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Abstract

This paper estimates a New Keynesian model of the U.S. economy over the period following the 2001 slump, a period for which the adequacy of monetary policy is intensely debated. We find that only when measuring inflation with core PCE does monetary policy appear to have been reasonable and sufficiently active to rule out indeterminacy. We then relax the assumption that inflation in the model is measured by a single indicator and re-formulate the artificial economy as a factor model where the theory's concept of inflation is the common factor to the empirical inflation series. CPI and PCE provide better indicators of the latent concept while core PCE is less informative. Finally, we estimate an economy that distinguishes between core and headline inflation rates. This model comfortably rules out indeterminacy.

******JEL codes* **E32**, **E52**, **E58**. *Keywords*: Indeterminacy, Taylor Rules, Great Deviation.

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1 Introduction

It has become prevalent to think of monetary policy in terms of nominal interest rate feedback rules. In certain situations, for example, loose monetary policy, these rules may introduce indeterminacy and sunspot equilibria into otherwise stable economic environments. Lubik and Schorfheide (2004) and many others suggest that, empirically, such sunspots-based instability was confined to the seventies and that the post-Volcker years can ostensibly be characterized by determinacy. The current paper extends this analysis to more recent data leading up to the Great Recession.

The issue of loose monetary policy during the 2000s is closely related to Taylor (2007, 2012), who asserts that the Federal Reserve kept the policy rate too low for too long following the recession of 2001. While Taylor does not touch the issue of indeterminacy, he nevertheless argues that this loose policy created an environment that ultimately brought the economy close to the brink. To bolster his thesis of an extra easy monetary policy, Taylor constructs an artificial path for the Federal Funds rate that follows his proposed rule. He characterizes this counterfactual rate's loose fitting to the actual rate as

> "[...] the biggest deviation, comparable to the turbulent 1970s." [Taylor, 2007, 2]

His view is disputed by many. Amongst them, Bernanke (2010) argues that Taylor's use of the headline consumer price index (CPI) to measure inflation in the Federal Reserve's reaction function is misleading. In fact, the Federal Reserve switched the inflation measures that inform its monetary policy deliberations several times over the last two decades. In particular, it moved away from the CPI to the personal consumption expenditure deflator (PCE) in early 2000. In turn, PCE was abandoned midway through 2004 in favor of the core PCE deflator (which excludes food and energy prices).[1] Bernanke (2015) revisits Taylor's exercise and constructs his own counterfactual Federal Funds rate using core PCE. Bernanke's verdict of the Federal Reserve's policy during the 2000s is inimical to Taylor's and he says that

> "[...] the predictions of my updated Taylor rule and actual Fed policy are generally quite close over the past two decades. In particular, it is no

[1] See Mehra and Sawhney (2010).

longer the case that the actual funds rate falls below the predictions of the rule in 2003-2005." [Bernanke, 2015]

Our paper sheds further light on this debate. It takes as a point of departure Taylor's claim of an analogy between the 1970s and the 2000s as well as one of the key recommendations for monetary policy that has emanated from New Keynesian modelling: interest rates should react strongly to inflation movements to not destabilize the economy. Phrased alternatively, if the central bank's response to inflation is tuned too passively in a Taylor rule sense, multiplicity and endogenous instability may arise. In fact, the U.S. economy of the 1970s can be well represented by an indeterminate version of the New Keynesian model as was shown by Lubik and Schorfheide (2004). Along these lines, the current paper turns Taylor's *too low for too long* story into questioning whether the Federal Reserve operated on the indeterminacy side of the rule after the 2001 slump.

The empirical plausibility of a link between monetary policy and macroeconomic instability was first established by Clarida, Gali and Gertler (2000). They estimate variants of the Taylor rule and their research suggests that the Federal Reserve's policy may have steered the economy into an indeterminate equilibrium during the 1970s. Yet, they also find that the changes to policy which have taken place after 1980 – essentially a more aggressive response to inflation – brought about a stable and determinate environment. Lubik and Schorfheide (2004) reinforce this point but they refrain from using a single equation approach. They recognize that indeterminacy is a property of a rational expectations system and apply Bayesian estimation techniques to a general equilibrium model. Their results parallel the earlier findings that the U.S. economy veered from indeterminacy to determinacy around 1980 – largely as the result of a more aggressive response of monetary policy towards inflation.

Moreover, this monetary policy change had perhaps an even greater influence on the economy: the transformation from the Great Inflation of the 1970s to the Great Moderation is often conjoined to the conduct of monetary policy.[2] Yet, the Great Moderation came to an end sometime during the 2000s, and it was followed by

[2]See, for example, Benati and Surico (2009), Bernanke (2012), Coibion and Gorodnichenko (2011), Arias, Ascari, Branzoli and Castelnuovo (2014) and Hirose, Kurozumi and Van Zandweghe (2015).

enormous economic volatility. Our aim is to examine the possible connection between this transformation and an alteration in the Federal Reserve's monetary policy. In particular, we concentrate on the effects of a possibly too easy monetary policy after the 2001 slump. We frame our analysis from the perspective of (in-)determinacy and conduct it under the umbrella of the Bernanke versus Taylor dispute by considering the measures of inflation that repeatedly occur in the discussion: CPI, PCE and core PCE.

Accordingly, we estimate a small-scale New Keynesian model allowing for indeterminacy over the period between the 2001 slump and the onset of the Great Recession, thus, the NBER-dated 2002:I-2007:III window to be precise. To test for indeterminacy, we employ the method of Lubik and Schorfheide (2004) to compute the posterior probabilities of determinacy and indeterminacy. We take as starting point the same basic New Keynesian model, priors and observables as Lubik and Schorfheide (2004). This strategy allows us to create a continuity between their and our results, which is important given the shortness of our period of interest.

We establish a number of new insights regarding recent U.S. central bank policy. For example, we can indeed expose a violation of the Taylor principle for most of the 2000s when using CPI to measure inflation. This finding supports the visual inspection checks based on single equations in Taylor (2012) who coined the phrase *Great Deviation* to refer to this period. Hence, the 2002:I to 2007:III period would appear to be best described by an indeterminate version of the New Keynesian model. Our upshot is different when basing the analysis on PCE data: we can neither rule in nor rule out indeterminacy. Finally, the evidence in favor of indeterminacy altogether vanishes when we use core PCE. Monetary policy then appears to have been quite appropriate. This conclusion parallels the insight from Bernanke's (2015) counterfactual Federal Funds rate. We thus establish that tests for indeterminacy are susceptible to the data used in the estimation.

We next consider whether our results are an artifact of the six year sample of data. To address this issue, we re-estimate the model on rolling windows of fixed length (23 quarters to match the length of the 2002:I-2007:III period) starting in the mid-1960s and focussing on the same inflation measure as Lubik and Schorfheide (2004) namely CPI inflation. The outcomes of the indeterminacy test performed on rolling windows are highly plausible. In particular, we identify only two broad periods (i.e. several consecutive windows) in which a passive policy has likely led to indeterminacy: the 1970s and the post-2001 period. The first period, which coincides with the span of the Burns and Miller chairmanships, exactly matches the indeterminacy duration, as well as the timing of the switch to determinacy in 1980, that Coibion and Gorodnichenko (2011) document. We take this analogy as a reassuring validation of our small sample approach, i.e. even though our period of interest is quite short, it is possible to infer meaningful information from it.[3]

We then attend the issue of how best to measure inflation in the New Keynesian model. We address the ambiguity between the theoretical concept and the empirical inflation proxies by employing the DSGE-factor methodology proposed by Boivin and Giannoni (2006). Accordingly, we combine various measures of inflation in the measurement equation and re-estimate our model. CPI and PCE emerge as better indicators of the concept of inflation than core PCE and indeterminacy cannot be ruled out.

However, the finding that indeterminacy cannot be ruled out may well hinge on the fact that the baseline three-equations New Keynesian model features a single concept of inflation. To address this question, we finally turn toward an artificial economy that distinguishes explicitly between core and headline inflation. We find that the Federal Reserve was responding mainly to core PCE and was sufficiently active, hence, in this model setup we can comfortably rule out indeterminacy.

Perhaps most closely related to our work are Belongia and Ireland (2015) who, like us, evaluate the Federal Reserve's monetary policy during the 2000s.[4] Belongia and Ireland (2015) estimate a time-varying VAR to track the evolution of Federal Reserve policy that occurred through the 2000s. They find evidence of a change in the Federal Reserve's behavior away from stabilizing inflation towards stabilizing output and also of persistent deviations from the estimated policy rule. While similar in spirit to our results they do not address issues of indeterminacy.

Bianchi (2013) examines the Federal Reserve's policy post-WWII taking a Markov

[3] Judd and Rudebusch (1998) is another example of an evaluation of monetary policy over similarly short sample periods.

[4] See Fackler and McMillin (2015), Fitwi, Hein and Mercer (2015), Groshenny (2013) and Jung and Katayama (2014) for related exercises.

switching rational expectations approach with two monetary policy regimes (i.e. *Hawk* and *Dove*). Bianchi characterizes monetary policy in the early 2000s as Hawkish and identifies a switch to a Dove regime after 2005. His approach to deal with the issue of passive monetary policy is by requiring a linear representation of the Markov switching model to have a unique solution. Phrased alternatively, the regime transitions do not imply moving from determinacy to indeterminacy as both regimes are determinate. Hence, Bianchi's model cannot address questions involving sunspot equilibria as in our paper.

The remainder of the paper evolves as follows. The next section sketches the baseline model and its solution. Section 3 presents the econometric strategy and baseline results. Robustness checks are conducted in section 4. Section 5 relaxes the assumption that model inflation is properly measured by a single empirical indicator. In section 6 we consider an economy that features more than one inflation rate. Section 7 concludes.

2 Baseline model

The familiar three linearized equations summarize our basic New Keynesian model:

$$y_t = E_t y_{t+1} - \tau(R_t - E_t\pi_{t+1}) + g_t \qquad \tau > 0 \tag{1}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa(y_t - z_t) \qquad \kappa > 0,\ 0 < \beta < 1 \tag{2}$$

$$R_t = \rho_R R_{t-1} + (1-\rho_R)(\psi_\pi \pi_t + \psi_y [y_t - z_t]) + \epsilon_{R,t} \qquad 0 \le \rho_R < 1. \tag{3}$$

Here y_t stands for output, R_t denotes the nominal interest rate and π_t symbolizes inflation. E_t represents the expectations operator. Equation (1) is the dynamic IS relation reflecting an Euler equation. Equation (2) describes the expectational Phillips curve. Finally, equation (3) represents monetary policy, i.e. a Taylor-type rule in which $\psi_\pi > 0$ and $\psi_y > 0$ are chosen by the central bank and echo its responsiveness to inflation and the output gap, $y_t - z_t$. The term $\epsilon_{R,t}$ denotes an exogenous monetary policy shock whose standard deviation is given by σ_R. The other fundamental disturbances involve exogenous shifts of the Euler equation which are captured by the process g_t and shifts of the marginal costs of production captured by z_t. Both variables follow AR(1) processes:

$$g_t = \rho_g g_{t-1} + \epsilon_{g,t} \qquad 0 < \rho_g < 1$$

and

$$z_t = \rho_z z_{t-1} + \epsilon_{z,t} \qquad 0 < \rho_z < 1.$$

We denote by σ_g and σ_z the standard deviations of the innovations $\epsilon_{g,t}$ and $\epsilon_{z,t}$. Finally, the term $\rho_{g,z}$ denotes the correlation between the demand and supply innovations. Then, the vector of model parameters entails

$$\theta \equiv \left[\psi_\pi, \psi_y, \rho_R, \beta, \kappa, \tau, \rho_g, \rho_z, \rho_{g,z}, \sigma_R, \sigma_g, \sigma_z\right]'.$$

Indeterminacy implies that fluctuations in economic activity can be driven by arbitrary, self-fulfilling changes in people's expectations (i.e. sunspots). Concretely, in our simple New Keynesian model, indeterminacy occurs when the central bank passively responds to inflation changes, i.e. when $\psi_\pi < 1 - \psi_y (1 - \beta) / \kappa$.

To solve the model, we apply the method proposed by Lubik and Schorfheide (2003) in which case the full set of rational expectations solutions takes on the form

$$\varrho_t = \Phi(\theta)\varrho_{t-1} + \Phi_\varepsilon(\theta, \widetilde{\mathbf{M}})\varepsilon_t + \Phi_\zeta(\theta)\zeta_t \tag{4}$$

where ϱ_t is a vector of model variables,

$$\varrho_t \equiv \left[y_t, R_t, \pi_t, E_t y_{t+1}, E_t \pi_{t+1}, g_t, z_t\right]',$$

ε_t denotes a vector of fundamental shocks and ζ_t is a non-fundamental sunspot shock.[5] The coefficient matrices $\Phi(\theta)$, $\Phi_\varepsilon(\theta, \widetilde{\mathbf{M}})$ and $\Phi_\zeta(\theta)$ are related to the structural parameters of the model. The sunspot shock satisfies $\zeta_t \sim i.i.d.\mathsf{N}(0, \sigma_\zeta^2)$. Indeterminacy can manifest itself in two ways: (i) through pure extrinsic non-fundamental shocks, ζ_t (a.k.a sunspots), disturbing the economy and (ii) through affecting the propagation mechanism of fundamental shocks via $\widetilde{\mathbf{M}}$.

[5] Under determinacy, the solution (4) boils down to $\varrho_t = \Phi^D(\theta)\varrho_{t-1} + \Phi_\varepsilon^D(\theta)\varepsilon_t$.

Table 1 - Prior and posteriors of DSGE parameters.

		Priors		Posterior Mean [5th pct, 95th pct]	
Name	Range	Density	Prior Mean (Std. Dev.)	CPI Indeterminacy	Core PCE Determinacy
ψ_π	$\mathbb{R}^+$	Gamma	1.10 (0.50)	0.84 [0.61,0.98]	3.01 [1.97,4.17]
ψ_y	$\mathbb{R}^+$	Gamma	0.25 (0.15)	0.19 [0.05,0.41]	0.28 [0.07,0.64]
ρ_R	[0,1)	Beta	0.50 (0.20)	0.83 [0.74,0.90]	0.76 [0.64,0.85]
π^*	$\mathbb{R}^+$	Gamma	4.00 (2.00)	3.28 [1.27,6.01]	1.99 [1.67,2.31]
r^*	$\mathbb{R}^+$	Gamma	2.00 (1.00)	1.15 [0.47,2.01]	1.40 [0.84,2.01]
κ	$\mathbb{R}^+$	Gamma	0.50 (0.20)	0.91 [0.51,1.41]	0.71 [0.31,1.19]
τ^{-1}	$\mathbb{R}^+$	Gamma	2.00 (0.50)	1.66 [1.00,2.49]	1.62 [0.95,2.48]
ρ_g	[0,1)	Beta	0.70 (0.10)	0.60 [0.45,0.73]	0.80 [0.72,0.87]
ρ_z	[0,1)	Beta	0.70 (0.10)	0.80 [0.68,0.89]	0.61 [0.49,0.74]
ρ_{gz}	[-1,1]	Normal	0.00 (0.40)	-0.28 [-0.72,0.17]	0.86 [0.57,0.97]
$M_{R\zeta}$	$\mathbb{R}$	Normal	0.00 (1.00)	-0.57 [-1.90,1.00]	
$M_{g\zeta}$	$\mathbb{R}$	Normal	0.00 (1.00)	-1.99 [-2.92,-1.05]	
$M_{z\zeta}$	$\mathbb{R}$	Normal	0.00 (1.00)	0.41 [0.05,0.83]	
σ_R	$\mathbb{R}^+$	IG	0.31 (0.16)	0.16 [0.12,0.21]	0.16 [0.12,0.21]
σ_g	$\mathbb{R}^+$	IG	0.38 (0.20)	0.28 [0.18,0.40]	0.19 [0.14,0.25]
σ_z	$\mathbb{R}^+$	IG	1.00 (0.52)	0.74 [0.54,1.03]	0.62 [0.47,0.82]
σ_ζ	$\mathbb{R}^+$	IG	0.25 (0.13)	0.20 [0.12,0.30]	

Notes: The inverse gamma priors are of the form $p(\sigma|\upsilon,\varsigma) \propto \sigma^{-\upsilon-1}e^{-\frac{\upsilon\varsigma^2}{2\sigma^2}}$, where $\nu = 4$ and ς equals $0.25, 0.3, 0.6$ and 0.2, respectively. The prior predictive probability is 0.527.

