

# ACCEPTED VERSION

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1       **Three-dimensional modeling of geocell-reinforced straight and curved**  
2                                       **ballast embankments**

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8  
9       **ABSTRACT**

10       This paper outlines a three-dimensional modeling study conducted on straight  
11       and curved geocell-reinforced embankments. The study uses the discrete  
12       element method to represent varying angularities of ballast infill and models  
13       their mechanical response under monotonic and cyclic loading conditions. The  
14       simulation results show good agreement with test results and the case studies  
15       indicate that the geocell enhances embankment stiffness under monotonic  
16       loading and improves its resilience when subjected to cyclic loading. The  
17       geocell more evenly distributes stresses within the ballast embankments. The  
18       reinforced ballast embankments also exhibit less vertical displacement and  
19       lateral spreading than the unreinforced ballast embankments do.

20       **Keywords:** discrete element, railway embankment, ballast, geocell, cyclic.

21

## 22 1. INTRODUCTION

23 As time progresses, trains travel faster, railways become longer and convey  
24 heavier goods, and more stringent safety standards mandate a higher level of  
25 below-rail alignment for longer design periods. However, the main below-rail  
26 ballast layer, which is referred to in the present study as the ballast  
27 embankment, eventually becomes misaligned due to ballast breakage and  
28 rearrangement [1-4]. As a result, the embankment is prone to subsidence and  
29 lateral spreading, which undermines the safety of the tracks. The damage to the  
30 embankment is more pronounced on sharp track curves where the train creates  
31 large centrifugal forces, which can result in significant settlement in the track  
32 embankment, which exacerbates rail misalignment. Poor track geometry results  
33 in significant expenditure due to ballast inspection, maintenance and sometimes  
34 reconstruction. For example, in the year ending 30 June 2016, the Australian  
35 Rail Track Corporation (ARTC) – one of Australia’s largest rail network owners  
36 – expended more than \$AUD188 million on railway infrastructures maintenance  
37 work, accounting for 22.3% of their total revenue in the same year [5]. To  
38 minimize this expenditure, studies [6-9] have successfully applied geosynthetics  
39 to reinforce embankments. Of the suite of available geosynthetics, geocells  
40 provide a promising means to reinforce railway embankments [7, 8].

41

42 The geocell, as shown in Fig. 1, is a cellular confinement system developed to  
43 reinforce granular infills. The system is supplied in a folded form and, when in  
44 use, outstretched into a honeycomb-like, three-dimensional (3D) panel. The  
45 stretched panel provides a space to accommodate and confine the infill

46 materials and facilitates the joining of individual cell panels into an integrated  
47 mattress. When fully outstretched, the panel usually measures a couple of  
48 meters in width and up to 20 meters in length, with an individual cell space of  
49 around 250 mm square, in width, and between 75 to 200 mm deep. The panel  
50 size and the cell space can be varied as part of the manufacturing process to  
51 suit individual requirements. The cell wall, which is around 5 mm thick,  
52 commonly consists of high-density polyethylene (HDPE) or other polymer  
53 material, and is perforated to allow water drainage, facilitate root growth  
54 between cells and provide interlocking with the infill.

55

56 Geocell panels have been widely used in a variety of infrastructures, such as  
57 foundations and subbases [10-16], slopes [17], retaining structures [18] and  
58 embankments [19, 20]. All of these studies have shown that using geocells  
59 improves performance of the infrastructures by reinforcing the granular infill  
60 materials. More recently, Leshchinsky and Ling [7, 8] conducted a prototype test  
61 and a finite element (FE) analysis on a geocell-reinforced railway embankment.  
62 Their studies confirmed the superiority of the geocell in reinforcing the  
63 embankment. Similar approaches were attempted in other studies [21-23]. In  
64 parallel with the FE method, Liu *et al.* [24] employed the discrete element  
65 method (DEM) to examine the performance of straight, geocell-reinforced  
66 embankments. As a further step, this study extends the DEM approach to  
67 curved embankments. Additional work includes the advanced contact model  
68 used to simulate the geocell and the examination on geocell embedment depth.

69

70 The DEM possesses the capability to represent, with appropriate engineering  
71 accuracy, distinct ballast particles and to simulate particle motion [25]. The  
72 method does not rely on a constitutive model for continuum media; rather, it  
73 incorporates a contact model developed between the individual particles. The  
74 method is also able to replicate variable angularities of the ballast, and similarly  
75 reflects variable material micro-properties, such as stiffness and friction [6, 26,  
76 27]. More importantly, it enables 3D modeling. This is particularly important for  
77 the accurate simulation of the 3D geocell panel, as 2D modeling neglects, or at  
78 least simplifies, the interaction between cells and so underestimates the  
79 performance of the geocell panel. However, an additional calibration stage is  
80 required in order to yield simulated behavior substantially similar to that  
81 observed in reality. Further, it is not possible to simulate a full-scale structure as  
82 replacing a continuum with particle assemblies is computationally intensive.  
83 Thus, the simulation of a full-scale railway structure in DEM is beyond current  
84 computational capacity and the scope of this study.

85

86 This study adopts the commercially-available DEM program, Particle Flow Code  
87 in 3 Dimensions (PFC3D) version 4.0 [28], to simulate a geocell-reinforced  
88 embankment. The railway embankment examples included in the paper are  
89 established in accordance with the relevant codes of practice, which are  
90 discussed later. The paper aims to establish a DEM-based framework for  
91 modeling railway ballast and to evaluate the performance of incorporating  
92 geocells in ballasted embankments. Chen *et al.* [6] adopted DEM to simulate  
93 geogrid-reinforced railway ballast and they successfully demonstrated the

94 capability of using DEM in modeling geosynthetics-reinforced ballast. The  
95 methodologies used in [6], such as material generation, have inspired the  
96 framework proposed in the current study. Improvements have also been made  
97 in the geometric complexity of ballast model as well as in the behaviors and  
98 contact models of geocell and ballast in DEM.

99

100

## 101 **2. MODEL DEVELOPMENT**

102 This section outlines the development of the ballast-geocell model in PFC3D  
103 and provides details of the particle contact and the calibration of the geocell and  
104 ballast assemblage.

105

### 106 **2.1 Particle contact**

107 DEM simulation is governed by the physical contact between particles. The  
108 contacts are present as a combination, in series and/or parallel, of the following  
109 basic physical elements: a bond, slider, spring and dashpot. When applying an  
110 external force to an assemblage of particles, the contacts between them  
111 determine how individual particles will respond and where they will travel at  
112 each time step in the simulation. PFC3D incorporates the contact mechanism  
113 and allows the user to encode a material-oriented, contact, constitutive model.  
114 Once validated, the model is implemented to reproduce the mechanical  
115 response of the material used in any desirable field application. The model  
116 usually defines a set of material micro-properties, such as particle stiffness,

117 bond strength and friction coefficient, which are determined through material  
118 calibration tests.

119

## 120 **2.2 Material calibration**

121 In this section the procedures for calibrating the input parameters for the geocell  
122 and railway ballast in PFC3D are discussed.

123

### 124 **2.2.1 Geocell**

125 The geocell material was calibrated by conducting a tensile strength test. The  
126 test setup is shown in Fig. 2. A geocell strip was cut from a full panel and  
127 cropped into a standard specimen shape for tensile strength testing, in  
128 accordance with AS 1145.3 [29]. The specimen was tested using an Instron  
129 mechanical device [Fig. 2(a)] and three replicates, as the one illustrated in Fig.  
130 2(a), were tested to obtain representative results. The stress–displacement  
131 relationship of the averaged results was then compared with the DEM  
132 simulation. The DEM simulation involved discretizing the specimen strip into 32  
133 equal-sized spheres – an object in PFC3D for modeling materials [26]. The 32  
134 spheres are arranged in two columns, forming a strip [Fig. 2 (b) and (c)]. Each  
135 sphere is assigned an equivalent diameter of 5 mm, and so the sphere-based  
136 strip (5 mm thick × 10 mm wide × 80 mm long) is equal in size to the specimen  
137 section, which is elongated during the test.

138

139 Table 1 shows the material micro-properties used to simulate the behavior of  
140 the geocell. The properties were determined using the formulation proposed by

141 Potyondy and Cundall [30] and the stress–displacement results presented in  
142 Fig. 3. As can be seen, close agreement is obtained between the simulation  
143 and test results. Both sets of results show a very close peak strength, a clear  
144 elongation process and similar residual strength. The agreement was achieved  
145 by encoding a ductile model [26] to provide a softening slope. A previous study  
146 [24] used conventional linear parallel-bond which can only provide a linear-  
147 elastic stress-strain response before reaching peak tensile strength. The ductile  
148 model is a modification, rather than a replacement to the contact-bond, and it is  
149 invoked when brittle failure occurs in bonded particles, so that the geocell model  
150 does not experience sudden failure when it reaches its peak tensile strength.  
151 Instead, the bond reduces its strength to behave like HDPE; the material from  
152 which the geocells used in this study are manufactured from. As can also be  
153 observed, there is a disparity between the simulation and experimental results  
154 in the elastic regions. This phenomenon can be attributed to the nature of the  
155 parallel-bond, which is essentially designed to model linear-elastic behavior.  
156 The model incorporates three contacts: stiffness (i.e. springs), a parallel bond  
157 and a slip. As a further note, the micro-properties shown in Table 1 were  
158 attained using an iterative approach – harmonizing the simulations with the test  
159 results [26]. Whilst this approach is somewhat indirect, satisfactory outcomes  
160 are obtained. The geocell model obtained a yielding strain  $\epsilon_y=11.02\%$  and a  
161 failure strain  $\epsilon_f=46.7\%$ ; identified as points A and B respectively in Fig. 3.

162



### 163 **2.2.2 Ballast**

164 Railway ballast is usually produced by blasting and/or fragmenting a rock mass,  
165 and hence exhibits variable angularities. Past studies [3, 31, 32] have  
166 demonstrated the importance of accurately modeling the particle angularities,  
167 and suggested that reflecting angularities in simulations better reproduces the  
168 actual behavior of the ballast. To achieve this, four ‘clump templates’ were  
169 developed: trapezoidal, triangular, rectangular and hexagonal (Table 2), which  
170 account for the major geometric shapes of ballast infills. Clumps are groups of  
171 ‘slaved’ spheres that are firmly bonded together. In the modeling undertaken in  
172 the present study, debonding within the clump is prohibited, so as to focus on  
173 the motion of the ballast and eliminate the possibility of problems associated  
174 with breakage.

175

176 The calibration of the ballast is similar in concept to that of the geocell. Lim and  
177 McDowell [32] suggested the use of a triaxial test simulation to calibrate the  
178 ballast in PFC3D, and test results by Indraratna *et al.* [4] were used for this  
179 purpose. As suggested by Lim and McDowell [32] and Lu and McDowell [33],  
180 the interlocking of the clumps was represented by applying a weak and  
181 breakable parallel bond between two contacting clumps. The bond can  
182 reconstitutes at a new contact if particles rearrange. In addition, the membrane  
183 used to confine a sample is represented as a wall and assumed to be  
184 frictionless [6]. As PFC adopts the lower friction coefficient of two contacting  
185 entities, the friction between the clumps and the membrane is ignored. This  
186 approach is also adopted in subsequent ballast embankment models, which

187 helps focus on the mechanical response of the geocell-reinforced ballast.  
188 Similarly, the sleepers situated on the top of the embankment act merely as  
189 loading platens and the friction between the sleepers and the ballast is ignored.  
190  
191 The test setup, as shown in Fig. 4, comprises a cylindrical cell of 300 mm in  
192 diameter x 600 mm high. The cell is initially filled with a number of spheres of  
193 varying diameters, 20 mm to 50 mm [Fig. 4(a)], in accordance with the ballast  
194 grading characteristics specified by Indraratna *et al.* [4]. The spheres are then  
195 replaced [Fig. 4(b)], in equal volume, with the clump templates shown in Table  
196 2. The replacement is conducted in equal allocations among the four templates,  
197 and at random orientations within the cell. It is important to note that particle  
198 overlap occurs when assigning the clump templates to the spheres due to the  
199 created clump angularities. To negate this effect, as well as a prestressing  
200 problem, the top cap of the cell is allowed to move upward at an extremely slow  
201 rate of 0.1 mm/s until an equilibrium of inter-clump contact forces is achieved  
202 [32]. The equilibrium is determined by the ratio of the average mechanical solve  
203 ratio, defined as unbalanced force over the average value of the sum of contact  
204 forces, body forces and applied forces over all particles. The ratio is set as  
205  $1 \times 10^{-3}$ , which is small enough to signal the equilibrium. The specimen porosity  
206 at equilibrium is 0.39, which is the average measured by two spheres. The  
207 spheres, 300 mm diameter each, are inscribed in the triaxial chamber. The  
208 spheres sit edge-to-edge, enabling the most occupation of the chamber space.  
209 The inscribing avoids possible boundary effect of the chamber. A total of 632  
210 clumps (i.e. 7,584 spheres) are incorporated in the specimen.

211

212 The specimens are then subjected to triaxial compression tests at 6 different  
213 confining pressures: 15, 30, 60, 90, 120 and 240 kPa. The loading is achieved  
214 by moving top wall downward at a rate of 0.045 mm/s and the tests continue  
215 until an axial strain of 20% is attained. It should be noted that all loading rates  
216 used in this study have been selected by trial and error to achieve desirable  
217 numerical stability while reasonable computational effort is spent. A numerical,  
218 servo-control algorithm [26] is incorporated in the simulation to maintain a  
219 constant confining pressure throughout the respective loading phases. The top  
220 loading wall is assigned with following micro-properties: a normal stiffness of  $1 \times$   
221  $10^{10}$  N/m; shear stiffness of  $1 \times 10^{10}$  N/m and a friction coefficient of 0.5 (i.e.  $\tan$   
222  $27^\circ$ ). The wall stiffnesses are higher than the ballast stiffness in order to prevent  
223 ballast penetration. Fig. 5 shows the simulation and test results of the triaxial  
224 tests. The simulation was achieved by encoding a linear contact model [26] and  
225 using the micro-properties provided in Table 3, which were obtained through  
226 trial and error. The micro-properties show that the model, similar to that for the  
227 geocell, also incorporates the three contacts: stiffness, a parallel bond and a  
228 slip. Similarly close agreement is found across the entire series of confining  
229 pressures. The accuracy of the simulations is further validated by the dilation  
230 observed under lower confining pressures and contraction under higher ones.  
231 These results demonstrate that the material properties and encoded models are  
232 capable of appropriately modeling the mechanical behavior of the ballast.

