Conceptual Problems in The Causal Theory of Quantum Mechanics

A Treatise on the Foundations of Quantum Theory

by

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Abstract

The standard theory of non-relativistic quantum mechanics is essentially an algorithm for obtaining statistical predictions and a prescription for avoiding fundamental questions. A better understanding of the foundations of non-relativistic quantum mechanics is needed than is provided by the standard theory if we are to explain physical reality. The way to achieve this is to specify both the ontology and the laws that govern the quantum realm. The Causal Theory of Quantum Mechanics (which has been steadily developing since the early 1950s) does this by describing micro-phenomena in terms of entities and processes in space and time.

In this thesis, solutions will be presented to some unresolved conceptual problems of the Causal Theory and related theoretical problems. A conceptual problem is generated when a theory is in conflict with another well-established belief. The thesis will focus especially on finding solutions to conceptual problems about the nature of energy, the conservation of energy, and the Exclusion Principle within the context of the Causal Theory of Quantum Mechanics.

STATEMENT

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

I give consent to this copy of my thesis being made available in the University Library.

(Peter J. Riggs)
December, 2007

PREFACE AND ACKNOWLEDGEMENTS

There has been a great deal of ill-informed commentary about quantum mechanics since its beginnings. Much of this has been speculation and/or confusion based upon improper interpretations of the Uncertainty Relations, or by identifying non-deterministic solutions with an absence of physical causality. These speculations and confusions have led to the abandonment of some concepts and principles that were strongly held prior to the advent of quantum mechanics. I would argue that we should not give away established physical concepts and principles until such time as they are clearly shown to be inappropriate, not applicable, or simply false. In this respect, I have always thought that Orthodox Quantum Theory (i.e. the Copenhagen Interpretation) 'throws the baby out with the bath water'. There are certainly many, new features to be learnt about the microworld and which quantum mechanics can inform about. This does not necessarily require renouncing physical principles and ontological concepts that have been previously developed – more a case of modifying most of these to suit new knowledge. This is a better way to proceed if we are to gain a fuller understanding of the foundations of quantum mechanics.

The Causal Theory of Quantum Mechanics is not well known within the general physics community and many physicists who do know of it are generally dismissive in their attitudes. I certainly do not pretend to have all the answers in regard to the problems of quantum mechanics. However, I hope that this thesis may contribute to changing the opinions of those who reject the Causal Theory out of hand. One of the most influential figures in physics in the second half of the twentieth century is Professor John A. Wheeler of Princeton University. He once remarked that doing scientific research means that you "stick up for something". In this thesis, I take Wheeler's advice and make an unashamed stand on a number of issues.

This is a thesis that was nearly never finished, despite the strength of conviction that I personally feel for the subject matter. The reasons are many, both professional and personal. However, the over-riding reason was my effective retrenchment from academia with the consequence of having to gain employment in areas totally removed from academic life.

I acknowledge the assistance and support of both my principal supervisor, Dr. Peter Szekeres of the Department of Physics, University of Adelaide, and of my co-supervisor, Professor Graham Nerlich of the Department of Philosophy, University of Adelaide.

Extracts from this thesis has been published in refereed academic journals as the following articles:

- 'Quantum Phenomena in Terms of Energy-Momentum Transfer', *Journal of Physics A: Mathematical and General* **32** (1999): 3069-74.
- 'Reflections on the deBroglie-Bohm Quantum Potential', *Erkenntnis* (In-print).

During part of this doctoral candidature, my family and I had the support of an Australian Postgraduate Award, for which we are duly appreciative.

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PART I

INTRODUCTION AND AIMS

General Introduction

I am now convinced that theoretical physics is actual philosophy.

—- Max Born [1]

Science in effect creates philosophy.

— Gaston Bachelard [2]

The subject of this thesis is the Causal Theory of (Non-Relativistic) Quantum Mechanics. The names under which this theory has developed have varied somewhat in the literature and include: Causal Interpretation of Quantum Mechanics; Bohm Interpretation; deBroglie-Bohm Interpretation; Bohm's Theory; deBroglie-Bohm Theory; the Bohm Formulation, Ontological Interpretation; Quantum Theory of Motion; and Bohmian Mechanics. We shall use the designation – The Causal Theory of Quantum Mechanics (or Causal Theory, for short). The principal rationale for preferring this title is that the Causal Theory of Quantum Mechanics is *a theory in its own right* rather than merely being just an interpretation of the formalism of quantum mechanics. This claim is justified for the following reasons:

(i) The Causal Theory is a mature scientific endeavour with more than fifty years of progress;

(ii) The axioms of the Causal Theory are not identical to those of Orthodox Quantum Theory (or Copenhagen Interpretation) which is accepted by the majority of physicists. See Chapter 2 for the former and Appendix I for the latter;

(iii) The conceptual structures postulated in the Causal and Orthodox quantum theories are radically distinct. This is because the underlying models of the Causal and Orthodox views are diametrically opposed.[3]

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1 Born, M., *My Life and My Views* (Scribner's Sons, New York, 1968) p.48.
2 Bachelard, G., *The New Scientific Spirit* (Beacon Press, Boston, 1984) p.3.

Why use the name 'Causal'? Orthodox Quantum Theory is known for doing away with causality in the sense of event by event causality in space and time.[4] The Causal Theory, on the other hand, embraces causality by explaining micro-phenomena in terms of entities and processes in space and time. In the Causal Theory, the future state of a quantum system is determined by the dynamics of the system and its interactions with the surrounding environment.

There are many advantages to be found in accepting the Causal Theory over Orthodox Quantum Theory. In particular, the Causal Theory has the following benefits:

- a definite ontology in terms of entities in space and time;
- no arbitrary division between classical and quantum realms;
- a single, continuous dynamics;
- no problems involving measurements;
- no need to postulate 'ad-hoc' mechanisms designed to overcome difficulties inherent in Orthodox Quantum Theory;
- all the quantum paradoxes are solvable or do not occur; and
- Heisenberg's Uncertainty Relations do not have ontological implications.

Despite these desirable features, the Causal Theory's reception from its beginning to the present day has been most unfavourable. The general assessment by the physics community of the Causal Theory is plagued with a variety of 'myths' and widely-held misconceptions. The alleged flaws of the Causal Theory that are most commonly found in the literature are:

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[3] Freistadt, H., 'The Causal Formulation of the Quantum Mechanics of Particles', *Supplemento Nuovo Cimento* V (1957) p.3.
[4] Bunge, M, *Causality and Modern Science* (Dover, New York, 1979) p.14; Cushing, J.T., *Philosophical Concepts in Physics: The Historical Relation Between Philosophy and Scientific Theories* (Cambridge University Press, Cambridge, 1998) pp.290 & 298.

① It is a return to classical physics.

② It contains 'hidden variables'.

③ It has been disproved by the various impossibility theorems.

④ It is pure metaphysics.

⑤ It has been refuted by experiments on Bell-type inequalities.

⑥ It is inconsistent.

⑦ It cannot incorporate intrinsic angular momentum (spin).

These alleged flaws are quite commonly held by physicists who are aware of the existence of the Causal Theory (although many are not). However, they have either not been properly substantiated or have been shown to be false, as noted by the late American physicist and philosopher of science James T. Cushing:

> Most physicists do not really know much about Bohm's [Causal] theory and those who are even vaguely aware of its existence usually "know" that it is wrong, although they are not exactly certain just why. … the folklore-wisdom charges against Bohm's program … can be seen to be either specious or inconclusive …[5]

Since the above alleged flaws continue to have *strong currency* in the physics community and that the 'disarming' of them may be subsumed under Principal Aim I (below), it will be shown (or otherwise indicated) during the presentation of this thesis why they are ill-founded and/or unwarranted.

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[5] Cushing, J.T., 'The Causal Quantum Theory Program' in Cushing, J.T., Fine, A. and Goldstein, S. (eds), *Bohmian Mechanics and Quantum Theory: An Appraisal* (Kluwer, Dordrecht, 1996) pp.5-6.

Principal Aims

There are two principal aims for this dissertation which may be stated as follows:

(I) To show that the Causal Theory of Quantum Mechanics is a viable theory that provides realistic explanations for physical phenomena;

and

(II) To offer solutions to some unsolved problems of the Causal Theory of a conceptual nature and associated theoretical problems.

In order to achieve these aims, we shall to have take on board a few points about scientific theories in general and assess (to some degree) the various merits of the Causal Theory versus Orthodox Quantum Theory. It is important when comparing rival scientific theories to acknowledge that they are postulated in order to meet a number of needs. A scientific theory must be empirically successful, i.e. the predictions of the theory must be borne out by experiment to within the range of the accuracy available. However, empirical success is *not all* that is required from scientific theories. Of perhaps equal standing with empirical success is a theory's ability to describe why events occur and why our instruments record the numerical results that they do.

The following criteria for the assessment of the virtues of rival scientific theories has a large degree of consensus in both the scientific and philosophical communities:

- empirical adequacy;
- explanatory success;
- predictive power;
- consistency; and
- conceptual coherence.

The sometimes used criteria of simplicity and aesthetic appeal will not be entertained as explicit criteria for theory choice as both are very subjective.[6] Nature need not conform to a specific account of simplicity that one finds attractive nor to particular notions of what constitutes 'beauty' in a physical theory.

As with all physical theories, the Causal Theory embodies a mathematical model. It is an uncontroversial point that no model captures all aspects of the phenomena under study. A scientific theory then, should not be taken literally in all respects. Nor is it the case that any one theory (or version thereof) is the final word regarding the phenomena described. However, it shall be the contention here that some suitably developed and tested scientific theories, when interpreted realistically, have features that 'mirror' aspects of objective (i.e. observer independent) reality.[7]

The Distinction between Conceptual and Theoretical Problems

Physics is a discipline where theoretical and conceptual problems are always to be found. Depending on the context, the definitions of these two types of problems is not always made clear and, in some instances, there can be overlap between them. In order to avoid any confusion, theoretical and conceptual problems will now be defined for the purposes of this dissertation.

We shall follow general usage in relation to defining theoretical problems. A theoretical problem relates to some unsolved technical aspect of the theory under examination and requires further formal development of the theory for its solution.

In respect to conceptual problems, we shall use the definition formulated by Laudan. A conceptual problem is generated either when a theory contains internal

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6 Giere, R.N., *Explaining Science: A Cognitive Approach* (University of Chicago Press, Chicago, 1988) pp.224-225.

7 Bunge, M, *Philosophy of Physics* (Reidel, Dordrecht, 1973) pp.7-8; Penrose, R., *The Road to Reality: A Complete Guide to the Laws of the Universe* (Jonathon Cape, London, 2004) p.508.

inconsistencies (internal conceptual problem), or where a theory is in conflict with another well-established theory or doctrine (external conceptual problem).[8]

The criterion for internal consistency is fairly obvious since any theory that is not internally consistent will contain within itself, logical contradictions. However, the notion of conflict with another well-established theory or doctrine needs elaboration. Such conflict might be due to one theory being logically inconsistent with another theory. More frequently though, external conceptual problems involve a different form of conflict.[9] A well known example should suffice to illustrate this point. In 1917, the initial relativistic model of the cosmos invented by Albert Einstein was not static. This conflicted with the prevailing view of the era that the universe was neither expanding nor contracting.[10] The conflict with this prevailing view generated an external conceptual problem for Einstein's theory. Consequently, Einstein introduced another term into the field equations of General Relativity (called the Cosmological Constant) which resulted in a static universe model and thereby resolved the external conceptual problem faced by the theory at that time. (This turned out, of course, to be a step that was not necessary, as Edwin Hubble announced in 1929 that his observations of distant galaxies showed that the universe was expanding.)

Finding solutions to external conceptual problems is not generally as straight forward as solving theoretical ones. This is due to a number of factors. The solution to an external conceptual problem may involve the generation of new ideas and/or new interpretations of existing concepts. Indeed, the process of finding a solution to a given (external) conceptual problem may not require any formal theoretical development of the theory at all !

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8 Laudan, L., *Progress and its Problems* (University of California Press, Los Angeles, 1977) p.49.
9 *ibid.*, p.51.
10 Cushing, J.T., Philosophical Concepts in Physics, *op. cit.*, p.262.

A well known example from the history of science that illustrates this point is Tycho Brahe's model of the universe. This model avoided the conceptual problem inherent in the Copernican model, i.e. that of not having the Earth at the centre of the universe. It did so by postulating that although the Sun revolved around the Earth, all the other planets revolved around the Sun. Brahe's model was mathematically equivalent to the Copernican model (and so had all of its technical advantages) but without the unacceptable conflict of the time of having the centre of the universe at a point other than the centre of the Earth.[11]

The solution of conceptual problems may lead to theoretical advances, i.e. the resolution of conceptual difficulties in a physical theory may open up new avenues for the solution of previously unsolved (or perhaps unknown) theoretical problems. Conceptual problems may also arise from new theoretical advances and it is not uncommon for the two to go hand in hand. Stephen Hawking's theorems about the singular nature of the origin of the universe is a paradigm example. Hawking showed that, based on some very reasonable physical assumptions, the General Theory of Relativity implied that there must have been a beginning to the whole universe at a finite time in the past and that this point was a singularity in spacetime (i.e. a point of infinite density and zero volume) commonly called the 'Big Bang' origin.[12] This created a host of both theoretical and conceptual problems, e.g. the presence of singularities is deemed by many theoreticians as a blemish on a physical theory that needs to be removed (a theoretical problem); the 'Big Bang' origin is a point where the laws of physics apparently break down (a conceptual problem). Such examples as

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11 Dreyer, J.L.E., *A History of Astronomy from Thales to Kepler* (Dover, New York, 1953) pp.363-364.

12 Hawking, S.W., 'Sixty Years in a Nutshell' in Gibbons, G.W., Shellard, E.P.S. and Rankin, S.J. (eds), *The Future of Theoretical Physics and Cosmology* (Cambridge University Press, Cambridge, 2003) p.111.

this indicate the significance of conceptual problems in physical theories and also the importance of resolving them.

PART II

THE QUANTUM REALM

CHAPTER 1

PRELIMINARIES

I think I can safely say that nobody understands quantum mechanics.

— Richard Feynman [1]

1.1 Orthodox Quantum Theory and its Mathematical Formalism

The early days of quantum theory were a period of great puzzlement and disillusion for those involved in trying to formulate a consistent theoretical scheme of atomic phenomena. This scheme had to provide an empirically satisfactory account of diverse sets of experimental data, such as the Photoelectric and Compton Effects, atomic spectral lines, and electron diffraction. Werner Heisenberg expressed the frustration experienced when he wrote:

> … an intensive study of all questions concerning the interpretation of quantum theory in Copenhagen finally led to a complete … clarification of the situation. But it was not a solution which one could easily accept. … I repeated to myself again and again the question: *Can nature possibly be as absurd as it seems to us in these atomic experiments?* [2]

Heisenberg's worries about the apparent absurdity of quantum physics are just as relevant to the contemporary debate on the foundations of quantum theory, as they were when he first conceived them. Yet, as absurd as it may sometimes appear, quantum mechanics is one of the two best experimentally confirmed theories in the whole history of physics (the other being, of course, Relativity). There exists little disagreement about the mathematical apparatus of quantum mechanics, but what does this formalism tell us about the nature of the quantum realm? The version of

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[1] Feynman, R.P., *The Character of Physical Law* (MIT Press, Cambridge, MA., 1965) p.129.

quantum theory originally due to Neils Bohr and Werner Heisenberg, and progressed by Max Born, Wolfgang Pauli, John von Neumann, Paul Dirac, and others has achieved dominance amongst working physicists. This is called the 'Copenhagen Interpretation' (for historical reasons) and is a *broad interpretive framework* adhered to by a majority of physicists.[3] In this treatise, the title of 'Orthodox Quantum Theory' will be used in preference to 'Copenhagen Interpretation'. The alternative designations found in the literature include: 'Standard Quantum Theory', 'Orthodox Interpretation', 'Usual Interpretation', 'Conventional Quantum Mechanics', and 'Quantum Orthodoxy'. These shall be taken as synonymous with 'Orthodox Quantum Theory'. The formal aspects of Orthodox Quantum Theory are defined by the set of axioms in Appendix I.

Wave-particle duality is the expression of the apparent dual nature of quantum entities as manifest in either wave-like or particle-like behaviour but not both in a single experimental arrangement.[4] The notion of wave-particle duality is an essential component of Orthodox Quantum Theory. This is evidenced by the number of in-depth discussions of the topic by the founders of quantum theory and by the number of textbooks that present it as such.[5] (There are, however, some physicists who are clearly not keen to acknowledge this any more.[6]) Bohr embodied this notion into his Principle of Complementarity which he considered to be at the heart of

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[2] Heisenberg, W., *Physics and Philosophy* (Penguin, London, 1989) p.30 (italics mine).

[3] Jammer, M., *The Conceptual Development of Quantum Mechanics* (McGraw-Hill, New York, 1966) p.361; Baggott, J., *The Meaning of Quantum Theory* (Oxford University Press, Oxford, 1992) p.82; Cushing, J.T., *Philosophical Concepts in Physics* (Cambridge University Press, Cambridge, 1998) p.289.

[4] Heisenberg, W., Physics and Philosophy, *op. cit.*, p.37.

[5] Examples include: Enge, H.A., Wehr, M.R. and Richards, J.A., *Introduction to Atomic Physics* (Addison-Wesley, Reading, M.A., 1972) pp.142-143; Eisberg, R. and Resnick, R., *Quantum Physics of Atoms, Molecules, Solids, Nuclei, and Particles* (Wiley, New York, 1985) pp.62-63; Rae, A.I.M., *Quantum Mechanics* (IOP Publishing, Bristol and Philadelphia, 1992) p.242.

[6] Griffiths, D.J., *Introduction to Quantum Mechanics* (Pearson Prentice-Hall, New Jersey, 2005) p.420 note 1.

quantum mechanics. Although Bohr's statements on this matter suffered from a lack of clarity, the Principle may be broadly stated as follows:[7]

- *Principle of Complementarity*

  Any application of a classical concept precludes the simultaneous use of other classical concepts which in a different connection are equally necessary for the description of phenomena.

Bohr also proposed a principle designed to 'bridge' the gap between classical and quantum realms. This is called the Correspondence Principle of Orthodox Quantum Theory:[8]

- *Correspondence Principle*

  Quantum states and measurements will tend to the corresponding classical case in the limit of large quantum numbers.

Two other fundamental features of Orthodox Quantum Theory are the roles of measurement and quantum indeterminacy. It has become fairly standard in presentations of Orthodox Quantum Theory to claim that the quantum realm is one where physical quantities do not have determinate values unless they are measured. This is inferred on the basis of the Uncertainty Relations (see Section 1.2 below). The process of observing/ measuring physical systems is specified in the axioms of Orthodox Quantum Theory thereby endowing a special status on measurement within the theory[9] (see also Section 1.3 below). These features are, of course, major departures from classical physics.

The formalism of Orthodox Quantum Theory can be represented in several different ways. The term 'quantum mechanics' is usually taken to encompass

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[7] Jammer, M., *The Philosophy of Quantum Mechanics: The Interpretations of Quantum Mechanics in Historical Perspective* (Wiley, New York, 1974) p.95.
[8] Gasiorowicz, S., *Quantum Physics* (Wiley, New York, 1974) p.19.
[9] Sudbery, A., *Quantum Mechanics and the Particles of Nature: An Outline for Mathematicians* (Cambridge University Press, Cambridge, 1986) p.185.

Schrödinger's wave mechanics, Heisenberg's matrix mechanics and abstract generalisations of them. The axioms of Orthodox Quantum Theory, as presented in Appendix I, specify that the pure state of a quantum system is 'represented' by a vector in a (mathematical) Hilbert space, a formalism that has proved high successful at the empirical level. Mixed states (see definition in Appendix I) will not be considered in this thesis as these will not be of assistance in resolving the stated conceptual problems. Basically, Hilbert space is a complex vector space (see State Vectors axiom, Appendix I) on which an inner product is defined[10] (see Inner Product axiom). This is very different from the way that the state of a physical system is characterised in Classical Mechanics where the system's state may be represented by a point in a phase space.

Why have a physical theory that uses abstract (Hilbert) vector spaces? Reasons for adopting the Hilbert space formalism for quantum mechanics include:

- the failure of Classical Mechanics and the old quantum theory of Bohr and Sommerfeld to successfully account for empirical results at the atomic level;
- the need for a mathematical space in which the represented form of quantum states have certain desirable properties (e.g. being normalisable);[11]
- representing the uncertainty in measurement in terms of a non-commutative algebra.[12]

However, the representation of physical systems by means of a Hilbert space has always been somewhat odd and counter-intuitive, as noted by Carroll:

> Perhaps no subject has been the focus of as much mystery as "classical" quantum mechanics (QM) even though the standard Hilbert space framework provides an eminently satisfactory vehicle ... So why all the fuss? The erection of

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10 Wallace-Garden, R., *Modern Logic and Quantum Mechanics* (Adam Hilger, Bristol, 1984) p.7.
11 Penrose, R., *The Road to Reality: A Complete Guide to the Laws of the Universe* (Jonathon Cape, London, 2004) p.534.

> the Hilbert space edifice … has an air of magic. It works but exactly why it works and what it really represents remain shrouded in ambiguity.[13]

In Orthodox Quantum Theory, a vector that 'represents' a particular state of a physical system is called a state vector. No physical reality is ascribed to a state vector.[14] The state vector is taken as containing all possible information about a quantum system (Completeness axiom). Consequently, any questions relating to a quantum system that go beyond what can be found from the state vector are considered *meaningless* in Orthodox Quantum Theory.[15] Examples of such questions include the exact nature of quantum entities, and the paths of quantum particles when they are not being observed.

It is usual for a given state of a quantum system to be in a superposition of other states. This follows from the Principle of Linear Superposition which applies to classical as well as quantum waves and fields:

♦ *Principle of Linear Superposition*

When several individual states are superimposed, the resultant state is the vector addition of the individuals.

When a quantum system is in a superposition, its state is described by a sum of state vectors. This appears formally in Orthodox Quantum Theory as the Linear Superposition axiom. Moreover, if not specifically prepared and left 'unobserved', a quantum system will be in a *superposition of eigenstates.* An eigenstate may be defined as a state pertaining to a particular Hermitian operator (call it **A**) defined on the Hilbert space. The eigenstate is described by a state vector (call it ϕ) such that the

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12 Bub, J., 'Complementarity and the Orthodox (Dirac-Von Neumann) Interpretation of Quantum Mechanics' in Clifton, R.(ed.), *Perspectives on Quantum Reality* (Kluwer, Dordrecht, 1996) p.211.
13 Carroll, R., 'Remarks on the Schrödinger Equation', *arXiv:quant-ph/0401082* (22 Mar 2004) p.1.
14 Penrose, R., The Road to Reality, *op. cit.*, p.805.
15 Jammer, M., The Conceptual Development of Quantum Mechanics, *op. cit.*, p.330; Bohm, D., *Causality and Chance in Modern Physics* (Routledge & Kegan Paul, London, 1957) p.92.

equation: $\mathbf{A}\phi = \alpha\phi$ holds, with α being a real number (Eigenstate axiom). The time evolution of a state vector is governed deterministically by the time-dependent Schrödinger equation (Time Development axiom):

$$i\hbar\ (\partial\Psi/\partial t) = \mathbf{H}\Psi$$

where ψ is the state vector, $\hbar$ is Planck's Constant divided by 2π, $i = \sqrt{-1}$ and $\mathbf{H}$ is the Hamiltonian operator.

Orthodox Quantum Theory is principally about the results that can be obtained when the value of some physical parameter (or quantity) of a quantum system, i.e. an observable, is measured. The result of measuring an observable is always an eigenvalue of the Hermitian operator corresponding to the observable (Quantisation Algorithm). However, unless a quantum system is already in an eigenstate of an Hermitian operator rather than a superposition of states, a measurement (or more generally, an 'observation') of some physical quantity will instantaneously 'reduce' the superposition to a single eigenstate (Projection Postulate). This is a process that is inherently *non-deterministic* and for which the theory only can predict the *probability* that a particular value for the relevant physical quantity will be found on measurement. Probability in Orthodox Quantum Theory is supposed to convey a notion of objective chance. The interpretation of probability in quantum mechanics has been the subject of much debate but at a basic level, probability may be understood in the sense of a relative frequency. The probability for a particular value of an observable to be found on a single measurement is taken as the fraction of the total number of results that yield this value in a large number of such measurements which are repeated under identical conditions.[16] There are,

[16] Ballentine, L.E., *Quantum Mechanics: A Modern Development* (World Scientific, Singapore, 1998) p.35; Gibbins, P., *Particles and Paradoxes* (Cambridge University Press, Cambridge, 1987) p.8.

therefore, two types of evolution of quantum states, i.e. evolution in conformity with the Schrödinger equation and evolution on measurement.[17]

It should be clear from the above discussion that Orthodox Quantum Theory is both an algorithm for obtaining statistical predictions for the results of experiments and a prescription for avoiding fundamental questions. In other words, Orthodox Quantum Theory is essentially an *instrumental* theory.[18]

1.2 Uncertainty at the Quantum Level

The common account of the uncertainty relations is that they are an *in-principle* limitation on the precision of simultaneous measurements of some quantities of a quantum system, such as position and momentum. Is this claim necessitated (in some sense) by the mathematics? In order to address this question, the general uncertainty relation will be derived. In mathematical statistics, the mean value of a quantity A is the average value obtained from a large set of individual values of A. This mean is defined by:

$$\langle A \rangle = \sum_{i=1}^{N} A_i \Big/ N \quad \text{.... (1-1)}$$

where the A_i are individual values of A of which the total number of values is N. In quantum mechanics, the expression for the expectation value (or mean) of a quantity A in a given state may be derived from its axioms. Measurements of the value of an observable A of a system in a state with state vector $\psi = \sum_i c_i\phi_i$ will yield eigenvalues α_i with frequency $|c_i|^2$. The expectation value will then be given by

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[17] Ballentine, L.E., 'The Statistical Interpretation of Quantum Mechanics', *Reviews of Modern Physics* **42** (1970), p.369; Penrose, R., The Road to Reality, *op. cit.*, pp.528-532.
[18] Smart, J.J.C., *Between Science and Philosophy: An Introduction to Philosophy of Science* (Random House, New York, 1968) p.159.

summing all the products of individual eigenvalues with their associated frequency,[19] i.e. $\sum_i \alpha_i |c_i|^2$. This provides agreement with the statistical definition (equation (1-1)). Since $\mathbf{A}\phi_n = \alpha_n\phi_n$ where **A** is the Hermitian operator corresponding to an observable A, it follows that: $\mathbf{A}\psi = \sum_i c_i \mathbf{A} \phi_i = \sum_i c_i\alpha_i\phi_i$ and $(\psi, \mathbf{A}\psi) = \sum_i (c_i\phi_i, c_i\alpha_i\phi_i) = \sum_i \alpha_i|c_i|^2 (\phi_i, \phi_i) = \sum_i \alpha_i |c_i|^2$ if $(\phi_i, \phi_i) = 1$, i.e. if ϕ_i is normalised. More generally, the expectation value of an observable A in a state described by a state vector ψ is given by:

$$\langle A\rangle_\psi = (\psi, \mathbf{A}\psi) / (\psi, \psi) \quad .... (1\text{-}2)$$

if ψ is not normalised. The variance (or dispersion) of a quantity A, denoted $(\Delta A)^2$, is defined by:[20]

$$(\Delta A)^2 = \langle (A - \langle A\rangle)^2 \rangle \quad .... (1\text{-}3)$$

Variance is a measure of the spread or scatter of values about the mean, i.e. the width of the statistical distribution of the value of A.[21]

The derivation of the general uncertainty relation uses a version of the Schwarz Inequality as applied to inner products. This derivation appears in some of the literature in varying forms.[22] Consider the inner product: (uv + w, uv + w) where v and w are arbitrary state vectors and u is a number. Using the rules set out in the Inner Product axiom (Appendix 1), we have:

$$(uv + w, uv + w) = (uv + w, uv) + (uv + w, w)$$
$$= u^2 (v, v) + u \{(w, v) + (v, w)\} + (w, w)$$

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19 Boas, M.L., *Mathematical Methods in the Physical Sciences* (Wiley, New York, 1966) p.697.
20 Ballentine, L.E., 'The Statistical Interpretation of Quantum Mechanics', *loc. cit.*, p.364.
21 Boas, M.L., Mathematical Methods, *op. cit.*, p.697.
22 Examples include: Redhead, M., *Incompleteness, Nonlocality, and Realism* (Clarendon, Oxford, 1987) pp.59-61; Sakurai, J.J., *Modern Quantum Mechanics* (Addison-Wesley, Redwood City, California, 1985) pp.34-36; and Griffiths, D.J., *Introduction to Quantum Mechanics*, *op. cit.*, pp.110-111.

$$\Rightarrow (uv + w, uv + w) = au^2 + bu + c$$

where the coefficients $a = (v, v)$, $b = \{(w, v) + (v, w)\}$, and $c = (w, w)$ must be non-negative numbers. The condition for this quadratic expression to be non-negative (which it must be as the left-hand side is an inner product) is ($b^2 \leq 4ac$).[23] Therefore

$$(v, v)(w, w) \geq \tfrac{1}{4} \{(w, v) + (v, w)\}^2 \quad \ldots. (1\text{-}4)$$

Now let $v = (\mathbf{A} - a)\chi$ and $w = i(\mathbf{B} - b)\chi$, where **A** and **B** are non-commuting Hermitian operators, χ is a normalised state vector, $a = \langle \mathbf{A} \rangle$, $b = \langle \mathbf{B} \rangle$, and $i = \sqrt{-1}$.

