Interdisciplinary Pedagogy of the Meter Visualising Mathematical Music Theory Through the Ski-hill Graph

Andrea M. Calilhanna
Western Sydney University, Australia

Abstract
This paper aims to convey how and why teaching the fundamentals of the meter through interdisciplinary Ski-hill graph pedagogy of the meter brings benefits to school-age students. Arguably, the lack of a comprehensive theory of the meter has contributed to problems in fields as diverse as music education, mathematics, acoustics (psychoacoustics) and physics. However, the Ski-hill graph and modern theory of the meter (Cohn, 2020), provides solutions for inclusive meter theory in diverse music education. Presented in this paper is a music educator's response to modern meter theory's emphasis on the subjective psychoacoustic - mind and body processes of the meter (acoustics as music) and the importance of its' accurate representation. The paper explores the flexible three-step approach (Calilhanna, 2018; 2020) of Ski-hill graph pedagogy to enable school-age students to accurately visualise, conceptualise, and apply to performance, the details of their quantification of all the pulses and all the meters and their relations from listening to music. The Ski-hill graph augments theories of the meter as time signatures and groups of beats in a measure through mathematical music theory to visualise each pair of adjacent pulses in a relation of ratio 2:1 (duple meter) and or 3:1 (triple meter). Represented in the mathematics of cyclic hierarchies, the detailed visualizations of the meters through the Ski-hill graph provide a visceral and dynamic map of the metric space - listener's temporal information of patterns often overlooked, over-simplified, or forgotten in analyses. The paper situates Cohn's Ski-hill graph and theorem as a bridge between the arts and sciences.

Keywords: Ski-hill graph, meter, mathematics, psychoacoustic, inclusion.

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Pedagogía interdisciplinaria del metro visualizando la teoría de la música matemática a través del gráfico Ski-hill

Resumen
Este documento tiene como objetivo transmitir cómo y por qué enseñar los fundamentos del medidor a través de la pedagogía interdisciplinaria del medidor de gráficos de ski-hill brinda beneficios a los estudiantes en edad escolar. Podría decirse que la falta de una teoría integral del metro ha contribuido a problemas en campos tan diversos como la educación musical, las matemáticas, la acústica (psicoacústica) y la física. Sin embargo, el gráfico ski-hill y la teoría moderna del metro (Cohn, 2020) proporcionan soluciones para la teoría inclusiva del metro en la educación musical diversa. En este artículo se presenta la respuesta de un educador musical al énfasis de la teoría moderna del metro en los procesos psicoacústicos subjetivos: mente y cuerpo del medidor (acústica como música) y la importancia de su representación precisa. El documento explora el enfoque flexible de tres pasos (Calilhanna, 2018; 2020) de la pedagogía gráfica ski-hill para permitir a los estudiantes en edad escolar representar con precisión los detalles de la cuantificación de todos los pulsos y todos los metros y sus relaciones al escuchar música. El gráfico Ski-hill aumenta las teorías convencionales basadas en la notación del medidor como firmas de tiempo y grupos de latidos en una medida a través de la teoría musical matemática para visualizar cada par de pulsos adyacentes en una relación de relación 2: 1 (medidor doble) y o 3: 1 (metro triple). Representadas en las matemáticas de las jerarquías cíclicas, las visualizaciones detalladas de los metros a través del gráfico Ski-hill proporcionan un mapa visceral y dinámico del espacio métrico: la información temporal del oyente de patrones a menudo pasados por alto, demasiado simplificados u olvidados en los análisis. El artículo sitúa el grafo y teorema de Ski-hill de Cohn como un puente entre las artes y las ciencias.

Palabras clave: Gráfico Ski-hill, metro, matemáticas, psicoacústica, inclusión.

Introduction

This paper provides a teacher-researcher's perspective of applying modern meter theory with school age students to explore the benefits of Ski-hill graph pedagogy (Calilhanna 2018, 2020) inspired by the music theories of Cohn (2001, 2016, 2017, 2018, 2020). For decades cognitive neuroscience has researched the meter and rhythm and examined how and why humans respond to music indicating the meter is experienced. (Nozaradan *et al.*, 2011) reports the neural activity of the meter is hierarchical with spiking in ratios of 2:1 and 3:1 to match the low meter frequencies invoked temporal processes of duple and triple meter. The results support a hypothesis that the meter is a central system for processing music, as is tonality, and requires pedagogy through accessing the same neural processes (cognitive fit) to explore and represent the reasons behind the psychoacoustic (mind and body) experience of the meter.