3 Estimation and baseline results

3.1 Data and priors

We employ Bayesian techniques for estimating the parameters of the model and test for indeterminacy using posterior model probabilities. The measurement equation relating the elements of ϱ_t to the three observables, x_t, is given by

$$x_t = \begin{bmatrix} 0 \\ r^* + \pi^* \\ \pi^* \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 0 \end{bmatrix} \varrho_t \tag{5}$$

where π^* and r^* are the annualized steady-state inflation and real interest rates respectively. Equation (4) and (5) provide a state-space representation of the linearized model that allows us to apply standard Bayesian estimation techniques. The technical appendix provides further details.

We use HP-filtered per capita real GDP and the Federal Funds Rate as our observable for output and the nominal interest rate. These choices follow Lubik and Schorfheide (2004) and make our baseline empirical analysis comparable to theirs in all dimensions but the sample period. To draw up our analysis in the Bernanke versus Taylor debate, we consider in turn three different measures of inflation: CPI, PCE deflator and core PCE (all expressed in annualized percentage changes from the previous quarter). The data covers the period between the 2001 slump and the onset of the Great Recession, i.e. 2002:I to 2007:III.

Our baseline priors are identical to the ones in Lubik and Schorfheide (2004) and they are reported in Table 1. Following Lubik and Schorfheide (2004) we replace $\widetilde{\mathbf{M}}$ in equation (4) with $\mathbf{M}^*(\theta) + \mathbf{M}$ where $\mathbf{M} \equiv [M_{R\zeta}, M_{g\zeta}, M_{z\zeta}]'$. We select $\mathbf{M}^*(\theta)$ such that the responses of the endogenous variables to fundamental shocks are continuous at the boundary between the determinacy and the indeterminacy regions. We set the prior mean for $\mathbf{M}$ equal to zero.

3.2 Testing for indeterminacy

For each measure of inflation, we estimate the model over the two different regions of the parameter space, i.e. determinacy and indeterminacy. To assess the quality of the model's fit to the data we present marginal data densities and posterior model probabilities for both parametric zones. We approximate the data densities using Geweke's (1999) modified harmonic mean estimator. Table 2 reports our results.

Following Lubik and Schorfheide (2004) and Taylor (2007, 2012), we begin by using headline CPI to measure inflation. In this case, the data favors the indeterminate model: the posterior probability of indeterminacy is 0.90. This result suggests that Taylor's characterization of the Federal Reserve's monetary policy as *too low for too long* is in fact consistent with indeterminacy and potentially has veered the economy into instability.

Table 2: Determinacy versus Indeterminacy

	Log-data density		Probability	
Inflation measure	Determinacy	Indeterminacy	Determinacy	Indeterminacy
CPI	-95.48	-93.28	0.10	0.90
PCE	-85.42	-85.75	0.58	0.42
Core PCE	-64.60	-71.58	1	0

Notes: According to the prior distributions, the probability of determinacy is 0.527.

Yet, the upshot differs depending on which measure of inflation we employ in the estimation. Take Bernanke's (2015) suggestion that Taylor's counterfactual experiment should have been performed with core PCE. When making this choice, the posterior probability for our sample concentrates all of its mass in the determinacy region. This result flags that the Federal Reserve had not been responding passively to inflation during this period.

However, the Humphrey-Hawkins reports to Congress document that the Federal Reserve based monetary policy deliberations on headline PCE from the beginning of 2000 until mid-2004. Since Taylor is particularly critical of the monetary policy from 2002 to 2004, we next measure inflation using headline PCE data. We repeat the estimation and the finding is now ambiguous: the probability of determinacy is 0.58. Phrased alternatively, we cannot dismiss the possibility of indeterminacy.

Table 1 also reports the posterior estimates of the model parameters of the respectively favored models for CPI and for core PCE inflation.[6] The estimated policy

[6]The appendix reports results for parameter estimates when using headline PCE inflation data.

rule's response to inflation, ψ_π, which essentially governs the indeterminacy, differs significantly depending on the way we measure inflation. In particular, when basing the estimation on CPI, the posterior mean equals 0.84 (with 90-percent interval $[0.61, 0.98]$). This result indicates that monetary policy violated the Taylor principle over the 2002-2007 period or in the words of Taylor:

> "[t]he responsiveness appears to be at least as low as in the late 1960s and 1970s." [Taylor, 2007, 8]

The opposite result ensues when using core PCE. In that case, the posterior mean of ψ_π is well above one at 3.01 (with 90-percent interval $[1.97, 4.17]$).

3.3 How important are sunspots and what drives the results?

Indeterminacy can manifest itself through pure extrinsic non-fundamental shocks and/or by affecting the propagation mechanism of fundamental shocks. Given our above results, the question of how important sunspot fluctuations were during the 2000s comes up naturally. To answer to this question, we study the propagation of shocks and the unconditional forecast error variance decomposition. A more detailed analysis can be found in the Appendix' sections 2.4 and 2.5. Based on our estimation using CPI data, sunspots played only a marginal role with the most significant contribution being seven to eight percent in explaining the variances of the policy rate and inflation. However, indeterminacy qualitatively altered the propagation of demand shocks by changing the sign of the inflation response.

In sum, we find that indeterminacy outcomes are dependent on the inflation measure that is used. What is the intuition behind this result and which features of the data stand behind it? Headline inflation (both CPI and PCE) is more volatile than core inflation over the relevant period. In fact, headline inflation tends to be more volatile than core inflation measures that exclude the most volatile components, particularly in periods of persistent commodity price shocks. This volatility feature of the data partly drives our findings through its influence on the estimates of the Taylor rule. With core PCE as the preferred measure of inflation, the monetary authority reacts to relatively small movements in inflation. In that case, any policy response to inflation has to be substantially larger for the estimation procedure to fit the Federal

Funds rate data. In contrast, when measuring inflation with CPI, the estimated responsiveness to inflation turns out to be smaller due to the larger fluctuations of the inflation gap. As monetary policy fails to guarantee a unique rational expectations equilibrium whenever it is insufficiently active with respect to inflation, the posterior probability of indeterminacy is higher with headline than with core inflation.

Beyond the difference in the volatility of the inflation measures, another feature of the data that drives our (in)-determinacy results is a disconnect between core and headline measures of inflation in face of persistent oil price shocks. Indeterminacy affects the propagation of demand shocks. The parameter $M_{g\zeta}$ redirects the transmisson of this disturbance, making it look similar to a cost-push shock. This mix of disturbances helps the model fit the joint behavior of headline inflation (especially CPI), real activity and monetary policy during the 2002-2007 episode.

4 Sensitivity analysis

We now investigate the sensitivity of our results in various directions. The robustness checks involve (i) testing for indeterminacy on rolling windows, (ii) estimating the policy parameters only, (iii) alternative priors for ψ_π, (iv) alternative measures of output and inflation including real-time data, (v) serially correlated monetary policy shocks, and (vi) trend inflation.

Rolling windows The size of our sample is undeniably short. So first and foremost, we want to assess the extent to which our results might be an artifact of the small sample. To do so, we re-estimate the model on rolling windows starting in the mid-1960's, and keeping the size of the windows fixed at 23 quarters to match the number of observations in our period of interest. Thus the first window is 1966:I-1971:III. We move the window forward one quarter at a time, and re-estimate all parameters each time.[7] Here we just consider CPI inflation as the Federal Reserve only began to base its monetary policy deliberations on PCE and core PCE in the 2000s. Moreover, do-

[7] This approach to estimate linear DSGE models was recently promoted by Canova (2009), Canova and Ferroni (2011a) and Castelnuovo (2012a,b). Rolling window estimation provides two benefits. It allows us to uncover time-varying patterns of the model's parameters, in particular, of the monetary policy coefficients. At the same time, the procedure permits us to remain within the realm of linear models and apply standard Bayesian methods.

ing so makes our results directly comparable to Lubik and Schorfheide (2004). Figure 1 presents the evolution of the posterior probability of determinacy for the U.S. economy from 1966:I to 2008:III. The end point is chosen to avoid obvious complications that emanate from hitting the zero lower bound. The graph suggests that the U.S. economy was likely in a state of indeterminacy during the 1970s. Thereafter, beginning with the Volcker disinflation policies, the economy shifted back to a determinate equilibrium. These findings are consistent with related studies such as Clarida, Gali and Gertler (2000), Lubik and Schorfheide (2004) and Coibion and Gorodnichenko (2011).[8] We take this correspondence as a justification for estimating our model on a short window.[9] Our paper documents a second shift after the 2001 slump now from determinacy to indeterminacy.

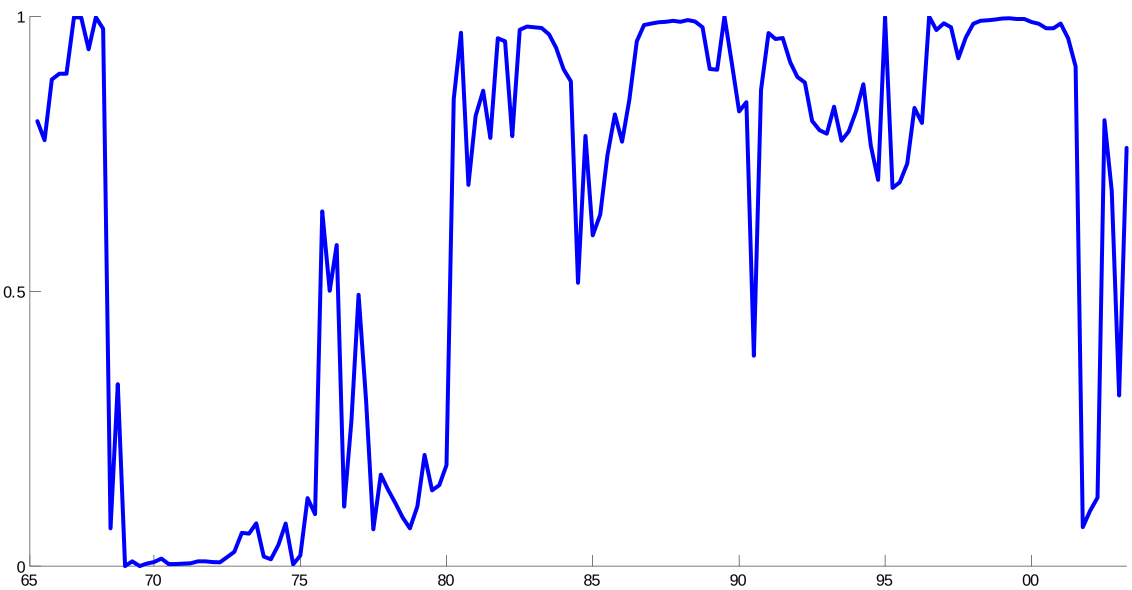

Figure 1: Probability of determinacy using rolling window estimation. The figure plots the probability at the first quarter of a window.

[8]Figure 1 is comparable to Coibion and Gorodnichenko (2011, Figure 4). They report a moving average of the determinacy probability which makes their series smoother than ours. Coibion and Gorodnichenko (2011) use a model with trend inflation. We will explore such model in section 6.

[9]We furthermore experimented with the window length and the results appear to be robust.

Table 3. Determinacy versus Indeterminacy (Robustness)

Inflation measure		Log-data density		Probability	
		Det.	Indet.	Det.	Indet.
CPI	Policy parameters only	-99.97	-95.50	0.01	0.99
	Alternative prior for ψ_π	-95.04	-93.58	0.19	0.81
	CBO output gap	-97.89	-95.85	0.12	0.88
	Output growth	-93.29	-89.58	0.02	0.98
	AR(1) policy shocks	-89.51	-85.68	0.02	0.98
	Trend Inflation	-91.38	-87.13	0.02	0.98
PCE	Policy parameters only	-99.36	-88.79	0.07	0.93
	Alternative prior for ψ_π	-85.04	-85.98	0.72	0.28
	CBO output gap	-88.08	-88.18	0.53	0.47
	Output growth	-82.89	-81.80	0.25	0.75
	Real-time data	-83.32	-83.06	0.44	0.56
	AR(1) policy shocks	-77.59	-77.25	0.42	0.58
	Trend Inflation	-81.54	-82.01	0.62	0.38
Core PCE	Policy parameters only	-63.49	-69.49	1	0
	Alternative prior for ψ_π	-64.47	-71.74	1	0
	CBO output gap	-68.53	-73.63	0.99	0.01
	Output growth	-62.54	-67.58	1	0
	Real-time data	-65.85	-70.24	0.99	0.01
	AR(1) policy shocks	-53.91	-62.09	1	0
	Trend Inflation	-61.13	-64.53	0.97	0.03

Estimating the policy parameters only As a further robustness check to address the small sample issue, we only estimate the policy parameters over the 2002-2007 period. More concretely, we exclusively estimate the three Taylor rule parameters along with the standard deviation of the monetary policy shock (as well as the sunspots related parameters, i.e. the **M**s and σ_ζ, for the indeterminacy version of the model). As for the other parameters, all were calibrated at the posterior means obtained from estimating the determinate model over the period 1991:II to 2001:IV. The reason for beginning right after the 1990-91 recession is closely connected to Figure 1: it comfortably rules out indeterminacy even for "short" periods. Table 3 reports strong evidence for indeterminacy not only when we measure inflation with

CPI but also with PCE. However, as before, the posterior probability puts all its weight on determinacy when inflation is measured using Core PCE.

Alternative priors One possible drawback to using a small sample size is that the prior might *speak louder* than the data. To make our empirical analysis transparent, the priors we employ in our baseline estimation (Table 1) were set identical to the ones used by Lubik and Schorfheide (2004). Accordingly, our baseline specification implies a prior probability of determinacy equal to 0.53. To assess the sensitivity of our results to the priors, we alter the prior distribution for the key parameter that drives indeterminacy. Specifically, we change the prior mean of ψ_π from 1.1 to 1.3 and in doing so we ramp up the prior probability of determinacy from 0.53 to 0.7. Thus, the indeterminacy test will now find it harder to favor indeterminacy. Table 3 reports the posterior probabilities of (in-)determinacy under this alternative prior for each measure of inflation. The results remain largely unaltered. For example, the odds of indeterminacy versus determinacy are still five to one when estimating the model using CPI inflation. This finding provides some further support for our results.