Then substituting for v gives:

$$(v, v) = ((\mathbf{A} - a)\chi, (\mathbf{A} - a)\chi) = (\chi, (\mathbf{A} - a)^2 \chi) = \langle (\mathbf{A} - a)^2 \rangle = \langle (\mathbf{A} - \langle \mathbf{A} \rangle)^2 \rangle$$

Similarly, we have:

$$(w, w) = \langle (\mathbf{B} - b)^2 \rangle = \langle (\mathbf{B} - \langle \mathbf{B} \rangle)^2 \rangle$$

and

$$\{(w, v) + (v, w)\} = i \{\langle \mathbf{AB} \rangle - \langle \mathbf{BA} \rangle\} = \langle i [\mathbf{A}, \mathbf{B}] \rangle$$

where $[\mathbf{A}, \mathbf{B}] = \mathbf{AB} - \mathbf{BA}$. The product of the variances of the two operators **A** and **B** may now be found from the above results using equations (1-3) and (1-4):

$$(\Delta \mathbf{A})^2 (\Delta \mathbf{B})^2 \geq \tfrac{1}{4} \langle i [\mathbf{A}, \mathbf{B}] \rangle^2 = \tfrac{1}{4} \langle \mathbf{C} \rangle^2 \quad \ldots. (1\text{-}5)$$

where $[\mathbf{A}, \mathbf{B}] = i\mathbf{C}$. This inequality shows that the product of the statistical variances of **A** and of **B** has a lower bound.

The uncertainty of a quantity (also known as the root mean square deviation or the standard deviation) is defined as the positive square root of its statistical variance.[24] Dispersionless states are those for which the root mean square deviation is zero. The general uncertainty relation follows from this definition and the inequality (1-5):

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[23] Sudbery, A., Quantum Mechanics, *op. cit.*, p.59.

$$(\Delta \mathbf{A})\,(\Delta \mathbf{B}) \geq \tfrac{1}{2}\,|\langle \mathbf{C} \rangle| \quad \ldots.\ (1\text{-}6)$$

The interpretation of this general uncertainty relation is if a system is prepared in a pure state, then repeated measurements of A in an ensemble of identically prepared systems will yield a standard deviation $\Delta\mathbf{A}$ around the mean value $\langle \mathbf{A} \rangle$. Likewise, measurements of **B** will yield a standard deviation $\Delta\mathbf{B}$.[25] If **A** and **B** are canonically conjugate operators, then $[\mathbf{A}, \mathbf{B}] = i\hbar$ (Canonical Commutation axiom) and the expression for the lower bound (1-6) reduces to the recognisable form of Heisenberg's uncertainty inequality: $(\Delta\mathbf{A})\,(\Delta\mathbf{B}) \geq \hbar/2$.

Many critiques emphasise that the common account of the uncertainty relations is often misunderstood and misrepresented. Karl Popper, for example, argued continually against this common account:

> ... Heisenberg's famous formulae ... are, beyond all doubt, validly derivable *statistical formulae* of the quantum theory. But they have been *habitually misinterpreted* by those quantum theorists who said that these formulae can be interpreted as determining some upper limits to the *precision of our measurements* ...[26]

Other analyses of the uncertainty relations have made similar conclusions.[27] What the general uncertainty relation asserts is that there are limitations on the preparation of dispersion-free states for all observables.[28] Based on its formal statistical origin alone, the uncertainty relations would not constitute an in-principle limitation on the precision of *simultaneous* measurements on 'conjugate' observables of a quantum

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24 Boas, M.L., Mathematical Methods, *op. cit.*, p.697.
25 Redhead, M., Incompleteness, Nonlocality, and Realism, *op. cit.*, p.61.
26 Popper, K.R., *Quantum Theory and the Schism in Physics* (Unwin & Hyman, London, 1982) pp.53-54 (his italics).
27 Margenau, H., *The Nature of Physical Reality* (McGraw-Hill, New York, 1950) pp.375-377; Ballentine, L.E., 'The Statistical Interpretation ...', *loc. cit.*, p.365.
28 Redhead, M., Incompleteness, Nonlocality, and Realism, *op. cit.*, p.62.

system. Such a limitation would need a further assumption.[29] (In any case, the simultaneous measurement of different observables in the same quantum system is not possible in practice). Nor do the uncertainty relations indicate the absence of possessed values for some physical quantities. The mathematics does not necessitate these claims. (It can be argued that these are perhaps demanded by other considerations but this is, of course, a different issue.) Further discussion of the nature of the uncertainty relations will appear in Chapter Two, in the context of the Causal Theory.

1.3 The Measurement Problem and Quantum Paradoxes

The concept and meaning of 'measurement' has always held central stage in discussions of the foundations of quantum mechanics. This is certainly the case in Orthodox Quantum Theory which provides little more than predictions of the results that can be obtained if one *were to measure* some physical parameter of a quantum system. Measurement holds a privileged place in Orthodox Quantum Theory as it appears in the theory's axioms. This special status is afforded to measurement on the basis that its effects cannot be made arbitrarily small and that a system's evolution on measurement is different from its non-measurement evolution (as noted in Section 1.1).

Much has been written about the Measurement Problem in quantum mechanics and the various quantum paradoxes. Indeed, whole books and dissertations have been devoted to these topics. It is not the intention here to try to do justice to these extensive discussions. Since there are many detailed descriptions of these in the literature, a level of familiarity will be assumed and only a brief outline is provided.

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[29] Popper, K.R., *The Logic of Scientific Discovery* (Hutchinson, London, 1975) p.216.

The Measurement Problem

What exactly is the 'Measurement Problem'? The problem (according to some accounts[30]) arises by simultaneously holding firm to the following propositions: (i) the state vector ψ is a complete description of any system's physical state; (ii) the state vector evolves deterministically in time according to: $\psi = \mathbf{U}\psi_o$, where $\mathbf{U}$ is a linear unitary operator; (iii) to each observable quantity there corresponds a linear operator $\mathbf{A}$ with at least one nonzero eigenvector; (iv) the quantity measured will always be an eigenvalue of $\mathbf{A}$; (v) the probability that a measurement will yield a specific eigenvalue α is: $|(\psi, \psi_j)/(\psi, \psi)|^2$, where ψ_j are all eigenvectors of $\mathbf{A}$ with eigenvalue α. These propositions are merely restatements of the relevant axioms of Orthodox Quantum Theory. Although the assertion that the Measurement Problem is due to holding the above propositions simultaneously is correct, this presentation of the problem does not illuminate its essential features.

In order to bring out these features, we shall need to consider a particular case. The spin state of a quantum system is an example typically used for this purpose.[31] The quantity known as the intrinsic angular momentum (or spin) of a quantum system along a particular reference direction can be 'up' or 'down' with respect to a given coordinate system and is described by a state vector. Let the 'spin-up' state vector be ψ_1 and the 'spin-down' state vector be ψ_2. The apparatus with which a measurement is to be made has an initial state described by the state vector ϕ_o. The combination of quantum system and apparatus would then have an initial state described by either the state vector $(\psi_1\phi_o)$ or $(\psi_2\phi_o)$. The coupling of the quantum system with the measuring apparatus allows for a correlation between the

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30 For example, Stone, A.D., 'Does the Bohm Theory Solve the Measurement Problem?', *Philosophy of Science* **61** (1994) p.250.

state of the system and the state of the apparatus, i.e. this coupling allows the apparatus to give a 'readout' (a result of measurement).

Let ϕ_1, ϕ_2 be the state vectors of the apparatus that correspond to a readout of 'spin-up' or 'spin-down' respectively. The initial state of the coupled quantum system plus apparatus will evolve linearly into a state described by the state vector (a $\psi_1\phi_1$) or by (b$\psi_2\phi_2$), where a and b are numbers. If the quantum system is not initially prepared in a particular spin state, i.e. if not in an eigenstate of either 'spin-up' or 'spin-down', then the quantum system will be in the superposition given by: $\psi = a\psi_1 + b\psi_2$, where $|a|^2 + |b|^2 = 1$. In this case, the initial state of the quantum system plus apparatus will evolve linearly from a state described by the state vector: $(a\psi_1 + b\psi_2)\phi_o$ into a state described by the state vector: $(a\psi_1\phi_1 + b\psi_2\phi_2)$, which is also a superposition. According to Orthodox Quantum Theory, the apparatus (as a part of the coupled combination) will be in a superposition after measurement, i.e. the result of the measurement should be something 'smeared out'. The Measurement Problem resides in accepting that the measurement apparatus is governed by the laws of quantum mechanics and that superpositions of macroscopic objects do not occur, i.e. after an actual measurement the apparatus is only in a state that corresponds to a readout of a single result, e.g. 'spin-up' or 'spin-down'.

The Measurement Problem constitutes an (external) conceptual problem for Orthodox Quantum Theory since a 'smeared out' measurement result conflicts with the prevailing belief that superpositions of macroscopic objects (such as instrument pointers) do not occur. Orthodox Quantum Theory does not solve the Measurement Problem but merely circumvents it by applying the Projection Postulate, i.e. a measurement instantaneously 'projects' the superposition ψ into eigenstate ψ_1 with

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[31] Rae, A.I.M., Quantum Mechanics, *op. cit.*, pp.238-240; Cushing, J.T., *Philosophical Concepts in*

probability $|(\psi, \psi_1)|^2 = |a|^2$ or eigenstate ψ_2 with probability $|(\psi, \psi_2)|^2 = |b|^2$, since $(\psi, \psi) = 1$. In Chapter Two, it will be shown that the Measurement Problem does not occur within the context of the Causal Theory.

The paradoxes of Orthodox Quantum Theory are indicative of problems in the foundations of the theory. The two best known quantum paradoxes, Schrödinger's Cat and the EPR Paradox, are outlined below.

Einstein-Podosky-Rosen (EPR) Paradox

EPR is not a paradox in the formal sense, i.e. it does not contain an explicit logical contradiction. Instead, it is a thought experiment originally designed to show that Orthodox Quantum Theory is incomplete.[32] EPR's necessary condition for completeness is that every element of physical reality must have a counterpart in physical theory.[33]

According to EPR, elements of physical reality are discovered by experiment. In quantum mechanics, if the state vector ψ of a system is an eigenstate of an operator **A** (which corresponds to a physically measurable quantity A) then $\mathbf{A}\psi = \alpha\psi$, i.e. the physical quantity A will have value α with certainty. Then for the state described by the state vector ψ, EPR claim that there is an element of reality which corresponds to the physical quantity A. They wrote:

> *If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value*

Physics (Cambridge University Press, Cambridge, 1998) pp.309-311.

32 Pais, A., *'Subtle is the Lord ...': The Science and the Life of Albert Einstein* (Oxford University Press, Oxford, 1984) p.456.

33 Einstein, A., Podosky, B. and Rosen, N., 'Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?', *Physical Review* **47** (1935) p.777.

> *of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.*[34]

The EPR argument is based on an assumption of realism (i.e. the existence of objective 'elements of reality') and a Principle of Locality. The Principle of Locality used in the EPR paper is sometimes referred to as "Einstein Locality",[35] and may be stated as follows:

♦ *Principle of Locality*

Elements of reality pertaining to one system cannot be affected by measurements performed at a space-like distance on another system, even if the systems previously interacted.

The argument proceeds by considering two quantum systems (denoted I and II) each consisting of one particle whose state vector is known. The systems interact for a short period, then separate. (Once interaction has occurred, these systems are said to be 'entangled'.) Orthodox Quantum Theory allows that state of the combined system (I + II) to be calculated at any subsequent time after interaction. When the systems have separated, a measurement on one system cannot affect the other (by the Principle of Locality). If we measure a physical quantity of system I, say the particle's momentum, quantum mechanics allows an inference to be made on the value of the momentum of the other particle (system II) which, by EPR's assumption of realism, constitutes an 'elements of reality'. However, the measurement of system I could just as easily been made on its particle's position from which the position of the other particle would likewise constitute an 'element of reality'.

This leads to a conflict with the Completeness axiom of Orthodox Quantum Theory as the position and momentum of a particle are quantities represented by non-

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[34] *ibid*. (their italics).
[35] Redhead, M., Incompleteness, Nonlocality, and Realism, *op. cit*., p.61.

commuting operators and therefore (as conventionally interpreted) cannot simultaneously both have values predicted with certainty. On this basis, position and momentum cannot simultaneously be 'elements of reality'.[36]

Given the assumption of realism and the Principle of Locality, the conclusion of the EPR argument is that the completeness assumption of Orthodox Quantum Theory is false.

Schrödinger's Cat Paradox

"Schrödinger's Cat" is the most famous of the quantum paradoxes[37] and is a graphic example of the Measurement Problem in Orthodox Quantum Theory. Schrödinger apparently proposed the Cat Paradox after corresponding with Einstein over the EPR paper.[38] Schrödinger's aim was also to show that Orthodox Quantum Theory is incomplete. Here the cat is in a sealed box and its life or death depends on a (random) quantum event happening, such as a radioactive decay. If the event occurs, the cat dies. If it does not occur, the cat lives. We do not know the result until we make a measurement (e.g. look into the box). If we assume that the cat's state is described by a state vector, then prior to the observation Orthodox Quantum Theory dictates that the cat's state vector is in a superposition corresponding to the cat being both alive and dead! Although a microscopic quantum entity being described by a superposition may be palatable in the view of most physicists, such a result is unacceptable for macroscopic objects (such as cats).[39]

Further, *if* the state vector gives a complete description of the state of the cat, then the observation (i.e. measurement) will project the cat's (superposition) state vector into either the eigenstate where the cat is alive or the eigenstate where it is

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36 Einstein, A., et al., 'Can Quantum-Mechanical Description ...', *loc. cit.*, p.780.
37 Schrödinger, E., 'Die Gegenwärtige Situation in der Quantenmechanik', *Die Naturwissenschaften* **23** (1935): 807-812, 824-828, 844-849.
38 Cushing, J.T., Philosophical Concepts in Physics, *op. cit.*, p.311.

dead. Something appears wrong here for a cat will either be alive or dead and it seems highly implausible that a simple act of observation could alter the state of the cat.

Solutions of these two paradoxes within the context of the Causal Theory will be presented in Chapter Two.

1.4 'Hidden Variable' Theories and Impossibility Proofs

Quantum mechanical 'hidden variables' were originally proposed to be those variables that determine the values of measurable quantities but which are not themselves accessible to empirical investigation.[40] A (so-called) 'hidden-variable' quantum theory (also known as a 'hidden-variable' extension to quantum theory) is a recasting of quantum theory into some classical (or classical-like) form. There have been various proofs of the impossibility of such 'hidden-variable' theories (the 'no-go' theorems) advanced in the literature since the 1930s. The first of these theorems was derived by John von Neumann in 1932.[41] This theorem was later strongly endorsed by Neils Bohr himself.[42] It took several decades before von Neumann's Theorem was shown to have premises that are too wide.[43] In any case, the existence of a consistent counter-example to von Neumann's Theorem (i.e. Bohm's Causal Theory) indicated that its conclusion cannot hold for all 'hidden-variable' theories. However, this consistent counter-example seemed to make little difference to most

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39 Penrose, R., The Road to Reality, *op. cit.*, pp.804-805.

40 Hughes, R.I.G., *The Structure and Interpretation of Quantum Mechanics* (Harvard University Press, Cambridge, MA., 1989) p.172.

41 von Neumann, J., *Mathematische Grundlagen der Quantenmechanik* (Springer, Berlin, 1932). English translation by R.T. Beyer, *Mathematical Foundations of Quantum Mechanics* (Princeton University Press, Princeton, New Jersey, 1955).

42 Selleri, F., *Quantum Paradoxes and Physical Reality* (Kluwer, Dordrecht, 1990) p.35.

43 Hughes, R.I.G., The Structure and Interpretation ..., *op. cit.*, p.173.

researchers who knew about it as they assumed that there must be something wrong with Bohm's theoretical arguments.[44]

Kochen and Specker Theorem

Of even more significance than von Neumann's Theorem is the theorem of Kochen and Specker. This theorem purports to show that extending quantum mechanics by the addition of 'hidden-variables' is not possible because the algebraic structure of self-adjoint operators on a Hilbert space cannot be 'embedded' (in a sense defined below) into the commutative algebra of real-valued functions on a phase space.[45] Kochen and Specker stated the problem of making a 'classical reinterpretation' of quantum theory (i.e. what they referred to as a 'hidden-variable' extension of quantum mechanics) as follows:

> The proposals … for a classical reinterpretation usually introduce a phase space of hidden pure states in a manner reminiscent of statistical mechanics. The attempt is then shown to succeed in the sense that the quantum mechanical average of an observable is equal to the phase space average. However, this statistical condition does not take into account the algebraic structure of the quantum mechanical observables. A minimum such structure is given by the fact that some observables are functions of others. This structure … should be preserved in a classical reinterpretation … this is not provided for by the above statistical condition …[46]

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44 Cushing, J.T., 'The Causal Quantum Theory Program' in Cushing, J.T., Fine, A. and Goldstein, S. (eds), *Bohmian Mechanics and Quantum Theory: An Appraisal* (Kluwer, Dordrecht, 1996) p.5.
45 Kochen, S. and Specker, E.P., 'The Problem of Hidden Variables in Quantum Mechanics', *Journal of Mathematics and Mechanics* **17** (1967): 59-67. Reprinted in Hooker, C.A. (ed.), *The Logico-Algebraic Approach to Quantum Mechanics* (Reidel, Dordrecht, 1975).
46 Kochen, S. and Specker, E.P. in Hooker, C.A. (ed.), The Logico-Algebraic …, *ibid.*, p.293.

Before proceeding any further, let's define a 'hidden-variable' extension of quantum mechanics (as accepted by Kochen and Specker). This will also provide a more precise meaning for their term 'embedding':[47]

(1) Each individual quantum system is specified by a (statistical) state function ψ and additional 'hidden' states denoted by the parameter λ. The totality of hidden states is the phase space Γ. The result of measuring any observable of the quantum system (i.e. the value of the observable) is determined by both ψ and λ.

(2) Each state function ψ is associated with a probability measure $\rho_\psi(\Lambda)$ on Γ. The measure $\rho_\psi(\Lambda)$ is the probability that the state (i.e. the phase point of the system) lies in Λ, where Λ is a measurable subset of Γ.

(3) An observable A (represented by a self-adjoint operator **A** on a Hilbert space) is interpreted as denoting an attribute of a physical object.[48] Each observable A is associated with a single-valued, real-valued function $f_\mathbf{A} : \Gamma \rightarrow \Re$, i.e. $f_\mathbf{A}$ maps Γ into the set of reals $\Re$.

(4) Let $\mathcal{M}$ be a measurable subset of $\Re$ and let $\mu_\psi{}^A$ be the quantum mechanical probability measure such that $\mu_\psi{}^A(\mathcal{M})$ is the probability that the value of A lies in $\mathcal{M}$. Then, the measure of the set of phase space points in Γ that are mapped by $f_\mathbf{A}$ onto the set $\mathcal{M}$ is equal to the measure of the set $\mathcal{M}$ specified by quantum theory, i.e.

$$\mu_\psi{}^A(\mathcal{M}) = \rho_\psi[f_\mathbf{A}{}^{-1}(\mathcal{M})].$$

This sense of 'embedding' means that the statistical (quantum) theory is expressible in terms of a more fundamental one whose states are *not statistically* related to the

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47 Jammer, M., The Philosophy of Quantum Mechanics, *op. cit.*, p.262.

48 Bub, J., 'What is a Hidden Variable Theory of Quantum Phenomena?', *International Journal of Theoretical Physics* **2** (1969) p.102.

physical parameters.[49] This is, of course, not the only possible general definition of a hidden-variable extension of quantum theory. (Jammer labels this as *Definition II* of a hidden-variable theory.[50])

A 'hidden-variable' theory is constituted by the parameters (or hidden variables) λ, the space Γ, the set of measures $\{\rho\}$ and the set of functions $\{f_i\}$which satisfy the above four constraints. If a measurement system is in a state given by ψ and λ, precise predictions could be made about the result of any measurement if the values of λ were known. Alternatively, if the probability distribution of the parameters λ are known, then the obtainable statistical results will be in accord with those of quantum mechanics.[51] Also, such a theory is *non-contextual* in that the result is not dependent on the context of the performed measurement, i.e. f_A does not depend on whether any other observables of the system are measured simultaneously.[52]

Whether such an 'embedding' is possible depends on the algebraic structure of the statistical theory involved (in this case, quantum mechanics). The minimum algebraic structure of the quantum mechanical observables is taken into account by 'embedding' the partial algebra of *comeasurable* observables into the commutative algebra of real-valued functions on a phase space. This is known as the Kochen and Specker Condition.[53] Kochen and Specker defined the term 'comeasurable' to mean that for a set of observables A_i, $i \in I$, (represented by operators $\mathbf{A_i}$) there exists another observable B (represented by the operator **B**) and functions f_i such that $\mathbf{A_i}$ =

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49 *ibid.*, pp.102-103.
50 Jammer, M., The Philosophy of Quantum Mechanics, *op. cit.*, p.262.
51 *ibid.*
52 *ibid.*, p.263.
53 *ibid.*, p.323.

$f_i(\mathbf{B})$. Redhead calls this the Functional Composition Principle.[54] The value of any of the observables A_i can be ascertained simply by measuring the value of B and applying the function f_i. These form a partial algebra if the following conditions apply:[55]

If $\mathbf{A_1} = f_1(\mathbf{B})$, $\mathbf{A_2} = f_2(\mathbf{B})$ and r_1, r_2 are real numbers, then $\mathbf{A_1 A_2} = f_1 f_2(\mathbf{B})$

$$\text{and } r_1\mathbf{A_1} + r_2\mathbf{A_2} = (r_1 f_1 + r_2 f_2)(\mathbf{B})$$

The Kochen and Specker Condition imposes some restrictions on the functions f_i. Suppose that $f_\mathbf{A}$ equals some eigenvalue of the operator $\mathbf{A}$ (corresponding to an observable A), e.g. $f_\mathbf{A} = \alpha_n$ where $\mathbf{A}\psi = \alpha_n\psi$, with ψ being an eigenvector of a system in an eigenstate of $\mathbf{A}$. Further suppose that the operator $\mathbf{B}$ (corresponding to observable B) which commutes with $\mathbf{A}$ is given by: $\mathbf{B} = g(\mathbf{A})$, where g is a real-valued function. Since physical parameters (as represented by Hermitian operators) are supposed to denote the physical attributes of objects, it is assumed that the value of $g(\mathbf{A})$ is equal to $g(\alpha_n)$. Then, in the state ψ, the measured value of observable B is $g(\alpha_n)$ where $g(\alpha_n) = g(f_\mathbf{A}) = f_\mathbf{B}(\lambda)$. Alternatively, we could write that if $f_\mathbf{A}(\lambda) = \alpha_n$ then $f_{g(\mathbf{A})}(\lambda) = g(\alpha_n)$, or $f_{g(\mathbf{A})} = g(f_\mathbf{A})$.

The functions $f: \Gamma \rightarrow \mathfrak{R}$ preserve the partial algebra P of 'comeasurable' operators.[56] If Σ is the set of all real-valued functions on Γ which constitute a commutative algebra, then a 'hidden-variable' extension exists only if P can be 'embedded' into Σ. In other words, the Kochen and Specker condition requires the existence of a homomorphism (i.e. a structure preserving mapping) h_λ for each $\lambda \in \Gamma$

54 Redhead, M., Incompleteness, Nonlocality, and Realism, *op. cit.*, p.121.
55 Kochen, S. and Specker, E.P. in Hooker, C.A. (ed.), The Logico-Algebraic ..., *op. cit.*, pp.299-300.
56 Bub, J., 'What is a Hidden Variable Theory of Quantum Phenomena?', *loc. cit.*, p.104.

defined as: h_λ (**A**) = $f_{\mathbf{A}}(\lambda)$, which maps P into $\Re$.[57] Such a homomorphism associates a value with every physical parameter simultaneously.[58] Another way of putting this is that the algebraic structure of 'comeasurable' self-adjoint operators should be reflected in the possessed values of the observables.[59] The Kochen-Specker Theorem proves that no homomorphism h_λ exists if the Hilbert space has more than two dimensions.[60] Their proof was unnecessarily complicated involving a set of *117 observables* which were associated with the components of the square of an angular momentum operator.[61]

Kochen and Specker did provide, however, an example to illustrate the correctness of their theorem. The example concerned exciting Orthohelium (i.e. causing an energy perturbation by applying an electric field) and then measuring the emitted photon energy which corresponds to the change in energy levels. They suggested that all the components of the square of the spin angular momentum of Orthohelium could be inferred from such measurements. Each of the components commutes with the others and so meets the criterion of being comeasurable. The energy perturbation can be achieved by applying an electric field of rhombic symmetry to the atom.[62] Given that the components are simultaneously measurable, it should be possible in a 'hidden-variable' extension to define a function which assigns values which would be obtained on measurement of the energy perturbation. In terms of their assumptions, Kochen and Specker showed by means of a geometrical argument that this assignment function cannot be defined.[63]

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57 Jammer, M., The Philosophy of Quantum Mechanics, *op. cit*, p.323.
58 Bub, J., 'What is a Hidden Variable Theory ...?', *loc. cit.*, p.104.
59 Redhead, M., Incompleteness, Nonlocality, and Realism, *op. cit.*, p.121.
60 *ibid.*
61 *ibid.*, pp.121-130; Ballentine, L.E., Quantum Mechanics: A Modern Development, *op. cit.*, p.607.
62 Kochen, S. and Specker, E.P. in Hooker, C.A. (ed.), The Logico-Algebraic ..., *op. cit.*, pp.308-312.
63 Jammer, M., The Philosophy of Quantum Mechanics, *op. cit*, p.324.

What is of particular importance in the assumptions of Kochen and Specker (and in keeping with the definition of a non-contextual 'hidden-variable' extension given above) is that the manner in which the measurement is done has no effect on the result of the measurement.[64] Therefore, the Kochen-Specker Theorem does show that *non-contextual* 'hidden-variable' extensions to quantum theory (with a Hilbert space of more than two dimensions) cannot exist. It does not prove the impossibility of all 'hidden-variable' extensions since the theorem is not applicable to contextual 'hidden-variable' theories.