233

234 **3. MODELING PROCEDURE**

235 A full-scale embankment simulation is computationally, extremely time-  
236 consuming, owing to the large number of spheres needed to simulate the  
237 geocell and ballast infills, and is beyond current and available computer  
238 capability. This concern has been confirmed in a similar simulation study [6].  
239 Therefore, the embankment is scaled down by a factor of five in terms of its  
240 crest and base width with regards to the actual dimensions specified by ARTC  
241 [34, 35]. In this context, there are still approximately 78,000 spheres  
242 incorporated in the reinforced embankment. The scaling does not significantly  
243 influence performance comparison made between the reinforced and  
244 unreinforced embankments, as both embankments are subject to the same  
245 level of scaling. Moreover, the scaled embankment is comparable in size with  
246 the one adopted in a prototype test [7] and so provides an opportunity to  
247 validate the simulation results against those from the test. In order to focus on  
248 the contribution of the geocell to embankment stability, a simplified track  
249 assemblage is adopted, where only sleepers are included in the DEM model  
250 and rails, fastenings and anchors are excluded.

251

252 **3.1 Straight embankment**

253 The straight rail embankment is summarized in Fig. 6. A crest width of 500 mm,  
254 base width of 1,080 mm, height of 300 mm and a length of 1,000 mm are  
255 adopted. The gradient of its shoulder slope is approximately 1:1. Six sleepers,  
256 each 50 mm wide and 500 mm long, are founded on the crest at an edge-to-  
257 edge spacing of 120 mm. The sleepers were simulated using stiff walls – an

258 object in PFC3D for materials with line segments [26], which exhibit dimensions  
259 of actual, heavy-duty, prestressed concrete sleepers. As the contact forces  
260 between two contacting objects are governed by their stiffnesses, the sleepers  
261 are assigned with the micro-properties used for the loading wall in the triaxial  
262 simulation, enabling a consistent stress–strain behavior of the ballast assembly.  
263 Considering the 2D nature of the embankment (i.e. no longitudinal movement of  
264 the infill), the front and rear cross-sections were simulated using non-movable  
265 walls, with normal and shear stiffnesses of  $1 \times 10^{10}$  N/m, and a higher friction  
266 coefficient of 1.0 (i.e.  $\tan 45^\circ$ ) to reflect the ballast-to-ballast friction along the  
267 section boundaries. In order to reflect embankment subsidence caused by the  
268 underlying subgrade, the subgrade was also represented by a wall, with lower  
269 normal and shear stiffnesses of  $1 \times 10^8$  N/m, and a friction coefficient of 0.5.

270

271 The role of the geocell in the stability of rail embankments is examined by  
272 placing the geocell at two different levels within the ballast layer: at the base of  
273 the embankment [Fig. 7(a)] and 50 mm above the base [Fig. 7(b)]. At each  
274 level, as shown in Fig. 7(c), the geocell panel is centered within the ballast-filled  
275 embankment. The panel [Fig. 7(d)] includes 8 cells and measures 748 mm  $\times$   
276 480 mm edge-to-edge. Each cell is 75 mm deep and 175 mm  $\times$  175 mm wide.  
277 The long and short sides of the panel are aligned with the embankment's  
278 longitudinal and transverse directions, respectively. The short side is less than  
279 the width of embankment crest, so that a 10 mm margin is present along the  
280 embankment crest edges. In the longitudinal direction, the panel length is  
281 252 mm shorter than the extension of the embankment, which negates

282 boundary effects associated with the panel. The geocell panel is longitudinally  
283 divided into two halves: *A* and *B*. Representative cell junctions are marked as *a*  
284 to *g* for subsequent displacement analysis. As is required by PFC3D, the  
285 geocell material is also simulated by a layer of spheres. The spheres are  
286 aligned and bonded together contiguously using the micro-properties shown  
287 previously in Table 1. A total of 12,762 spheres are used to generate the entire  
288 geocell panel.

289

290 The ballast infill is generated using the procedures similar to those used in the  
291 ballast triaxial calibration. Temporary walls are generated first on the  
292 embankment slopes and crest as boundaries. The geocell and associated  
293 bonds are then generated within the pre-defined boundaries, followed by  
294 generation of ballast and corresponding parallel-bond. The geocell can deform  
295 freely and it is breakable during this process. It should be noted that the ballast  
296 is generated in three layers (i.e. 100 mm thick each). As contact forces between  
297 clumps are created due to overlapping during clumps generation, additional  
298 time steps are permitted between the generations of each layer, so that  
299 previous layers can reach equilibrium (i.e. release contact forces). The  
300 temporary walls prevent the escape of clumps due to the contact forces and  
301 they are permitted to move slowly outward until the inter-clump contact forces  
302 dissipate, upon which they are removed. During the ballast generation process,  
303 no constraint is applied to the interaction between the geocell and ballast. This  
304 is to reflect the actual placement of ballast in the field. A total of 4,002 clumps  
305 (i.e. 56,083 spheres) are used for the infill in the situations where a geocell

306 panel is used. For the unreinforced embankment, similar numbers of clumps  
307 (4,106) and spheres (57,479) are generated for the infill.

308

### 309 **3.2 Curved embankment**

310 A horizontally-curved embankment has its outer rail elevated to provide a  
311 banked curvature. This super-elevation, also known as a cant, serves the  
312 purpose of providing a centripetal force to balance the centrifugal force exerted  
313 by the train's motion, which in turn allows the train to negotiate bends at higher  
314 speed. Fig. 8 shows a diagram of the curved embankment used in this study.  
315 The diagram is similar to that for the straight embankment except for the 5%  
316 gradient adopted at the crest. This gradient is set in accordance with ARTC [35]  
317 and the value corresponds to the typical limit of super-elevation for an intrastate  
318 line in Australia. Compared with the straight embankment, the curved  
319 embankment uses the same geocell arrangements and material micro-  
320 properties, and a similar number of spheres for the geocell and ballast.

321

### 322 **3.3 Monotonic and cyclic loading**

323 This sub-section describes the monotonic and cyclic loading adopted in the  
324 study. The aim of the monotonic loading is to determine the embankment  
325 subsidence in response to a slowly increasing vertical load and is similar in  
326 nature to a plate load test. For the straight embankment, the numerical model  
327 constrains the sleepers to move in a downward direction along a trajectory  
328 normal to the crest. The sleepers advance at a rate of 0.1 mm/s to cause the  
329 embankment to settle at the desired strain of 20% (60 mm). The modest value

330 of the loading rate improves the simulation accuracy by allowing sufficient time  
331 to calculate the inter-particle contact forces. The strain-limiting value is  
332 consistent with that used in the triaxial calibration and helps predict the load  
333 extremes that the embankments can sustain. The monotonic loading applied to  
334 the curved embankment acted at an angle of inclination  $\theta$  (i.e.,  $54.5^\circ =$   
335  $\arctan(P_L/P_V)$ ) [Fig. 9], where  $P_V$  is the vertical applied load and is calculated to  
336 equal 125 kPa for a 30-tonne axle load of a heavy haul train wagon [34];  $P_L$  is  
337 the lateral load acting on the sleepers and equal to the centrifugal force as:  
338

$$P_L = \frac{mv^2}{R} \quad (1)$$

339  
340 where  $m$  is the axle load,  $v$  is the speed of the train, and  $R$  is the horizontal  
341 curve radius. ARTC [34] specifies  $R = 200$  m as the minimum allowable  
342 horizontal curve radius for a heavy haul line. Thus,  $P_L$  is approximately 175 kPa  
343 when the haul train wagon passes through the curve at the ARTC's design  
344 speed of 60 km/h [36]. The values for the vertical load, radius and design speed  
345 are adopted to reflect adverse situations in practice and so amplify the loading  
346 conditions and expedite the simulation process. To achieve a displacement  
347 direction at the angle  $\theta$ , the sleepers advance at a lateral rate of 0.14 mm/s and  
348 vertical rate of 0.1 mm/s; that is, at a velocity ratio of 1.4, which is equivalent to  
349 the  $P_L/P_V$  ratio.

350

351 Cyclic loading, on the other hand, is of higher significance in regard to the  
352 assessment of the long-term serviceability of railway embankments. For the



353 straight embankment, a vertical load of  $P_v = 125$  kPa, which reflects a full-scale  
354 25-tonne heavy freight train passing through, was applied normal to the  
355 sleepers in the form of loading-complete unloading-reloading cycles. Although  
356 the geometry of the railway structure and geocell is downscaled, the strength  
357 and mechanical behavior are calibrated against laboratory and full-scale  
358 experimentation, therefore no scale factor is applied to the loading values. The  
359 load applied has been shown to be frequency-independent, as reported by  
360 Shenton [37]. Due to the long computational time when performing the  
361 simulation, a total of 20 loading cycles were performed for each simulation.  
362 Even with this somewhat modest number, the simulations utilized the full  
363 capability of the PC hardware (Intel core i7-4500U, 8GB DDR3L 1333 RAM with  
364 integrated Intel HD Graphics 4400) and the entire modeling process took  
365 approximately two months to complete. Albeit with the constraint of  
366 computational time, the simulations provide indicative observations of  
367 embankment subsidence and the performance of geocells in the early stages of  
368 the cyclic loading. Similar simulations were applied to the curved embankment,  
369 except for the load applied. The resultant force ( $P_R$ ) of the vertical ( $P_v$ ) and  
370 lateral ( $P_L$ ) loads was calculated as 215 kPa and acted at an angle of  $\theta$  with  
371 respect to the vertical direction (Fig. 9). It is worth mentioning that for both  
372 straight and curved embankment subject to cyclic loading cases, all sleepers  
373 advance simultaneously at the same rates. No lag is applied to the sleepers to  
374 reflect train passage as the freight can pass the sleepers gap in an extremely  
375 short period of time over a 1-meter embankment.  
376

377 Local damping was activated for ballast clumps only to absorb the vibration  
378 energy generated in the cyclic loading process. The clumps tend to rebound  
379 and occasionally escape from the embankment boundaries during the  
380 unloading phases, as a result of accumulated internal forces. The introduction  
381 of a damping coefficient,  $\zeta$ , facilitates the dissipation of these forces in the  
382 agitated clumps and allows the ballast assembly to cease oscillating more  
383 rapidly [26]. In this study, the local damping ratio was set to 1.0.

384

## 385 **4. RESULTS AND DISCUSSION**

### 386 **4.1 Straight embankment**

387 Fig. 10 shows the vertical displacement of the sleepers plotted against the  
388 applied vertical load for the straight embankments under monotonic loading,  
389 where the results of the numerical simulations from this study are compared  
390 with the test results presented by Leshchinsky and Ling [7]. The simulated  
391 vertical displacement is the average of the 6 sleepers and the load is the  
392 average resistance measured at the base of the sleepers [Fig. 6(b)]. The  
393 boundary effects caused by the walls in longitudinal direction are neglected in  
394 this study as the individual data set for each sleeper shows insignificant  
395 differences in axial stress value. Unlike traditional FE analysis, the results of the  
396 DEM modeling show a somewhat irregular curve with slight fluctuations. These  
397 are associated with the rearrangement of clumps as the applied load increases.  
398 Overall, the vertical displacement rises with increased load for the three design  
399 cases, without defined yielding for the range of loads applied. It is clear that  
400 using a geocell panel has a noticeable influence on the vertical displacement of

401 the embankment. With the same applied load, the geocell-reinforced  
402 embankment exhibits less vertical displacement than that of the unreinforced  
403 embankment. Specifically, given a load of 125 kPa, the vertical embankment  
404 displacements are 18.9 mm, when the geocell is located 50 mm above the  
405 base, 27.9 mm when the geocell is founded at the base, and 29.5 mm when the  
406 embankment is unreinforced. As shown in Fig. 10, the performance of the  
407 geocell reinforcement is in agreement with the test results presented by  
408 Leshchinsky and Ling [7], who conducted a similar monotonic loading test on a  
409 geocell-reinforced ballast embankment. This implies that incorporating a geocell  
410 panel in a railway embankment will reduce vertical displacement, and placing it  
411 50 mm above the base, yields superior performance to that when the geocell is  
412 placed at the base. The superiority can be attributed to the position of geocell.  
413 The suspended geocell limits the loading propagating into the bottom 50 mm  
414 layer, which minimizes the settlement and lateral spreading of the bottom layer.  
415  
416 The monotonic loading curves, given in Fig. 10, can be subdivided into two  
417 zones: *A* and *B*, which correspond, respectively, to vertical displacements of  
418 less than 10 mm and those beyond 10 mm. In Zone *A*, the early stages of  
419 vertical embankment displacement, the sleepers displace in a similar fashion  
420 across the three cases examined and exhibit largely equal stiffness. This  
421 implies that the ballast skeleton supports the majority of the load when the load  
422 remains at a relatively low level, and the geocell is 'at rest' and contributes little  
423 to the embankment stiffness. In Zone *B*, where the vertical displacement  
424 exceeds 10 mm, the geocell demonstrates a strain-hardening effect. It aids in

425 reinforcing the ballast skeleton and increases the stiffness of the embankment.  
426 As a result, for an equal vertical displacement, the geocell-reinforced  
427 embankment is able to support a higher load than the unreinforced  
428 embankment. Due to the curves fluctuation, however, there is a section  
429 disagreeing the comparison. Where the vertical load falls into 165 to 220 kPa,  
430 the reinforced embankment with geocell at base experiences slightly higher  
431 vertical displacement than the unreinforced does, with a maximum difference of  
432 2.3 mm. The curves fluctuation is caused by the DE simulation attaining  
433 convergence at some time steps. In addition, placing a geocell 50 mm above  
434 the base provides an improved stiffness response than placing it at the base.  
435

436 Fig. 10 also presents a comparison of the stiffness development between the  
437 simulation results and the prototype test results presented by Leshchinsky and  
438 Ling [7] who placed geocell at 100 mm above base. The inclusion of this set of  
439 experimental data is not for making quantitative comparison against the results  
440 obtained from this study (place geocell at 50 mm above base). The intention is  
441 to claim that by suspending geocell within ballast embankment, further  
442 improvements can be made, and it has been validated by previous  
443 experimentation. As can be seen, both sets of results show a short segment of  
444 low stiffness, in the early stages of monotonic loading, followed by a more  
445 prolonged development of improved stiffness. Once the results enter Zone *B*,  
446 placing geocell at 100 mm above the base becomes more advantageous in  
447 reducing sleeper's displacement than placing geocell at 50 mm does. The  
448 displacement difference is up to 5.2 mm when the vertical load reaches 285

449 kPa. From this point onward, the reinforcing effect decreases and the two  
450 curves cross over where the vertical load increases to 498 kPa. Afterward,  
451 placing geocell at 50 mm offers better performance until the end of simulation.  
452 Overall, both studies indicate that suspending geocell within the ballast  
453 embankment can yield better load-bearing performance. This agreement,  
454 however, is not observed with the unreinforced embankments. Strain-softening  
455 was observed in the test embankment, whereas the simulated embankment  
456 exhibits strain-hardening throughout. Therefore, the unreinforced test  
457 embankment yields a lower secant stiffness than in the simulation: 2,916 kPa/m  
458 for the test and 7,975 kPa/m for the simulation, at a vertical displacement of  
459 60 mm.