Alternative measures of output To make our baseline analysis comparable with Lubik and Schorfheide (2004), we used HP-filtered GDP to measure output fluctuations. However, as argued by Canova (1998), Gorodnichenko and Ng (1998) among others, HP-filtered data may induce spurious results. Accordingly, we now consider two alternative ways to gauge real economic activity. First, we replace HP-trend output by the Congressional Budget Office's estimate of potential output as in Belongia and Ireland (2015) and others. Table 3 suggests that, again, our results remain robust. Second, we use output growth instead of an output gap measure. To this end, we assume that the artificial economy now features trend-stationary technology – it follows a deterministic trend as in Mattesini and Nisticò (2010) or Ascari, Castelnuovo and Rossi (2011).[10] Also, we no longer estimate the intertemporal rate of substitution, $1/\tau$, and instead set it equal to one to make the model consistent with balanced growth. Then, Table 3 shows that when using output growth, the case for

[10] The measurement equation now writes $\gamma_{yt}^{obs} = \gamma^* + \Delta\widehat{y}_t$ where γ_{yt}^{obs} is the observed growth rate of output, γ^* stands for the steady state growth rate and $\Delta\widehat{y}_t$ is the first-differenced logarithm of detrended model output. The prior distribution of γ^* is $\mathsf{N}(0.5, 0.1)$.

indeterminacy becomes even stronger for CPI and PCE, yet, it remains unchanged when measuring inflation via core PCE data.[11]

GDP deflator While not mentioned in the Humphrey-Hawkins reports to have informed Federal Reserve's policy deliberations during the 2000s, we lastly re-do the analysis with the GDP deflator as the inflation measure (as in Smets and Wouters, 2007). Then, the log-data densities are very close at -73.26 for determinacy and -74.16 for indeterminacy. Phrased differently, the posterior probabilities of determinacy and indeterminacy are 71% versus 29% and again we cannot rule out indeterminacy.

Real-time data One important distinction between CPI and PCE price indices is that the former are not revised (except for seasonal adjustments), whereas the latter go through repeated rounds of revision as more information becomes available. In particular, the PCE-based measure of inflation in Bernanke's (2010) speech is a real-time measure, which, as he argues, may exhibit considerable differences relative to the revised PCE data. Hence, like Orphanides (2004), our paper takes into account that monetary policymakers make decisions based on contemporaneously available information. Therefore, our estimation uses contemporaneously realized rates of output growth and both real-time PCE and core PCE inflation in turn from the Real-Time Data Set for Macroeconomists provided by the Federal Reserve Bank of Philadelphia. Table 3 confirms that our findings remain robust. That we can confidently rule out indeterminacy when basing our estimation on core PCE while when using PCE instead there is a possibility that indeterminacy might have plated some role.

Serially correlated monetary policy shocks Our findings so far have lend some support to the conjecture that monetary policy was extra easy following the 2001 recession. Our exercise interprets this view as a reduction in the Federal Reserve's systematic response to the inflation gap (thereby leading to indeterminacy of the rational expectations equilibrium). However, alternatively, extended periods of low interest rates could also arise due to discretionary deviations from the monetary policy

[11]Given the indicated issues of HP-filtered data and the essentially unchanged results when employing output growth, the remainder of this paper will concentrate on output growth.

rule (see also Rudebusch, 2002, Groshenny, 2013, and Belongia and Ireland, 2016). To assess the robustness of our interpretation, we next allow the monetary policy shocks to be serially correlated. Specifically, we assume that the policy shocks follow the AR(1) process

$$\epsilon_{R,t} = \rho_{\epsilon_R}\epsilon_{R,t-1} + v_t \qquad 0 \leq \rho_{\epsilon_R} < 1$$

where v_t is $i.i.d.\mathsf{N}(0, \sigma_v^2)$ and jointly estimate the autocorrelation parameter, ρ_{ϵ_R}, and the standard deviation of the shock, σ_v^2, along with the other parameters of the model.[12] Table 3 confirms that our results remain unaltered: we still cannot rule out passive responsiveness to inflation and thereby the possibility of indeterminacy.

Trend inflation So far, our analysis had assumed that the U.S. economy is reasonably approximated by the standard New Keynesian model linearized around a zero inflation steady state. However, the Federal Reserve's implicit inflation target as well as the average inflation rate during the Great Moderation period was around two to three percent (depending on the chosen price index). Thus, we extend the baseline model to allow for positive trend inflation. This extension becomes meaningful for at least two further reasons as (i) positive trend inflation alters the determinacy properties of the model and (ii) as the determinate plain-vanilla New Keynesian model features a poor internal propagation mechanism, the posterior mass might be biased toward the indeterminacy region[13], however, trend inflation generates more endogenous persistence of inflation and output even in the determinacy case.

Our estimation is based on a version of Ascari and Sbordone's (2014) Generalized New Keynesian model.[14] In the Generalized New Keynesian model the usual $\psi_\pi > 1$ condition is no longer a sufficient condition for local determinacy of equilibrium and it is not possible to analytically derive the indeterminacy conditions. To continue solving the model via Lubik and Schorfheide's (2004) continuity solution, we resort to numerical methods. In particular, we follow Hirose's (2014) numerical solution strategy for finding the boundary between determinacy and indeterminacy by perturbing the parameter ψ_π in the monetary policy rule.[15] As before we use the growth

[12] The AR(1) coefficient of the policy shock follows a beta prior with mean 0.5 and standard deviation 0.2.

[13] See the discussion between Beyer and Farmer (2007) and Lubik and Schorfheide (2007).

[14] The model's presentation is delegated to the Appendix.

[15] See also Justiniano and Primiceri (2008).

rate of GDP, the Federal Funds rate and the three measures of inflation sequentially. Table 3 provides the marginal data densities along with the posterior model probabilities. The emerging results parallel our earlier findings. When basing the estimation on CPI, the U.S. economy was very likely in an indeterminacy region, however, the opposite holds, again, under core PCE.

5 Which measure of inflation to choose?

Our baseline estimations have delivered mixed evidence regarding the probability of indeterminacy for the 2002:I to 2007:III period. The results are consistently dependent on the specific inflation measure used in the estimation – only with core PCE series can we comfortably rule out indeterminacy. However, each inflation proxy may only provide an imperfect indicator of the model concept. Put differently, all three measures of inflation may contain relevant information. In this line of thinking, we will now depart from the assumption that model inflation is measured by a single series and draw on Boivin and Giannoni's (2006) *data-rich environment* application of dynamic factor analysis to DSGE models.[16] In a nutshell, we want to exploit the information from all the inflation series in the estimation to deliver more robust results. We treat the model concept of inflation as the unobservable common factor for which data series are imperfect proxies.

More concretely, the estimation involves the transition equation (4)

$$\varrho_t = \Phi(\theta)\varrho_{t-1} + \Phi_\varepsilon(\theta, \widetilde{M})\varepsilon_t + \Phi_\zeta(\theta)\zeta_t$$

or its determinacy equivalent

$$\varrho_t = \Phi^D(\theta)\varrho_{t-1} + \Phi_\varepsilon^D(\theta)\varepsilon_t$$

and the measurement equation

$$\begin{bmatrix} \Delta GDP_t \\ FFR_t \\ \mathbf{X}_t \end{bmatrix} = \begin{bmatrix} \gamma^* \\ r^* + \pi^* \\ \underset{4\times 1}{\mathbf{0}} \end{bmatrix} + \begin{bmatrix} \mathbf{I}_2 & \underset{2\times 4}{\mathbf{0}} \\ \underset{4\times 2}{\mathbf{0}} & \mathbf{\Lambda} \end{bmatrix} \begin{bmatrix} \Delta y_t \\ 4R_t \\ \boldsymbol{\pi}_t \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \mathbf{u}_t \end{bmatrix}. \tag{6}$$

Here ΔGDP_t stands for the growth rate of per-capita real GDP, FFR_t denotes the Federal Funds rate, $\mathbf{X}_t \equiv [\Delta CPI_t, \Delta PCE_t, \Delta corePCE_t, \Delta DEF_t]'$ is the vector of

[16] Canova and Ferroni (2011b) and Castelnuovo (2013) are recent applications.

empirical inflation proxies,[17] $\mathbf{\Lambda}$ =diag$(\lambda_{CPI}, \lambda_{PCE}, \lambda_{corePCE}, \lambda_{DEF})$ is a 4×4 diagonal matrix of factor loadings relating the latent model concept of inflation to the four indicators, $\boldsymbol{\pi}_t \equiv 4[\pi_t, \pi_t, \pi_t, \pi_t]'$ and $\mathbf{u}_t = [u_t^{CPI}, u_t^{PCE}, u_t^{corePCE}, u_t^{DEF}]' \sim i.i.d.(\mathbf{0}, \mathbf{\Sigma})$ is a vector of serially and mutually uncorrelated indicator-specific measurement errors, with $\mathbf{\Sigma}$ =diag$(\sigma^2_{CPI}, \sigma^2_{PCE}, \sigma^2_{corePCE}, \sigma^2_{DEF})$.

Table 4: Determinacy versus Indeterminacy (DSGE-Factor)

Log-data density		Probability	
Determinacy	Indeterminacy	Determinacy	Indeterminacy
-162.50	-161.83	0.34	0.66

Notes: The prior predictive probability of determinacy is 0.527.

We jointly estimate the parameters $(\mathbf{\Lambda}, \mathbf{\Sigma})$ of the measurement equation (6) along with the structural parameters θ. We calibrate π^* equal to 2.5 percent - a value roughly in line with the average of the sample means of the inflation series. We standardize the four indicators to have mean zero and unit variance. This standardization permits us to interpret the factor loadings, $\lambda_j s$, as correlations between the latent theoretical concept of inflation and the respective observables.[18] Our prior distribution for the loadings and measurement errors are $\lambda_j \sim Beta(0.50, 0.25)$ and $u_t^j \sim Inverse\ Gamma(0.10, 0.20)$ respectively. By employing a beta distribution, the support of the λ_j is restricted to the open interval $(0, 1)$ which is a necessary sign restriction.

Table 4 reports the resulting log-data densities which are -162.50 for determinacy and -161.83 for indeterminacy. Phrased differently, the posterior probabilities of determinacy and indeterminacy are 34% versus 66%, hence, we cannot rule out indeterminacy.[19]

[17] DEF is the acronym for the GDP Deflator.

[18] See Geweke and Zhou (1996) and Forni, Hallin, Lippi and Reichlin (2000).

[19] We also replicated Lubik and Schorfheide (2004) with the DSGE factor model approach. The outcomes of the indeterminacy test for the pre-Volcker and post-1982 sample periods remain unaltered to this extension.

Table 5 - Parameter Estimation Results (DSGE-Factor)

	Determinacy		Indeterminacy	
	Mean	[5th pct, 95thpct]	Mean	[5th pct, 95th pct]
ψ_π	2.13	[1.29,3.13]	0.80	[0.61,0.98]
ψ_y	0.30	[0.07,0.65]	0.21	[0.05,0.45]
ρ_R	0.81	[0.72,0.88]	0.81	[0.73,0.88]
r^*	1.00	[0.45,1.67]	1.23	[0.57,2.00]
κ	0.74	[0.41,1.15]	1.00	[0.57,1.49]
γ^*	0.53	[0.45,0.62]	0.51	[0.44,0.58]
ρ_g	0.79	[0.68,0.87]	0.60	[0.45,0.74]
ρ_z	0.68	[0.50,0.85]	0.70	[0.54,0.84]
ρ_{gz}	0.14	[-0.33,0.70]	-0.31	[-0.74,0.15]
$M_{R\zeta}$			-0.31	[-1.53,1.17]
$M_{g\zeta}$			-1.77	[-2.59,-0.95]
$M_{z\zeta}$			0.30	[0.01,0.62]
σ_R	0.18	[0.13,0.25]	0.16	[0.12,0.21]
σ_g	0.19	[0.14,0.27]	0.28	[0.18,0.42]
σ_z	0.69	[0.50,0.94]	0.73	[0.53,1.00]
σ_ζ			0.18	[0.12,0.27]
λ_{CPI}	0.76	[0.55,0.93]	0.57	[0.37,0.79]
λ_{PCE}	0.79	[0.59,0.95]	0.59	[0.40,0.82]
$\lambda_{CorePCE}$	0.28	[0.07,0.52]	0.21	[0.06,0.40]
λ_{DEF}	0.53	[0.31,0.77]	0.41	[0.23,0.64]
σ_{CPI}	0.31	[0.20,0.43]	0.32	[0.22,0.43]
σ_{PCE}	0.18	[0.10,0.31]	0.18	[0.10,0.29]
$\sigma_{CorePCE}$	0.91	[0.72,1.14]	0.91	[0.72,1.14]
σ_{DEF}	0.71	[0.56,0.90]	0.70	[0.56,0.88]

Notes: The table reports posterior means and 90 percent probability intervals of the DSGE-Factor model parameters.

Table 5 reports the posterior estimates of the model parameters along with the factor loadings (i.e. the correlations between the latent factor and the proxies) as well as the standard deviations of the measurement errors. Conditional on both determinacy and indeterminacy the loadings on CPI and PCE are about three times as large as the loading on core PCE. Furthermore, there is evidence of substantial indicator-specific component for core PCE as evident in the high standard deviation of its measurement error. These results imply that CPI and PCE provide better indicators of the latent concept of inflation, while core PCE, despite being promoted by Bernanke (2015), is less informative. In other words, while core PCE might better fit the Federal Reserve's behavior in isolation, the other inflation measures are more consistent with the New Keynesian model as a whole.

In sum, when taking the considered variants of the New Keynesian model, indeterminacy cannot be ruled out. What these model versions have in common though is that they all feature only one measure of inflation. In the next section we turn to an economy that explicitly differentiates between core and headline inflation rates.

6 An economy that distinguishes between core and headline inflation

Our baseline results on the issue of equilibrium determinacy were clearly dependent on the particular measure of inflation used in the estimation, thus leaving us with essentially the same dilemma that Taylor and Bernanke originally posed: should we measure inflation with CPI or Core PCE? In the previous section we have attempted to resolve this ambiguity by taking an econometric approach that draws on the DSGE-Factor analysis. Our findings there suggest that the latent concept of inflation in the basic New Keynesian model is more strongly correlated with broad indicators such as CPI and PCE than with narrower proxies such as core PCE. The immediate corollary implication of this finding is that the indeterminate version of the canonical New Keynesian model fits better than its determinate analogue.

However, the finding that indeterminacy cannot be ruled out may well hinge on the fact that the simple three-equation New Keynsian model features a single concept of inflation. Indeed, our DSGE-Factor approach forces the central bank to respond to the exact same measure of inflation (i.e. same combination of indicators) as the one that households consider in their consumption-spending decisions. But what (would be the consequences for equilibrium determinacy) if the Federal Reserve was actually focusing on core inflation in its conduct of monetary policy, as claimed by Bernanke (2015), while private-sector agents were looking at a different, broader, measure of inflation?

To address this question, we now turn toward a structural approach by employing an artificial economy that distinguishes explicitly between core and headline inflation, i.e. both inflation concepts simultaneously appear in the model.

