AUTOMATION OF BUS CREW ROSTERS

by

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SUMMARY

This thesis is concerned with the problem of rostering operating crews for a city transportation system and in particular with the mathematical formulation of constructing an automated week day operator's roster.

The problem of constructing late p.m. straight shifts is discussed and formulated as a heuristic programme. Methods for forming straight shifts consisting of two or three pieces of work are derived.

The other problem considered is the formulation of a basic framework which defines the structure of the a.m. straight shifts and broken shifts. A technique for classifying the type of shifts necessary over the off-peak period is also discussed.

SIGNED STATEMENT

This thesis contains no material which has been accepted for the award of any other degree or diploma in any University. To the best of my knowledge and belief, the thesis contains no material previously published or written by any other person, except where due reference is made in the text of the thesis.

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(R.M. POTTER)

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CHAPTER I

INTRODUCTION

The movement of vehicles and the allocation of duties of operating staff of many transport industries must be planned in advance. A time-table defines the vehicle movement and a roster specifies the duties.

The logical sequence in which bus operations are usually planned is as follows. First, the time-table is developed from consideration of desirable route structure, public demand, capital available for investment in equipment and operations and the policy to be adopted in meeting the demand and carrying out the operations.

When the time-table is developed and rules for operation have been formulated it is possible to produce a schedule of shifts to be worked by the drivers and conductors.

An operator shift schedule is determined by run-cutting. It is necessary to cut the runs defined by the time-table into pieces of work which can be combined to form man-shifts. The two usual types of shift allowed are broken and straight shifts. A broken shift consists of two pieces of work, one covering the a.m. peak period and the other covering the p.m. peak period. A straight shift however consists of one piece of work broken only to allow time for a crib or meal.

For a large transportation enterprise, the scheduling problem is very formidable and although data processing techniques are commonly used as an aid in the preparation of time-tables and rosters, only little progress has been reported in the completely automated preparation of the operator's week-day roster. A significant challenge is to construct a computer programme which will produce in a short time schedules which are better or at least as good as those constructed by the schedule officers. These men base their methods on years of experience and their knowledge is hard to define explicitly and hard to surpass.

There is very little literature available on the automatic construction of daily work rosters. Elias [2], [3] describes a computer simulation of clerical procedures used by a few American Transit Companies with mixed success. Martinello and Biffignandi [9] report on the computer procedure used in Turin and Moss [10] describes the use in Hamburg of a mathematical technique which ensures, if possible, that shifts consisting of two sections satisfy the regulations governing working conditions.

This thesis describes methods used to prepare an automated operator's week-day roster for the bus system in the city of Adelaide.*

* A description of this bus system and the agreements which must be satisfied is given in Appendix I.

The following steps are used in the run-cutting procedure:

(1) construction of the late finishing straight work schedule;

(2) calculation of the minimum number and type of work schedules necessary to man runs left from (1);

(3) construction of morning straight work schedules;

(4) allocation of the remaining duty to broken work schedules.

The construction of the late finishing straight-work schedules, i.e. p.m. straight shifts, can be divided into the following stages:

(i) grouping the runs so that the minimum number of extra men are needed to cover the crib periods;

(ii) forming late operators' shifts consisting of two pieces of work;

(iii) forming late operators' shifts consisting of three pieces of work.

The contents of the chapters of this thesis are as follows. In Chapter 2 the problem of choosing the least number of men to cover the late operator cribs is formulated and a sequential stepping algorithm is discussed. The computer preparation of completed p.m. straight shifts consisting of two or three pieces of work is described in Chapters 3 and 4.

The assessment of the number and type of staff needed to cover the work left over from the preparations of p.m. straight shifts is discussed in Chapter 5.

Chapter 6 contains the method used to produce the most efficient off-peak roster.

The thesis is concluded with a short discussion chapter.

CHAPTER II

THE SEQUENTIAL STEPPING PROCEDURE

The agreement (8) of Appendix 1 states that operators must have a meal break or crib within five hours of starting or finishing their duty. Therefore, if buses are to be kept in continuous operation, provision must be made for replacing men taking cribs or meal breaks. This can be done economically by arranging for a man finishing a crib to replace a man starting a crib. This second man at the end of his crib would then replace an operator starting his crib, and so on. This procedure is termed sequential stepping and is illustrated in figure 2.1.

2.1 Use of Broken Shift Operators

An important consideration in sequential stepping is the use of broken shift men (illustrated in figure 2.2). Provided that the crib of a straight shift operator does not finish after the time limit of agreement (29) for broken shifts, viz. 8 p.m., the crib of a straight shift operator can be covered by a broken shift operator before he goes off duty. However, if the straight shift crib ended later than the limit of broken shifts, it would have to be covered by a straight shift driver. Use of broken shift men to cover cribs facilitates preparation of rosters, and the sequential stepping procedure is started for runs in which the crib ends after the broken shift limit.

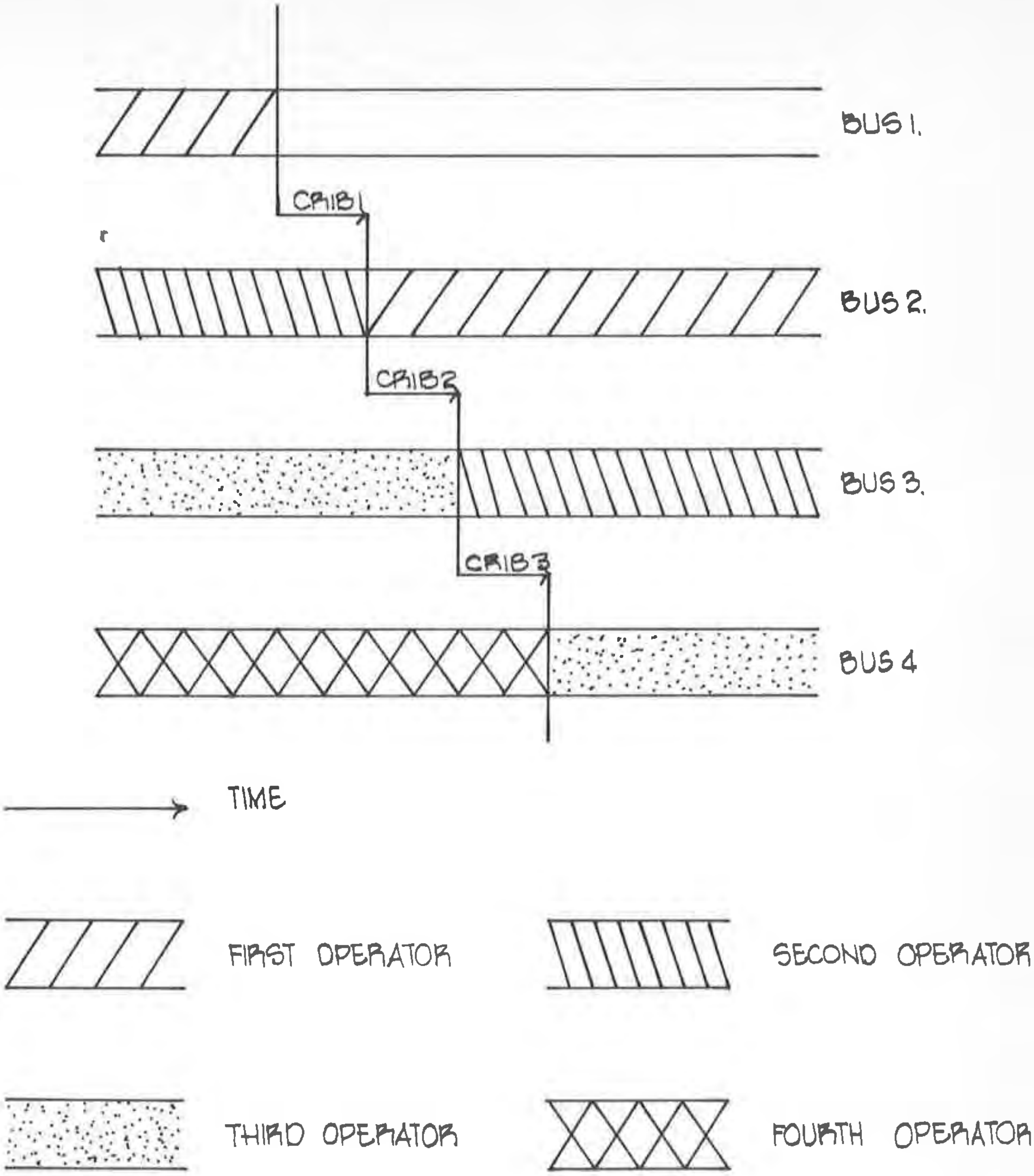

FIGURE 2.1 THE SEQUENTIAL STEPPING PROCEDURE

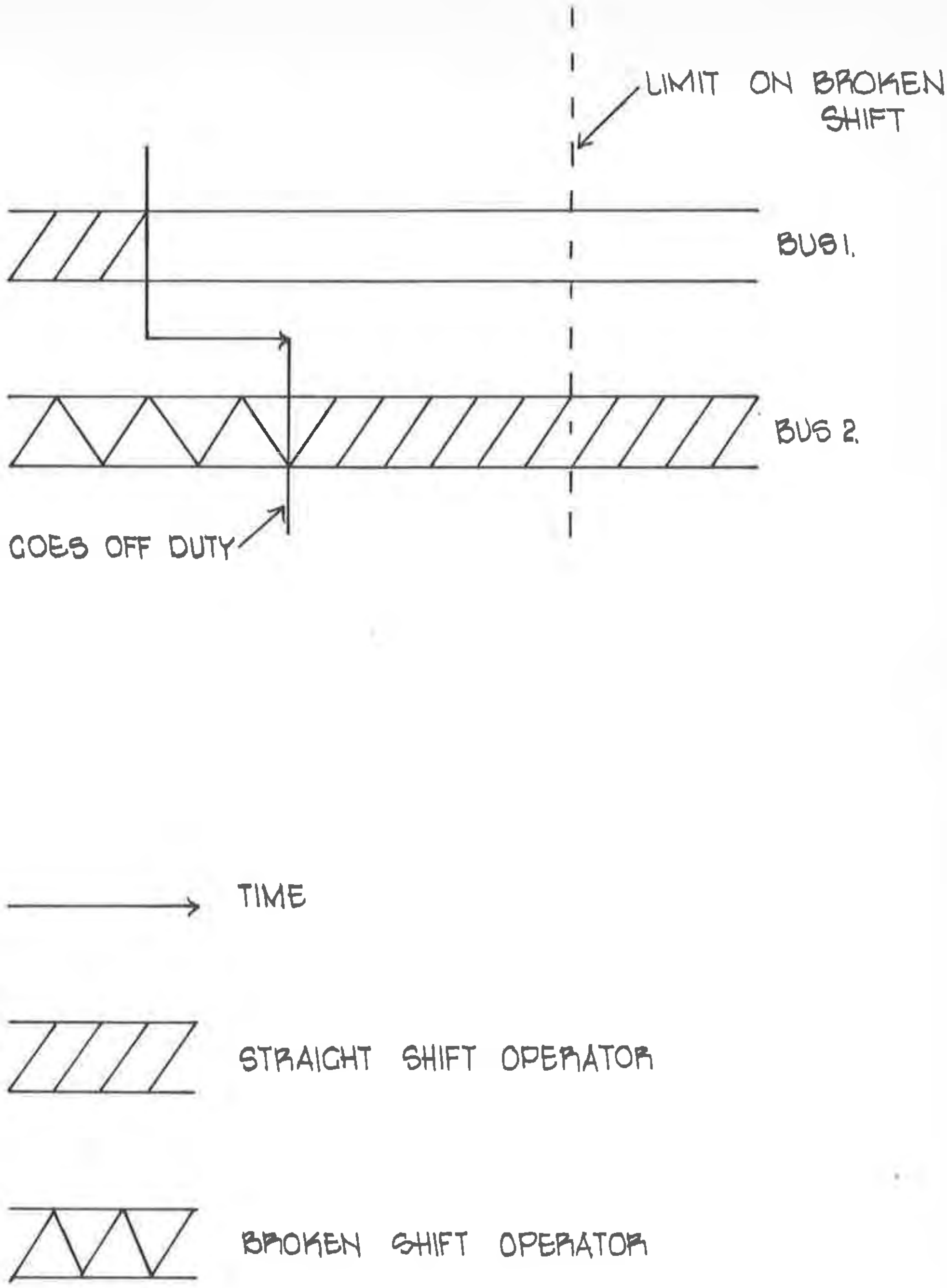

FIGURE 2.2 THE USE OF A BROKEN SHIFT OPERATOR IN SEQUENTIAL STEPPING

2.2 Classification of the Sequential Groups

The considerations given above lead to the requirement that the initial problems in schedule mechanization shall be the formation of groups of sequentially stepped runs for the evening straight shifts.

Bus runs are divided into three groups depending on the proximity of their relief points to the three depots as follows:-

Group A:

Runs with Hackney relief points (relief points 1,2)

Group B:

Runs with Port relief points (relief points 14,15,...,20)

Group C:

Runs with City relief points (relief points 5,6,...,13).

The number of steps in a sequential group is limited on account of the length of crib and that there is a minimum amount of duty which must be paid for in a straight shift. In practice there are never more than nine steps. The smallest number of groups possible so far has been found to be 14. (The fewer the groups the less extra men needed to be rostered to cover the p.m. straight shift cribs).

2.3 Selection of the Initial Relief Time

The first task in sequential stepping is to find the relief time for the run on which the first group of steps will be based. The following conditions must be met:

(i) the point at which it is intended to relieve the operator must be one of the specified relief points for the group being considered,

i.e. $rp_{min,m} \leqslant rp_{ij} \leqslant rp_{max,m}$ (2.3.1)

where $rp_{min,m}$: smallest relief point specified for group m,

$rp_{max,m}$: largest relief point specified for group m,

$m = 1,2,\ldots,14$;

(ii) the relief time, rt_{ij}, associated with the relief point rp_{ij} defined in (2.3.1) must be such that the remainder of the shift including the sign off allowance does not exceed the five hour maximum of agreement (8),

i.e. $rt_{ij} \geqslant at_i + SOF - 300 + tt_{cd,rp_{ij}}$ (2.3.2)

where SOF = 25 mins., from the agreement (24) and cd is the depot closest to rp_{ij};

(iii) the relief time, rt_{ij}, must be later than the latest sign off time of a broken shift operator,

i.e. $rt_{ij} \geqslant 8$ p.m. $- SOF_1$ (2.3.3)

where $SOF_1 = 15 + tt_{cd,rp_{ij}}$, from the agreement (26).

Let A(i) be a list of first relief times of runs which satisfy the equations (2.3.1), (2.3.2) and (2.3.3), where i is a count of such runs.

Thus when the list A has been completed, the earliest time is selected as the initial relief time of the first group, i.e. define relief time, rt_z, of the initial run, rn_z, by

$$rt_z = \min_i A(i) \qquad (2.3.4)$$

where z is the position of this run in list L_{ijk}.

If there is a run which satisfies equations (2.3.1) and (2.3.2) but not equation (2.3.3) its crib can be covered by a broken shift operator.

The values of rt_{ij}, rp_{ij} and rn_i for the runs satisfying equations (2.3.1) and (2.3.2) are stored in $JT(\ell)$, $JP(\ell)$, $JG(\ell)$ respectively where ℓ is a count of all such runs.

A flow diagram setting out the logic used is shown in figure 2.3.

