashow - Weinberg Model

The GW model starts with the symmetry breaking Hamiltonian given as

$$H = H_o + \epsilon_0 u_0 + \epsilon_8 u_8 \tag{2.22}$$

where ϵ_0 and ϵ_8 are the symmetry breaking parameters. The $\epsilon_0 u_0$ has been added to H_{SB} using the same arguments as for $\epsilon_8 u_8$. More generally, it was assumed the symmetry breaking component of the Hamiltonian transformed like $(\bar{3},3)+(3,\bar{3}) = (8,1)+(1,8)$. The operators u_a and v_a obey the following :

$$[Q_i, u_a] = i f_{iab} u_b, \quad [Q_i, v_a] = i f_{iab} v_b,$$

$$[Q_i^5, u_a] = -i d_{iab} v_b, \quad [Q_i^5, v_a] = -i d_{iab} u_b, \tag{2.23}$$

for $i = 1..8$ and $a, b = 0, 1...8$. H_0 is invariant under the $SU(3) \otimes SU(3)$ group of transformations. The formalism that yields physical predictions for this model is based on the relevant Ward identities for the matrix elements for the $SU(3) \otimes SU(3)$ currents, which are derived from H. The technique is not without its flaws, because certain assumptions have to be made in evaluating the integral equations associated with the Ward identities, namely that they are dominated by the pion polc (saturation), and that the integrals are slowly varying near the pole (smoothness). In this respect χPT is a far more effective method. The model does, however, provide a good description of symmetry breaking, although the assumptions made during evaluation of the integral equations lead to poor quantitive results.

The Gell-Man - Oakes - Renner Model

The difference between the GMOR model and the GW technique outlined in the previous section is that the behaviour of nature in the chiral limit is given more consideration in the GMOR model. The Hamiltonian is the same, however, the assumption is made that in the chiral limit, that is $\epsilon_{0,8} \to 0$, H is $SU(3) \otimes SU(3)$ invariant and while the corresonding vector charges annihilate the vacuum, the the axial changes do not. Applying Coleman's theorem and Goldstone's theorem, we find that the physical states are classified as a fundamental representation of $SU(3)_V$ and the pseudo-scalar mesons are Goldstone bosons with zero mass.

An alternative approach to the breaking of $SU(3) \otimes SU(3)$

Another way of looking at the breakdown of chiral symmetry is to rewrite H_{SB} as

$$H_{\text{SB}} = \epsilon_0(u_0 + cu_8) \tag{2.24}$$

with $c = \epsilon_8/\epsilon_0$, initially set at $-\sqrt{2}$. When $c = -\sqrt{2}$, H_{SB} breaks $SU(3) \otimes SU(3) \longrightarrow SU(2) \otimes SU(2)$, the strange mesons acquire mass and the octet baryons are no longer degenerate. When c deviates from $-\sqrt{2}$, $SU(2) \otimes SU(2)$ is broken and the pions acquire a mass. Using χPT, $c = -1.25$ [11], reinforcing the notion that $SU(2) \otimes SU(2)$ is a very good symmetry of hadronic physics, second only to isospin.

2.2.2 How good is $SU(3) \otimes SU(3)$?

There are a number of ratios that can be extracted from the light meson masses to give an indication of how 'good' a symmetry is. If we consider the first order meson mass formulae,

$$M_{\pi^+}^2 = (m_u + m_d)B$$

$$M_{K^+}^2 = (m_u + m_s)B$$

$$M_{K^0}^2 = (m_d + m_s)B, \tag{2.25}$$

where $B =< 0|\bar{q}q|0 >$, we can extract the following ratios [12]

$$\frac{m_u}{m_d} = 0.66, \quad \frac{m_s}{m_d} = 20.1, \quad \frac{m_s}{\hat{m}} = 24.2. \tag{2.26}$$

The charged mesons are surrounded by a photon cloud, thus leading to corrections of $O(e^2)$, which alter the ratios slightly. However, the above ratios will be altered further when χPT is used to calculate all next to leading order contributions to the meson masses. The ratio, R,

$$R = \frac{m_s - \hat{m}}{m_d - m_u} = 43.5 \pm 3.2 \tag{2.27}$$

which measures the level of breaking of $SU(3)_V$ relative to $SU(2)_V$, can be extracted from the values of $M_p - M_n$, $M_{\Sigma^+} - M_{\Sigma^-}$ and $M_{\Xi^0} - M_{\Xi^-}$. The most recent calculation of the mass ratios is [12]

$$\frac{m_u}{m_d} = 0.56 \pm 0.006, \quad \frac{m_s}{m_d} = 20 \pm 2.0, \quad \frac{m_s}{\hat{m}} = 25.6 \pm 2.0. \tag{2.28}$$

These ratios are in agreement with eq. (2.27). The errors arise from the uncertainties in the constants determined in χPT.

The exact values for the quark mass ratios are far from resolved, as any calculations involving higher order corrections will possess some degree of model dependence. As a consequence, one must examine as many constraints as possible. However, recent work suggests that the above ratios are very close to the actual values [12].

We saw earlier that $SU(2) \otimes SU(2)$ was accurate to 7 %. Similiarly, we can derive the ratio

$$\frac{m_d}{m_u} = 1.8 \pm 0.2. \tag{2.29}$$

This ratio is a measure of isospin breaking. As it turns out, it is a rather poor estimate. For a long while it was thought that strong interactions were exactly isospin symmetric, and that any mass differences in hadrons were due to electromagnetic effects. However, the sign of the proton-neutron mass difference (amongst others) suggested otherwise. Isospin is broken because $m_u \neq m_d$. The ratios (2.29) and (2.27) are not a good indication of the magnitude of the symmetry breaking. This is because QCD disguises the current quark masses rather well, and the actual contribution to the baryon mass from the current quark mass is quite small. More sophisticated techniques are required to measure the exact level of symmetry breaking. To determine whether or not we can treat a quark mass as a perturbation we can compare the mass/mass differences involved with a given energy scale, say, M_ρ. It is clear that $SU(2)_V$ and $SU(2) \otimes SU(2)$ are 'good' enough to treat the symmetry breaking in perturbation theory. It would be worth considering how accurate the $SU(3)_V$ and $SU(3) \otimes SU(3)$ symmetries are.

A slightly crude technique would be to look at the ratio

$$R' = \frac{< N|H^{I}_{\text{SB}}|N >}{< N|H^{II}_{\text{SB}}|N >} \tag{2.30}$$

where H^{I}_{SB} and H^{II}_{SB} are the components of the Hamiltonian breaking $SU(3)_V$ and $SU(2)\otimes SU(2)$ respectively. Neglecting isospin breaking terms, we would have

$$R' = \frac{c_8 u_8}{\hat{m}(\bar{u}u + \bar{d}d)} = \frac{9c_8u_8}{\sqrt{3}(\sqrt{2}c_0 + c_8)(\sqrt{6}u_0 + \sqrt{3}u_8)2}. \tag{2.31}$$

We know $< N|u_8|N >= 166 \pm 10$ [9], however $< N|u_0|N >$ has not been determined. It is reasonable to assume that the magnitude of breaking of $SU(3) \otimes SU(3)$ is of the same order as

$SU(3)_V$, thus we can work with

$$\frac{< N|u_0|N >}{< N|u_8|N >} = p = 1.5 \pm 0.5. \tag{2.32}$$

We also know $\frac{c_8}{c_0} = -1.25$, thus we find

$$|R'| = 7.2 \pm 1.2. \tag{2.33}$$

This is by no means an exact value, merely a guide to the difference in the level of breaking that occurs between $SU(3)$ and $SU(2)$.

2.3 The Quark Model

Following the discovery of quarks, a new formalism for the treatment of chiral symmetry ensued. The challenge was to find a consistent mathematical treatment of the quark mass term in the QCD Lagrangain, viz

$$\mathcal{L}_I = -[m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s]. \tag{2.34}$$

Equation (2.34) can be written

$$\begin{aligned}\mathcal{L}_I = -[&\frac{m_u + m_d + m_s}{3}(\bar{u}u + \bar{d}d + \bar{s}s) + \frac{m_u - m_d}{2}(\bar{u}u - \bar{d}d) \\ &\frac{1}{3}(\frac{m_u + m_d}{2} - m_s)(\bar{u}u + \bar{d}d - 2\bar{s}s)].\end{aligned} \tag{2.35}$$

If we define $u_i(x) = \bar{q}(x)\lambda_i q(x)$, for $i = 0, 1..8$ with λ_i the Gell-Mann matrices, then equation (2.35) is written

$$\mathcal{L}_I = -[c_0 u_0 + c_3 u_3 + c_8 u_8]. \tag{2.36}$$

Clearly then, the pre-quark treatment was was close, in terms of the symmetry breaking components of the Lagrangian (if quarks **are** the only source of chiral symmetry breaking). The $c_3 u_3$ term is responsible for the breaking of charge symmetry. The $c_8 u_8$ term represents the mass splitting between the light quarks, (i.e. $m_u = m_d \neq m_s$), and the $c_0 u_0$ term is turned on when any light quark mass is not zero. This fits in the historical view that u_0, which transforms as a singlet, breaks the $SU(3) \otimes SU(3)$ group, and u_8 breaks $SU(3)_V$ symmetry.

2.4 The Mass Formula

In the chiral limit, and in the absence of self-energy processes, the octet baryons are degenerate in mass. The mass formulas are derived by treating the mass terms perturbatively, with

$$H_I = \int d^3x (m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s), \tag{2.37}$$

for the nucleon we have the resulting shift

$$\delta(M_N^2) = m_u B_u + m_d B_d + m_s B_s + \text{ higher order terms} \tag{2.38}$$

where $B_q = \langle p|\bar{q}q|p\rangle$. Similarly, we could produce a mass formula for the other octet baryons, with new constants, $B_q^{\Sigma} = \langle\Sigma|\bar{q}q|\Sigma\rangle$ for example. However, from a computational point of view, it would be prudent to express the baryon masses in terms of as few parameters as possible. In the chiral limit the octet baryons are degenerate in mass, hence the value of M_0 will be the same for all baryons. The $SU(3)$ symmetry allows us to relate the matrix elements of $\bar{q}q$ for all octet baryons to those of the nucleon. In the absence of higher order corrections we have

$$\begin{aligned}
M_N^2 &= M_0^2 + \hat{m}(B^u + B^d) + m_s B^s \\
M_\Xi^2 &= M_0^2 + \hat{m}(B^d + B^s) + m_s B^u \\
M_\Lambda^2 &= M_0^2 + \frac{\hat{m}}{3}(B^u + 4B^d + B^s) + \frac{m_s}{3}(2B^u - B^d + 2B^s) \\
M_\Sigma^2 &= M_0^2 + \hat{m}(B^s + B^d) + m_s B^u
\end{aligned} \tag{2.39}$$

Some terms of higher order in the expansions are non-analytic in the quark masses. For example, the next term in the quark mass expansion is $O(\hat{m}^{3/2})$, not $O(\hat{m}^2)$ [14], as determined by improved chiral perturbation theory (ICPT). ICPT also suggests that the the leading non-analytic term (corresponding to the one loop process) dominates the higher order terms. The assumption that we are making is that the term in the quark mass expansion is the same as for the one-loop process in an effective Lagrangian (e.g. using the CBM). Finally there are the kinematic contributions to the mass formula, terms of order $\hat{m}^2$. Ignoring terms of order m_q^2 we have

$$M_N^2 = M_0^2 + \hat{m}(B^u + B^d) + m_s B^s + \delta M_N^2$$

$$M_{\Xi}^2 = M_0^2 + \hat{m}(B^d + B^s) + m_s B^u + \delta M_{\Xi}^2$$

$$M_{\Lambda}^2 = M_0^2 + \frac{\hat{m}}{3}(B^u + 4B^d + B^s) + \frac{m_s}{3}(2B^u - B^d + 2B^s) + \delta M_{\Lambda}^2$$

$$M_{\Sigma}^2 = M_0^2 + \hat{m}(B^s + B^d) + m_s B^u + \delta M_{\Sigma}^2 \tag{2.40}$$

where $\delta M_B^2 = 2M_B \delta M_B$, δM_N is the difference between the loop corrections with massive mesons and the loop corrections in the chiral limit[4].

Unfortunately, the quark masses cannot be extracted from the baryon mass formula. Indeed only 4 pieces of information can be gleaned from these equations. As we shall see later, the treatment of the non-analytic corrections requires further assumptions regarding the properties of B^u and B^d in order to determine M_0.

We are working with the CBM to determine the numerical value of the leading non-analytic contributions to the baryon mass. Our interest lies in the analytic structure of the loop corrections to the baryon mass. The pion loop correction to the nucleon mass, δM_N as a function of $\hat{m}$ is shown on figure (2.1). One contentious issue is the role played by decuplet baryons in the loop corrections. For this reason the octet and decuplet contributions are shown seperately. The expression used to calculate δM_N is found in section (4.5).

[4]In chapter 4, δM_B refers to the loop correction to baryon B for massive mesons. In the remainder of this report, this correction is denoted ΔM_B, and δM_B is as defined in section (2.4).

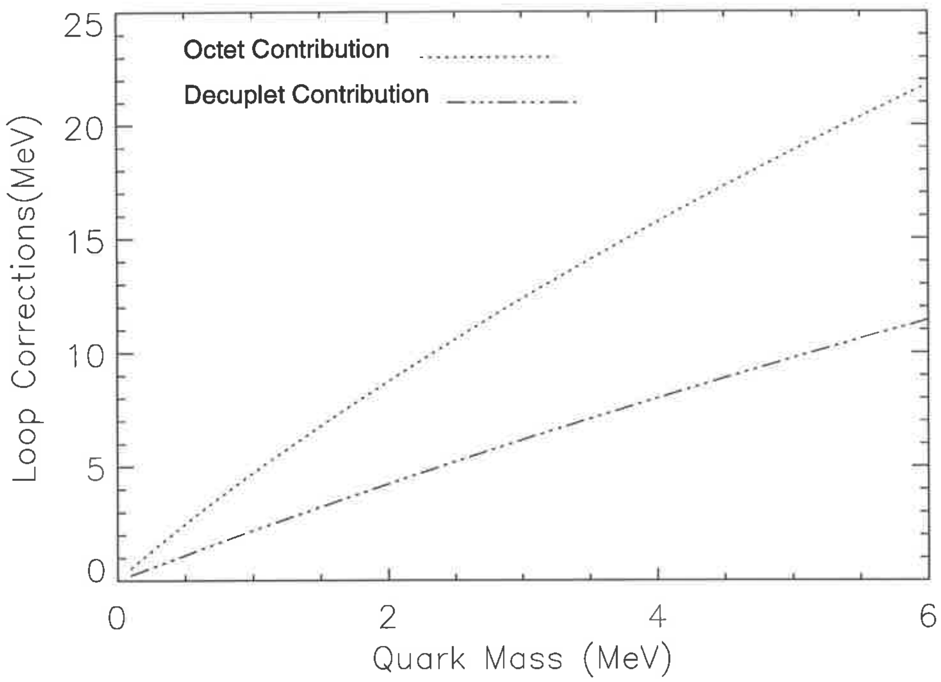


Figure 2.1: The analytic behaviour of the pion loop corrections to the nucleon mass.

2.5 The Quark Masses

The ratios in eq. (2.26) do not provide sufficient information to derive the actual values of the quark masses. Using sum rules, the non-strange quark mass can be determined - for example we cite two values, $\hat{m} = 5.4$ MeV [13] and $\hat{m} = 7 \pm 2$ MeV [14]. If the sigma term is determined correctly, it should provide information regarding the accuracy of our value for $\hat{m}$. We will use

$$\hat{m} = 5.5 \pm 2, \quad \text{and} \quad m_s = 130 \pm 50. \tag{2.41}$$

2.6 Chiral Lagrangians

One of the principal justifications for the use of effective field theories is that it is an easy method of examining low energy physics. The need for doing so will come about either because the underlying model is not known or is inappropriate for the given energy levels. The starting point is the Chiral Lagrangian, that is, the Lagrangian that contains all terms allowed by chiral symmetry. The resulting theory is non-renormalisable, however, as it is a low energy theory, this is not a problem. Or, more to the point, the problems which result from non-renormalisability do not occur at the energy levels considered.