6.1 Model

The artificial economy builds on Blanchard and Gali (2010) and Blanchard and Riggi (2013) who introduce imported oil into an otherwise standard New Keynesian model. We present the key aspects of the linearized model here and delegate the full description to the Appendix. Our exposition draws heavily on Blanchard and Gali (2010).

Oil is used by firms in production and by households in consumption. In particular, technology is given by a Cobb-Douglas production function that uses labor, n_t, and oil, m_t :

$$q_t = \alpha m_t + (1-\alpha) n_t \qquad 0 < \alpha < 1 \tag{7}$$

where q_t stands for gross output. Similarly, final consumption, c_t, is made up of domestically produced good, $c_{q,t}$, and imported oil, $c_{m,t}$:[20]

$$c_t = (1-\chi) c_{q,t} + \chi c_{m,t} \qquad 0 < \chi < 1. \tag{8}$$

Denoting the price of domestic output and the price of consumption by $p_{q,t}$ and $p_{c,t}$ respectively, and letting $p_{m,t}$ be the nominal price of oil, the following relationship arises between consumption-price inflation $\pi_{c,t}$ and domestic output-price inflation $\pi_{q,t}$:

$$\pi_{c,t} = \pi_{q,t} + \chi \Delta s_t \tag{9}$$

where s_t is the real price of oil, $s_t \equiv p_{m,t} - p_{q,t}$, which is exogenous. Following Aoki (2001) and Blanchard and Gali (2010), we interpret $\pi_{c,t}$ and $\pi_{q,t}$ as headline and core inflation respectively. Utility maximization by the household yields the standard intertemporal optimality condition

$$c_t = E_t c_{t+1} + E_t z_{t+1} - R_t + E_t \pi_{c,t+1} + d_t - E_t d_{t+1} \tag{10}$$

[20] If the shares α and χ are set to zero, the economy boils down to a simple three-equation New Keynesian model, similar to the one we have used in the previous sections.

and the intratemporal leisure-consumption trade-off

$$w_t - p_{c,t} = \gamma(w_{t-1} - p_{c,t-1}) + (1-\gamma)[\varphi n_t + c_t]. \tag{11}$$

Here R_t denotes the nominal interest rate, d_t is a discount-factor shock, z_t is a shock to the growth rate of technology, w_t denotes the nominal wage and φ stands for the inverse Frisch elasticity. The parameter $\gamma \in [0,1]$ captures the extent of real wage rigidity where larger values indicate higher degrees of rigidity. Notice in the household's Euler equation (10) that the model-consistent real interest rate that drives consumption dynamics involves headline consumption price inflation. Domestic firms are monopolistic competitors facing nominal rigidities à la Calvo. Firms' profit-maximizing pricing decisions result in the familiar aggregate New Keynesian Phillips curve which governs the dynamics of domestic-good sticky-price inflation (i.e. core inflation):

$$\pi_{q,t} = \beta E_t \pi_{q,t+1} - \kappa\mu_t \tag{12}$$

where the slope coefficient $\kappa \equiv \frac{(1-\xi)(1-\beta\xi)}{\xi}$, ξ denotes the probability of not being able to reset prices, β represents the household's discount factor and μ_t is the price markup over nominal marginal costs. Cost minimization by firms gives rise to the following demand for oil:

$$m_t = q_t - \mu_t - s_t. \tag{13}$$

The requirement that trade be balanced (as oil is imported) delivers the following relationship between final consumption and domestic output:

$$c_t = q_t - \chi s_t + \eta\mu_t \tag{14}$$

where $\eta \equiv \frac{\alpha}{\mathcal{M}^P - \alpha}$ and $\mathcal{M}^P$ denotes the steady-state gross markup. Value added (i.e. GDP), denoted by y_t, is given by:

$$y_t = q_t + \frac{\alpha}{1-\alpha}s_t + \eta\mu_t. \tag{15}$$

Monetary policy follows a Taylor rule which reacts to inflation, deviations of GDP from the balanced-growth path and the growth rate of GDP, $gy_t \equiv y_t - y_{t-1} + z_t$:

$$R_t = \rho_R R_{t-1} + (1-\rho_R)[\psi_\pi\{\omega\pi_{c,t} + (1-\omega)\pi_{q,t}\} + \psi_y y_t + \psi_{gy} gy_t] + \epsilon_{R,t}$$

where the monetary policy shock $\epsilon_{R,t}$ is i.i.d. $\mathsf{N}(0, \sigma_R^2)$. Notice that the central bank responds to a convex combination of headline and core inflation (with the parameter ω governing the relative weights; setting ω to zero implies that the central bank responds to core inflation only). As we have seen, the controversy between Taylor and Bernanke essentially boils down to the choice of the inflation measure in the monetary policy rule. By estimating ω, we will let the data speak as to whether the Federal Reserve was actually focusing on headline (Taylor, 2007) or core inflation (Bernanke, 2015). Lastly the structural disturbances s_t, z_t, and d_t are assumed to follow independent stationary $AR(1)$ processes:

$$s_t = \rho_s s_{t-1} + \varepsilon_{st} \qquad z_t = \rho_z z_{t-1} + \varepsilon_{zt} \qquad \text{and} \qquad d_t = \rho_d d_{t-1} + \varepsilon_{dt}.$$

We find that the *Taylor Principle* continues to hold in the Blanchard-Gali model.[21] In line with Carlstrom, Fuerst and Ghironi (2006), the indeterminacy condition is not dependent on any particular measure of inflation: as long as the central bank sets this coefficient greater than unity to either headline inflation, core inflation or a combination of the two, such policy will ensure equilibrium determinacy.

6.2 Econometric strategy and results

To address typical identification issues, we calibrate a subset of the model parameters. We set the discount factor β to 0.99, the steady-state markup at ten percent, and the inverse of the labor-supply elasticity φ to one. Following the computations in Blanchard and Gali (2010) for their post-1984 sample period, we calibrate the shares of oil in production and consumption to $\alpha = 0.012$ and $\chi = 0.017$. Furthermore, we assume that shocks to the growth rate of technology are i.i.d., i.e. $\rho_z = 0$. We estimate the remaining parameters with Bayesian techniques. We use a loose Beta distribution centered at 0.5 to place an agnostic prior on both the wage-rigidity parameter, γ, and the weight on headline inflation in the monetary policy rule, ω. The other priors are similar to the ones we have used in the earlier sections and are reported in Table 6.

For our purpose, the main appeal of the Blanchard-Gali model is that it offers a micro-founded distinction between core and headline inflation which permits us

[21] Figure A5 in the Appendix shows the determinacy region for combinations of ψ_π with the other policy parameters as well as with the degree of real wage rigidity γ.

to use both headline and core inflation data in the estimation. This approach will hopefully resolve some of the ambiguity that characterized our previous results. At first, however, to maintain a continuity with our earlier findings, we estimate the new model using the exact same dataset with only three observables: the quarterly growth rate of real GDP per-capita, the Federal Funds rate and two alternative inflation rates (CPI and core PCE).

Since we are initially using one inflation data at a time, the weight ω in the Taylor rule is not well identified. Hence when using CPI data, we calibrate this parameter to one, so that the central bank responds solely to headline inflation as in Taylor (2007). Table 6 reports the posterior estimates while Table 7 gives the log-data densities. In line with all our previous results based on CPI, the data favours the indeterminate version of the model.

However, Bernanke (2015) suggests that the Federal Reserve targeted core PCE hence we next calibrate ω to zero, so that the central bank responds to core inflation only. Again, Tables 6 and 7 present the results. As before, when estimating the model based on core PCE, the data favours determinacy. Comparing the augmented economy with the baseline model shown in Table 3 (the row labelled 'Output Growth'), we see that the marginal data densities are of similar magnitude thus the additional micro-foundations of the Blanchard-Gali model are not rejected by the data.

We can now move on to the next exercise: treating both headline and core inflation as observables to properly identify the exogenous stochastic process that governs the commodity price inflation. To do so, we use observations on PCE and Core PCE inflation rates in our dataset which now includes four variables. Also, we are now in a position to estimate the parameter ω governing the relative weights between core and headline inflation in the policy rule. Table 7 shows that the posterior probability concentrates all of its mass in the determinacy region. Looking at the posterior mean estimates in Table 6, the weight on headline PCE inflation, ω, is 0.25. Our estimation therefore provides some empirical support for Bernanke's (2015) claim that the Federal Reserve was actively reacting to core inflation (as opposed to headline) during this period. Moreover, as anticipated, the parameters pertaining to the commodity-price shock are now well identified: the posterior mean estimates of ρ_s and σ_s are 0.88 and 18.04 respectively, both significantly higher than the estimates we had obtained when using only three observables.

A key parameter in the Blanchard and Gali (2010) model is the degree of real wage rigidity, γ. To sharpen the identification of this feature, we finally add real wage data, i.e. we ultimately employ five observables to estimate the model. We use observations on hourly compensation for the non-farm business sector for all persons as a measure of nominal wages. We then divide this proxy by the PCE price deflator to get real wages. To circumvent the issue of stochastic singularity, we add a labor supply shock, ν_t.[22] As a result, the labor supply equation (11) becomes:

$$w_t - p_{c,t} = \gamma(w_{t-1} - p_{c,t-1}) + (1-\gamma)[\varphi n_t + c_t] + \nu_t. \tag{16}$$

Our main finding, that the data favours determinacy in this extended model with both core and headline inflation, remains unchanged. The parameter estimate of γ, however, turns out to be higher when using real wage data. It suggests significant wage rigidity which contrasts with Blanchard and Riggi (2013) who find that real wages were flexible during the Great Moderation period. This divergence might be due to the different estimation strategy we employ. While Blanchard and Riggi (2013) adopt a limited-information approach that matches the DSGE model's impulse responses to a commodity price shock to the corresponding impulse responses inferred from a structural VAR, we use a full-information Bayesian estimation with multiple shocks.

In summary, our estimation of the Blanchard-Gali model provides evidence that the Federal Reserve's monetary policy in the aftermath of the 2001 slump was responding to core PCE and was sufficiently active to ensure equilibrium determinacy. These results line up with Bernanke's (2015) account.[23]

[22] As in Smets and Wouters (2007) and Justiniano, Primiceri and Tambalotti (2010), we normalize the labor supply shock such that it enters the household's intratemporal optimality condition with a unit coefficient. This procedure improves the identification of the standard deviation of the labor supply disturbance and facilitates convergence of the MCMC algorithm.

[23] We have also estimated the extended model using data on CPI/core CPI and CPI/core PCE to measure headline and core inflation. Our finding that the Federal Reserve was actively reacting to core inflation, thereby ensuring determinacy, remains robust. Furthermore we have used real-time data on per-capita real GDP growth rate, PCE and core PCE inflation. Our results remain robust and are reported in the Appendix.

Table 6: Priors and posteriors for DSGE parameters.

Name	Range	Density	Prior Mean (Std. Dev.)	Posterior Mean [5th pct, 95th pct] Three obs (CPI, ω=1)	Three obs (Core PCE, ω=0)	Four obs	Five obs
ψ_π	$\mathbb{R}^+$	Gamma	1.10 (0.50)	0.85 [0.63,0.98]	3.00 [2.01,4.14]	2.91 [1.94,4.03]	2.61 [1.57,3.86]
ψ_y	$\mathbb{R}^+$	Gamma	0.25 (0.15)	0.22 [0.06,0.46]	0.28 [0.07,0.61]	0.30 [0.08,0.64]	0.11 [0.03,0.26]
ψ_{gy}	$\mathbb{R}^+$	Gamma	0.25 (0.15)	0.47 [0.17,0.81]	0.28 [0.08,0.55]	0.29 [0.08,0.58]	0.62 [0.21,1.15]
ρ_R	[0,1)	Beta	0.70 (0.10)	0.79 [0.70,0.86]	0.72 [0.61,0.81]	0.73 [0.62,0.82]	0.78 [0.66,0.88]
ω	[0,1)	Beta	0.50 (0.20)		–	0.25 [0.08,0.47]	0.32 [0.10,0.59]
κ	$\mathbb{R}^+$	Gamma	0.50 (0.10)	0.61 [0.45,0.80]	0.54 [0.39,0.72]	0.52 [0.38,0.70]	0.38 [0.25,0.53]
γ	[0,1)	Beta	0.50 (0.20)	0.23 [0.07,0.46]	0.26 [0.07,0.50]	0.14 [0.04,0.28]	0.50 [0.30,0.68]
π^*	$\mathbb{R}^+$	Gamma	4.00 (2.00)	2.92 [1.12,5.42]	1.96 [1.55,2.39]	1.99 [1.59,2.42]	1.95 [1.37,2.53]
r^*	$\mathbb{R}^+$	Gamma	2.00 (1.00)	1.06 [0.43,1.85]	1.30 [0.72,1.99]	1.17 [0.64,1.75]	1.17 [0.59,1.84]
γ^*	$\mathbb{R}$	Normal	0.50 (0.10)	0.51 [0.39,0.64]	0.48 [0.38,0.60]	0.48 [0.37,0.59]	0.50 [0.37,0.64]
ρ_s	[0,1)	Beta	0.70 (0.10)	0.70 [0.53,0.85]	0.70 [0.53,0.85]	0.88 [0.80,0.94]	0.88 [0.80,0.94]
ρ_d	[0,1)	Beta	0.70 (0.10)	0.68 [0.52,0.81]	0.87 [0.79,0.93]	0.82 [0.72,0.91]	0.78 [0.66,0.88]
ρ_ν	[0,1)	Beta	0.70 (0.10)	–	–	–	0.58 [0.39,0.81]
σ_z	$\mathbb{R}^+$	IG	0.50 (∞)	0.61 [0.46,0.80]	0.43 [0.34,0.55]	0.42 [0.33,0.54]	0.68 [0.52,0.89]
σ_R	$\mathbb{R}^+$	IG	0.50 (∞)	0.17 [0.12,0.24]	0.17 [0.12,0.23]	0.17 [0.12,0.24]	0.16 [0.11,0.24]
σ_s	$\mathbb{R}^+$	IG	0.50 (∞)	0.30 [0.15,0.59]	0.43 [0.16,1.00]	18.04 [14.07,22.85]	18.17 [14.08,23.36]
σ_d	$\mathbb{R}^+$	IG	0.50 (∞)	0.61 [0.26,1.08]	0.80 [0.53,1.21]	0.64 [0.42,0.99]	0.77 [0.48,1.17]
σ_ν	$\mathbb{R}^+$	IG	0.50 (∞)	–	–	–	0.62 [0.44,0.85]
σ_ς	$\mathbb{R}^+$	IG	0.50 (∞)	0.20 [0.13,0.33]	–	–	–
$M_{z\varsigma}$	$\mathbb{R}$	Normal	0.00 (1.00)	−0.36 [−0.63,−0.11]	–	–	–
$M_{R\varsigma}$	$\mathbb{R}$	Normal	0.00 (1.00)	−0.17 [−1.12,0.90]	–	–	–
$M_{s\varsigma}$	$\mathbb{R}$	Normal	0.00 (1.00)	0.01 [−0.71,0.76]	–	–	–
$M_{d\varsigma}$	$\mathbb{R}$	Normal	0.00 (1.00)	−1.20 [−1.66,−0.87]	–	–	–

Notes: The inverse gamma priors are of the form $p(\sigma|\upsilon,\varsigma) \propto \sigma^{-\upsilon-1}e^{-\frac{\upsilon\varsigma^2}{2\sigma^2}}$, where $\nu = 2$ and $\varsigma = 0.282$. The prior predictive probability is 0.51.