460

461 This disagreement arises mainly from the unconfined nature (in both  
462 longitudinal and transverse directions of the embankment) of the prototypical  
463 test conducted by Leshchinsky and Ling [7]. The ballast can move freely in both  
464 directions, whereas the longitudinal movement is prohibited in the current  
465 models by installing two boundary walls. In addition, the difference between the  
466 test and simulated ballast infill, as well as other factors such as embankment  
467 geometry, loading plate size, geocell strength and boundary conditions, may  
468 also contribute to the significant difference in vertical displacement. The gravel  
469 that was used in the test is smaller on average than the ballast used in the  
470 simulation ( $D_{50} = 15.5$  mm and 35 mm, respectively) and so yields a lower  
471 shear strength. This is confirmed by the respective triaxial test results; for  
472 example, a shear strength of approximately 400 kPa for the gravel in the test [7]

473 and 700 kPa for the coarser aggregate in the simulation, when subjected to the  
474 same confining pressure of 90 kPa. The lower shear strength for the gravel  
475 leads to its strain-softening behavior and lower stiffness. It is interesting to note  
476 that the discrepancy occurred with the unreinforced embankment, whose  
477 behavior is dissimilar to that of the reinforced embankment. This implies that the  
478 use of a geocell panel is able to mitigate potentially 'weak' properties of the  
479 ballast infill and increase stiffness through its reinforcement effects.

480

481 Fig. 11 shows sleeper's vertical displacement plotted against the number of  
482 load cycles for the straight embankment under cyclic loading. It is evident that  
483 the geocell is effective in reducing vertical displacement associated with cyclic  
484 loading. During the initial 5 loading cycles (Zone A), all three cases exhibit a  
485 high displacement rate. Similar behavior is observed in a previous study [6]  
486 where geogrid is used. The early-stage quick displacement also agrees with the  
487 results obtained by Selig and Waters [38] who found that the relatively rapid  
488 displacement in the early stage is associated with the poorly consolidated  
489 nature of infills. In Zone A, the vertical displacement is reduced due to the use  
490 of geocell. However, no noticeable difference is observed between placing  
491 geocell at base and 50 mm above the base. The role of geocell becomes more  
492 pronounced as the cycle number increases which is suggested by the  
493 noticeably slower displacement rates in Zone B (5<sup>th</sup> to 20<sup>th</sup> loading cycle). This  
494 phenomenon can be attributed to the passive-confinement mechanism of  
495 geocell. Where cyclic loading continues, the infills is further compacted,  
496 stiffening the geocell mattress, which in turn provides better reinforcement to

497 the ballast embankment. In Zone *B*, placing geocell at 50 mm above base  
498 outperforms placing geocell at base. The reinforcing effect improves slightly  
499 along with the increase of load cycle number, resulting in a final vertical  
500 displacement of 45.5 mm versus 52.3 mm if placing geocell at base.

501 Interestingly, Chen *et al.* [6] who installed geogrid in ballast embankment as  
502 reinforcement at 50, 100 and 150 mm concluded otherwise. Their study  
503 reported that placing geogrid at lower levels (i.e. 50 mm above subgrade) better  
504 prevents the displacement. There is no clear reason to this disagreement, but  
505 the two geosynthetic materials work in different modes: cell confinement by the  
506 geocell and grid-particle friction by the geogrid. It is suggested that the  
507 confinement matter works better if placed next to the load on ground; the  
508 geogrid is placed at a lower level where the load becomes spread and reduced.

509

510 Comparison to the past study [7] has been made in the final vertical  
511 displacement only as the original displacement versus loading cycle relationship  
512 is unavailable. After the 20<sup>th</sup> cycle, the simulations show higher vertical  
513 displacement than that indicated by tests. The vertical displacement is 67.5 mm  
514 for the simulation and approximately 48 mm for the unreinforced embankment  
515 test; and 52.3 mm when placing geocell at 50 mm above base for the simulation  
516 and approximately 31 mm for the test of the embankment incorporating the  
517 geocell at the 100 mm above base. In addition to the compaction effort, other  
518 factors that may contribute to the final settlement difference are the size effect  
519 at the plate-infill interface and the geocell types used. The simulations use a  
520 sleeper of 50 mm × 500 mm and infill of  $D_{50} = 35$  mm, and the test used a

521 square plate, 356 mm × 356 mm in size, and infill of  $D_{50} = 15.5$  mm. The smaller  
522 sleeper-infill size ratio for the simulations results in the sleepers ‘punching’ to a  
523 greater extent into the infill than the test does. This punching effect likely  
524 reduces with depth as the lateral resistance (arching) of the infill between  
525 neighboring sleepers increases, and the displacement stabilizes. On the other  
526 hand, Leshchinsky and Ling [7] adopted Novel Polymeric Alloy (NPA) geocell  
527 which exhibits higher stiffness and tensile strength (27 MPa) than typical HDPE  
528 geocell [14, 31]. The material strength difference can also prevent embankment  
529 settlement.

530

531 In order to gain a greater insight into the force distribution and transmission  
532 mechanism of unreinforced and reinforced ballast embankments, as shown in  
533 Fig. 12, contact forces are drawn at the same scale for the straight  
534 embankments after the 20<sup>th</sup> cycle. The contact forces are observed through the  
535 front cross section of the respective embankments. It can be seen that the  
536 contact forces develop in different patterns between the unreinforced and  
537 reinforced embankments. The unreinforced embankment shows an uneven  
538 distribution of contact forces. The forces adjacent to the base of the  
539 embankment are more concentrated than elsewhere in the embankment. In  
540 contrast, the contact forces for the geocell-reinforced embankments are  
541 distributed more evenly. This even distribution of contact forces helps eliminate  
542 overstressing of the infill and reduces the likelihood of localized displacement  
543 and/or failure, thus improving the resilience of the embankment. In addition, an  
544 increase in the maximum and average contact forces within the ballast are



545 recorded among the three cases simulated. The unreinforced case exhibits the  
546 lowest contact force value comparing to the two reinforced cases. This  
547 difference can be attributed to the higher internal contact forces induced by a  
548 reduced settlement. The internal stress caused by loading cannot dissipate  
549 through particle movement as it is restricted by the geocell panel. The highest  
550 contact force is observed where the geocell panel is placed 50 mm above the  
551 base, which implies less ballast movement should be expected. This  
552 observation agrees with results shown in Fig. 10 (monotonic loading case). That  
553 is, at the same settlement, the reinforced cases sustain loads greater than the  
554 unreinforced case does.

555

556 Fig. 13 shows the total particle displacement vectors (i.e. the combination of  
557 vertical and lateral displacement) of the ballast after the 20<sup>th</sup> cycle, again drawn  
558 at the same scale as that shown previously to allow visualization of the  
559 microstructure strain evolution of the embankments. Fig. 13 (c) is tilted by 5  
560 degrees for better visualization of the displacement vectors, which causes the  
561 vectors appear slightly denser and longer. Apart from the reduced particle  
562 displacement, the major difference between the unreinforced and the reinforced  
563 embankments lies in the direction of the ballast displacement. The infill in the  
564 reinforced embankments [Fig. 13 (b and c)] displace mainly toward the base,  
565 whereas the infill in the unreinforced embankment [Fig. 13 (a)] tends to move  
566 laterally. This can be better visualized in Fig. 13 (d-f) which provide zoomed-in  
567 views of the left-hand-side unreinforced sections of three embankments. These  
568 observations confirm the ability of the geocell panel to prevent the ballast infill

569 from spreading. That is, the geocell panel helps restrain the confined infill  
570 equivalent to that of a relatively rigid pad. In this way, the pad effectively  
571 absorbs overlying loads and transfers them downward, avoiding or reducing  
572 lateral spreading. This is consistent with the distribution of contact forces shown  
573 previously in Fig. 12(b and c), where the contact force concentration is less  
574 significant at the base of the embankments and thus reduces embankment  
575 displacement. The central part of the elevated geocell panel [Fig. 13 (c)]  
576 undergoes modest subsidence (approximately 10 mm), which suggests slight  
577 lateral movement of the infill underlying the panel.

578

579 Fig. 14 shows the total displacement vectors for the geocell panels after the 20<sup>th</sup>  
580 loading cycle, as well as the maximum displacements and their approximate  
581 locations. These displacement vectors are scaled up by a factor of 50 in order  
582 to achieve better visualization. As can be seen, the panels undergo a limited  
583 amount of displacement and they hence remain effectively in their original  
584 configuration after repetitive loading, demonstrating their strength. In the case  
585 where the geocell is placed at the base [Fig. 14 (a)], the maximum displacement  
586 occurs at the bottom-left of the panel. This location shifts upward when the  
587 panel is located 50 mm above the base. The relocation implies that the geocell  
588 panel settles noticeably (10 mm approximately) together with the ballast  
589 assembly. In addition, the displacement is not position-dependent. All cell walls,  
590 at the center and along the edges, undergo a similar level of deformation. This  
591 behavior aids in evening out the stresses acting on the panel, eliminating local  
592 failures, maintaining its long-term reinforcement capability and, more

593 importantly, accommodating the displacement of the infill and harmonizing the  
594 particle contact forces.

595

## 596 **4.2 Curved embankment**

597 The vertical and horizontal displacements plotted against the corresponding  
598 loads of the curved embankment that was subjected to the resultant load,  $P_R$ ,  
599 (Fig. 9) are shown in Fig. 15. The load-vertical displacement curves [Fig. 15(a)]  
600 develop in a form similar to those observed with the straight embankment [Fig.  
601 10]. Non-yielding is clearly evident upon the load of 600 kPa. The three curves  
602 exhibit largely equal stiffness when the displacement is low (i.e. less than  
603 10 mm), where the vertical displacement mainly arises from rearrangement of  
604 the uncrushable infill (in the DEM model, in any case) and the geocell provides  
605 a marginal contribution to stiffness. The geocell's reinforcement effect becomes  
606 clear when the displacement exceeds 10 mm. It can be seen that the geocell-  
607 reinforced embankments obtain stiffness higher than that of the unreinforced  
608 embankment, and so support a greater load, given the same vertical  
609 displacement. Placing the geocell 50 mm above the base yields a higher  
610 stiffness. Similar improvement occurs in the lateral direction [Fig. 15(b)], where  
611 the sleepers of the reinforced embankments displace less than the sleepers of  
612 the unreinforced embankment, with an equal resultant load. This is attributed to  
613 the geocell enhancing the interlocking of the infill and so restraining the  
614 rearrangement and rotation of the ballast particles. In the later stages of loading  
615 (i.e. > 40 mm lateral displacement), lateral yielding occurs in all simulations,  
616 showing a marked displacement in response to the cyclic loading. This is a

617 result of the sleepers having partially moved out of the region influenced by the  
618 geocell, and thus having to rely on the shoulder ballast to provide lateral  
619 resistance. This observation is valid for all simulations performed for curved  
620 embankments. Although this phenomenon is unlikely to occur in actual railways,  
621 as catastrophic accidents can be caused due to de-railing, the results are  
622 presented for the purpose of demonstrating the improvements derived from  
623 placing geocell in railway ballast embankments.

624

625 Fig. 16 shows the vertical and lateral displacement of the sleepers due to cyclic  
626 loading. As was evident with monotonic loading, the geocell-reinforced  
627 embankments outperform the unreinforced embankment. The reinforced  
628 embankments exhibit less vertical and lateral displacements than those  
629 observed in the unreinforced embankment. Placing a geocell 50 mm above the  
630 base, again, better controls displacement in both the vertical and horizontal  
631 directions. The vertical displacement [Fig. 16(a)] is more pronounced over the  
632 first 5 cycles, and then shows a decreased rate over the remaining cycles. The  
633 lateral displacement of the sleepers is relatively high, given the low number of  
634 cycles [Fig. 16(b)]. This is likely caused by the unrestrained nature of the  
635 sleepers, where the restraining influence of the track structure, such as the rails,  
636 rail anchors and fastenings, were not taken into account in the simulations, as  
637 mentioned earlier. As a result, the sleepers are able to displace more freely  
638 than would occur in the field.