The Standard Model (SM) is thought to contain the correct description of strongly interacting particles. However, thus far, no one has been able to show analytically, that the SM predicts quark confinement (an observed property). Furthermore, a description of many nucleon systems using quark degrees of freedom would be far too complex, analytically at least. Because of the above mentioned problems it is, and has been, a profitable method of examining hadronic interaction problems using effective field theory. The Chiral Lagrangian is the best starting point, because it is likely that, if treated correctly, it incorporates all of the features of the SM. The complete theory incorporates a more complete perturbative treatment, Chiral Perturbation Theory (χPT). χPT involves the expansion in terms of the momenta and the meson masses. What makes χPT such a complete theory is that is incorporates the idea that pions are Goldstone bosons, and that their interactions are momentum dependent. χPT includes the chiral anomaly, PCAC and the chiral Ward identities. The problem with χPT is that the expansion of the Lagrangian possesses constants, which are not controlled by the underlying symmetries of the system. These constants need to be determined from the phenomenology of related systems.

The global symmetry of the QCD Lagrangian is $SU(3)_L \otimes SU(3)_R \otimes U(1)_V \otimes U(1)_A$. The $U(1)_V$ symmetry is simply the conservation of baryon number, and the $U(1)_A$ symmetry is only an exact symmetry at the classical level. As a result, the starting point for the Chiral Lagrangian is the $SU(3)_L \otimes SU(3)_R$ symmetry group, with the $U(1)$ symmetries being treated seperately. A model independent Chiral Lagrangian for the meson field (to lowest order) is given as [15]

$$\mathcal{L} = \frac{f^2}{4} < \partial_\mu U^\dagger \partial^\mu U >, \tag{2.42}$$

where $U(\Phi)$ is a parameterisation of the Goldstone boson fields, viz

$$U(\Phi) = \exp(i\sqrt{2}\Phi/f). \tag{2.43}$$

$\Phi(x)$ is the meson field matrix, and is given as

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \overline{K}^0 & -2\frac{\eta}{\sqrt{6}} \end{pmatrix}. \tag{2.44}$$

We can expand $U(\Phi)$ in terms of Φ to obtain meson-meson interaction terms, and all values can be expressed in terms of f. Next we introduce external fields, by generalising the Lagrangian in the presence of external sources, i.e.

$$\mathcal{L} = \frac{f^2}{4} < D_\mu U^\dagger D^\mu U + U^\dagger \varphi + \varphi^\dagger U >, \tag{2.45}$$

where

$$\varphi = 2B_0(s + ip),$$

$$D_\mu U = \partial_\mu U - ir_\mu U + iUl_\mu,$$

and

$$D_\mu U^\dagger = \partial_\mu U^\dagger + ir_\mu U^\dagger - iU^\dagger l_\mu \tag{2.46}$$

with r, l, s and p external fields. We then obtain the Green functions as functional derivatives of

$$\exp[iZ] = \int \mathcal{D}U(\Phi)\exp i \int d^3x\mathcal{L}. \tag{2.47}$$

At lowest order, it is straightforward to extract predictions from this Lagrangian. Many such results are shown in reference [15]. At $O(p^4)$ the most general Lagrangian has 10 coupling constants that cannot be determined by symmetry. In principle, these constants can be determined from various processes. When calculating loop effects, the resultant chiral logarithms are a function of f, m_π, m_η, and m_K only [15].

It has become clear that higher order processes in χPT give non-trivial contributions to certain processes. The analytic structure of these higher order contributions has not yet been determined. We are not using chiral perturbation theory, so as well as calculating the level of chiral symmetry breaking with the CBM, we will examine the analytic behaviour of our results to gain a better idea of the validity of our phenomenology. There are numerous reviews on this subject and we mention [15, 9, 16, 12, 11] as reasonable starting points for further investigation.

Chapter 3

The Sigma Term

3.1 The Sigma Term

There are two considerations when studying the pion-nucleon sigma term. Firstly there is the challenge of relating a pion-nucleon scattering amplitude to the 'experimental' value for $\sigma_{\pi N}$. The second is to provide an independent estimate of $\sigma_{\pi N}$ using a particular model. The latter consideration is the area that is least agreed upon, and is the focus of our investigations.

The pion-nucleon sigma term, a measure of chiral symmetry breaking in QCD, is defined as follows

$$\sigma_{\pi N}(t) = \frac{1}{3}\sum_{i=1}^{3} < N(p')|[Q_i^5, [Q_i^5, H]]|N(p) >, \tag{3.1}$$

where H is the strong interaction Hamiltonian,

$$H = H_0 + H_{SB} \tag{3.2}$$

with H_{SB} the part of the Hamiltonian that breaks chiral symmetry, and $|N(p)>$ is the nucleon wave function, with normalisation scheme

$$< N(p)|N(p') >= (2\pi)^3 2p^0 \delta^3(p'-p). \tag{3.3}$$

If we take the view that the only source of chiral symmetry breaking is the quark masses, then H_{SB} is

$$H_{SB} = \int d^3x[m_u \overline{u}u + m_d \overline{d}d + m_s \overline{s}s]. \tag{3.4}$$

We could, of course, choose another measure of chiral symmetry breaking to work with, however $\sigma_{\pi N}$ can be associated with an experimental value derived from pion-nucleon scattering. Furthermore, as H_0 commutes with Q_i^5, $\sigma_{\pi N}$ depends only on the symmetry breaking part of the Hamiltonian, which is good as it requires less assumptions than an expression requiring a knowledge of H_0.

For the purpose of evaluating $\sigma_{\pi N}$ we could ignore iso-spin breaking effects and write

$$H_{\mathrm{SB}} = c_0 u_0 + c_8 u_8, \tag{3.5}$$

as was discussed in the previous chapter. Using

$$[Q_i^5, u_j] = -i d_{ijk} v_k, \quad [Q_i^5, v_j] = i d_{ijk} u_k \tag{3.6}$$

we find

$$[Q_i^5,[Q_i^5,H_{\rm SB}]] = c_0[d_{i01}d_{i1l} + d_{i02}d_{i2l} + d_{i03}d_{i3l}]u_l$$
$$+c_8[d_{i81}d_{i1l} + d_{i82}d_{i2l} + d_{i83}d_{i3l}]u_l. \quad (3.7)$$

Summing over all contracted indices for $i = 1, 2, 3$ we have

$$[Q_i^5,[Q_i^5,H_{\rm SB}]] = \sqrt{3}(\sqrt{2}c_0 + c_8)(\bar{u}u + \bar{d}d). \quad (3.8)$$

Given the values for c_8 and c_0,

$$c_0 = \frac{1}{\sqrt{6}}(2\hat{m} + m_s) \quad (3.9)$$

and

$$c_8 = \frac{1}{\sqrt{3}}(\hat{m} - m_s) \quad (3.10)$$

we are left with

$$\sigma_{\pi N} = \hat{m} < N|\bar{u}u + \bar{d}d|N >, \quad (3.11)$$

where $\hat{m} = \frac{(m_u+m_d)}{2}$. There are various schemes to calculate a numerical value for this expression, and we consider them next.

3.2 Calculating $\sigma_{\pi N}$

We know

$$c_0 u_0 + c_8 u_8 \approx \left(\frac{m_u + m_d}{2}\right)(\bar{u}u + \bar{d}d) + m_s \bar{s}s, \tag{3.12}$$

which is the symmetry breaking component (of the Hamiltonian) in QCD. Given our understanding of the nucleon, we can assume that the strange quark content of the nucleon is very small relative to the non-strange quark content, i.e.

$$\hat{m}\left\langle N|\bar{u}u + \bar{d}d|N\right\rangle \approx \hat{m}\left\langle N|\bar{u}u + \bar{d}d - 2\bar{s}s|N\right\rangle, \tag{3.13}$$

thus

$$\begin{aligned} \sigma_{\pi N} &\approx \hat{m} < N|\bar{u}u + \bar{d}d - 2\bar{s}s|N > && (3.14)\\ &= \frac{\sqrt{3}\hat{m}}{c_8} < N|c_8 u_8|N > && (3.15)\\ &= \frac{3\hat{m}}{\hat{m} - m_s} < N|c_8 u_8|N > . && (3.16) \end{aligned}$$

To evaluate (3.16), we write [17]

$$\langle B|c_8 u_8|B\rangle = \alpha Tr(\bar{B} u_8 B) + \beta Tr(\bar{B} B u_8) \tag{3.17}$$

where

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -2\frac{\Lambda}{\sqrt{6}} \end{pmatrix}, \tag{3.18}$$

and noting that $\langle B|c_8 u_8|B\rangle$ is the mass shift from $SU(3)$ breaking for baryon B, we can determine α and β, giving us [18]

$$\sigma_{\pi N}(0) = \left(\frac{3(m_u + m_d)}{m_u + m_d - 2m_s}\right)(M_\Lambda - M_\Xi) \tag{3.19}$$

or

$$\sigma_{\pi N}(0) = 3\left(\frac{M_\Xi - M_\Lambda}{m_s/\hat{m} - 1}\right). \tag{3.20}$$

This gives a value for $\sigma_{\pi N}(0)$ of 26.5 MeV.

As we have neglected the $m_u - m_d$ term, this expression measures the level of symmetry breaking when the up and down quarks acquire the same non-zero mass. In deriving equation (3.20), we have assumed that the strange quark content of the nucleon is very small. Consider the quantity y which is a measure of the strange quark content of the nucleon,

$$y = \frac{< N|\bar{s}s|N >}{< N|\bar{u}u + \bar{d}d - 2\bar{s}s|N >}. \tag{3.21}$$

Instead of writing (3.13) we write

$$\sigma_{\pi N} = \hat{m} < N|\bar{u}u + \bar{d}d - 2\bar{s}s|N > \left(1 + \frac{< N|2\bar{s}s|N >}{< N|\bar{u}u + \bar{d}d - 2\bar{s}s|N >}\right). \tag{3.22}$$

This is an exact equation. If we do not assume y is zero, then we find

$$\sigma_{\pi N}(0) = 3\left(\frac{M_\Xi - M_\Lambda}{m_s/\hat{m} - 1}\right)(1+2y). \tag{3.23}$$

If the value for $\sigma_{\pi N}(0)$ is compared with its experimental value of 45 MeV, then y equals 0.4. This value seems very large, however we have not yet considered the non-analytic contributions to the sigma term. We shall see that when these terms are included, the sigma term can be understood without appealing to a large strange quark content of the nucleon.

3.3 The experimental value of $\sigma_{\pi N}$

We will use a more sophisticated technique for determining $\sigma_{\pi N}$. Before we do this, we shall consider the 'experimental' value with which our theoretical estimate will be compared.

Consider the pion-nucleon scattering process

$$N(p) + \pi_i(q) \rightarrow N(p') + \pi_j(q'). \tag{3.24}$$

The S-matrix for this process is

$$S = I - i(2\pi)^4\delta(p+q-p'-q')(q^2-m_\pi^2)(q'^2-m_\pi^2)$$

$$\int \exp(-iq'\cdot z) < p'|T(\phi_j(z)\phi_i(0))|p > \tag{3.25}$$

$$= I - i(2\pi)^4\delta(p+q-p'-q')T_{ji}(\nu, t; q^2, q'^2), \tag{3.26}$$

with $t = (p - p')^2$ and

$$\nu = \frac{-(p + p') \cdot q}{2M_N}. \tag{3.27}$$

Using PCAC,

$$T_{ji}(\nu, t; q^2, q'^2) = \frac{(q^2 - m_\pi^2)}{f_\pi m_\pi^2} \frac{(q'^2 - m_\pi^2)}{f_\pi m_\pi^2} \int d^4 z \exp(-iq' \cdot z) < p' | T(\partial_\mu A_j^\mu(z) \partial_\nu A_i^\nu(0) | p > .$$

Evaluating (3.28) we have

$$\begin{aligned} T_{ji}(\nu, t; q^2, q'^2) = \frac{(q^2 - m_\pi^2)}{f_\pi m_\pi^2} \frac{(q'^2 - m_\pi^2)}{f_\pi m_\pi^2} \int d^4 z \exp(-iq' \cdot z) \\ < p' | q_\mu q_\nu T[A_j^\mu(z), A_i^0(0)]_+ + iq'_\mu \delta(z_0)[A_j^\mu(z), A_i^0(0)] \\ -\delta(z_0)[A_j^\mu(z), \partial_\mu A_i^0(0)] | p > . \end{aligned} \tag{3.28}$$

Using

$$\partial_\nu A_i^\nu(0) = -i[Q_i^5, H(0)], \tag{3.29}$$

we see that

$$\begin{aligned} T_{ji}(0,0,0,0) &= -\frac{1}{f_\pi^2} < p' | [Q_i^5, [Q_j^5, H(0)]] | p > \\ &= -\frac{\Sigma_{\pi N}(0)}{f_\pi^2}, \end{aligned} \tag{3.30}$$

where $\Sigma_{\pi N}(0)$ is the sigma term.

T_{ji} can be determined from experiment. However, it will not correspond to $T_{ji}(0,0,0,0)$ as this is an off mass-shell amplitude. Various schemes can be used to relate $T_{ji}(0,0,0,0)$ to a physical value and we will mention some of them now.

Cheng and Dashen considered the isospin even amplitude, T^+ to arrive at

$$\begin{aligned} T^+(0,2m_\pi^2,m_\pi^2,m_\pi^2) &= -T(0,0,0,0)+O(m_\pi^4) \\ &= \frac{\Sigma_{\pi N}(2m_\pi^2)}{f_{\pi^2}}, \end{aligned} \tag{3.31}$$

which is an unphysical point. We can extrapolate back to this point from the observed amplitude by using a broad area subtraction relation, with the result

$$T^+(0,2m_\pi^2,m_\pi^2,m_\pi^2) = (1.2\pm 0.1)m_\pi^{-1} \tag{3.32}$$

which gives $\Sigma_{\pi N}(2m_\pi^2) = 77\pm 6$ MeV.

More general schemes have been considered, with a more general ansatz for the amplitude, and more precise extrapolation techniques. The latest value of $\Sigma_{\pi N}(2m_\pi^2)$ is 60 MeV. This value is not in agreement with any credible estimates of $\sigma_{\pi N}(0)$. This is not surprising as there is no reason to assume the t dependence of $\sigma_{\pi N}(t)$ would be weak. If we consider the dispersion relation [19]

$$\sigma(t)-\sigma(0) = \frac{t}{\pi}\int_{4m_\pi^2}^{\infty}\frac{dt'}{t'(t'-t-i\epsilon)}\mathrm{Im}[\sigma(t')]. \tag{3.33}$$

In the region $4m_\pi^2 < t < 16m_\pi^2$,

$$Im[\sigma(t)] = \frac{3}{2}\frac{\Gamma_\pi^*(t) f_\pi^0}{4m_\pi^2 - t}(1 - \frac{4m_\pi^2}{t}), \tag{3.34}$$

where $\Gamma_\pi^*(t) =< \pi^0(p')|\hat{m}(\bar{u}u + \bar{d}d)|\pi(p) >$. The energy dependence of $\Gamma_\pi(t)$ is determined in ref. [20]. If we use the Omnes function

$$\Delta_0(t) = \frac{t}{\pi}\int_{4m_\pi^2}^{t_1} \frac{dt'}{t'}\frac{\delta_0^0(t')}{t' - t - i\epsilon}, \tag{3.35}$$

where $\delta_0^0(t)$ is the IJ $= 00$ $\pi - \pi$ phase shift, then we can find $f_\pi^0(t)$ by considering

$$\tilde{f}(t) = \frac{exp[-\Delta_0(t)] f_+^0}{(4M_N^2 - t)}, \tag{3.36}$$

which can be determined by using a dispersion relation. As t is varied, various processes will become important, such as $\pi\pi \rightarrow N\bar{N}$, clearly giving δ_0^0 a significant energy dependence. When δ_0^0 is determined from $\pi - \pi$ scattering data, we find

$$\Delta_\sigma = \sigma(2M_\pi^2) - \sigma(0) = 15.2 \pm 0.4 \text{ MeV}. \tag{3.37}$$

The uncertainties for Δ_σ arise from the calculation of δ_0^0.