2.4 Construction of the First Sequential Group

Having selected the initial relief time, the next stage is to set up the sequential group associated with this relief time. Runs which meet the following conditions are possibilities for the second run of the first sequential group:

(i) the point at which it is intended to relieve the operator must be one of the specified relief points for the group being considered,

i.e. $$rp_{min,m} \leqslant rp_{ij} \leqslant rp_{max,m} \; ; \qquad (2.4.1)$$

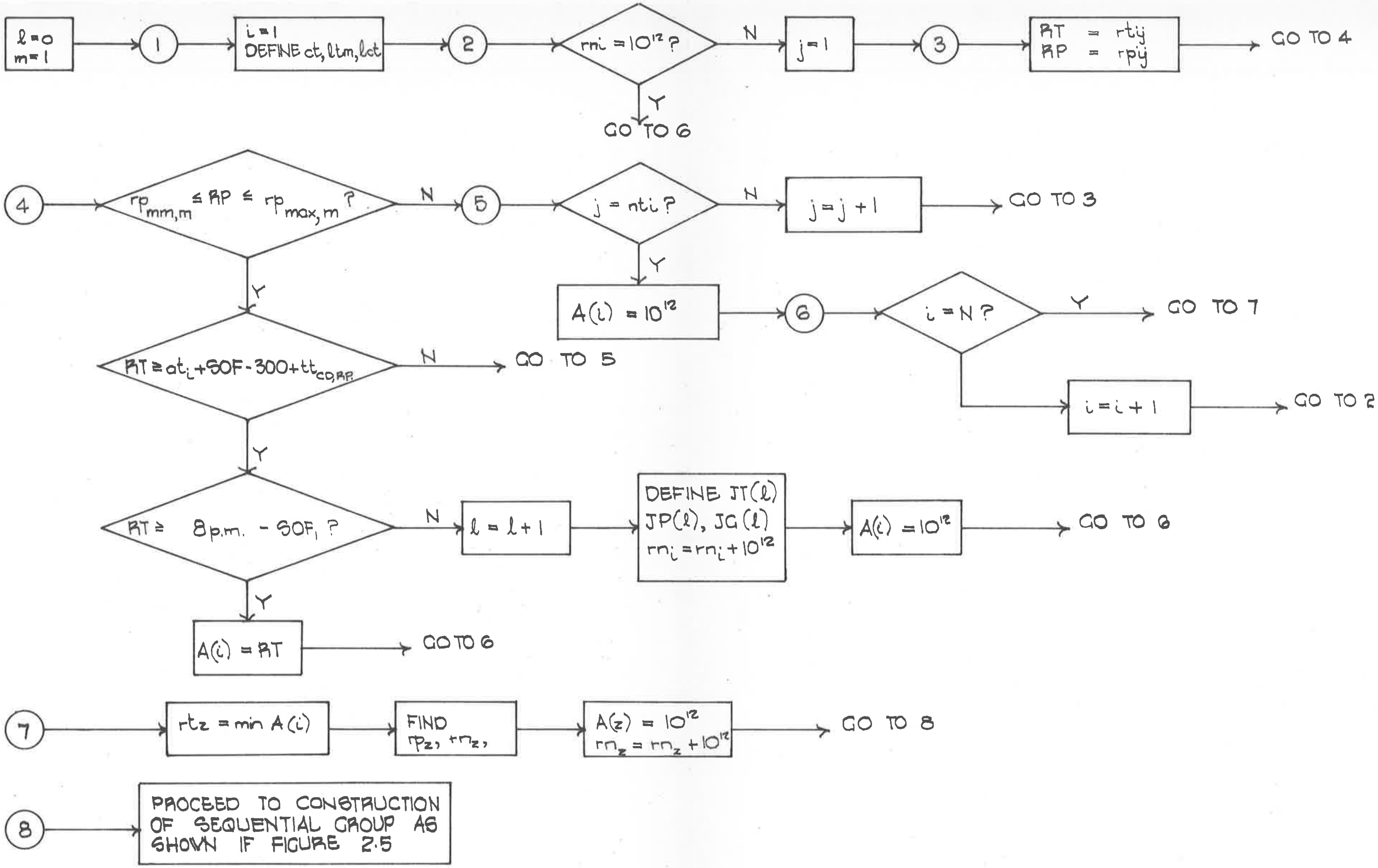

FIGURE 2.3 THE LOGIC FOR DETERMING THE FIRST RELIEF TIMES.

(ii) the length of the crib must be greater than the minimum allowable crib time plus the time needed to travel from the relief point of the first run to the nearest depot plus the time needed to travel from this depot to the relief point on the second run,

i.e. $rt_{ij} - rt_z \geqslant ct + tt_{cd,rp_z} + tt_{cd,rp_{ij}}$ (2.4.2)

where ct = 20 mins., from the agreement (10);

(iii) the run must not be one of those already chosen for relief by broken shift runs, or already used. Once a run has been used, a switching device defined by the function irn_i is used, where

$irn_i < 10^{12}$: bus i has not been used
(2.4.3)
$irn_i \geqslant 10^{12}$: bus i has been used.

Initially $irn_i = rn_i$ and irn_i is set equal to $rn_i + 10^{12}$ when bus i has been used;

(iv) in order that the maximum number of runs may be used to form a sequential group it was found that those runs which end relatively early, should not be used at the beginning of the sequential group,

i.e. if $b = 2$ then $at_i \geqslant \ell t_m$ (2.4.4)

where ℓt_m is the latest sign off time for runs in group m. It should be noted that this time ℓt_m is completely arbitrary and may be varied from one group of sequential runs to another.

Let B(i) be a list of the first relief times, rt_{ij}, of runs which satisfy the equations (2.4.1), (2.4.2), (2.4.4), where i is a count of all such runs.

When the list B has been completed, the earliest time is selected as the second relief time of the first group.

i.e. define relief time, rt_y, of the next run, rn_y, by

$$rn_y = \min_i B(i)$$

where y gives the position of the run in list L_{ijk}.

The next step is to identify the third relief point. This is done by replacing the relief time and relief point of the first run by the corresponding quantities for the second run and repeating the procedure above except that equation (2.4.4) need not be satisfied. With the third relief point identified there is no difficulty in obtaining the fourth and later relief points by further iterations of the same process. Indeed, the problem is not how to go on, but when to stop and this problem is dealt with in the next section.

2.4.1 <u>Identification of the Final Run</u>

One unwritten agreement that the M.T.T. has with the Union is that the latest starting time for a Port operator's crib is 9.15 p.m. and for a Hackney operator's crib is 10.00 p.m.,

i.e.
$$\ell ct = \begin{cases} 1275 \text{ mins. if Port operator} \\ 1320 \text{ mins. if Hackney operator.} \end{cases} \qquad (2.4.5)$$

Thus as soon as rt_z is greater than one of these two values, depending at which depot the operator signs off, the process must stop. However it is sometimes possible to combine runs as illustrated in figure 2.4. Operator on run 1 transfers to run 2 where the time between the two relief points may be less than the crib allowance. Never-the-less, it is possible to break run 1 at such a time before the time limit for broken shift operators and the length of the work is still less than the maximum of five hours.

2.5 Procedure for Ending the First Sequential Group

Let the last relief time before ℓct, found by the method described previously, be rt_x and its corresponding run number and relief point be rn_x, rp_x respectively.

For any run i, not already used by the sequential stepping it may be possible to find a time rt_{ij} and relief point rp_{ij} satisfying the following conditions:

$$rp_{min,m} \leqslant rp_{ij} \leqslant rp_{max,m} \qquad (2.5.1)$$

and $$rt_{ij} - rt_x \underset{\geq}{\nless} tt_{rp_x, rp_{ij}} \qquad (2.5.2)$$

where $tt_{rp_x, rp_{ij}}$ is the time needed to travel between the relief points rp_x and rp_{ij}.

However the following conditions must also be satisfied by some relief time, rt_{xj}, and relief point rp_{xj} on run rn_x:

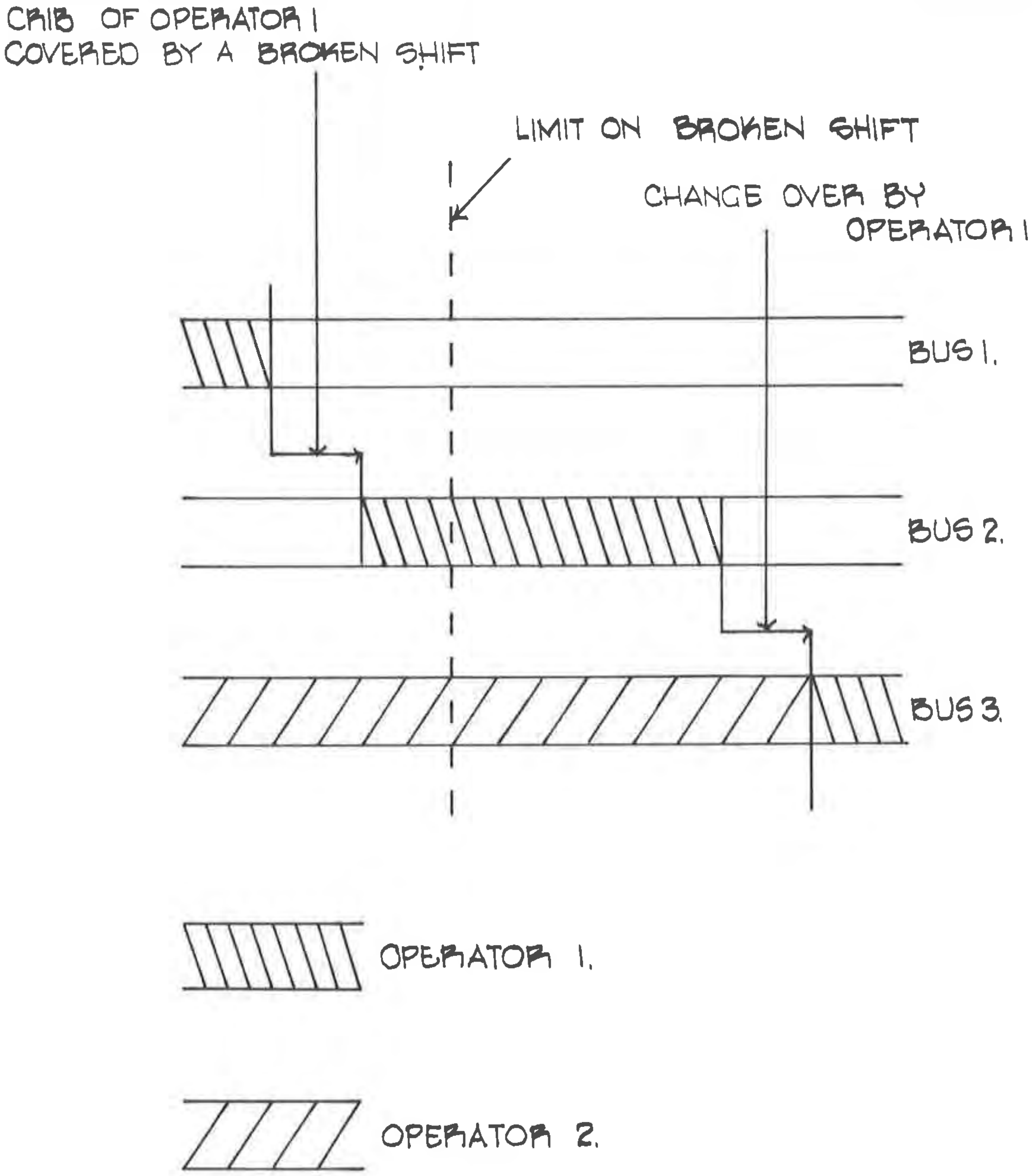

FIGURE 2.4 LATE RELIEFS IN SEQUENTIAL STEPPING

$$rt_{xj} \geqslant at_{x_i} + SOF - (300 - \tau) \qquad (2.5.3)$$

where SOF = 25 mins., by the agreement (24)

and $\tau = \min(tt_{1,rp_{xj}}, tt_{2,rp_{xj}}, tt_{3,rp_{xj}})$;

$$rt_{xj} \leqslant 8 \text{ p.m.} - SOF_1 \qquad (2.5.4)$$

where $SOF_1 = 15 + \tau$, by the agreement (26) ;

$$rp_{min,m} \leqslant rp_{xj} \leqslant rp_{max,m}. \qquad (2.5.5)$$

Let B(i) be a list of the first relief times of runs satisfying equations (2.5.3), (2.5.4) and (2.5.5).

When list B has been completed, the earliest time is selected. This time then is the value of rt_x and this procedure is repeated until no run can be found satisfying the equations (2.5.3), (2.5.4) and (2.5.5); the last such run found is the final run in the sequential group. The next stage is to form the remaining sequential groups.

2.6 Construction of Further Sequential Groups

The list of first relief times i.e. A(i), in which the values corresponding to the runs already used have been removed, is examined and the earliest time at a relief point within a specified range is selected. Then using the same procedures as before, another sequential group is constructed.

This logic is used to form all the groups. Two groups of Hackney runs were initially formed, then five Port groups and finally seven City groups.

2.7 Summary of Results

Denote the grouping of the runs formed by sequential stepping thus,

$G(i,j)$: j^{th} run number in the i^{th} group (2.7.1)

$T(i,j)$: relief time at which the j^{th} run in i^{th} group was broken (2.7.2)

$P(i,j)$: relief point corresponding to $T(i,j)$. (2.7.3)

A flow diagram setting out the logic used in the group construction is shown in figure 2.5. A set of results obtained from the computer are shown in Tables 1(a) and 1(b).

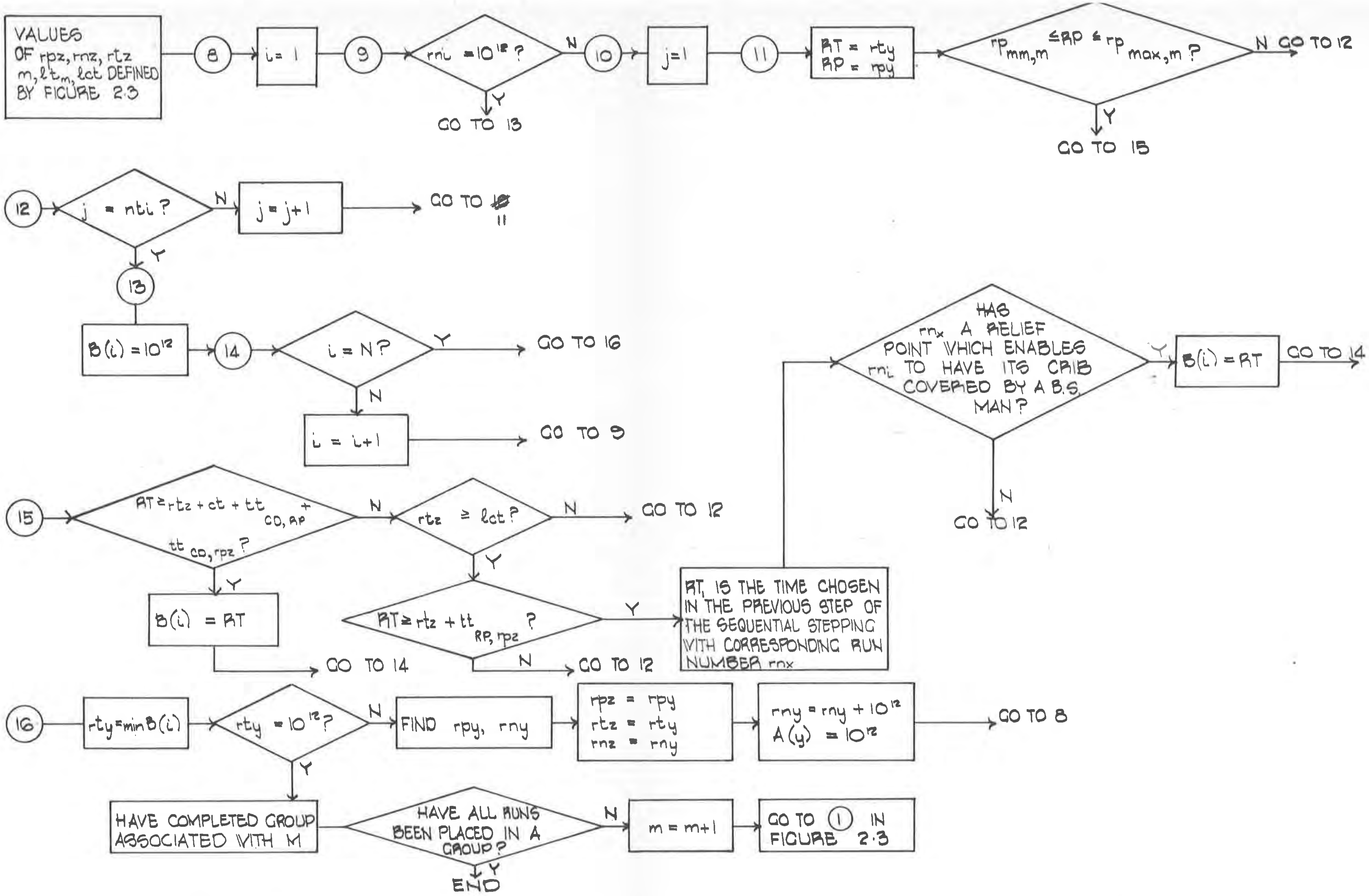

FIGURE 2.5 THE LOGIC FOR THE CONSTRUCTION OF THE SEQUENTIAL GROUPS

GROUP NUMBER	RUN NUMBER	RELIEF TIME	RELIEF POINT	RELIEF GROUP
1	337	19 42	2	HACKNEY
	182	20 13	2	
	142	20 46	2	
	184	21 13	1	
	327	21 47	2	
	5	22 19	2	
	181	22 55	1	
2	319	20 24	2	HACKNEY
	183	20 50	2	
	336	21 28	1	
	143	21 58	1	
	1	22 39	2	
	7	23 03	2	
3	389	19 47	15	PORT
	426	20 14	15	
	427	20 39	15	
	388	21 06	15	
	406	21 28	14	
	386	21 51	15	
	407	22 18	15	
	475	22 41	20	
	30	23 01	17	
4	396	19 56	15	PORT
	408	20 30	15	
	405	20 57	15	
	397	21 24	15	
	391	22 00	15	
	423	22 45	15	
5	412	20 7	14	PORT
	472	20 41	20	
	392	21 15	15	
	393	21 42	15	
	417	23 3	15	
6	257	19 48	19	PORT
	31	20 31	17	
	76	21 01	17	
	401	21 33	15	
	61	22 01	17	
	242	22 28	19	
7	401	19 48	16	PORT
	462	20 18	16	
	70	20 46	17	
	71	21 16	17	
	57	21 46	17	
	65	22 31	17	

TABLE I(a).