639

640 Fig. 17 shows the inter-particle contact forces drawn at the same scale after the  
641 20<sup>th</sup> load cycle. As for the straight embankments, the geocell panels also  
642 appear to promote an even stress distribution for the curved embankments.  
643 This is in agreement with the particle displacement vectors shown in Fig. 18,  
644 where reduced spreading is observed for the reinforced embankments, when  
645 compared with the unreinforced embankment. Moreover, comparing the  
646 displacement vectors with those for the straight embankments (Fig. 12) implies  
647 that the geocell panels in the curved embankments are similarly effective in  
648 forming a relatively rigid platform and to mitigate ballast spreading.

649

650 Fig. 19 shows the total displacement vectors for the geocell panels after the 20<sup>th</sup>  
651 cycle. The panels maintain their respective initial shape and demonstrate the  
652 geocell's capability to sustain the lateral load for the curved embankments. The  
653 geocell walls, in particular the walls adjacent to the longitudinal centerlines,  
654 deflect to the right – in line with the direction of the resultant forces. The  
655 concurrent deflection of the walls helps counteract the lateral load, confine the  
656 lateral load within the area of the panel, and reduce spreading of the infill along  
657 the edges. The panel situated 50 mm above the base appears to deflect slightly  
658 more than does the panel at the base. This is consistent with the geometric  
659 deformation which occurs in a 'suspended' panel [Fig. 18 (c)], and suggests it is  
660 likely to degrade sooner than the panel located at the base. This can be  
661 examined through additional case studies, such as increasing load cycles and  
662 placing panels at higher levels in the embankment. This is, however, beyond  
663 the scope of the present paper.

664

665 To gain a further insight into the deflection of the geocell panel, geocell strains  
666 are captured. As illustrated in Figure 20, for a pair of neighboring spheres of  
667 interest, the strain,  $\epsilon$ , is defined as the edge-to-edge distance after  
668 displacement,  $D_1 - D_0$ , to the initial center-to-center distance,  $D_0$ . The strain  
669 values at locations of interest are summarized in Table 4. These include  
670 junctions *a* to *g*, panel halves *A* and *B*, as shown in Fig. 7(d), and locations of  
671 maximum strain for the geocell panels at the base and 50 mm above the base,  
672 subjected to the monotonic and cyclic loading scenarios. Panel halves *A* and *B*  
673 rest on the lower and the higher side of the embankment, respectively.

674

675 The initial center-to-center distance is 5 mm, as shown in Fig. 2(b). The strain at  
676 a junction is calculated as the average strain of all spheres within 20 mm (i.e.  
677 11.4% of the cell side) to the junction. The selected percentage is intended to  
678 reflect the strain in the proximity of the junction. The strain for either half panel  
679 is the average strain of all the spheres belonging to that half panel. The strain  
680 values in Table 4 show that the geocell deforms at every junction with varying  
681 magnitude, for instance, ranging from 24.1% to 41.6% for the geocell at the  
682 base when subjected to monotonic loading. Where the sleepers advance less  
683 under the cyclic loading, noticeably lower strains of 14.1% on average occur to  
684 the junctions. There is a clear difference in strain between the panel halves *A*  
685 and *B*, where all other design details remain the same. For instance, the  
686 average strain is 18.5% for panel half *A* and 24.9% for panel half *B* under  
687 monotonic loading. This implies that greater deflection occurs at the part of the

688 geocell that provides direct reaction to the inclined train load  $P_R$ . Lower strains  
689 occur to both halves where the geocell is placed 50 mm above the base than  
690 the geocell placed at the base, which agrees with the embankment  
691 displacement results shown in Figs. 11 and 16. Under the monotonic loading,  
692 the entire panel is subject to a maximum strain of 39.7%, if placed at the base,  
693 and 45.6%, when placed 50 mm above. If subjected to the cyclic loading, the  
694 panel shows a maximum strain of 28.4%, when at the base, and 23.4%, when  
695 50 mm above. The magnitude of these strains indicates that the geocell panel  
696 remains at the pre-failure state for the load levels simulated. The approximate  
697 locations, L1 to L4, where maximum strains were recorded, are highlighted in  
698 Figure 21; i.e. L1 for 39.7% and L2 for 45.6% under the monotonic loading  
699 scenario, and L3 for 28.4% and L4 for 23.4% under the cyclic loading scenario.  
700 There is no clear pattern to the locations of maximum strain, however, as can  
701 be seen, they are all consistent with the center of a cell-wall. This indicates that  
702 cell-walls undergo greater deflection than the junctions do, as one might expect.

703

## 704 **5. CONCLUSIONS**

705 This study assesses the use of geocells in reinforcing railway ballast  
706 embankments. Discrete element modeling has been conducted, using clumped  
707 particles to simulate angular ballast, to evaluate bearing capacity, vertical  
708 displacement and lateral spreading of the embankment, as well as providing  
709 insights into the micro-behavior of the ballast infill and the geocell, including  
710 contact forces and displacements. Straight and curved embankments have  
711 been subjected to monotonic and cyclic loading conditions and the modeling

712 results have been compared with previous, published test results. The  
713 conclusions of this study are as follows:

- 714 1. The simulation results for the straight, reinforced embankment are in  
715 reasonably good agreement with the test results. This suggests that the  
716 discrete element modeling is valid and is an appropriate method to assess  
717 the mechanical response of railway embankments.
- 718 2. For the unreinforced, straight embankment, however, simulation results  
719 show modest agreement with the past test results. The suboptimal  
720 agreement may be attributed to the differences in the particle size  
721 distribution, embankment geometry and loading magnitude. These factors  
722 influence the embankment performance where reinforcement is not used.
- 723 3. The presence of a geocell within the ballast stiffens both straight and curved  
724 embankments. Geocell-reinforced embankments exhibit less vertical  
725 displacement and lateral spreading compared with unreinforced  
726 embankments and so aid in maintaining a safer track alignment in the longer  
727 term. The embankments with a geocell suspended 50 mm above the base  
728 are stiffer than the embankments with a geocell located at the interface  
729 between the ballast and the subgrade. The former, however, deflects more  
730 than the latter and so risks having a reduced operational life. The geocell  
731 embedment depth results disagree with results in Chen *et al.* [6] which used  
732 geogrid to reinforce straight embankment. Their study suggests that placing  
733 geogrid at a higher level causes less vertical displacement than placing it  
734 close to the subgrade.



735 4. The geocell constrains the displacement of the encased ballast infill to form  
736 a relatively solid mattress. The mattress helps absorb overlying loads,  
737 increase the stiffness of the embankment, reduce spreading of the infill and  
738 balance forces in the embankment.

739

740 Whilst the study proposes a valid approach to demonstrate and examine the  
741 effects of reinforcing railway ballast with geocell, a number of limitations and  
742 assumptions were adopted to undertake successfully the DEM simulation:

743 1. The geocell model was calibrated solely against a series of tensile strength  
744 tests. Other properties such as puncture resistivity, flexural stiffness and  
745 torsion stiffness were not considered in the current study. Attempts will be  
746 made to incorporate these material properties in future studies to improve  
747 the reliability of the modeling framework.

748 2. Whilst the use of clumps provides a more accurate representation of ballast  
749 angularity, when compared with the adoption of entirely spherical particles,  
750 their shape does not fully reflect actual ballast angularities and, hence, have  
751 limited capability to simulate accurately ballast interlock and inter-particle  
752 friction. Defining the clumps as non-breakable in the simulation, may also  
753 result in overestimating the long-term performance of the embankment. It is  
754 plausible to conduct a 3D simulation of the embankment, but the scaled-  
755 down embankment may compromise the simulation accuracy.

756 3. In simulation, the ballast is calibrated against the monotonic test results. The  
757 calibration can possibly improve where cyclic loading test results are  
758 available and used. However, as stated in previous study [33], the

759 calibration against cyclic test results can be extremely time consuming. Due  
760 to this reason, this calibration step was neglected, enabling a focus on the  
761 simulation of ballast embankments.

762 4. Due to the limited number of load cycles applied to the embankment, the  
763 results presented may not accurately reflect the long-term performance of  
764 the ballast embankment. Along with advancement in PFC3D and  
765 computational capacity, this issue can be resolved in future studies. In  
766 addition, the number and location of inter-clump parallel-bond breakage,  
767 which can provide in-sight on the ballast re-arrangement, was not recorded.  
768 It will be taken into consideration in our future studies when ballast breakage  
769 is incorporated into the modeling framework.

770

#### 771 **Acknowledgement**

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773 assistance in this research.

774

#### 775 **Notation**

776  $D_0$  center-to-center distance of neighboring spheres, before displacement

777  $D_1$  center-to-center distance of neighboring spheres, after displacement

778  $D_{50}$  diameter of particles 50% finer by weight

779  $k_n$  normal stiffness

780  $k_s$  shear stiffness

781  $\bar{k}_n$  parallel-bond normal stiffness

782  $\bar{k}_s$  parallel-bond shear stiffness

783	$\hat{k}_s$	softening stiffness
784	$\hat{k}_n$	normal stiffness in tension
785	$P_L$	lateral load
786	$P_R$	resultant load
787	$P_V$	vertical load
788	$R$	track horizontal curve radius
789	$\bar{R}$	bond radius
790	$v$	train velocity
791	$\mu$	friction coefficient
792	$\theta$	angle of inclination
793	$\rho$	density
794	$\bar{\sigma}_c$	parallel-bond normal strength
795	$\bar{\sigma}_t$	tensile strength
796	$\bar{\tau}_c$	parallel-bond shear strength
797	$\zeta$	local damping coefficient
798	$\varepsilon$	geocell strain
799	$\varepsilon_y$	geocell yielding strain
800	$\varepsilon_f$	geocell failure strain

801

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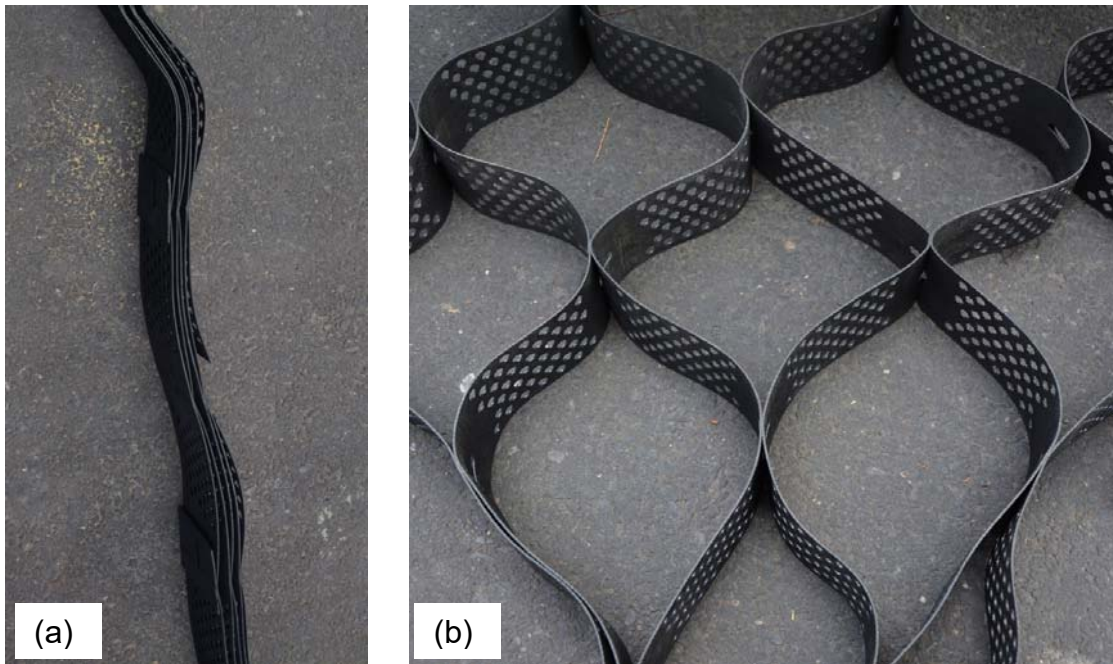
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896 **Figures**

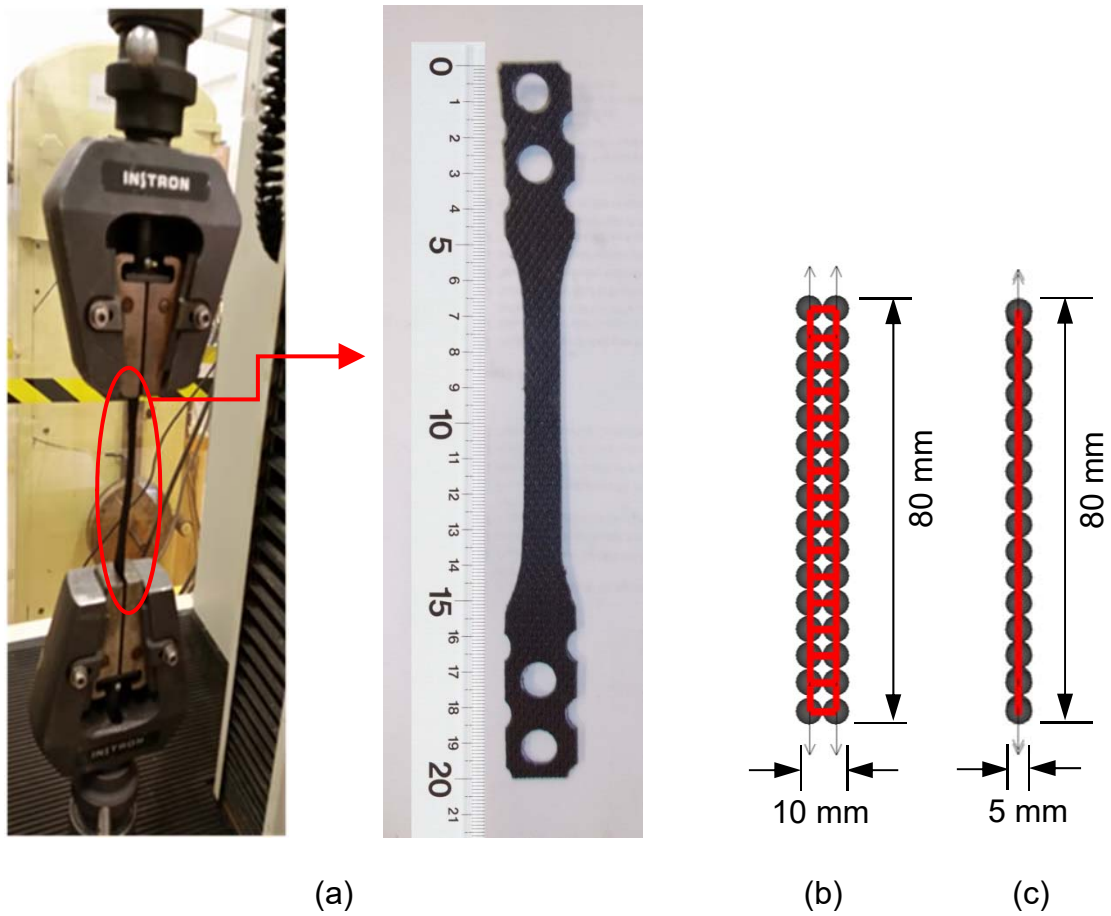


897 Fig. 1. Information on cell size and wall depth: (a) folded and (b) outstretched

898 ( $250 W \times 250 L \times 100 D$  mm for a cell).