Table 7: Determinacy versus Indeterminacy

	Log-data density		Probability	
Inflation measure	Determinacy	Indeterminacy	Determinacy	Indeterminacy
Three obs (CPI)	−93.98	−88.06	0	1
Three obs (Core PCE)	−61.14	−67.33	1	0
Four obs	−111.55	−123.16	1	0
Five obs	−156.30	−161.86	1	0

Notes: According to the prior distributions, the probability of determinacy is 0.51.

7 Concluding remarks

Using the Taylor rule as a benchmark for evaluating the Federal Reserve's interest-rate setting decisions, some commentators have argued that monetary policy was too accommodative during the 2002-2005 period. Along these lines, this paper estimates a New Keynesian model of the U.S. economy for the time following the 2001 slump. Our assessment of the Federal Reserve's performance varies with the measure of inflation that is put into the model estimation. When measuring inflation with CPI or PCE, we find some support for the view that monetary policy during these years was extra easy and led to equilibrium indeterminacy. Instead, if the estimation involves core PCE, monetary policy comes out as active and the evidence for indeterminacy dissipates. This result remains very robust to several extensions of the model including trend inflation. Our take on these diverging results is that each inflation series only provides an imperfect proxy for the model's concept of inflation. We re-formulate the artificial economy as a factor model where the theory's concept of inflation is the common factor to the alternative empirical inflation series. Again, extra easy monetary policy as well as indeterminacy cannot be ruled out. Finally, we move to an economy that explicitly distinguishes between headline and core inflation. We find that the Federal Reserve was responding mainly to core PCE and was sufficiently active to comfortably rule out indeterminacy.

We choose to make these arguments while staying in a relatively standard model. This choice enables to establish a bridge from existing research to our study which we believe is important given the short data sample that we consider. We specifically do not introduce asset markets into the model or in the estimation. Thus, in terms of possible extensions, it would be worthwhile to introduce housing into the model and in the econometric analysis. It is our intention to pursue these lines of research in the near future.

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Appendix for "Monetary Policy and Indeterminacy after the 2001 Slump" (not for publication)*

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Mark Weder‡

December 5, 2016

1 Roadmap

This Appendix presents several extensions and robustness checks to our paper. Section 2 describes the plain-vanilla New Keynesian model used in our baseline analysis, the solution method under indeterminacy as well as the data and estimation strategy. The section also discusses about the propagation of shocks, both fundamental and sunspots, and also the unconditional forecast error variance decomposition of shocks along with some extra results. Section 3 extends the baseline model to allow for positive trend inflation. Section 4 provides some additional results with respect to using core CPI as a measure of inflation. Finally, Section 5 describes in details an artificial economy that distinguishes between core and headline inflation. The theoretical model in that section builds on Blanchard and Gali (2010) and Blanchard and Riggi (2013).

2 Framework of the structural analysis

2.1 Baseline New Keynesian Model

The artificial economy can be summarized in terms of the familiar linearized three equations of the plain-vanilla New Keynesian (NK) model:

$$y_t = E_t y_{t+1} - \tau(R_t - E_t \pi_{t+1}) + g_t \tag{1}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa(y_t - z_t) \tag{2}$$

$$R_t = \rho_R R_{t-1} + (1 - \rho_R)(\psi_\pi \pi_t + \psi_y [y_t - z_t]) + \varepsilon_{R,t}. \tag{3}$$

* *JEL codes* **E32**, **E52**, **E58**. *Keywords*: Great Deviation, Indeterminacy, Trend inflation, Taylor Rules.

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Here y_t stands for the output, R_t denotes the interest rate and π_t symbolizes the inflation rate. E_t represents the expectations operator. Equation (1) is the dynamic IS-relation reflecting an Euler equation in which τ can be interpreted as the intertemporal elasticity of substitution. Equation (2) describes the expectational Phillips curve where $0 < \beta < 1$ is the agents' discount factor. Finally, equation (3) describes monetary policy, i.e. a Taylor-type nominal interest rate rule in which $\psi_\pi$and ψ_y are chosen by the central bank and echo its responsiveness to inflation and the output gap, $y_t - z_t$. $0 < \rho_R < 1$ is the usual smoothing term. $\epsilon_{R,t}$ denotes an exogenous monetary policy shock whose standard deviation is given by σ_R. Fundamental disturbances involve exogenous shifts of the Euler equation captured by the process g_t and shifts of the marginal costs of production captured by z_t. Both variables follow AR(1) processes:

$$g_t = \rho_g g_{t-1} + \epsilon_{g,t} \qquad 0 < \rho_g < 1 \tag{4}$$

$$z_t = \rho_z z_{t-1} + \epsilon_{z,t} \qquad 0 < \rho_z < 1. \tag{5}$$

The standard deviations for the demand and supply shocks are denoted by σ_g and σ_z. We allow for a non-zero correlation, $\rho_{g,z}$, between the demand and supply innovations.

Indeterminacy implies that fluctuations in economic activity can be driven by arbitrary, self-fulfilling changes in people's expectations (i.e. sunspots). Concretely, in the above New Keynesian model this can occur if the central bank only irresolutely responds to inflation changes. The precise analytical condition for indeterminacy corresponds to $\phi_\pi < 1 - \phi_y (1-\beta)/\kappa$.

2.2 Rational-expectations solution under indeterminacy

Heree we will outline the solution to this model which follows Lubik and Schorfheide (2003). Let us denote by η_t the vector of one-step ahead expectational errors. Moreover, define ϱ_t as the vector of endogenous variables and ε_t as vector of fundamental shocks. Then, the linear rational expectation system can be compactly written as

$$\Gamma_0(\theta)\varrho_t = \Gamma_1(\theta)\varrho_{t-1} + \Psi(\theta)\varepsilon_t + \Pi(\theta)\eta_t \tag{6}$$

where $\Gamma_0(\theta)$, $\Gamma_1(\theta)$, $\Psi(\theta)$, and $\Pi(\theta)$ are appropriately defined coefficient matrices. We follow Sims' (2002) solution algorithm that was revisited by Lubik and Schorfheide (2003). This has the advantage of being general and explicit in dealing with expectation errors since it makes the solution suitable for solving and estimating models which feature multiple equilibria. In particular, under indeterminacy η_t will be a linear function of the fundamental shocks and the purely extrinsic sunspot disturbances, ζ_t. Hence, the full set of solutions to the LRE model entails

$$\varrho_t = \Phi(\theta)\varrho_{t-1} + \Phi_\varepsilon(\theta, \widetilde{M})\varepsilon_t + \Phi_\zeta(\theta)\zeta_t \tag{7}$$

where $\Phi(\theta)$, $\Phi_\varepsilon(\theta, \widetilde{M})$ and $\Phi_\zeta(\theta)$[1] are the coefficient matrices.[2] The sunspot shock satisfies $\zeta_t \sim i.i.d.\mathsf{N}(0, \sigma_\zeta^2)$. Accordingly, indeterminacy can manifest itself in one

[1] Lubik and Schorfheide (2003) express this term as $\Phi_\zeta(\theta, M_\zeta)$, where M_ζ is an arbitrary matrix. For identification purpose, they impose the normalization such that $M_\zeta = I$.

[2] Under determinacy, the solution boils down to $\varrho_t = \Phi^D(\theta)\varrho_{t-1} + \Phi_\varepsilon^D(\theta)\varepsilon_t$.

of two different ways: (i) pure extrinsic non-fundamental disturbances can affect model dynamics through endogenous expectation errors and (ii) the propagation of fundamental shocks cannot be uniquely pinned down and the multiplicity of equilibria affecting this propagation mechanism is captured by the arbitrary matrix $\widetilde{M}$.

Following Lubik and Schorfheide (2004) we replace $\widetilde{M}$ with $M^*(\theta) + M$ and in the subsequent empirical analysis set the prior mean for M equal to zero. The particular solution employed in their paper selects $M^*(\theta)$ by using a least squares criterion to minimize the behaviour of the model under determinacy and indeterminacy by assuming that it remains unchanged across the boundary. "Behaviour" needs be described in some meaningful way and we follow them by choosing $M^*(\theta)$ such that the response of the endogenous variables to fundamental shocks, $\partial \varrho_t / \partial \varepsilon_t'$, are continuous at the boundary between the determinacy and the indeterminacy region.

2.3 Data

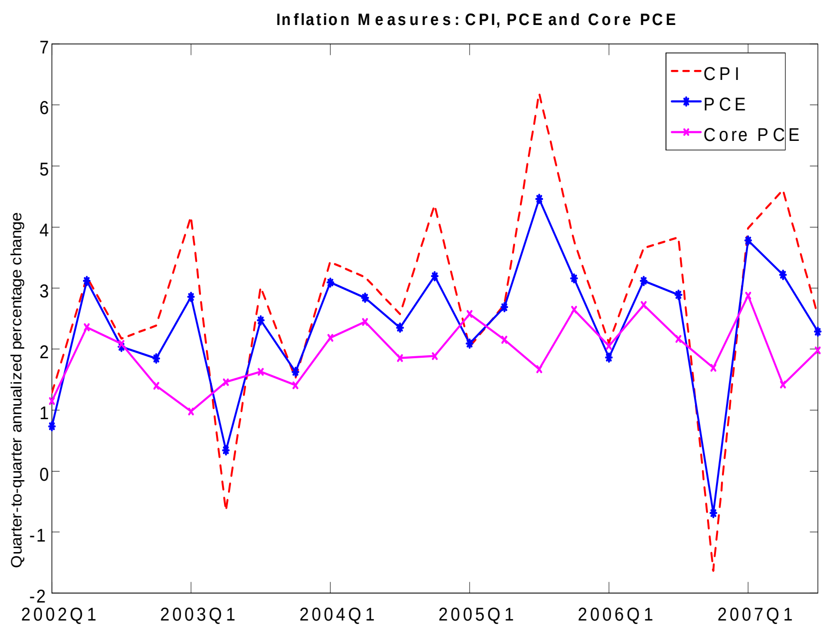

Figure A1

Figure A1 plots the three different measures of inflation, namely, CPI, PCE and core PCE. Headline inflation (both CPI and PCE) is more volatile than core inflation over the relevant period. In fact, headline inflation tends to be more volatile than core inflation measures that exclude or downweight the most volatile components, particularly in periods of persistent commodity price shocks.

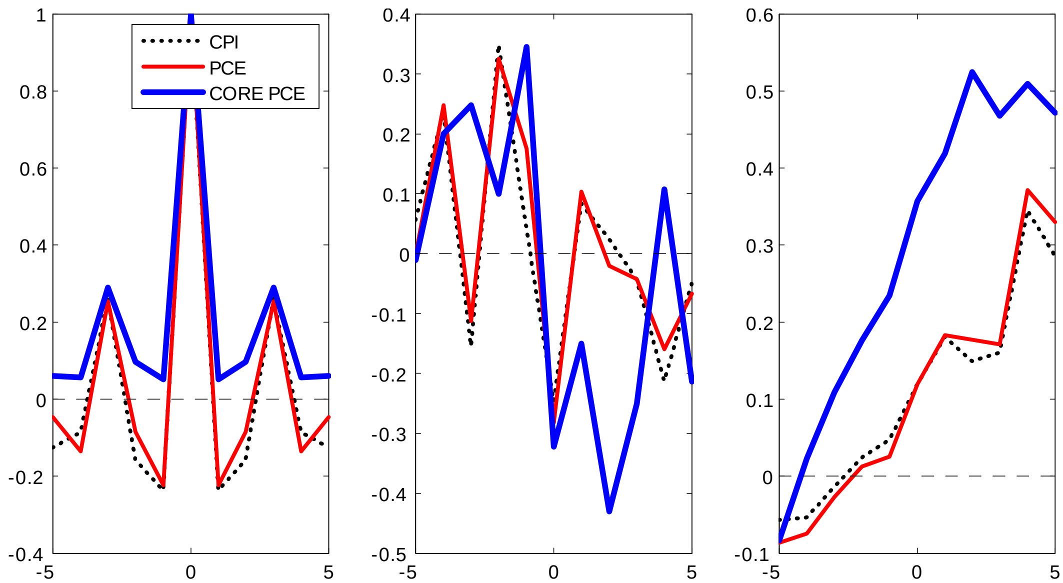

Figure A2

Figure A2 plots the autocorrelation pattern (with five leads and lags) of the three diferent measures of inflation along with their cross-correlation with the growth rate of GDP and the Federal Funds rate. As seen in the figure, the cross-correlation patterns of headline inflation measures (CPI and PCE) on the one hand, and of core PCE on the other hand, with the other two observables are notably different during our period of interest.

2.4 Estimation Strategy

We employ Bayesian techniques for estimating the parameters of the model and test for indeterminacy using posterior model probabilities. In order to construct a likelihood function the DSGE model is turned into a Bayesian model. Toward that purpose we need to define a set of measurement equations that relate the elements of ϱ_t to a set of observables x_t which is given by

$$x_t = \begin{bmatrix} \gamma^* \\ r^* + \pi^* \\ \pi^* \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 0 \end{bmatrix} \varrho_t \tag{8}$$

where π^*, $r^*$and γ^* are annualized steady state inflation, annualized steady state real interest rates and quarterly steady state growth rate of real GDP per capita respectively.[3] Equations (7) and (8) provide a state-space representation for the linearized DSGE model that allows us to continue to apply standard Bayesian methodologies.

First priors are described by a density function of the form

[3]When using HP-filtered data to measure real activity γ^* is set to zero.

$$p(\theta_S|S)$$

where $S \in \{D, I\}$ stands for a specific model, θ_S represents the parameter of the model S, $p(.)$ stands for probability density function. Next, the likelihood function describes the density of the observed data:

$$\mathcal{L}(\theta_S|X_T, S) \equiv p(X_T|\theta_S, S)$$

where X_T are the observations until period T. By using Bayes theorem we can combine the prior density and the likelihood function to get the posterior density:

$$p(\theta_S, X_T, S) = \frac{p(X_T|\theta_S, S)p(\theta_S|S)}{p(X_T, S)}$$

where $p(X_T|S)$ is the marginal marginal density of the data conditional on the model which is given by

$$p(X_T|S) = \int_{\theta_S} p(\theta_S; X_T) d\theta_S.$$

Finally, the posterior kernel corresponds to the numerator of the posterior density:

$$p(\theta_S|X_T, S) \propto p(X_T|\theta_S, S)p(\theta_S|S) \equiv \kappa(\theta_S|X_T, S).$$

We maximize the posterior kernel and find the posterior mode in the two regions of the parameter space using Sims' `csminwel`. The inverse Hessian is calculated at the posterior mode.[4] Next for each region of the parameter space we estimate the likelihood function with the help of the Kalman filter and generate 250,000 draws with a random-walk Metropolis Hastings algorithm. The algorithm is tuned to achieve 25 to 30 percent acceptance rate. Half of the parameter draws are discarded to ensure convergence and the remaining draws are used to generate our results. The marginal data densities for the two regions are computed with Geweke's (1999) modified harmonic mean estimator.

2.5 Propagation of Shocks

Here we study the propagation of sunspots as well as of fundamental shocks. Figure A3 depicts the impulse responses of output, inflation and the nominal interest rate under determinacy (the model being estimated using core PCE inflation) while Figure A4 graphs the responses under indeterminacy (using CPI inflation). Solid lines track the posterior means while the shaded areas cover the 90 percent probability intervals.