Ski-hill graph pedagogy is best understood through the conceptual framework of Cohn's (2020) definition of the meter, "A meter is an inclusionally related set of distinct, notionally isochronous time-point sets." Critically, Cohn's (2020) is the first comprehensive theory of the meter to acknowledge that the meter is a psychoacoustic, subjective, and spectral experience of acoustics that at no point defers to notational understandings of the meter. For instance, conventional theories of the meter define the meter as time signatures, groups of beats in a bar, and simple and compound time. However, students

experience (quantify) and report many more pulses and meters (mathematics: sets and subsets) than those represented by time signatures (Cohn, 2015, 2016, 2018, 2017, 2020; Hilton, Calilhanna and Milne, 2018; Milne and Calilhanna, 2019; Calilhanna, 2018) and teachers and students struggle to find satisfactory meter theory and pedagogy (Calilhanna and Webb, 2018).

Accuracy in the visualisation (Sanocki, 2003) of the meter is critically important because both music and mathematics are experienced temporally and share the same property of quantification of patterns. Quantification is core to executive functions and most other mind and body systems humans depend on to survive and navigate living. For instance, the visualizations of the meter, rhythm, and pitch through Ski-hill pedagogy embed the core geometric skills common to music and mathematics (Rogers *et al.*, 2016). These skills of quantification impact spatial reasoning (Cooper *et al.*, 1996) for survival and daily living, and other cognitive benefits such as analogical encoding, and transfer (Gentner *et al.*, 2003).

In other words, human quantification is paramount to the information necessary to inform the decisions behind the analysis and accurate representation of the meter. However, underlying conventional notation-based assumptions about meter are a default position even though the same pedagogy and research points to the fact that the meter is experience not notation. For example, musical motif (patterns) may be considered a generation of gesture and motion, yet preceding motif is thought through concepts generated processes such as memory, emotions, and critically by the often overlooked processes of quantification – mind first then body responds.

The Ski-hill graph to teach the meter

Since 2015, I have taught my piano, saxophone, and music theory students most primary and high school-age, also teachers, and undergraduates the meter, rhythm, and pitch through Ski-hill graph pedagogy (Calilhanna, 2018, 2020) a flexible three-step approach to teaching the meter, rhythm and pitch inspired by Cohn's (2001, 2016, 2017, 2018, 2020) psychoacoustic approach to mathematical music theory and definition of the meter. I attribute the majority of benefits for my students to the visual-spatial layout of the Ski-hill graph's mathematical representations which match my students' experiences of the pulses and the meters.

Cohn introduced the Ski-hill graph around the 1990's originally as a means of representing hemiolas to provide a more intuitive visual model than purely numeric models (1992, 2001). As noted by Cohn (2016), interest in visualising music through graphs has grown since the 1990's, in inter- and transdisciplinary fields such as music education, mathematics, physics, engineering, and cognitive neurosciences.

By representing the meter through the Ski-hill graph, the longest pulses are mapped towards the apex and each duple meter division is twice as fast descending to the left, and or three times as fast triple meter, when mapped descending to the right. See Figure 2a) a Ski-hill graph representing a set of pulses in ratio 2:1 or a minimal duple meter <2> (the least amount of meter possible); 2b) minimal triple meter pulses in ratio 3:1 <3>; 2c) a deep pure duple meter: adjacent pulses in sets and subsets of minimal meters each set in ratio 2:1 summarised as <2>; 2d) a Ski-hill graph where the listener has mapped a summary of the meter duple, duple, triple or mixed meter <2,2,3> from listening to a piece of music notated in 12/8.