Bell's Theorem

The EPR Paradox was aimed at showing that Orthodox Quantum Theory is incomplete given that the Principle of Locality holds. It was, however, nearly another thirty years before the late John S. Bell showed that predictions based on the assumption of the Principle of Locality are inconsistent with some predictions of quantum mechanics.[65] He did this by deriving an inequality which quantum mechanics violates. There are several versions of this inequality which are collectively referred to as Bell's Inequalities.[66] The version reproduced below follows Bell's 1971 argument.[67]

Suppose we have two particles and two measuring apparatuses that each can register the value of a particular variable associated with each of the particles. Further suppose that this particular variable has only two possible values which can be chosen to be ± 1 in some appropriate units. The measuring apparatuses, however,

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64 Belinfante, F.J., *A Survey of Hidden-Variable Theories* (Pergamon, Oxford, 1973) p.43.

65 Bell, J.S., 'On the Einstein-Podolsky-Rosen Paradox', *Physics* **1** (1964): 195-200. Reprinted in his *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press, Cambridge, 1987).

66 Clauser, J. F. and Shimony, A., 'Bell's theorem: Experimental tests and implications', *Reports on Progress in Physics* **41** (1978) p.1889.

67 Bell, J.S., 'Introduction to the Hidden-Variable Question' in *Proceedings of the International School of Physics 'Enrico Fermi', Course IL: Foundations of Quantum Mechanics* (Academic, New York,

have a range of possible settings, i.e. possible ways in which an individual apparatus may be configured. Let 'a' denote the settings for the first apparatus and 'b' for the second. The outcome of any measurements may depend on these settings together with unknown (and uncontrollable) variables which are associated with the particles and/or the apparatuses. These will be collectively denoted by λ. Then suppose there is a continuous function which determines the outcome of measurements on the first particle. This function (which shall be denoted A) depends on the variables λ and a, i.e. $A = A(a, \lambda)$. Likewise, we can suppose that there is a continuous function (denoted B) which determines the outcome of measurements on the second particle and depends on the variables λ and b, i.e. $B = B(b, \lambda)$.

We shall assume that A and B provide average values so that $|A(a,\lambda)| \leq 1$ and $|B(a,\lambda)| \leq 1$. If the Principle of Locality holds and the two measuring apparatuses are spatially well separated, then A cannot depend on the variable b and B cannot depend on the variable a. However, both A and B will depend on the unknown variables λ. We also shall assume that these variables have a distribution described by a probability function $\rho(\lambda)$, where

$$\rho(\lambda) \geq 0 \quad \text{and} \quad \int \rho \, d\lambda = 1 \quad \ldots. \ (1\text{-}7)$$

This distribution does not depend on the type of measurements made on the particles. Now let E(a,b) be the expectation value of the quantity AB which is defined by:

$$E(a,b) = \int A(a,\lambda) \, B(a,\lambda) \, \rho(\lambda) \, d\lambda \quad \ldots. \ (1\text{-}8)$$

where an integral rather than a simple summation is used as A and B are continuous functions. E(a,b) is a function that gives a measure of the correlation between the variables that are measured. Let a, a' be two different settings for the first apparatus.

1971). Reprinted in Bell, J.S., *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press, Cambridge, 1987).

Likewise, let b, b′ be two different settings for the second apparatus. Then we can form the difference between two correlation functions as follows:

$$E(a,b) - E(a,b') = \int [A(a,\lambda)\, B(b,\lambda) - A(a,\lambda)\, B(b',\lambda)]\, \rho(\lambda)\, d\lambda$$

This may be expressed in a more useful form by a little reorganisation to yield:

$$E(a,b) - E(a,b') = \int \{A(a,\lambda)\, B(b,\lambda)\, [1 \pm A(a',\lambda)\, B(b',\lambda)]\}\, \rho(\lambda)\, d\lambda$$

$$- \int \{A(a,\lambda)\, B(b',\lambda)\, [1 \pm A(a',\lambda)\, B(b,\lambda)]\}\, \rho(\lambda)\, d\lambda$$

$$\Rightarrow \ |E(a,b) - E(a,b')| \le \int \{[1 \pm A(a',\lambda)\, B(b',\lambda)] + [1 \pm A(a',\lambda)\, B(b,\lambda)]\}\, \rho(\lambda)\, d\lambda$$

Now

$$[1 \pm A(a',\lambda)\, B(b',\lambda)] + [1 \pm A(a',\lambda)\, B(b,\lambda)] = [2 \pm \{A(a',\lambda)\, B(b',\lambda) + A(a',\lambda)\, B(b,\lambda)]$$

Using the properties of the probability function ρ (relations (1-7)) and the definition of E (equation (1-8)), we find:

$$\int \{[1 \pm A(a',\lambda)\, B(b',\lambda)] + [1 \pm A(a',\lambda)\, B(b,\lambda)]\}\rho(\lambda)\, d\lambda = 2 \pm \{E(a',b') + E(a',b)\}$$

Thus,

$$|E(a,b) - E(a,b')| + |E(a',b') + E(a',b)| \le 2 \quad \ldots. (1\text{-}9)$$

This is Bell's Inequality.[68] No use has been made of the formalism of quantum mechanics in deriving this inequality. Quantum mechanics, however, indicates that there is greater correlation between the particles than would be expected on the basis of assuming that the Principle of Locality holds.[69] Some results of quantum mechanics readily show that the quantity $|E(a,b) - E(a,b')| + |E(a',b') + E(a',b)| >$

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68 Bell, J.S., 'Introduction to the Hidden-Variable Question' in Speakable and Unspeakable in Quantum Mechanics, *ibid.*, pp.36-37.
69 Sudbery, A., Quantum Mechanics, *op. cit.*, p.200.

2.[70] The theorem that inequality (1-9) conflicts with the predictions of quantum mechanics is called Bell's Theorem.[71]

Since Bell's original derivation (published in 1964), there have been assumptions other than the Principle of Locality identified in his theorem. This has been an significant advance as it has allowed further proofs of Bell's Theorem to be produced which do not depend on these assumptions. Of particular importance in this regard has been to eliminate dependence on the assumptions of: determinism; probability factorisation; counterfactual definiteness; and the presence of 'hidden variables'.[72] These issues are well covered in the literature and so will not be discussed here. The over-riding result of these further proofs has been to show that the Bell's Inequalities crucially depend on assuming the Principle of Locality.

We shall see in Chapter Two that the Causal Theory is not refuted by experiments that show Bell's Inequalities are violated.

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70 Clauser, J. F. and Shimony, A., 'Bell's theorem ...', *loc. cit.*, pp.1893-1894.
71 Ballentine, L.E., Quantum Mechanics: A Modern Development, *op. cit.*, p.590.
72 *ibid.*, p.608.

CHAPTER 2

EXPOSITION OF THE CAUSAL THEORY OF QUANTUM MECHANICS

This idea [the deBroglie-Bohm Causal Theory] seems to me so natural and simple … that it is great mystery to me that it was so generally ignored.

— J.S. Bell [1]

2.1 Motivations for the Causal Theory

In the Preface of this thesis, it was stated that speculations and ill-informed commentary regarding what quantum mechanics asserts about nature have led to the abandonment of some previously held physical concepts and principles. This has arisen as part of Orthodox Quantum Theory's interpretive framework. Orthodox Quantum Theory is a scientific theory that does not address fundamental questions – its axioms are principally 'geared' to provide predictions for results that could be obtained if a measurement were performed on a quantum system. Established physical concepts and principles (such as the Principle of Causality – see Section 2.3) should not be given away until it has been clearly demonstrated that the relevant concepts or principles no longer apply, or are false. In order to account for quantum phenomena and avoid the problems in Orthodox Quantum Theory, alternative interpretations of the quantum formalism have either postulated bizarre entities (including infinitely many parallel universes, wavicles, smearons), or resorted to rather odd and problematic mechanisms (such as spontaneous wavefunction collapse), or have invoked non-standard logic. Such moves are not particularly justifiable based on the formalism and experiment.

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[1] Bell, J.S., 'Six Possible Worlds of Quantum Mechanics' in his *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press, Cambridge, 1987) p.191.

Would it not be preferable to have a realist theory of quantum phenomena that not only gives correct predictions but also solves the outstanding problems and paradoxes without the undesirable aspects present in Orthodox Quantum Theory and the alternative interpretations of quantum mechanics? It is accepted by most scientists and philosophers of science that *one* aim of science is to provide explanations of physical events. The way to achieve this in the context of the quantum realm is to specify both the ontology and the laws that govern the realm in addition to those rules by which we predict the outcome of experiments.

The hypothesis that quantum particles are directed by some sort of guiding field (as postulated in the Causal Theory) offers perhaps the best current possibility of providing an empirically adequate, realist theory. The axioms of the Causal Theory (see Section 2.2. below) provide, *inter alia*, a basis for realistic explanations of quantum phenomena. A main attraction of the Causal Theory is its ontology – quantum entities (i.e. particles and fields) are prescribed to have a real existence in spacetime. Louis de Broglie is generally credited with formulating the first causal interpretation of quantum mechanics in the 1920s.[2] A consistent causal theory was postulated independently of de Broglie's ideas by David Bohm in the 1950s which also answered the principal objections levelled at de Broglie's original interpretation.[3] A further motivation for attempting to comprehend the fundamentals of non-relativistic quantum mechanics in realist terms is that without such comprehension, it is unlikely that we will ever arrive at a coherent understanding of relativistic quantum field theory and of quantum gravity.

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[2] de Broglie, L., 'Ondes et quanta', *Comptes Rendus des Seances l'Academie de Sciences* (Paris) **177** (1923): 507-10.
[3] Bohm, D., 'A Suggested Interpretation of the Quantum Theory in Terms of "Hidden" Variables' I & II, *Physical Review* **85** (1952): 166-79, 180-93.

This chapter is devoted to providing a formal exposition of the Causal Theory of Quantum Mechanics. However, before embarking on the technical details, there is an important point to be noted. The historical selection of Orthodox Quantum Theory over the Causal Theory was *not* dictated by empirical results. John Bell sought, in several published papers, to dispel the idea that Orthodox Quantum Theory had been chosen because it was in better agreement with experiment. He advocated the pilot wave picture (i.e. the Causal Theory) as a legitimate alternative to Orthodox Quantum Theory:

> Why is the pilot wave picture [Causal Theory] ignored in the text books? Should it not be taught, not as the only way, but as an antidote to prevailing complacency? To show that vagueness, subjectivity, and indeterminism, are not forced on us by experimental facts, but by theoretical choice? [4]

2.2 An Axiomatic Foundation

In this section, a set of axioms will be stated which will serve as a foundation of the Causal Theory of Quantum Mechanics. We begin with some initial remarks about the axiomization of physical theories. Any axiomatic treatment is necessarily limited and cannot present all the concepts and technicalities required for a mathematical theory. Max Jammer presented a rather barren view of axiomizations in his well-known exposition on the different interpretations of quantum mechanics. He writes:

> … an axiomization cannot dispense with undefined primitive concepts and relations whose concrete meaning can be conveyed only in terms of the language of ordinary experience. Since an axiomization of quantum mechanics is intended to clarify the latter it is not only sterile, as

[4] Bell, J.S., 'On the Impossible Pilot Wave', *Foundations of Physics* **12** (1982): 989-999. Reprinted in his *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press, Cambridge, 1987) p.160.

> axiomizations usually are, but also necessarily circular; it can, at best, serve as a test for the consistency of reasoning …[5]

Although Jammer's point about undefined concepts and relations (be they primitives or not) cannot be disregarded, it is also the case that axiomizations that attempt to cover all relevant concepts become impossibly long and/or hopelessly complicated. There always remains much that is assumed in any axiomization of a physical theory,[6] such as the rules of the propositional logic, the basis of geometry, the operations of vector algebra, the foundations of the differential calculus, and so forth. Despite obvious practical limitations, an axiomization of a physical theory has distinct advantages. In particular, the following possible benefits may be gained:

- assistance in verifying the theory's consistency;
- minimisation of the semantic content needed to present the theory;
- revealing gaps in any previous renderings of the theory;[7] and
- assistance in showing any mutual dependence of various parts of a physical system.[8]

It also will be beneficial to note relevant comments about axiomizations due the mathematical physicist J.L. Synge:

> Physical concepts, being by their nature vague, cannot be treated with logical rigour. … it would seem right that any systematic treatment … should start with axioms, carefully laid down, on which the whole structure would rest as a house rests on its foundations.
>
> The analogy to a house is, however, a false one. Theories are created in mid-air, so to speak, and develop upward and downward. Neither process is ever completed. …

[5] Jammer, M., *The Philosophy of Quantum Mechanics* (Wiley, New York, 1974) p.472.
[6] Bunge, M, *Philosophy of Physics* (Reidel, Dordrecht, 1973) p.9.
[7] van Fraassen, B.C., *Quantum Mechanics: An Empiricist View* (Oxford, Clarendon, 1991) p.5.
[8] Popper, K.R., *The Logic of Scientific Discovery* (Hutchinson, London, 1975) p.72.

> To a physicist ... there is an element of artificiality in the creation of a complete axiomatic base, for he knows that the axioms will be chosen to fit the theory ... .[9]

We will have cause to refer again to these comments by Synge later in this chapter.

Throughout the rest of this thesis, the configuration space representation in the Schrödinger picture of quantum mechanics where it is appropriate to describe a quantum system by its wavefunction Ψ instead of the state vector ψ. The wavefunction Ψ is given by the scalar product: (x,ψ) where x is the position observable. The axiomization presented below will make explicit reference to the wavefunction of a quantum system. Similar axiomizations appear in the literature.[10]

The axioms of the single particle Causal Theory of Quantum Mechanics are:

I

PARTICLE

A particle is a point-like object localised in (three-dimensional) Galilean space with an inertial mass.

II

WAVE FIELD

A wave field is a physical process that propagates in (three-dimensional) Galilean space over time. A wave field is described by its wavefunction Ψ which is a continuous, bounded function of the spacetime coordinates.

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[9] Synge, J.L., 'Classical Dynamics' in Flügge, S. (ed.), *Encyclopedia of Physics* Vol. 2 (Springer, Berlin, 1960) p.5.

[10] Examples include: Bohm, D., 'A Suggested Interpretation ... I', *loc. cit.*, pp.169-171; Freistadt, H., 'The Causal Formulation of the Quantum Mechanics of Particles', *Supplemento Nuovo Cimento* V (1957) pp.9-13; Holland, P.R., *The Quantum Theory of Motion: An Account of the deBroglie-Bohm Causal Interpretation of Quantum Mechanics* (Cambridge University Press, Cambridge, 1993) pp.66-68.

III

QUANTUM SYSTEM

A single particle quantum system consists of a particle and an accompanying wave field, i.e. the set {Ψ, x} where x is the particle's position.

IV

LAGRANGIAN DENSITY

A single particle quantum system has a Lagrangian density $\mathcal{L}$ which is expressed in terms of the wavefunction Ψ:

$$\mathcal{L} = \tfrac{1}{2}\, i\hbar(\Psi^* \dot{\Psi} - \dot{\Psi}^* \Psi) - (\hbar^2/2m)(\nabla\Psi^*)\cdot(\nabla\Psi) - V\,\Psi^*\Psi$$

where $i = \sqrt{-1}$, Ψ^* is the complex conjugate of Ψ, $\dot{\Psi}$ is the partial derivative of Ψ with respect to time, V is an external (classical) potential, $\hbar$ is Planck's Constant divided by 2π, ∇ is the standard three-dimensional differential operator, and m is the particle's inertial mass.

V

GUIDANCE CONDITION

A quantum particle is guided by its wave field in accordance with the condition:

$$\frac{dx}{dt} = \left(\frac{\hbar}{2im}\right) \nabla \left[\log\left(\frac{\Psi}{\Psi^*}\right)\right]$$

where 'log' denotes the natural (Naperian) logarithm, and other terms are as defined above.

VI

QUANTUM EQUILIBRIUM CONDITION

The probability density ρ (x) of possible values of the initial particle position in an ensemble of similarly prepared quantum systems satisfies the condition:

$\rho = |\Psi|^2$.

2.3 One Particle States

Axiom I states that a quantum particle is 'point-like'. (The term 'quantum particle' will be used to make an explicit distinction from a classical particle.) This characterisation of being 'point-like' is, of course, an abstraction for it is assumed that the particle's size is negligible and that it can be treated as if all its mass were concentrated at a single point. However, 'point-like' particles are a feature of many mathematical models and as such, simply having a point-like particle is not, in itself, a valid criticism of the model.

Axiom II states that there exists a physically real quantum field. This field is commonly called the wave field for historical reasons. Other names include: de Broglie wave, pilot wave, Schrödinger wave field, and matter wave. We shall follow the usage of 'wave field'. The wave field is described mathematically by its wavefunction which is defined on a configuration space and is usually a single-valued, square-integrable, complex function. In the case of a single particle, the quantum system's configuration space coincides with ordinary three-dimensional space. The wave field is not a theoretical fiction, it is postulated to have an *objective* existence in space and time. Neither is the wave field an expression of our knowledge (or lack of knowledge) of the particle's state nor merely a mathematical device for calculating the results of experiments, as in some interpretations of quantum mechanics.

The manifestation of both wave-like and particle-like behaviours in experiments at atomic dimensions offers a partial justification for accepting Axioms I and II since such results taken literally indicate the co-existence of both wave and particle. Axiom III states that the particle and wave field together constitute a quantum system. Indeed, they are physically inseparable aspects of a single quantum entity. However, different traits of these two aspects can be given a limited, individual description. A quantum particle can possess characteristics usually associated with classical particles, i.e. mass, energy, and (in principle) localisability. Given the co-existence of particle and wave field, it is conceivable from the perspective of the Causal Theory, that there may be experimental circumstances in which the Principle of Complementarity fails.

Without any loss of generality, the wavefunction may be expressed in polar form, i.e. as an amplitude multiplied by a phase factor: $\Psi = \mathrm{R}e^{iS/\hbar}$ where $i = \sqrt{-1}$, $\mathrm{R}\ (\geq 0)$ and S are both real-valued functions of the spacetime coordinates. If the wavefunction Ψ is single-valued (as usually assumed), so must be its amplitude R. However, the value of the wave field's phase $(S/\hbar)$ may change by an integral multiple of 2π. Also, as Ψ is bounded (from Axiom II), it must tend to zero with increasing distance from the quantum particle.

Axiom IV gives the system's Lagrangian density in terms of its wavefunction:

$$\mathcal{L} = \tfrac{1}{2}\, i\hbar(\Psi^* \dot{\Psi} - \dot{\Psi}^* \Psi) - (\hbar^2/2m)\,(\nabla\Psi^*)\cdot(\nabla\Psi) - \mathrm{V}\,\Psi^*\Psi$$

Substitution of $\mathrm{R}e^{iS/\hbar}$ for Ψ into this Lagrangian density gives the following expression:

$$\mathcal{L} = -\mathrm{R}^2\left(\frac{\partial S}{\partial t}\right) - \left(\frac{\mathrm{R}^2}{2m}\right)|\nabla S|^2 - \left(\frac{\hbar^2}{2m}\right)|\nabla \mathrm{R}|^2 - \mathrm{R}^2\mathrm{V}$$

We can then apply the Principle of Stationary Action. This Principle has the advantage in that it allows a range of different physical systems to be treated in a uniform manner.[11] The Principle may be stated as follows:[12]

- *Principle of Stationary Action*

  The change in the total Action for each infinitesimal variation of the state of a physical system is zero.

In the current context, the Principle of Stationary Action requires that the variation of the integral of the Lagrangian density is zero, i.e. $\delta \int \mathcal{L}\, d^4\mathbf{x} = 0$. If we first vary the parameter R and let $x_1 = x$, $x_2 = y$, $x_3 = z$, $x_4 = t$, then this is equivalent to applying the Euler-Lagrange equation:[13]

$$\sum_{\mu=1}^{4} \frac{\partial}{\partial x_\mu} \left[\frac{\partial \mathcal{L}}{\partial(\partial R / \partial x_\mu)} \right] - \left(\frac{\partial \mathcal{L}}{\partial R}\right) = 0$$

$$= -\left(\frac{\hbar^2}{m}\right) \nabla^2 R - \left\{ -2R \left(\frac{\partial S}{\partial t}\right) - \left(\frac{R}{m}\right) |\nabla S|^2 - 2RV \right\}$$

Dividing by 2R and rearranging terms gives:

$$-\frac{\partial S}{\partial t} = \frac{(\nabla S)^2}{2m} + V(\mathbf{x},t) - \frac{\hbar^2}{2m}\left(\frac{\nabla^2 R}{R}\right) \quad \text{.... (2-1)}$$

where $(\nabla S)^2 = (\nabla S) \cdot (\nabla S) = |\nabla S|^2$ is introduced to conform with the notation in the literature. Equation (2-1) is sometimes referred to as a modified Hamilton-Jacobi equation as it differs from its classical counterpart by the presence of the last term. A more appropriate name is the Quantum Hamilton-Jacobi Equation.[14]

Axiom V tells us that a quantum particle is guided by its wave field in accordance with the condition:

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[11] Doughty, N.A., *Lagrangian Interaction: An Introduction to Relativistic Symmetry in Electrodynamics and Gravitation* (Addison-Wesley, Sydney, 1990) p.5.

[12] Weyl, H., *Space-Time-Matter* (Dover, New York, 1952) pp.210-211.

[13] Sakurai, J.J., *Advanced Quantum Mechanics* (Addison-Wesley, Redwood City, 1982) p.5.

[14] Bohm, D. and Hiley, B.J., *The Undivided Universe: An Ontological Interpretation of Quantum Theory* (Routledge, London and New York, 1993) p.29.

$$\frac{d\mathbf{x}}{dt} = \left(\frac{\hbar}{2im}\right) \nabla \left[\log\left(\frac{\Psi}{\Psi^*}\right)\right]$$

Since $\Psi = R\, e^{iS/\hbar}$, $\log(\Psi/\Psi^*) = \log[(R\, e^{iS/\hbar}) / (R\, e^{-iS/\hbar})] = 2iS/\hbar$. Therefore

$$\frac{d\mathbf{x}}{dt} = \left(\frac{\hbar}{2im}\right) \nabla \left[\log\left(\frac{\Psi}{\Psi^*}\right)\right] = \left(\frac{\nabla S}{m}\right) \quad \ldots. (2\text{-}2)$$

In other words, the guidance condition requires that the momentum $\mathbf{p}$ of a single quantum particle be equal to (∇S). Note that these are continuous (possessed) values of momentum.

The justification for the guidance condition is as follows. First, in classical Hamilton-Jacobi theory, the momentum of a classical particle is given by the same equation, i.e. $\mathbf{p} = \nabla S$, with S being Hamilton's Principal Function. Thus, by direct analogy, it would be reasonable to expect that (∇S) is the particle's momentum in the case of the Quantum Hamilton-Jacobi Equation. Second, by taking $\mathbf{p} = \nabla S$, the Causal Theory provides correct empirical predictions.[15] This is, of course, an essential ingredient if the theory is to be accepted and may be considered sufficient justification in itself. Recall the apt remark of J.L. Synge (quoted in Section 2.2) that the choice of axioms is made to fit the theory! In Orthodox Quantum Theory, it is denied that the quantity (∇S) is the particle's momentum on the basis that this would violate the Uncertainty Principle.[16]

If $\mathbf{p} = \nabla S$, then the term $[(\nabla S)^2/2m]$ is the particle's kinetic energy and equation (2-1) is an energy equation for the quantum particle, where $[-(\partial S/\partial t)]$ is the total energy available to the particle. The last term in equation (2-1) is called the Quantum Mechanical Potential Energy Q (or just quantum potential, for short):

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[15] Bohm, D., 'A Suggested Interpretation ... I', *loc. cit.*, p.170.

[16] Sakurai, J.J., *Modern Quantum Mechanics* (Addison-Wesley, Redwood City, 1985) pp.102-103.

$$Q\,(\mathbf{x},t) = -\frac{\hbar^2}{2m}\left(\frac{\nabla^2 R}{R}\right) \quad .... (2\text{-}3)$$

The presence of the quantum potential accounts for most of the differences between classical and quantum physics. The mathematical form of the quantum potential is independent of the amplitude of the wave field. This can readily be seen by multiplying the amplitude R by some (real) constant b, say:

$$Q = -\frac{\hbar^2}{2m}\left(\frac{\nabla^2 (bR)}{bR}\right) = -\frac{\hbar^2}{2m}\left(\frac{\nabla^2 R}{R}\right)$$

i.e. the value of Q remains unchanged. The effect of the quantum potential is to produce highly non-classical behaviour in quantum systems. The form of the quantum potential will be further considered in Chapter Four.

The Hamilton-Jacobi formalism is the *most* appropriate representation for the Causal Theory. This idea can be traced back to de Broglie's original conception, as Cushing has remarked:

> De Broglie did believe that *one* theory should best conform to nature. He felt that the classical Hamilton-Jacobi formalism provided an embryonic theory of the union of waves and particles, all in a manner consistent with a realist conception of matter.[17]

Also, the Hamilton-Jacobi formalism allows both quantum and classical mechanics to be assessed using the same terminology.[18] Of course, other representations are possible (minimalist positions) but these exclude the quantum potential.[19] There are, however, important advantages in retaining the quantum potential. These advantages will be addressed in Chapters Three and Four.

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[17] Cushing, J.T., 'Why Local Realism?' in van der Merwe, A. and Garuccio, A. (eds), *Waves and Particles in Light and Matter* (Plenum Press, New York and London, 1994) p.224 (his italics).

[18] Holland, P.R., The Quantum Theory of Motion, *op. cit.*, p.78.

[19] See, for example, Dürr, D., Goldstein, S. and Zanghi, N., 'Quantum Equilibrium and the Origin of Absolute Uncertainty', *Journal of Statistical Physics* **67** (1992): 843-907.

The state of a quantum system at a given time is specified by both the wavefunction and the particle's position.[20] Since the form of the wavefunction is influenced by the system's surroundings (i.e. how the shape of the wave field is altered as it propagates), the quantum state has a holistic dependence on its environment.[21] This is called state dependence and is a feature not found in the paradigm of classical physics. This is contrary to the contention of alleged flaw ① in the General Introduction.

In the General Introduction, alleged flaw ② states that the Causal Theory contains 'hidden variables'. In Section 1.4, it was stated that quantum mechanical 'hidden variables' are those variables that determine the values of measurable quantities but are not themselves measurable. A 'hidden-variable' quantum theory contains such variables. The term 'hidden variables' can be given two meanings in this context. The first refers to the set of quantum theories with variables whose values cannot be known at all times (even though they may have values at all times). The Causal Theory is historically a member of this set of theories and is still labelled as such in much of the literature. The second meaning refers to variables that are forever hidden from empirical investigation (or alleged to be so). It is in this second sense that 'hidden variables' is used to imply that any theory containing such variables is flawed and therefore not worth serious attention. In the Causal Theory, the position of a particle is the (so-called) 'hidden variable'. Although there are restrictions on the measurement of particle positions, it is clearly possible to measure position in the Causal Theory. Therefore, alleged flaw ② when used in the second meaning is not applicable. John Bell made strong protests against the Causal Theory being labelled in this way. He wrote:

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[20] Holland, P.R., The Quantum Theory of Motion, *op. cit.*, p.75.

[21] *ibid.*, p.79.