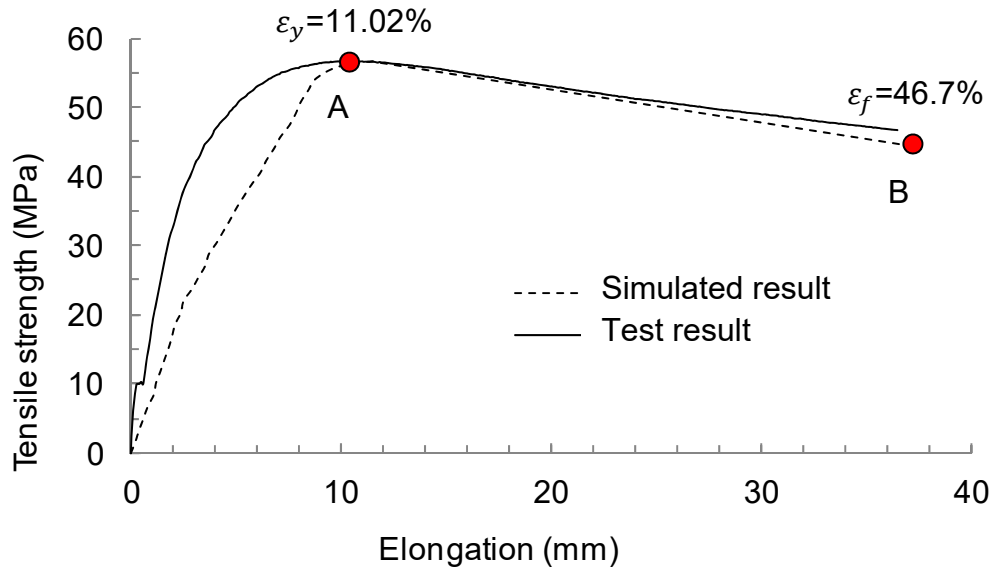
899





900 Fig. 2. Geocell tensile strength test: (a) setup and detail of representative tested  
 901 specimen; (b) front view in DE simulation; (c) side view in DE simulation.

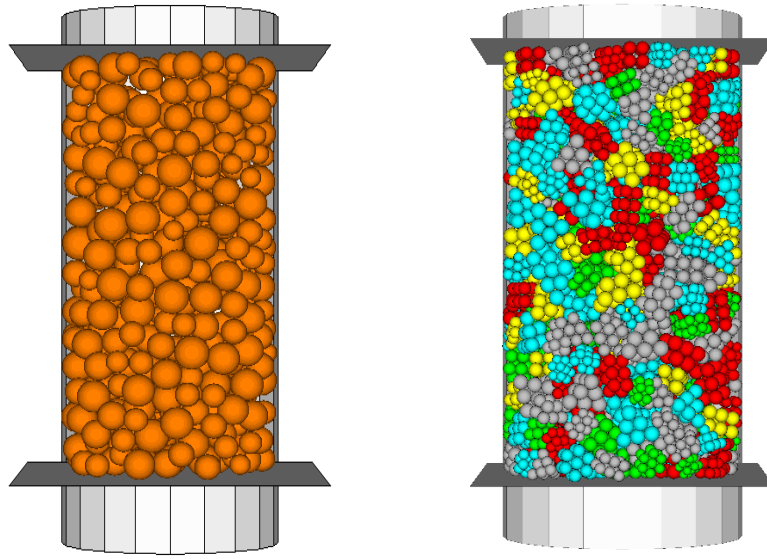
902



903

904 Fig. 3. Tensile strength of geocell specimen.

905

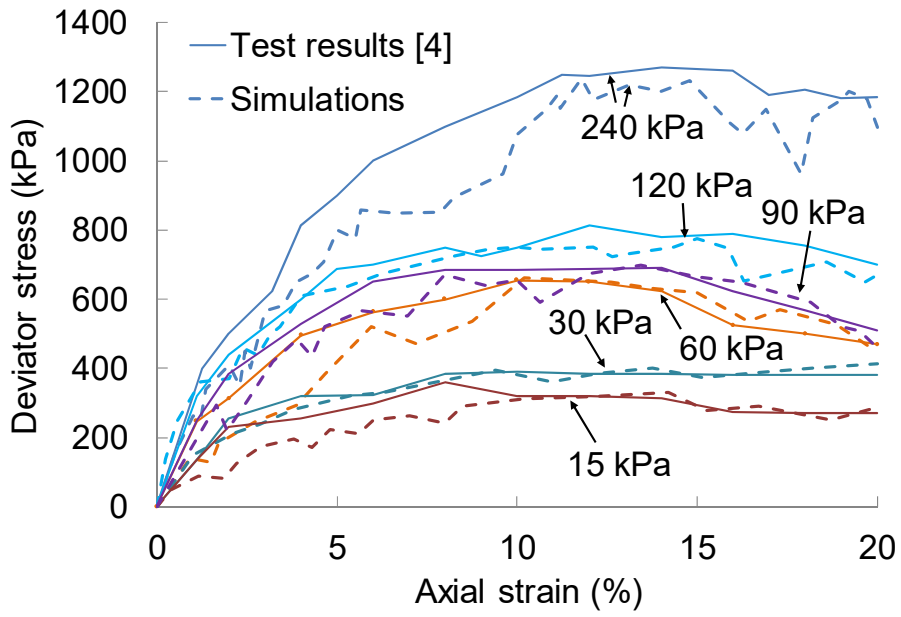


(a)

(b)

906 Fig. 4. Triaxial test specimen simulated by: (a) spheres; (b) clumps.

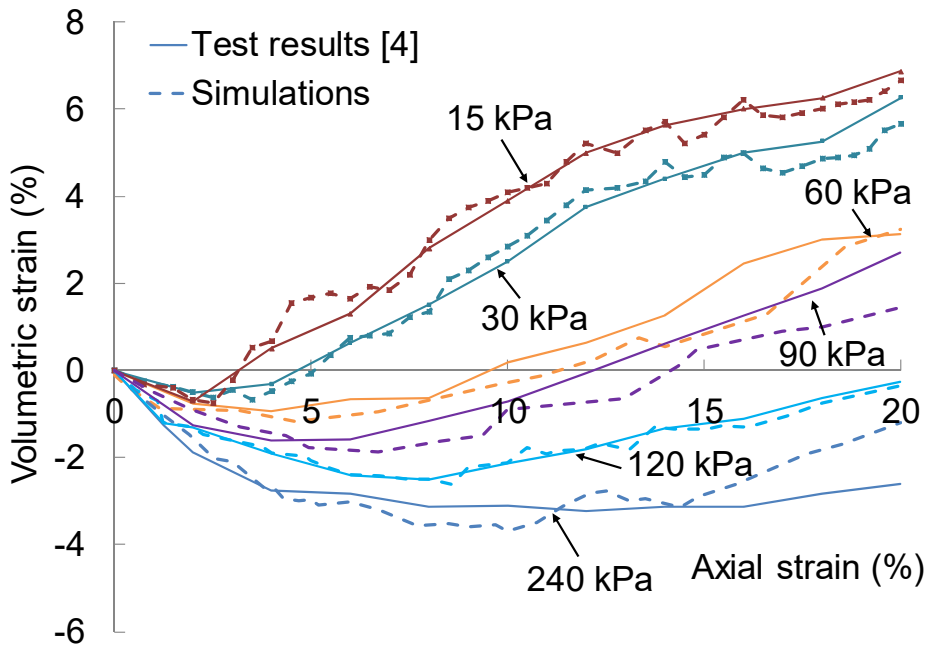
907



908

909

(a)



910

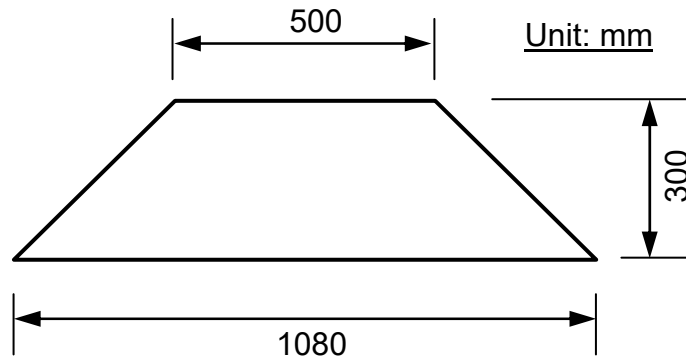
911

(b)

912 Fig. 5. Triaxial compression test results: (a) deviator stress vs. axial strain; (b)

913 volumetric strain vs. axial strain.

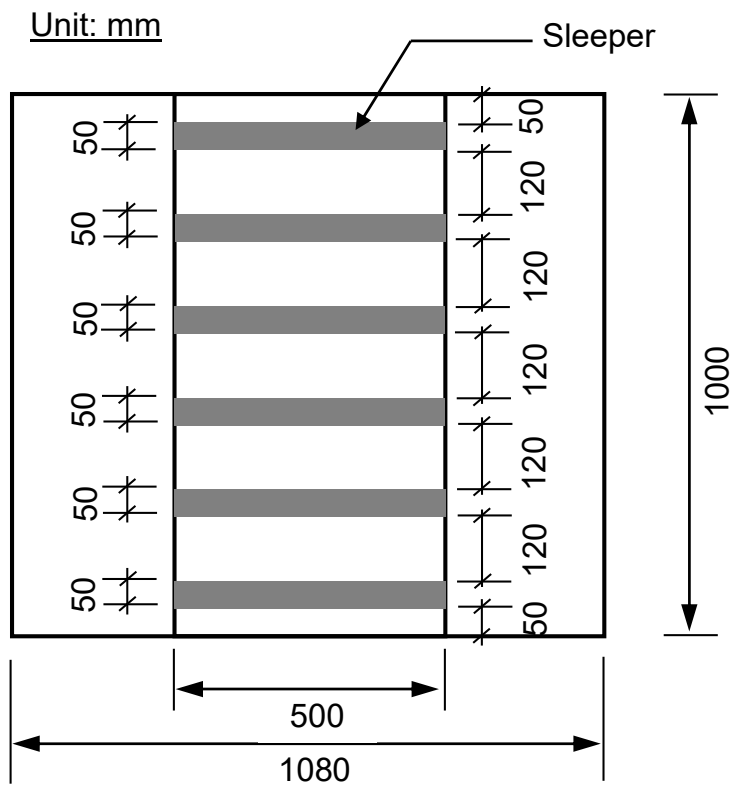
914



915

916

(a)



917

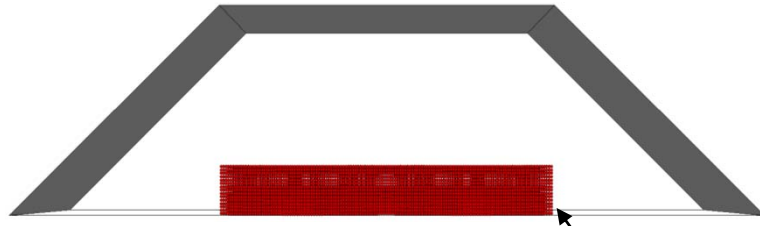
918

(b)

919 Fig. 6. Straight embankment: (a) cross section; (b) plan view.

920

921



922

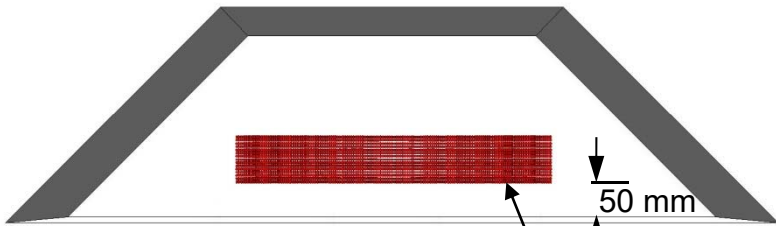
Geocell panel

923

(a)