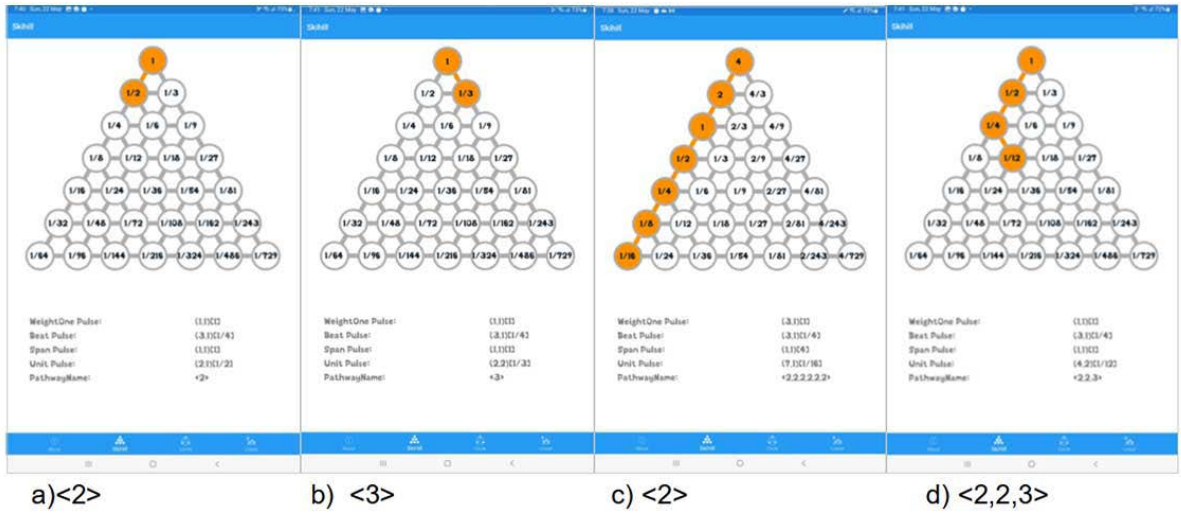

Figure 1. Ski-hill graph Stereo Metronome (Fan and Calilhanna, 2021 beta version)

The Ski-hill provides a solution when the denominator such as '8' doesn't match the division 1/8 (mathematics) experienced by listeners in instances where the division is a third 1/3, sixth 1/6, or a twelfth 1/12 as in the case of much Jazz music and several genres of traditional music of the Igbo people, Nigeria (see Figure 1d). For instance, students choose the fraction they experience as a pulse (acoustical vibration) in relation to the adjacent pulses as the meter. Figure 2 is an example of the music Basie's Blues transcribed from a notation in 4/4 with a time signature of 12/12 to match the mathematics of the Ski-hill graph to the listening experience:

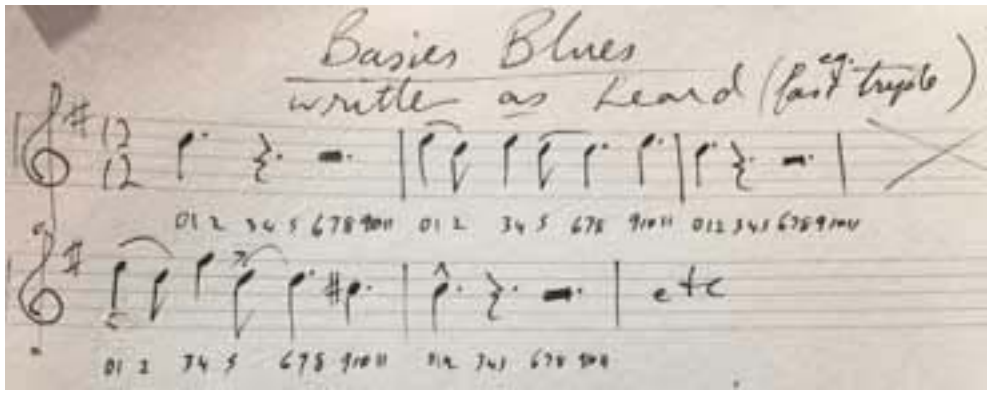

Figure 2. Basie's Blues by Jim Snidero transcribed by Andrea Calilhanna

Representing the meter including hemiolas through the Ski-hill graph provides students with mathematics visualised which provides answers for why music sounds a certain way but also enables the student to articulate how this occurs. For example, the Ski-hill graph in Figure 3a) illustrates a simple hemiola (Cohn, 2001) mapped by a listener to represent two pulses in ratio 2:3 (one metric level), and Figure 3b) a complex hemiola, with three metric levels in ratio 2:3 as evidenced in examples of traditional Igbo *Ogene Bunch* music, Nigeria and works of Frédéric Chopin, Johannes Brahms Antonin Dvořák. Visualising mathematics through the Ski-hill graph assists students to accelerate the time it takes to calculate the mathematics underpinning the rhythmic complexity of the two meters (duple and triple) contributing to hemiolas.