> Absurdly, such theories are known as 'hidden variable' theories. Absurdly, for there it is not in the wavefunction that one finds an image of the visible world, and the results of experiments, but in the complementary 'hidden'(!) variables.[22]

The specification of the position of a particle, in conjunction with equation (2-2), allows well-defined trajectories for a quantum particle $\mathbf{x} = \mathbf{x}(t)$, to be determined. Trajectories can be calculated by specifying the wavefunction (as ∇S is found from the wavefunction) and the particle's initial position. A unique trajectory can then be found by integrating equation (2-2), i.e.

$$\mathbf{x}(t) = \int \left(\frac{\nabla S}{m}\right) dt + \mathbf{x}_o$$

where $\mathbf{x}_o$ is the initial particle position. In the Causal Theory, therefore, a quantum particle has a distinct worldline in Galilean spacetime with an attached inertial mass. Note that the existence of particle trajectories is specifically ruled out in Orthodox Quantum Theory *by fiat*. Although $S(\mathbf{x}, t)$ is a multi-valued function, (∇S) is single-valued and is not defined at nodal points or surfaces, i.e. where $R = 0$. This requires that trajectories do not pass through the nodes of the wavefunction.[23] This is reflected in the dynamics of the Causal Theory which ensure that quantum particles cannot pass through nodes.

An expression for the total time rate of change of momentum (i.e. the net force acting) of a quantum particle can also be derived. The total derivative of the momentum $\mathbf{p}$ with respect to time is:

$$\frac{d\mathbf{p}}{dt} = \sum_{i=1}^{3} \frac{\partial \mathbf{p}}{\partial x_i}\frac{dx_i}{dt} + \frac{\partial \mathbf{p}}{\partial t}$$

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[22] Bell, J.S., 'Are there quantum jumps?' in his *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press, Cambridge, 1987) p.201.

[23] Holland, P.R., The Quantum Theory of Motion, *op. cit.*, p.85.

Since $\mathbf{p} = \nabla S$, we have

$$\frac{d\mathbf{p}}{dt} = \sum_{i=1}^{3} \frac{p^i}{m} \frac{\partial \nabla S}{\partial x_i} + \frac{\partial \nabla S}{\partial t}$$

$$= \frac{(\nabla S)}{m} \cdot \nabla (\nabla S) + \frac{\partial \nabla S}{\partial t}$$

$$= \nabla \Big[\frac{(\nabla S)^2}{2m} + \frac{\partial S}{\partial t}\Big]$$

Using equations (2-1) and (2-3), we can make the following substitution:

$$\frac{(\nabla S)^2}{2m} = \Big[\frac{\partial S}{\partial t} + V(\mathbf{x},t) + Q(\mathbf{x},t)\Big]$$

then

$$\Big(\frac{d\mathbf{p}}{dt}\Big) = \nabla \Big(-\Big[\frac{\partial S}{\partial t} + V(\mathbf{x},t) + Q(\mathbf{x},t)\Big] + \frac{\partial S}{\partial t}\Big)$$

It can be seen now that the total time rate of change of momentum of a quantum particle (net force) is given by:

$$(d\mathbf{p}/dt) = -\nabla (V + Q) \quad .... (2\text{-}4)$$

The term $(-\nabla Q)$ is sometimes called the 'quantum mechanical force'. It is clear from equation (2-4) that the motion of a quantum particle cannot be, in general, derived entirely from the classical potential V. Since $R = |\Psi|$ the force acting on the particle depends (in part) on a function of the absolute value of the wavefunction, evaluated at the particle's position.[24]

The existence of well-defined trajectories for quantum particles together with the explicit recognition of the role of causal agents (e.g. the 'quantum mechanical force') is consistent with event by event causality in spacetime and subject to the Principle of Causality:

[24] Bohm, D., 'A Suggested Interpretation … I', *loc. cit.*, p.170.

Principle of Causality

> The same cause always produces the same effect or effects (other things being equal) and the cause temporally precedes, or is simultaneous with, its effects.

2.4 Statistical Predictions

If we again substitute $Re^{iS/\hbar}$ for Ψ into the equation for the Lagrangian density and apply the Principle of Stationary Action in the form: $\delta \int \mathcal{L} \, d^4\mathbf{x} = 0$, by varying S, then we have:

$$\sum_{\mu=1}^{4} \frac{\partial}{\partial x_\mu} \left[\frac{\partial \mathcal{L}}{\partial(\partial S / \partial x_\mu)} \right] - \left(\frac{\partial \mathcal{L}}{\partial S}\right) = 0$$

which gives the equation:

$$\nabla \cdot \left\{ R^2 \frac{(\nabla S)}{m} \right\} + \left(\frac{\partial R^2}{\partial t}\right) = 0 \quad .... (2\text{-}5)$$

Let $\rho(\mathbf{x})$ be a function called the quantum equilibrium distribution. The Quantum Equilibrium Condition (Axiom VI) states that when a system has a wavefunction Ψ, the probability density $\rho(\mathbf{x})$ of possible values of the initial particle position in an ensemble of similarly prepared quantum systems is equal to $|\Psi|^2$. (Note that this is also assumed in Orthodox Quantum Theory.) The status of Axiom VI will be reviewed later in this chapter. Since $R^2 = |\Psi|^2 = \rho$, we can rewrite equation (2-5) in the alternative form:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \frac{\nabla S}{m}\right) = 0 \quad .... (2\text{-}6)$$

This is an equation of continuity for ρ which ensures that if the initial value is $|\Psi|^2$ then it will remain so at all subsequent times.[25] In Orthodox Quantum Theory, the quantity $(\rho \nabla S/m)$ is known as the probability current density $\mathbf{j}$. In the Causal Theory,

$\mathbf{j}(\mathbf{x}, t) = \rho\, \mathbf{v}$ (where $\mathbf{v} = \nabla S/m$), i.e. this 'current' lies on the tangent to each point on a trajectory $\mathbf{x}(t)$.[26]

It should be clear now that the function R plays a *dual* role in the Causal Theory: (i) R represents the amplitude of the wave field and therefore (in part) determines the value of the quantum potential. Hence R helps to determine individual particle motion; and (ii) $R^2 = |\Psi|^2$ describes the ensemble quantum state.[27] Although the particle trajectories are causally determined, they depend on initial conditions. Initial particle positions that fluctuate in a random manner will be unknown. The Quantum Equilibrium Condition (Axiom VI) requires that we cannot know the possible distribution of a particle's position better than given by $|\Psi|^2$. We may then interpret R^2 as a probability density such that the probability that a particle's position lies in an interval between $\mathbf{x}$ and $(\mathbf{x} + d\mathbf{x})$ at a given time t is $R^2(\mathbf{x}, t)\, d^3\mathbf{x}$.[28] Probability, therefore, is not inherent to the Causal Theory but merely expresses a lack of knowledge of initial conditions. This is a situation similar to the use of probability in Classical Statistical Mechanics. In the absence of exact knowledge of a system's initial conditions, we can make statistical predictions if we have the system's wavefunction and the initial value of the amplitude R.

Incidentally, equation (2-5) combined with the polar form of the wavefunction allows the equation of propagation of the wave field to be obtained. Since we now have $(\partial S/\partial t)$ and $(\partial R^2/\partial t)$, we might expect to find $(\partial\Psi/\partial t)$ as a first step.

$$\frac{\partial \Psi}{\partial t} = R\, \frac{\partial}{\partial t}\left(e^{iS/\hbar}\right) + \left(e^{iS/\hbar}\right) \frac{\partial R}{\partial t}$$

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25 Holland, P.R., The Quantum Theory of Motion, *op. cit.*, p.99.
26 *ibid.*, p.75.
27 *ibid.*, p.100.
28 *ibid.*, p.67.

$$= \left[\left(\frac{i}{\hbar}\right) R \frac{\partial S}{\partial t} + \frac{\partial R}{\partial t}\right] (e^{iS/\hbar})$$

$$= \left[\left(\frac{i}{\hbar}\right) R \left\{- \frac{(\nabla S)^2}{2m} + \left(\frac{\hbar^2}{2m}\right)\left(\frac{\nabla^2 R}{R}\right) - V(\mathbf{x},t)\right\} + \frac{\partial R}{\partial t}\right] (e^{iS/\hbar})$$

where we have substituted from equation (2-2) for ($\partial S/\partial t$). It can be shown readily that equation (2-5) is equivalent to the following equation:

$$\frac{\partial R}{\partial t} = \left(\frac{-1}{2m}\right) \{R\nabla^2 S + 2(\nabla R)\cdot(\nabla S)\} \quad .... (2\text{-}7)$$

Substituting for ($\partial R/\partial t$) in the expression for ($\partial\Psi/\partial t$) gives:

$$\left[\left(\frac{iR}{\hbar}\right)\left\{- \frac{(\nabla S)^2}{2m} + \left(\frac{\hbar^2}{2m}\right)\left(\frac{\nabla^2 R}{R}\right) - V(\mathbf{x},t)\right\} - \left(\frac{1}{2m}\right)\{R\nabla^2 S + 2(\nabla R)\cdot(\nabla S)\}\right] e^{iS/\hbar}$$

Rearranging gives: $(\partial\Psi/\partial t) =$

$$\left[\left(\frac{-iR}{2m\hbar}\right)(\nabla S)^2 - \left(\frac{R}{2m}\right)(\nabla^2 S) + \left(\frac{i\hbar}{2m}\right)\nabla^2 R - \frac{(\nabla R)\cdot(\nabla S)}{m} - \left(\frac{i}{\hbar}\right) RV\right] (e^{iS/\hbar})$$

Now if we multiply by $(i\hbar)$ then we get: $(i\hbar)\,(\partial\Psi/\partial t) =$

$$\left[\left(\frac{R}{2m}\right)(\nabla S)^2 - \left(\frac{i\hbar R}{2m}\right)(\nabla^2 S) - \left(\frac{\hbar^2}{2m}\right)\nabla^2 R - i\hbar \frac{(\nabla R)\cdot(\nabla S)}{m}\right] (e^{iS/\hbar})$$

$$+ RV(e^{iS/\hbar}) = -(\hbar^2/2m)\, \nabla^2 (R\, e^{iS/\hbar}) + (R\, e^{iS/\hbar})\, V$$

This is just the time-dependent Schrödinger Equation:

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Psi + V\Psi \quad .... (2\text{-}8)$$

which is the equation of propagation of the wave field.

2.5 Dynamic Theory of Measurement

We saw in Section 1.3 that a special status is afforded to 'measurement' in Orthodox Quantum Theory. On the question of this status, John Bell wrote:

> It would seem that the [Orthodox Quantum] theory is exclusively concerned with 'results of measurement' and has nothing to say about anything else. ... And does not any

> *analysis* of measurement require concepts more *fundamental* than measurement? And should not the fundamental theory be about these more fundamental concepts? [29]

Bell is certainly correct about physical theories not just being about predicting the outcomes of experiments. What does the 'measurement process' entail within the Causal Theory? Measurement is not fundamental in the Causal Theory as can be seen with reference to its stated axioms (as presented in Section 2.1). Measurement is merely a special type of interaction between two systems (the quantum system under investigation and a measurement apparatus). One principle that is frequently discussed in quantum measurement theory is the Principle of Faithful Measurement:[30]

- *Principle of Faithful Measurement*

  The result of measurement is numerically equal to the value possessed by an observable immediately prior to measurement.

In detailing what the 'measurement process' entails, we will answer the question of whether the Principle of Faithful Measurement generally holds in the Causal Theory.

An exposition of the measurement process will now be presented which will closely follow the version in Holland's text[31] rather than in Bohm's original paper (as the former is clearer and the latter contains a number of errors). Let $\psi(\mathbf{x},t)$ be the single-particle wavefunction of a quantum system. An hermitian operator $\mathbf{A}$ corresponds to an observable A of the system. In an ensemble of similar systems with the wavefunction ψ, the initial value A_o is given by:

$$A_o = (\mathrm{Re}\ \psi_o^*)\,\mathbf{A}\psi_o / |\psi_o|^2$$

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[29] Bell, J.S., 'Quantum Mechanics for Cosmologists' in his *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press, Cambridge, 1987) pp.117-18 (his italics).

[30] Redhead, M., *Incompleteness, Nonlocality, and Realism* (Clarendon, Oxford, 1987) p.89.

[31] Holland, P.R., The Quantum Theory of Motion, *op. cit.*, pp.339-341.

where only the real part contributes due to the operator **A** being hermitian.[32] The quantum system interacts with a measurement apparatus which has an initial wavefunction $\phi_o(y)$. This wavefunction is a packet with coordinate y which is its 'readout' (or pointer display). The measurement of A is an interaction which is assumed to be impulsive such that any independent evolution of apparatus or quantum system is negligible. The interaction Hamiltonian is given by:

$$\mathbf{H}_I = g\,\mathbf{A}\,\mathbf{p}_y \quad \text{.... (2-9)}$$

where g is a coupling constant and $\mathbf{p}_y$ is the momentum operator conjugate to **y**.

The total initial wavefunction for the combination of quantum system and apparatus is the product of their initial individual wavefunctions:

$$\Psi_o(\mathbf{x},y) = \psi_o(\mathbf{x})\,\phi_o(y)$$

During the time of the interaction, Schrödinger evolution with the Hamiltonian given by equation (2-9) requires:

$$i\hbar\frac{\partial\Psi}{\partial t} = \mathbf{H}_I\Psi = g\mathbf{A}\mathbf{p}_y\Psi = -i\hbar\, g\mathbf{A}\frac{\partial\Psi}{\partial y} \quad \text{.... (2-10)}$$

where Ψ is the total combined wavefunction for time $t > 0$ and $\mathbf{p}_y\Psi = (-i\hbar)(\partial\Psi/\partial y)$. The wavefunction Ψ can be expanded into a complete set of (orthonormal) eigenfunctions $\psi_\alpha(\mathbf{x})$ of the operator **A**. (Here 'orthonormal' means that these wavefunctions are normalised and their inner product is zero.) This expansion has coefficients $f_\alpha(y,t)$ where $\mathbf{A}\psi_\alpha = \alpha\psi_\alpha$ and α is an eigenvalue, i.e.

$$\Psi(\mathbf{x},y,t) = \sum_\alpha f_\alpha(y,t)\,\psi_\alpha(\mathbf{x}) \quad \text{.... (2-11)}$$

Substitution of equation (2-11) into equation (2-10) and applying the orthonormal conditions of the eigenfunctions ψ_α we find that the partial derivatives of the coefficients f_α are related by:

[32] *ibid.*, p.92.

$$\frac{\partial f_\alpha(y,t)}{\partial t} = -\, g\alpha \frac{\partial f_\alpha(y,t)}{\partial y} \quad .... \text{(2-12)}$$

Using a standard method (such as separation of variables), the partial differential equation (2-12) can be shown to have the solution:

$$f_\alpha(y,T) = f_{\alpha o}(y - gAT) \quad .... \text{(2-13)}$$

where the $f_{\alpha o}$ are initial values and T is the period of the impulse. Now if we let

$$\psi_o(\mathbf{x}) = \sum_\alpha c_\alpha(y,t)\, \psi_\alpha(\mathbf{x})$$

then

$$\Psi(\mathbf{x},y,0) = \sum_\alpha f_\alpha(y,0)\, \psi_\alpha(\mathbf{x}) = \Psi_o(\mathbf{x},y) = \sum_\alpha c_\alpha\, \psi_\alpha(\mathbf{x})\, \phi_o(y)$$

$$\Rightarrow \quad f_{\alpha o} = c_\alpha\, \phi_o(y) \quad .... \text{(2-14)}$$

Substitution of equation (2-14) into (2-13) and then the result into equation (2-11) provides the wavefunction at the termination of the interaction:

$$\Psi(\mathbf{x},y,t) = \sum_\alpha c_\alpha\, \phi_o(y - g\alpha T)\, \psi_\alpha(\mathbf{x}) \quad .... \text{(2-15)}$$

The wave field is split into non-overlapping packets which move off independently. These are represented by the summands in equation (2-15). Only one of these represents the packet in which the particle is present. Since there is no overlap, the other packets will have no further effect on the particle and consequently are not relevant to subsequent system evolution.[33] The wavefunction then effectively becomes:

$$\Psi = c_\alpha\, \psi_\alpha(\mathbf{x})\, \phi_o(y - g\alpha T) \quad .... \text{(2-16)}$$

In the Causal Theory, the wave field divides into separate parts which continue to exist albeit as empty quantum waves. (The term ‘empty wave’ is to be understood in the sense that it does not include a quantum particle but still possesses energy and

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[33] *ibid.*, pp.341.

momentum.[34]) There is *no 'collapse'* of the wavefunction and the particle has a definite position at all times.

The measurement apparatus will give a 'readout' which will be a single value of y as the combined wavefunction is effectively given by equation (2-16), i.e. the apparatus will be in a definite state. Thus, there is no Measurement Problem. The initial value A_o of the observable A will have evolved into a value which would be identified in Orthodox Quantum Theory as an eigenvalue of the operator **A**.[35] Since the measurement apparatus has an enormous number of degrees of freedom, once the measurement interaction is over, the process is essentially irreversible.[36] Occasionally one finds in the literature, claims that the Causal Theory fails to solve the 'Measurement Problem'.[37] These claims have been successfully answered by Maudlin and by Lewis.[38]

In general, the measurement process introduces an uncontrollable (and unpredictable) disturbance to the wave field of the quantum system. The interaction with a measurement device transforms the wavefunction of the system into an eigenfunction of the observable being measured. Since the wave field is a real, physical entity there is no sudden collapse on measurement. Instead there is a change in the wave field which may alter the particle's momentum, position, energy, etc. The statistical results of measurement coincide with the probabilistic predictions for the measured values of physical quantities, i.e. eigenvalues of the associated operator,

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[34] *ibid.*, p.86.

[35] *ibid.*, pp.342-343.

[36] *ibid.*, p.348; Cushing, J.T., *Quantum Mechanics: Historical Contingency and the Copenhagen Hegemony* (Chicago University Press, Chicago, 1994) p.51.

[37] For example: Stone, A.D., 'Does the Bohm Theory Solve the Measurement Problem?', *Philosophy of Science* **61** (1994): 250-266; Zeh, H.D., 'Why Bohm's Quantum Theory?', *Foundations of Physics Letters* **12** (1999): 197–200; Brown, H.R. and Wallace, D., 'Solving the Measurement Problem: de Broglie-Bohm loses out to Everett', *Foundations of Physics* **35** (2005): 517–540.

[38] Maudlin, T., 'Why Bohm's Theory Solves the Measurement Problem', *Philosophy of Science* **62** (1995): 479–483; Lewis, P.J., 'Empty Waves in Bohmian Quantum Mechanics', available at: http://philsci-archive.pitt.edu/archive/00002899/.

and not necessarily the statistical distribution of possessed values.[39] Therefore the measurement of physical observables, according to the Causal Theory, does not provide necessarily the (pre-existing) possessed value of the observable prior to measurement. The exception is measurements of a particle's position which does yield pre-measurement values.[40] Since measurement is a dynamic process (in the sense that the measured value of a physical quantity need not be identical with its possessed value prior to the measurement process), the Principle of Faithful Measurement cannot be generally upheld in the Causal Theory.[41] Further consideration of physical measurement within the Causal Theory will appear in Chapter Four.

It was stated in Chapter One that the Kochen and Specker Theorem does not apply to contextual 'hidden variable' theories. The context dependence aspect of measurement in the Causal Theory requires that the value of an observable obtained on measurement depends on the evolution of the quantum state and this, in turn, depends on the system's Hamiltonian. Measurements of 'incompatible' variables will alter the wavefunction resulting in a different outcome from what would be found otherwise. Therefore the measured value depends on what other observables are measured. This context dependence excludes the Causal Theory from the conclusion of Kochen and Specker.[42] This suffices to dispose of alleged flaw ③ in the General Introduction, viz., that the Causal Theory has been disproved by impossibility theorems.

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39 Bohm, D.J., 'A Suggested Interpretation … II', *loc. cit.*, pp.182-183; Holland, P.R., The Quantum Theory of Motion, *op. cit.*, p.343.

40 Holland, P.R., *ibid.*, p.351; .

41 Dewdney, C. and Malik, Z., 'Angular-Momentum Measurement and Nonlocality in Bohm's Interpretation of Quantum Theory', *Physical Review A* **48** (1993) pp.3522-3533.

42 *ibid.*, p.3523.

It was also argued in Chapter One that the Heisenberg uncertainty relations specify a lower bound of the variance of two kinds of ('incompatible') measurements made on an ensemble of similarly prepared quantum systems. The existence of definite, sharp values of possessed quantities is not inconsistent with the uncertainty relations. This position has been explicitly stated by several commentators, including Dewdney and Malik:

> There is no contradiction with Heisenberg's uncertainty relations in the assumption of definite values for both the position and momentum of the particle (or other sets of noncommuting observables), since the uncertainty relations simply refer to the inevitable statistical scatter in the values obtained for complementary variables in an ensemble of measurements.[43]

This "inevitable statistical spread" is explained by the Causal Theory as due to the change in the wave field (and therefore to the quantum potential) caused by the measurement process.[44]

We should also note that the labelling of the Causal Theory as 'pure metaphysics' is invalid. In the General Introduction, alleged flaw ④ is the statement that the Causal Theory is pure metaphysics. The term 'pure metaphysics' applied in this context implies that the Causal Theory is devoid of physical content. This is because it has been claimed that no experimental test can discriminate between it and Orthodox Quantum Theory and that the particle paths described by the Causal Theory cannot be observed. The so-called 'pure metaphysics' criticism can be immediately dismissed as the Causal Theory is no more metaphysics than is any other physical theory that postulates the existence of entities or processes that, at present, cannot be directly observed, e.g. quarks, black holes, dark matter, event horizons, etc. This

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[43] *ibid.*, p.3513.

'pure metaphysics' criticism is used as an emotive condemnation of the Causal Theory despite not being a legitimate criterion for theory rejection. John Bell explicitly commented on this:

> [the opponents of the Causal Theory] ... could produce no more devastating criticism of Bohm's [Causal] version [of Quantum Theory] than to brand it 'metaphysical' and 'ideological'.[45]

Indeed, Orthodox Quantum Theory itself includes a postulate that cannot be experimentally tested – the Completeness axiom, i.e. that the state vector contains all information about the quantum state.[46] If the 'pure metaphysics' criterion is accepted then Orthodox Quantum Theory would also qualify as a piece of metaphysical speculation!

2.6 Many Particle States and the Non-Locality Aspect

In a system consisting of N 'point-like' particles (where N is an integer > 1), if each particle is unconstrained then its position can be given by assigning it three coordinates (not necessarily Cartesian ones). The minimum number of coordinates (variables) required to specify the positions of all the particles in an unconstrained system at a given time, i.e. the configuration of the system, must be 3N. An N-particle quantum system may be considered as a generalisation from the single particle case in which the wavefunction is a field on a 3N-dimensional configuration space. All the single particle quantities and equations have many-particle analogues. However, there is only one guiding wave field described by a wavefunction $\Psi =$

[45] Bell, J.S., 'On the Impossible Pilot Wave' in his Speakable and Unspeakable in Quantum Mechanics, *op. cit.*, p.160.
[46] Holland, P.R., The Quantum Theory of Motion , *op. cit.*, p.25.

$\Psi(\mathbf{x}_1, \ldots, \mathbf{x}_N, t)$ where each of the $\mathbf{x}_i$ is a set of Cartesian coordinates.[47] This wavefunction evolves according to the many-particle Schrödinger equation:

$$i\hbar \frac{\partial \Psi}{\partial t} = \sum_{i=1}^{N} \left(\frac{-\hbar^2}{2m_i}\right) \nabla_i^2 \Psi + V\Psi$$

where ∇_i^2 is the Laplacian evaluated at the position of the i^{th} particle.

Let's consider, for simplicity, a two-particle system where the particles have equal mass m. We denote the particles by numerical subscripts 1 and 2. At a given time t, let particle 1 have coordinates $\mathbf{x}_1$ and particle 2 have coordinates $\mathbf{x}_2$. As before, the wavefunction of the system may be written as: $\Psi = \mathrm{R}e^{iS/\hbar}$, then the two-particle equivalent of equation (2-1) is:

$$-\frac{\partial S}{\partial t} = \frac{(\nabla_1 S)^2}{2m} + \frac{(\nabla_2 S)^2}{2m} - \frac{\hbar^2}{2m}\left[\frac{(\nabla_1^2 + \nabla_2^2)R}{R}\right] + V$$

This is the quantum Hamilton-Jacobi equation in 6-dimensional configuration space. The subscripts on the Laplacian operators refer to explicit dependence on the coordinates of the individual particle. The two-particle equivalent of equation (2-5) is:

$$\frac{\partial R^2}{\partial t} + \nabla \cdot \left(R^2 \frac{\nabla_1 S}{m}\right) + \nabla \cdot \left(R^2 \frac{\nabla_2 S}{m}\right) = 0$$

The two-particle quantum potential, $Q = Q(\mathbf{x}_1, \mathbf{x}_2, t)$, is given by:

$$Q = -\left(\frac{\hbar^2}{2mR}\right)\left(\nabla_1^2 R + \nabla_2^2 R\right)$$

The respective momenta of the two particles is:

$$\mathbf{p}_1 = \nabla_1 S \quad \text{and} \quad \mathbf{p}_2 = \nabla_2 S$$

The respective momentum of each particle will, in general, depend on the position of the other. This is a manifestation of the state dependence which, in a many-particle

[47] *ibid.*, p.277.

system, finds expression as a holistic, non-local connection between the particles of the system. The quantum mechanical force **f** for a two-particle system is given by:

$$\mathbf{f} = -(\nabla_1 Q + \nabla_2 Q)$$

The above two-particle system equations are easily generalised to their many-particle equivalents (which will be used in later chapters).

The reality of the wave field is not placed in jeopardy just because its mathematical description is in terms of a multi-dimensional configuration space. Nor does the use of such a description imply that a multi-dimensional space has an existence in the same sense that physical three-dimensional space may be said to exist. There is an acceptance by some researchers that the multi-dimensional configuration space of the many-particle Causal Theory is a real aspect of nature, as Holland claims:

> ... an *individual physical system resides in a multidimensional (configuration) space*. While the particles each move in 3-space, the guiding wave is, in general, irreducibly defined in $3n$-space. Since we conceive of the wave as a physical influence on the particles, *we ascribe to configuration space as much physical reality as we do to three-dimensional Euclidean space in the one-body theory.*[48]

This position is rejected unequivocally as it confuses the formal machinery of the model with the reality that the model represents. This is an important distinction. The mathematical technique of using a multi-dimensional (mathematical) space to model physical phenomena is well established in classical mechanics. In the configuration space description of a many-particle system in classical mechanics, the system is represented by a *single* point in the space. The empirical predictions of classical mechanics are correct within its domain even though this space is not physical space.

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[48] *ibid.*, pp.277-278 (his italics).

The use of a multi-dimensional mathematical space to model phenomena does not necessarily require an ontological commitment to the physical existence of such a space.

Although the configuration space representation in Classical Mechanics, is a convenient summary of the positions of all the particles in a system, the situation is different in quantum mechanics for there is information in the configuration space wavefunction that is not present in the individual wavefunctions for the particles. The problem is then that we can't represent the physics in terms of a wavefunction (or even wavefunctions) in three-dimensional space. So the argument goes that either we accept the multi-dimensional space as a real physical space or we don't accept the wavefunction as representing a real field. Yet, as stated above, the *use* of a multi-dimensional mathematical space to model phenomena does not necessarily require an ontological commitment to the physical existence of a multi-dimensional space. A position similar to Bohm's 1952 account is proposed in this thesis, i.e. that the wavefunction in 3N-dimensional space is taken to be a *mathematical representation* of an objectively real field in three-dimensional space. Is this a coherent position to take? In defence of this position, it was stated that a scientific model should not be taken literally in all respects but this does not directly address the issue that we can't describe a system by wavefunctions in three-dimensional space without a loss of information. The justification for claiming that the wavefunction represents a real field in three-dimensional space is as follows.