It can be shown using current algebra [21] that

$$f_\pi^2 A^+(0, 2m_\pi^2) = \sigma_{\pi N}(2m_\pi^2) + \text{higher order corrections.} \tag{3.38}$$

Comparing this with the analysis of [22]

$$A^{+}(0, 2m_{\pi}^{2}) = \frac{\Sigma_{\pi N}(2m_{\pi}^{2})}{f_{\pi}^{2}}, \tag{3.39}$$

we can see

$$\Sigma_{\pi N}(2m_{\pi}^{2}) = \sigma_{\pi N}(2m_{\pi}^{2}) + \Delta_{R}. \tag{3.40}$$

Δ_R has been calculated using χPT, and was found to be 0.35 MeV [23]. Given this, the latest value of $\sigma_{\pi N}$ is

$$\sigma_{\pi N}(0) = 45 \pm 12 MeV. \tag{3.41}$$

3.4 Non-analytic contributions to the sigma term

The Feynman-Hellman theorem [14] gives us

$$2M_N \sigma_{\pi N} = \hat{m} \frac{\partial M_N^2}{\partial \hat{m}}. \tag{3.42}$$

We can determine the LNAC to the sigma term by evaluating equation (3.42) with and without the loop corrections included in the mass formula for M_N^2.

The first order mass formula was derived in chapter 2 by considering the lowest order perturbation from

$$H_I = \int d^3x(m_u\overline{u}u + m_d\overline{d}d + m_s\overline{s}s). \tag{3.43}$$

If we apply the Feynman-Hellman theorem to M_N^2, as described by the first order mass formula, and use eq. (2.39) to determine B^u and B^d, we would find

$$2M_N\sigma_{\pi N}(0) = \frac{2\hat{m}}{2\hat{m}+m_s}[M_N^2 + \frac{3}{2}\frac{m_s}{m_s-\hat{m}}(M_\Xi^2 - M_\Lambda^2) - M_0^2]. \tag{3.44}$$

However, there are higher order corrections that appear in the second order mass formula, resulting from loop processes. Using the Feynman-Hellman theorem with M_N^2 described by the second order mass formula, we have

$$2M_N\sigma_{\pi N} = \hat{m}(B^u + B^d) + \hat{m}\frac{\partial}{\partial\hat{m}}\delta M_N^2. \tag{3.45}$$

We note that if we applied the Feynman-Hellman theorem using the first order nucleon mass formula we would have

$$2M_N\sigma_{\pi N}(0) = \hat{m}(B^u + B^d). \tag{3.46}$$

Equations (3.46) and (3.45) are the same formula, however the meaning of the term B^q is slightly different in each case. In the first order mass formula for the nucleon,

$$M_N^2 = M_0^2 + \hat{m}(B_0^u + B_0^d) + m_sB_0^s \tag{3.47}$$

can be used to express the nucleon mass in terms of B_0^u, B_0^d, and B_0^s. When the non-analytic terms are considered in the expansion of the mass formula, we can consider adding the LNAC outright, or consider new values for B^q, viz

$$B_0^q = B^q + \delta B^q, \tag{3.48}$$

hence $\delta M_N^2 = \hat{m}(\delta B^u + \delta B^d)$, with similiar expressions for the other baryons.

Using the second order baryon mass formulae to solve for B^u and B^d we have

$$\begin{aligned} 2M_N\sigma_{\pi N}(0) = & \frac{2\hat{m}}{2\hat{m}+m_s}[M_N^2 + \frac{3}{2}\frac{m_s}{m_s-\hat{m}}(M_\Xi^2 - M_\Lambda^2) - M_0^2] \\ & -\frac{2\hat{m}}{2\hat{m}+m_s}[\delta M_N^2 + \frac{3}{2}\frac{m_s}{m_s-\hat{m}}(\delta M_\Xi^2 - \delta M_\Lambda^2)] \\ & +M_\pi^2[\frac{\partial}{\partial M_\pi^2} + \frac{1}{2}\frac{\partial}{\partial M_K^2} + \frac{1}{3}\frac{\partial}{\partial M_\eta^2}]\delta M_N^2. \end{aligned} \tag{3.49}$$

Comparing (3.44) and (3.49), we can identify the third part of equation (3.49) as the LNAC to the Sigma Term. Chiral perturbation theory tells us

$$M_\pi^2 = B\hat{m} + \text{h.o.t}, \tag{3.50}$$

so

$$\hat{m}\frac{\partial M_N^2}{\partial \hat{m}} = M_\pi^2\frac{\partial M_N^2}{\partial M_\pi^2}. \tag{3.51}$$

However, the light quark also appears in the mass formula for the K and η mesons as well, thus we have

$$\hat{m}\frac{\partial M_N^2}{\partial \hat{m}} = M_\pi^2(\frac{\partial}{\partial M_\pi^2} + \frac{1}{2}\frac{\partial}{\partial M_K^2} + \frac{1}{3}\frac{\partial}{\partial M_\eta^2})M_N^2, \tag{3.52}$$

which is the third component in (3.49).

Historically, equation (3.49) was evaluated with δM_B determined using chiral perturbation theory. The LNAC from χPT actually decreased the value of $\sigma_{\pi N}$. This was because the corrections to the nucleon were too large to treat in such a way. To avoid this problem we use the CBM, which has been very successful in calculating baryonic properties in the past. The value of δM_B refers to the difference between corrections to M_B for the real pion mass, and corrections in the chiral limit. We can see this by considering :

$$\text{in the chiral limit } M_0 = \mathcal{M}_{\text{bare}} + \Delta M_N^0, \tag{3.53}$$

$$\text{with finite quark masses } M_N = \mathcal{M}_{\text{bare}} + \sum_q b_q \bar{q}q + \Delta M_N, \tag{3.54}$$

where ΔM_N and $\Delta^0 M_N$ are the meson loop contributions to the nucleon mass for massive and massless mesons repectively, $b_q = B^q/2M_N$ and $\mathcal{M}_{\text{bare}}$ is the nucleon mass in the absence of quark mass and meson corrections. The difference between equations (3.53) and (3.54) gives the linear version of the nucleon mass formula.

In static approximation for the baryons, the loop corrections to the nucleon mass are given as [24, 7]

$$\delta M_N = \frac{3f_{NN\pi}^2}{\pi m_\pi^2}\int_0^\infty \frac{k^4u^2(k)dk}{\omega_k(M_N-\omega_k-M_N)} + \frac{4f_{\Delta N\pi}^2}{3\pi m_\pi^2}\int_0^\infty \frac{k^4u^2(k)dk}{\omega_k(M_N-\omega_k-M_\Delta)}. \tag{3.55}$$

Therefore, including for the moment only the pionic contribution, the LNAC is found to be [1]

$$\frac{m_\pi^2}{3\pi\mu^2}\int\frac{dk}{\omega_\pi(k)^2}k^4u^2(kR)\left[\frac{9f_{NN\pi}^2}{\omega_\pi(k)^2}+\frac{2f_{N\Delta\pi}^2}{(M_\Delta-M_N+\omega_\pi(k))^2}+\frac{2f_{\Delta N\pi}^2}{\omega_\pi(k)(M_\Delta-M_N+\omega_\pi(k))}\right] \tag{3.56}$$

The first two terms in (3.49) are the valence quark contribution (VQC) to the pion-nucleon sigma term. The only unknown parameter in the VQC is the value of the baryon bare mass, M_0, and that is easily determined.

Expression (3.56) is identical to the corrected version of the LNAC to the sigma term determined in [18]. The corrected version appeared in [25] and was found by evaluating

$$\sigma_{\pi N}^{\pi}(0) = \frac{m_\pi^2}{2}\int d^3x < N|\underline{\phi}^2|N> . \tag{3.57}$$

[1] $f_{N\Delta\pi} = 2f_{\Delta N\pi}$

3.5 The Baryon Bare Mass

If we are to use expression (3.49) to calculate the sigma term, then a procedure needs to be found to determine M_0. In the work of Gasser and Leutwyler [16], two approaches are mentioned as possibilities. One is to consider a range of values of M_0, and the other is to appeal to independent evaluations of y_0, which is a measure of the strangeness content of the proton. Either method will determine $\sigma_{\pi N}^{\mathrm{vq}}$, although we feel neither method is satisfactory.

Using our notation, we have the first term in equation (3.49),

$$2M_N\sigma_{\pi N}^{vq}(0) = \frac{2\hat{m}}{2\hat{m}+m_s}[M_N^2 + \frac{3}{2}\frac{m_s}{m_s-\hat{m}}(M_\Xi^2 - M_\Lambda^2) - M_0^2], \tag{3.58}$$

which can be re-written

$$2M_N\sigma_{\pi N}(0) = \frac{\hat{m}}{m_s-\hat{m}}(D^p - 3F^p)\frac{1}{1-y_0}, \tag{3.59}$$

with

$$y_0 = \frac{2B^s}{B^u+B^d}, \tag{3.60}$$

$$F^p = \frac{1}{2}(\hat{m}-m_s)(B^u - B^s), \tag{3.61}$$

and

$$D^p = \frac{1}{2}(\hat{m}-m_s)(B^u + B^s - 2B^d). \tag{3.62}$$

For various values of y_0, equation (3.59) can be determined.

Gasser and Leutwyler do not commit themselves to the method for determining M_0, or y_0, and write in ref. [16] (on the disrepancy between the theoretical and experimental value of $\sigma_{\pi N}$)[2].

> ... *We do not intend to argue about the possible value of M_0 or y_0. Instead we determined those values of $\sigma_{\pi N}$ which are compatible with the meson and baryon spectrum, leaving open the actual value of M_0 or y_0.* ...

They then go on to conclude that the bare mass must be less than a certain value for the sigma term to be compatable with data. This will be discussed more later.

This method is unsatisfactory, not least because it is not self consistent. More importantly, if one is restricted to determining $\sigma_{\pi N}$ by a fitting procedure with empirical values, then one cannot gain a good measure of the accuracy of one's treatment. The method we will use to determine M_0 (hence the valence quark contribution to $\sigma_{\pi N}$, $\sigma_{\pi N}^{vq}$) involves two assumptions

1. The higher order corrections to the baryon mass formulae are dominated by the loop corrections, as described by the Cloudy Bag Model,
2. The up quark content of the proton is twice that of the down quark content ($B^u = 2B^d$).

This method is far more self consistent, and allows us to examine more critically the role of decuplet baryons in value of the sigma term.

[2] In [16] they use y to denote the term we label y_0.

We can rewrite the mass formulae for the N, Ξ and Λ baryons with $B^u = 2\ B^d$, giving

$$M_N^2 = M_0^2 + \hat{m}3B^d + m_s B^s + 2M_N\delta M_N$$

$$M_\Xi^2 = M_0^2 + \hat{m}(B^d + B^s) + 2m_s B^d + 2M_\Xi\delta M_\Xi$$

$$M_\Lambda^2 = M_0^2 + \frac{\hat{m}}{3}(2B^d + 4B^d + B^s) + \frac{m_s}{3}(2B^d - B^d + 2B^s) + 2M_\Lambda\delta M_\Lambda. \tag{3.63}$$

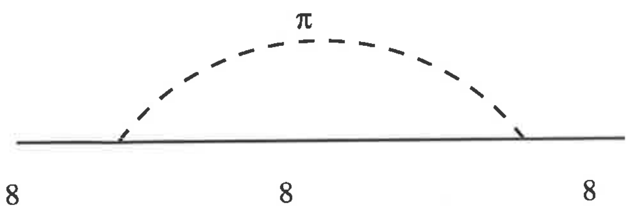
This is now a simple set of linear equations. Solving for M_0, B^s and B^d we have

$$M_0^2 = -\frac{6\delta M_\Xi M_\Xi\hat{m} - 3M_\Xi^2\hat{m} - 18\delta M_\Lambda\hat{m}M_\Lambda + 9\hat{m}M_\Lambda^2 + 10\delta M_N\hat{m}M_N - 5\hat{m}M_N^2}{-\hat{m} + m_s}$$
$$-\frac{6\delta M_\Xi M_\Xi m_s - 3M_\Xi^2 m_s - 12\delta M_\Lambda M_\Lambda m_s + 6M_\Lambda^2 m_s + 8\delta M_N M_N m_s - 4M_N^2 m_s}{-\hat{m} + m_s} \tag{3.64}$$

The motivation for this work, which will be discussed in greater detail later, was that the effect of including certain processes in the calculation of the LNAC was not certain. The two dominant contributions to δM_B, hence the LNAC to $\sigma_{\pi N}$ are shown on fig. (3.1). It has been remarked that within the framework of chiral perturbation theory, contributions to $\sigma_{\pi N}$ from process (b) in fig. (3.1) are absorbed into a redefinition of B^u and B^d and have no effect on the value of $\sigma_{\pi N}$. We compare the values of $\sigma_{\pi N}$ with and without process (b) in the following section.

Our final expression for the valence quark contribution to the sigma term is then

$$\sigma_{\pi N}^{vq} = \frac{3}{2M_N}\frac{\hat{m}}{m_s - \hat{m}}[3M_\Lambda^2 - M_\Xi^2 - 2M_N^2 +$$



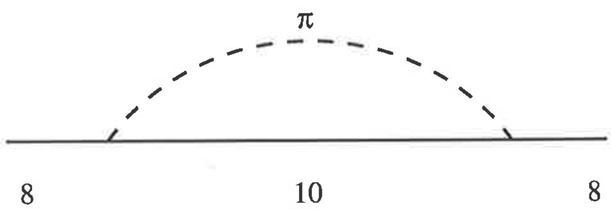


Figure 3.1: Loop processes that contribute to LNAC to $\sigma_{\pi N}$.

$$\delta M_{\Xi}^{2} + 2\delta M_{N}^{2} - 3\delta M_{\Lambda}^{2}]. \tag{3.65}$$

The second half of equation (3.65) should have a small component with non-analytic structure. This is a consequence of our method to determine M_0 and should be quite small, and for the moment warrants no further investigation.

3.6 Results

The details of the loop corrections to the octet baryons are discussed in the next chapter. Using the CBM to determine δM_N, δM_{Ξ}, δM_{Λ}, (the discussion of this is found in section (4.5)), and the CBM to determine the LNAC to the sigma term, we found for the CBM radius [0.6, 1.2] fm the results shown in table (3.1), with

$$\sigma_{\pi N} = \sigma_{\pi N}^{I} + \sigma_{\pi N}^{II}, \tag{3.66}$$

with $\sigma_{\pi N}^{I}$ being given by equation (3.65), and $\sigma_{\pi N}^{II}$ the third component of eq. (3.49). The sigma term will consist of both analytic and non-analytic terms. $\sigma_{\pi N}^{I}$ has been considered analytic in the quark masses, and is known as the valence quark contribution to the sigma term.

Quantity	With Decuplet	Without Decuplet
δM_N	[55.3, 18.5]	[34.5,12.8]
δM_Λ	[29.4,8.6]	[12.7,4.2]
δM_Ξ	[7.4,2.3]	[1.4,0.5]
$\sigma^I_{\pi N}$	[19.5,18.4]	[20.6,18.7]
$\sigma^{II}_{\pi N}$	[47.9,13.7]	[29.1,9.1]
$\sigma_{\pi N}$	[67.4,32.1]	[49.7,27.7]

Table 3.1: Results for pion loop processes only.

3.6.1 Pion Loops

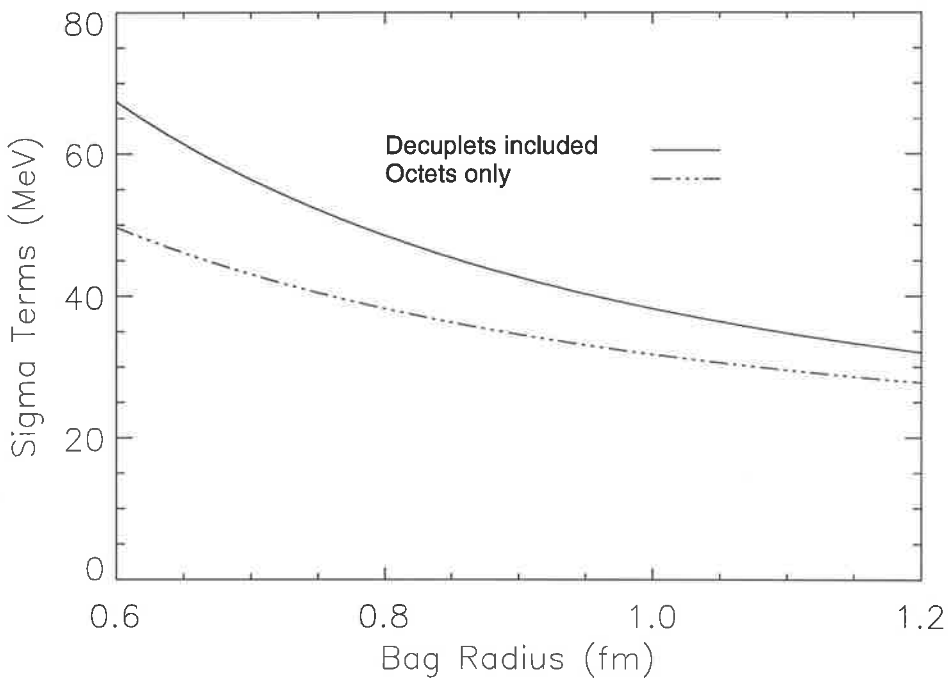


Figure 3.2: A comparison of $\sigma_{\pi N}$ with and without decuplet contributions(pion loops only).