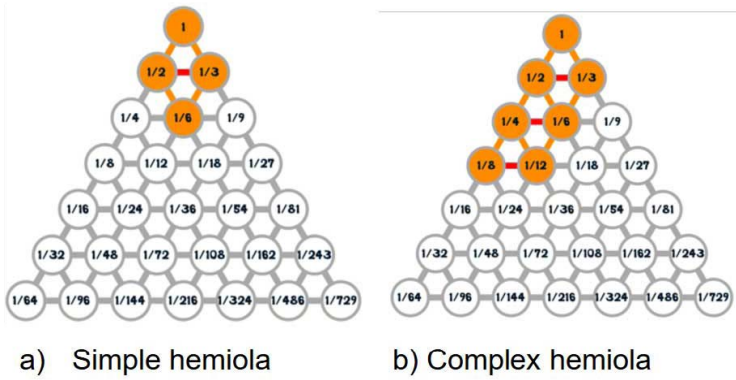

Figure 3. Ski-hill graph a) Simple hemiola 2:3 b) Complex hemiola

In other words, from my observations as a teacher-researcher, the Ski-hill graph enables students to engage in their classes and develop critical thinking and meta-cognition skills by applying real world mathematics relevant to their projects. Students learn how and why the mathematics of the meter informs decisions about structure, musical expression, and the fine details of genre, style and more. Ski-hill graph pedagogy facilitates and empowers teacher-student collaborations, peer-to peer learning, and independent learning through workshop-style lessons. Unlike conventional meter pedagogy, the Stereo Metronome enables students to articulate accurate representations of the meter tailored to each piece of music which reduces cognitive load (Sweller, 2010) by providing cognitive fit to the mathematics of both symbol to sound and sound to symbol. The meter represented accurately through the mathematics of the Ski-hill graph closes the cognitive gaps between the mathematics of time signatures and the experience of the meter and provides an opportunity to bridge the gap between the arts and science.

Ski-hill graph pedagogy

Ski-hill graph pedagogy is a flexible three-step psychoacoustic approach to visualise mathematical music theory where students actively listen to music, map all the pulses and meters through the two-dimensional matrix of the Ski-hill layout. Students then apply the data to the graphics of circle and linear graphs to annotate and decode the meter, rhythm, and pitch of their music notation, apply to performance, or to compose music. In addition to teaching the meter, in collaboration with Nigerian scholars, we have also successfully analysed traditional music of the Igbo people through the same flexible three-step process (Calilhanna, Onwubiko and Adeogun, 2019).

a) Ski-hill graph view *b) Linear graph view* *c) Circle graph view*

Figure 4 Stereo Metronome (Fan and Calilhanna, 2021 beta version)

Figure 1a)-c) illustrates the three views of the *Stereo Metronome* (Fan and Calilhanna, 2021 beta version) with a student's mapping to represent the pulses and meter from active listening to music. The representation includes pulses physically sounded through acoustics and those pulses and meter imagined. Figure 1a) Ski-hill graph view: All the pulses and meter in a cyclic hierarchy – a summary of the whole metric space (depth and relations of timepoints and timespans). Sets and subsets of the pulses and meter as fractions to visualise the cardinality of each pulse in a relation of inclusion as duple and or triple meter and to determine the cycle sizes and beat-class. Figure 1b): Linear graph view: All timepoints and pulses (depth and relation) – beat-class integers and numbers for counting the meter (measuring) – linear array of the cycles. Figure 1c): Circle graph view: Each pulse is a polygon with timepoints, spans, and beat-class to represent the cardinality of each pulse in relation to other pulses. Hypermetric pulses are nested in '0' for a single cycle. Rhythms are mapped as polygons. A fourth view is currently under development for pitch {0-11} such as scales and chords. We use 0-based counting from the first lessons to measure the meter and cycles to establish the timing and expression of the rhythm and pitch, and 1-based counting for scale degrees and for the number of notes in a triad or scale and so on.