Let us begin with the model's reaction to sunspots. The bottom panels of Figure A3 display the reaction to an inflationary sunspot shock. The impulse responses show

[4] For our rolling windown approach, if for a particular sample a region of the parameter space does not have a local mode, we use the inverse Hessian obtained from the nearest previous sample for that region.

that the shock reduces the expected real return which subsequently increases current consumption and hence output. The Phillips curve then translates this into a rise of inflation thereby creating a self-fulfilling cycle: higher inflation expectations lead to higher actual inflation.

Fundamental shocks follow next. The first and second rows of Figures A3 and A4 plot the responses to monetary policy and cost-push shocks. The patterns of the key model variables look similar for both the indeterminate and the determinate versions of the model. This contrasts with the responses to aggregate demand shocks. While at impact we observe an increase of output in both regimes, the responses of inflation are quite different. The determinate model's response of inflation is conventional: it increases which is matched by the central bank tightening its policy – the nominal interest rate rises. However, inflation falls under indeterminacy which appears to reflect the alternative propagation of fundamental shocks in model versions that feature indeterminacy. These propagation dynamics are captured by the elements of the matrix $\mathbf{M}$. In particular, the posterior estimate of $M_{G\zeta}$ is far from zero at -1.99 and as such qualitatively alters the dynamics of a demand shock.

2.6 Variance Decomposition

The unconditional forecast error variance decomposition at the posterior mean for output (deviations from trend), inflation and interest rates are reported in Table A1. The ε_{gt} and ε_{zt} shocks are orthogonalized such that the cost-push shock only affects ε_{zt} and the demand shock affects both ε_{gt} and ε_{zt}. The rationale is that demand shocks will affect the labor supply decisions, hence, the firms' cost function.

The main message we take from this exercise is that in the indeterminacy regime, cost-push shocks cause over 80 percent of output fluctuations whereas in determinacy case aggregate demand disturbances are the main driver of aggregate fluctuations. Sunspot shocks play only a marginal role with the most significant contribution being eight percent in explaining the variance decomposition of the policy rate. This is in line with the results reported above. In conclusion, using different measures of inflation results in drastically different interpretations of the potential causes of output fluctuations.

Table A1: Variance Decomposition

	Variables\Shocks	ε_R	ε_g	ε_z	ζ
CPI (Indet.)	y	9.44	7.47	82.37	0.71
	π	21.82	54.53	16.45	7.2
	R	1.29	74.28	16.24	8.20
Core PCE (Det.)	y	1.99	83.57	14.43	-
	π	39.25	31.03	29.72	-
	R	7.51	69.37	23.12	-

Variance decompositions are performed at the mean of the posterior distribution of the model's parameters.

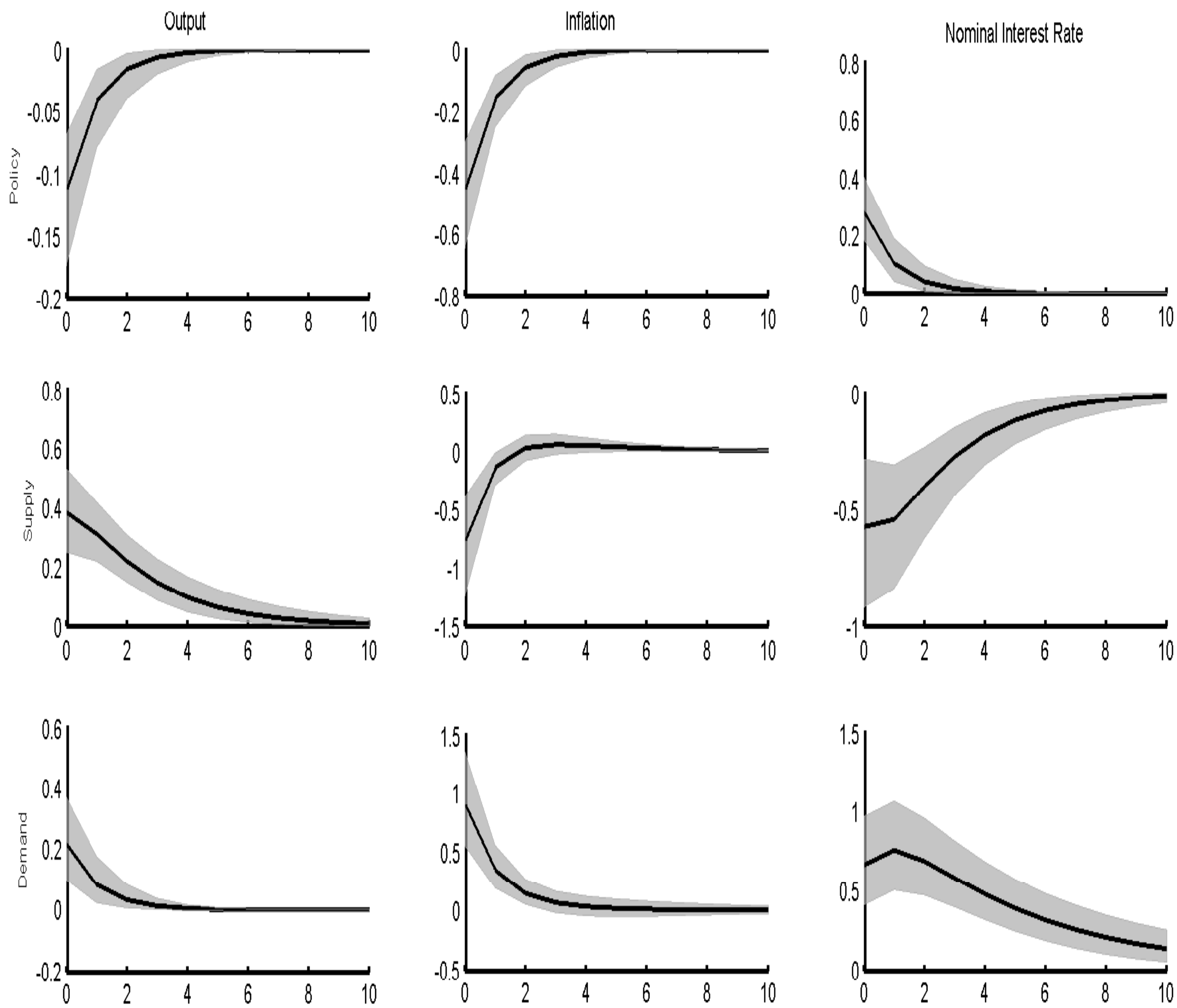

Figure A3: Impulse responses under determinacy from the model estimated over the period 2002:I - 2007:III using Core PCE inflation.

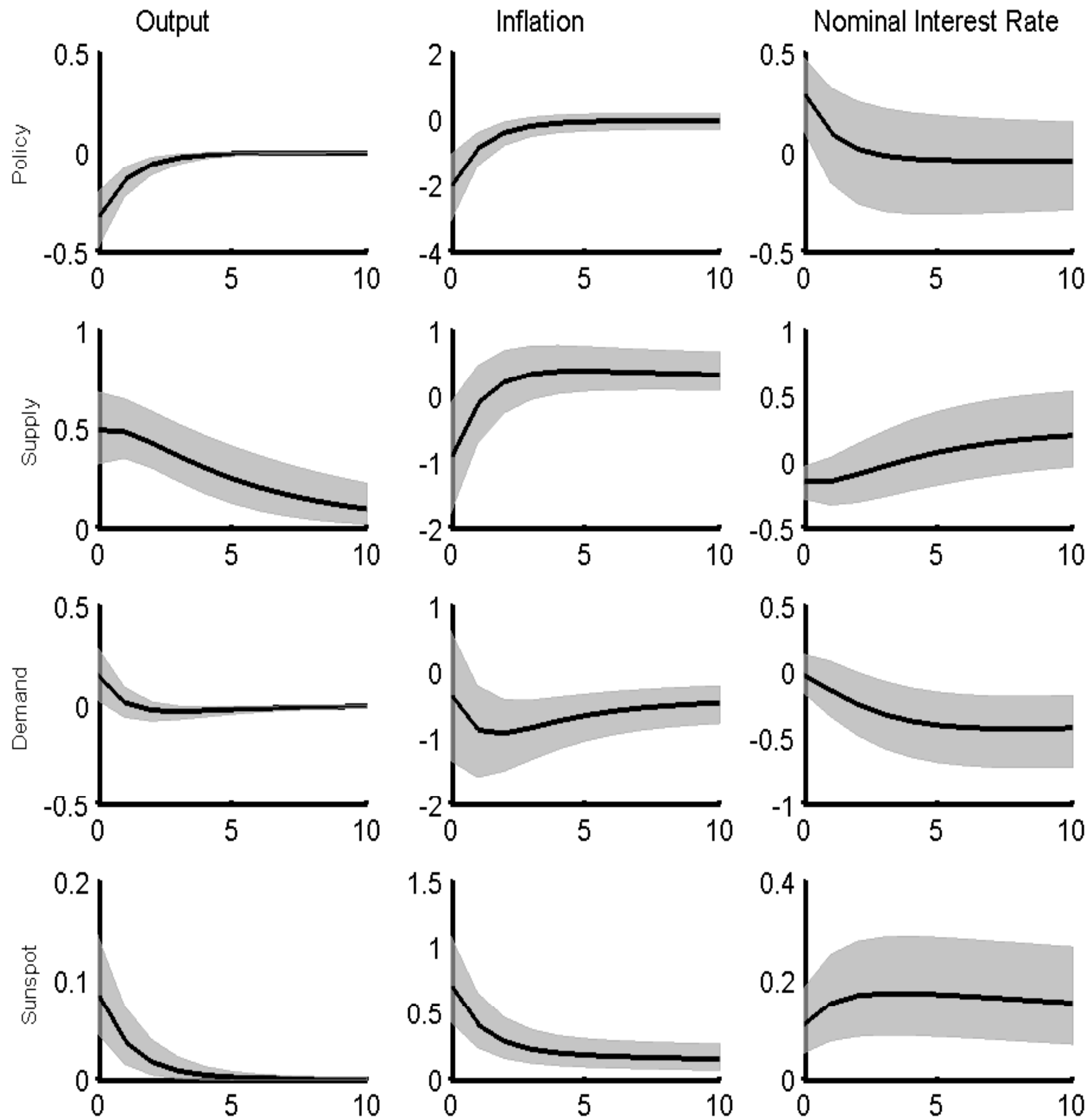

Figure A4: Impulse responses under indeterminacy from the model estimated over the period 2002:I - 2007:III using CPI inflation.

2.7 Habit formation

It is well known that the determinate New Keynesian model features a poor internal propagation mechanism while the model potentially exhibits richer dynamics under indeterminacy. Accordingly, the posterior mass might be biased toward the indeterminacy region.[5] Hence, following Lubik and Schorfheide (2004), we extend the model by adding consumption habits. Log-data densities for the habit specification conditional on determinacy are reported in Table A2: the habit model fits better than the no-habit specification restricted to determinacy. The last column of Table A2 compares the respective posterior probabilities of the baseline model under indeterminacy and the habit model under determinacy. For example, when measuring inflation with CPI, the data favors the benchmark model under indeterminacy over the habit specification restricted to determinacy. Again, the results carry over from the benchmark exercise i.e. Table 2 in the paper.

Table A2: Benchmark Model versus Determinate Model with Habit

		Log-data density		
Inflation measure	Specification	Det.	Indet.	Probability
CPI	Benchmark	-95.48	-93.28	0.87
	Habit	-95.18		0.13
PCE	Benchmark	-85.42	-85.75	0.26
	Habit	-84.70		0.74
Core PCE	Benchmark	-64.60	-71.58	0
	Habit	-62.73		1

[5] See the discussion between Beyer and Farmer (2007) and Lubik and Schorfheide (2007).

2.8 Estimation Results under PCE

According to the semi-annual monetary policy reports to Congress (Humphrey-Hawkins reports), the Federal Reserve has also been looking at headline PCE inflation from 2000 to 2004. Hence, we employ PCE to measure inflation while estimating our model and the evidence is mixed at best, the probability of determinacy is 0.58. Phrased alternatively, we can neither exclude nor rule in indeterminacy.Table A3 reports posterior estimates of the model parameters under both determinacy and indeterminacy.

Table A3 - Parameter Estimation Results

	PCE (Indeterminacy)		PCE (Determinacy)	
	Mean	90-percent interval	Mean	90-percent interval
ψ_π	0.82	[0.58,0.97]	2.13	[1.30,3.09]
ψ_y	0.21	[0.05, 0.45]	0.27	[0.06,0.59]
ρ_R	0.83	[0.74, 0.90]	0.85	[0.77,0.91]
π^*	3.36	[1.30, 6.21]	2.24	[1.63,2.84]
r^*	1.26	[0.55, 2.10]	1.17	[0.56,1.90]
κ	0.73	[0.40, 1.16]	0.75	[0.39,1.22]
τ^{-1}	1.69	[1.02 2.50]	1.83	[1.09,2.72]
ρ_g	0.60	[0.45, 0.73]	0.79	[0.70,0.86]
ρ_z	0.81	[0.70, 0.90]	0.62	[0.46,0.78]
ρ_{gz}	-0.27	[-0.72, 0.25]	0.64	[0.23,0.92]
$M_{R\zeta}$	-0.16	[-1.51, 1.40]		
$M_{g\zeta}$	-1.91	[-2.80, -1.01]		
$M_{z\zeta}$	0.43	[0.09, 0.81]		
σ_R	0.15	[0.12, 0.20]	0.16	[0.12,0.21]
σ_g	0.26	[0.17, 0.38]	0.19	[0.14,0.27]
σ_z	0.69	[0.50, 0.94]	0.70	[0.51,0.96]
σ_ζ	0.19	[0.12, 0.28]		

Notes: The table reports posterior means and 90-percent probability intervals of the model parameters. The posterior summary statistics are calculated from the output of the Metropolis Hastings algorithm.

3 Trend Inflation

The estimation is based on a version of Ascari and Sbordone's (2014) Generalized New Keynesian model (GNK). Unlike Ascari and Sbordone, we assume deterministic growth and we replace their labor supply disturbance by a discount factor shock, d_t, as our stand-in for demand shocks. Also, our Taylor rule involves responses to the output gap instead of log-deviations from the steady state. This then makes our setup similar to Hirose, Kurozumi and Van Zandweghe (2015).[6] The log-linearized (detrended) model consists of the Euler equation

$$y_t = E_t y_{t+1} - (R_t - E_t\pi_{t+1}) + d_t - d_{t+1}$$

where we have set the intertemporal rate of substitution equal to one to make the model compatible with balanced growth as well as the Taylor rule

$$R_t = \rho_R R_{t-1} + (1-\rho_R)(\psi_\pi \pi_t + \psi_y[y_t - z_t]) + \epsilon_{R,t} \qquad 0 \leq \rho_R < 1$$

to capture the central bank's behavior. The supply side is no longer summarized by a single Phillips curve expression but rather it consists of the following three equations for inflation, an auxiliary variable, ψ_t, and price dispersion, s_t:

$$\begin{aligned}
\pi_t &= \varkappa E_t \pi_{t+1} + \vartheta\left[\varphi s_t + (1+\varphi)y_t - (1+\varphi)z_t\right] - \varpi E_t\psi_{t+1} + \varpi d_t \\
\psi_t &= (1-\xi\beta\pi^\varepsilon)\left[\varphi s_t + (1+\varphi)(y_t - z_t) + d_t\right] + \xi\beta\pi^\varepsilon\left[E_t\psi_{t+1} + \varepsilon E_t \pi_{t+1}\right] \\
s_t &= \varepsilon\xi\pi^\varepsilon\left(1 - \frac{1-\xi\pi^\varepsilon}{\pi - \xi\beta\pi^\varepsilon}\right)\pi_t + \xi\pi^\varepsilon s_{t-1}
\end{aligned}$$

where $\vartheta \equiv (1-\xi\pi^{\varepsilon-1})(1-\xi\beta\pi^\varepsilon)/\xi\pi^{\varepsilon-1}$, $\varkappa \equiv \beta\left[1+\varepsilon(\pi-1)(1-\xi\pi^{\varepsilon-1})\right]$, and $\varpi \equiv \beta(1-\pi)(1-\xi\pi^{\varepsilon-1})$. The term ξ denotes the Calvo-parameter and β stands in for the steady state discount factor. We set the Frisch elasticity of labor supply, φ, equal to one and calibrate the elasticity of substitution $\varepsilon = 11$ such that the steady state mark-up equals ten percent.