GROUP NUMBER	RUN NUMBER	RELIEF TIME	RELIEF POINT	RELIEF GROUP
8	566	19 59	9	CITY
	568	20 37	9	
	503	21 07	9	
	565	21 37	9	
	253	22 14	11	
9	507	20 7	9	CITY
	508	20 47	9	
	564	21 17	9	
	506	21 47	9	
	567	22 17	9	
10	248	19 50	10	CITY
	114	20 23	5	
	62	20 51	9	
	251	21 30	11	
	563	22 06	8	
	94	22 56	5	
11	93	20 08	5	CITY
	4	20 35	13	
	121	21 11	5	
	125	21 41	5	
	122	22 13	6	
12	98	19 53	5	CITY
	297	20 24	11	
	294	21 04	11	
	326	21 58	8	
	504	22 32	7	
13	272	19 59	10	CITY
	273	20 39	10	
	118	21 28	6	
	284	22 01	11	
14	244	20 10	10	CITY
	277	20 56	10	
	505	21 35	7	

RUNS 144,145,179,180 ARE COVERED BY BROKEN SHIFT MEN.

TABLE I(b).

CHAPTER III

COMPLETION OF STRAIGHT SHIFTS - TWO PIECES OF WORK

3.1 Introduction

The aim of Chapter III is to indicate how p.m. straight shifts are formed by adding pieces of work to those chosen by the Sequential Stepping procedure. This addition is most desirable as one extra piece of work but two or even three pieces may be needed to form a shift of acceptable length.

Figure 2.1 illustrates the various types of runs in a sequential group, viz., first, last and intermediate runs. Different algorithms are used to complete the p.m. straight shifts depending on the type of run ending the shift.

3.2 Notation

The essence of forming straight shifts is the splitting up of each bus run into several pieces of work and recombining the pieces into shifts satisfying the agreements. The following data must be known for each piece of work:

(i) the run number,

(ii) the start and finish times,

(iii) the start and finish places.

Let Rn_1, Rn_2, Rn_3,... define the run numbers of the first, second, third,... pieces of work needed to complete a straight shift.

Let T_2, T_1 be the start and finish times for run Rn_1 and

P_2, P_1 be the start and finish places for run Rn_1 .

Similarly

let T_4, T_3 be the start and finish times for run Rn_2 and

P_4, P_3 be the start and finish places for run Rn_2, etc.

Thus a p.m. straight shift consisting of two pieces of work is completely defined when $T_4, T_3, T_2, T_1, P_4, P_3, P_2, P_1, Rn_1, Rn_2$ and allowances for signing off, signing on and travelling are known. Such a shift is illustrated in figure 3.1. Similarly a p.m. straight shift consisting of three pieces of work is completely defined when $T_6, T_5, T_4, T_3, T_2, T_1, P_6, P_5, P_4, P_3, P_2, P_1, Rn_3, Rn_2, Rn_1$ and corresponding allowances are known.

3.3 Classification of Shifts

Various types of shifts are formed depending on the position of the runs in the sequential groups.

(i) Shifts with one run number defined.

The first and last run in each sequential group have only one piece of work defined as indicated in figures 3.2(a) and 3.2(b). Let the shifts which are formed from runs depicted in figure 3.2(a) be called F type shifts. The following variables for F type shifts are already known:

$Rn_1 = G(K,1)$

T_1 = time bus with run number $G(K,1)$ returns to depot

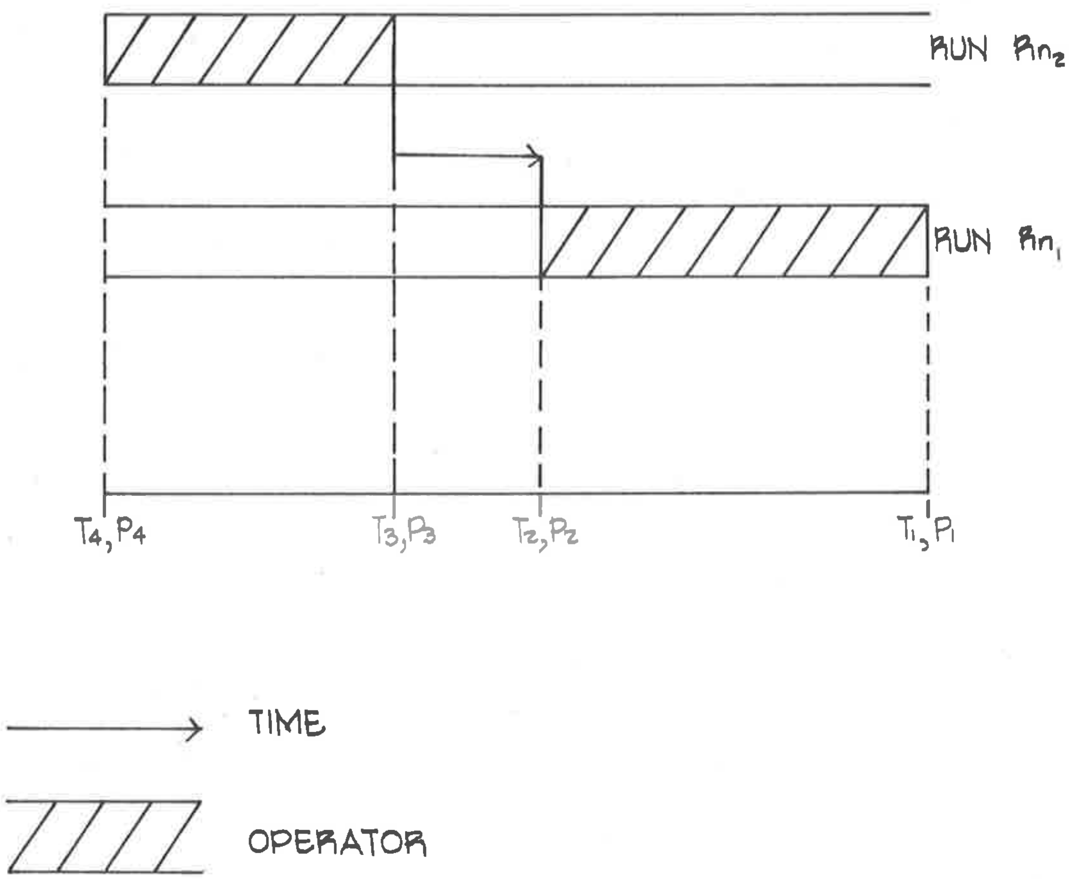

FIGURE 3.1 A P.M. STRAIGHT SHIFT CONSISTING OF TWO PIECES OF WORK.

P_1 = arrival depot

T_2 = T(K,1)

P_2 = P(K,1).

Let the shifts which are formed from runs depicted in figure 3.2(b) be called L type shifts.

The following variables for L type shifts are already defined:

Rn_1 = G(K,L)

T_1 = T(K,L)

P_1 = P(K,L)

K = 1,2,...,14

L is number of runs in Group K.

A method for finding values of T_2 and P_2 for L type shifts is described in a later section.

To complete F or L type shifts one or two new pieces of work must be chosen. An algorithm for choosing suitable new pieces is given in section 3.8.

(ii) Shifts with two run numbers defined.

Intermediate runs in the sequential groups have two pieces of work defined as illustrated in figures 3.3(a) and 3.3(b). Let the shifts which are formed from runs depicted in figure 3.3(a) be called I_1 type shifts.

The following variables for I_1 type shifts are already defined:

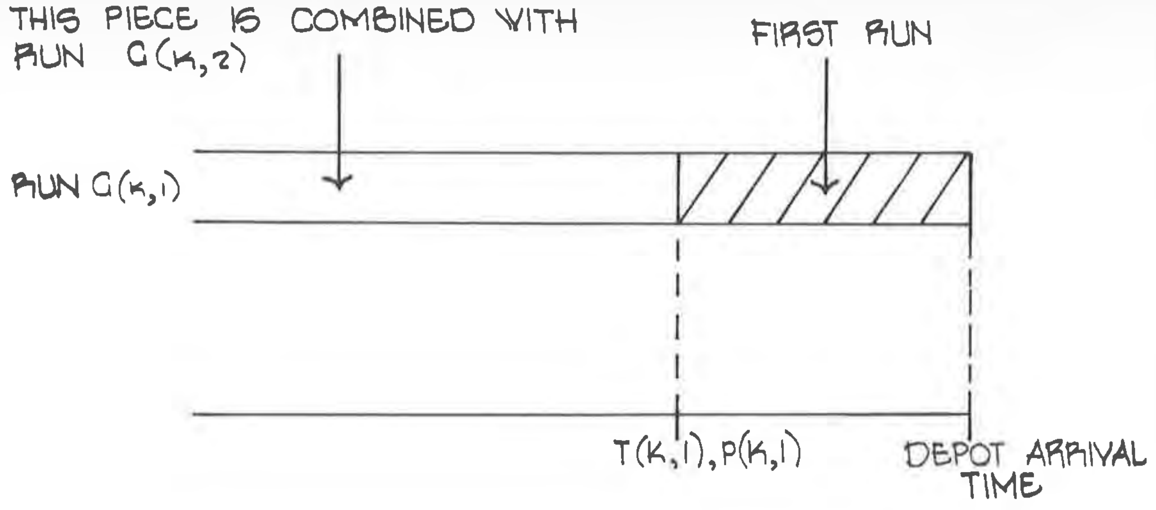

FIGURE 3.2(a) THE FIRST RUN IN A SEQUENTIAL GROUP WITH ONE PIECE OF WORK COMPLETELY DEFINED

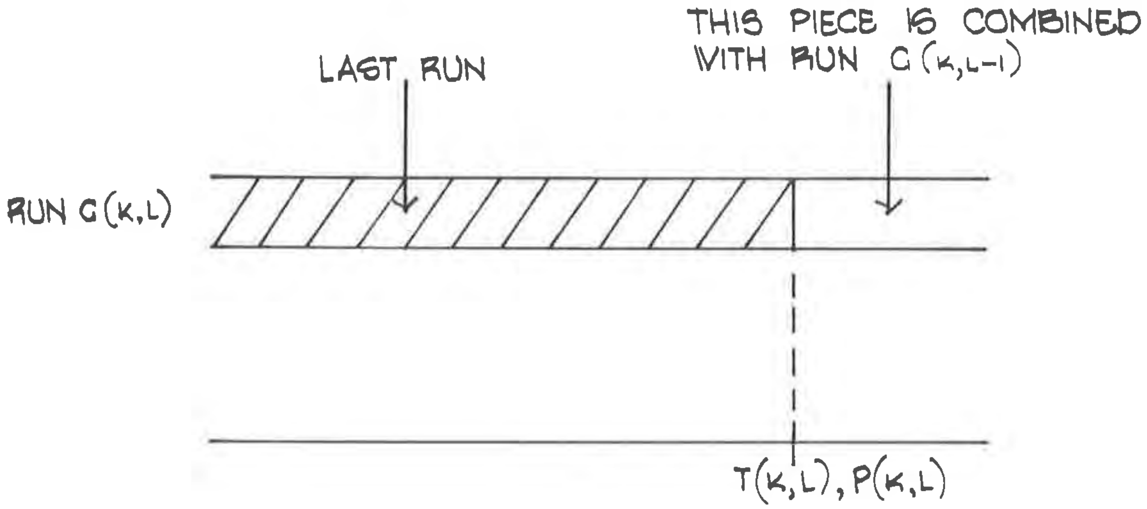

FIGURE 3.2 (b) THE LAST RUN IN A SEQUENTIAL GROUP WITH ONE PIECE OF WORK PARTIALLY DEFINED.

$Rn_1 = G(K,r+1) \quad , \quad K = 1,2,\ldots,14; \quad 2 \leq r+1 \leq L$

P_1 = arrival depot

T_1 = depot arrival time for bus with run number Rn_1

$T_2 = T(K,r+1) \quad , \quad K = 1,2,\ldots,14; \quad 2 \leq r+1 \leq L$

$P_2 = P(K,r+1) \quad , \quad K = 1,2,\ldots,14; \quad 2 \leq r+1 \leq L$ (3.3.1)

$T_3 = T(K,r) \quad , \quad K = 1,2,\ldots,14; \quad 1 \leq r \leq L$

$P_3 = P(K,r) \quad , \quad K = 1,2,\ldots,14; \quad 1 \leq r \leq L$

$Rn_2 = G(K,r) \quad , \quad K = 1,2,\ldots,14; \quad 1 \leq r \leq L.$

Let the shifts which are formed from runs depicted in figure 3.3(b) be called I_2 type shifts.
The variables for the I_2 type shifts are defined by the equation (3.3.1).

Values for T_4 and P_4 must be found for both I_1 and I_2 type shifts. The method for doing this is described later in this chapter.

I_2 type shifts must have at least one new piece of work chosen whereas I_1 types may or may not need an extra piece of work to be complete shifts.

3.4 T_2,P_2 Values - L Type Shifts

It was mentioned in Chapter II that the last runs in a sequential group have a crib prior to the time T_2. Since these runs end at approximately 10.30 p.m. and the agreement (28) specifies a maximum of five hours work after a crib, the earliest time that a crib could be taken would be approximately 5.30 p.m., i.e. the crib would fall during the p.m. peak period. This is undesirable, therefore T_2

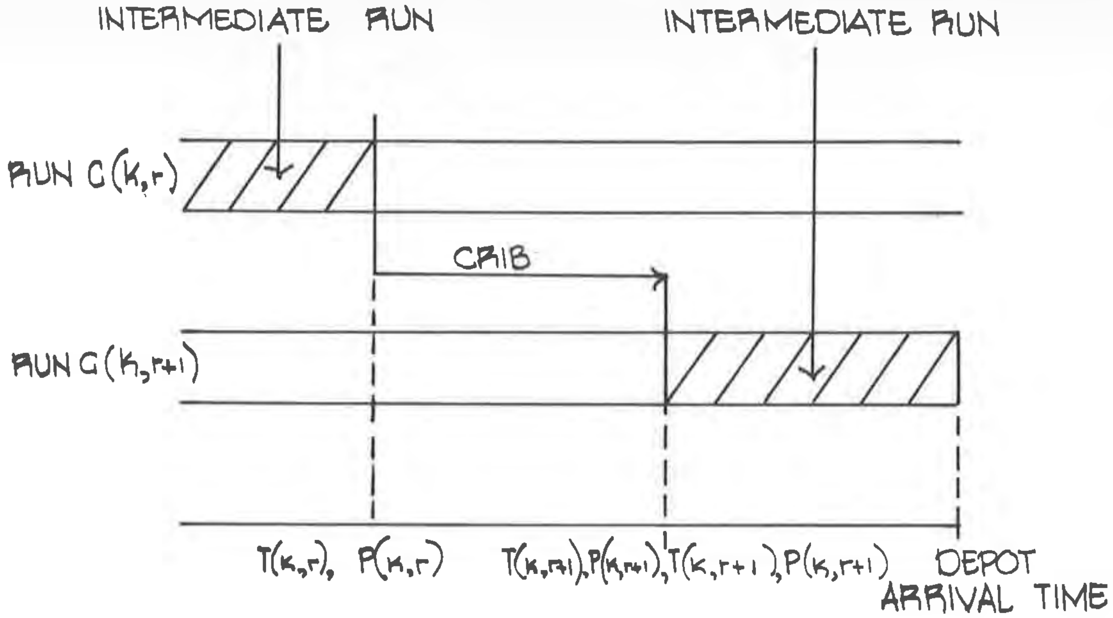

FIGURE 3.3(a) INTERMEDIATE RUNS OF SEQUENTIAL GROUP WITH A CRIB ALLOWED.