First, a simply connected three-dimensional space cannot describe holistic quantum connectiveness that is a feature of multi-particle quantum systems. Instead, this is done formally by employment of a multi-dimensional configuration space without an ontological commitment being made to a multi-dimensional space since the problems associated with such an ontological commitment are considerable and include:

– needing at least three separate dimensions for every particle in the universe;

– the total number of dimensions in the universe varying from moment to moment along with the creation and annihilation of particles;

– the extra dimensions always being completely unnoticeable at macroscopic scales;

and

– a complete lack of any experimental evidence for the existence of multi-dimensional physical spaces.

These are good reasons for not having an ontological commitment to a physical multi-dimensional space in the same sense as we do for physical three-dimensional space. Second, we do not know the 'means' by which quantum non-local connections (see below) are actualised. Further, this is not because of the non-relativistic context for non-locality is present in relativistic versions of quantum theory.

In the absence of information about such 'means' and the strong reasons against taking multi-dimensional space as real, it is coherent to take the wavefunction in 3N-dimensional configuration space to be a mathematical representation of a real field in three-dimensional space. The empirical predictions of quantum mechanics are some of the best confirmed in the whole history of physics even though its multi-dimensional configuration space is not physical space. The multi-dimensional configuration space of quantum mechanics is an artefact of the mathematics and not the physical space in which quantum particles and wave fields exist. Hopefully, when we have figured out (or have a model of) the 'means' by which quantum non-local connections are actualised then we will be able to describe this in three-dimensional space (e.g. it might need a multi-connected three-dimensional space).

When there are N particles in the Causal Theory, their trajectories are traced out in 3N-dimensional configuration space. However, as has been shown elsewhere,[49] this is not a problem for describing the motion of the quantum particles as there exists a natural mapping from trajectories in 3N-dimensional configuration space to trajectories in three-dimensional space.

Consider now the issue of locality in quantum theory. In the Causal Theory, a many-particle quantum system exhibits non-local effects as its quantum potential allows for a strong and direct interconnection between the particles. In particular, the non-local influence on a particle depends on the positions of all other particles in the system. This provides a physical explanation for the motions of quantum particles in a many-particle system.[50] Although this is consistent with the Principle of Causality, it violates the Principle of Locality. Fortunately, parts of the universe are sufficiently separable (i.e. do not constantly exhibit non-local behaviours) that we can still use established methods of scientific investigation and analysis to obtain knowledge of the physical world.[51]

Non-locality is another of the various criticisms that has been laid at the feet of the Causal Theory.[52] The experimental tests of the various Bell Inequalities have come down 'fair and square' on the side of non-locality, i.e. experiments continue to confirm that the Bell Inequalities are indeed violated,[53] as predicted by the formalism

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49 Dewdney, C., 'Nonlocally Correlated Trajectories in Two-Particle Quantum Mechanics', *Foundations of Physics* **18** (1988): 867-86; 'Illustrations of the Causal Interpretation of One and Two Particle Quantum Mechanics' in Kostro, L. *et al.* (eds), *Problems in Quantum Physics, Gdan'sk '87: Recent and Future Experiments and Interpretations* (World Scientific, Singapore, 1988).

50 Bohm, D. and Hiley, B.J., 'On the Intuitive Understanding of Nonlocality as Implied by Quantum Theory', *Foundations of Physics* **5** (1975) p.99.

51 Bohm, D. and Hiley, B.J., The Undivided Universe, *op. cit.*, p.59; Cushing, J.T., Quantum Mechanics, *op. cit.*, p.185.

52 Rae, A.I.M., *Quantum Physics: Illusion or Reality?* (Cambridge University Press, Cambridge, 1986) p.27.

53 See, for example, Aspect, A., Dalibard, J. and Roger, G., 'Experimental Test of Bell's Inequalities Using Time-Varying Analyzers', *Physical Review Letters* **49** (1982): 1804-1807; Grangier, P., Roger, G. and Aspect, A., 'Experimental Evidence for a Photon Anticorrelation Effect on a Beam Splitter: A

of quantum mechanics (cf. Section 1.4). Although non-locality is explicit in the Causal Theory, the continued criticism on this issue is a little surprising. This is because it has become clear that Orthodox Quantum Theory also requires some kind of non-locality (i.e. action-at-a-distance), as Redhead concludes in his detailed analysis of quantum theory:

> ... some sort of action-at-a-distance or ... nonseparability seems to be built into a reasonable attempt to understand the quantum view of reality.[54]

The emergence of non-locality (by the formal means of a multi-dimensional configuration space) is the expression in the model of an holistic quantum connectiveness. Yet, it is a curious result that neither the Causal Theory (nor Orthodox Quantum Theory) violates the Special Theory of Relativity in the sense that the connections between quantum systems cannot be used for the purposes of signalling (i.e. the transmission of information) or the transfer of energy faster than the speed of light in vacuum.[55]

There is a widespread belief that the confirmation that the Bell Inequalities are violated has decided the question about (so-called) 'hidden-variable' theories in the negative (alleged flaw ⑤ in the General Introduction). Unfortunately, this 'myth' continues to be promulgated in the literature.[56] Given the assumptions behind Bell's Theorem (as set out in Section 1.4), these experiments only show that the class of

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New Light on Single-Photon Interferences', *Europhysics Letters* **1** (1986): 173-179; Weihs, G. et al., 'Violation of Bell's Inequality under Strict Einstein Locality Conditions', *Physical Review Letters* **81** (1998): 5039-5043; Rowe, M. et al., 'Experimental violation of a Bell's inequality with efficient detection', *Nature* **409** (2001): 791-794.

[54] Redhead, M., Incompleteness, Nonlocality, and Realism, *op. cit.*, p.169.

[55] Ballentine, L.E. and Jarrett, J.P., 'Bell's Theorem: Does Quantum Mechanics Contradict Relativity?', *American Journal of Physics* **55** (1987) p.696; Maudlin, T., *Quantum Nonlocality, and Relativity: Metaphysical Intimations of Modern Physics* (Blackwell, Oxford, 1993) p.125.

[56] An example at the time of writing is found in: Demtröder, W., *Atoms, Molecules and Photons: An Introduction to Atomic-, Molecular- and Quantum-Physics* (Springer, Berlin, 2006) p.497.

local, realistic 'hidden-variable' theories are ruled out.[57] This disposes of alleged flaw ⑤, as the Causal Theory is non-local.

2.7 Resolution of the Quantum Paradoxes

The two 'paradoxes' of Orthodox Quantum Theory that were summarised in Chapter One may now be shown to be readily solvable using the Causal Theory.

EPR (Einstein-Podosky-Rosen)

The EPR Paradox is resolved within the Causal Theory by the existence of a non-local connection between the particles and by rejecting the Completeness Axiom of Orthodox Quantum Theory. A change in the wave field (and therefore the quantum potential) results from a measurement on one of the particles. The quantum potential allows for a direct connection between the particles which depends on the state of both. The connection between the particles via the quantum potential is instantaneous, but as noted in the previous section, Special Relativity is not violated.[58]

Schrödinger's Cat

The root of the problem here is the Completeness axiom of Orthodox Quantum Theory. If this axiom is rejected (as in the Causal Theory) then the solution is straight-forward. Whether the radioactive decay occurs (which leads directly to the cat dying) will depend on the position of the relevant particle in the radioactive source. The initial position of the particle together with the many-particle wavefunction of the source determines its future behaviour (i.e. decay or not). The usual expected events will then follow. The cat is not, of course, ever in a

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[57] Hardy, L., 'Contextuality in Bohmian Mechanics' in Cushing, J.T., Fine, A. and Goldstein, S. (eds), *Bohmian Mechanics and Quantum Theory: An Appraisal* (Kluwer, Dordrecht, 1996) pp.68-69.

[58] Bohm, D. and Hiley, B.J., 'On the Intuitive Understanding …', *loc. cit.*, p.107.

superposition of live and dead states. It is either alive or dead, but this will not be known until an observation is made.

2.8 Transition to the Classical Realm

It is commonly asserted in textbooks on Orthodox Quantum Theory that the transition to Classical Mechanics arises either:

(i) in the limit of large quantum numbers (as $n \to \infty$); or

(ii) when allowing $\hbar \to 0$.

The former criterion is an application of the Correspondence Principle where quantum states tend to classical ones in the limit of large quantum numbers. What exactly constitutes 'large quantum numbers' is, however, not rigorously specified. Indeed, there are examples where the quantum number specifying the energy of a state (n) can be made arbitrarily large with the system still being governed by quantum mechanics.[59]

The latter criterion is used to cover situations where a system grows to macroscopic proportions and thereafter develops according to the laws of Classical Mechanics. This criterion (strictly speaking) is nonsense, for $\hbar$ is a physical constant (with units of energy-time) and not a variable parameter.

The formal transition from quantum to classical realms has nothing to do with the above two criteria. Classical Mechanics emerges naturally when the value of the quantum potential becomes negligible with respect to the other terms in the Quantum Hamilton-Jacobi equation (equation (2-1)). When this occurs, the Quantum Hamilton-Jacobi equation tends to the classical version and the wave field no longer affects particle motions. Therefore, the problem of having an arbitrary boundary

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[59] Holland, P.R., The Quantum Theory of Motion, *op. cit.*, p.221.

between classical and quantum realms (as in Orthodox Quantum Theory) does not arise.

2.9 The Quantum Equilibrium Condition

The Quantum Equilibrium Condition (Axiom VI) is necessary in order for the statistical predictions of the Causal Theory to agree with experiment. Therefore one might again cite Synge's comment (quoted in Section 2.2) that the choice of axioms is made to fit the theory, as sufficient justification for the axiom.

Whether the Quantum Equilibrium Condition should be of axiomatic status has been the subject of dispute. In 1952, Keller suggested that if Bohm's interpretation was to bear a similar relationship to Orthodox Quantum Theory that Classical Mechanics has to Classical Statistical Mechanics, then the probability density $P(\mathbf{x},t) = |\Psi(\mathbf{x},t)|^2$ would have to be derivable from the other assumptions.[60] In response to this and other criticisms, Bohm attempted to show that the initial probability density $\rho(\mathbf{x}) = P(\mathbf{x},0) = |\Psi|^2$ was a theorem, for then the equation of continuity (equation 2-6) would ensure that $P(\mathbf{x},t) = |\Psi(\mathbf{x},t)|^2$ holds at all subsequent times.[61] Bohm's 1953 proof was not successful. Indeed, Hans Freistadt was later to point out that the mathematics in Bohm's argument was somewhat suspect.[62] Bohm also tried an alternative approach in a paper with Vigier in 1954 where a fluctuating 'sub-quantum realm' was assumed.[63] There was no general acceptance of this approach either.

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[60] Keller, J.B., 'Bohm's Interpretation of the Quantum Theory in Terms of "Hidden" Variables', *Physical Review* **89** (1953) p.1040.

[61] Bohm, D., 'Proof that Probability Density Approaches $|\psi|^2$ in Causal Interpretation of Quantum Theory', *Physical Review* **89** (1953): 458-466.

[62] Freistadt, H., 'The Causal Formulation ... ', *loc. cit.*, p.29.

[63] Bohm, D. and Vigier, J.-P., 'A Causal Interpretation of Quantum Theory in Terms of a Fluid with Irregular Fluctuations', *Physical Review* **96** (1954): 208-216.

There have been more recent attempts too. Antony Valentini has claimed to have done what Bohm failed to do in 1953.[64] Dürr, Goldstein and Zanghi also claimed to have shown that $P(\mathbf{x},t) = |\Psi(\mathbf{x},t)|^2$ using different assumptions.[65] What's more, they assert that Valentini's derivation is not only unnecessary, it is mathematically incorrect![66] Who one believes depends on what premises are found acceptable and whether the derivations are judged to be mathematically rigorous. It is not the intention here to attempt to decide this question. The status of the Quantum Equilibrium Condition and its possible derivation is a continuing area of research within the Causal Theory. A generally accepted proof of $P(\mathbf{x},t) = |\Psi(\mathbf{x},t)|^2$ would be a boost to the fortunes of the Causal Theory and one can only hope that a proof which gains general acceptance will be forthcoming in the near future.

We might also put to rest, in this last section, the claim that the Causal Theory is inconsistent (alleged flaw ⑥ in the General Introduction). If this were the case with the Causal Theory then one or more logical contradictions would be apparent (i.e. there would be obvious internal conceptual problems). However, the formalism, as presented in this chapter, can be seen to be a fully consistent mathematical scheme. Further, Bohm's papers are now over fifty years old and the occasional claims of inconsistency have never been established – indeed the opposite has been the case.[67] This suffices to dispose of alleged flaw ⑥. Alleged flaw ⑦ (i.e. that the Causal Theory cannot incorporate spin) will be addressed in Chapter Five.

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64 Valentini, A., 'Signal-Locality, Uncertainty, and the Subquantum H-Theorem'. I & II, *Physics Letters A* **156** (1991): 5-11 and **158** (1991): 1-8.

65 Dürr, D., Goldstein, S. and Zanghi, N., 'Quantum Equilibrium and the Origin of Absolute Uncertainty', *loc. cit.*, pp.856-858.

66 Dürr, D., Goldstein, S. and Zanghi, N., 'Quantum Mechanics, Randomness, and Deterministic Reality', *Physics Letters A* **172** (1992), p.11.

67 Hiley, B.J. and Peat, F.D., 'The Development of David Bohm's Ideas from the Plasma to the Implicate Order' in Hiley, B.J. and Peat, F.D. (eds), *Quantum Implications: Essays in Honour of David Bohm* (Routledge, London and New York, 1987) pp.7-8.

PART III

CONCEPTUAL AND ASSOCIATED THEORETICAL PROBLEMS

CHAPTER 3

ENERGY AND THE WAVE FIELD

… the principle of energy in its generality … is nowadays no longer disputed.

— Max Planck [1]

3.1 The Wave Field and the Concept of Energy

Since the initial development of quantum mechanics the wave field has been the subject of different views as to its role and ontological status. The name 'wave field', although of historical origin, is appropriate since it obeys the Principle of Linear Superposition.[2] In the early days of quantum theory, the mathematical formalism came first then its interpretation (or rather interpretations) ultimately resulting in acceptance by the majority of the physics community of what is now known as the Copenhagen Interpretation (i.e. Orthodox Quantum Theory). It is interesting to note the early thoughts of Max Born in regard to the wave field. Born initially ascribed some kind of reality to both particles and waves but thought that the waves did not carry energy or momentum.[3] Born later changed his conception of wave fields to that of 'waves of probability' and postulated that the square of the wavefunction provides a probability density for finding a particle.[4] There have been, of course, several other accounts postulated since Born's era. These range from the subjective view where the wave function merely represents an observer's knowledge of a quantum system,

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[1] Planck, M., *Treatise on Thermodynamics* (Longmans Green, London, 1927) p.41.

[2] Holland, P.R., *The Quantum Theory of Motion* (Cambridge University Press, Cambridge, 1993) p.69.

[3] Born, M., Letter to Renninger, quoted in Jammer, M., *The Philosophy of Quantum Mechanics* (Wiley, New York, 1974) p.495.

[4] Pais, A., *'Subtle is the Lord ...': The Science and the Life of Albert Einstein* (Oxford University Press, Oxford, 1984) p.442.

through to Everett's universal wave function which gave rise to the Many-Universes interpretation of quantum mechanics.[5]

Even amongst adherents to the Causal Theory, there is no unanimous agreement on the nature of the wave field. Franco Selleri considers the wave field to be real but, like Born's original view, he argues that it has zero energy content.[6] Another view appears in the later writings of David Bohm wherein the role of the wave field is presented in terms of his notion of 'active information'. Here the quantum potential is seen only as an 'information potential'.[7] The Active Information Hypothesis will be examined later in this chapter where it will be shown to be unacceptable in several respects. P.R. Holland furnishes yet another description of the wave field in his comprehensive text on the Causal Theory in which he not only argues that wave field carries energy, momentum and angular momentum through space but can do so far from the particle's location.[8] Holland's position is very close to that taken in this dissertation.

The physics literature pays little or no attention to elucidating a general definition of energy. Further, it is occasionally flagged that the quantity that is the same for all inertial frames is not energy but relativistic energy-momentum.[9] This implies that it is energy-momentum that is objectively real. However, in a non-relativistic context, we have to make do with energy and momentum separately. Given this, we may take energy as the relevant (real) aspect of system's energy-momentum in the non-relativistic domain.

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[5] A suitable summary of these views may be found in: Sudbery, A., *Quantum Mechanics and the Particles of Nature* (Cambridge University Press, Cambridge, 1986) pp.212-224.

[6] Selleri, F., 'On the Direct Observability of Quantum Waves', *Foundations of Physics* **12** (1982): 1087-1112.

[7] Bohm, D. and Hiley, B.J., *The Undivided Universe* (Routledge, London and New York, 1993) p.32.

[8] Holland, P.R., The Quantum Theory of Motion, *op. cit.*, p.84.

[9] Rindler, W., *Introduction to Special Relativity* (Pergamon, Oxford, 1982) pp.78-81.

In Classical Mechanics, energy is defined as the capacity of a physical system to perform work. The definition of energy then becomes dependent on the definition of work. This is usually given by an integral of the scalar product of force and displacement, i.e. $\int \mathbf{F} \cdot d\mathbf{x}$. This definition suffices for purely mechanical systems but has severe limitations in other contexts. Typically, different forms of energy (e.g. kinetic, gravitational, heat, etc.) are defined in each specific domain of physics. There is, however, no quantitative definition which covers all aspects of energy. Instead, there is a well known general concept of energy that draws on particular examples in order to illustrate itself.

Conservation of energy, on the other hand, is either postulated as a law (as in Thermodynamics) or derived as a theorem (e.g. Noether's Theorem) from a set of axioms depending on the area of physics involved. Given the importance in physics of both the concept of energy and its conservation, it is a little surprising that chemistry texts tend to discuss these issues in somewhat more detail than do most physics texts. One undergraduate chemistry textbook, for example, makes the following statement:

> No single theory of physics is more widely accepted or more generally useful [than conservation of energy], yet the statement [energy is conserved] refers to an abstract concept about a quantity never directly measured. We measure velocity and mass to calculate energy of motion. We measure an altitude … to determine energy of position. We measure moles of a substance to infer its chemical energy. We measure the change in the density of mercury to infer transfer of heat. Frequently, the main evidence for the existence of a

> quantity or type of energy is that apparently energy is not conserved unless some unseen energy is assumed.[10]

The concept of energy is abstract *only* in the sense that energy is not measured directly. Energy differences, however, provide a way of keeping track of changes in physical systems and assist in gaining an understanding of physical processes that would otherwise be unintelligible. It must also be acknowledged that energy is a real attribute of all physical systems.

This general concept and the characteristics of energy (e.g. exists in different forms, is able to be stored, transferred, transformed, etc.) are sufficiently well understood that these will be referred to without requiring further elaboration. Accounts of the wave field that mention 'energy' take the meaning of the term to be fully understood or at least understood from the context. However, throughout this thesis, the general concept of energy as outlined above will be used extensively. It will be useful, however, to explicitly state what is meant by 'conservation of energy' as a principle within the non-relativistic context:

- ♦ *Principle of the Conservation of Total Energy*

  The energy of a physical system is neither created nor destroyed, but may be transformed from one kind of energy into another, such that it is always theoretically possible to account for the total energy of a system.

The Principle of the Conservation of Total Energy is widely accepted and is one of the most empirically confirmed principles of physics, i.e. it is based on an enormous and extensive experimental basis.[11] In addition, it is not only at the 'observational

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[10] Pimentel, G.C. and Spratley, R.D., *Understanding Chemistry* (Holden-Day, San Francisco, 1971) p.248

[11] *McGraw-Hill Concise Encyclopedia of Physics* (McGraw-Hill, New York, 2005), p.116.

level' that we have reason to believe in energy conservation. There are also theoretical reasons for accepting energy conservation (such as Noether's Theorem).[12]

If the Conservation of Total Energy is construed as a law of nature applicable to individual processes (as is usually the case), then there is a conceptual problem for the Causal Theory in accounting for the energy of isolated quantum systems.

3.2 The Active Information Hypothesis

One influential rendering of the wave field's nature is due to David Bohm and Basil Hiley. This account incorporates their idea of 'active information'. Bohm's original description of causal quantum phenomena included a contribution to the total force exerted on a quantum particle from the quantum potential.[13] Later in his work with Hiley, Bohm abandoned the view that the wave field exerts a 'force' on quantum particles in favour of one in which the quantum potential becomes only an *information potential*. Bohm and Hiley postulated the existence of what they called 'active information' where the quantum potential is interpreted as representing information that encodes details relevant to the whole of a given experimental arrangement or environment. The information becomes 'active' upon entering an entity that can process the information (such as a quantum particle). Their basic hypothesis is that information carried by something with only a small amount of energy can direct something else with much greater energy.[14]

Why did Bohm abandon his earlier view of the quantum potential in favour of the Active Information Hypothesis? Consider the following analogy offered by Bohm and Hiley about a cork bobbing up and down as water waves travel past. The energy

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[12] Ho-Kim, Q., Kumar, N. and Lam, C.S., *Invitation to Contemporary Physics* (World Scientific, New Jersey, 2004) p.428.

[13] Bohm, D., 'A Suggested Interpretation ... I', *Physical Review* **85** (1952) p.170.

[14] Bohm, D. and Hiley, B.J., The Undivided Universe, *op. cit.*, p.35.

of the cork depends on the intensity of the water wave (where intensity is proportional to the square of the wave's amplitude). The greater the distance the cork is away from the cause of the water oscillations, the smaller will be the effect on the cork. The quantum potential effects do not, however, depend on the wave field's intensity since multiplication of the amplitude R by a constant cancels out in the expression for Q (as noted in Section 2.3). Bohm and Hiley described this as follows:

> … the effect of the quantum potential is independent of the strength (i.e., the intensity) of the quantum field but depends only on its *form*. By contrast, classical waves, which act mechanically (i.e., to transfer energy and momentum, for example, to push a floating object) always produce effects that are more or less proportional to the strength of the wave.[15]

Bohm and Hiley seemed to be very conscious that the effect of the quantum potential is independent of the intensity of the wave field, whereas a classical wave has effects that are (more or less) proportional to the strength of the wave. In light of this, they appear to have inferred that the wave field must act in a totally non-mechanical way, which depends only on the form of the wave field. (For a discussion of why Bohm might have changed his opinion, see Marcello Guarini's article.[16])

In the current context, the 'active information' (which is carried by the wave field and represented by the quantum potential) determines a quantum particle's path and its velocity by using the particle's own energy. Bohm and Hiley illustrated this idea with an analogy concerning a ship being automatically guided by a radio signal. The effect of the signal on the ship does not depend on its intensity, for a weak signal

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[15] Bohm, D. and Hiley, B.J., 'An Ontological Basis for the Quantum Theory I', *Physics Reports* **144** (1987) p.326 (their italics).

[16] Guarini, M., 'Bohm's Metaphors, Causality, and the Quantum Potential', *Erkenntnis* **59** (2003): 77-95.

will do just as well as a strong one (provided the radio signal is received properly). What is important is the form of the signal for this carries information which, when processed by the ship's autopilot, determines how the ship's own energy will be utilised. The information is described as 'active' when it has entered something which exploits its form (i.e. the information is processed):

> … the effect of the radio waves is independent of their intensity and depends only on their form. The essential point is that the ship is moving with its own energy, and that the *form* of the radio waves is taken up to direct the much greater energy of the ship.[17]

Bohm and Hiley argued that the quantum potential works in a similar manner – by 'informing' a quantum particle about how it will move under its own energy. The other illustrations provided by Bohm and Hiley are not especially helpful to their case and will not be discussed here.

The Active Information Hypothesis opens up a whole host of questions and issues that are extremely problematic. Consider first the difficulties encountered with particle structure. Quantum particles would require complex internal structures with which the 'active information' is processed in order that the particle be directed through space. Bohm and Hiley readily acknowledge this:

> The fact that the particle is moving under its own energy, but being guided by the information in the quantum field suggests that an electron or other elementary particle has a complex and subtle inner structure (e.g. perhaps even comparable to that of a radio).[18]

It has not been specified what these complex structures consist of or how they might be arranged within elementary particles. Nor has it been suggested how the actual

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[17] Bohm, D. and Hiley, B.J., The Undivided Universe, *op. cit.*, p.32 (their italics).
[18] *ibid.*, p.37.

processing of the 'active information' could occur. Bohm and Hiley's account is presented solely by way of alluding to a number of indirect analogies (e.g. portable radios, computers, DNA) and not by detailed and specific arguments. What's more, it seems likely that at least some fundamental particles do not have the kind of structure necessary. Electrons, which are a prime example for Bohm and Hiley, do not seem to have any constituent parts[19] and therefore cannot have a complex internal structure.

Second, consider the difficulties with satisfying physical laws. If information carried by something with only a small amount of energy is to direct something else with much greater energy, where does this greater energy come from in the case of quantum particles? Marcello Guarini has also expressed this question, stating:

> Radios have batteries or some other power source to draw on. Metaphorically speaking, where are the electron's batteries?[20]

Energy conservation necessitates that either the quantum particle would have to have an internal energy content to draw on or that energy be transferred to the particle from a source external to itself. Further, in the case of a particle that is increasing its speed, it would need a continuous supply of energy during periods of (positive) acceleration. After a large number of such speed increases (which might be interspersed with periods of deceleration), any internal energy content would become depleted. The particle would not be able to 'speed up' thereafter. If the particle's energy comes from an external source, what is it? Other than a mere conjecture about vacuum fluctuations as a possible reservoir of energy,[21] there is no explanation provided of where the required energy might originate from or how such energy might be 'tapped into'.

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[19] Veltman, M.J.G., *Facts and Mysteries in Elementary Particle Physics* (World Scientific, New Jersey, 2003) pp.54-55.

[20] Guarini, M., 'Bohm's Metaphors ... ', *loc.cit.*, p.82.

[21] Bohm, D. and Hiley, B.J., The Undivided Universe, *op. cit.*, p.48.

The Law of Inertia requires that there be some change made to a body's momentum for its path to be altered. If the Active Information Hypothesis is correct, then a quantum particle would have to be deviated from its initial trajectory (i.e. its momentum changed) as a result of the internal processing of the 'active information'. If we relate this to the ship analogy, a ship can have the highest quality radio receiver, a state-of-the-art autopilot, a large reserve of fuel (i.e. energy content), but if it has no engines then none of these other components will affect any change in the ship's momentum. We might ask, metaphorically, what constitutes the electron's engines? Bohm and Hiley give no indication as to how the interior make-up of a quantum particle can possibly affect its momentum.

Serious and substantial flaws have been highlighted in the Active Information Hypothesis. In summary, it is clear that the Active Information Hypothesis:

- leaves too many questions unanswered about its operation;
- cannot be applied to some elementary particles;
- would seem to require violations of the Law of Inertia; and
- does not provide a proper account of energy conservation.

These problems of the Active Information Hypothesis are sufficiently severe that they warrant its abandonment.

3.3 A Non-Interactive Approach to the Wave Field

An approach to the wave field that might be labelled 'non-interactive' has been proposed by Parmenter and DiRienzo. In their assessment, the Causal Theory has several attractive features which includes the possibility of addressing fundamental questions of quantum phenomena. Most of the familiar features of the Causal Theory are present in their account but the wave field does *not* exert any direct influence on quantum particles. Parmenter and DiRienzo pose the following questions:

> There are, however, weaknesses in the original [deBroglie-Bohm Causal] theory. One of the most obvious of these relates to the quantum potential Q: What is its source? Typically in physics a force, and its associated potential, have a source. However, nowhere in the literature is this fundamental question addressed in a physically reasonable way.[22]

Parmenter and DiRienzo provide their own answer to the origin of the force associated with the quantum potential (i.e. the quantum mechanical force).