924

925



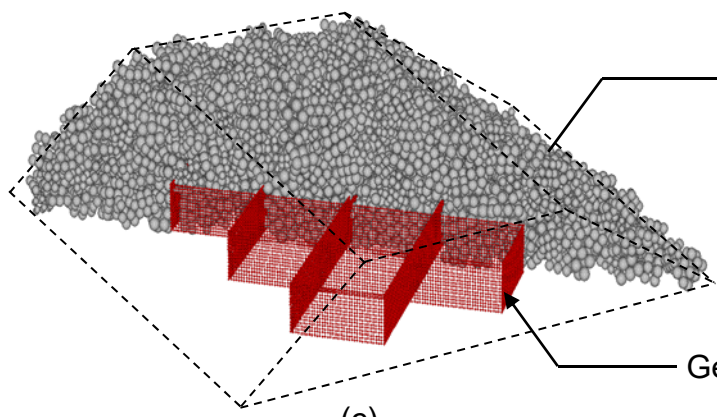
926

Geocell panel at  
50 mm level  
above base

927

(b)

928

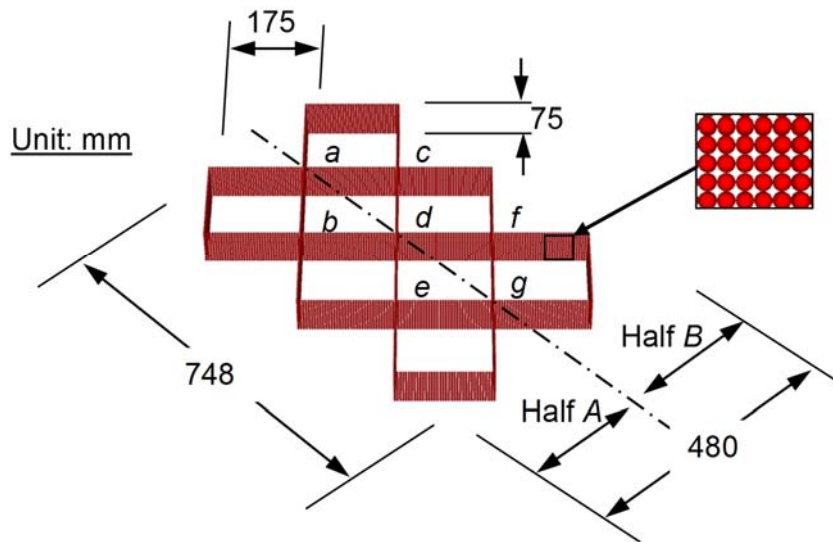


929

Ballast infills

Geocell panel

(c)



(d)

930

931

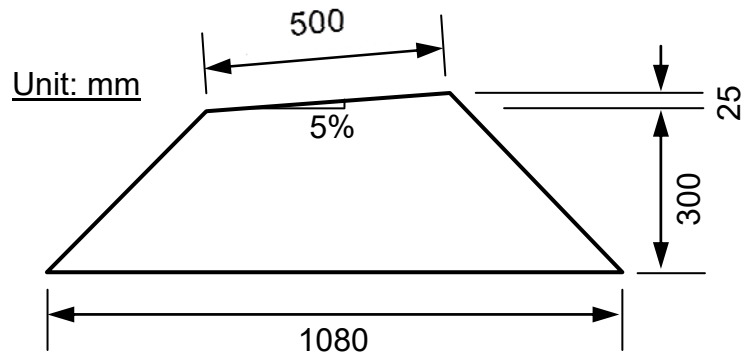
932 Fig. 7. Geocell panel: (a) at embankment base; (b) 50 mm above the base; (c)

933 3D perspective: infilled with ballast; and (d) 3D perspective: simulated using

934 spheres.

935

936



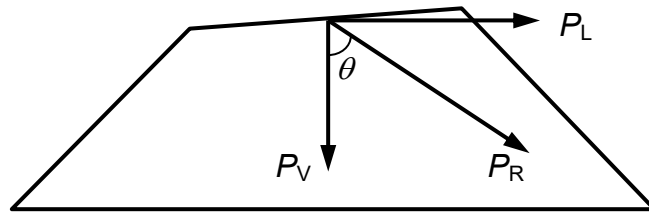
937

938 Fig. 8. Curved embankment cross section.

939



940

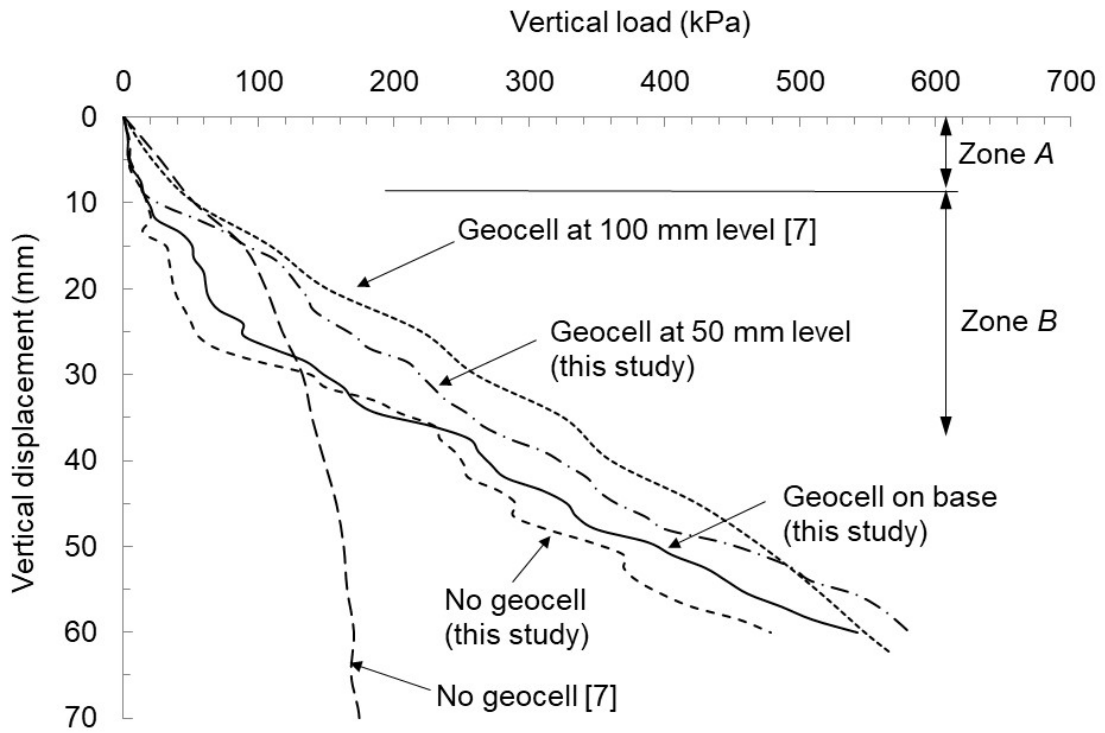


941

942 Fig. 9. Forces acted on curved embankment.

943

944

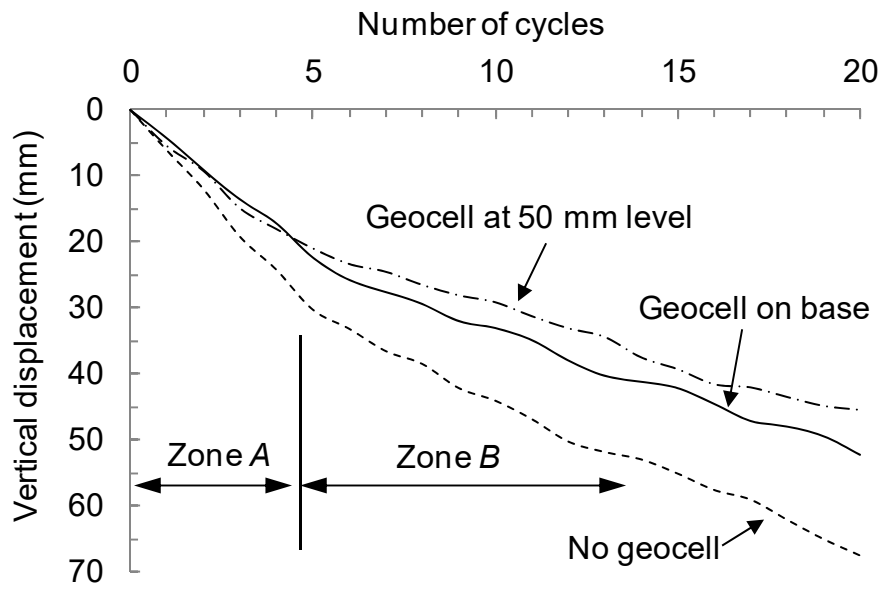


945

946 Fig. 10. Vertical displacement for straight embankment under monotonic

947 loading.

948

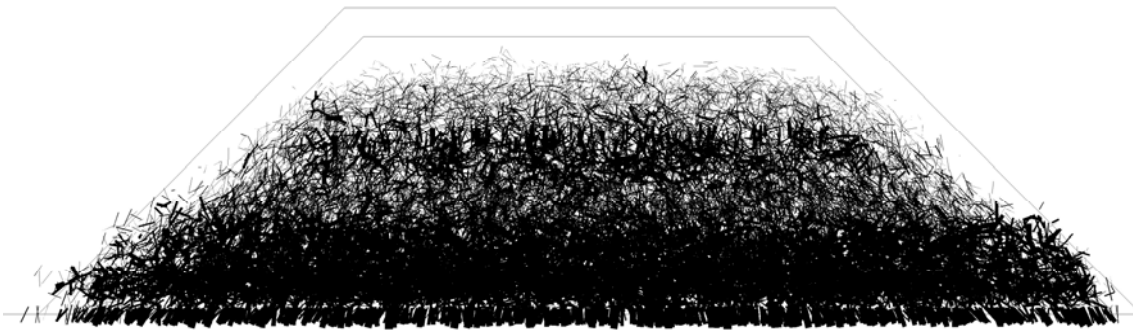


949

950 Fig. 11. Vertical displacement for straight embankment under cyclic loading.

951

952

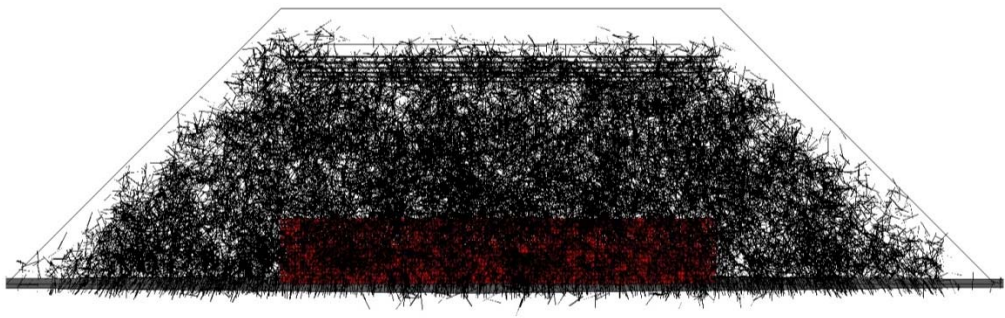


Max. Contact Force:  $7.55 \times 10^6$  N    Average Contact Force:  $4.77 \times 10^5$  N

953

(a)

954

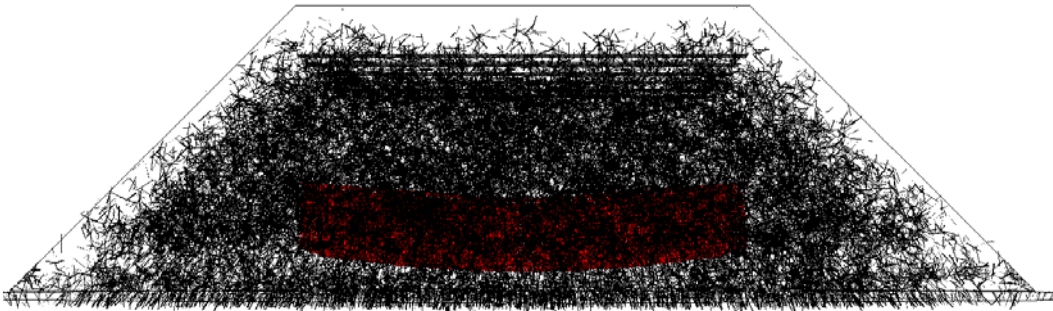


Max. Contact Force:  $8.74 \times 10^6$  N    Average Contact Force:  $5.34 \times 10^5$  N

955

(b)

956



Max. Contact Force:  $9.95 \times 10^6$  N    Average Contact Force:  $6.28 \times 10^5$  N

957

(c)

958

Fig. 12. Contact forces drawn at the same scale for straight embankment after

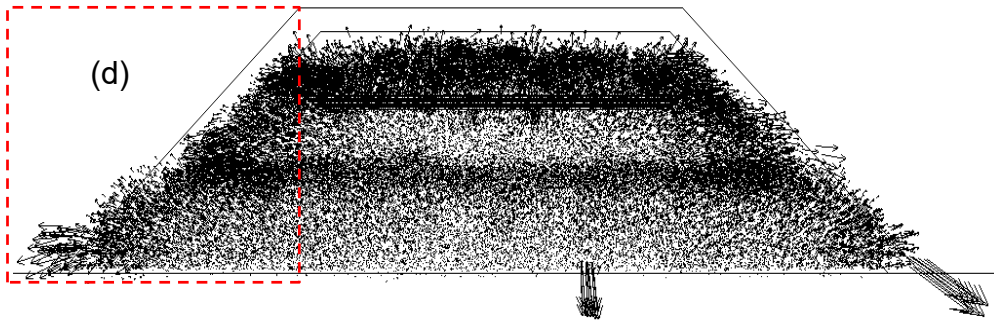
959

the 20<sup>th</sup> cycle: (a) unreinforced; (b) geocell at base; and (c) geocell 50 mm

960

above the base.