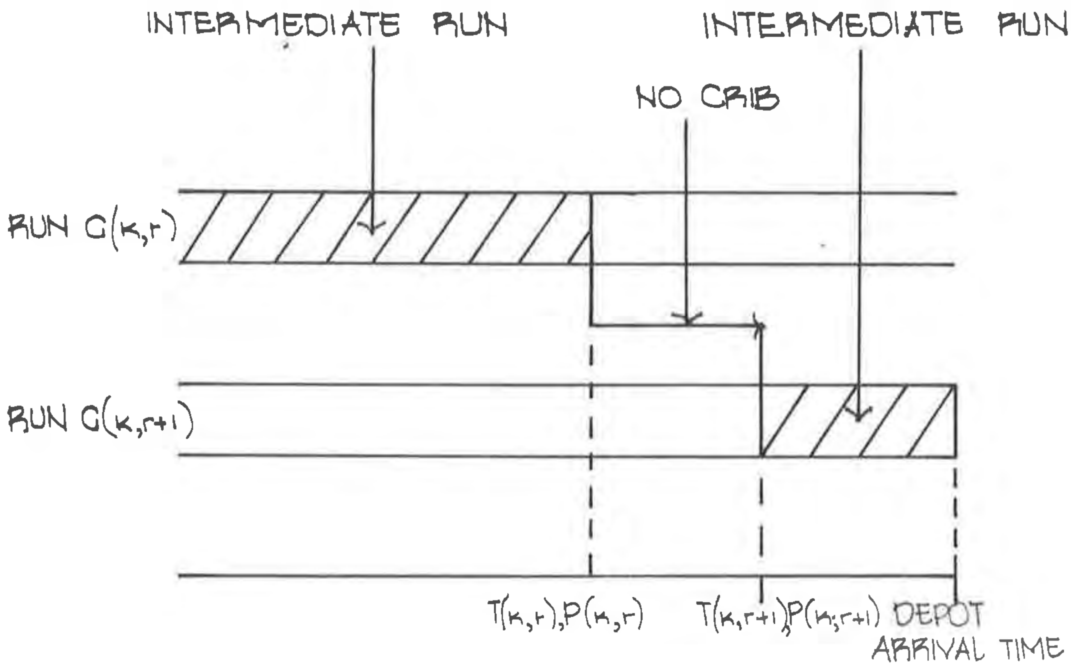

FIGURE 3.3(b) INTERMEDIATE RUNS OF A SEQUENTIAL GROUP WITHOUT A CRIB ALLOWED

is chosen as the most appropriate time after the peak, i.e. after 6.30 p.m.

Let T_2' and P_2' be the first relief time and its corresponding relief point on the run with run number Rn_1 such that *

$$8 \text{ p.m.} - SOF \geq T_2' \geq 6.30 \text{ p.m.} \qquad (3.4.1)$$

$$P_2' = \text{'a'} \qquad (3.4.2)$$

where $SOF = 15 + tt_{C,P_2'}$ by the agreement (26).

The values of T_2' and P_2' satisfying equations (3.4.1) and (3.4.2) are defined to be the T_2 and P_2 values for the L type shifts.

3.4.1 Residues

When the run with run number Rn_1 is cut at T_2, the residual work on this run is stored in the array D where:

$D_{\ell 1} = i$ i.e. the position of the run in the list L_{ijk} (see Appendix 2)

---

* P_x = 'a' means

(i) P_x must be a Hackney relief point if there are any on the run, if not
(ii) P_x must be a Port relief point if there are any on the run, if not
(iii) P_x must be a City relief point if there are any on on the run.

[This order of choice simplifies the formation of the broken shifts.]

$D_{\ell 2} = T_2$

$D_{\ell 3} = P_2$

$$\ell = 1,2,\ldots$$

where ℓ is a count of the number of residual pieces of work.

3.5 T_4, P_4 Values - I_1 Type Shifts

When values for T_4 and P_4 are defined for I_1 type shifts, a completed p.m. straight shift may have been formed, i.e. the length of the shift is greater than $8\frac{1}{4}$ hours.

$$\text{Let} \quad t_1 = T_1 + SOF - 540 \qquad (3.5.1)$$

i.e. t_1 is the earliest starting time of the shift, ensuring that its length is less than the 9 hours mentioned in the agreement (7).

$$\text{Let} \quad t_2 = T_3 + tt - 300 \qquad (3.5.2)$$

where tt is the time needed to travel from P_3 to its nearest depot. Thus t_2 is the earliest starting time for the shift, ensuring that the length of the piece of work before the crib is less than the 5 hours mentioned in the agreement (8).

$$\text{Let} \quad t = \max(t_1, t_2). \qquad (3.5.3)$$

For a valid shift, T_4 must satisfy

$$T_4 \geqslant t + SON \qquad (3.5.4)$$

where the signing on allowance, SON, depends on

(i) whether the bus is 2 man or 1 man operated, or

(ii) whether P_4 is a relief point or a depot, or

(iii) whether the operator has to obtain a ticket outfit before the commencement of the shift or between T_2 and T_3.

The appropriate value for SON is calculated from the agreement (14).

Since P_1 is a depot for all I_1 type shifts, SOF mentioned in equation (3.5.1) has the value 25 mins., defined by the agreement (24).

For a valid shift, P_4 must satisfy*

$$P_4 \sim P_1. \qquad (3.5.5)$$

The actual values of SON, mentioned in equation (3.5.4),

---

* $P_x \sim P_y$ means either

(i) P_x is the same depot as P_y (3.5.6)

or (ii) P_x belongs to the group of relief points closest to P_y

or (iii) P_y belongs to the group of relief points closest to P_x

[If P_y is a Port relief point or Port depot, $P_x \sim P_y$ is extended to mean:
P_x is initially tested to see if it satisfies any of the conditions given in (3.5.6); if it does not then P_x may be Hackney depot or a Hackney relief point and all allowances associated with P_x are increased by 50 mins., i.e. the travel time from Port Adelaide to Hackney.]

are

(i) 20 mins if P_4 is a depot; from the agreement (24) (3.5.7)

(ii) $10 + tt_{C,P_4}$ if P_4 is a relief point; from the agreement (26) (3.5.8)

where C is the depot $\sim P_4$.

If the departure time, T_D, and departure depot, P_D, for the run with run number Rn_2 satisfy the equations (3.5.4) and (3.5.5) the T_4 and P_4 values for I_1 type shifts are defined to be T_D and P_D respectively.

If T_4 satisfies

$$T_4 - SON > t + 45 \qquad (3.5.9)$$

it is possible that a third piece of work may be added to produce a shift approximately nine hours in length. The procedure for finding such third pieces of work, if they exist, is discussed in Chapter 4.

If the equation (3.5.9) is not satisfied then the I_1 type shift is completely defined.

However if T_D and P_D do not satisfy the equations (3.5.4) and (3.5.5), let T_4' be the first time and P_4' the corresponding relief point satisfying these equations. For maximum service to the public, men are rostered so that they work as much of the p.m. peak as possible. This means that shifts starting between 5.10 p.m. and 6.30 p.m. are not permitted, i.e. if

$$T_4' < 5.10 \text{ p.m.} \qquad (3.5.10)$$

the T_4 and P_4 values for I_1 type shifts are defined to be T_4' and P_4' respectively. If T_4 does not satisfy equation (3.5.9), the shift is considered complete, otherwise a search for a third piece of work must be made. If the equation (3.5.10) is not satisfied by T_4', a value for T_4, not falling in the peak period, must be found.

Let T_4'' be the first time and P_4'' its corresponding relief point satisfying the following conditions:

$$T_4'' \geq 6.30 \text{ p.m.} \qquad (3.5.11)$$

$$T_4'' \leq 8 \text{ p.m.} - \text{SOF} \qquad (3.5.12)$$

$$P_4'' = \text{'a'} \qquad (3.5.13)$$

where SOF is calculated from the agreement (26).

When values of T_4'' and P_4'' are found satisfying the equations (3.5.11) to (3.5.13), these values are taken as the T_4 and P_4 values for the I_1 type shifts.

3.6 T_4,P_4 Values - I_2 Type Shifts

Since no crib was allowed between the times T_3 and T_2, it must be given prior to the time T_4. Therefore all I_2 type shifts must consist of at least three pieces of work.

Let T_4'' be the first time and P_4'' its corresponding relief point satisfying the equations (3.5.11), (3.5.12), (3.5.13), then the T_4 and P_4 values for I_2 type shifts are defined to be T_4'' and P_4'' respectively.

3.7 Residues

When P_4 is a relief point, there is residual work left on the run with run number Rn_2. This work is stored in the array D defined thus:

$D_{\ell 1} = i$, where i is the position of the run in list L_{ijk}

$D_{\ell 2} = T_4$

$D_{\ell 3} = P_4$

$$\ell = 1,2,\ldots$$

where ℓ is a count of the number of residual pieces. The flow chart in figure 3.4 indicates the method used for finding the T_4 and P_4 values for I_1 and I_2 type shifts.

3.8 Fixed Runs

Some runs on the original headway sheet have no associated list of relief times. These are called fixed work runs because the work defined by:

(i) the depot arrival time and the corresponding depot,

(ii) the depot departure time and the corresponding depot,

cannot be cut at any relief point. Such runs could have been included in L_{ijk} with $nt_i = 0$, but it is more convenient to consider them separately and define them using the array B discussed in Appendix 2.

3.9 Second Piece of Work - L and F Type Shifts

Values of T_4, P_4, T_3, P_3 and Rn_2 which define the second piece of work for these shifts must be determined. Possible pieces of work may be contained in the arrays B and D.

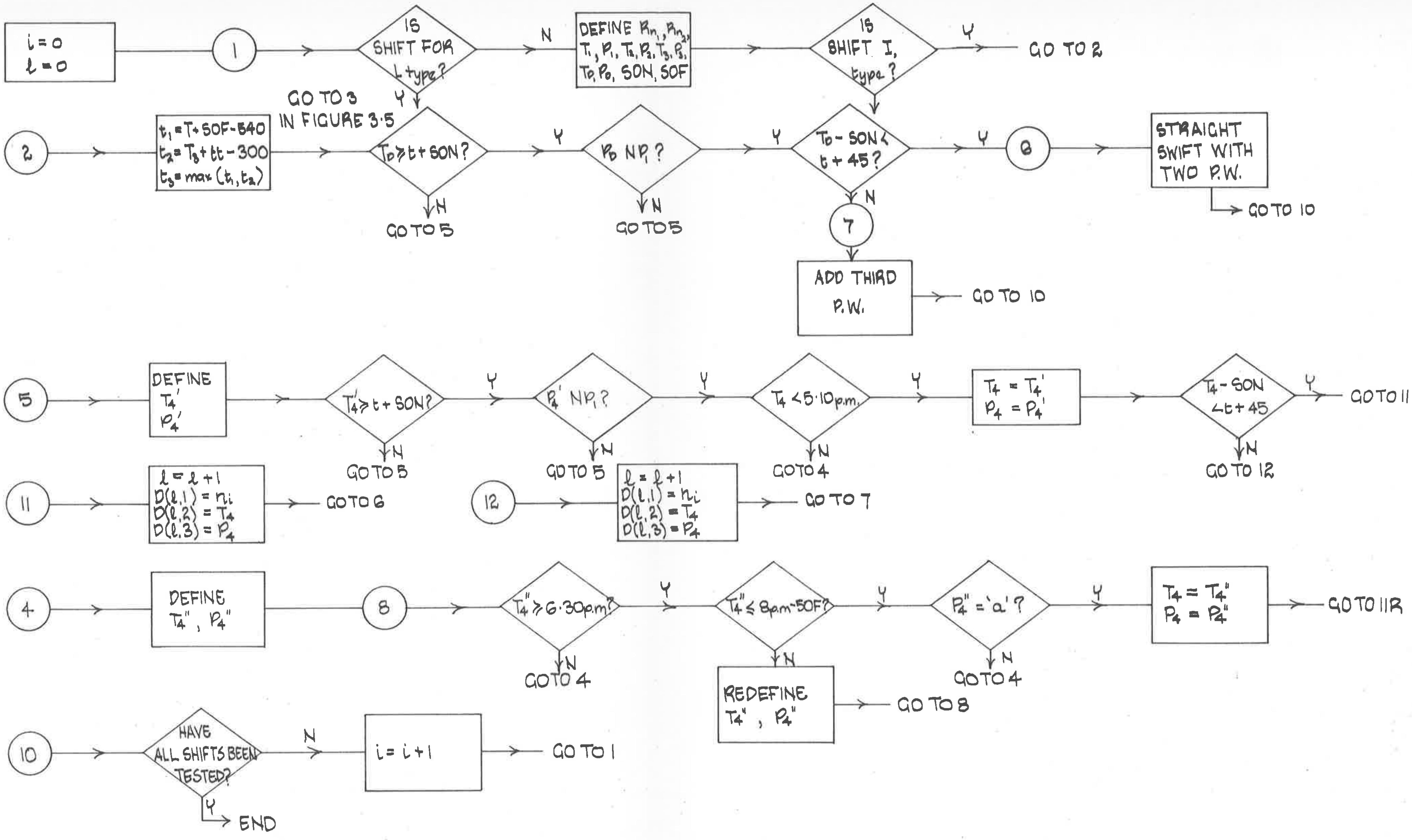

FIGURE 3·4 T_4, P_4 VALUES FOR I_1 AND I_2 TYPE SHIFTS.

3.9.1 "Suitable" Fixed Runs

Each run in B is tested for "suitability". Suppose the j^{th} run is being tested. The variables $T_4, P_4, T_3, P_3, T_{CL}, P_{CL}$ and Rn_2 for this run are defined as follows:

$$T_4 = B_{j2},\ P_4 = B_{j3},\ Rn_2 = B_{j1},\ T_3 = B_{j4},$$
$$P_3 = B_{j5}, T_{CL} = B_{j8},\ P_{CL} = B_{j9} \qquad (3.9.1)$$

where T_{CL} is the time that the conductor leaves the bus and P_{CL} is the corresponding relief point.

Allowance must be made for the operator to obtain a ticket outfit between the times T_3 and T_2 if the following conditions hold:

$$T_3 < T_{CL} \qquad (3.9.2)$$

$$P_3 \sim P_1. \qquad (3.9.3)$$

Most fixed work buses automatically satisfy these equations because the conductor returns to depot with the bus.

Let A_ℓ be the minimum permissible period between T_3 and T_2 viz:

$$A_\ell = ct + SOF_1 + tt_{CD,P_2} \qquad (3.9.4)$$

where ct is the crib allowance defined by the agreement (10),

CD is the depot nearest the relief point P_2,

SOF_1 is the sign off allowance defined by the agreement (17).

If an outfit is to be collected between the times T_3 and T_2, A_ℓ must be increased by 5 minutes and the sign on allowance must be reduced by 10 minutes thereby satisfying the agreement (20).

The values of t_1 and t_2 for these shifts are calculated by:

$$t_1 = T_1 + SOF - 540 \quad (3.9.5)$$

$$t_2 = T_3 + SOF_1 - 300 \quad (3.9.6)$$

where SOF is the sign off allowance defined by the agreement (24).

The earliest possible starting time, t, satisfies the condition:

$$t = \max(t_1, t_2). \quad (3.9.7)$$

The j^{th} run in list B is considered "suitable" if the values T_4, P_4, T_3, P_3 defined by the equation (3.9.1) satisfy the following conditions:

$$T_4 - SON \geq t \quad (3.9.8)$$

$$T_4 < 5.10 \text{ p.m.} \quad (3.9.9)$$

$$6 \text{ p.m.} \leq T_3 \leq T_2 - A_\ell \quad (3.9.10)$$

$$P_3 \sim P_2 \quad (3.9.11)$$

$$P_4 \sim P_1 \quad (3.9.12)$$

where SON is calculated from either equation (3.5.7) or (3.5.8).