They begin with an isolated, many-particle quantum system which has a quantum potential $Q = Q(x_1, x_2, x_3, ..., x_N, t)$ given by:

$$Q = -\sum_{i=1}^{N} \left(\frac{\hbar^2}{2m_i R}\right) \nabla_i^2 R$$

such that for the *i*-th particle:

$$(d\mathbf{p}_i/dt) = -\nabla_i V - \nabla_i Q$$

where $\mathbf{p}_i$ is the momentum of the *i*-th particle, $-\nabla_i V$ is the sum of all the classical forces on the *i*-th particle, and $-\nabla_i Q$ is interpreted as the quantum force on the *i*-th particle. The total momentum $\mathbf{p}$ of an N-particle system is:

$$\mathbf{p} = \sum_{i=1}^{N} \mathbf{p}_i$$

For a classically free system (i.e. where V = 0) we have:

$$\frac{d\mathbf{p}}{dt} = -\sum_{i=1}^{N} \nabla_i Q$$

Parmenter and DiRienzo assume that for an isolated quantum system, $(d\mathbf{p}/dt) = 0$.

This condition requires that:

$$-\sum_{i=1}^{N} \nabla_i Q = \sum_{i=1}^{N} \mathbf{F}_i = 0 \quad .... (3\text{-}1)$$

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22 Parmenter, R. H. and DiRienzo, A.L., 'Reappraisal of the causal interpretation of quantum

where $\mathbf{F}_i$ is the quantum force on the *i*-th particle. If we let N = 2, then $\mathbf{F}_1 = -\mathbf{F}_2$. This suggests to Parmenter and DiRienzo that the source of the quantum mechanical force on one particle is just the other particle. More generally, they conclude that the quantum mechanical force on a given particle is a force of constraint with its origin being all the other particles in the N-particle system.[23] The exact nature of the 'quantum force' is *unspecified* with the quantum potential Q acting as an intermediary which is also unspecified by them, except to hypothesise that Q results from a non-holonomic constraint on the system,[24] i.e. a constraint that cannot be expressed as an equation in the form $f(x_1, x_2, x_3, ..., x_N, t) = 0$, which relates the coordinates of the particles and time.[25]

Parmenter and DiRienzo use the rather curious argument that *if* Q is removed from the Quantum Hamilton-Jacobi equation, *then* the Schrödinger equation would be modified by the addition of the term:

$$\frac{\hbar^2}{2m}\left(\frac{\nabla^2|\Psi|}{|\Psi|}\right)\Psi = -Q\Psi$$

which would make the Schrödinger equation non-linear. There would, of course, be many consequences (both mathematical and empirical) that would follow from such a non-linear equation. One consequence would be a different Hamiltonian. This Hamiltonian suggests to Parmenter and DiRienzo that there is a possibility of deterministic chaos in the time development of different wavefunctions.[26] They then hypothesised that the quantum potential Q results from a constraint which prevents such deterministic chaos for wavefunctions, presumably because when Q is present there is no possibility of the kind of deterministic chaos envisaged. The argument for

mechanics and of the quantum potential concept', *arXiv:quantum-ph/0305183* (22 May 2004) p.2.

[23] *ibid.*, p.7.

[24] *ibid.*, pp.4 & 7.

[25] Goldstein, H., *Classical Mechanics* (Addison-Wesley, Reading, M.A., 1980) p.12.

[26] Parmenter, R. H. and DiRienzo, A.L., 'Reappraisal ... ', *loc. cit.*, p.9.

the existence of this constraint is invalid for it is a not legitimate approach to simply 'pluck-out' a term in an equation of physics without detailed and careful justification. This cannot be the case with the quantum potential for Q is not just another potential function that is added to the classical Hamilton-Jacobi equation.[27] Nor can Q be adjusted to zero as can some types of externally imposed potentials. Further, the notion of a mathematical constraint is that it restricts the possible solutions of the equation governing the phenomenon under study, not that it can add or subtract terms from the governing equation. In any case, the removal of the quantum potential from the Quantum Hamilton-Jacobi equation formally changes this to its classical counterpart, i.e. the subject matter is no longer quantum mechanics!

The overall solution of Parmenter and DiRienzo must also be seen to be unsuccessful for it cannot explain the motion of a single quantum particle such as occurs in the Double Slit experiment when only one particle is present between the slits and the screen at any particular time. The defect in their argument occurs in the assumption of equation (3-1). In the one-particle Double Slit arrangement $(N = 1)$, if $\mathbf{F} = -\nabla Q = (d\mathbf{p}/dt) = 0$, then it would follow that the momentum $\mathbf{p}$ = constant. In other words, the sole quantum particle will execute rectilinear motion and consequently, the familiar two slit diffraction pattern cannot be formed. This example clearly shows that quantum mechanical force cannot be due to the particles of a quantum system.

The 'non-interactive' approach to the wave field of Parmenter and DiRienzo, like the Active Information Hypothesis, needs to be abandoned in favour of a more promising line of development. This will be presented in Chapter Four in terms of the physical characteristics of the wave field.

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[27] Kyprianidis, A., 'Hamilton-Jacobi Theory and Quantum Mechanics I. The Free Particle Case', *Physics Letters A* **131** (1988) p.412.

3.4 The Physical Nature of Potential Energy

Before proceeding to an alternative account of the motion of quantum particles, it will be necessary to deal with a 'thorny' issue of the physical underpinings of potential energy, as this will be pivotal to subsequent discussion. In undergraduate studies of (classical and quantum) mechanical systems, the energy of a system is divided into kinetic and potential quantities. Potential energy is introduced to account for the ability of a physical entity to perform work on its surroundings (where work is given by the product of force and displacement) and for energy conservation. The formal potential energy term is (explicitly or implicitly) defined as the potential energy of a particle or object. In electrostatics for example, a (point) particle with an electric charge q_1 at a distance r from another particle with an electric charge q_2 is defined to have a potential energy V given by:

$$V = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r} \quad \ldots (3\text{-}2)$$

where ε is the electric permittivity constant. Explicit statements that particles possess potential energy may be found in many introductory texts.[28] Such definitions of potential energy are drummed into students to the extent that it is (almost universally) accepted that potential energy is a particle characteristic which depends on its position. This is, in a strict sense, *incorrect*. The labelling of potential energy as a particle attribute is only a convenient description which is a statement of convention and not a matter of physical reality.

The familiar potential energy term is a potential energy function that represents an amount of field energy that is available to a particle situated within the

[28] Explicit examples include: Stephenson, R.J., *Mechanics and the Properties of Matter* (Wiley, New York and London, 1969) p.70; Giancoli, D.C., *Physics: Principles with Applications* (Prentice-Hall, New Jersey, 1985) p.100; Nolan, P.J., *Fundamentals of College Physics* (W.C. Brown, Dubuque, Iowa, 1993) p.189.

field. In other words, potential energy is energy properly associated with fields, not particles. Hans Freistadt stressed this very point in an article written in the 1950s:

> V [potential energy] is merely a shorthand way of writing …
> an energy which really resides in the field.[29]

A field, for example, may be present in a spatial region which is totally devoid of any particles because fields can propagate enormous distances into otherwise empty regions of space, regions which might be many cubic light-years in size. Yet, despite the absence of particles, such a spatial region possesses a (potential) energy density due to the presence of a field.[30]

The fact that it is physical fields that are repositories of potential energy (and not particles) was emphasised by D.W. Theobald in his classic treatise, *The Concept of Energy*:

> … the field is characterised by the presence of energy … A field is nothing more than a spatial distribution of energy which varies with time. [The concept of] Energy has thus been freed from its dependence upon physical vehicles such as particles …[31]

Potential energy being a field attribute is rarely stated in the physics literature and indeed, assigning potential energy to particles is a standard and unquestioned practice. This is because, in the majority of physical contexts (such as particle mechanics) it makes no difference to the final result by assigning potential energy to a particle. Further, regarding potential energy as a particle property is easier to use and simpler for students to assimilate for this treatment acts as a kind of 'shortcut' to the actual location of potential energy in a field. This 'shortcut', however, *is not*

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[29] Freistadt, H., 'The Causal Formulation of the Quantum Mechanics of Particles', *Supplemento Nuovo Cimento* V (1957) p.17.
[30] Jackson, J.D., *Classical Electrodynamics* (Wiley, New York, 1975) p.46.
[31] Theobald, D.W., *The Concept of Energy* (Spon Publishers, London, 1966) p.98.

possible in all physical situations, in particular those involving non-linear interactions.[32]

A notable exception to labelling potential energy as a particle attribute appears in the works of the respected physicist Wolfgang Rindler. He writes:

> In classical mechanics, a particle moving in an electromagnetic (or gravitational) field is often said to possess potential energy, so that the sum of its kinetic and potential energies remains constant. This is a useful "book-keeping" device, but energy conservation can also be satisfied by debiting the *field* with an energy loss equal to the kinetic energy gained by the particle.[33]

This is a theme that is repeated throughout Rindler's writings and one that he has emphasised in a number of texts (although the message seems to have 'fallen on deaf ears'). In another textbook, he states:

> … [particle] potential energy, which is really nothing but a useful 'book-keeping' device. But physically it is more satisfactory to credit the field *itself* with whatever momentum or energy is required to 'balance the books'.[34]

Surely then, this is the critical point – a physically satisfactory account of the nature of potential energy in both linear and non-linear interactions requires that fields, *not particles*, possess potential energy.[35]

In order to illustrate this, consider the following two examples. The first example concerns the everyday supply of household electricity. In most industrialised countries, electricity is supplied by large power generation stations through heavy duty metallic cables using a form of alternating current, i.e. current that changes

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[32] Freistadt, H., 'The Causal Formulation … ', *loc. cit.*, p.17.

[33] Rindler, W., *Essential Relativity: Special, General, and Cosmological* (Springer, Berlin, 1977) p.83 (his italics).

[34] Rindler, W., *Introduction to Special Relativity* (Pergamon, Oxford, 1982) p.132 (his italics).

direction over a short time interval (typically with a frequency of 50-60 Hertz). The regular change in the polarity of the electricity requires the electrons in the cables to oscillate back and forth about equilibrium positions. Consequently, there is no net electron flow along the cables from the generator to the consumer. The electrons cannot, therefore, transport the electrical energy since they do not travel from source to user. Instead, the electrical energy is transferred as potential energy in the generated electric field. It is the field and not the particles that possess potential energy.

A second example may bring this into sharper focus. Consider an electrically charged particle placed in an external electric field. Such an external field may be produced by applying an electrical potential difference to two (usually parallel) metal plates. Also assume that this is done in inter-galactic space so that the effects of gravity and air resistance will be totally negligible. If the charged particle is released at rest between the plates before they become charged, the particle remains at rest. However, if the particle is released at rest between the plates when they are charged, the particle will immediately accelerate. The electric field between the charged plates imparts energy to the particle as it had no kinetic energy initially. This energy is gained at the expense of some (but not all) of the potential energy stored in the field between the charged plates, i.e. by a small fraction of the potential energy contained within the external electric field. Moreover, if we were to 'shoot' the charged particle in a direction towards the plate of similar charge to itself, the particle would decelerate and then come to a (momentary) stop. The particle's kinetic energy would then be instantaneously zero. If at the instant when the particle stops, we arrange for the electric field between the plates to be zero, then the value of the potential energy would also be zero. If potential energy is taken to be a particle property, then *all* the

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[35] Rindler, W., *Relativity: Special, General, and Cosmological* (Oxford University Press, Oxford,

particle's energy (i.e. kinetic and potential) would have just disappeared from existence! This situation is inexplicable. The loss of potential energy when the external field is turned off can only be accounted for in a physically reasonable way if potential energy is contained in the field.

It is also important to distinguish between the potential energy available to a particle situated in a field and the total energy stored in the field. In the current example, if the plates are the same size and shape, are parallel, and the particle is a perpendicular distance y from the plate of opposite charge, then the former energy is given by (qEy), where q is the particle's electric charge and E is the strength of the electric field. The total energy stored in the field is given by ($\frac{1}{2}\varepsilon AdE^2$) where A is the surface area of one plate, and d is the separation of the plates.[36] The amount of potential energy available to the particle depends on a number of factors such as the particle's location in the field and how the field's amplitude varies. In this respect, the physicist and mathematician Hermann Weyl wrote:

> Not only the field as a whole, but every portion of the field
> has a definite amount of potential energy ...[37]

The proper characterisation of potential energy as field energy will permit the solution of a significant conceptual problem of the Causal Theory, viz. energy conservation in quantum systems. This will be done in Chapter Four.

3.5 The Existence and Characteristics of the Wave Field

If quantum entities consist of both particles and waves, then it should not only be not surprising that atomic and elementary particle experiments show particle and wave

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2001) p.113.

[36] Johnk, C.T.A., *Engineering Electromagnetic Fields and Waves* (Wiley, New York, 1975), p.210.

[37] Weyl, H., *Space-Time-Matter* (Dover, New York, 1952) p.70.

aspects, it should be expected. The objective existence of the wave field is an essential characteristic of the Causal Theory, as John Bell has commented:

> *No one can understand this [Causal] theory until he is willing to think of* ... [the wave field] *as a real objective field rather than just a 'probability amplitude'.*[38]

In this section will be discussed some of the reasons for accepting the existence of the wave field and for holding to the quantum potential approach. In doing so, we shall review three empirically significant phenomena.

There are 'minimalist' accounts of the Causal Theory that postulate only the Schrödinger and guidance equations, the Quantum Equilibrium Condition and the existence of quantum particles with definite positions.[39] Mathematically speaking, it is the case that these accounts will produce all the predictions of quantum mechanics in addition to the trajectories of quantum particles. However, they will also fall short of a full causal explanation of quantum behaviour because such accounts are primarily *kinematic* descriptions. Just as in Classical Mechanics a complete explanation of physical phenomena requires the explication of the dynamics of the system under study, so too in the microscopic realm, the dynamics of a quantum system is required. We shall see that such an explanation is provided by the existence and function of the quantum potential. In particular, the quantum potential is essential to account for the conservation of energy.[40] This is, of course, one reason why the concept of potential energy was *originally introduced* into physics. Indeed, solutions to problems in theoretical chemistry and solid state physics within the

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[38] Bell, J.S., 'Quantum Mechanics for Cosmologists' in his *Speakable and Unspeakable in Quantum Mechanics* (Cambridge, Cambridge University Press, 1987) p.128 (his italics).

[39] E.g. Dürr, D., Goldstein, S. and Zanghi, N., 'Bohmian Mechanics as the Foundation of Quantum Mechanics' in Cushing, J.T., Fine, A. and Goldstein, S. (eds), *Bohmian Mechanics and Quantum Theory: An Appraisal* (Kluwer, Dordrecht, 1996).

[40] Holland, P.R., The Quantum Theory of Motion, *op. cit.*, p.78.

context of the Causal Theory *require* application of the quantum potential approach.[41]

In regard to the question of the energy content of the wave field, it should be recognised that to claim something to be real and yet possess no energy at all would go against long established physical results. This notion is embodied in physics (for example by the laws of Thermodynamics) and will be stated as the following general principle:

♦ *Principle of Energy Content*

Every physically real entity in the universe contains some finite quantity of energy.

In accordance with this principle, the wave field will always possess some amount of energy, although it may be exceedingly small at times in comparison to the kinetic energy of the accompanying quantum particle.

Consider now the three examples of empirically significant phenomena mentioned above. Two of these (the Double Slit Experiment and the Aharonov-Bohm Effect) are exemplars wherein the assumption of an objectively existing wave field provides a coherent, realistic and causal explanation. The third (the manipulation of matter waves) is an example of a phenomenon made possible only by recent developments in laser technology, the causal explanation of which also requires accepting the wave field as physically real.

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[41] Examples include: Garashchuk, S. and Rassolov, V.A., 'Energy Conserving Approximations to the Quantum Potential: Dynamics with Linearized Quantum Force', *Journal of Chemical Physics* **120** (2004): 1181-1190; 'Quantum Dynamics with Bohmian Trajectories: Energy Conserving Approximation to the Quantum Potential', *Chemical Physics Letters* **376** (2003): 358-363; and Grubin, H.L., Kreskovsky, J.P., Govindan, T.R. and Ferry. D.K., 'Uses of the Quantum Potential in Modelling Hot-Carrier Semiconductor Devices', *Semiconductor Science and Technology* **9** (1994): 855-858.

The Double Slit Experiment

The classic Double Slit Experiment with light is credited to Thomas Young in 1803. The experiment was first performed with electrons by Claus Jönsson in 1959.[42] The behaviour of a single quantum particle passing through a double slit arrangement can be found using the quantum potential approach. At a large distance from the slits, the value of the wave field's amplitude R is constant. The quantum potential, therefore, is zero since $\nabla^2 R = 0$ for constant R. The particle's trajectory, at this time, will be as predicted by Classical Mechanics.

It is well known that the diffraction pattern obtained in the Double Slit experiment is different when two slits are open from when only one is open. The Causal Theory gives a explanation for this behaviour. When both slits are open, a quantum particle will pass through one, and only one, of the slits (or will impact on the barrier in which the slits are cut). The wave field, however, passes through *both* slits and the emergent waves interfere with each other. When a particle passes through a slit, it experiences rapidly varying values of the quantum potential, as the value of R changes with position due to self-interference of the wave field.[43] Bohm provided a succinct description:

> ... A particle is incident on this system, along with its quantum wave. While the particle can only go through one slit or the other, the wave goes through both. On the outgoing side of the slit system, the waves interfere to produce a complex quantum potential which does not in general fall off with distance from the slits.[44]

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[42] Jönsson, C., *Zeitschrift für Physik* **161** (1961): 454-474. English reprint: 'Electron Diffraction at Multiple Slits', *American Journal of Physics* **42** (1974): 4-11.
[43] Bohm, D., 'A Suggested Interpretation ... I', *Physical Review* **85** (1952) p.174.
[44] Bohm, D.J. and Hiley, B.J., 'An Ontological Basis for the Quantum Theory I: Non-relativistic Particle Systems', *Physics Reports* **144** (1987), p.326.

Particles that pass through the slits will have their trajectories altered from a straight path by the action of the quantum mechanical force in a manner such that the familiar two slit interference pattern emerges if sufficient numbers of particles are allowed to pass through the slits, as shown in Figure 1 (below).

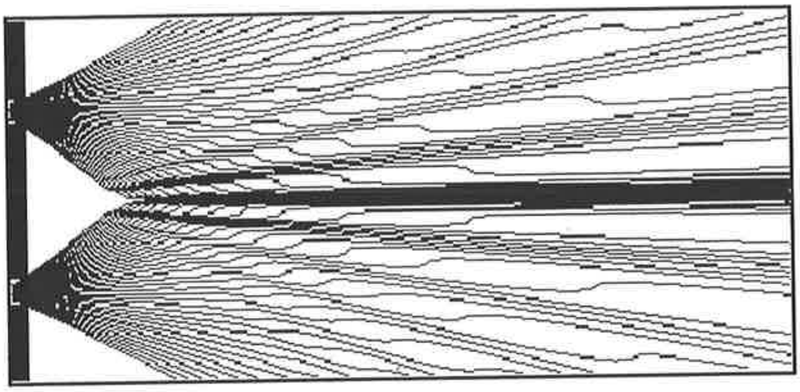

Figure 1: Trajectories for the two-slit experiment [45]

Since the probability density is $P(\mathbf{x}) = |\Psi|^2$, it may be concluded that the particle cannot be found at any point where the wavefunction vanishes. What's more, the particle trajectories do not cross the line of symmetry between the slits, so that particles that are incident on the left (right) side of the screen passed through the left (right) slit. These trajectories are similar to those calculated using electromagnetic energy flow lines from a two slit arrangement.[46] The calculated particle trajectories in the Double Slit Experiment constitute an example of what Bohr, Heisenberg, and Feynman (to name a few) explicitly declared to be impossible. Their attitude is summarised in Feynman's well known textbook, *The Feynman Lectures on Physics*:

> We choose to examine a phenomenon [the double-slit experiment] which is impossible, *absolutely* impossible, to explain in any classical way, and which has in it the heart of quantum mechanics.[47]

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[45] Philippidis, C., Dewdney, C. and Hiley, B.J., 'Quantum Interference and the Quantum Potential', *Il Nuovo Cimento B* **52** (1979) pp.21-23.

[46] Prosser, R.D., 'Quantum Theory and the Nature of Interference', *International Journal of Theoretical Physics* **15** (1976) p.190.

[47] Feynman, R.P., Leighton, R. B. and Sands, M., *The Feynman Lectures on Physics* (Addison-Wesley, 1963) Vol. 1, p.37-2 (their italics).

It is clear from the context of Feynman's comments that by "classical way", he meant an explanation in terms of the particles having well-defined trajectories through space from the slits to the screen.

In respect to trajectories calculated using the Causal Theory, the research group led by B.-G. Englert has alleged that the Double Slit trajectories are "surrealist", i.e. not physically real.[48] This claim has been explicitly addressed by Detlef Dürr and his colleagues and independently by Basil Hiley and his associates.[49] The latter response is very detailed and concludes that the trajectories provide a deep insight into quantum processes.

The Aharonov-Bohm Effect

In 1959, Yakir Aharonov and David Bohm calculated that there would be a shift in the fringes of a double slit arrangement with electrons when an energised cylindrical solenoid is placed in the geometric shadow of the two electron beams emanating from the slits.[50] This is the Aharonov-Bohm Effect. The calculated trajectories for electrons affected by the Aharonov-Bohm Effect are shown in Figure 2.

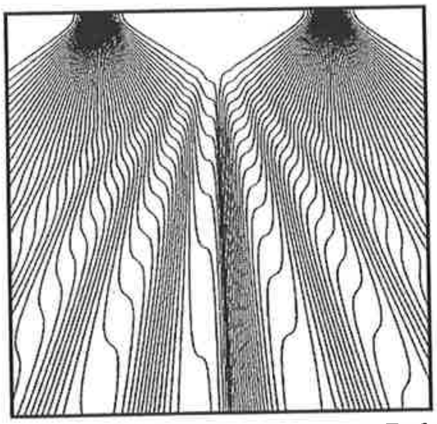

Figure 2: Trajectories for the Aharonov-Bohm Effect [51]

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[48] Englert, B., Scully, M., Sussman, G. and Walter, H., 'Surrealistic Bohm Trajectories', *Zeitschrift für Naturforschung A* **47** (1992): 1175-1186.

[49] Dürr, D., Fusseder, W. and Goldstein, S., 'Comment on Surrealistic Bohm Trajectories', *Zeitschrift für Naturforschung A* **48** (1993): 1261-1262; Hiley, B.J., Callaghan, R.E. and Maroney, O.J.E., 'Quantum Trajectories, Real, Surreal or an Approximation to a Deeper Process? ', *arXiv:quant-ph/0010020* (2000).

[50] Aharonov, Y. and Bohm, D., 'Significance of Electromagnetic Potentials in the Quantum Theory', *Physical Review* **115** (1959): 485-491.

[51] Philippidis, C., Bohm, D. and Kaye, R.D., 'Bohm-Aharonov Effect and the Quantum Potential', *Il Nuovo Cimento B* **71** (1982) p.84.

The Aharonov-Bohm Effect has been experimentally confirmed.[52] The electromagnetic vector potential **A** related to a magnetic field may be defined (up to a gauge transformation) by: $\mathbf{B} = (\nabla \times \mathbf{A})$, where **B** is the (classical) magnetic induction. The Schrödinger equation including the electromagnetic vector potential takes the form:

$$i\hbar \frac{\partial \Psi}{\partial t} = \frac{1}{2m} (-i\hbar \nabla - \frac{e}{c}\mathbf{A})^2 \Psi \quad .... (3\text{-}3)$$

where e is the electronic charge and c is the speed of light in vacuum.[53] The interesting feature of the Aharonov-Bohm Effect is that the electrons only pass through space where there is no magnetic field strength (i.e. regions where $\mathbf{B} = 0$). The solenoid produces a closed loop of magnetic flux which is zero outside the solenoid. The vector potential, however, cannot be zero in these regions or the enclosed magnetic flux loop would also be zero. The presence of the vector potential **A** in equation (3-3) induces a phase shift in the wavefunctions of the electrons emerging from both slits from what they would be with $\mathbf{A} = 0$. This phase shift alters the interference pattern that results when the two electron beams combine. The shift of the interference fringes is evident if one compares the pattern of trajectories as shown in Figure 1 (no phase shift) with the pattern shown in Figure 2.

The Causal Theory provides an understanding of why the electron trajectories change (and therefore the interference pattern) in terms of the quantum potential Q. The quantum mechanical force, i.e. $(d\mathbf{p}/dt) = -\nabla Q$, is present even if the magnetic field and the (classical) Lorentz Force (i.e. $\mathbf{F} = e\mathbf{v} \times \mathbf{B}/c$, where **v** is the electron's velocity) are zero. The usual Double Slit pattern is explained by the variation of the quantum potential due to self-interference of the wave field (as outlined above). The

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[52] Tonomura, A., Osakabe, N., Matsuda, T., Kawasaki, T., Endo, J., Yano, S. and Yamada, H., 'Evidence for Aharonov-Bohm Effect with Magnetic Field Completely Shielded from Electron Wave', *Physical Review Letters* **56** (1986) p.792.

additional phase shift changes the values of the quantum potential from those that occur when the vector potential is absent. The quantum mechanical force is therefore also changed resulting in different electron trajectories and an altered interference pattern. Bohm and Hiley described these circumstances as follows:

> We see then that in general … there is a quantum force which is present even when the magnetic field is zero. … It is clear that Q [the quantum potential] will be large only where … the two beams overlap … The phase shift will then alter the quantum potential in a significant way and this will explain the origin in the shift of the interference pattern.[54]

The Aharonov-Bohm Effect is therefore explained if the reality of the electromagnetic vector potential **A** is accepted and its effect on electron trajectories through the modification of the quantum potential.[55]

Matter Wave Manipulation

There is also mounting evidence for the existence of wave fields from the new experimental area of atom optics, where the term 'matter waves' is used in preference to 'wave fields'. Applications of laser technology have now made possible the control of atoms consistent with the manipulation of their matter waves.[56] The atoms of a gas can be put into the same quantum state by the process of Bose-Einstein Condensation.[57] This is done using 'laser cooling' which reduces the temperature of the gas to a fraction of a degree above Absolute Zero. Once in this state, the de Broglie wavelength of the atoms is approximately equal to the spacing between individual atoms. The atoms then have a dominant wave behaviour that allows

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53 Bohm, D. and Hiley, B.J., The Undivided Universe, *op. cit.*, p.51.

54 *ibid.*, p.52.

55 Holland, P.R., The Quantum Theory of Motion, *op. cit.*, p.195.

56 Milburn, G.J., *Quantum Technology* (Allen & Unwin, Sydney, 1996) Chapter 3.

57 Strecker, K.E., Partridge, G.B., Truscott, A.G. and Hulet, R.G., 'Formation and Propagation of Matter Wave Soliton Trains', *Nature* **417** (May 2002): 150-153.

manipulation by laboratory atom-optical devices.[58] Although the matter wave (i.e. wave field) is not directly observable, the fact that significant quantities of matter can be diffracted, focussed, reflected, etc., using essentially optical devices is a powerful indication that the matter waves are physically real.

Recent experiments of 'matter wave amplification' offer further evidence for the existence of wave fields.[59] The term 'matter wave amplification' refers to the production of an output of atoms with particular properties from a holding reservoir of atoms (an atom trap) using a process similar to the stimulated emission of light in a laser. A initial matter wave is created and then amplified by using the (Bose-Einstein condensate) atoms in the reservoir as a gain medium. This produces atoms with the desired properties in large numbers. The particular properties that the output atoms acquire is that they have the same momentum and phase relations as the atoms used as input. This process has been described by one of the experimental groups conducting research into 'matter wave amplification' as follows:

> … we report the observation of phase-coherent amplification of atomic matter waves. The active medium is a Bose-Einstein condensate … An atomic wave packet is split off the condensate by diffraction from an optical standing wave, and then amplified. We verified the phase coherence of the amplifier by observing interference of the output wave with a reference wave packet.[60]

A matter wave is not directly observable but the coherence of the matter wave produced in the output has been established by using interferometers. However, if a matter wave really can be 'amplified' then it logically follows that it must exist in

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58 Helmerson, K., 'Giving a Boost to Atoms', *Nature* **402** (9 Dec. 1999), p.587.