As mentioned above, the GNK model exhibits richer dynamics and the usual Taylor principle ($\psi_\pi > 1$) is no longer a sufficient condition for local determinacy of equilibrium. Due to the higher-order dynamics of the GNK model and our assumption of a unit Frisch elasticity of labor supply, it is not possible to analytically derive the indeterminacy conditions. To continue solving the model via Lubik and Schorfheide's (2004) continuity solution (where $\mathbf{M}^*(\theta)$ is selected such that the responses of the endogenous variables to the fundamental shocks are continuous at the boundary between the determinacy and indeterminacy region) one needs to resort to numerical methods. In particular, we follow Hirose's (2014) numerical solution strategy for finding the boundary between determinacy and indeterminacy by perturbing the parameter ψ_π in the monetary policy rule.[7]

As before we use the growth rate of GDP, the Federal Funds rate and the three measures of inflation sequentially. Table A4 provides the marginal data densities

[6] They, however, assume firm-specific labor as well as stochastic growth.

[7] See also Justiniano and Primiceri (2008).

along with the posterior model probabilities while Table A5 reports the priors and the posterior estimates. The emerging results parallel our earlier findings. When basing the estimation on CPI, the U.S. economy was very likely in an indeterminacy region, however, the opposite holds, again, under core PCE. Notably, as mentioned above, the posterior estimate of trend inflation under CPI is higher than under core PCE while the Calvo parameter is smaller implying more flexible prices under CPI.

Lastly, we investigate the sensitivity of our results to Coibion and Gorodnichenko's (2011) Taylor rule that allows for interest rate smoothing of order two, as well as a response to inflation, output growth, and the output gap. Coibion and Gorodnichenko document a shift in the Federal Reserve's response from output gap to output growth for the Great Moderation period and also show that the two lags of interest rate are required to remove the serial correlation in the monetary policy shocks. Thus, we re-estimate the GNK model by replacing the standard policy rule with the following formulation:

$$R_t = \rho_{R_1} R_{t-1} + \rho_{R_2} R_{t-2} + (1 - \rho_{R_1} - \rho_{R_2})(\psi_\pi \pi_t + \psi_y [y_t - z_t] + \psi_{gy} \Delta y_t) + \epsilon_{R,t}.$$

Even though the posterior probabilities of indeterminacy are now lower across the board, Table A4 shows that the only case in which we can confidently rule out the possibility of indeterminacy is when we use core PCE. Apart from the parameter estimates of the responsiveness to output growth, ψ_{gy}, and the interest rate lags, ρ_{R1} and ρ_{R2}, all other parameter estimates remain essentially unchanged.

Table A4: Determinacy versus Indeterminacy (Trend Inflation)

		Log-data density		Probability	
Inflation measure		Det.	Indet.	Det.	Indet.
CPI	Standard Taylor rule	-91.38	-87.13	0.02	0.98
	Alternative Taylor rule	-85.16	-83.25	0.13	0.87
PCE	Standard Taylor rule	-81.54	-82.01	0.62	0.38
	Alternative Taylor rule	-75.79	-77.41	0.83	0.17
Core PCE	Standard Taylor rule	-61.13	-64.53	0.97	0.03
	Alternative Taylor rule	-56.68	-60.75	0.98	0.02

Notes: The prior predictive probability is 0.539 for the standard rule and 0.503 for the alternative rule.

Table A5: Priors and posteriors for DSGE parameters.

				Posterior Mean [5th pct, 95th pct]			
				Standard TR		Alternative TR	
Name	Range	Density	Prior Mean (Std. Dev.)	CPI Ind.	CorePCE Det.	CPI Ind.	CorePCE Det.
ψ_π	$\mathbb{R}^+$	Gamma	1.40 (0.50)	0.93 [0.82,1.00]	2.65 [1.66,3.77]	0.94 [0.84,1.00]	2.53 [1.64,3.55]
ψ_y	$\mathbb{R}^+$	Gamma	0.25 (0.15)	0.25 [0.07,0.54]	0.35 [0.09,0.72]	0.25 [0.07,0.52]	0.30 [0.07,0.65]
ψ_{gy}	$\mathbb{R}^+$	Gamma	0.25 (0.15)			0.33 [0.11,0.59]	0.35 [0.10,0.70]
ρ_R	[0,1)	Beta	0.50 (0.20)	0.73 [0.63,0.82]	0.76 [0.64,0.85]		
ρ_{R1}	$\mathbb{R}$	Normal	1.00 (0.20)			1.09 [0.85,1.32]	1.13 [0.88,1.36]
ρ_{R2}	$\mathbb{R}$	Normal	0.00 (0.20)			−0.35 [−0.55,−0.14]	−0.35 [−0.57,−0.12]
π	$\mathbb{R}^+$	Gamma	2.50 (1.00)	2.21 [1.03,3.73]	1.89 [1.52,2.26]	2.12 [1.02,3.49]	1.93 [1.54,2.35]
r	$\mathbb{R}^+$	Gamma	2.00 (1.00)	1.01 [0.45,1.71]	1.27 [0.69,1.91]	0.93 [0.40,1.59]	1.22 [0.63,1.87]
γ	$\mathbb{R}$	Normal	0.50 (0.10)	0.49 [0.44,0.53]	0.55 [0.48,0.62]	0.49 [0.45,0.54]	0.55 [0.48,0.62]
ξ	[0,1)	Beta	0.70 (0.10)	0.26 [0.19,0.34]	0.64 [0.51,0.74]	0.31 [0.23,0.40]	0.65 [0.53,0.74]
ρ_d	[0,1)	Beta	0.70 (0.10)	0.71 [0.54,0.85]	0.82 [0.72,0.90]	0.73 [0.56,0.87]	0.82 [0.73,0.90]
ρ_z	[0,1)	Beta	0.70 (0.10)	0.70 [0.58,0.82]	0.76 [0.61,0.89]	0.69 [0.55,0.81]	0.75 [0.59,0.88]
$M_{R\zeta}$	$\mathbb{R}$	Normal	0.00 (1.00)	−0.74 [−1.82,0.42]		−0.84 [−1.97,0.39]	
$M_{d\zeta}$	$\mathbb{R}$	Normal	0.00 (1.00)	−1.47 [−2.57,0.13]		−1.15 [−2.62,0.61]	
$M_{z\zeta}$	$\mathbb{R}$	Normal	0.00 (1.00)	1.98 [1.36,2.65]		1.68 [1.01,2.39]	
σ_R	$\mathbb{R}^+$	IG	0.50 (∞)	0.19 [0.13,0.26]	0.15 [0.11,0.21]	0.18 [0.12,0.26]	0.14 [0.10,0.20]
σ_d	$\mathbb{R}^+$	IG	0.50 (∞)	0.36 [0.15,0.82]	0.74 [0.51,1.09]	0.25 [0.13,0.47]	0.70 [0.46,1.04]
σ_z	$\mathbb{R}^+$	IG	0.50 (∞)	0.46 [0.35,0.59]	0.56 [0.37,0.85]	0.48 [0.36,0.63]	0.58 [0.48,0.62]
σ_ζ	$\mathbb{R}^+$	IG	0.50 (∞)	0.27 [0.15,0.47]		0.31 [0.16,0.56]	

Notes: The inverse gamma priors are of the form $p(\sigma|\upsilon,\varsigma) \propto \sigma^{-\upsilon-1} e^{-\frac{\upsilon\varsigma^2}{2\sigma^2}}$, where $\nu = 2$ and $\varsigma = 0.282$.

Table A6 below displays the estimation results under PCE for both the standard Taylor rule and the alternative rule following Coibion and Gorodnichenko (2011). Most of our parameter estimates are in line with the results from the previous table.

Table A6 - Prior and posteriors for structural parameters.

	Posterior Mean [5th pct, 95th pct]			
Name	Stand. TR Indeterminacy	Stand. TR Determinacy	Alt. TR Indeterminacy	Alt. TR Determinacy
ψ_π	0.94 [0.83,1.00]	2.17 [1.46,3.03]	0.95 [0.83,1.04]	1.97 [1.31,2.83]
ψ_y	0.26 [0.07,0.55]	0.28 [0.07,0.59]	0.28 [0.07,0.60]	0.25 [0.06,0.55]
ψ_{gy}			0.36 [0.11,0.67]	0.42 [0.12,0.81]
ρ_R	0.73 [0.64,0.81]	0.79 [0.66,0.87]		
ρ_{R1}			1.11 [0.87,1.35]	1.20 [0.95,1.43]
ρ_{R2}			−0.37 [−0.58,−0.15]	−0.40 [−0.62,−0.17]
π^*	2.13 [1.01,3.49]	2.19 [1.58,2.81]	2.09 [1.02,3.43]	2.23 [1.57,2.91]
r^*	1.11 [0.51,1.79]	1.06 [0.51,1.70]	1.05 [0.49,1.69]	1.03 [0.49,1.66]
γ	0.49 [0.45,0.53]	0.55 [0.48,0.63]	0.51 [0.46,0.59]	0.55 [0.48,0.63]
θ	0.30 [0.22,0.39]	0.49 [0.36,0.61]	0.40 [0.26,0.62]	0.50 [0.38,0.62]
ρ_d	0.69 [0.53,0.84]	0.84 [0.76,0.92]	0.75 [0.58,0.89]	0.84 [0.75,0.91]
ρ_z	0.70 [0.59,0.81]	0.78 [0.64,0.89]	0.70 [0.56,0.83]	0.76 [0.61,0.88]
$M_{R\zeta}$	−0.29 [−1.55,1.06]		−0.43 [−1.90,1.02]	
$M_{d\zeta}$	−1.45 [−2.79,0.30]		−0.89 [−2.66,0.96]	
$M_{z\zeta}$	2.34 [1.62,3.14]		1.78 [−0.17,2.85]	
σ_R	0.17 [0.13,0.24]	0.18 [0.13,0.27]	0.17 [0.12,0.24]	0.17 [0.11,0.25]
σ_d	0.30 [0.14,0.59]	0.73 [0.48,1.08]	0.28 [0.14,0.54]	0.65 [0.40,0.99]
σ_z	0.46 [0.35,0.59]	0.58 [0.40,0.85]	0.55 [0.37,0.90]	0.61 [0.41,0.92]
σ_ζ	0.30 [0.16,0.53]		0.34 [0.16,0.65]	

4 Additional Results

4.1 Estimation Results under Core CPI

Table A7: Determinacy versus Indeterminacy (Core CPI)

Inflation measure	Specification	Log-data density		Probability	
		Det.	Indet.	Det.	Indet.
Core CPI	Benchmark	-62.06	-69.64	1	0
	Policy parameters only	-63.64	-66.52	0.95	0.05
	Alternative prior for ψ_π	-61.61	-70.07	1	0
	CBO Output Gap	-65.60	-71.61	1	0
	Output Growth	-59.73	-64.68	0.99	0.01
	AR(1) policy shocks	-51.30	-58.78	1	0
	Trend Inflation with standard TR	-58.78	-62.40	0.97	0.03
	Trend Inflation with alternative TR	-53.88	-57.64	0.98	0.02

5 A micro-founded distinction between core and headline inflation

The artificial economy is a variant of Blanchard and Gali (2010) and Blanchard and Riggi (2013) and so our description of the model below draws heavily from their exposition. It is a New Keynesian economy with a commodity product which they interpret as oil. This model offers a micro-founded setup that naturally features various inflation rates. The economy consists of monopolistically competitive wholesale firms who produce differentiated goods using labor and oil. These goods are bought by perfectly competitive firms (retailers) that weld them together into the final good that can be consumed. People rent out their labor services on competitive markets. Firms and people are price takers on the market for oil.

5.1 People

The representative agent's preferences depend on consumption, C_t, and hours worked, H_t, and they are represented by the expected utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t d_t u(C_t, N_t) \qquad 0 < \beta < 1$$

which the agent acts to maximize. Here, E_0 represents the expectations operator. The term d_t stands for a shock to the discount factor, β, which follows the stationary autoregressive process

$$\ln d_{t+1} = \rho_d \ln d_t + \epsilon_{d,t+1}$$

where $\epsilon_{d,t+1}$ is a zero-mean, serially uncorrelated innovation that is normally distributed with standard deviation σ_d. The period utility is additively separable in consumption and hours worked and it takes on the functional form

$$u(C_t, N_t) = \ln C_t - \phi \nu_t \frac{N_t^{1+\varphi}}{1+\varphi} \qquad \phi > 0,\ \varphi \geq 0.$$

Logarithmic utility is the only additive-separable form consistent with balanced growth. The term φ is the inverse of the Frisch labor supply elasticity and ϕ governs the disutility of working in steady state. ν_t denotes a shock to the disutility of labor and it follows

$$\ln \nu_{t+1} = \rho_\nu \ln \nu_t + \epsilon_{\nu,t+1}$$

where $\epsilon_{\nu,t}$ is $N(0, \sigma_\nu^2)$. The overall consumption basket, C_t, is a Cobb-Douglas bundle of output of domestically produced goods, $C_{q,t}$, and the imported oil, $C_{m,t}$. In particular, we assume that

$$C_t = \chi^{-\chi}(1-\chi)^{-(1-\chi)} C_{m,t}^{\chi} C_{q,t}^{1-\chi} \qquad 0 < \chi < 1.$$

The parameter χ equals the share of energy in total consumption.

Retail firms combine the domestically-produced intermediate varieties $C_{q,t}(i)$, where $i \in [0,1]$, using a Dixit-Stiglitz aggregator to produce the consumption bundle $C_{q,t}$:

$$C_{q,t} = \left(\int_0^1 C_{q,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

Here, the term ε measures the elasticity of demand for each intermediate good.

The agent sells labor services to the wholesale firms at the nominal wage W_t and has access to a market for one-period riskless bonds, B_t, at the interest rate R_t. Any generated profits, Π_t, flow back to the representative household. Thus, the period budget is constrained by

$$W_t N_t + R_t B_{t-1} + \Pi_t \geq P_{q,t} C_{q,t} + P_{m,t} C_{m,t} + B_t$$

where $P_{q,t}$ denotes the domestic output-price index

$$P_{q,t} = \left(\int_0^1 P_{q,t}(i)^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}$$

with $P_{q,t}(i)$ the price of intermediate good i.