961

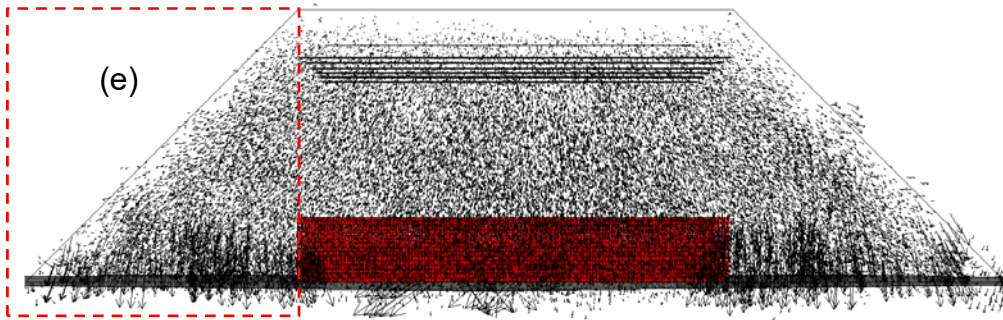


962

Max. Displacement: 33.1 mm Average Displacement: 18.1 mm

963

(a)

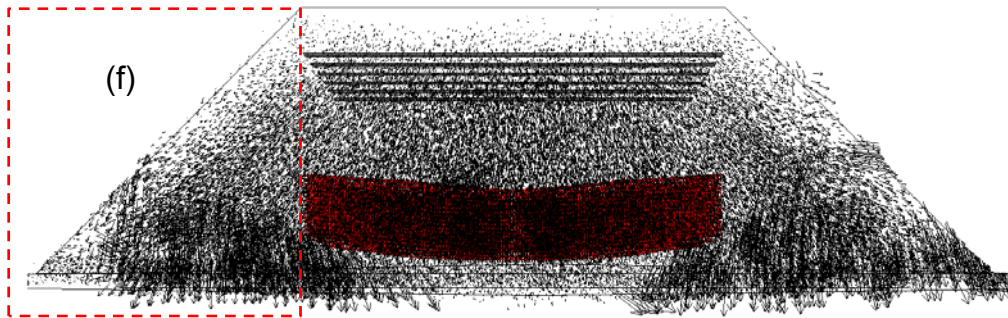


964

Max. Displacement: 16.5 mm Average Displacement: 9.4 mm

965

(b)

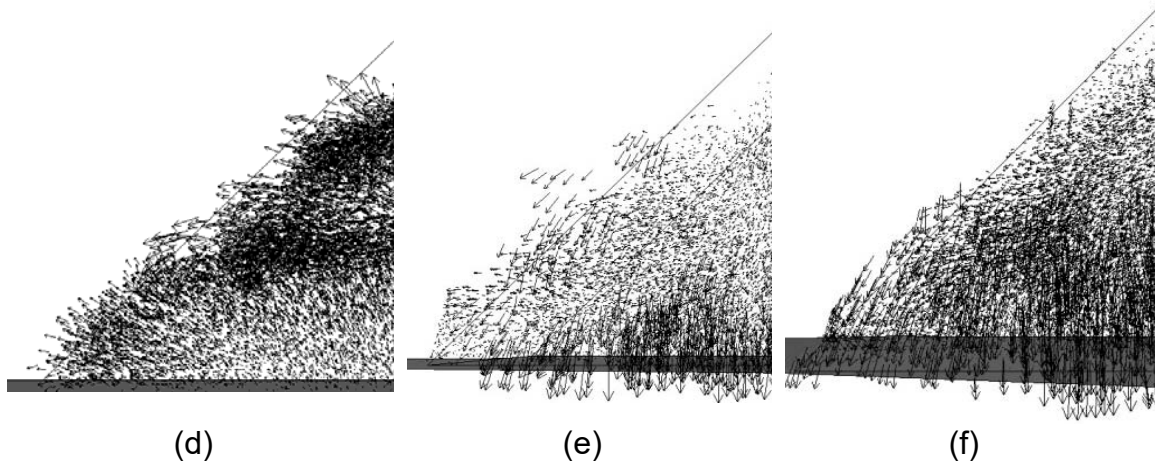


966

Max. Displacement: 13.4 mm Average Displacement: 7.2 mm

967

(c)



968 Fig. 13. Total displacement vectors drawn at the same scale for straight  
 969 embankment after the 20<sup>th</sup> cycle: (a) unreinforced; (b) geocell at base; (c)  
 970 geocell 50 mm above the base; (d–f) zoomed-in views of the left-hand-side  
 971 unreinforced sections of three embankments.

972

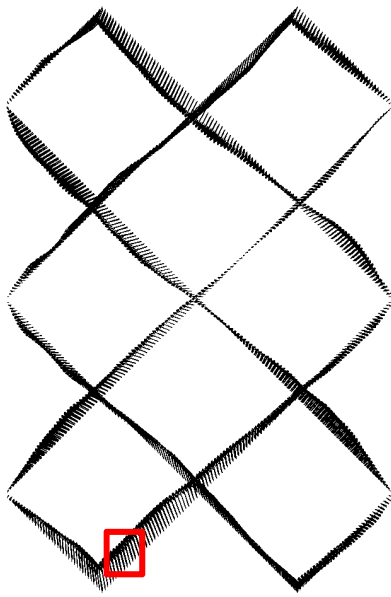
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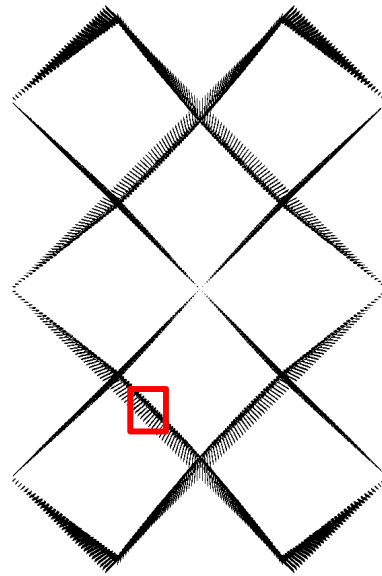
976

977



Max. Displacement: 8.5 mm

(a)



Max. Displacement: 10.5mm

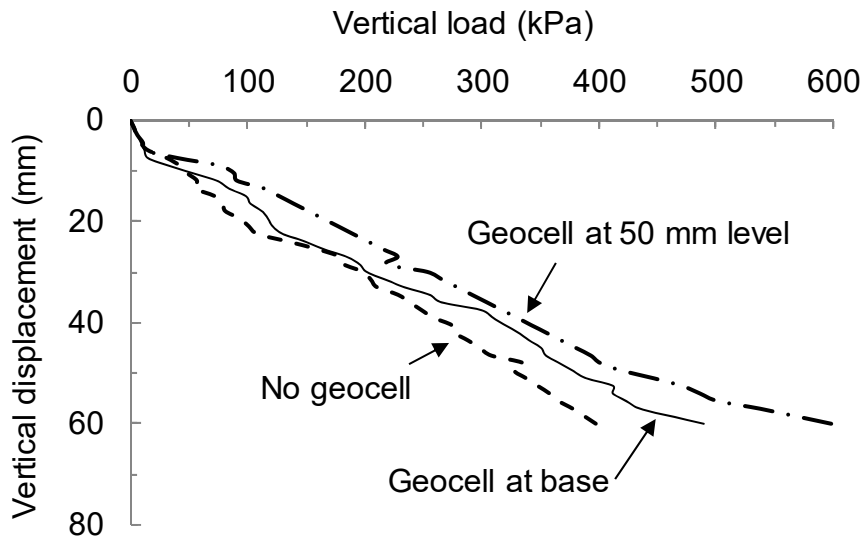
(b)

978

979

980 Fig. 14. Total displacement vectors drawn at the same scale for geocell panel  
 981 after the 20<sup>th</sup> cycle: (a) geocell on base; (b) geocell at 50 mm above the base.

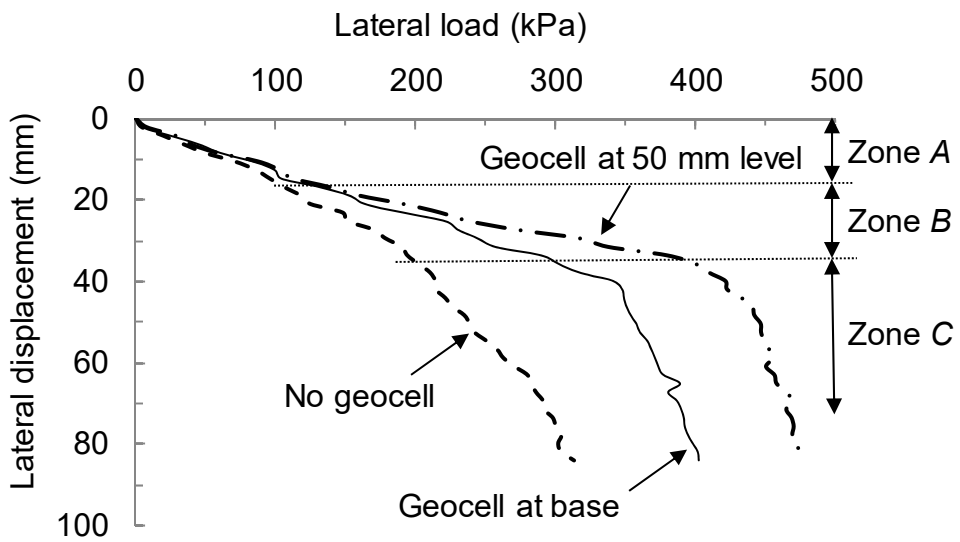
982



983

984

(a)



985

986

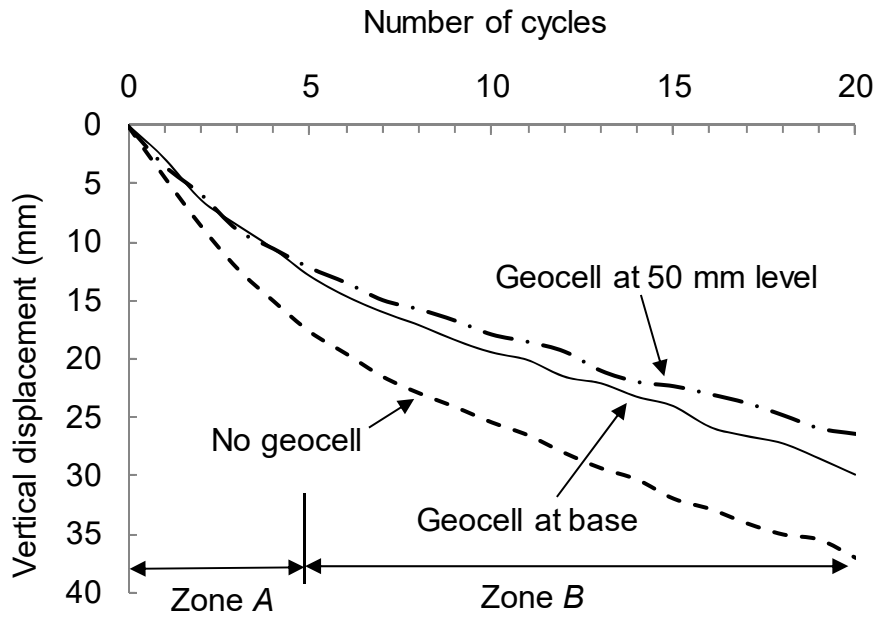
(b)

987 Fig. 15. Monotonic loading-induced sleepers movement in curved embankment:

988 (a) Vertical displacement, (b) lateral displacement.

989

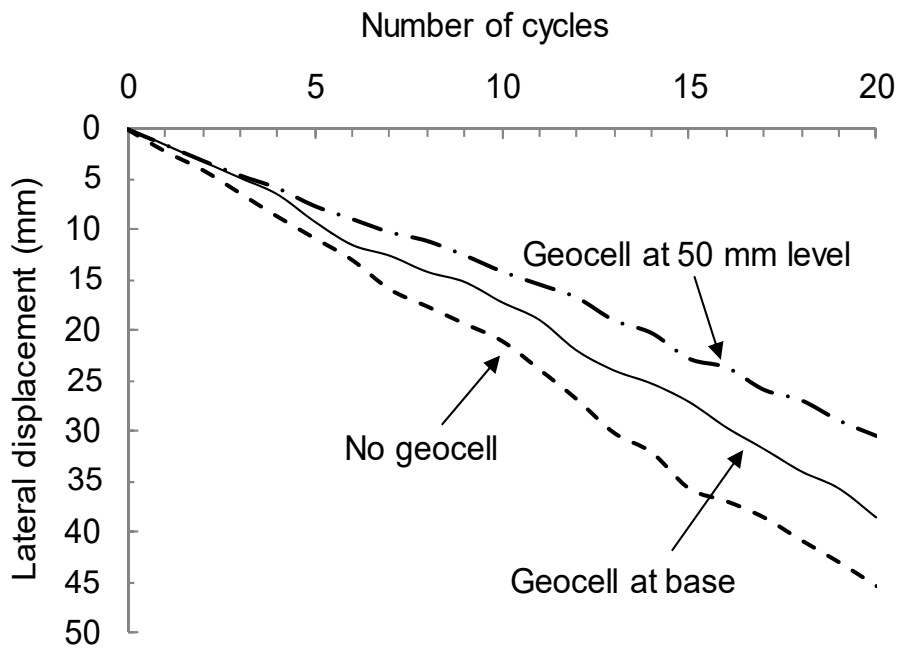




990

991

(a)



992

993

(b)

994 Fig. 16. Cyclic loading-induced sleepers movement in curved embankment: (a)

995 vertical displacement; (b) lateral displacement.