3.9.2 "Suitable" Residual Runs

Each run in D is tested for "suitability". Suppose the j^{th} run in D is being tested. The variables $n, T_x, P_x, T_3, P_3, T_{CL}, P_{CL}$ and Rn_2 for this run are defined as follows:

$n = D_{j1}$, $P_x = L_{n,-7,0}$, $T_x = L_{n,-8,0}$, $Rn_2 = L_{n,-9,0}$,

(3.9.13)

$T_3 = D_{j2}$, $P_3 = D_{j3}$, $T_{CL} = L_{n,-2,0}$, $P_{CL} = L_{n,-1,0}$,

where n is the position of this run in the list L_{ijk}. The allowance A_ℓ, permitted between T_3 and T_2, is calculated from the equation (3.9.4) and is increased by 5 minutes if the equations (3.9.2) and (3.9.3) are satisfied. The earliest starting time, t, is calculated from the equations (3.9.5), (3.9.6) and (3.9.7) where SOF_1 is defined by the agreement (22).

The j^{th} run in list D is considered "suitable" if its values of T_x and P_x satisfy the equations (3.9.8) to (3.9.11). The values of T_4 and P_4 are then taken as T_x and P_x respectively.

If T_x and P_x satisfy equations (3.9.9) to (3.9.11) only, it may still be possible to cut the run at a suitable relief point.

Let T_x' be the first time and P_x' its corresponding relief point which satisfy the following conditions:

$$T_x' - SON \geqslant t \qquad (3.9.14)$$

$$P_x' \sim P_1 \qquad (3.9.15)$$

$$T_x' < T_3 \qquad (3.9.16)$$

where SON is calculated from the agreement (20). If values of T_x' and P_x' are found satisfying these equations the j^{th} run in list D is still considered "suitable" and the T_4, P_4 values for the shift are defined to be T_x' and P_x' respectively.

3.9.3 "Most Suitable" Run.

The "most suitable" run chosen from all "suitable" runs in lists B and D is considered to be that run for which the length of the second piece of work is a maximum, i.e. $T_3 - T_4$ is a maximum.

If the shift length is now greater than $8\frac{1}{4}$ hours the shift is said to be complete. However, if it is less than $8\frac{1}{4}$ hours a third piece of work may be added to produce a still feasible shift. The method for finding such third pieces of work, if they exist, is discussed in the next chapter.

Once a run from list B is added to a shift, it may not be tested for "suitability" again. Although a portion of a run from list D may be added to a shift, the residual work may still be reconsidered for "suitability". Therefore if part of the j^{th} run in list D had been used, its corresponding values for D_{j2} and D_{j3} then satisfy:

$$D_{j2} = T_4$$

$$(3.9.17)$$

$$D_{j3} = P_4.$$

Agreement (31) implies that only limited runs from lists B and D may be tested for "suitability" if P_1 is a Port relief point or Port depot.

The flow chart used by the computer program to choose the second piece of work for F and L type shifts is illustrated in Figure 3.5.

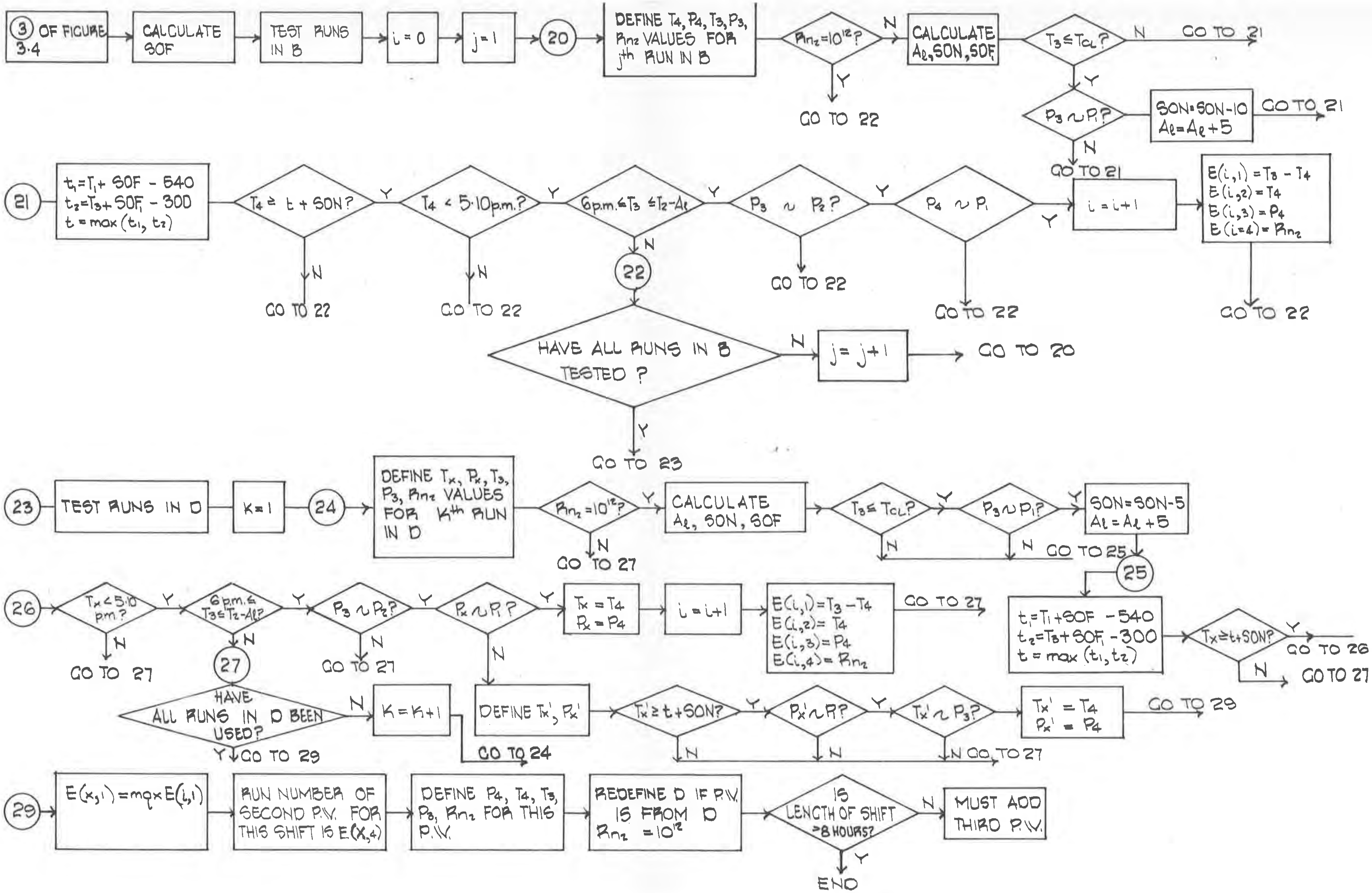

FIGURE 3.5 THE LOGIC FOR ADDING A SECOND PIECE OF WORK TO L AND F TYPE SHIFTS

CHAPTER IV

COMPLETION OF STRAIGHT SHIFTS - THREE PIECES OF WORK

4.1 Introduction

Third pieces of work may be chosen to complete the following type shifts, L,F,I_1 and I_2. Each type of shift will be treated separately and the method for choosing the third piece of work will be discussed.

4.2 Third Piece of Work - I_1 Type Shifts

The form of an I_1 type shift consisting of three pieces of work is illustrated in the figure 4.1. The values of T_6,P_6,T_5,P_5 and Rn_3 which define the third piece of work for this shift type must be determined for the "most suitable" run.

4.2.1 "Suitable" Fixed Runs

Each run in list B is tested for "suitability". Suppose the j^{th} run is being tested. The variables $T_6,P_6,Rn_3,T_5,P_5,T_{CL}$ and P_{CL} for this run are defined as follows:

$$T_6 = B_{j2},\ P_6 = B_{j3},\ Rn_3 = B_{j1},\ T_5 = B_{j4}, \qquad (4.2.1)$$

$$P_5 = B_{j5},\ T_{CL} = B_{j8},\ P_{CL} = B_{j9}.$$

Allowance must be made for the operator to obtain his ticket outfit between the times T_5 and T_4 if these times satisfy the relations:

$$T_5 \leqslant T_{CL} \qquad (4.2.2)$$

$$P_5 \sim P_1. \qquad (4.2.3)$$

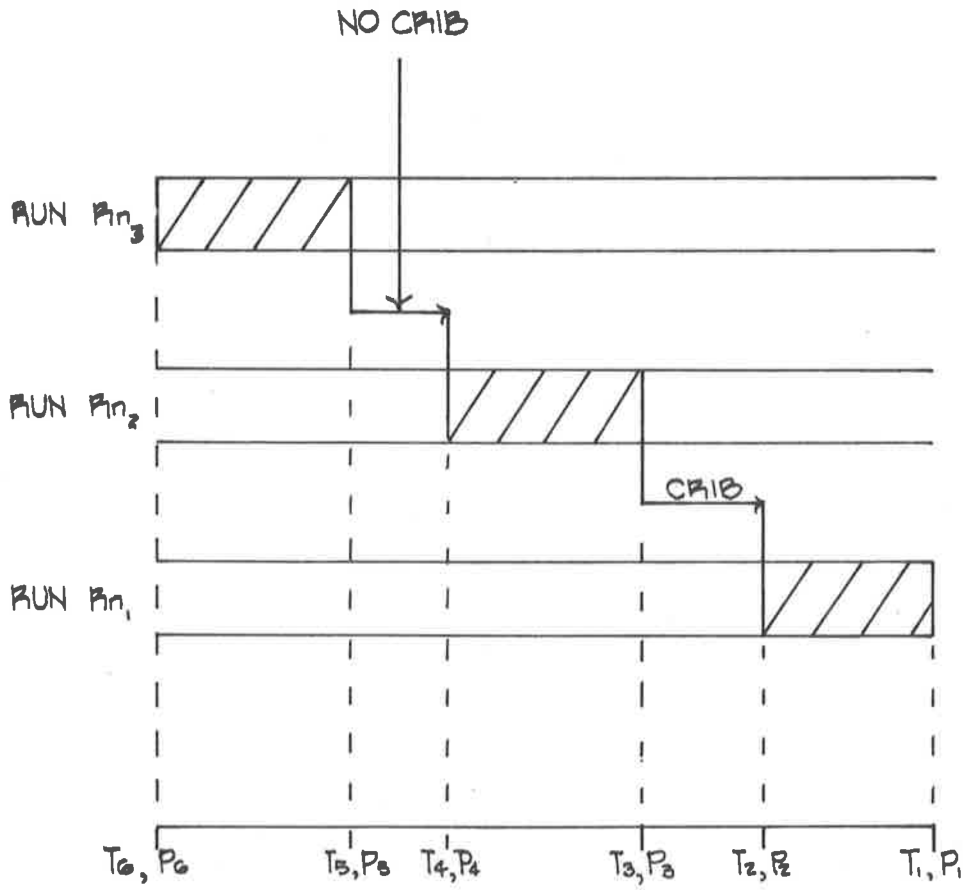

FIGURE 4.1 AN I, TYPE SHIFT WITH THREE PIECES OF WORK DEFINED

Let A_ℓ be the minimum period allowed between the times T_5 and T_4. The value of A_ℓ is calculated using the condition:

$$A_\ell = SOF_2 + tt_{CD,P_4} \qquad (4.2.4)$$

where SOF_2 is the sign off allowance, determined by the agreement (17), for the run with run number Rn_3, and CD is the depot nearest the relief point P_4. If the equations (4.2.2) and (4.2.3) are satisfied, A_ℓ is increased by 5 minutes and the sign on allowance is reduced by 10 minutes, thereby satisfying the agreement (20).

The earliest starting time of the shift allowed by the agreement (7) is t_1, and the earliest starting time of the shift allowed by the agreement (8) is t_2. The values of t_1 and t_2 for this shift are calculated by:

$$t_1 = T_1 + SOF - 540 \qquad (4.2.5)$$

$$t_2 = T_3 + tt_{C,P_3} - 300 \qquad (4.2.6)$$

where C is the depot nearest P_3 and SOF is the sign off allowance for the run with the run number Rn_1.

The earliest possible starting time, t, satisfies the condition:

$$t = \max\,(t_1, t_2). \qquad (4.2.7)$$

The j^{th} run in list B is considered "suitable" if the values T_6, P_6, T_5, P_5 defined by the equation (4.2.1) satisfy

$$T_6 - SON \geqslant t \qquad (4.2.8)$$

$$T_5 \leqslant T_4 - A_\ell \qquad (4.2.9)$$

$$T_6 < 5.10 \text{ p.m.} \qquad (4.2.10)$$

$$P_6 \sim P_1 \qquad (4.2.11)$$

$$P_5 \sim P_4 \qquad (4.2.12)$$

where SON is calculated from either equation (3.4.7) or (3.4.8).

4.2.2 "Suitable" Residual Runs

Each run in D is tested for "suitability". Suppose the j^{th} run in D is being tested. The variables $n, P_x, T_x, T_5, P_5, Rn_3, T_6, P_6, T_{CL}$ and P_{CL} are defined as follows:

$$n = D_{j1},\ P_x = L_{n,-7,0},\ T_x = L_{n,-8,0},\ Rn_3 = L_{n,-9,0} \qquad (4.2.13)$$

$$T_5 = D_{j2},\ P_5 = D_{j3},\quad T_{CL} = L_{n,-2,0},\ P_{CL} = L_{n,-1,0},$$

where n is the position of this run in the list L_{ijk}.

The allowance A_ℓ, permitted between T_5 and T_4, is calculated from the equation (4.2.4). Since a crib is not given between the times T_5 and T_4, no allowance need be made for the operator to collect a ticket outfit in this period. The earliest possible starting time, t, for the shift is calculated from the equations (4.2.5), (4.2.6) and (4.2.7).

The j^{th} run in D is considered "suitable" if the values of T_x, P_x, T_5 and P_5 satisfy the equations

(4.2.8) to (4.2.12) where SOF_2, needed for calculating A_ℓ, is determined by the agreement (18). The values of T_6 and P_6 are then defined to be T_x and P_x respectively.

If the equations (4.2.9) and (4.2.12) are satisfied but equation (4.2.8) is not, it may still be possible to cut the j^{th} run at a suitable relief point and produce a "suitable" run. Let T_x' be the first time with a corresponding relief point P_x' which satisfy the following conditions:

$$T_x' - SON \geq t \qquad (4.2.14)$$

$$P_x' \sim P_1 \qquad (4.2.15)$$

$$T_x' < T_5 \qquad (4.2.16)$$

$$T_x' < 5.10 \text{ p.m.}. \qquad (4.2.17)$$

If there exist values of T_x' and P_x' which satisfy these conditions, the j^{th} run in D is considered "suitable". These values are then taken as the values of T_6 and P_6 for the shift.

4.2.3 "Most Suitable" Run

The "most suitable" run is selected from all "suitable" runs contained in lists B and D, by the method described in section 3.9.3, providing equation (3.9.17) is replaced by

$$D_{j2} = T_6$$
$$D_{j3} = P_6. \qquad (4.2.18)$$

4.3 Third Piece of Work - I_2 Type Shifts

The form of an I_2 type shift consisting of three pieces of work is illustrated in the figure 4.2. The values for T_6, P_6, T_5, P_5 and Rn_3 which define the third piece of work for this shift type must be determined for the "most suitable" run.

4.3.1 "Suitable" Fixed Runs

Each run in list B is tested for "suitability". The variables $T_6, P_6, Rn_3, T_5, P_5, T_{CL}$ and P_{CL} for the j^{th} run are calculated from the equation (4.2.1). If the equations (4.2.2) and (4.2.3) are satisfied, allowance must be made for the operator to collect a ticket outfit between the times T_5 and T_4.

The minimum period, A_ℓ, permitted between the times T_5 and T_5 is defined by the following equation:

$$A_\ell = ct + SOF_2 + tt_{CD,P_4} \qquad (4.3.1)$$

where ct is the crib allowance specified by agreement (10) and the SOF_2, CD values are determined from section 4.2.1.