A comparison of $\sigma_{\pi N}$ with and without decuplet baryons is shown on figure (3.2).

The error associated with the value for $\sigma_{\pi N}$ as a result of the uncertainty in the values for $\hat{m}$ and m_s was about 14 MeV. Regardless of the errors, clearly the including the decuplet processes in δM_B and $\sigma^{II}_{\pi N}$ improves the value of $\sigma_{\pi N}$. The sigma term with errors is represented graphically on fig. (3.3).

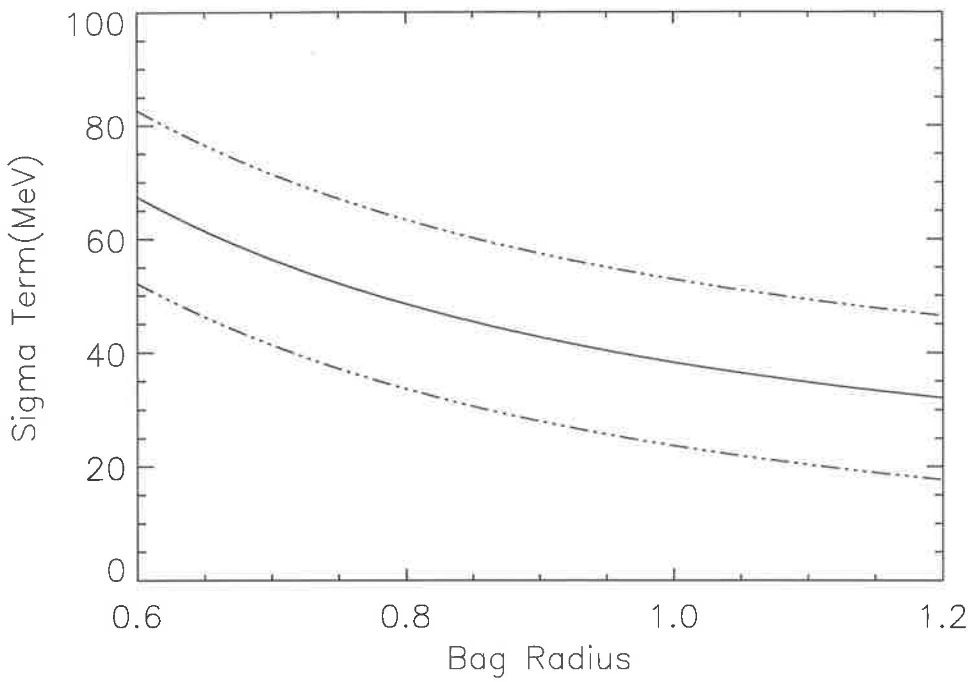


Figure 3.3: Pion-Nucleon Sigma Term with errors.

Quantity	With Decuplet	Without Decuplet
δM_N	[111.9,29.1]	[73.3,20.3]
δM_Λ	[303.3,75.5]	[249.8,64.3]
δM_Ξ	[354.0,87.2]	[251.2,67.9]
$\sigma^I_{\pi N}$	[-30.8,5.5]	[-34.1,4.8]
$\sigma^{II}_{\pi N}$	[49.1,13.8]	[29.9,9.2]
$\sigma_{\pi N}$	[18.3,19.3]	[-4.9,14.0]

Table 3.2: Results for all meson loop processes.

3.6.2 Inclusion of all light mesons

The inclusion of the K, $\overline{K}$ and η mesons decreases the value of $\sigma^I_{\pi N}$ quite significantly. The results are shown on table (3.2).

A comparison of the pion nucleon sigma term with and without decuplet baryons, for processes involving all pseudo-scalar mesons is shown on figure (3.4).

It is apparent that the inclusion of all SU(3) mesons destroys the accuracy of the method used to determine the sigma term. However, it is also clear that the theoretical value for $\sigma_{\pi N}$ is closer to the experimental value when decuplet processes are included in the calculation. This will be discussed later.

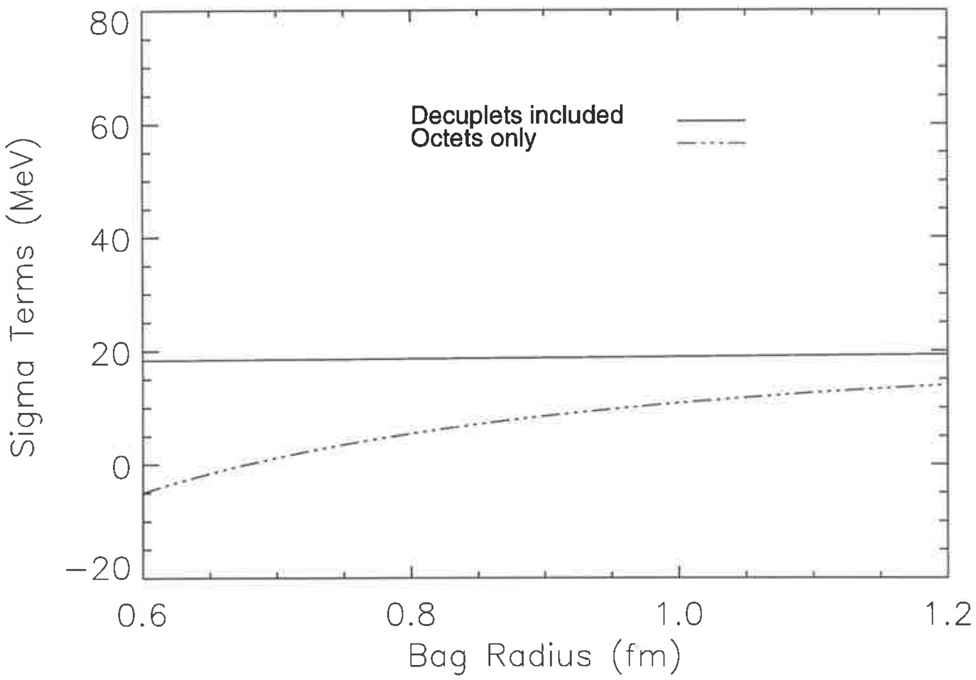


Figure 3.4: Sigma Terms with all meson loop processes included.

3.7 Conclusion

Our analysis of the sigma term has produced the following

- The LNAC to the sigma term increases when processes involving decuplet baryons are included in the calculation.
- When processes only involving pions are considered, the theoretical value is very close to the experimental value, although because of the uncertainties involved, including decuplet processes is not crucial to reconcile theory with experiment.

- The inclusion of processes involving all SU(3) mesons increased the value of $\sigma_{\pi N}^{II}$ slightly. There was a rather substantial decrease in the value of $\sigma_{\pi N}^{I}$, resulting in the total value for the sigma term falling well below that of the experimental value. The inclusion of processes involving decuplet baryons produced a value closer to the experimental value, however this value (18.5 MeV) is far too small to be considered satisfactory. It is likely that when perturbing about $m_s = 0$, truncating the expansion after the δM_B^2 term is not sufficient. This is currently under investigation.

- As we have seen, an evaluation of the $\sigma_{\pi N}$ term within the constraints of the CBM gives a different value with and without the decuplet baryons. This is in disagreement with the work of Gasser and Leutwyler [27]. Their contention is that the inclusion of the decuplet baryons will be absorbed into the redefinition of the constants B_u and B_d, as the contribution will be linear in the quark mass. This will be discussed more later.

Chapter 4

The Cloudy Bag Model

4.1 Introduction

While QCD has had many successes, most notably in the use of QCD sum rules [26], and in lattice calculations, it still yet to be shown that QCD predicts the confinement of colour. In many situations, it is acceptable to use effective theories to model strongly interacting particles. While these theories may not be useful for all situations, they are easier to work with, and readily yield testable results.

4.2 The MIT Bag Model

The first bag model was developed by Boglioubov in 1967. In this model, the quarks are assumed to be massless particles inside the bag, and infinitely massive particles outside it i.e. [7],

$$W(r) = -m, \qquad r \leq R$$

$$W(r) = 0, \qquad r > R \tag{4.1}$$

where $W(r)$ is the scalar potential and R is the bag radius. In the limit $m \to \infty$ we achieve confinement. Solving the Dirac equation for the potential (4.1), and imposing the boundary condition that the quark current should vanish at $r = R$, leads to the quark wave function [7]

$$\psi_{n,-1}(\vec{r}) = \frac{N_{n,-1}}{(4\pi)^{\frac{1}{2}}} \begin{pmatrix} j_0(\frac{\omega r}{R}) \\ i\vec{\sigma}.\hat{\mathbf{r}} j_1(\frac{\omega r}{R}) \end{pmatrix} \chi^{\mu}_{\frac{1}{2}}. \tag{4.2}$$

This wave function can be used to calculate various properties of the model. The problems with this model were numerous, however it was never intended to be a serious attempt to model baryons, rather a starting point for further investigations. The MIT bag model modified Boglioubov's model by introducing more consistent boundary conditions, thus confining the quarks in a Lorentz invariant way. Boglioubov left R as a free parameter, to be determined by a comparison with empirical values. In contrast, the MIT bag model's constraint upon the value of R came about as a result of an additional term in the Lagrangian (or the energy-stress tensor). That is,

$$T^{\mu\nu}_{\text{MIT}} = (T^{\mu\nu}_{\text{Bog.}} + Bg^{\mu\nu})\Theta_V. \tag{4.3}$$

Now, if we demand conservation of energy, $(\partial_\mu T^{\mu\nu}_{\text{MIT}} = 0)$, then it is straight forward to show that B represents a pressure term which stabilises the system, and that the radius is determined as a consequence of the value of B. The value of B is assumed to be the same for all baryons - an untested assumption. However, in light of the successes of more sophisticated bag models based on the MIT version, it is reasonable to assume that this is a good approximation.

The Lagrangian density is given as

$$\mathcal{L}_{\mathrm{MIT}}(r) = \left[\sum_{i=1}^{3} \frac{i}{2}\bar{q}_i \overleftrightarrow{\partial\!\!\!/}\, q_i - m_i\bar{q}_i q_i\right]\Theta_V - B\Theta_V - \frac{1}{2}\sum_{i=1}^{3}\bar{q}_i q_i \Delta_S, \tag{4.4}$$

where

$$\Theta_V = 1 \text{ insidebag}$$
$$\Theta_V = 0 \text{ outside bag} \tag{4.5}$$

and the surface delta function, Δ_S, satisfies

$$\partial^\mu \Theta_V = n^\mu \Delta_S. \tag{4.6}$$

The MIT bag model is mentioned here as an historical prelude to the chiral bag models. A more detailed discussion can be found in references [7, 28–30].

4.3 Chiral Symmetry and the Bag model

A fatal problem with the MIT bag model is that it does not possess chiral symmetry. If we consider the infinitesimal chiral transformations

$$q \rightarrow q - i(\underline{\tau}.\underline{\epsilon}/2)\gamma_5 q \tag{4.7}$$

$$\bar{q} \to \bar{q} - i\bar{q}\gamma_5(\underline{\tau}.\underline{\epsilon}/2) \tag{4.8}$$

we find

$$\mathcal{L}_{\text{MIT}} \to \mathcal{L}_{\text{MIT}} + \frac{1}{2}\bar{q}[\gamma^\mu, \gamma_5]_+\vec{\partial}_\mu \frac{\underline{\tau}.\underline{\epsilon}}{2} q\Theta_V + \frac{i}{2}\bar{q}\underline{\tau}.\underline{\epsilon}\gamma_5 q\delta_S. \tag{4.9}$$

Obviously the MIT bag model is not chirally symmetric, which is a problem given that one of the features of massless QCD is chiral symmetry. For the moment we will restrict ourselves to the $SU(2) \times SU(2)$ group, which has 3 corresponding Goldstone Bosons, the pion iso-multiplet. The obvious starting point for the development of a chiral bag model is the sigma model. The Lagrangian for the sigma model is

$$\mathcal{L}_\sigma = \frac{i}{2}\bar{\psi}\vec{\partial}\psi - g\bar{\psi}(\sigma + i\underline{\tau}\cdot\underline{\pi}\gamma_5)\psi + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\underline{\pi})^2 - U(\sigma,\underline{\pi}) + c\sigma \tag{4.10}$$

or

$$\mathcal{L}_\sigma = \mathcal{L}_0 - U(\sigma,\underline{\pi}) + c\sigma. \tag{4.11}$$

$(\sigma, \underline{\pi})$ are the boson fields, ψ is the nucleon iso-doublet. $U(\sigma,\underline{\pi})$ is a the potential, and is given as

$$U(\sigma,\underline{\pi}) = \frac{\lambda}{4}(\sigma^2 + \underline{\pi}^2) + \frac{\mu^2}{2}(\sigma^2 + \pi^2). \tag{4.12}$$

$\mathcal{L}_0 - U(\sigma,\underline{\pi})$ is invariant under a chiral transformation. A detailed discussion of the sigma model is given in ref. [31]. It describes the symmetry features of hadron physics, but is not good enough to be a serious candidate for a model of low energy physics. Fatally, it predicts the existence of a massive scalar meson, which has not been observed. This problem was corrected in the non-linear sigma model, where the boson fields are defined (making use of $\sigma^2 + \pi^2 = f_\pi^2$)

$$\begin{aligned} \sigma &= f_\pi \cos(\phi/f_\pi) \\ \underline{\pi} &= f_\pi \hat{\phi} \sin(\phi/f_\pi). \end{aligned} \tag{4.13}$$

The term in which we are interested (for the purposes of a chiral bag model), is the nucleon-boson interaction term. Under the above substitution we get

$$g\bar{\psi}(\sigma + i\underline{\tau}\cdot\underline{\pi}\gamma_5)\psi \longrightarrow -g f_\pi \bar{\psi} \exp(i\frac{\underline{\tau}\cdot\underline{\phi}}{f_\pi}\gamma_5)\psi. \tag{4.14}$$

4.4 The Cloudy Bag Model

It was the coupling term given in equation (4.14) that resolved the problem of chiral symmetry breaking in the MIT bag model. After a low order expansion in powers of the pion field, the Lagrangian for the Cloudy Bag Model (CBM) is given as

$$\mathcal{L}_{\mathrm{CBM}} = \mathcal{L}_{\mathrm{MIT}} + \mathcal{L}_\pi + \mathcal{L}_{\mathrm{I}}, \tag{4.15}$$

where

$$\mathcal{L}_{\mathrm{MIT}} = (\frac{i}{2}\bar{q} \overleftrightarrow{\not{D}}\, q - B)\Theta_V - \frac{1}{2}\bar{q}q\Delta_S, \tag{4.16}$$

$$\mathcal{L}_\pi = \frac{1}{2}(\partial_\mu \pi)^2 - \frac{1}{2}m_\pi^2 \pi^2, \quad \text{and} \tag{4.17}$$

$$\mathcal{L}_{\mathrm{I}} = \frac{-i}{2f_\pi}\bar{q}\underline{\tau} \cdot \underline{\pi}\gamma_5 q\Delta_S. \tag{4.18}$$

We can think of the cloudy bag model describing the nucleon as a bag containing 3 confined quarks, surrounded by a pion cloud. The pions are 'allowed' to enter the bag, however the only interaction between quarks and pions is at the bag surface. It is the surface term (4.18) that restores chiral symmetry. In deriving the CBM Lagrangian, we have to assume that the pion field, ϕ/f_π is relatively small, so expanding $\exp(i\frac{\underline{\tau}\cdot\phi}{f_\pi}\gamma_5)$ to first order is a good approximation. Furthermore, we have to assume that the quark wave function is not perturbed to any significant degree by the pion field so the CBM quark wave function is the same as the MIT quark wave function.