Meter confuses students because of the mathematics of time signatures which relays very little information about the music and can bear no relation to the experience of the music nor the notation. From introducing the *Stereo Metronome* (Fan and Calilhanna, 2021) I have consistently observed student engagement and increased confidence through their interaction with the representations of the meter, rhythm, and pitch applied to their performances. The aim of the user-interface is to simply the process of learning without compromising on pedagogical quality or the integrity of knowledge and learning. In just a few steps students have articulated the information they require visualised and with audio to apply to the performance of a new piece of music. The student's intrinsic temporal experience of the meter is easily forgotten, however, Ski-hill graph pedagogy develops spectral awareness skills captured through the extrinsic mathematical representations of the Ski-hill, circle, and linear graphs. The student's intrinsic information is in a sense transformed into a meaningful and useful extrinsic conceptual map. The meter is no longer a static notation but a dynamic relation of pulses and meters experienced at the micro level of pulses through to macro levels of the broad metric structure. The *Stereo Metronome* is suitable to use for most teaching styes and resonates with students' different learning styles with its multi-modal and multi-sensory properties.

The mathematical music theory of Ski-hill graph pedagogy with its application to circle and linear graphs, students also develop foundational skills in mathematics such as fractions, set theory, counting, geometry, equivalence, ratio, proportion, modular arithmetic, and more, quantifications which all impact on executive functions and many other human functions.

Further implications

Arguably, a major contributor to the underrepresentation and cultural appropriation of much of the world's music such as the traditional music of the Igbo people, Nigeria is as a result of conventional Western notation-based music theory applied through colonialisation. Much traditional music around the world has already become extinct, taking with it important cultural heritage DNA (identity) pointing to immense implications for cultural heritage infrastructure policy as regards temporal data to protect at risk music and cultural identity. Ski-hill pedagogy, however, enables accuracy of records to transmit into the future through training in active listening – psychoacoustic analysis of music. Analyses of the meter solely through computational (White, 2021) and statistical methods is compromised through its limited capacity to distinguish the complexities of meter's maximally even sets and subsets (Clough

and Douthett, 1991; Cohn, 2017, 2020) of pulses (most of which are imagined). In other words, music analysis still requires the human auditory system, mathematics and accurate representation made possible through the mathematical music theory of the Ski-hill graph.

Conclusion

Visualising temporal data is an ancient practice and the meter is no exception. As the saying goes, a picture says a thousand words and graphs are visceral where words are not. Music educators are positioned to adopt modern meter theory (Cohn, 2020) with their students at all levels of the curriculum. This paper discusses why I chose to teach interdisciplinary mathematical music theory visualized through the mathematical representations of Ski-hill graph pedagogy (Calilhanna, 2018, 2020). From the perspective of teaching the fundamentals of the meter with school-age students, the paper includes a discussion of the benefits and implications of adopting and adapting Cohn's theories for meter pedagogy with a focus on the importance of the accurate representation of the meter through active listening and critical thinking.

Active listening and learning of spectral awareness through Ski-hill graph pedagogy and the *Stereo Metronome* is not a quick fix – learning mathematical music theory takes time - but the results indicate students learn meaningful understandings of the meter, authentically engage in learning, learn about their learning, and problem-solve. Opportunities to declutter curriculum arise through learning shared concepts across music and mathematics. The paper also introduces how and why Ski-hill graph as a means of teaching the meter provides an inclusive and ethical approach to the pedagogy and accurate representation of the meter in diverse music education. Finally, this current critical turning point in music theory history situates Cohn's Ski-hill graph as a bridge between the sciences and music to both augment the notational understandings of time signatures and to provide immense potential for music education and interdisciplinary approaches to science and arts education more broadly.

References

Calilhanna, A., Webb, M. (2018, Unpublished). Teaching time: A survey of music educators' approaches to meter, presented at the *Meter Symposium 3*, Sydney, 2018.