The values of t_1 and t_2 for this shift are calculated by:

$$t_1 = T_1 + SOF - 540 \qquad (4.3.2)$$

$$t_2 = T_5 + SOF_2 - 300 \qquad (4.3.3)$$

where SOF_2 is the sign off allowance for the run with run number Rn_3. The value of SOF_2 is determined by agreement (17) or (18). The earliest possible starting

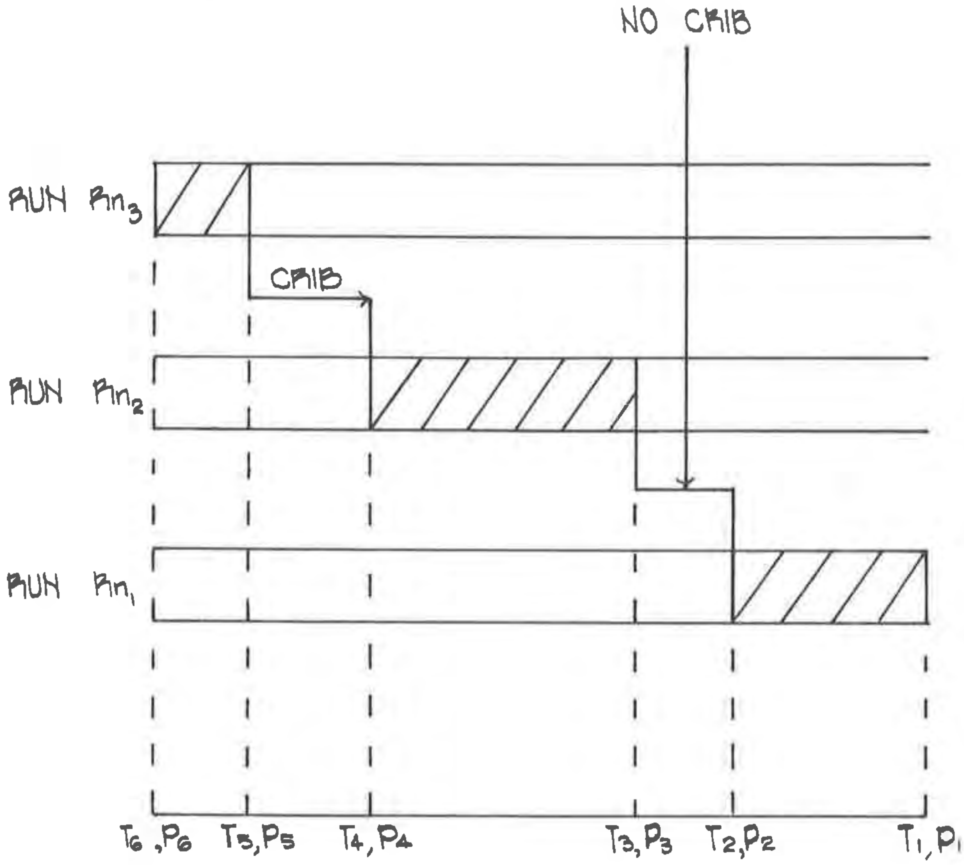

FIGURE 4.2 AN I_2 TYPE SHIFT WITH 3 PIECES OF WORK DEFINED.

time, t, satisfies the condition:

$$t = \max(t_1, t_2) \qquad (4.3.4).$$

The j^{th} run in list B is considered "suitable" if the values T_6, P_6, T_5, P_5 satisfy the equations (4.2.8) to (4.2.12).

4.3.2 <u>"Suitable" Residual Runs</u>

The method discussed in section 4.2.2, with equations (4.2.4) to (4.2.7) replaced by the equations (4.3.1) to (4.3.4), is repeated to find the "suitable" runs contained in list D.

If the equations (4.2.2) and (4.2.3) are satisfied, A_ℓ is increased by 5 minutes, whereas SON is reduced by 5 minutes if agreement (21) applies or it is reduced by 10 minutes if agreement (22) applies.

4.3.3 <u>"Most Suitable" Run</u>

The "most suitable" run is selected from all "suitable" runs, contained in lists B and D, by the method described in section 4.2.3.

4.4 <u>Third Piece of Work - L or F Type Shifts</u>

The method needed for choosing the "most suitable" third piece of work for L or F type shifts is the same as that discussed in section 4.2. If the equations (4.2.2) and (4.2.3) are satisfied by runs from both lists, allowance must be made for the operator to collect a ticket outfit.

4.5 Completed P.M. Straight Shifts

Methods for finding straight shifts comprising two or three pieces of work have now been discussed. If it had been possible for the addition of yet a fourth piece of work, and the shift still a feasible length, the method described in this chapter could be extended to find a "most suitable" run with a run number Rn_4.

The flow chart used for the computer programme to choose the third piece of work for the p.m. straight shifts is illustrated in figure 4.3.

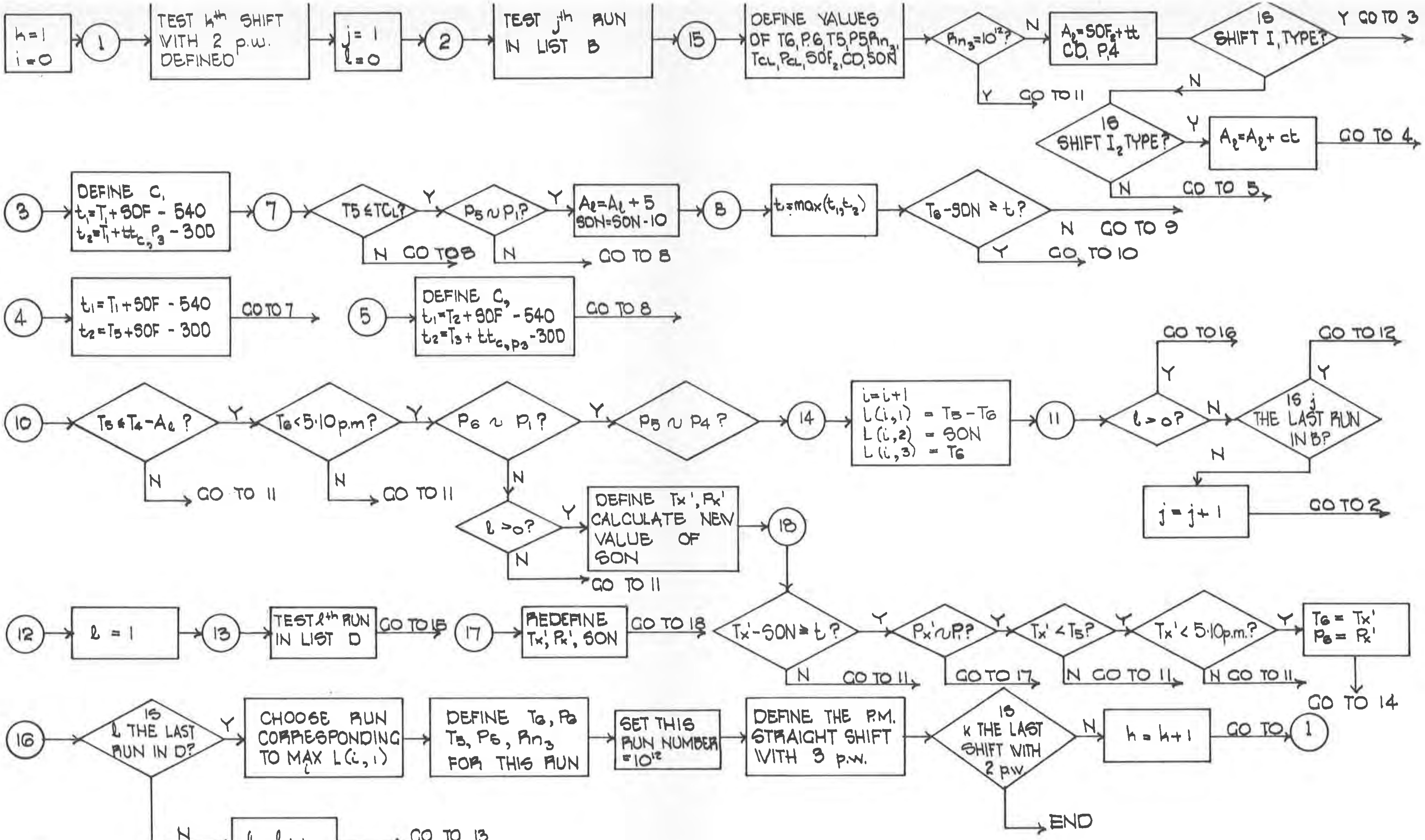

FIGURE 4.3 THE LOGIC USED FOR CHOOSING THE THIRD PIECE OF WORK FOR A P.M. STRAIGHT SHIFT.

CHAPTER V

THE BASIC FRAMEWORK

5.1 Introduction

Having determined the staff needed to man the evening services and to provide their late cribs, an assessment of the men required for the remaining day's work must be made. Thus, the number of broken operator and conductor shifts, a.m. straight operator and conductor shifts, and early p.m. straight operator and conductor shifts for each depot must be determined. The construction of the basic framework, which defines the number and type of shifts needed to cover both peak periods, is discussed in this chapter.

5.2 Conditions

The conditions which govern the staff requirement are the following:

(i) broken shifts can not commence before 5.50 a.m., (5.2.1)

(ii) broken shifts can not finish after 8 p.m., (5.2.2)

(iii) the spread of hours on a broken shift must not exceed 12 hours. (5.2.3)

5.3 Sign-off Lists

A list of all sign off times for both operators and conductors may be constructed from all runs which were defined by the headway sheet and which were not used in a p.m. straight shift. If the run has been cut at a relief point, the sign off allowance needed for calculating the sign off time, is obtained from agreement (25) or (26). The appropriate conductor's sign off allowance is also calculated from these agreements. However, if either the conductor or operator return to a depot travelling on a bus, the sign off allowance has the value specified by the agreement (23) or (24).

Any sign off time occurring before 5 p.m. is not included in this list, because the piece of work ending at this time does not cover the p.m. peak period.

Since agreement (28) requires all employees to sign off and on at the same depot, six sub-lists are constructed and defined in the following way:

group 1, denoted by $C1j$; increasing sign off list for City operators,

group 2, denoted by $C2j$; increasing sign off list for City conductors,

group 3, denoted by $C3j$; increasing sign off list for Port operators,

group 4, denoted by $C4j$; increasing sign off list for Port conductors,

group 5, denoted by $C5j$; increasing sign off list for Hackney operators,

group 6, denoted by $C6j$; increasing sign off list for Hackney conductors, where $j = 1,2,\ldots,n_i$ and n_i is the number of elements in group i.

5.4 Sign-on Lists

A corresponding list of sign on times for operators and conductors may also be formed for the runs not used by the p.m. straight shifts. The sign on allowance necessary for calculating the sign on times of the conductors and operators is obtained from agreement (15) or (19).

Any sign on time occurring after 9.30 a.m. is not included in this list, because the piece of work starting at this time does not cover the a.m. peak period.

The sub-lists corresponding to this list of sign on times are defined as follows:

group 7, denoted by C7j; increasing sign on list for City operators,

group 8, denoted by C8j; increasing sign on list for City conductors,

group 9, denoted by C9j; increasing sign on list for Port operators,

group 10, denoted by C10j; increasing sign on list for Port conductors,

group 11, denoted by C11j; increasing sign on list for Hackney operators,

group 12, denoted by C12j; increasing sign on list for Hackney conductors, where $j = 1,2,\ldots,n_i$ and n_i is the number of elements in group i.

5.5 Pairing Sign-on and Sign-off Times

An ideal pairing situation would exist if there were the same number of sign off times as sign on times in corresponding sub-lists. Since this is not the case in practice, allowance is made for adjusting the sign on and sign off times, thus permitting a man to start his duty at one depot, finish it at another depot or a relief point not associated with this depot and then travel back to the sign on depot. Allowance is also made for "shandy" shifts. These are formed if a man is initially rostered as an operator but finishes his duty as a conductor, or vice versa. When this occurs the shift worked is classified as an operator's shift; that is, paid at the higher rate. These allowances enable each of the p.m. sub-lists a choice of two or four a.m. sub-lists from which possible pairing may occur.

Table II illustrates the order of preference, allowed by the computer programme, for each p.m. sub-list.

5.5.1 Notation

Let the a.m. sub-lists to be tested against the p.m. list, Cij, be denoted by $P^k_{i\ell}$, where k is the preference Cij has for the a.m. list; i.e., $k = 1,2,\ldots,4$; and $\ell = 1,2,\ldots,m_k$ where m_k is the number of elements in list P^k_{ij}.

P.M. GROUPS	POSSIBLE A.M. GROUPS [IN ORDER OF PREFERENCE]			
C1j	C7j	C8j	C11j	C12j
C2j	C8j	C7j	C12j	C11j
C3j	C9j	C10j	C11j	C12j
C4j	C10j	C9j	C12j	C11j
C5j	C11j	C12j	-	-
C6j	C12j	C11j	-	-

TABLE II

5.5.2 Early p.m. Straight Shift and Broken Shift Structure

Let RT be the last sign off time in the p.m. sub-list Cij, thus

$$RT = Cin_1 \qquad (5.5.1).$$

The earliest possible starting time for a broken shift with RT as its sign off time is 12 hours before RT.

If RT is greater than 8 p.m. then it must be the sign off time of an early ending p.m. straight shift, since condition (5.2.2) states that no broken shift may end after 8 p.m. Let the last sign off time in list $P\overset{k}{i}\ell$ be RT_k, thus

$$RT_k = P\overset{k}{i}\ell_k \qquad (5.5.2)$$

$$k = 1,2,\ldots,4.$$

Since the spread of a broken shift must be less than 12 hours, RT and RT_k will define the basic framework for a broken shift, if

$$RT - 720 \geq RT_k \qquad (5.5.3)$$

$$k = 1,2,\ldots,4.$$

Every sign on time in $P\overset{k}{i}\ell$ is less than RT_k, therefore RT will not define a broken shift with any time in $P\overset{k}{i}\ell$ if it does not with RT_k. However, if RT_k has already been used, the new value of RT_k is taken as the latest unused sign on time in $P\overset{k}{i}\ell$.

If RT does not satisfy the equation (5.5.3) for any value of RT_k, $k = 1,2,\ldots,4$, it must be the sign off time of an early ending p.m. straight shift. The basic structure of a broken shift or an early p.m. straight shift is produced from each element in the list Cij, where $i = 1,2,\ldots,6$; $j = n_i, n_{i-1}, \ldots, 1$. Sometimes in practice, there are more sign off times in a p.m. sub-list than there are sign on times in the corresponding first preference group. Suppose list C1j has 100 sign off times and $P^1_{1\ell}$ has only 95 sign on times. If the previous procedure is performed, the 4th sign off time in C1j may be paired with the latest sign on time in $P^2_{i\ell}$. Therefore a broken shift with a spread of perhaps only 9 hours may be defined. This means that when the pairing of late times in the p.m. sub-list which has $P^2_{1\ell}$ as its first preference is attempted with sign on times in $P^2_{1\ell}$, all the late sign on times may have been unnecessarily paired with sign off in C1j. To prevent this from happening, the following procedure is used.

Let RT be the latest unused sign off time in the group Cij. If RT satisfies the following conditions:

$$RT - 720 \leqslant P^1_{i\ell} \qquad (5.5.4)$$

$$\text{where } \ell = 1,2,\ldots,n_i$$

$$RT < 6 \text{ p.m.} \qquad (5.5.5),$$

let RT_k be the smallest unused sign on time in list $P^k_{i\ell}$, $k = 2,3,4$. It is extremely likely that the value of RT_k defined in this way does not satisfy the equation (5.5.3), but each element larger than RT_k in $P^k_{i\ell}$, $k = 2,3,4$, is tested in turn, until one sign on time is found which satisfies equation (5.5.3). This time then defines the structure of a broken shift with a sign off time of RT.

One of the procedures, depending on the number of elements in the groups, is used to produce the broken shift or early p.m. straight shift structure. Figure 5.1 shows the flow chart of the computer programme for constructing these shifts.

5.5.3 A.M. Straight Shift Structure

Each time a member of the lists $P_{i\ell}^{k}$ was used to construct a broken shift, its value was set equal to 10^{12}. This dummy value prevented the sign on time from being used again. Let rt be a time in the a.m. list Cij, i = 7,8,...,12. Since a broken shift can not start before 5.50 a.m., rt must be the sign on time of an a.m. straight shift if

$$rt < 5.50 \text{ a.m.} \qquad (5.5.6).$$

Since each element in the lists $P_{i\ell}^{k}$, i = 1,2,...,6; k = 1,2,...,4, corresponds to an element in the lists Cij, i = 7,8,...,12, any sign on time not used from these lists will start an a.m. straight shift. Thus, rt defines the basic structure of an a.m. straight shift if it is not equal to 10^{12} or if it satisfies the equation (5.5.6).