The equations of motion are found to be [32]

$$i\,\not{\partial} q = 0 \text{ inside the bag,} \tag{4.19}$$

$$i\gamma_\nu n^\nu q = q, \tag{4.20}$$

$$B = -\frac{1}{2} n \cdot \partial[\bar{q}q] \quad \text{and} \tag{4.21}$$

$$(\partial^2 + m_\pi^2)\underline{\pi}(x) = \frac{i}{2f_\pi} \bar{q}\gamma_5 \underline{\tau} q \Delta_S. \tag{4.22}$$

The first three equations are the equation of motion and boundary conditions for the MIT bag model. Equation (4.22) is the Klein-Gordon equation in the presence of an external source.

This is a general summary of the CBM. A more rigorous and complete discussion can be found in [7, 32–34, 24]. It is not our intention to enlarge on the achievements of the CBM. Our interest lies in using it to model the contributions to baryons masses from loop processes. We will examine this in more detail now.

4.5 Loop Corrections

4.5.1 The Hamiltonian Formulation

The easiest way to examine the CBM treatment of one-loop processes is to use a Hamiltonian formulation. We know

$$H = \int d^3x T^{00}_{\text{CBM}}(x) \tag{4.23}$$

where

$$T^{\mu\nu}_{\mathrm{CBM}}(x) = \frac{\partial \mathcal{L}_{\mathrm{CBM}}}{\partial(\partial_\mu q)}\partial^\nu - g^{\mu\nu}\mathcal{L}_{\mathrm{CBM}}(x). \tag{4.24}$$

We can quantise the pion field

$$\phi_j(\vec{r},t) = \int \frac{d^3k}{(2\omega(k)(2\pi)^3)^{1/2}}\left[a_j(\vec{k})e^{i\vec{k}\cdot\vec{r}} + a_j^\dagger(\vec{k})e^{-i\vec{k}\cdot\vec{r}}\right] \tag{4.25}$$

with $a(\vec{k}), a^\dagger(\vec{k})$ satisfying

$$[a_j(\vec{k}), a_{j'}(\vec{k'})] = [a_j^\dagger(\vec{k}), a_{j'}^\dagger(\vec{k})] = 0 \text{ and} \tag{4.26}$$

$$[a_j(\vec{k}), a_{j'}^\dagger(\vec{k'})] = [a_j(\vec{k}), a_{j'}^\dagger(\vec{k})] = \delta_{jj'}\delta(\vec{k} - \vec{k'}). \tag{4.27}$$

Substituting

$$\psi_{n,-1}(\vec{r}) = \frac{N_{n,-1}}{(4\pi)^{\frac{1}{2}}}\begin{pmatrix} j_0(\frac{\omega r}{R}) \\ i\vec{\sigma}.\hat{\mathbf{r}}j_1(\frac{\omega r}{R}) \end{pmatrix}\chi^\mu_{\frac{1}{2}} \tag{4.28}$$

into (4.24) we get H_{CBM}. $\mathcal{L}_I$ is given as

$$\mathcal{L}_I = \frac{i}{2f_\pi}\bar{q}\underline{\tau}\cdot\underline{\phi}\gamma_5 q\Delta_S \tag{4.29}$$

and we can show

$$\bar{q}\vec{\tau}\gamma_5 q = \chi^\dagger\left(j_0(\frac{\omega r}{R}) \quad - i\vec{\sigma}.\hat{\mathbf{r}}j_1(\frac{\omega r}{R})\right)\gamma_0\gamma_5\underline{\tau}\cdot\underline{\phi}\begin{pmatrix} j_0(\frac{\omega r}{R}) \\ i\vec{\sigma}.\hat{\mathbf{r}}j_1(\frac{\omega r}{R}) \end{pmatrix}\chi. \tag{4.30}$$

Noting that the Bessel functions are real, and that γ_5 and γ_0 in the Pauli-Dirac representation are

$$\gamma_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \text{ and } \gamma_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{4.31}$$

we get

$$H_I = 2j_1(\frac{\omega r}{R})j_0(\frac{\omega r}{R})\vec{\sigma} \cdot \hat{\mathbf{r}}\underline{\tau} \cdot \underline{\phi}\chi^\dagger \chi. \tag{4.32}$$

We now have

$$H = H_{\text{MIT}} + H_\pi + H_I \tag{4.33}$$

where

$$H_{\text{MIT}} = \sum_\alpha \alpha^\dagger M_\alpha \alpha. \tag{4.34}$$

Strictly speaking, H_{MIT} is found to be

$$H_{\text{MIT}} = \int d^3x \left[\bar{q}(-i\gamma \cdot \nabla)q + B + \frac{1}{2}\sum_{a=1}^{8}(E_a^2 - B_a^2) \right] \Theta_V. \tag{4.35}$$

However, as the non-exotic bag states, α, should be eigenstates of H_{MIT} we use expression

(4.34). The pionic hamiltonian is, as expected,

$$H_\pi = \sum_i \int d^3k\omega(k)a_j^\dagger(\vec{k})a_j(\vec{k}) \tag{4.36}$$

and

$$PH_IP = \sum_{\alpha\beta} \frac{d^3k}{(2\pi)^{3/2}}(\nu_{\vec{k}i}^{\beta\alpha}\beta^\dagger\alpha a_{\vec{k}i} + \text{ hermitian conjugate}) \tag{4.37}$$

where (in a spherically symmetric bag)

$$\nu_{\vec{k}i}^{\beta\alpha} = \frac{i}{2f_\pi}\frac{1}{(2\omega(k))^{1/2}}\int d^3x\exp(i\vec{k}\cdot\vec{x})\delta(r-R) < \beta|\bar{q}\tau_i\gamma_5 q|\alpha > . \tag{4.38}$$

P is the projection operator onto non-exotic states,

$$P = \sum_{\text{all non-exotic states}} |\alpha>< \alpha|. \tag{4.39}$$

Given (4.32), and

$$N^2 = \frac{1}{8\pi R^3 j_0^2(\Omega)}\frac{\Omega}{\Omega-1} \tag{4.40}$$

we get

$$\nu_{\vec{k}i}^{\beta\alpha} = \frac{1}{2f_\pi}\frac{1}{(2\omega_k)^{1/2}}\frac{1}{4\pi R^3}\frac{\Omega}{\Omega-1}\int d^3x\delta(x-R) < \beta|\vec{\sigma}\cdot\hat{r}\tau_i|\alpha > . \tag{4.41}$$

Now,

$$\exp(i\vec{k}\cdot\vec{R}) = 4\pi\sum_{l}\sum_{m} i^l j_l(kR) Y^*_{lm}(\theta',\phi') Y_{lm}(\theta,\phi) \tag{4.42}$$

and

$$\vec{\sigma}\cdot\hat{r} = \sqrt{\frac{4\pi}{3}}\sum_{\mu}(-1)^{\mu}\sigma_{-\mu}Y_{1\mu}(\theta',\phi') \tag{4.43}$$

where (θ',ϕ') are the angles for r, R, while (θ,ϕ) are co-ordinates associated with $\hat{k}$. Using equations (4.43,4.42,4.41, 4.40), we have

$$\int d\Omega\vec{\sigma}\cdot\hat{r}a_j(\vec{k})\exp(i\vec{k}\cdot\vec{R}) =$$

$$\int d\Omega 4\pi\sqrt{\frac{4\pi}{3}}\sum_{lm} i^l j_l(kR) Y^*_{lm}(\theta',\phi') Y_{lm}(\theta,\phi)\sum_{\mu}(-1)^{\mu}\sigma_{-\mu}Y_{1\mu}(\theta',\phi'). \tag{4.44}$$

Making use of

$$\int d\Omega Y^*_{lm}(\theta',\phi') Y_{l'm'}(\theta',\phi') = \delta_{l'l}\delta_{m'm}, \tag{4.45}$$

we get

$$\int d\Omega\vec{\sigma}\cdot\hat{r}a_j(\vec{k})exp(i\vec{k}\cdot\vec{R}) = 4\pi j_1(kR)\vec{\sigma}\cdot\hat{k}a(\vec{k}). \tag{4.46}$$

Multiplying equation (4.41) by k/k we have

$$\nu_{\vec{k}i}^{\beta\alpha} = \frac{i}{2f_\pi}\frac{j_1(kR)}{kR}\frac{1}{(2\omega(k))^{1/2}}\frac{\Omega}{\Omega-1} < \beta_{s-f}|\sum_{a=1}^{3}\lambda_{ai}\vec{\sigma}_a\cdot\vec{k}|\alpha > . \tag{4.47}$$

Now, it is straightforward to show that for pionic transitions

$$< N_{s-f}|\sum_{a=1}^{3}\lambda_{ai}\vec{\sigma}_a\cdot\vec{k}|N_{s-f} >= \frac{5}{3} < N|\lambda_i\vec{\sigma}\cdot\vec{k}|N > \tag{4.48}$$

where the $s-f$ subscript denotes the baryon spin-flavour wave function. Thus, by writing

$$u(k) = \frac{3j_1(kR)}{kR} \tag{4.49}$$

and

$$g_A^{\text{bag}} = \frac{5}{9}\frac{\Omega}{\Omega-1}, \tag{4.50}$$

we have

$$\nu_{\vec{k}i}^{NN} = \frac{i}{(2\omega(k))^{1/2}}\frac{g_A}{2f_\pi}u(k)\tau_i\vec{\sigma}\cdot\vec{k}. \tag{4.51}$$

We do this in order to compare the CBM with other baryon-meson theories, such a static interaction

$$\nu_{\vec{k}i} = i\sqrt{4\pi}\frac{1}{(2\omega(k))^{1/2}}\frac{f_{NN\pi}}{m_\pi}v(k)\tau_i\vec{\sigma}\cdot\vec{k}, \tag{4.52}$$

where $v(k)$ is a simple "ad hoc" form factor. This illustrates the beauty of the CBM - the form factor is a consequence of the phenomenology.

4.6 The General Baryon Vertex

We have the general-baryon baryon vertex

$${}^p\nu^{\alpha\beta}_{\vec{k}j}(k) = i\sqrt{4\pi}\frac{f^0}{M_p}\vec{S}^{\alpha\beta}\cdot\vec{k}T^{\alpha\beta}\frac{u(k)}{[2\omega_p(k)(2\pi)^3]^{1/2}}, \tag{4.53}$$

which can be simplified by defining

$$\vec{S} = \sum_{m=-1}^{+1} S^{(1)}_m \hat{s}^*_m, \tag{4.54}$$

$$\vec{\sigma}_a = \sum_{m=-1}^{+1} \sigma^{(1)}_{am}\hat{s}^*_m, \tag{4.55}$$

$$\underline{T} = \sum_{r=-1}^{+1} T^{(p)}_r \hat{t}^{p*}_r, \tag{4.56}$$

and

$$\underline{\lambda}^a = \sum_{r=-1}^{+1} \lambda^{a(p)}_r \hat{t}^{p*}_r, \tag{4.57}$$

with $\hat{t}^*_m$ and $\hat{s}^*_m$ unit vectors in a spherical basis and a the quark label.

The various meson transitions are represented by the various Gell-Mann matrices, viz

$$\lambda^p_r = \mp\frac{1}{\sqrt{2}}(\lambda_1 \pm i\lambda_2), \lambda_3 \text{ for } p = 1, \tag{4.58}$$

$$\lambda^p_r = \mp\frac{1}{\sqrt{2}}(\lambda_4 \pm i\lambda_5), \mp\frac{1}{\sqrt{2}}(\lambda_6 \pm i\lambda_7) \text{ for } p = 1/2, \tag{4.59}$$

and

$$\lambda^p_r = \lambda_8 \text{ for } p = 0. \tag{4.60}$$

The values of $p = 1, 1/2, 0$ correspond to pion, kaon (and anti-kaon) and eta meson transitions respectively. For the kaon transitions, the operators in equations (4.56) and (4.57) will be defined in terms of two unit vectors, $\hat{t}^{\frac{1}{2}*}_{\pm 1}$, and for the eta transitions, these operators will be defined in terms of one unit vector, $\hat{t}^{0*}_0$.

Now, the Wigner-Eckart Theorem states [35] that the matrix elements of an irreducible tensor, $T^{(\mu)}_\nu$ between two states, $|\phi^{\mu_1}_{\nu_1}>$ and $|\phi^{\mu_2}_{\nu_2}>$ is written (in the notation of ref. [35])

$$< \phi^{\mu_2}_{\nu_2}|T^{(\mu)}_\nu|\phi^{\mu_1}_{\nu_1} >= \sum_\gamma \begin{pmatrix} \mu_1 \mu \mu_2 \gamma \\ \nu_1 \nu \nu_2 \end{pmatrix} < \mu_2||T^\mu||\mu_1 >_\gamma, \tag{4.61}$$

where

$$\begin{pmatrix} \mu_1 \mu \mu_2 \gamma \\ \nu_1 \nu \nu_2 \end{pmatrix} \tag{4.62}$$

is the Wigner 3j symbol and the reduced matrix element depends only on the representation involved, i.e. is independent of the azimuthal quantum numbers. The reduced matrix element does not need to be evaluated explicitly. Given

$$\binom{\mu_1 \mu \mu_{2\gamma}}{\nu_1 \nu \nu_2} = (-1)^{\mu_{2\gamma}+\nu_2-2\mu_1}(2\mu_{2\gamma}+1)^{-1/2}C^{\nu_1\nu\nu_2}_{\mu_1\mu\mu_{2\gamma}}, \tag{4.63}$$

for the $\alpha\beta\phi_p$ coupling we can write

$$S^{\alpha\beta}_m =< S_\alpha s_\alpha|S^{(1)}_m|S_\beta s_\beta >=< S_\alpha||S^{(1)}_m||S_\beta > C^{s_\beta m s_\alpha}_{S_\beta 1 S_\alpha}(\frac{1}{2S_\alpha+1})^{1/2} \tag{4.64}$$

and

$$T^{(p)\alpha\beta}_r =< T_\alpha t_\alpha|T^{(p)}_r|T_\beta t_\beta >=< T_\alpha||T^{(p)}_r||T_\beta > C^{t_\beta r t_\alpha}_{T_\beta p T_\alpha}(\frac{1}{2T_\alpha+1})^{1/2} \tag{4.65}$$

If we write the reduced matrix element as $\mathcal{C}^{\alpha\beta 1}$, then we have

$$< S_\alpha s_\alpha T_\alpha t_\alpha|T_n S_m|S_\beta s_\beta T_\beta t_\beta >= \mathcal{C}^{\alpha\beta}C^{s_\beta m s_\alpha}_{S_\beta 1 S_\alpha}C^{t_\beta r t_\alpha}_{T_\beta p T_\alpha}. \tag{4.66}$$

We can now see that

$$\vec{S}^{\alpha\beta}\cdot\vec{k}T^{\alpha\beta}_j = \sum_m\sum_r \mathcal{C}^{\alpha\beta}C^{s_\beta m s_\alpha}_{S_\beta 1 S_\alpha}C^{t_\beta r t_\alpha}_{T_\beta p T_\alpha}(\hat{t}^{p*}_r\cdot e_j)(\hat{s}^*_m\cdot\vec{k}). \tag{4.67}$$

---

[1] $\mathcal{C}^{\alpha\beta}$ incorporates the factor of $(\frac{1}{2S_\alpha+1})^{1/2}$

Comparing baryon-baryon vertex using quark and baryon degrees of freedom, we find

$$\frac{f^0}{M_p} C^{\alpha\beta} C^{s_\beta m s_\alpha}_{S_\beta 1 S_\alpha} C^{t_\beta r t_\alpha}_{T_\beta p T_\alpha} = \frac{g_A}{2f_p} < \beta_{s-f} | \sum_{a=1}^{3} \lambda_r^{a(p)} \vec{\sigma}_a | \alpha_{s-f} > \frac{3}{5}, \tag{4.68}$$

and if we define the $\alpha\beta\phi_p$ coupling constant to be

$$f^{\alpha\beta p} = C^{\alpha\beta} f^0, \tag{4.69}$$

and use the symmetry properties of the spin flavour wave function of the baryon, then

$$f^{\alpha\beta p} = 3 g_p g_A \frac{< \alpha_{s-f} | \sigma^{(1)}_{3m} \lambda_r^{3(p)} | \beta_{s-f} >}{C^{s_\beta m s_\alpha}_{S_\beta 1 S_\alpha} C^{t_\beta r t_\alpha}_{T_\beta p T_\alpha}} \frac{3}{5}, \tag{4.70}$$

where $g_p = \frac{M_p}{2f_p}$. We have calculated the ratio $3f^{\alpha\beta j}/5(g_j g_A)$ using mathematica, and the results appear on tables (4.1) to (4.4). If we compare equation's (4.51) and (4.52), we can see that

$$g_A = \sqrt{4\pi} \frac{2f_\pi}{M_\pi} f_{NN\pi}. \tag{4.71}$$

We can now relate all coupling constants to the value of $f_{NN\pi}$, i.e.

$$f^{\alpha\beta p} = 3\sqrt{4\pi} \frac{g_p}{g_\pi} \frac{< \alpha_{s-f} | \sigma^{(1)}_{3m} \lambda_r^{3(p)} | \beta_{s-f} >}{C^{s_\beta m s_\alpha}_{S_\beta 1 S_\alpha} C^{t_\beta r t_\alpha}_{T_\beta p T_\alpha}} f_{NN\pi} \frac{3}{5} \tag{4.72}$$

The value for $f_{NN\pi}$ has been determined to be $f^2_{NN\pi} = 0.081$ [7].