The Euler equation is given by

$$\frac{d_t}{P_{c,t} C_t} = \beta E_t \frac{R_t d_{t+1}}{P_{c,t+1} C_{t+1}},$$

where $P_{c,t}$ is the price of the overall consumption basket.

The intratemporal optimality condition is described by

$$\frac{W_t}{P_{c,t} C_t} = \phi \nu_t N_t^{\varphi}.$$

In the optimal allocation, we have

$$P_{q,t}C_{q,t} = (1-\chi)P_{c,t}C_t$$

and

$$P_{m,t}C_{m,t} = \chi P_{c,t}C_t$$

where $P_{c,t} \equiv P_{m,t}^{\chi}P_{q,t}^{1-\chi}$ and $\mathrm{P}_{m,t}$ is the nominal price of oil. Also note that $P_{c,t} \equiv P_{q,t}S_t^{\chi}$, where $S_t \equiv \frac{P_{m,t}}{P_{q,t}}$ is the real price of oil.

5.2 Monopolistically competitive wholesale firms

Intermediate goods are produced using labor, $N_t(i)$, and oil, $M_t(i)$, both supplied on perfectly competitive factor markets. Each firm i produces output according to the production function

$$Q_t(i) = \left[A_t N_t(i)\right]^{1-\alpha} M_t(i)^{\alpha} \qquad 0 < \alpha < 1.$$

Here, α is the share of oil in production and A_t stands for labor augmenting technological progress whose growth rate, $z_t \equiv \frac{A_t}{A_{t-1}}$, follows an exogenous process

$$\ln z_t = \ln z + \epsilon_{z,t}$$

with $z > 1$ and $\epsilon_{z,t}$ is $N(0, \sigma_z^2)$. Each intermediate good-producing firm's nominal marginal costs are given by

$$\psi_t(i) = \frac{W_t}{(1-\alpha)Q_t(i)/N_t(i)} = \frac{P_{m,t}}{\alpha Q_t(i)/M_t(i)}$$

and the markup, $\mu_t(i)$, equals

$$\mu_t(i) = \frac{P_{q,t}(i)}{\psi_t(i)}.$$

The intermediate goods producers face a constant probability, $0 < 1-\xi < 1$, of being able to adjust prices to a new optimal one, P_t^*, in order to maximize expected discounted profits

$$E_t \sum_{\tau=0}^{\infty} \xi^{\tau} \Lambda_{t,t+\tau} Q_{t+\tau|t} \left[P_t^* - \mathcal{M}^p \psi_{t+\tau|t}(i)\right] = 0,$$

where Q_t denotes gross output, $\Lambda_{t,t+\tau}$ denotes the household's stochastic discount factor and $\mathcal{M}^p \equiv \frac{\varepsilon}{\varepsilon-1}$ is the desired gross markup.

The domestic price level evolves as

$$P_{q,t} = \left[\xi P_{q,t-1}^{1-\varepsilon} + (1-\xi)P_t^{*1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}.$$

Consumption and gross output are related as

$$P_{c,t}C_t = \left(1 - \frac{\alpha}{\mathcal{M}^p}\right) P_{q,t}Q_t$$

and the production function becomes

$$Q_t = (A_t N_t)^{1-\alpha} M_t^{\alpha}$$

Moreover,

$$M_t = \frac{\alpha}{\mathcal{M}^p} \frac{Q_t}{S_t}.$$

Value added (or GDP), Y_t, is given by

$$P_{y,t} Y_t = \left(1 - \frac{\alpha}{\mathcal{M}^p}\right) P_{q,t} Q_t$$

where $P_{y,t}$ is the GDP deflator defined via $P_{q,t} \equiv P_{y,t}^{1-\alpha} P_{m,t}^{\alpha}$.

Finally, the growth of the real price of oil follows an AR(1) process

$$\ln S_{t+1} = (1 - \rho_s) \ln S + \rho_s \ln S_t + \epsilon_{s,t+1}.$$

where the innovation $\epsilon_{s,t}$ is i.i.d. $N\left(0, \sigma_s^2\right)$.

5.3 Log-linearized equations

Here we present the detailed log-linearized equations of the model. Lowecase letters are proportional deviations from steady state.

Production is given by a Cobb-Douglas production function in labor and oil[8]:

$$q_t = \alpha m_t + (1 - \alpha) n_t, \tag{9}$$

Consumption is given by a Cobb-Douglas consumption function in output and oil:

$$c_t = (1 - \chi) c_{q,t} + \chi c_{m,t}, \tag{10}$$

The relationship between consumption price inflation and the domestic output price inflation is given by

$$\pi_{c,t} = \pi_{q,t} + \chi \Delta s_t, \tag{11}$$

where $\pi_{c,t} \equiv p_{c,t} - p_{c,t-1}$ is headline inflation and $\pi_{q,t} \equiv p_{q,t} - p_{q,t-1}$ is core inflation.

Note that if we set α and χ to zero, the Blanchard and Gali (2010) model boils down to a simple New Keynesian model similar to the one used in the previous sections of the paper.

The behaviour of households is characterized by two equations. The first one is an inter-temporal Euler equation:

$$c_t = d_t - E_t d_{t+1} + E_t c_{t+1} + E_t z_{t+1} - \{R_t - E_t \pi_{c,t+1}\}, \tag{12}$$

[8] We assume that firms operate under constant returns to labor and oil. So, $1 - \alpha$ is then the share of labor in output.

where z_t is a shock to the growth rate of technology. Note that to be compatible with balanced growth we assume that the intertemporal elasticity of substitution is 1.

The second condition characterizes labor supply and is given by[9]:

$$w_t - p_{c,t} = \gamma(w_{t-1} - p_{c,t-1}) + (1-\gamma)[\varphi n_t + c_t] + \nu_t, \tag{13}$$

where $\gamma \in [0,1]$ captures the extent of real wage rigidity. When $\gamma = 0$, the supply wage is equal to the marginal rate of substitution. The higher the value of γ, the higher the degree of real wage rigidity.

Domestic goods are imperfect substitutes in consumption, and firms are thus monopolistic competitors. Given the production function, cost minimization implies that the firms' demand for oil is given by:

$$m_t = -\mu_t - s_t + q_t. \tag{14}$$

Using this expression to eliminate m_t in the production function (9) gives a reduced-form production function:

$$q_t = n_t - \frac{\alpha}{1-\alpha} s_t - \frac{\alpha}{1-\alpha}\mu_t. \tag{15}$$

Combining the cost minimization conditions for oil and for labor with the aggregate production function yields the following factor price frontier:

$$(1-\alpha)(w_t - p_{c,t}) + (\alpha + (1-\alpha)\chi)s_t + \mu_t. \tag{16}$$

Firms are assumed to set prices à la Calvo (1983). The resulting inflation dynamics are described by the following expectational Phillips curve:

$$\pi_{q,t} = \beta E_t \pi_{q,t+1} - \kappa\mu_t, \tag{17}$$

where $\kappa \equiv \frac{(1-\xi)(1-\beta\xi)}{\xi}$ is the slope of the Phillips curve.

Balanced trade gives us a relation between consumption and output:

$$c_t = q_t - \chi s_t + \eta\mu_t, \tag{18}$$

where $\eta \equiv \frac{\alpha}{\mathcal{M}^P - \alpha}$, with $\mathcal{M}^P$ denoting the steady state gross markup.

Combining the reduced form production function (15) with the above equation gives a relationship between consumption and employment:

$$c_t = n_t - (\frac{\alpha}{1-\alpha} + \chi) + (\eta - \frac{\alpha}{1-\alpha})\mu_t. \tag{19}$$

The characterization of the equilibrium does not require us to introduce valued (or GDP). But it is needed to undertake the estimation of the model where we use

[9] As in Smets and Wouters (2007) and Justiniano, Primiceri and Tambalotti (2010), the labor supply shock is re-normalized such that it enters the labor supply equation with a coefficient of one as seen here. In this way, it is easier to choose a reasonable prior for its standard deviation denoted by σ_ν.

GDP growth data. The definition of value added, combined with the demand for oil, yields the following relation between GDP and gross output:

$$y_t = q_t + \frac{\alpha}{1-\alpha} s_t + \eta \mu_t. \tag{20}$$

The shocks s_t, ν_t and d_t are assumed to follow independent stationary AR(1) processes:

$$s_t = \rho_s s_{t-1} + \varepsilon_{st}, \qquad \nu_t = \rho_\nu \nu_{t-1} + \varepsilon_{\nu t}, \qquad d_t = \rho_d d_{t-1} + \varepsilon.$$

Lastly, to close the model, the central bank's policy is described by a Taylor rule

$$R_t = \rho_R R_{t-1} + (1-\rho_R)[\psi_\pi \{\omega \pi_{c,t} + (1-\omega)\pi_{q,t}\} + \psi_y y_t + \psi_{gy} gy_t] + \epsilon_{R,t}, \qquad 0 \leq \rho_R < 1, \tag{21}$$

where $\epsilon_{R,t}$ is $N(0, \sigma_R^2)$ and $gy_t \equiv y_t - y_{t-1} + z_t$ stands for the growth rate of detrended output.[10] The central bank responds to a convex combination of headline and core inflation with the parameter ω governing the relative weights. For instance, setting ω to zero implies that the central bank responds to core inflation only.

5.4 Determinacy region

Figure A5 below shows the determinacy region for combinations of ψ_π with the other policy parameters as well as with the degree of real wage rigidity γ. As can be seen from the figure, the *Taylor Principle* continues to hold in this micro-founded model with a distinction between core and headline inflation. In line with the findings of Carlstrom, Fuerst and Ghironi (2006), equilibrium determinacy criterion does not imply a preference to any particular measure of inflation. As long as the central bank responds with a coefficient greater than unity to either headline inflation, core inflation or a combination of the two, then such policy will ensure equilibrium determinacy.

[10] Unlike Blanchard and Gali (2010) and Blanchard and Riggi (2013), we assume that the central bank is perfectly credible. While these authors allow for a role of central bank credibility to explain the reduced impact of oil shocks in the 2000s, they restrict their attention to determinacy only. Our purpose in this present paper is to specifically test for indeterminacy due to passive monetary policy during our period of interest while allowing for a distinction between headline and core inflation. Hence, we assume that the central bank is perfectly credible.

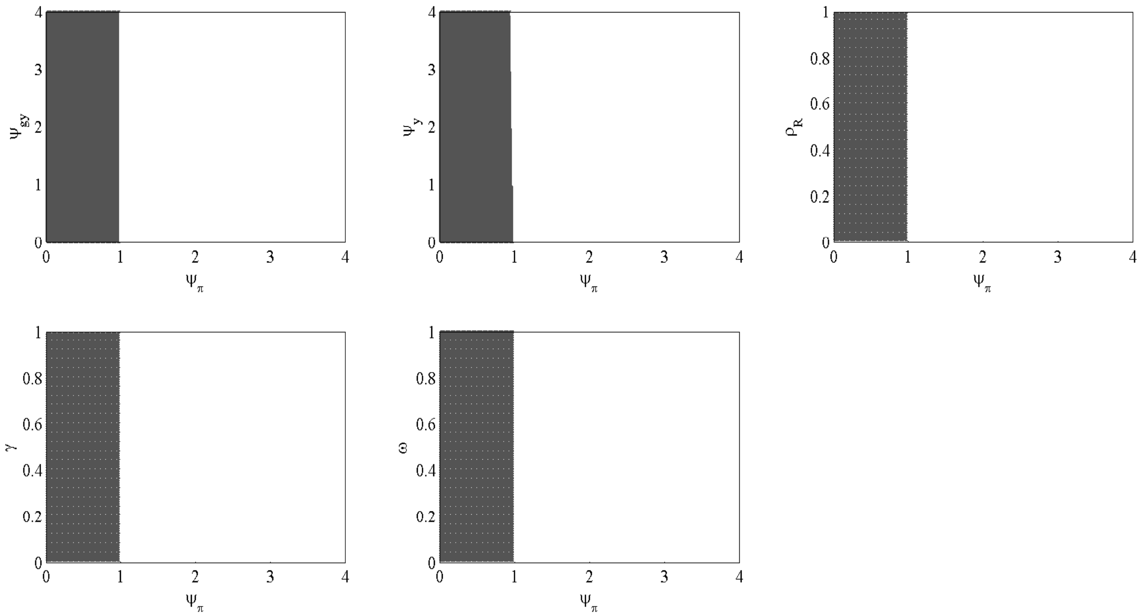

Figure A5: Determinacy region for the Blanchard and Gali (2009) model

Table A8: Determinacy versus Indeterminacy

	Log-data density		Probability	
Inflation measure	Determinacy	Indeterminacy	Determinacy	Indeterminacy
Real-time data	-173.66	-179.39	1	0
CPI&CoreCPI	-177.12	-183.41	1	0
CPI&CorePCE	-174.66	-181.61	1	0

Notes: According to the prior distributions, the probability of determinacy is 0.51.

Table A9: Posterior for DSGE parameters

	Posterior Mean [5th pct, 95th pct]		
Name	Real-time data	CPI&CoreCPI	CPI&CorePCE
ψ_π	2.37 [1.31,3.54]	2.41 [1.47,3.62]	2.76 [1.69,4.03]
ψ_y	0.05 [0.01,0.10]	0.09 [0.02,0.20]	0.07 [0.01,0.16]
ψ_{gy}	0.76 [0.20,1.44]	0.84 [0.31,1.44]	0.69 [0.23,1.24]
ρ_R	0.74 [0.63,0.84]	0.78 [0.68,0.86]	0.79 [0.70,0.87]
π^*	1.86 [1.17,2.61]	2.05 [1.35,2.74]	1.99 [1.40,2.58]
r^*	1.36 [0.71,2.12]	1.13 [0.54,1.79]	1.20 [0.59,1.87]
γ^*	0.50 [0.37,0.63]	0.51 [0.38,0.65]	0.53 [0.39,0.66]
κ	0.44 [0.26,0.65]	0.36 [0.23,0.52]	0.40 [0.25,0.57]
γ	0.48 [0.18,0.77]	0.53 [0.35,0.69]	0.43 [0.24,0.60]
ρ_s	0.90 [0.83,0.95]	0.90 [0.83,0.95]	0.91 [0.85,0.96]
ρ_d	0.81 [0.70,0.90]	0.77 [0.65,0.88]	0.77 [0.64,0.87]
ρ_ν	0.78 [0.52,0.94]	0.62 [0.42,0.84]	0.71 [0.50,0.90]
ω	0.32 [0.11,0.58]	0.22 [0.07,0.42]	0.21 [0.06,0.41]
σ_s	20.91 [16.13,27.16]	28.47 [22.23,36.45]	29.21 [22.77,37.40]
σ_z	0.79 [0.60,1.01]	0.75 [0.57,0.97]	0.69 [0.53,0.89]
σ_R	0.19 [0.13,0.28]	0.16 [0.11,0.24]	0.16 [0.11,0.22]
σ_d	0.70 [0.37,1.13]	0.74 [0.43,1.13]	0.72 [0.45,1.07]
σ_ν	0.82 [0.49,1.23]	0.68 [0.49,0.94]	0.80 [0.59,1.09]