The number of a.m. straight operator and conductor shifts is the number of elements not used for constructing

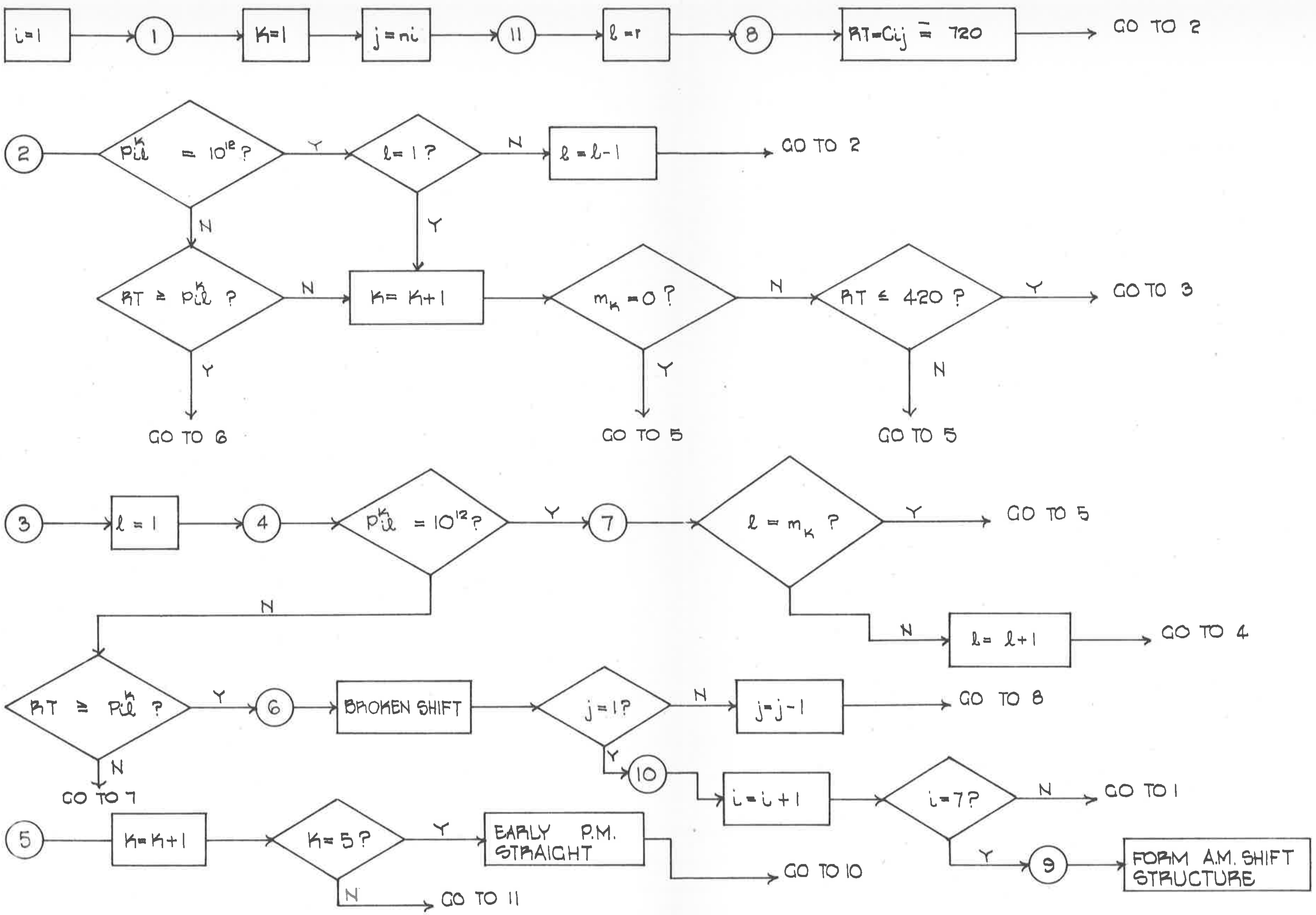

FIGURE 5.1 THE LOGIC FOR PAIRING THE SIGN-ON AND SIGN-OFF TIMES.

broken shifts from the a.m. groups. The flow chart used for constructing the basic framework of the a.m. straight shifts is given in the figure 5.2.

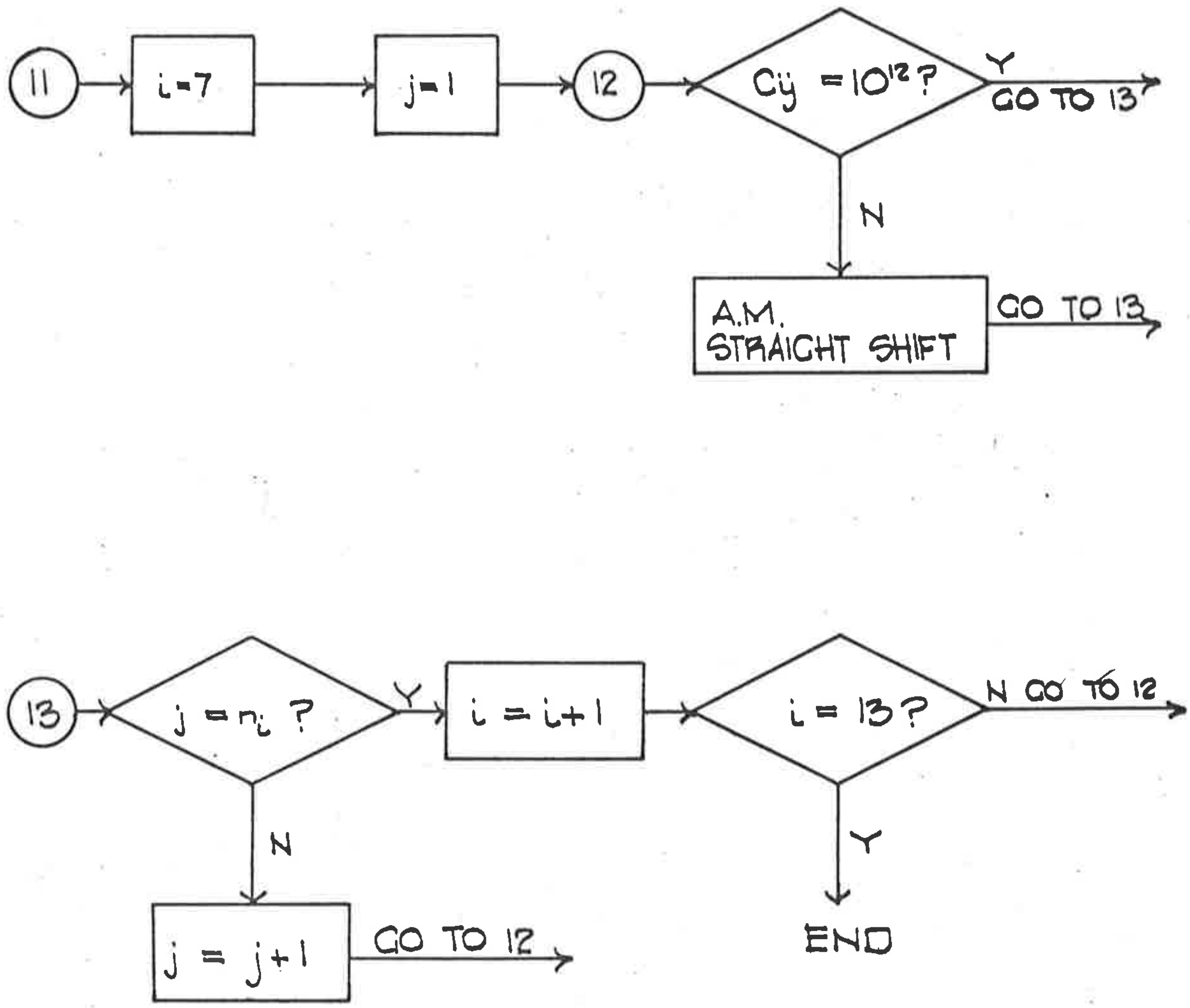

FIGURE 5.2 THE LOGIC FOR THE FORMATION OF A.M. STRAIGHT SHIFT STRUCTURE

CHAPTER VI

THE MOST EFFICIENT OFF-PEAK ROSTER

6.1 Introduction

There are three periods during a week-day which are classified as off-peak periods, namely:

(1) early morning:- before 7.00 a.m.,

(2) mid-day:- between 9.30 a.m. and 4.45 p.m.,

(3) late evening:- after 6.30 p.m.

The mid-day off peak period is called the intermediate period. The number and type of shifts for both operators and conductors at each depot during or before the a.m. peak and during or after the p.m. peak was determined by the basic framework, defined in the preceding chapter. However, the number and type of shifts which are to be worked during the intermediate period are not known. This chapter will discuss the classification of operator shifts over this period of the day and how this affects the formation of a.m. straight and broken operator shifts.

The time table determines the number of buses on the road at any time of the day. Figure 6.1 is a graph of the number of buses on the road during a typical week day, where

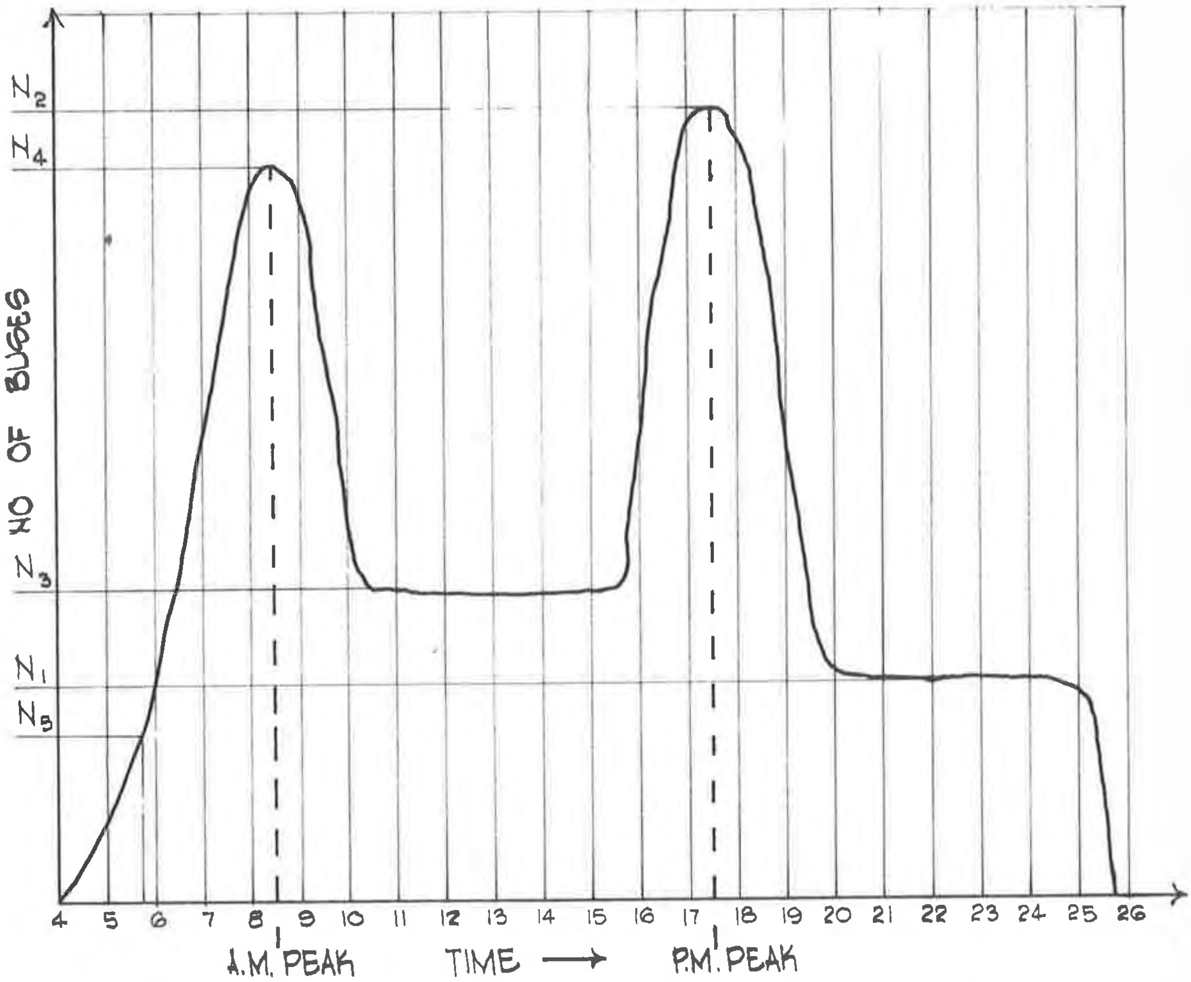

FIGURE 6.1 THE NUMBER OF BUSES ON THE ROAD DURING A TYPICAL WEEK-DAY.

N_1 is the number of late p.m. buses,

N_2 is the maximum number of buses operating at one time during the p.m. peak,

N_3 is the maximum number of buses operating at one time during the intermediate period,

N_4 is the maximum number of buses operating at one time during the a.m. peak,

N_5 is the number of early a.m. buses.

6.2 The P.M. Peak Roster

The late p.m. straight shifts were constructed in such a way that all the operators were rostered over the p.m. peak. Let δ be the number of extra men needed to cover the cribs given after 8 p.m. The number, n_1 of late p.m. straight shifts is the sum of N_1 and δ, i.e.

$$n_1 = N_1 + \delta \qquad (6.2.1).$$

The number, n_2, of men needed to operate the remaining p.m. peak buses is the difference between N_2 and n_1, i.e.

$$n_2 = N_2 - n_1 \qquad (6.2.2).$$

The most economic use of staff is obtained if these n_2 operators are also rostered over the a.m. peak, i.e. if there are n_2 broken shifts.

6.3 The A.M. Peak Roster

If it were possible to form n_2 broken shifts from the basic framework, the number, n_3, of extra men needed to operate the remaining a.m. peak buses is the difference between N_4 and n_2, i.e.

$$n_3 = N_4 - n_2 \qquad (6.3.1).$$

Thus the number of straight shifts which cover the a.m. peak is n_3.

The most efficient operator's roster is therefore produced if the number of p.m. straight shifts, broken shifts and a.m. straight shifts are n_1, n_2 and n_3 respectively. If in practice, N_5 is greater than n_3, and there are some early p.m. straight shifts, the number of possible broken shifts is less than n_2.

6.4 The Intermediate Period Roster

To prevent extra men being rostered during the intermediate period, the buses running during this period must be operated by all the a.m. straight shift and some of the broken shift men.

The agreements (7) and (9) limit the latest sign off time of the a.m. straight shifts to between 1.30 p.m. and 2.30 p.m. Therefore only n_3 of the intermediate runs can be operated until approximately 2 p.m. by a.m. straight shift men. The latest limiting sign off time for the morning piece of any broken shift is between 12.15 p.m. and 1 p.m. due to the agreements (10) and (32). The remaining (N_3-n_3) intermediate buses must be operated by broken shift men. In order that the maximum amount of work is covered, (N_3-n_3) broken shifts having morning portions which finish near mid-day are chosen. The residual work after approximately 2 p.m. for n_3 runs and after approximately mid-day for the other runs must be covered by large afternoon portions of broken shifts.

A summary of the most efficient rostering of the week-day buses of figure 6.1, is given in figure 6.2.

6.5 Conditions

The following two conditions arise from the most economic way of rostering the intermediate period:

(i) a.m. straight shifts must end as late as possible, i.e. between 1 p.m. and 2 p.m.,

(ii) (N_3-n_3) broken shifts should be constructed in such a way that their morning portions end between 12.15 p.m. and 1 p.m.

6.6 Meals

The method used to provide the meal, necessary by the agreement (9), for the a.m. straight shift operators is similar to the sequential stepping algorithm discussed in chapter 2.

6.7 Optimum Broken Shifts

When all the a.m. straight shifts are produced, the remaining pieces of work are added to existing pieces of broken shift work. No emphasis is placed on how economical the resultant broken shift is, but providing the conditions

(1) no break without pay in a day's duty shall be less than 2 hours,

(2) the maximum length of any portion of a broken shift is 6 hours,

are satisfied, the broken shift is acceptable. An optimum pairing of the a.m. and p.m. pieces of work for all these broken shifts is accomplished by using Bennett's pairing algorithm [1].