4.6.1 The Baryon Self Energy

We now have the simplified vertex function

$$ {}^{p}\nu_{\vec{k}j}^{\alpha\beta} = i\frac{f^{\alpha\beta p}}{M_p}\sum_{m,r} C_{T_\beta p T_\alpha}^{t_\beta r t_\alpha} C_{S_\beta 1 S_\alpha}^{s_\beta m s_\alpha} (\hat{s}_m^* \cdot \vec{k})(\hat{t}_r^{p*} \cdot e_j^p)\frac{u(k)}{(2\omega_p(k)(2\pi)^3)^{1/2}} \tag{4.73} $$

Given the vertex function, we can calculate the loop correction to the baryon mass, $\delta M_\alpha(E_\alpha)$, where

$$ \delta M_\alpha(E_\alpha) =< \alpha|H_I\frac{1}{(E_\alpha - H_0)}H_I|\alpha >, \tag{4.74} $$

where H_I is written

$$ H_I = \sum_p\sum_j \int d^3k \left[V_i(\vec{k})a_i(\vec{k}) + \text{ hermitian conjugate}\right]. \tag{4.75} $$

Using the commutation properties of H_0, we find

$$ \delta M_\alpha(E_\alpha) = \sum_p\sum_j \int d^3k < \alpha|V_j(k)\frac{1}{E_\alpha - H_0 - \omega_p(k)}V_j^\dagger(k)|\alpha > . \tag{4.76} $$

Using $\sum_\beta |\beta >< \beta| = I$ and $\nu_j^{\alpha\beta} =< \alpha|V_j|\beta >$, we have

$$ \delta M_\alpha(E_\alpha) = \sum_j\sum_\beta \int d^3k \frac{\nu_j^{\alpha\beta}(\nu_j^{\alpha\beta})^*}{E_\alpha - M_\beta - \omega_p(k)}. \tag{4.77} $$

If we substitute eq. (4.73) into eq. (4.77) we will have

$$ \delta M_\alpha(E_\alpha) = \sum_j\sum_p\sum_\beta \int d^3k \left(\frac{f^{\alpha\beta p}}{m_p}\right)^2 \frac{u^2(k)k^4}{16\pi^3\omega_p(k)(E_\alpha - M_\beta - \omega_p(k))}AB, \tag{4.78} $$

where

$$A = \sum_{m,m'} \sum_{s_\beta} \int d\Omega C^{s_\beta 1 s_\alpha}_{S_\beta m S_\alpha} C^{S_\beta m' S_\alpha}_{s_\beta 1 s_\alpha} (\hat{s}^*_{m'} \cdot \vec{k})(\hat{s}_m \cdot \vec{k}) \tag{4.79}$$

and

$$B = \sum_{r,r'} \sum_{t_\alpha} \sum_{j} C^{t_\beta r t_\alpha}_{T_\beta p T_\alpha} C^{t_\beta r' t_\alpha}_{T_\beta p T_\alpha} (\hat{\underline{t}}^*_{r'} \cdot \underline{e}_j)(\hat{\underline{t}}_r \cdot \underline{e}_j) \tag{4.80}$$

Given that an arbitary function can be expanded into a series of spherical harmonics, i.e. [36]

$$f(\theta, \phi) = \sum_{l=0}^{\infty} a_{lm} \mathcal{Y}_{lm}(\theta, \phi), \tag{4.81}$$

where

$$a_{lm} = \int_0^{2\pi} d\phi \int_0^{\pi} sin\theta \mathcal{Y}^*_{lm} f(\theta, \phi), \tag{4.82}$$

we can write

$$\hat{s}^*_m \cdot \hat{k} = \sqrt{\frac{4\pi}{3}} \sum_{\mu} (-1)^{\mu} s^*_{m-\mu} \mathcal{Y}_{1\mu}(\theta, \phi). \tag{4.83}$$

We now have

$$\frac{4\pi}{3}\int d\Omega \sum_{t_\beta}\sum_{m,m'} C^{S_\beta m S_\alpha}_{S_\beta 1 S_\alpha} C^{s_\beta m' s_\alpha}_{S_\beta 1 S_\alpha} \sum_{\mu,\nu} s_{m-\nu} s^*_{m'-\mu} \mathcal{Y}^*_{lm}(\theta,\phi)\mathcal{Y}_{lm}(\theta,\phi), \tag{4.84}$$

which can be simplfied by using [36]

$$\int d\Omega \mathcal{Y}^*_{l'm'}(\theta,\phi)\mathcal{Y}_{lm}(\theta,\phi) = \delta_{ll'}\delta_{mm'}, \tag{4.85}$$

$$C^{s_\beta m s_\alpha}_{S_\beta 1 S_\alpha} C^{s_\beta m' s_\alpha}_{S_\beta 1 S_\alpha} = [C^{s_\beta m s_\alpha}_{S_\beta 1 S_\alpha}]^2 \delta_{mm'}, \tag{4.86}$$

and

$$\sum_{s_\beta, m} C^{s_\beta m s_\alpha}_{S_\beta 1 S_\alpha} C^{S_\beta m' S_\alpha}_{S_\beta 1 S_\alpha} = \delta_{mm'}, \tag{4.87}$$

which gives $A = \frac{4\pi}{3}$. Now, if we consider

$$B = \sum j, t_\beta \sum_{r,r'} C^{t_\beta r t_\alpha}_{T_\beta r T_\alpha} C^{t_\beta r' t_\alpha}_{T_\beta p T_\alpha} (\hat{t}^*_r \cdot e_j)(\hat{t}_{r'} \cdot e_j), \tag{4.88}$$

we can use (4.86), (4.87) and

$$\sum_{j,r} \hat{t}^*_r \cdot e_j \hat{t}_r \cdot e_j = 1, \tag{4.89}$$

to give B = 1.

ratio	N	Σ	Λ	Ξ	Δ	Ξ^*	Σ^*	Ω
N	5	0	0	0	$4\sqrt{2}$	0	0	0
Σ	0	$4\sqrt{6}/3$	-2	0	0	0	$-4\sqrt{3}/3$	0
Λ	0	$2\sqrt{3}$	0	0	0	0	$2\sqrt{6}$	0
Ξ	0	0	0	-1	0	$-2\sqrt{2}$	0	0

Table 4.1: Unrenomalised π α β coupling constants.

We now have a simplified form for the baryon self energy, at $E_\alpha = M_\alpha$,

$$\delta M_\alpha = \sum_p \sum_\beta (\frac{f^{\alpha\beta p}}{M_p})^2 \frac{1}{3\pi} \int_0^\infty \frac{dk k^4 u(k)^2}{\omega_p(k)(M_\alpha - M_\beta - \omega_p(k))}, \tag{4.90}$$

where $f^{\alpha\beta p}$ is the $\alpha\beta p$ coupling constant as defined in eq.(4.70). For the nucleon self energy corresponding to the process shown in fig. (3.1) we would have

$$\delta M_N = \frac{3f_{NN\pi}^2}{\pi m_\pi^2} \int_0^\infty \frac{k^4 u^2(k) dk}{\omega_k(M_N - \omega_k - M_N)} + \frac{32}{25} \frac{3f_{NN\pi}^2}{\pi m_\pi^2} \int_0^\infty \frac{k^4 u^2(k) dk}{\omega_k(M_N - \omega_k - M_\Delta)} \tag{4.91}$$

which is in agreement with the expression for δM_N given in ref. [7].

Equation (4.70) allows us to relate the couplings for all SU(3) mesons to the nucleon-nucleon-pion coupling constant. The ratio given in eq. (4.70) for the 4 mesonic transitions is given on tables (4.1), (4.2), (4.3) and (4.4). Given this, we can find the self energy for all baryon-pseudoscalar loops. The mass correction corresponding to the proccess shown in figure (4.1) is found to be

$$\delta M_N^\eta = \frac{1}{3\pi} (\frac{f_{NN\pi}}{m_\pi})^2 \frac{9}{25} (\frac{f_\pi}{f_\eta})^2 \int_0^\infty \frac{k^4 u^2(k) dk}{\omega_\eta(k)(M_N - \omega_\eta(k) - M_N)}, \tag{4.92}$$

with $\omega_\eta(k) = \sqrt{k^2 + m_\eta^2}$. The contribution, for this case, is clearly negligible when compared with the pionic corrections.

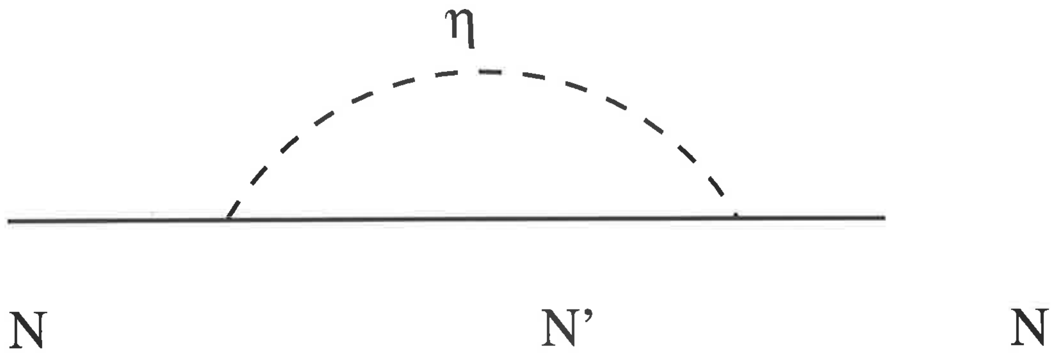


Figure 4.1: $NN\eta$ self energy process.

ratio	N	Σ	Λ	Ξ	Δ	Σ^*	Ξ^*	Ω
N	1	0	0	0	0	0	0	0
Σ	0	2	0	0	0	-2 $\sqrt{2}$	0	0
Λ	0	0	-2	0	0	0	0	0
Ξ	0	0	0	-3	0	0	-2$\sqrt{2}$	0

Table 4.2: Unrenomalised η α β coupling constants.

ratio	N	Σ	Λ	Ξ	Δ	Ξ^*	Σ^*	Ω
N	0	1	-3	0	0	0	-2$\sqrt{2}$	0
Σ	0	0	0	-5 $\sqrt{6}/3$	0	$4/\sqrt{3}$	0	0
Λ	0	0	0	-$\sqrt{2}$	0	-4	0	0
Ξ	0	0	0	0	0	0	0	4

Table 4.3: Unrenomalised K α β coupling constants.

ratio	N	Σ	Λ	Ξ	Δ	Σ^*	Ξ^*	Ω
N	0	0	0	0	0	0	0	0
Σ	-$\sqrt{2/3}$	0	0	0	$8/\sqrt{3}$	0	0	0
Λ	3$\sqrt{2}$	0	0	0	0	0	0	0
Ξ	0	-5	1	0	0	-2$\sqrt{2}$	0	0

Table 4.4: Unrenomalised $\overline{K}$ α β coupling constants.

Chapter 5

Checks of calculation

5.1 Overview

There are a number of known quantities that can be used as a check of our results. The Fortran program that we used to calculate our theoretical values involved coupling constants that were determined by a Mathematica program. The expression for the self energy of the nucleon as a result of pion loops has been derived many times, however the treatment of processes involving $SU(3)$ transitions is slightly more complex. To check this we reformulated the anti-kaon loop processes into SU(2) (V-spin) transitions, to gain a comparison.

5.2 Coupling Constants

In section (4.6) we derived an expression for the general baryon-baryon-meson coupling. We compared the coupling constants derived with those derived from an OBE potential for baryon-

α	β	p	$f^{\alpha\beta p}/M_p(\times 10^{-3})$	$g_{\alpha\beta}/(M_\alpha + M_\beta)(\times 10^{-3})$
N	N	π	2.12	1.97
Λ	Σ	π	1.47	1.46
Σ	Σ	π	1.39	1.40
N	N	η	0.80	0.96
Σ	Σ	η	1.68	1.87
Λ	N	$\overline{K}$	4.33	1.94
Σ	N	$\overline{K}$	0.84	0.57

Table 5.1: Comparison of results derived using the SU(3) quark model and those derived from hyperon-hyperon scattering in the OBE model.

baryon interactions. The interaction is characterised by the Hamiltonian density

$$\mathcal{H}_I^{\alpha\beta p} = iC\bar{\psi}\gamma_\mu\gamma_5\psi\partial^\mu\phi, \tag{5.1}$$

where ψ and ϕ are the baryon and meson fields and C is the coupling constant, which can be written $f^{\alpha\beta p}/M_p$ or $g_{\alpha\beta}/(M_\alpha + M_\beta)$. $f^{\alpha\beta p}$ can be determined from equation (4.70) and $g_{\alpha\beta}$ is reproduced from [37]. A comparison is shown on table (5.1)[1].

5.3 Kaon Transitions

Using the method outlined in chapter 4, we would have

$$\delta M_N^K = \frac{3}{\pi}(\frac{f_{NN\pi}}{m_\pi})^2\frac{1}{25}\int\frac{dkk^4u^2(k)}{\omega_K(M_N - M_\Sigma - \omega_K)} + \frac{3}{\pi}(\frac{f_{NN\pi}}{m_\pi})^2\frac{9}{25}\int\frac{dkk^4u^2(k)}{\omega_K(M_N - M_\Lambda - \omega_K)}. \tag{5.2}$$

The procedure used to evaluate δM_B using SU(2) is well documented. The treatment of loop corrections with SU(3), as is done in this report, is not. Of course, the loop corrections will be the same, but it is not unreasonable to compare.

[1] It is likely that the OBE coupling constants, $g_{\alpha\beta}$, are constrained by SU(3) relations. The discrepancy between the two columns is a consequence of using real baryon masses in column 3 (which would have different values in a theoretical "SU(3)-symmetric world").

Just as the octet baryons can be considered a super multiplet consisting of 2 isospin doublets ((n,p) and (Ξ^-, Ξ^0)) an isospin triplet $(\Sigma^-, \Sigma^0, \Sigma^+)$ and the isosinglet (Λ), it can also be thought of as a super multiplet of V-spin multiplets. The V-spin multiplets are as follows

$$
\begin{aligned}
(\Sigma^-, n), \quad (\Xi^0, \Sigma^+) \quad & \mathrm{V-spin}\ \ 1/2 \\
(\Xi^-, \Sigma^0, p) \quad & \mathrm{V-spin}\ \ 1 \\
\Lambda \quad & \mathrm{V-spin}\ \ 0.
\end{aligned}
\tag{5.3}
$$

Using the V-spin SU(2) subgroup we can then calculate the relative coupling constants of the baryon-baryon-kaon vertex, providing a useful check for the method used in chapter 3.

The operators for the SU(2) subgroup can be written in terms of the Gell-Mann matrices,

$$
\begin{aligned}
V_{\pm} &= \mp(\lambda_4 \pm i\lambda_5) \\
V_3 &= -(\frac{1}{4}\lambda_3 + \frac{3}{8\sqrt{2}}\lambda_8).
\end{aligned}
\tag{5.4}
$$

More simply, we can see that

$$
\begin{aligned}
V^+|s> &\rightarrow -|u>, \\
V^-|u> &\rightarrow |s>, \\
V_3|u> = \frac{1}{2}|u> \text{ and } V_3|s> &= -\frac{1}{2}|s> .
\end{aligned}
\tag{5.5}
$$

Using the same methods we used for the pion transitions, we can relate all baryon-baryon-kaon coupling to a particular coupling constant. The advantage of using SU(2) subgroups to do this is that the working is simplified somewhat.