59 Kozuma, M., Suzuki, Y., Torii, Y., Sugiura, T., Kuga, T., Hagley, E.W. and Deng, L., 'Phase-Coherent Amplification of Matter Waves', *Science* **286** (1999): 2309-2312; Inouye, S., Pfau, T., Gupta, S., Chikkatur, A.P., Görlitz, A., Pritchard, D. E. and Ketterle, W., 'Phase-coherent Amplification of Atomic Matter Waves', *Nature* **402** (9 Dec. 1999): 641-644; Schneble, D.,

order to be acted upon. This explanation requires accepting the wave field as physically real.

These three examples (Double Slit, Aharonov-Bohm Effect and matter wave manipulation) although not providing absolutely conclusive evidence, nevertheless lend strong support to the proposition that the wave field has an objective existence and, most importantly, is *causally efficacious* in bringing about observed quantum phenomena.

If the wave field is objectively real then it is to be expected that it will have characteristics in common with classical fields and waves. The wave field will, of course, also have non-classical features. The wave field and its quantum particle are *physically* inseparable aspects of a single quantum entity. Bohm himself, stressed that quantum theory needed *some* non-mechanistic descriptions and emphasised the importance of a holistic view.[61] These stipulations, however, do not prevent an *in-principle* analysis of the characteristics and causal function exhibited by wave fields.

The quantum potential performs similar roles to those of classical potentials. This is evident in situations where a quantum particle is subject to both classical and quantum potentials. Given the explanation of potential energy in Section 3.4, a (partial) answer to the question of what constitutes the quantum potential may be fleshed out in terms of Q being the potential energy function of the wave field (see Section 4.4 for more discussion on this issue). The quantum potential has some features in common with classical potentials for this reason, such as the relationship expressed by equation (2-5), viz., $(d\mathbf{p}/dt) = -\nabla(V + Q)$ which shows that classical and quantum potentials are on an 'equal footing' in regard to affecting the particle's

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Campbell, G.K., Streed, E.W., Boyd, M., Pritchard, D.E. and Ketterle, W., 'Raman Amplification of Matterwaves', *Physical Review A* **69** (2004): 041601-1 — 041601-4 (R).

[60] Inouye, S. et al., 'Phase-coherent Amplification of Atomic Matter Waves', *ibid.*, p.641.

[61] Bohm, D., *Quantum Theory* (Prentice-Hall, New York., 1951) pp.166-167.

motion.[62] However, Q is not completely equivalent to an external classical potential and could not be so for the following reasons. Classical potentials are due to fields which do not, in general, travel along with the particle, i.e. the particle is embedded in the wave and together they constitute a single quantum entity. Nor is a classical field that is externally imposed on a particle intrinsic to a physical state of the particle in the way that the wave field is to a quantum state. The quantum potential is also not a preassigned function of coordinates as are classical potentials.[63] Holland uses the term 'internal potential' to distinguish the quantum potential from potentials that are externally imposed on a quantum system.[64]

3.6 The Gaussian Wave Packet Representation of the Wave Field

In his original pilot wave theory, Louis de Broglie postulated that a quantum particle is always situated inside an envelope of waves that guides the particle.[65] Such a wave packet can be described mathematically by the superposition of an infinite number of monochromatic plane waves differing only slightly in wavelength. A wave packet description is consistent with Axiom II of the Causal Theory for if the wavefunction is bounded, then its amplitude tends to zero with increasing distance from the quantum particle, i.e. $\Psi \to 0$ as $r \to \infty$. However, it should be kept in mind that the superposition that forms the wave envelope is part of a model and although we take the wave field to be a real entity, the infinite number of plane waves used in the superposition description of the wave packet is a mathematical convenience only. This is an example of not taking a theory *literally in all respects* (as discussed in Chapter One).

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62 Holland, P.R., The Quantum Theory of Motion, *op. cit.*, p.74.
63 *ibid.*
64 *ibid.*, p.63.
65 de Broglie, L., 'A Tentative Theory of Light Quanta', *The London, Edinburgh and Dublin Philosophical Magazine and Journal of Science* 47 (1924) p.450.

Although there are several types of (mathematical) wave packets, one description of an envelope of waves which has particularly useful properties (such as being able to be solved exactly) is the Gaussian wave packet. Gaussian distribution functions are standardly employed in statistics and probability theory (such as depicted in Figure 3).[66] The curve has the same shape as the normal distribution with a standard deviation σ.

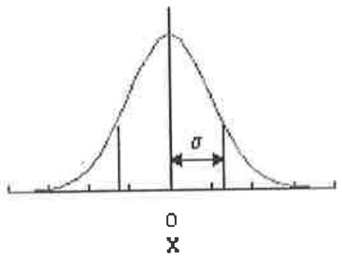

Figure 3: Gaussian distribution curve

The form of the Gaussian wave packet for a classically-free quantum particle and the associated Gaussian expressions will now be derived. These expressions will be used to illustrate important results and to solve some theoretical problems in Chapter Four.

We shall start with a physically acceptable (i.e. normalisable), configuration space wavefunction $\psi(x, t)$ which may be expressed by the following integral (in one-dimension):[67]

$$\psi(x, t) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \phi(p) \exp\left[\frac{ipx}{\hbar} - \frac{ip^2 t}{2m\hbar}\right] dp$$

where $\phi(p)$ are the corresponding momentum space wavefunctions. Then at time t = 0, we have:

$$\psi(x, 0) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \phi(p) \exp\left[ipx/\hbar\right] dp$$

A Fourier transform will give $\phi(p)$ if $\psi(x, 0)$ is specified, viz.

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[66] Boas, M.L., *Mathematical Methods in the Physical Sciences* (Wiley, New York, 1966) p.708.

$$\phi(p) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \psi(x', 0) \exp[-ipx'/\hbar]\, dx'$$

where position coordinate x' relates to the particle at time t = 0. Then it follows that:

$$\psi(x, t) = \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \psi(x', 0) \exp\left[\frac{ip(x - x')}{\hbar} - \frac{ip^2 t}{2m\hbar}\right] dx'\, dp$$

$$= \frac{m}{\sqrt{2\pi i\hbar t}} \int_{-\infty}^{\infty} \psi(x', 0) \exp\left[\frac{-im(x - x')^2}{2\hbar t}\right] dx' \quad \ldots (3\text{-}4)$$

where standard definite integrals (as listed in Appendix III) have been employed.

It is stated in the relevant literature that the initial form of a normalised Gaussian wave packet is given by:[68]

$$\Psi_0(\mathbf{x}) = \Psi(\mathbf{x}, 0) = (2\pi\sigma_0^2)^{-3/4} \exp\{i\mathbf{k}\cdot\mathbf{x} - (\mathbf{x}^2/4\sigma_0^2)\}$$

where σ_0 is the initial root-mean-square (RMS) width of the packet in each coordinate direction with $\sigma_0^2 = \langle x^2 \rangle = \langle y^2 \rangle = \langle z^2 \rangle$. The expression for $\Psi_0(x)$ is also called a minimum uncertainty wave packet because it yields the equality: $(\Delta p)(\Delta x) = (\hbar/2)$.[69] At times t > 0, the Gaussian wave packet will evolve according to the classically-free (i.e. V = 0) Schrödinger equation. In one dimension, the initial normalised Gaussian packet is:

$$\psi(x, 0) = (2\pi\sigma_0^2)^{-1/4} \exp\{(ipx/\hbar) - (x^2/4\sigma_0^2)\} \quad \ldots (3\text{-}5)$$

with $k = (p/\hbar)$.[70] If we replace x by x' in equation (3-5) and then substitute into equation (3-4), we get:

$$\psi(x, t) = \frac{m}{(2\pi)^{1/4}\sqrt{i\hbar t\sigma_0}} \int_{-\infty}^{\infty} \exp\left[\frac{ip\,x'}{\hbar} - \frac{x'^2}{4\sigma_0^2} - \frac{im}{2\hbar t}(x^2 - 2xx' + x'^2)\right] dx'$$

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67 Saxon, D.S., *Elementary Quantum Mechanics* (Holden-Day, San Francisco, 1968) pp.31 & 60.

68 Belinfante, F.J., *A Survey of Hidden-Variable Theories* (Pergamon, Oxford, 1973) p.194.

69 Schiff, L.I., *Quantum Mechanics* (McGraw-Hill Kogakusha, Tokyo, 1968) p.62; Sakurai, J.J., *Modern Quantum Mechanics* (Addison-Wesley, Redwood City, California, 1985) p.58.

70 Saxon, D.S., Elementary Quantum Mechanics, *op. cit.*, p.64; Ashby, N. and Miller, S.C., *Principles of Modern Physics* (Holden-Day, San Francisco, 1970) pp.181-182.

$$= \frac{m}{(2\pi)^{1/4}\sqrt{i\hbar t\sigma_o}} \exp\left[\frac{imx^2}{2\hbar t}\right]\int_{-\infty}^{\infty} \exp\left[-\left(\frac{1}{4\sigma_o^2} - \frac{im}{2\hbar t}\right)x'^2 - \left(\frac{imx}{\hbar t} - \frac{ip}{\hbar}\right)x'\right] dx'$$

$$= \frac{m}{(2\pi)^{1/4}\sqrt{i\hbar t\sigma_o}} \exp\left[\frac{imx^2}{2\hbar t}\right]\exp\left[\frac{im\hbar t\, x^2 + 4m\sigma_o^2\, ptx - 2\sigma_o^2 p^2 t^2}{2\hbar t\,(\hbar t - 2im\sigma_o^2)}\right]$$

Using u_1 to denote the initial velocity, with $u_1 = p/m = \hbar k_1/m$, the above expression then becomes:

$$\psi(x, t) = (2\pi s_t^2)^{-1/4} \exp[ik_1(x - ½\, u_1 t) - (x - u_1 t)^2/4\sigma_o s_t] \quad .... (3\text{-}6)$$

where $s_t = \sigma_o (1 + i\hbar\, t/2m\sigma_o^2)$.

In three dimensions, the normalised Gaussian packet is the product of wave packets in each of the coordinate directions:[71]

$$\Psi(\mathbf{x}, t) = \psi(x, t)\ \psi(y, t)\ \psi(z, t)$$

Using equation (3-6) and similar expressions for $\psi(y, t)$ and $\psi(z, t)$, we derive the following expression for the three dimensional wavefunction:

$$\Psi(\mathbf{x}, t) = (2\pi s_t^2)^{-3/4} \exp\{i\mathbf{k}\cdot(\mathbf{x} - ½\, \mathbf{u}t) - (\mathbf{x} - \mathbf{u}t)^2/4\sigma_o s_t\} \quad .... (3\text{-}7)$$

where Ψ is a solution of the Schrödinger equation, $\mathbf{k}\cdot\mathbf{x} = (k_1 x + k_2 y + k_3 z)$ and $\mathbf{k}\cdot\mathbf{u} = k_1 u_1 + k_2 u_2 + k_3 u_3$.

Gaussian expressions for R, S and other quantities are readily found in the literature where they are presented but *not explicitly* derived. Since $\Psi = Re^{iS/\hbar}$, the functions $R(\mathbf{x}, t)$ and $S(\mathbf{x}, t)$ for the normalised Gaussian wave packet may now be derived from equation (3-7). Looking at the first term of Ψ, i.e. $(2\pi s_t^2)^{-3/4}$, we can express the complex number s_t as follows:

$$s_t = \sigma_o (1 + i\hbar\, t/2m\sigma_o^2) = (a + ib), \text{ where } a = \sigma_o \text{ and } b = (\hbar\, t/2m\sigma_o)$$

or in polar form:

$$s_t = |s_t| \exp(i\varphi), \text{ where } |s_t| = (a^2 + b^2)^{½} \text{ and } \varphi = \arctan(b/a).$$

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[71] Holland, P.R., The Quantum Theory of Motion, *op. cit.*, p.158.

So

$$|s_t| = [\sigma_o^2 + (\hbar\, t/2m\sigma_o)^2]^{½} = \sigma_o [1 + (\hbar\, t/2m\sigma_o^2)^2]^{½}$$

and

$$\varphi = \arctan[(\hbar\, t/2m\sigma_o)/\sigma_o] = \arctan(\hbar\, t/2m\sigma_o^2).$$

In order to conform with notation in the recent literature, let $\sigma = |s_t|$ then

$$\sigma^2 = \sigma_o^2\ [1 + (\hbar^2 t^2/4m^2\sigma_o^4)]$$

where σ is the RMS width of the packet in each coordinate direction at time t,[72] with

$$\sigma^2 = \langle x^2 - \langle x\rangle^2\rangle = \langle y^2 - \langle y\rangle^2\rangle = \langle z^2 - \langle z\rangle^2\rangle.$$

Then

$$4m^2\sigma_o^2\sigma^2 = 4m^2\sigma_o^4 + \hbar^2 t^2$$

and

$$(2\pi s_t^2)^{-3/4} = [(2\pi\sigma^2)\ \exp(2i\varphi)]^{-3/4} = (2\pi\sigma^2)^{-3/4}\ \exp(-3i\varphi/2).$$

The second term of Ψ is: $\exp\{i\mathbf{k}\cdot(\mathbf{x} - ½\,\mathbf{u}t) - (\mathbf{x} - \mathbf{u}t)^2/4\sigma_o s_t\}$ which can also be separated into real and imaginary terms.

The factor $(1/\sigma_o s_t) = [\sigma_o^2 (1 + i\hbar\, t/2m\sigma_o^2)]^{-1} = 2m/(2m\sigma_o^2 + i\hbar\, t)$

$$= \frac{2m(2m\sigma_o^2 - i\hbar\, t)}{4m^2\sigma_o^4 + \hbar^2 t^2} = \frac{(4m^2\sigma_o^2 - 2im\hbar\, t)}{4m^2\sigma_o^2\sigma^2}$$

$$= (1/\sigma^2) - i\,(\hbar\, t/2m\sigma_o^2\sigma^2)$$

Thus

$$\exp\{-(\mathbf{x} - \mathbf{u}t)^2/4\sigma_o s_t\} = \exp\{-(\mathbf{x} - \mathbf{u}t)^2/4\sigma^2\}\ \exp\{(i\hbar\, t)(\mathbf{x} - \mathbf{u}t)^2/8m\sigma_o^2\sigma^2\}$$

The wavefunction Ψ is then expressed as:

$$(2\pi\sigma^2)^{-3/4}\exp\{-(\mathbf{x} - \mathbf{u}t)^2/4\sigma_o^2\}\exp\{i[\mathbf{k}\cdot(\mathbf{x} - ½\mathbf{u}t) + (\hbar\, t/8m\sigma_o^2\sigma^2)(\mathbf{x} - \mathbf{u}t)^2 - 3\varphi/2]\}$$

from which we can now identify the following expressions for R and S:

$$R(\mathbf{x}, t) = (2\pi\sigma^2)^{-3/4}\exp\{-(\mathbf{x} - \mathbf{u}t)^2/4\sigma^2\} \quad \ldots (3\text{-}8)$$

and

$$S(\mathbf{x},t) = -(3\hbar/2)\arctan(\hbar\, t/2m\sigma_o^2) + m\mathbf{u}\cdot(\mathbf{x} - ½\mathbf{u}t) + \hbar^2 t\,(\mathbf{x} - \mathbf{u}t)^2/8m\sigma_o^2\sigma^2 \quad ..(3\text{-}9)$$

where $\hbar\,\mathbf{k} = m\mathbf{u}$.

Now the gradient of $S(\mathbf{x}, t)$ gives the particle's (possessed) momentum:

$$\nabla S = m\,\nabla[\mathbf{u}\cdot(\mathbf{x} - \tfrac{1}{2}\mathbf{u}t)] + (\hbar^2 t/8m\sigma_o^2\,\sigma^2)\;\nabla\,[(\mathbf{x} - \mathbf{u}t)^2]$$

$$= m\mathbf{u} + (\hbar^2 t/4m\sigma_o^2\,\sigma^2)\,(\mathbf{x} - \mathbf{u}t) \quad \ldots.\ (3\text{-}10)$$

and

$$\nabla^2 S = (3\hbar^2 t/4m\sigma_o^2\sigma^2)$$

The particle's velocity is then:

$$\mathbf{v} = (\nabla S/m) = \mathbf{u} + (\hbar^2 t/4m^2\sigma_o^2\,\sigma^2)\,(\mathbf{x} - \mathbf{u}t) \quad \ldots.\ (3\text{-}11)$$

This expression can be used to find the particle's trajectory as a function of time $\mathbf{x}(t)$. Belinfante did so by making a change of variable.[73] Alternatively, the trajectory may be found by solving a first-order differential equation using the integrating factor method. Equation (3-11) may be rearranged as:

$$(d\mathbf{x}/dt) - (\hbar^2 t/4m^2\sigma_o^2\,\sigma^2)\,\mathbf{x} = [1 - (\hbar^2 t^2/4m^2\sigma_o^2\,\sigma^2)]\,\mathbf{u} = (\sigma_o^2/\sigma^2)\,\mathbf{u}$$

In this case, the integrating factor I is given by:

$$I = -(\hbar^2/4m^2\sigma_o^2)\int(t/\sigma^2)\,dt = -\log\sigma.$$

Then

$$\mathbf{x}(t) = e^{-I}\int[(\sigma_o^2/\sigma^2)\,\mathbf{u}\,e^{I}]\,dt + \mathbf{c}\,e^{-I} = (\mathbf{u}\,\sigma_o^2\sigma)\int(1/\sigma^3)\,dt + \mathbf{c}\,\sigma = \mathbf{u}\,t + \mathbf{c}\,\sigma$$

where $\mathbf{c}$ is a constant (vector) of integration. At $t = 0$, $\sigma = \sigma_o$, $\mathbf{x} = \mathbf{x}_o$ (initial position) and $\mathbf{c} = \mathbf{x}_o/\sigma_o$.

$$\Rightarrow \mathbf{x}(t) = \mathbf{u}t + (\sigma/\sigma_o)\,\mathbf{x}_o \quad \ldots.\ (3\text{-}12)$$

where the integrals and identities used in this derivation are presented in Appendix III. The quantum potential for a Gaussian wave packet is easily calculated as follows:

$$Q = -(\hbar^2/2m)\,(\nabla^2 R/R)$$

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72 *ibid.*, p.159.

73 Belinfante, F.J., A Survey of Hidden-Variable Theories, *op. cit.*, pp.196-197.

$$= -(\hbar^2/2m)\,\nabla^2\,[\exp\{-(\mathbf{x}-\mathbf{u}t)^2/4\sigma^2\}]/\exp\{-(\mathbf{x}-\mathbf{u}t)^2/4\sigma^2\}$$

Now

$$\nabla\{-(\mathbf{x}-\mathbf{u}t)^2/4\sigma^2\} = -(\mathbf{x}-\mathbf{u}t)/2\sigma^2 \quad \text{and} \quad \nabla^2\{-(\mathbf{x}-\mathbf{u}t)^2/4\sigma^2\} = -3/2\sigma^2$$

as $(\nabla\cdot\mathbf{x}) = 3$. We then find:

$$\nabla R = -(2\pi\sigma^2)^{-3/4}\,[(\mathbf{x}-\mathbf{u}t)/2\sigma^2]\,\exp\{-(\mathbf{x}-\mathbf{u}t)^2/4\sigma^2\}$$

and

$$\nabla^2 R = (2\pi\sigma^2)^{-3/4}\,\{\nabla^2\,[-(\mathbf{x}-\mathbf{u}t)^2/4\sigma^2] + (\nabla[-(\mathbf{x}-\mathbf{u}t)^2/4\sigma^2])^2\}\exp\{-(\mathbf{x}-\mathbf{u}t)^2/4\sigma^2\}$$

$$= (2\pi\sigma^2)^{-3/4}\,\{-(3/2\sigma^2) + [(-1/2\sigma^2)\,(\mathbf{x}-\mathbf{u}t)]^2\}\,\exp\{-(\mathbf{x}-\mathbf{u}t)^2/4\sigma^2\}$$

which yields:

$$Q = (\hbar^2/4m\sigma^2)\,\{3-(\mathbf{x}-\mathbf{u}t)^2/2\sigma^2\} \quad \ldots.\ (3\text{-}13)$$

Using equation (2-4) with V set to zero, the time rate of change of the particle's momentum in the classically-free case is then:

$$\frac{d\mathbf{p}}{dt} = -\nabla Q = -\frac{\hbar^2}{4m\sigma^2}\nabla\Big[3-\frac{(\mathbf{x}-\mathbf{u}t)^2}{2\sigma^2}\Big] = \frac{\hbar^2}{4m\sigma^4}(\mathbf{x}-\mathbf{u}t) \quad \ldots.\ (3\text{-}14)$$

In the classically-free case, $(dS/dt) = (T-Q) = \tfrac{1}{2}m|\mathbf{u}|^2 - (3\hbar^2 t/4m\sigma^2)$ $+ (\hbar^2 t/4m\sigma_0^2\sigma^2)\,[\mathbf{u}\cdot(\mathbf{x}-\mathbf{u}t)] + (\hbar^2/8m\sigma^4)\,[1+(\hbar^2 t^2/4m^2\sigma_0^4)]\,(\mathbf{x}-\mathbf{u}t)^2$

$$= \tfrac{1}{2}m|\mathbf{u}|^2 - (3\hbar^2 t/4m\sigma^2) + (\hbar^2 t/4m\sigma_0^2\sigma^2)\,[\mathbf{u}\cdot(\mathbf{x}-\mathbf{u}t)]$$
$$+ (\hbar^2/8m\sigma_0^2\sigma^2)\,(\mathbf{x}-\mathbf{u}t)^2 \quad \ldots.\ (3\text{-}15)$$

These Gaussian expressions will be extensively employed in Chapter Four.

CHAPTER 4

ENERGY-MOMENTUM TRANSFER AND THE QUANTUM POTENTIAL

Quantum mechanics is very impressive. But an inner voice tells me that it is not yet the real thing The theory produces a good deal but hardly brings us closer to the secret of the Old One. I am at all events convinced that *He* does not play dice. Waves in $3n$-dimensional space whose velocity is regulated by potential energy …

— Albert Einstein [1]

4.1 Energy Conservation in the Causal Theory

It has been stated in the literature on the Causal Theory that energy is not conserved for a quantum system as a whole (i.e. wave field and particle together). The non-conservation of energy is claimed because, although the wave acts on the particle, the particle does not appear to react back on the wave.[2] Both the apparent energy non-conservation and the absence of a back reaction constitute conceptual problems for the Causal Theory. In this chapter, we examine the role played by the quantum potential in the conservation of energy. We shall build on the account of the nature of potential energy given in Chapter Three in order to solve some conceptual and theoretical problems in connection with energy conservation, energy transfer, and action-reaction in quantum systems.

4.2 Energy-Momentum Exchange in Single Particle States

The indictment that the Causal Theory fails to conserve energy generates a conceptual problem as this conflicts with the Principle of the Conservation of Total Energy. However, the key to understanding energy conservation in the Causal Theory

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[1] Einstein, A., Reply to Max Born, quoted in Pais, A., *'Subtle is the Lord ...': The Science and the Life of Albert Einstein* (Oxford University Press, Oxford, 1984) p.443.

is accepting that the quantum potential is the potential energy function of the wave field. The wave field acts on its particle(s) via the quantum potential and, as such, it is the wave field that is the origin of the quantum mechanical force (i.e. the particle's rate of change of momentum with respect to time). Where then does the energy that is necessary for the wave field to act upon the particle come from? An isolated, one-particle quantum system provides the answer for, in such a system, the only possible repositories of energy are the wave field and its accompanying particle. In this case, the wave field may gain energy at the expense of the particle's kinetic energy or may lose energy to the particle, as may be seen from the following example of an isolated, classically-free (i.e. V = 0), one-particle system.

Consider a free (spinless) particle of mass *m* that is initially not subject to any force fields or barriers. Its corresponding wave field is represented by a plane wave with a constant amplitude. Since $\nabla^2 R = 0$ for a wave of constant amplitude, the value of its quantum potential Q is zero. The particle moves with a constant velocity as both V and Q are zero, i.e. all the particle's energy is kinetic. If we were to trap the particle in a sealed enclosure that had no classical force fields within, we find that the wave field takes up a stationary wave pattern due to its new boundary conditions. This is the case with the simple example of an 'infinite' well (also known as a particle in a box). In three dimensions, imagine a cubical well of side length L with zero classical potential inside and 'infinite' potential outside. If we take one corner of the well as the origin of a rectangular Cartesian coordinate system then the stationary state wavefunction Ψ for a (spinless) particle of mass *m* inside the well is given by:

$$\Psi = (2/L)^{3/2} \,|\sin(n_1\pi x/L)\sin(n_2\pi y/L)\sin(n_3\pi z/L)|\, e^{-iE_n t/\hbar} = R\, e^{iS/\hbar} \ \ldots.\ (4\text{-}1)$$

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[2] Holland, P.R., *The Quantum Theory of Motion* (Cambridge University Press, Cambridge, 1993) p.120.

The total energy is: $E = (n_1^2 + n_2^2 + n_3^2)(\pi^2\hbar^2/2mL^2)$, where the n are positive integers. In Orthodox Quantum Theory, the particle must be in motion – it cannot be at rest as this would violate the Uncertainty Principle.[3] Since $V = 0$, the particle has kinetic energy only. In the Causal Theory the situation is seen to be very different. Since $S = -Et$, $\nabla S = 0$, i.e. the particle's momentum is zero and therefore has zero kinetic energy!

In keeping with the Principle of the Conservation of Total Energy, we should be asking where has the particle's kinetic energy gone. The only possibility in this case for the location of the energy is the wave field. If we calculate the quantum potential corresponding to equation (4-1), we find:

$$Q = -(\hbar^2/2m)(\nabla^2 R)/R = (n_1^2 + n_2^2 + n_3^2)(\pi^2\hbar^2/2mL^2) \quad \ldots. (4\text{-}2)$$

This is the *same* magnitude as the particle's kinetic energy as given by Orthodox Quantum Theory. Clearly then, the particle has come to rest (as $\nabla S = 0$) and all its energy is taken up by the quantum potential, as shown by equation (4-2). Since the quantum potential is the potential energy function of the wave field, it is the wave field in which the energy is stored.[4] (Surprisingly, this explanation appears in Bohm's original papers.[5]) What's more, this energy will be returned to the particle if the wave field's stationary state is disturbed, e.g. if any side of the box is removed. This idea too, was suggested by Bohm when he wrote:

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[3] Saxon, D.S., *Elementary Quantum Mechanics* (Holden-Day, San Francisco, 1968) p.77.
[4] Riggs, P.J., 'Quantum Phenomena in Terms of Energy-Momentum Transfer', *Journal of Physics A* **32** (1999) p.3072.
[5] Bohm, D., 'A Suggested Interpretation of the Quantum Theory in Terms of "Hidden" Variables II', *Physical Review* **85** (1952) p.184.

> … the kinetic energy of the particle will come from the ψ field, which is able to store up even macroscopic orders of energy when its wave-length is small.[6]

Bohm did not develop these ideas any further, opting in later years for his Active Information Hypothesis.