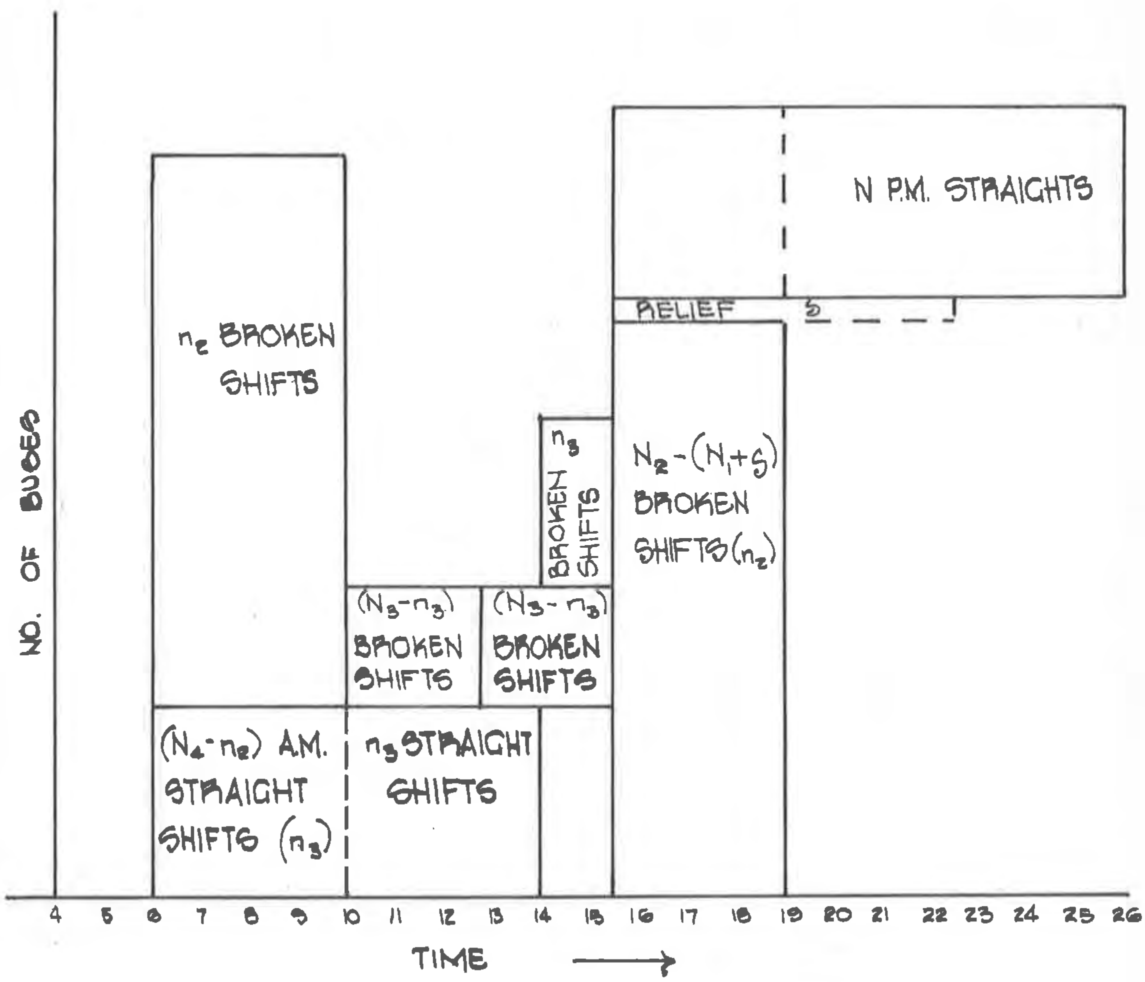

FIGURE 6.2 THE MOST EFFICIENT COVERING OF A WEEKDAY'S WORK SCHEDULE (DRAWN TO THE SAME SCALE AS FIGURE 6.1).

CHAPTER VII

CONCLUSIONS

Various attempts to automate scheduling procedures for transit companies have met with only limited success. The general methods developed by Elias [4], for example, have been widely tested in the U.S.A. but have not proved of direct practical benefit. The approach to the M.T.T. scheduling problem has been fundamentally different in that it has been recognised that it is preferable to subdivide the total complex problem into many sub-problems of manageable size and to base the computer algorithms on the ingenious manual methods which have been developed by the schedule officers. Figure 7.1 indicates this breakdown of the complete rostering problem into a sequence of interrelated sub-problems.

In solving the rostering problem step by step, it has proved advantageous to start with steps 5 and 11, which were automated by Bennett [1], and then to proceed with steps 1 and 2, which have been automated as described in this thesis. Completion of steps 3 and 4, at present under investigation, will then enable the whole M.T.T. roster to be produced automatically with intervention of the schedule officers at prescribed check points.

The methods used in this thesis are typical of those classified as heuristic programming. The aim has

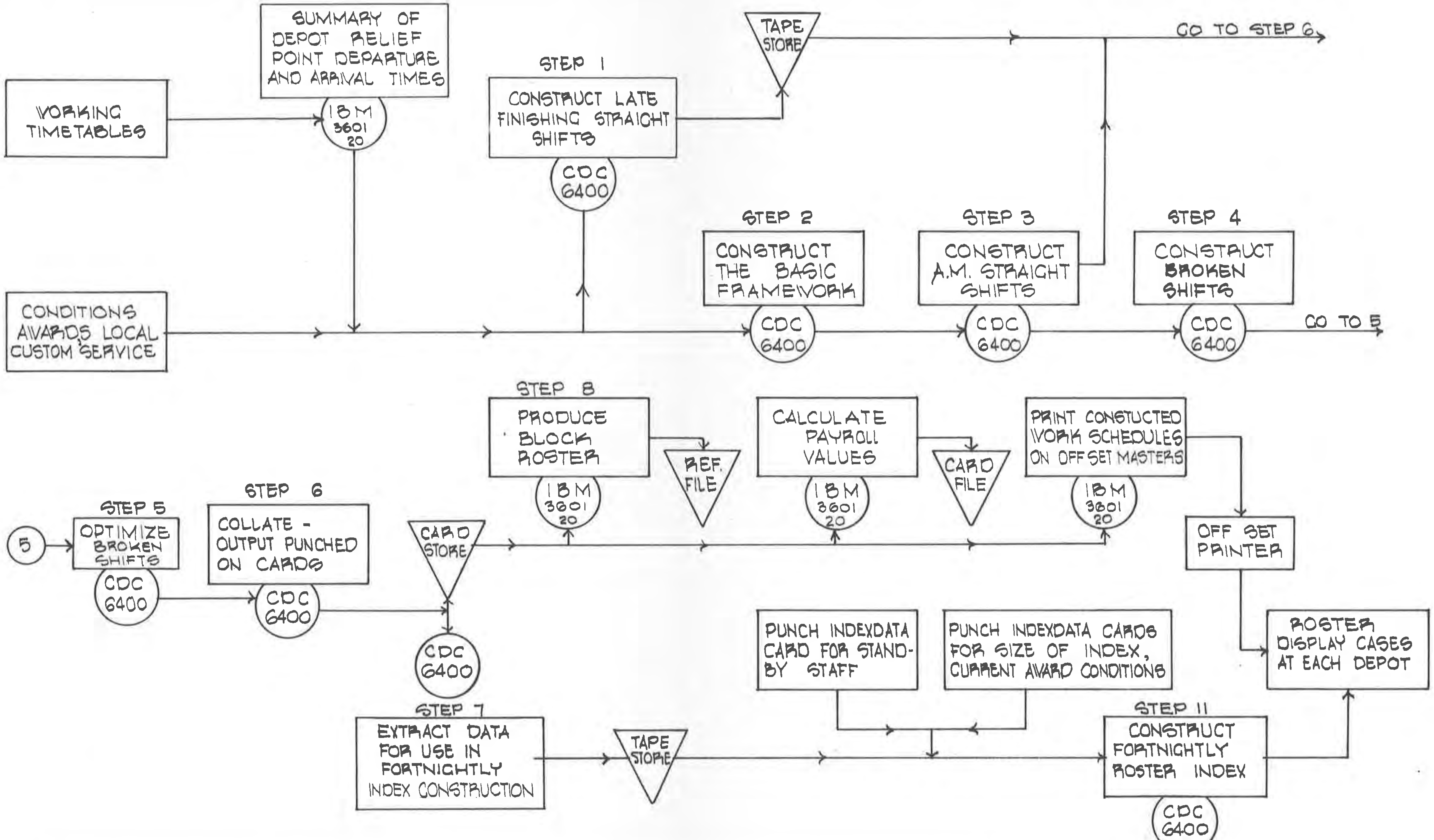

FIGURE 7.1 THE DIVISION OF THE COMPLETE ROSTER PREPARATION INTO SUB-PROBLEMS

not been optimization, partly because of the complexity of the problem and partly because of the impossibility of specifying the criteria explicitly. Instead, the purpose of the research has been to develop computer algorithms which give good acceptable rosters. By basing the approach on the manual methods, many latent constraints have been automatically satisfied without the necessity for their precise definition.

The automation of bus crew rosters has proved worthwhile from the management point of view and the following benefits have been derived.

(1) Frequent reviews of both time-tables and rosters are able to be made because the time needed to prepare new duty rosters has decreased from eight weeks to three weeks.

(2) Experimentation to test new ideas and a flexibility in approach have resulted from the reduction in roster preparation time.

(3) Top management has now a measure of control over traffic costs because

- (i) rosters are being prepared on a minimum cost basis,
- (ii) quick evaluation can be made of the effect or changes or proposed changes in industrial working conditions,

(iii) more accurate estimates of the effect of service adjustments, extensions of services or the introduction of new services, can be obtained.

Because of the general acceptance of the present work to the M.T.T., it is now intended to proceed with the automation and optimization of the time-table, a problem which will be challenging in concept and complexity but which should yield to the powerful heuristic programming techniques developed in this thesis.

APPENDIX I

Adelaide, the capital city of the State of South Australia has a population of 800,000.

The Municipal Tramways Trust (M.T.T.) is the authority for street public transport services in metropolitan Adelaide. The Trust operates most of the bus services itself, particularly over the heavily patronised routes, but also licenses private bus companies to serve some areas. The number of diesel buses run by the Trust is 350, and these buses are operated on a one-man/two-man basis. Two man crews, comprising driver and conductor, are used during times of heavy passenger loading - between 7.30 a.m. and 6.30 p.m. on week days and on Saturday mornings, whilst at all other times drivers are required to collect fares.

An Industrial Arbitration Court Award and agreements between the Employees Association and the M.T.T. determine the rules governing the formation of rosters. The award and agreements are changed occasionally. The methods developed in this thesis are based on the following agreements:

(1) An employee may be required to work broken shifts, provided that all duty performed on any day outside the spread of ten consecutive hours shall be paid for at the following rates -

(2) if the spread is between 10 and 11 hours - time and a half should be paid;

(3) if the spread is between 11 and 12 hours - double time should be paid.

(4) No break without pay in a day's duty shall be less than 2 hours.

(5) Employees shall not be signed off more than twice in any one day.

(6) The ordinary hours of duty shall not be less than 7 hours on any shift.

(7) The hours of duty on straight shifts shall not exceed 9 hours.

(8) Employees shall not be rostered to work for more than 5 hours without a meal relief or crib.

(9) Where an unpaid meal relief is allowed, a minimum of 40 mins. and a maximum of 55 mins. shall be allowed.

(10) Where a meal relief of at least 40 mins. is not provided, a crib shall be taken in the Trust's time. The minimum crib allowance is 20 mins.

(11) All time worked in excess of 8 hours 15 mins. shall be paid for at the rate of time and a half.

(12) When an employee finishes work on one bus to take over duties on another bus, the rate of pay described for the working of the former bus ceases immediately he terminates working it. The time taken to travel

to the other bus shall be payable for the working of such second bus.

(13) When an employee signs on duty as a one-man bus operator he shall be allowed sign on time irrespective of the period he is engaged as a one-man operator for such shift. Sign on time shall be paid at the rate applicable to the first one-man bus driven during the shift.

(14) The following sign on and sign off allowances must be allowed for a bus operator of a 2-man bus:

(15) When a bus is taken from a depot, the sign on allowance is 10 mins.

(16) When the operator takes charge of the bus in traffic, having signed on at a depot, the sign on allowance is 5 mins. + the time needed to travel from the depot to the relief point.

(17) When the bus is returned to a depot, the sign off allowance is 10 mins.

(18) When the operator is relieved in traffic and signs off at a depot, the sign off allowance is 5 mins. + the time needed to travel from the relief point to the depot.

The following sign on and sign off allowances must be allowed for a bus operator of a 1 man bus:

(19) When the bus is taken from a depot and a ticket outfit is obtained, the sign on allowance is 20 mins.

(20) When the operator takes charge of the bus in traffic, having signed on and obtained an outfit at a depot, the sign on allowance is 10 mins. + the time needed to travel from the depot to the relief point.

(21) When the bus is taken from the depot, the sign on allowance is 10 mins.

(22) When the operator takes charge of the bus in traffic, having signed on at a depot, the sign on allowance is 5 mins. + the time needed to travel from the depot to the relief point.

(23) When the bus is returned to a depot, the sign off allowance is 10 mins.

(24) When the bus is returned to a depot and the money collected paid in, the sign off allowance is 25 mins.

(25) When the operator is relieved in traffic and signs off at a depot the sign off allowance is 5 mins. + the time needed to travel from the relief point to the depot.

(26) When the operator is relieved in traffic and signs off and pays in the money collected, the sign off allowance is 15 mins. + the time needed to travel from the relief point to the depot.

(27) An employee signing on at any depot and subsequently directed to commence duty at some other point shall be allowed travelling time from the point at which he signed on to the point at which he commenced duty.

(28) All employees shall, each day, finally sign off at the depot at which they sign on.

(29) For broken shifts the sign on time shall not be before 5.50 a.m. and the sign off time shall not be after 8 p.m.

(30) Port operators may only drive buses with the following run numbers, 30 inclusive to 78, 242 inclusive to 267, 385 inclusive to 435, 471 inclusive to 479.

(31) A ticket outfit must be collected from the sign off depot.

(32) The maximum duty for a portion of a broken shift is 6 hours.

APPENDIX II

Data Manipulation

The amount of data associated with the problem is fairly voluminous, and all this data must be held in core store if the cost of the computer processing is to stay within reasonable bounds. The following are the items of data involved for each bus run i:

rn_i : run number assigned to bus i,

dt_i : depot departure time of bus i,

ddt_i : depot of departure of bus i,

at_i : depot arrival time of bus i,

dat_i : depot of arrival of bus i,

cj_i : time when conductor joins bus i,

dj_i : place where conductor joins bus i,

$c\ell_i$: time when conductor leaves bus i,

$d\ell_i$: place where conductor leaves bus i,

nt_i : number of relief points on run rn_i,

rp_{ij} : bus i's j^{th} relief point,

rt_{ij} : time corresponding to rp_{ij}.

The time needed to travel from depot k, to relief point ℓ is represented by $tt_{k\ell}$, where $k = 1,2,3;$ $\ell = 1,2,\ldots,20$.

Since there are some 400 runs each of which have between twenty and ninety pieces of information, some

45,000 store locations are involved. Lists L and B, were defined for handling the large amount of data. The variables given as data by the headway sheet define L in the following way:

$L_{i,-9,0} = rn_i$,

$L_{i,-8,0} = dt_i$,

$L_{i,-7,0} = ddt_i$,

$L_{i,-6,0} = at_i$,

$L_{i,-5,0} = dat_i$,

$L_{i,-4,0} = cj_i$,

$L_{i,-3,0} = dj_i$,

$L_{i,-2,0} = c\ell_i$,

$L_{i,-1,0} = d\ell_i$,

$L_{i,j,1} = rt_{ij}$; where $j > 0$

$L_{i,j,2} = rp_{ij}$; where $j > 0$.

For example, $L_{10,4,1}$ is the fourth relief time given by the headway sheet for the tenth run.

The variables given as data for fixed pieces of work, i.e. those pieces of work for which nt_i is zero, define the list B in the following way:

$B_{i1} = rn_i$,

$B_{i2} = dt_i$,

$B_{i3} = ddt_i$,

$B_{i4} = at_i$,

$B_{i5} = dat_i$,

$B_{16} = cj_1,$

$B_{17} = dj_1,$

$B_{18} = c\ell_1,$

$B_{19} = d\ell_1.$

For example, B_{42} is the depot arrival time of the fourth fixed work run.

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