It is not our intention to evaluate every kaon loop correction. We will look at the coupling for the $N \to \Lambda K \to N$ and $N \to \Sigma K \to N$ processes.

We know

$$^{\pi}\nu_{ki}^{NN} = \frac{i}{2f_\pi}\frac{\Omega}{\Omega - 1}\frac{j_1(kR)}{kR} < N_{s-f}|\sum_{a=1}^{3}\tau_{ai}\vec{\sigma}_a \cdot \vec{k}|N_{s-f} > . \tag{5.6}$$

Similarly, we will have

$$^{K}\nu_{ki}^{N\Sigma} = \frac{i}{2f_K}\frac{\Omega}{\Omega - 1}\frac{j_1(kR)}{kR} < N_{s-f}|\sum_{a=1}^{3}\tau_{ai}^{K}\vec{\sigma}_a \cdot \vec{k}|\Sigma_{s-f} >, \tag{5.7}$$

where τ_{ai}^{K} is the $SU(2)_{\text{V-spin}}$ Pauli matrix. Using the spin-flavour wave functions for N and Σ, equation (5.7) is found to be

$$^{K}\nu_{ki}^{N\Sigma} = i\frac{1}{5}\frac{g_A}{2f_K}\frac{u(k)}{(2\omega_K(k)(2\pi)^3)^{1/2}}\tau_{ai}^{K}\vec{\sigma}_a \cdot \vec{k}. \tag{5.8}$$

Using

$$\sum_i < N|\tau_{ai}^2|\Sigma >= 3, \tag{5.9}$$

we find the contribution to the nucleon mass from the ΣK loop process is

$$\delta M_N^{K\Sigma} = \frac{3}{\pi}\frac{1}{25}(\frac{f_{NN\pi}}{m_\pi})^2(\frac{f_\pi}{f_K})^2 \int \frac{dkk^4u^2(k)}{\omega_K(k)(M_N - M_\Sigma - \omega_K(k))}, \tag{5.10}$$

which is in agreement with our term derived in chapter 4.

We could use the above method to find

$${}^K\nu_{ki}^{N\Lambda} = i\frac{3\sqrt{3}}{5}\frac{g_A}{2f_K}\frac{u(k)}{(2\omega_K(k)(2\pi)^3)^{1/2}}\tau_{ai}^K\vec{\sigma}_a\cdot\vec{k}. \tag{5.11}$$

Using

$$\sum_i < N|\tau_i^2|\Lambda >= 1, \tag{5.12}$$

we would have the same expression as is given in equation (5.2).

Chapter 6

Discussion

6.1 Introduction

There are several areas that require discussion.

- The original motivation for this work was the role by the decuplet baryons in determining the value of the sigma term. We are comparing our value, determined by using the CBM, with a value determined using chiral perturbation theory. It is unlikely that this is the source of discrepancy between the two sets of results, however the relative complexities of the two methods suggests that the conclusion is not a closed case.

- The consequences of including all meson loop processes in the calculation highlights the problems with Gasser and Leutwyler's expression for the sigma term (eq. (3.49)). If we are to produce a credible estimate for the valence quark contribution to the sigma term then we must develop a new expression. If one is to use the octet baryon mass formula to estimate B^u and B^d, then one must consider perturbing about $m_s = 0$. This presents serious problems.

6.2 The role of the decuplet baryons

The challenge of Chiral Perturbation Theory is to provide a model of QCD, which yields physical predictions by treating QCD as an expansion in the masses of the 3 light quarks. The formalism for a given perturbation theory depends on the choice of expansion parameter, and as a result there are various perturbation schemes. The role of the decuplet baryons in the treatment of hadronic physics is not yet clear. Certainly there is a contribution, but it is not yet certain as to whether it is significant. The role of the decuplet baryons in the various forms of χPT is far from resolved.

In this section we review previous work involving the role of decuplet baryons in chiral perturbation theory. We conclude that there are reasonable grounds to assert that the processes involving decuplet baryons **do** contribute significantly to the pion-nucleon sigma term. Furthermore, this contribution should be observed when χPT is used, and should not be a relic of the phenomenology.

6.2.1 The Work of Gasser and Leutwyler

The original motivation for this work was comments made by Gasser, Leutwyler and Sainio [19]. Their contention was that the Δ-resonance does not alter the value of the sigma term at order q^2 in χPT. If we define

$$\sigma_{\pi N} = \Sigma_d + \Delta$$

$$\text{where } \Delta = \Delta_D - \Delta_\sigma - \Delta_R$$

$$\text{and } \Sigma_{\pi N} = \Sigma_d + \Delta_D. \tag{6.1}$$

$\Sigma_{\pi N}$ is the experimental value of the pion-nucleon sigma term, with

$$\Sigma_d = f_\pi^2(d_{00}^+ + 2\mu^2 d_{01}^+) \tag{6.2}$$

determined by considering the amplitude

$$\overline{D}^+(t) = d_{00}^+ + t d_{00}^+. \tag{6.3}$$

In [19] it was written

> *... the constants d_{00}^+ and d_{01}^+ must account for all analytic terms of order q^2. In particular they include the contributions of order q^2 generated by the singularity ... What the chiral representation for $\overline{D}^+(t)$ at first non-leading order does not account for is only what remains of the Δ-term after the piece of order q^2 is removed ...*

In [38] it is argued that the first observable effects of the intermediate decuplet states appears at order q^4, and that a closer look at higher order contributions is required.

While the comments in [19] may have been correct for order q^2 (indeed, it is shown that decuplet contributions **are** linear in $\hat{m}$ for order q^2 in [38]), it has become clear that treating χPT to this order is not sufficient.

Generally, the whole question of whether it is productive to perturb about $SU(3)_V$ symmetry remains unanswered.

6.2.2 Further Studies of the Decuplet Contributions

When considering the octet baryon mass formula, and the role played by intermediate decuplet states in the loop corrections [38], there are many higher order terms that give non-trivial contributions to the baryon mass. It is generally accepted that for a comprehensive analysis of the role of decuplet baryons in χPT, a calculation of higher order terms in the expansion is needed.

There is no doubt the decuplets play a crucial role in a complete treatment of χPT. For example, in [39], it was found that if a heavy particle effective Lagrangian is used, the LNAC to the baryon axial form factors is of the order of 100%. That is, the $SU(3)$ breaking corrections to the axial charge are approximately 100 % of the total value. This is much larger than expected, and it is found that the corrections are largely cancelled by including contributions from the decuplet baryons [40].

The role of decuplets in calculating corrections to the Gell-Mann - Okubo (GMO) mass formula, the Coleman - Glashow relation (CG) and the Σ equal spacing rule was investigated in reference [41]. It was found that including a decuplet term in the Chiral Lagrangian with a non-zero mass difference (between octet states) did improve the CG relation. The inclusion was insignificant in the calculation of the Σ equal spacing rule (as are all loop corrections), and did not provide information regarding the GMO mass formula as any results would have been model dependent.

In the entire body of this report, we have used the Cloudy Bag Model to evaluate the effect of loop processes on the value of the sigma term. The CBM is a highly successful, albeit simple, model of baryons at low energies. Gasser and Leutwyler use Chiral Perturbation Theory in all of their work [19,27,14]. Chiral perturbation theory is superior in that when all processes involved in the chiral expansion are considered, it should model Nature exactly (it should be noted that χPT does not yield readily to analysis for anything but the simplest of perturbation parameters). However, it is unlikely that the difference in models used is a satisfactory explanation for the difference between our results and those of Gasser and Leutwyler.

In an explicit calculation, Rawlinson et al [25] find that when the CBM is used to determine the loop corrections to the sigma term, 47 per cent of the total contribution by processes involving decuplet baryons is from the non-analytic component. Given that the sigma term, as described by the CBM, exhibits the same analytic structure as expected from χPT, we can be confident that the difference between our work and the work of Gasser and Leutwyler is not the result of using different techniques.

6.3 Meson Loop Corrections

As we saw in the last chapter, the inclusion of the K, $\overline{K}$ and η mesons loops destroyed the accuracy of the expression used to determine $\sigma_{\pi N}$. If one is to use equation (3.49), then all pseduo-scalar meson loops must be considered in the evaluation of δM_B, where δM_B is the difference between the value of loop corrections for massive and massless mesons. As the masses of the K, $\overline{K}$ and η are quite large, (≈ 500 MeV), the size of the loop corrections are quite large, therefore equation (3.49) of little use to evaluate $\sigma_{\pi N}$. The problem lies in the use of the mass formula to express B^q in terms of the octet baryon masses. As was mentioned in chapter 1, the mass formulae are constructed by using the fact that B^q for $q = u, d, s$ obey SU(3) relations. For this to be the case, the interaction Hamiltonian must contain $m_s \bar{s} s$.

The size of the strange quark mass makes it unproductive to use the point $m_s = 0$ in a perturbation scheme. This does not present an insurmountable problem, as what we are interested in is evaluating the mass shift to the nucleon when the up and down quarks are given mass.

One of the problems with using equation (3.49) is that the value of M_0 is not directly obtainable from empirical data (without further assumptions). After an extensive examination, Gasser and Leutwyler state that for the sigma term to be compatible with data, we must have $M_0 < 600$ MeV, or $y \geq 0.3$. This might be correct, however it is hardly a satisfactory conclusion. It should be noted that at the time of writing [16], it was generally believed that the experimental value of the sigma term was 60 MeV. Furthermore, Gasser and Leutwyler were working with a LNAC determined to be roughly 10 MeV. If Gasser and Leutwyler were working with an experimental value for the sigma term of 45 MeV, then they would demand $M_0 < 1200$ MeV.

We have determined M_0 to illustrate the problem with using eq. (3.49) to evaluate $\sigma_{\pi N}$. When all relevant processes are considered, we find $M_0 > 1200$ MeV and is therefore not compatible with the experimental value for $\sigma_{\pi N}$ of 45 MeV. The problem lies not with our treatment of the loop corrections, rather the reliance on the mass formula to determine $\hat{m}(B^u + B^d)$. The values for M_0 when various loop processes are considered are shown on table (6.1).

Processes	Octets only	Decuplets Included
Pions Only	[1057.5,1085.2]	[1066.3,1087.6]
All Mesons	[1475.7,1196.0]	[1330.7,1164.6]

Table 6.1: Value of the baryon bare mass for various loop processes.

A more straightforward demonstration of the inadequacy of Gasser and Leutwyler's formula is to simply consider the value of M_0. For $B^s = 0$ we have

$$M_N^2 = M_0^2 + 2M_N\sigma_{\pi N}^{vq} + 2M_N\delta M_N. \tag{6.4}$$

Obviously M_0 should be less than M_N. The fact that it is not is the result of M_0 being determined by a series of expressions which are incomplete. The only way in which the baryon mass formula, produced by considering $H_I = -(\hat{m}(\bar{u}u + \bar{d}d) + m_s\bar{s}s)$, can be of use is if the expansion includes terms higher than those corresponding to one loop processes.

The fact that M_0 is greater than M_N is a relic of trying to determine M_0 from an incomplete set of equations. Solving the octet baryon mass formulae gives $m_sB^s/(M_N^2) \approx -1/2$ when only pionic processes are considered and a slightly larger negative value when all pseudoscalar meson loop contributions are included.

Using equation (6.4), we can see that M_0 should be about 890 MeV.

6.4 Remark : The GMO Mass Relation

If we truncate the expansion of the baryon mass after one loop then,

$$\begin{aligned}\frac{3}{4}M_\Lambda^2 + \frac{1}{4}M_\Sigma^2 - \frac{1}{2}(M_N^2 + M_\Xi^2) &= \Delta GMO2 \\ &= \frac{3}{4}\delta M_\Lambda^2 + \frac{1}{4}\delta M_\Sigma^2 - \frac{1}{2}(\delta M_N^2 + \delta M_\Xi^2),\end{aligned} \tag{6.5}$$

or, using the linear mass formula,

$$\begin{aligned}\frac{3}{4}M_\Lambda + \frac{1}{4}M_\Sigma - \frac{1}{2}(M_N + M_\Xi) &= \Delta GMO \\ &= \frac{3}{4}\delta M_\Lambda + \frac{1}{4}\delta M_\Sigma - \frac{1}{2}(\delta M_N + \delta M_\Xi).\end{aligned} \tag{6.6}$$

This is the Gell-Mann Okubo mass relation. At first glance it appears that it may be a decent test of our phenomenology. However, the GMO does not depend on the loop corrections to the 4 octet baryons in the chiral limit, that is

$$\frac{3}{4}\Delta M_\Lambda^0 + \frac{1}{4}\Delta M_\Sigma^0 - \frac{1}{2}(\Delta M_N^0 + \Delta M_\Xi^0) = 0, \tag{6.7}$$

where ΔM_B^0 is the correction to the baryon bare mass from massless meson loops. As the contribution from kaon and eta mesons to the total loop correction for massive mesons (ΔM_B) is quite small compared with the contribution from just pion loops, the GMO mass relation will hold regardless of the size of the contribution from kaon and eta meson loops. Put more simply, ΔGMO will be small, regardless of the value of δM_B, where

$$\delta M_B = \Delta M_B - \Delta^0 M_B. \tag{6.8}$$

The values of ΔGMO for various loop processes are shown on table (6.2).

This section is included as a comment, and is not directly relevant to the study of the sigma term. It is worth noting for interests sake, as well as explaining why the inclusion of kaon

Processes	ΔGMO (MeV)
octets only, pions only	-5.6
decuplets inc. pions only	-3.4
octets only, all mesons inc.	12.7
decuplets inc. all mesons	-3.6
experimental value	7.4

Table 6.2: ΔGMO for various loop processes.

and eta mesons destroys the accuracy of expression (3.49), but not the accuracy of the GMO mass relation. Eq. (3.49) depends on the value of loop processes in the chiral limit.

6.5 A new expression for $\sigma_{\pi N}^{vq}$

Ideally, we should look for a method to evaluate the valence quark contribution to the sigma term that does not involve using the strange quark mass as a perturbation parameter. When the strange quark mass is used as such, it is relatively straightforward to express $\hat{m}(B^u + B^d)$ in terms of the Ξ, N and Λ baryon masses. In the absence of strange quark masses, a similar technique (as was used previously) would not be feasible as the bare mass, $\hat{M}_0$, that is, the mass of the nucleon in the limit $\hat{m} = 0$ ($m_s \neq 0$), would not be the same for the Ξ and Λ baryons. However, the Δ and N baryons have the same quark content, thus one might expect

$$M_N = \hat{M}_0 + \sigma_{\pi N} + \delta M_N + \alpha \Delta^h M \tag{6.9}$$

and

$$M_\Delta \approx \hat{M}_0 + \beta \sigma_{\pi N} + \delta M_\Delta + \gamma \Delta^h M, \tag{6.10}$$

where $(\alpha\Delta^h M - \gamma\Delta^h M)$ is the mass difference between the N and Δ resulting from the respective spin-spin interactions. Of course, it is no longer a simple matter of using $H_I = \hat{m}(\bar{u}u + \bar{d}d)$. Using the quark model, one can paramaterise the octet-decuplet baryon mass splittings. One such method is covered in great detail by Murpurgo [42, 43]. The problem with using such a parameterisation is that in order to obtain enough data to fit the unknown constants to, one must parameterise the Ξ and Λ baryon masses, which was just what we were trying to avoid. For example, if we modify Murpurgo's parameterisation to include non-strange quarks, we have

$$\begin{aligned} M_B = M_0^{\text{Mur.}} + B\sum_i P_i^s + C\sum_{i>k}(\sigma_i\cdot\sigma_k) + D\sum_{i>k}(\sigma_i\cdot\sigma_k)(P_i^s + P_k^s) + E\sum_{i\neq j\neq k,\ i>k}(\sigma_i\cdot\sigma_k)P_j^s + \\ F\sum_i(P_i^u + P_i^d) + G\sum_{i>k}(\sigma_i\cdot\sigma_k)(P_i^u + P_i^d + P_k^u + P_k^d) + H\sum_{i\neq j\neq k,\ i>k}(\sigma_i\cdot\sigma_k)(P_j^u + P_j^d) \end{aligned} \tag{6.11}$$

where P_i^q is the projection operator for quark number i $(q = u, d, s)$. $M_0^{\text{Mur.}}$ and $B, C...H$ are unknowns and need to be fitted to the baryon mass spectrum. It is reasonable to put $B/F = D/G = E/H = m_s/\hat{m}$, which gives us 5 unknowns. If we consider the N, Ξ, Λ, Δ and Ξ^* masses, then $\sigma_{\pi N}^{vq} = 24.5$ MeV. Still, this is really no more satisfactory than the method used in the previous chapter, and is only mentioned to illustrate that even if one is to consider the decuplet baryon mass formulae, it is still not possible to extract $\sigma_{\pi N}^{vq}$ from the baryon mass spectrum in a model independent way.