Calilhanna, A. M. (2020). Teaching musical meter to school-age students through the Ski-hill graph. *Proceedings of the178th Meeting of the Acoustical Society of America*, *39*. doi: https://doi.org/10.1121/2.0001234

Calilhanna, A., Onwubiko, S. G., Adeogun, A. (2019). Mathematical music theory of embodied acoustics of Ikoro music using beat-class theory. *The Journal of the Acoustical Society of America*, *146*(4). doi: https://doi.org/10.1121/1.5137386

Clough, J., Douthett, J. (1991). "Maximally Even Sets". *Journal of Music Theory* (35).

Cohn, R. (1992). Metric and Hypermetric Dissonance in the Menuetto of Mozart's Symphony in G minor, K. 550. Intégral, 6.

Cohn, R. (2001). Complex hemiolas, ski-hill graphs and metric spaces. *Music Analysis*, *20*(3). Retrieved from: http://www.jstor.org/stable/854473

Cohn, R. (2015). Why We Don' t Teach Meter, and Why We Should. *Journal of Music Theory Pedagogy*, *29*.

Cohn, R. (2016b). Graph-theoretic and geometric models of music. In Jordan, B.L., Smith, E., & Chew, G.A. (eds). *Mathemusical Conversations: Mathematics and Computation in Music Performance and Composition* (Lecture Notes Series, Institute for Mathematical Sciences, National University of Singapore). Singapore: World Scientific Publishing Company.

Cohn, R. (2018). Scaling up to Atonality: The Pedagogy of Small Cyclic Universes. In *Theoretical and Practical Pedagogy of Mathematical Music Theory: Music for Mathematics and Mathematics for Music, From School to Postgraduate Levels*. World Scientific.

Cohn, R. (2020). Meter. In Rehding, A. and Rings, S. (eds.), *The Oxford Handbook of Critical Concepts in Music Theory*. Oxford University Press.

Cooper, L. A., Lang, J. M. (1996) Imagery and visual—spatial representations. In Bjork, E. L. and Bjork, R. A. (eds.), *Memory*. Academic Press.

Fan, S., Calilhanna, A. (2021). *Stereo Metronome* (beta version).

Gentner, D., Loewenstein, J., Thompson, L. (2003). Learning and transfer: A general role for analogical encoding. *Journal of educational psychology*, *95*(2).

Hilton, C., Calilhanna, A., Milne, A. J. (2018). Visualizing and sonifying mathematical music theory with software applications: Implications of computer-based models for practice and education. In Montiel, M. and Gómez, F. (eds.), *Theoretical and Practical Pedagogy of Mathematical Music Theory. Music for Mathematics and Mathematics for Music, From School to Post-graduate Levels*. World Scientific. doi: https://doi.org/10.1142/9789813228351_0009

Milne, A. J., Calilhanna, A. M. (2019). Teaching music with mathematics: A pilot study. In Montiel, M., Gómez-Martin, F., and Agustín-Aquino, O. A., editors, Mathematics and Computation in Music. Springer, Berlin.

Nozaradan, S., Peretz, I., Missal, M., Mouraux, A. (2011). Tagging the neuronal entrainment to beat and meter. *Journal of Neuroscience*, *31*(28).

Peck, R.W. (2014). Mathematical Music Theory Pedagogy and the 'New Math'. *Journal of Mathematics and Music 8*(2). doi:10.1080/17459737.2014.927115.

Rogers, N., Clendinning, J.P., Ganley, S.H.C (2016). Specific mathematical and spatial abilities correlate with music theory abilities. In: Proceedings of the *14th International Conference on Music Perception and Cognition.*

Sanocki, T. (2003). Representation and perception of scenic layout. *Cognitive Psychology*, *47*(1).

Sweller, J. (2010). Cognitive load theory: Recent theoretical advances. In Plass, J. L., Moreno, R. & Brünken, R. (Eds.), *Cognitive load theory*. Cambridge University Press. doi: https://doi.org/10.1017/CBO9780511844744.004

White, C. W. (2021). Some observations on autocorrelated patterns within computational meter identification. *Journal of Mathematics and Music*, *15*(2). doi: https://doi.org/10.1080/17459737.2021.1923843