996

997

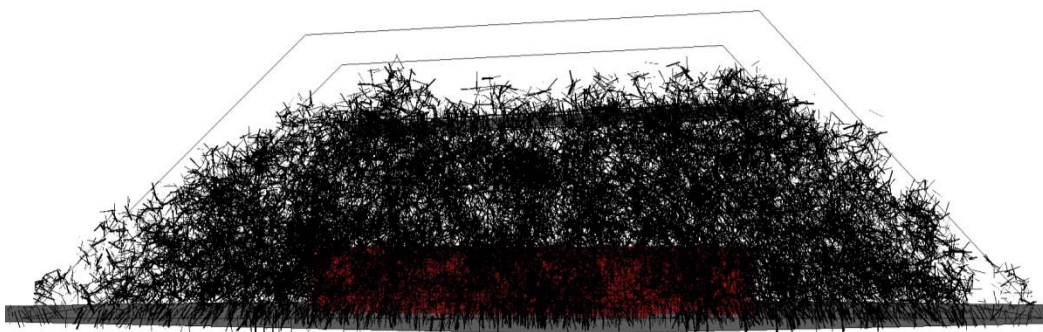


Max. Contact Force:  $8.73 \times 10^6$  N    Average Contact Force:  $4.74 \times 10^5$  N

998

(a)

999



Max. Contact Force:  $9.38 \times 10^6$  N    Average Contact Force:  $6.28 \times 10^5$  N

1000

(b)

1001



Max. Contact Force:  $1.23 \times 10^7$  N    Average Contact Force:  $8.51 \times 10^6$  N

1002

(c)

1003

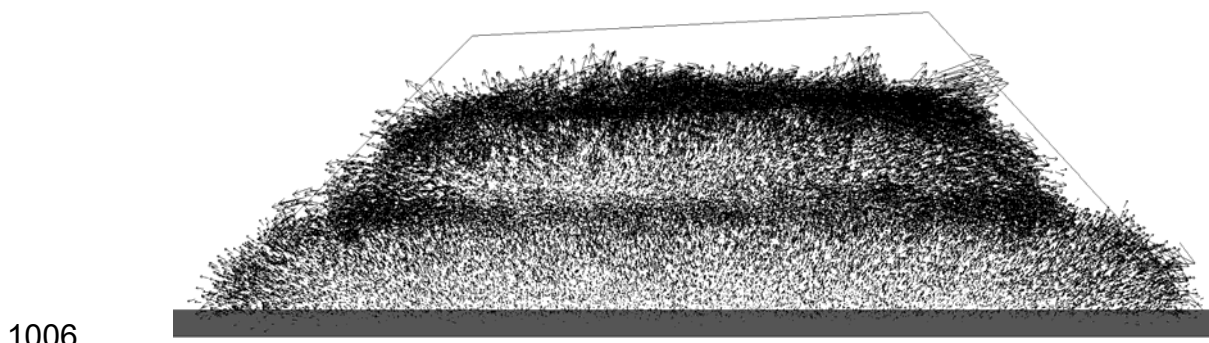
Fig. 17. Contact forces drawn at the same scale for the curved embankment

1004

after the 20<sup>th</sup> cycle: (a) unreinforced; (b) geocell at base; and (c) geocell 50mm

1005

above the base.

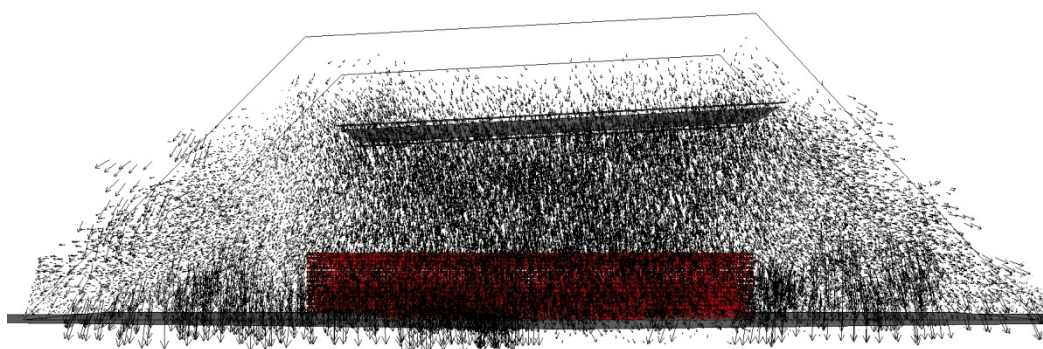


1006

Max. Displacement: 33.1 mm Average Displacement: 22.2 mm

1007

(a)

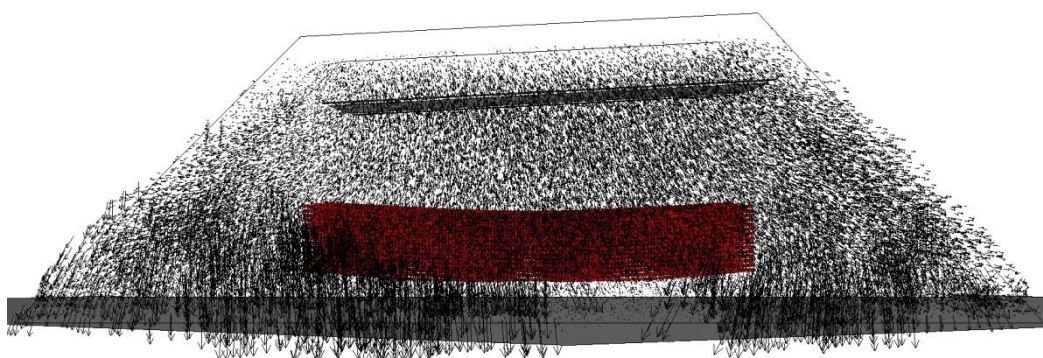


1008

Max. Displacement: 16.1 mm Average Displacement: 10.7 mm

1009

(b)



1010

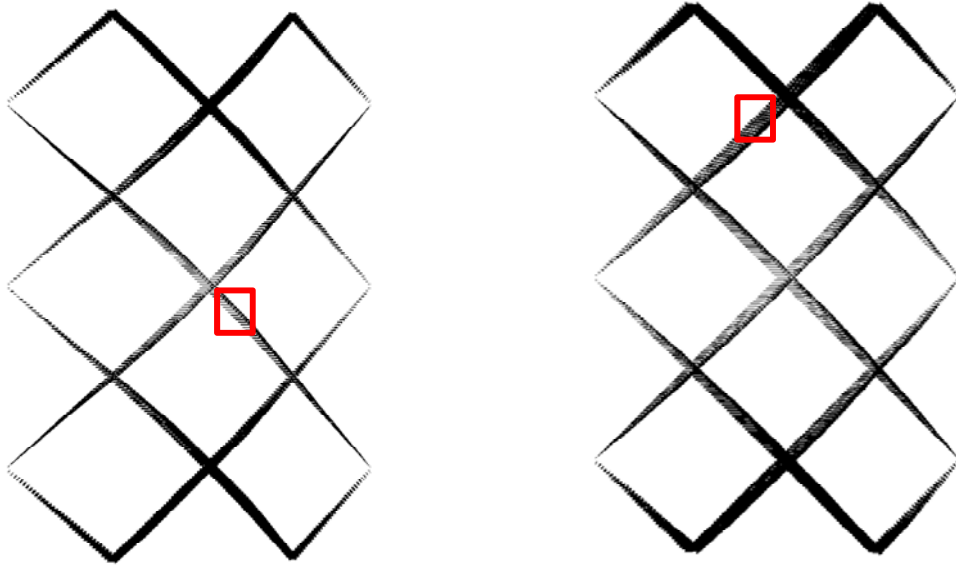
Max. Displacement: 14.3 mm Average Displacement: 9.6 mm

1011

(c)

1012 Fig. 18. Total displacement vectors drawn at the same scale for the curved  
 1013 embankment after the 20<sup>th</sup> cycle: (a) unreinforced; (b) geocell at base; and (c)  
 1014 geocell 50mm above the base.

1015



1016

Max. Displacement: 7.7 mm

Max. Displacement: 11.4 mm

1017

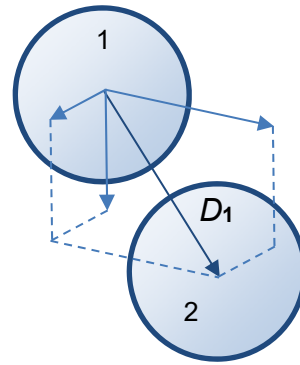
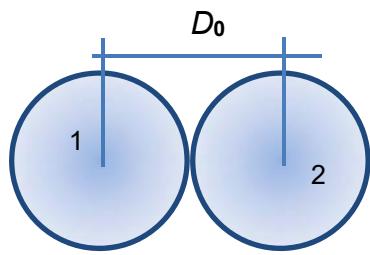
(a)

(b)

1018 Fig. 19. Total displacement vectors drawn at the same scale for geocell panel

1019 after the 20<sup>th</sup> cycle: (a) geocell on base; (b) geocell at 50 mm above the base.

1020



$$\varepsilon = \frac{(D_1 - D_0)}{D_0} \times 100\%$$

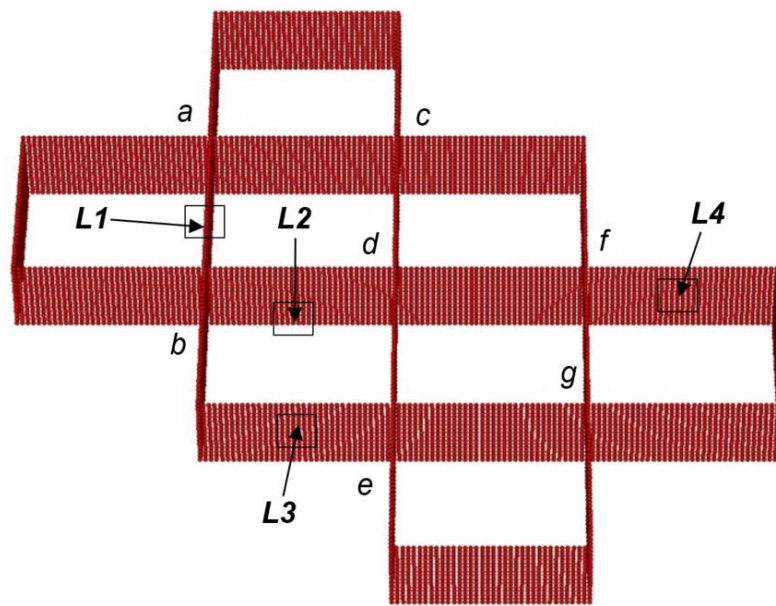
(a)

(b)

1021 Figure 20 Illustration on the calculation methodology of strain in geocell: (a)

1022 before displacement; (b) after displacement.

1023



1024

1025 Figure 21 Locations of maximum strain.

1026

1027

1028 **Tables**

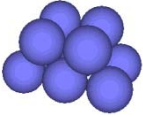
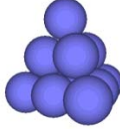
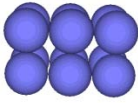
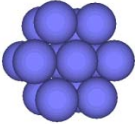
1029 Table 1. Micro-properties for geocell

Micro-property	Value
Density $\rho$ (kg/m <sup>3</sup> )	$1.0 \times 10^3$
Normal stiffness $k_n$ (N/m)	$3.2 \times 10^3$
Shear stiffness $k_s$ (N/m)	$3.2 \times 10^3$
Parallel-bond normal stiffness $\bar{k}_n$ (N/m <sup>3</sup> )	$2.8 \times 10^4$
Parallel-bond shear stiffness $\bar{k}_s$ (N/m <sup>3</sup> )	$4.5 \times 10^4$
Parallel-bond normal strength $\bar{\sigma}_c$ (N/m <sup>2</sup> )	$6.8 \times 10^4$
Parallel-bond shear strength $\bar{\tau}_c$ (N/m <sup>2</sup> )	$6.5 \times 10^4$
Parallel-bond radius $\bar{R}$ (mm)	2.5
Tensile strength $\bar{\sigma}_t$ (N/m <sup>2</sup> )	$5.598 \times 10^4$
Softening stiffness $\hat{k}_s$ (N/m <sup>3</sup> )	$2.75 \times 10^4$
Normal stiffness in tension $\hat{k}_n$ (N/m)	$3.2 \times 10^4$
Friction coefficient $\mu$	0.3

1030

1031

1032 Table 2. Clump templates developed for ballast

Clump template	Geometry	Number of spheres
Trapezoidal		10
Triangular		10
Rectangular		12
Hexagonal		14

1033

1034



1035 Table 3. Micro-properties for ballast clumps

Micro-property	Value
Density $\rho$ (kg/m <sup>3</sup> )	$2.5 \times 10^3$
Normal stiffness $k_n$ (N/m)	$5 \times 10^9$
Shear stiffness $k_s$ (N/m)	$5 \times 10^9$
Parallel-bond normal stiffness $\bar{k}_n$ (N/m <sup>3</sup> )	$1.8 \times 10^5$
Parallel-bond shear stiffness $\bar{k}_s$ (N/m <sup>3</sup> )	$1.8 \times 10^5$
Parallel-bond normal strength $\bar{\sigma}_c$ (N/m <sup>2</sup> )	$6 \times 10^{10}$
Parallel-bond shear strength $\bar{\tau}_c$ (N/m <sup>2</sup> )	$6 \times 10^{10}$
Parallel-bond radius $\bar{R}$ (mm)	1.0
Frictional coefficient $\mu$	1.0

1036

1037

1038 Table 4. Geocell panel strains

Position	Strain (%)			
	Monotonic loading		Cyclic loading	
	Geocell	Geocell 50 mm	Geocell	Geocell 50 mm
	at base	above base	at base	above base
Junction <i>a</i>	29.1	34.1	10.9	11.3
Junction <i>b</i>	39.2	39.8	15.6	14.8
Junction <i>c</i>	39.2	21.1	18.7	18.3
Junction <i>d</i>	24.1	29.7	13.3	13.4
Junction <i>e</i>	30.2	38.9	12.0	10.2
Junction <i>f</i>	41.6	41.2	18.0	15.3
Junction <i>g</i>	26.9	33.0	16.2	10.0
Panel half <i>A</i>	18.9	35.3	19.8	16.1
Panel half <i>B</i>	24.4	39.1	27.0	22.7
Maximum strain	39.7	45.5	28.4	23.4

1039