Clearly the expressions for M_N and M_Δ, given in (6.9) and (6.10), do not provide us with enough information to evaluate $\sigma_{\pi N}^{vq}$. The term $\Delta^h M$ is the spin-spin splitting between the quarks in the baryon, and can be calculated for the various baryons using the MIT Bag model [7]. A discussion of the hyperfine splitting is found in ref. [29]. There may also be similar processes that need to be considered, all of which could ultimately provide enough data to determine $\sigma_{\pi N}^{vq}$ from equations (6.9) and (6.10). However, if one is going to use the MIT bag model to calculate the effect of turning on the light quark masses, it would be much simpler to evaluate $\hat{m} < N|\bar{u}u + \bar{d}d|N >$, where $|N>$ is the nucleon MIT bag model state. Jameson et al have done this and find [45]

$$\sigma_{\pi N}^{vq} = 17.5 \pm 9 \text{ MeV}. \tag{6.12}$$

When this value is used to determine $\sigma_{\pi N}^{vq}$ $(= \sigma_{\pi N}^{I})$, we find that at $R = 0.84$, $\sigma_{\pi N} = 44.5 \pm 14$ MeV (when only pion loop processes are considered to determine $\sigma_{\pi N}^{II}$). When decuplet processes are omitted, $\sigma_{\pi N} = 34.6 \pm 13$ MeV.

The inclusion of the $K, \overline{K}$ and η meson loop processes increases $\sigma_{\pi N}$ by 0.6 MeV, giving a remarkable agreement between theory and experiment.

This method is not entirely satisfactory because of the large size of the error. However, as we saw earlier in this section, it is simply not possible to extract $\sigma_{\pi N}^{vq}$ from data when the baryon mass formulae are used to one loop or less.

6.6 Conclusion

In this chapter we have discussed our calculation of the sigma term, with particular emphasis on a comparison with the work of Gasser and Leutwyler. We have found that their technique to determine $\sigma_{\pi N}^{vq}$ is unworkable, when treated completely. That is, when the loop terms in the mass formulae include $K, \overline{K}$ and η processes. We have also observed that their treatment of $\sigma_{\pi N}^{II}$

was incomplete, and made references to other work that reinforced this notion. Finally, when we consider a method to determine $\sigma_{\pi N}^{vq}$ that does not involve the strange quark, and evaluate $\sigma_{\pi N}^{II}$ with decuplet processes included, we find $\sigma_{\pi N} = 45.1$ MeV, in excellent agreement with experiment.

One final note, in the body of this report, there have been a number of papers of which, while they have not been directly cited, were extremely useful. These publications, ref. numbers [46–61] are listed merely as a guide to the interested reader.

Chapter 7

Conclusion

In this thesis we have examined previous attempts to calculate the pion nucleon sigma term. We found that the attention given to the processes involving decuplet baryons and all SU(3) mesons has not been sufficient in the past. As a result we examined a method to calculate $\sigma_{\pi N}$ that did not involve strange mesons and found that the inclusion of processes involving the decuplet baryon gave a significant improvement to the theoretical value of $\sigma_{\pi N}$.

Appendix A

Fortran Program

This is the original version of the Fortran program used to evaluate the sigma term. Modifications were made to evaluate the $\hat{m}$ dependence. When calculating the sigma term for certain processes, the relevant couplings were removed.

```
C      CONSTANTS
C      mb(i) - Mass of baryon i.  mm(i) - Mass of meson i
C      fm(i) - Decay constant of meson i,
C      rad - RADIUS OF BAG mpi - mass of pion
C      fff(l,j,o) - coupling constant between baryon l and j, meson o
C      N Number of points sampled for each integration.
C      ALL MASSES ARE GIVEN IN INVERSE FERMI'S AND ANSWER IS GIVEN
C      IN MeV.

       PROGRAM final
```

```
C      Define constants

       implicit double precision (a-h,o-z)

       double precision ff(4,8,8) kc,h2

       double precision wt,i,mb(8),tot8,g7,mbare

       double precision mpi,rad,upp,factor(4),l,e1

       double precision k,ep(4),h,fm(4),o,e,g4,sum4,sum5

       double precision sum1,mm(4),c,d,sum2,sum3,g2,g3,g4,zz

       double precision tot1,tot2,tot3,tot4(4),tot5(4),tot6,tot7

       double precision mhat,mstr,sigma,tot9(4),tot0,mz,dz

       double precision p4,mn,ml,mc,dn,dl,dc,sig1,sig2

       double precision sam2,sam3,sam4,tat1,tat0,tat2,tat3,tat4(4)

       double precision tat5(4),tat6,tat7,tat9(4),h1,bs1,bd1

       real n,pp


C      Assign Values

       n = 1000

       mpi = 134.9739/197.3

       mm(1) = 134.9739/197.3

       mm(2) = 548.8/197.3

       mm(3) = 493.646/197.3

       mm(4) = 493.646/197.3

       fm(1) = 93/197.3

       fm(2) = 117/197.3

       fm(3) = 125/197.3
```

```
      fm(4) = 125/197.3

      factor(1) = 1

      factor(2) = 0.333333

      factor(3) = 0.5

      factor(4) = 0.5


      mhat = 5.5/197.3

      mstr = 130/197.3


      mb(1) = 938.4/197.3

      mb(2) = 1314.9/197.3

      mb(3) = 1115.6/197.3

      mb(4) = 1189.4/197.3

      mb(5) = 1232.0/197.3

      mb(6) = 1530/197.3

      mb(7) = 1385/197.3

      mb(8) = 1672.4/197.3


C     Coupling constants.  Certain processes removed by removing

C     relavent coupling constant, hence giving contribution equal to zero.


      ff(1,1,1) = 0.081*3

      ff(1,1,5) = (72*0.081/25)*4/3

c      ff(2,1,1) = (1*0.081*3/25)*(fm(1)/fm(2))**2
```

```
c       ff(3,1,4) = (1*0.081*3/25)*(fm(1)/fm(3))**2

c       ff(3,1,3) = (9*0.081*3/25)*(fm(1)/fm(3))**2

c       ff(3,1,7) = (8*0.081*3/25)*(fm(1)/fm(3))**2

      ff(1,4,4) = 48*0.081*3/(3*25)

      ff(1,4,3) = 4*0.081*3/25

      ff(1,4,7) = 16*0.081*3/(3*25)

c       ff(2,4,4) = (4*0.081*3/25)*(fm(1)/fm(2))**2

c       ff(2,4,7) = (8*0.081*3/25)*(fm(1)/fm(2))**2

      ff(1,3,4) = 12*0.081*3/25

      ff(1,3,7) = 24*0.081*3/25

c       ff(2,3,3) = (4*0.081*3/25)*(fm(1)/fm(2))**2

c       ff(3,3,2) = (2*0.081*3/3)*(fm(1)/fm(3))**2

c       ff(3,3,6) = (16*0.081*3/25)*(fm(1)/fm(3))**2

c       ff(4,3,1) = (18*0.081*3/25)*(fm(1)/fm(3))**2

      ff(1,2,2) = 1*0.081*3/25

      ff(1,2,6) = 8*0.081*3/25

c       ff(2,2,2) = (9*0.081*3/25)*(fm(1)/fm(2))**2

c       ff(2,2,6) = (8*0.081*3/25)*(fm(1)/fm(2))**2

c       ff(3,2,8) = (16*0.081*3/25)*(fm(1)/fm(3))**2

c       ff(4,2,4) = 1*0.081*3*(fm(1)/fm(3))**2

c       ff(4,2,3) = (1*0.081*3/25)*(fm(1)/fm(3))**2

c       ff(4,2,6) = (8*0.081*3/25)*(fm(1)/fm(3))**2


      pi = 3.141592653858
```

```
C      Open data files for output, modify names as required for
C      considering certain processes.

       open(1, file='din1.dat')
       open(2, file='din2.dat')
       open(4, file='din4.dat')
       open(6, file='din6.dat')
       open(7, file='din7.dat')
       open(8, file='din8.dat')
       open(9, file='din9.dat')

C      Start loop for a range of values of rad.

       do 35 zz = 1,31
       rad = 0.6 + (zz-1)*0.02
       sigma = 0
       p4 = 0
C      Start loop for each baryon considered.

       do 20 l = 1,4
       tot5(l) = 0
       tot4(l) = 0
       tat5(l) = 0
```

```
C       Start loop for each meson considered.

        do 17 o = 1,4
        tot0 = 0
        tot1 = 0
        tot2 = 0
        tot3 = 0
        tot7 = 0
        tot9(o) = 0
        tat6 = 0
        tat7 = 0
        tat8 = 0
        tat9(o) = 0

C       Start loop for each transition baryon
        do 15 j = 1,8
        sum2 = 0
        sum3 = 0
        sum4 = 0
        sam4 = 0
c       Begin Integration.
        do 10 i = 1,n-1
C       Define Gaussian Quadrature
```

```
      pp = -1 + 2*(i)/n
      if ((mod(i,2)).eq.0) then
   wt = 2
       else
wt = 4
      endif

      if ((i.eq.(n/2)).or.(i.eq.n-1)) then
wt = 1
      endif
C     For the massless pion processes, the principle value will need to
C     taken to allow the integral to be evaluated numerically.
C     The point upp represents the half way point in the integration
C     range.  If the principle value integration is not required, the
C     midpoint is chosen to be 10 inverse fermi.

      kc = mb(l)  - mb(j)
      if (mb(l).gt.mb(j)) then
        upp = kc
      else
       upp = 10
      endif
```

```
C     Notes on integration : g2,g3,g4 represent the two LNAC's to the
C     sigma term and the loop corrections to the baryon mass for massive
C     mesons respectively.

      k = (upp*(1+pp)/(1-pp))
      h = 9*((((sin(k*rad))**2)/(k*k*rad**6))-(2*(cos(k*rad)*
     & sin(k*rad))/(k*rad**5)) + (((cos(k*rad))**2)/(rad**4)))
      ep(o) = sqrt(k*k + mm(o)*mm(o))
      h1 = 9*((((sin(k*rad))**2)/(k*k*rad**6))-(2*(cos(k*rad)*
     & sin(k*rad))/(k*rad**5)) + (((cos(k*rad))**2)/(rad**4)))
      c = 2*(ep(o)*ep(o)*((mb(l) - mb(j) - ep(o))**2))
      d = 2*(ep(o)**3)*(mb(l) - mb(j) - ep(o))
      e = ep(o)*(mb(l) - mb(j) -ep(o))

      g2 = (2*upp/((1-pp)**2))*h/c
      g3 = (2*upp/((1-pp)**2))*h/d
      g4 = (2*upp/((1-pp)**2))*h/e

C     When the loop corrections are evaluated with massless mesons,
C     some processes require the integral to be evaluated using the
C     principle value method, as follows.

      kc = mb(l) - mb(j)
       if (mb(l).gt.mb(j)) then
```

```
C     The above determines whether PVI is needed or not.  If yes then -

         upp = kc
         h1 = 9*((((sin(k*rad))**2)/(k*k*rad**6))-(2*(cos(k*rad)*
     &   sin(k*rad))/(k*rad**5)) + (((cos(k*rad))**2)/(rad**4)))
         h2 = 9*((((sin(kc*rad))**2)/(kc*kc*rad**6))-(2*(cos(kc*rad)*
     &   sin(kc*rad))/(kc*rad**5)) + (((cos(kc*rad))**2)/(rad**4)))
         e1 = k*(mb(l) - mb(j) -k)
      if(k.eq.kc) then
         g7 = 0
      else
         g7 = (2*upp/((1-pp)**2))*(h1/e1 + 2*h2/(k**2 - kc**2))
      endif

C     Else, if the PVI is not needed, the integral is numerically evaluated
C     as normal.

      else
         h1 = 9*((((sin(k*rad))**2)/(k*k*rad**6))-(2*(cos(k*rad)*
     &   sin(k*rad))/(k*rad**5)) + (((cos(k*rad))**2)/(rad**4)))
         e1 = k*(mb(l) - mb(j) -k)
         g7 = (2*upp/((1-pp)**2))*h1/e1
      endif
```

```
      sum2 = sum2 + wt*(g2)*2/n

      sum3 = sum3 + wt*(g3)*2/n

      sum4 = sum4 + wt*(g4)*2/n


      sam4 = sam4 + wt*g7*2/n


10    continue

c     ------------------------------------------------------

      tot2 = sum2/3

      tot3 = sum3/3

      tot0 = ff(o,l,j)*(tot2 - tot3)/(mm(1)*mm(1)*pi)

      tot1 = (tot0)*197.3


C     The following line sums over all meson processes for the LNAC

C     to sigma term, the factors are included as required by expression

C     in report.

      tot4(l) = tot4(l) + factor(o)*tot1


C     The following evaluates the loop corrections with massless and massive

      mesons, summing over all meson loops for given baryons.


      tot7 = (sum4/3)*ff(o,l,j)/(pi*mm(1)*mm(1))

      tot9(o) = tot9(o) + (tot7)
```

```
      tat7 = (sam4/3)*ff(o,l,j)/(pi*mm(1)*mm(1))

      tat9(o) = tat9(o) + (tat7)


15    continue

      tot5(l) = tot5(l) + tot9(o)

      tat5(l) = tat5(l) + tat9(o)

      sigma = mm(1)*mm(1)*tot4(1)

17    continue


20    continue

C     mn, ml, mc and mz refer to the masses of the nucleon, lambda and

C     sigma baryons, the "d" preceding each term refers to the total

C     contribution from loop processes.


      mn = mb(1)

      ml = mb(3)

      mc = mb(2)

      mz = mb(4)

      dn = tot5(1) - tat5(1)

      dl = tot5(3) - tat5(3)

      dc = tot5(2) - tat5(2)

      dz = tat5(4) - tat5(4)


      bd1 = (2*dc*mc - mc**2 - 6*dl*ml + 3*ml**2 +
```

```
      & 4*dn*mn - 2*mn**2)/(-mhat + mstr)

C     sig1 one is equal to the valence quark contribuion to the sigma term.

      sig1 = 3*mhat*bd1/(2*mn)

C     output to files.
      write(1,*) rad, sigma
      write(2,*) rad, 197.3*(sig1)
      write(4,*) rad, sigma + 197.3*(sig1)
      write(6,*) rad, dn*197.3
      write(7,*) rad, dl*197.3
      write(8,*) rad, dc*197.3
      write(9,*) rad, 197.3*(0.75*dl + 0.25*dz - 0.5*(dn+dc))

35    continue
      close(1, status = 'keep')
      close(2, status = 'keep')
      close(4, status = 'keep')
      close(6, status = 'keep')
      close(7, status = 'keep')
      close(8, status = 'keep')
      close(9, status = 'keep')
```

```
stop

end
```

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Addendum

This section is included on the advice from one of the moderators for this thesis. The moderator recommended that the thesis be accepted without further modification, but felt the attention payed to the Goldberger Treiman relation (Sec. 2.1.2) was not sufficient.

The usual form of the Goldberger-Treiman relation is

$$g_{\pi NN} f_\pi = g_A m_N, \tag{A.1}$$

where $g_{\pi NN}$ is the pion-nucleon coupling constant, f_π is the pion decay constant, g_A is the axial charge and m_N is the nucleon mass. This corresponds to the use of pseudo-scalar coupling with coupling constant $g_{\pi NN}$. Alternatively we may use (as in Sec. 2.1.2), pseudo vector coupling, with

$$H_I = i\frac{f_{\pi NN}}{m_\pi}\gamma^\mu \gamma_5 \underline{\tau}\psi \cdot \partial_\mu \underline{\phi}, \tag{A.2}$$

where the coupling constant must have dimensions $E^{(-1)}$ and usually written in terms of the physics pion mass. One could use any other quantity with the same dimensions and the m_π used in this expression does not approach zero in the chiral limit.