It might be objected that the above explanation cannot be so in the case of the infinite well because the value of the quantum potential inside the well is independent of position, whereas the wave field varies from a maximum at its anti-nodes to zero at its nodes. This objection is ill-founded. In response, we note again that it is physical fields that are the repositories of potential energy. Second, we must ask what is it that the quantum potential represents in such a stationary state? In a stationary state the quantum potential does not give the potential energy at a particular point but instead gives the value of the *total* field energy of the system.[7]

Although the above example shows that the wave field may gain or lose energy to the quantum particle, it does not provide the exact mechanism for these energy transfers. However, this is also the case in classical theory, e.g. Newtonian Gravitation (which is also a non-local theory) does not give a mechanism for energy transfers between a massive particle and a (classical) gravitational field.[8] The explication of a mechanism for energy transfer in quantum systems would require a relativistic quantum field approach.

We shall now examine, in more generality, the features of energy transfer and storage for a one-particle quantum system as this will bring out the essential features under examination. The energy available to the particle (denoted E) is:

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[6] Bohm, D., 'A Discussion of Certain Remarks by Einstein on Born's Probability Interpretation of the ψ-Function' in *Scientific Papers Presented to Max Born* (Oliver & Boyd, Edinburgh and London, 1953) p.14.

[7] Riggs, P.J., 'Quantum Phenomena …', *loc. cit.*, p.3072.

[8] Doughty, N.A., *Lagrangian Interaction: An Introduction to Relativistic Symmetry in Electrodynamics and Gravitation* (Addison-Wesley, Sydney, 1990), p.123.

$$E = T + Q = -\frac{\partial S}{\partial t} = \frac{(\nabla S)^2}{2m} + Q$$

where T is the particle's kinetic energy. The time rate of change of E is given by:

$$\frac{dE}{dt} = \left(\frac{1}{2m}\right)\frac{d}{dt}(\nabla S)^2 + \frac{dQ}{dt} = (\nabla S)\cdot\left(\frac{-\nabla Q}{m}\right) + \frac{dQ}{dt} \quad \ldots. (4\text{-}3)$$

Now

$$\frac{dQ}{dt} = \sum_{i=1}^{3}\frac{\partial Q}{\partial x^i}\frac{dx^i}{dt} + \frac{\partial Q}{\partial t} = (\nabla Q)\cdot\left(\frac{\nabla S}{m}\right) + \frac{\partial Q}{\partial t} \quad \ldots. (4\text{-}4)$$

where $(\nabla S/m) = (d\mathbf{x}/dt)$. The term $[(\nabla Q)\cdot(\nabla S/m)]$ is equal to minus the rate of change of the particle's kinetic energy with respect to time, i.e. $(-\,dT/dt)$, as can be seen with reference to equation (4-3). Substitution of equation (4-4) into equation (4-3) yields:

$$\frac{dE}{dt} = (\nabla S)\cdot\left(\frac{-\nabla Q}{m}\right) + (\nabla Q)\cdot\left(\frac{\nabla S}{m}\right) + \frac{\partial Q}{\partial t} = \frac{\partial Q}{\partial t} \quad \ldots. (4\text{-}5)$$

When $(\partial Q/\partial t) = 0$, the energy available to the particle E is constant and changes in kinetic energy are exactly balanced by changes in the quantum potential. If $[(\nabla Q)\cdot(\nabla S/m)] > 0$, energy passes from particle to field. If $[(\nabla Q)\cdot(\nabla S/m)] < 0$, energy passes from field to particle. Thus any change in the particle's kinetic energy is a straight-forward energy conversion process, familiar from classical mechanics.

What if $(\partial Q/\partial t) \neq 0$? What does this term represent? Holland calls it "the quantum power".[9] This label is misleading, for if any single term should be called the quantum power, it should be $[-(\nabla Q)\cdot(\nabla S/m)]$ since this term is the scalar product of the quantum mechanical force and the particle's instantaneous velocity (i.e. the formal definition of instantaneous power). Holland also does not suitably characterise the role of $(\partial Q/\partial t)$. The quantum potential itself gives the potential energy available to the quantum particle at its specific position in the field but Q does not, in general, coincide with the total field energy. Acceptance of this point requires a separate mathematical expression for total field energy (to be shown at the end of Section 4.3).

In an isolated one-particle state, the wave field is the only repository of energy other than the particle itself. This being so, it follows that $(\partial Q/\partial t)$ gives the time rate of change of the quantum potential due to energy stored in the wave field other than at the particle's location. This indicates that the particle's energy will increase (decrease) with decreases (increases) in the amount of energy stored in the wave field as a whole. Therefore, we can account for the total energy of a classically-free single particle quantum system without recourse to external sources and without the need to conjecture about the existence of vacuum fluctuations and the like (as Bohm and Hiley did[10]).

It was noted in Section 2.6 that the Causal Theory does not violate Special Relativity in the sense that the connections between quantum systems cannot be used for the purposes of signalling (i.e. the transmission of information) or the transfer of energy faster than the speed of light in vacuum. The restriction on our knowledge of initial particle positions to $|\Psi|^2$ guarantees that the non-locality aspect of many-particle quantum systems cannot be used for signalling.[11] Clearly though, satisfying Special Relativity isn't merely a matter of ruling out superluminal transmission of information (signalling) or superluminal energy transfer. However, the latter is a larger question, i.e. that *satisfying* Special Relativity requires there not be a preferred frame of reference and that of superluminal causal influences (as distinct from superluminal signalling).

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[9] Holland, P.R., The Quantum Theory of Motion, *op. cit.*, p.119.
[10] Bohm, D. and Hiley, B.J., *The Undivided Universe* (Routledge, London and N.Y., 1993) p.38.
[11] Cushing, J.T., Quantum Mechanics, *op. cit.*, p.58.

In respect to the issue of a preferred frame, the effect on an individual quantum particle in a many-particle system depends on all particles in the system at a given instant (and therefore needs a preferred frame). This cannot be fixed in a non-relativistic theory and so is outside the scope of this thesis. In regard to the issue of superluminal causal influence, if it is meant that a measurement or other disturbance on one of a pair of spatially distant entangled particles produces an effect on the other, then Special Relativity is not satisfied in this respect. However, this is not disputed in the literature.[12] The issue of energy transfer is considered in Section 4.5. The further implications for Special Relativity of any kind of superluminal causal influence are serious but these are topics outside the current work.

The above account resolves the external conceptual problem of conflict with the Principle of the Conservation of Total Energy. It also provides a direction for further development of the Causal Theory, i.e. allows for the solution of related theoretical problems.

4.3 Wave Field Energy and its Transfer

Now that it has been established that quantum systems (as described by the Causal Theory) do conserve energy, we shall focus our attention in this section on theoretical problems relating to the energy content of the wave field and changes to it.

The Hamiltonian density $\mathcal{H}$ is defined as minus the 'time-time' component of the energy-momentum tensor $\mathcal{T}^{\mu}{}_{\nu}$ of a system, i.e. [$-\mathcal{T}^{0}{}_{0}$]. The single particle energy-momentum tensor is defined in terms of the Lagrangian density $\mathcal{L}$ (as defined in Section 2.2) and the functions S and ρ (= R^2):[13]

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[12] *ibid.*, p.337.

[13] Holland, P.R., The Quantum Theory of Motion, *op. cit.*, p.114.

$$\mathcal{T}'^{\mu}_{\ \nu} = \delta_{\mu\nu}\mathcal{L} - \frac{\partial\mathcal{L}}{\partial(\partial_\mu\rho)}\partial_\nu\rho - \frac{\partial\mathcal{L}}{\partial(\partial_\mu S)}\partial_\nu S$$

where $\delta^{\mu}_{\ \nu}$ is the Kronecker Delta. The Hamiltonian density is the total density of mass-energy in an observer's frame of reference.[14] In the non-relativistic context, the Hamiltonian density is the total energy density of the system. In the case of a classically-free quantum system, the Hamiltonian density is given by:[15]

$$\mathcal{H} = R^2(\nabla S)^2/2m + (\hbar^2/2m)(\nabla R)^2 \quad .... (4\text{-}6)$$

Let

$$H = \iiint_{-\infty}^{\infty} \mathcal{H}\, d^3\mathbf{x} \quad .... (4\text{-}7)$$

Integration of the classically-free Hamiltonian density given by equation (4-6) shows that H (as defined by equation (4-7)) is constant.[16] The quantity H may now be interpreted as the total energy of the isolated, classically-free system (i.e. wave field and particle) and not just the energy of the wave field alone for the following reasons:

- particle and wave field are intrinsic parts of a single quantum system (the particle is not an 'add-on' to, or an enlargement of, the system);
- the quantum particle receives energy from the wave field;[17]
- the quantum potential represents part of the wave field's energy;
- there are isolated, classically-free quantum systems where the field energy decreases (such as a Gaussian wave packet described below);
- in any isolated system, total energy is a conserved quantity.

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[14] Misner, C.W., Thorne, K.S. and Wheeler, J.A., *Gravitation* (Freeman & Co., San Francisco, 1973) p.137.

[15] Holland, P.R., The Quantum Theory of Motion, *op. cit.*, p.115.

[16] *ibid.*, p.116.

[17] *ibid.*, p.120.

It then follows that the energy of the wave field is (H – T). Now let the quantity U be defined as the energy content of the wave field (in a non-stationary state) minus that given by the quantum potential, i.e.

$$U = H - (T + Q) \quad .... (4\text{-}8)$$

Consequently,

$$\frac{dU}{dt} = \frac{dH}{dt} - \left(\frac{dT}{dt} + \frac{dQ}{dt}\right) = -\frac{dE}{dt} = -\frac{\partial Q}{\partial t} \quad .... (4\text{-}9)$$

using equation (4-5) and the fact that H is constant in the classically-free case.[18] It can be seen therefore, that a change of the energy content of the wave field appears as a change in the quantum potential.

Since Q represents the potential energy available to the particle at a specific position in the wave field, both deviations from inertial motion and conservation of energy can be accounted for in individual quantum processes provided the single particle state is isolated. Depending on the prevailing circumstances, some (or all) of a particle's energy-momentum can be transferred and temporarily stored in its wave field. Once stored in the field, energy-momentum can be returned to the particle if circumstances change. This transfer back and forth of energy-momentum affects the particle's motion for a change in the momentum of the particle over time is, by definition, the net force acting. Therefore the motion of a quantum particle need not be in a straight line even if there is no external field present. Since the particle is inseparable from its 'guiding' wave field, exchanges of energy-momentum occur between wave field and particle as they travel along together.

Equation (4-9) shows that ($\partial Q/\partial t$) gives the change of the quantum potential due to changes in U, i.e. the time rate of change of Q due to changes in the amount of energy stored in the wave field other than at the particle's position. Then, by equation

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[18] Riggs, P.J., 'Quantum Phenomena ...', , *loc. cit.*, p.3071.

(4-5), the particle's kinetic energy can be shown to increase (decrease) with decreases (increases) in the amount of energy stored in the wave field. Any change in the particle's kinetic energy is then explained by an energy conversion process, the concept of which is common to all branches of physics. Energy transfers, therefore, occur through a process whereby T ⇆ Q ⇆ U, with the direction of the arrows depending on whether the particle is losing or gaining energy. The quantum potential is the physical interface between particle and wave field and its role is to channel energy (or more generally, energy-momentum) from wave field to particle and back again. These conversions need to be registered when accounting for the total energy of an isolated, classically-free quantum system. A suitable summary of such energy exchange was provided by Hermann Weyl (albeit from another field context):

> The total energy … remains unchanged: they merely stream from one part of the field to another, and become transformed from field energy … into kinetic-energy … and *vice versa*.[19]

Unlike a classical field, the wave field's form has greater physical significance than its amplitude. The form of the wave field may be described with reference to its wavefronts. A wavefront is defined as a surface over which the phase of the wave ($S/\hbar$) is constant. Common examples of wavefronts include spherical waves which have expanding spheres as their wavefronts and plane waves whose wavefronts are flat planes perpendicular to the direction of wave propagation. The shape of a wavefront depends (in part) on what the wave field encounters, i.e. whether its initial shape has been altered by passing over or through an obstruction. It is generally the case that when a wave changes its shape there will be a change in its amplitude. The total rate of change of the amplitude R with respect to time is:

$$\frac{dR}{dt} = \sum_{i=1}^{3} \frac{\partial R}{\partial x_i}\frac{dx_i}{dt} + \frac{\partial R}{\partial t} = (\nabla R)\cdot\left(\frac{\nabla S}{m}\right) + \frac{\partial R}{\partial t}$$

What do the terms $[(\nabla R)\cdot(\nabla S/m)]$ and $(\partial R/\partial t)$ represent? The simple example of a uniformly expanding spherical wave field will be useful in illustrating this. The wavefunction for a spherical wave field is:

$$\Psi = (A/r)\exp[i(kr-\omega t)]$$

where A is a constant, k is the wave number, $S = \hbar(kr-\omega t)$, amplitude $R = (A/r)$, where $r = (x^2+y^2+z^2)^{1/2}$, and the other symbols have their usual meanings.[20] The term $[(\nabla R)\cdot(\nabla S/m)]$ gives the change in the value of R due to any changes in the size of a radius vector. However, as the wave expands, the value of R will decrease with time as r increases, thus $(\nabla R) \neq 0$. The term $(\partial R/\partial t)$ gives the rate of change of R explicitly due to changes over time in the shape of its wavefronts since changes in wavefront shape are generally accompanied by changes in amplitude. The spherical wavefront however, retains its shape as it expands so the amplitude of a spherical wave does not change due to any changes in the shape of the wave over time, i.e. $(\partial R/\partial t) = 0$.

It can be seen that (dR/dt), in general, will depend on changes in the wave field's shape. The explicit dependence of $(\partial Q/\partial t)$ on $(\partial R/\partial t)$ is given by:

$$\frac{\partial Q}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial}{\partial t}\left(\frac{\nabla^2 R}{R}\right) = -\frac{\hbar^2}{2mR}\nabla^2\left(\frac{\partial R}{\partial t}\right) - \frac{Q}{R}\left(\frac{\partial R}{\partial t}\right) \quad \text{.... (4-10)}$$

which clearly shows that $(\partial Q/\partial t) \neq 0$ over the time interval of a change in the shape of the wave field. The more pronounced the change in shape is, the greater will be the amount of energy exchanged between the particle and the wave field. Since $(dU/dt) = -(\partial Q/\partial t)$ from equation (4-9), it can be seen from equation (4-10) that the

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[19] Weyl, H., *Space-Time-Matter* (Dover, New York, 1952) p.168.
[20] Holland, P.R., The Quantum Theory of Motion, *op. cit.*, p.141.

condition for energy exchange between wave field and particle (and vice-versa) is $(\partial R/\partial t) \neq 0$.

The shape of the wave field depends, in large part, on whether it has encountered any obstructions which have distorted it. (In this limited sense only, the form of the wave field indirectly carries information about the surrounding environment.) The surrounding environment modifies the form of the wave field which, in turn, acts by altering the motion of the particle. The environment changes the shape of its wave field and it is changes in shape of the wave field that are a major determining factor of the extent of any energy-momentum exchanges. Thus both the form of the wave field and the energy stored within it depends on whether the wave has encountered any obstructions which have distorted its shape.

A free particle not being subject to any external barriers or force fields has a wave field *represented* by a plane wave with constant amplitude. However, it can be seen that such a plane wave is an idealisation for the following reasons. First, a plane wave is usually given as infinite in spatial extent as noted in de Broglie's original reasoning:

> The plane monochromatic wave must in a certain sense be considered as an abstraction, for it would fill the whole of space and last throughout all time. In practice a wave always occupies a limited region of space at a particular instant, and at any particular point it has a beginning and an end.[21]

Second, this would violate the *Principle of Energy Content* for something real not to possess any energy at all. This suggests that the wave field never divests itself completely of energy gained from its accompanying quantum particle or external interactions. The wave field retains a very small amount of energy at all times and the

[21] de Broglie, L., *An Introduction to the Study of Wave Mechanics* (Methuen, London, 1930) p.51.

shape of the wave field approaches a constant amplitude plane wave only as a limiting process.

In any real situation, the wave field will be of finite extent in all directions and initially localised about the particle. This is readily described by a Gaussian wave packet with the quantum particle located somewhere within the packet. Such a wave packet can be used to model a variety of physical phenomena such as diffraction by a slit with imperfect edges.[22] As shown in Section 3.6, the wavefunction for a Gaussian wave packet at any time t (> 0) is given by:

$$\Psi(\mathbf{x}, t) = (2\pi\sigma_t^2)^{-3/4} \exp\{i\mathbf{k}\cdot(\mathbf{x} - \tfrac{1}{2}\mathbf{u}t) - (\mathbf{x} - \mathbf{u}t)^2/4\sigma_o s_t\}$$

where σ_o is the initial root-mean-square (RMS) width of the packet in each coordinate direction, with $s_t = \sigma_o(1 + i\hbar t/2m\sigma_o^2)$, and $\mathbf{u}$ is the initial group velocity. The Hamiltonian H for a classically-free quantum system may be found by integrating the Hamiltonian density $\mathcal{H}$. From equations (4-6) and (4-7), we have:

$$H = \iiint_{-\infty}^{\infty} \mathcal{H}\, d^3\mathbf{x} = \iiint_{-\infty}^{\infty} [R^2(\nabla S)^2/2m + (\hbar^2/2m)(\nabla R)^2]\, d^3\mathbf{x}$$

The relevant functions for a classically-free Gaussian wave packet that were derived in Section 3.6 are as follows:

$$R^2 = (2\pi\sigma^2)^{-3/2} \exp[-(\mathbf{x} - \mathbf{u}t)^2/2\sigma^2]$$

$$(\nabla R)^2 = (2\pi\sigma^2)^{-3/2} [(\mathbf{x} - \mathbf{u}t)^2/4\sigma^4] \exp[-(\mathbf{x} - \mathbf{u}t)^2/2\sigma^2] = R^2 [(\mathbf{x} - \mathbf{u}t)^2/4\sigma^4]$$

The particle's kinetic energy $T = (\nabla S)^2/2m =$

$$\tfrac{1}{2}m|\mathbf{u}|^2 + (\hbar^2 t/4m\sigma_o^2\sigma^2)[\mathbf{u}\cdot(\mathbf{x} - \mathbf{u}t)] + (\hbar^4 t^2/32m^3\sigma_o^4\sigma^4)(\mathbf{x} - \mathbf{u}t)^2 \quad \text{.... (4-11)}$$

where $\sigma = |s_t| = \sigma_o[1 + (\hbar t/2m\sigma_o^2)^2]^{1/2}$ is the RMS width of the packet at time t. Thus

$$H = \iiint_{-\infty}^{\infty} [(R^2(\nabla S)^2/2m + (\hbar^2/2m)(\nabla R)^2]\, d^3\mathbf{x}$$

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[22] Holland, P.R., The Quantum Theory of Motion, *op. cit.*, p.163.

$$= (2\pi\sigma^2)^{-3/2} \iiint_{-\infty}^{\infty} \{a + b\,\mathbf{u}\cdot(\mathbf{x} - \mathbf{u}t) + (c + f)(\mathbf{x} - \mathbf{u}t)^2\} \exp[-(\mathbf{x} - \mathbf{u}t)^2/2\sigma^2]\, d^3\mathbf{x}$$

where $a = \tfrac{1}{2}m|\mathbf{u}|^2$, $b = (\hbar^2 t/4m\sigma_0^2\sigma^2)$, $c = (\hbar^4 t^2/32m^3\sigma_0^4\sigma^4)$ and $f = (\hbar^2/8m\sigma^4)$.

In one dimension, we find:

$$H = (2\pi\sigma^2)^{-1/2} \exp[-|\mathbf{u}|^2 t^2/2\sigma^2] \times$$

$$\int_{-\infty}^{\infty} \{(c + f)\,x^2 + (b\mathbf{u} - 2c\mathbf{u}t - 2f\mathbf{u}t)x + (a - b\mathbf{u}t + cu^2t^2 + fu^2t^2)\} \exp[-\alpha x^2 - \beta x]\, dx$$

where $\alpha = (1/2\sigma^2)$, $\beta = -(ut/\sigma^2)$. Using standard definite integrals (as listed in Appendix III) we get:

$$H = (c + f)(\sigma^2 + u^2t^2) + ut(b - 2cut - 2fut) + (a - but + cu^2t^2 + fu^2t^2) = (c + f)\sigma^2 + a$$

$$= (\hbar^4 t^2/32m^3\sigma_0^4\sigma^2) + (\hbar^2/8m\sigma^2) + \tfrac{1}{2}m|\mathbf{u}|^2$$

$$= (\hbar^2/8m\sigma^2)\{(\hbar^2t^2/4m^2\sigma_0^4) + 1\} + \tfrac{1}{2}m|\mathbf{u}|^2$$

The triple integration therefore yields:

$$H = (3\hbar^2/8m\sigma^2) + (3\hbar^4 t^2/32m^3\sigma_0^4\sigma^2) + \tfrac{1}{2}m|\mathbf{u}|^2$$

$$= (3\hbar^2/8m\sigma^2)\,[1 + (\hbar^2t^2/4\,m^2\sigma_0^4)] + \tfrac{1}{2}m|\mathbf{u}|^2 \quad > 0$$

or

$$H = \tfrac{1}{2}m|\mathbf{u}|^2 + (3\hbar^2/8m\sigma_0^2) \quad \ldots.\ (4\text{-}12)$$

which is the total energy of this isolated, classically-free quantum system. The first term on the right-hand side of equation (4-12) is obviously the particle's initial kinetic energy. The second term can be seen to be the initial field energy as it contains the initial RMS width of the wave packet and does not involve any time-dependent quantities or velocities.

The time dependence of σ shows that the packet will expand with increasing time t and be accompanied by a change in shape. Therefore, both the energy of the particle and the amount of energy stored in the wave field will be time-dependent. The total energy of the system, however, will remain constant. Transfer of energy-momentum will occur from the wave field to the quantum particle. The particle, in turn, will accelerate until such time as the value of the quantum potential drops effectively to zero. This can be shown quantitatively as follows. Consider a quantum particle positioned in the front of a wave packet, so that $(\mathbf{x} - \mathbf{u}t) > 0$. The quantum potential derived from the Gaussian wavefunction is (cf. Section 3.6):

$$Q = (\hbar^2/4m\sigma^2)\{3 - (\mathbf{x} - \mathbf{u}t)^2/2\sigma^2\} \quad \ldots. (4\text{-}13)$$

from which we find by partial differentiation:

$$\frac{\partial Q}{\partial t} = \frac{\hbar^4 t}{8m^3\sigma_o^2\sigma^6}(\mathbf{x} - \mathbf{u}t)^2 + \frac{\hbar^2}{4m\sigma^2}[\mathbf{u}\cdot(\mathbf{x} - \mathbf{u}t)] - \frac{3\hbar^4 t}{8m^3\sigma_o^2\sigma^4} \quad \ldots. (4\text{-}14)$$

and the total rate of change of the particle's momentum with respect to time:

$$\frac{d\mathbf{p}}{dt} = -(\nabla Q) = \frac{\hbar^2}{4m\sigma^4}(\mathbf{x} - \mathbf{u}t) \quad \ldots. (4\text{-}15)$$

Since $(\mathbf{x} - \mathbf{u}t) > 0$, $(d\mathbf{p}/dt) > 0$ which shows that the particle's momentum is increasing, implying that its kinetic energy is also increasing. This is confirmed by the rate of change of the particle's kinetic energy with respect to time:

$$(dT/dt) = -(\nabla Q)\cdot(\nabla S)/m$$

$$= (\hbar^2/4m\sigma^4)[\mathbf{u}\cdot(\mathbf{x} - \mathbf{u}t)] + (\hbar^4 t/16m^3\sigma_o^2\sigma^6)(\mathbf{x} - \mathbf{u}t)^2 > 0 \quad \ldots. (4\text{-}16)$$

The dominant terms for large values of time t will be those containing powers of σ, as $\sigma^2 = \sigma_o^2 + (\hbar^2 t^2/4m^2\sigma_o^2)$. Since the σ's are all denominator terms, Q, $(\partial Q/\partial t)$ and (dT/dt) will all tend to zero as time $t \to \infty$. Provided no further obstacles or disturbances are encountered, the wave field will expand extensively for large values of t, and correspondingly, the quantum potential and the energy contained in the wave field as a whole will rapidly approach zero, resulting in the system's energy becoming overwhelmingly kinetic. It is interesting to note that although Bohm and Hiley denied any explanation in which the wave field itself transferred energy, a reading of their account of the spread of a wave packet might lead one to question their consistency on this issue:

> It is clear then that the particles are accelerated … This acceleration is evidently a result of the quantum potential … the quantum potential decreases as the wave packet spreads, falling eventually to zero.
>
> The picture is then that as the wave packet spreads, the particle gains kinetic energy, the amount depending upon where it was initially in the packet. … *the energy represented by the quantum potential was turned into kinetic energy.*[23]

The time taken for the transfer of energy can be calculated in the case of a free Gaussian wave packet.[24] This is a theoretical problem which may be worked out in detail now that the conceptual problem involving energy conservation has been resolved and the equations governing energy transfer have been deduced. The kinetic energy of the quantum particle is:

$$T = \tfrac{1}{2} m|\mathbf{u}|^2 + (\hbar^2 t/4m\sigma_o^{\,2}\sigma^2)\,[\mathbf{u}\cdot(\mathbf{x}-\mathbf{u}t)] + (\hbar^4 t^2/32m^3\sigma_o^{\,4}\sigma^4)(\mathbf{x}-\mathbf{u}t)^2 \quad \ldots.\ (4\text{-}17)$$

$$\Rightarrow (T - T_i) = (\hbar^2 t/4m\sigma_o^{\,2}\sigma^2)\,[\mathbf{u}\cdot(\mathbf{x}-\mathbf{u}t)] + (\hbar^4 t^2/32m^3\sigma_o^{\,4}\sigma^4)(\mathbf{x}-\mathbf{u}t)^2 \quad \ldots.\ (4\text{-}18)$$

where $T_i = \tfrac{1}{2} m\,|\mathbf{u}|^2$. Using equation (4-16), equation (4-18) becomes:

$$(\sigma_o/\sigma)^2\,(T - T_i) = (\hbar^2 t/4m\sigma^4)\,[\mathbf{u}\cdot(\mathbf{x}-\mathbf{u}t)] + (\hbar^4 t^2/32m^3\sigma_o^{\,2}\sigma^6)(\mathbf{x}-\mathbf{u}t)^2$$

$$= t\,(dT/dt) - (\hbar^4 t^2/32m^3\sigma_o^{\,2}\sigma^6)(\mathbf{x}-\mathbf{u}t)^2 \quad \ldots.\ (4\text{-}19)$$

Now $(\hbar^4 t^2/32m^3\sigma_o^{\,2}\sigma^6)(\mathbf{x}-\mathbf{u}t)^2 = (\sigma_o/\sigma)^2\,(T - T_i) - (\hbar^2 t/4m\sigma^4)\,[\mathbf{u}\cdot(\mathbf{x}-\mathbf{u}t)]$, so equation (4-19) becomes:

$$(\sigma_o/\sigma)^2\,(T - T_i) = t\,(dT/dt) - \{(\sigma_o/\sigma)^2\,(T - T_i) - (\hbar^2 t/4m\sigma^4)\,[\mathbf{u}\cdot(\mathbf{x}-\mathbf{u}t)]\}$$

$$= t\,(dT/dt) - (\sigma_o/\sigma)^2\,(T - T_i) + t\,\mathbf{u}\cdot(d\mathbf{p}/dt)$$

where we have used equation (4-15).

Therefore

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[23] Bohm, D. and Hiley, B.J., The Undivided Universe, *op. cit.*, p.47 (italics mine).

[24] Riggs, P.J., 'Quantum Phenomena …', *loc. cit.*, pp.3073-3074.

$$t\,(dT/dt) = 2\,(\sigma_o/\sigma)^2\,(T - T_i) - t\,\mathbf{u}\cdot(d\mathbf{p}/dt)$$

Rearranging and integrating gives:

$$2\sigma_o^2 \int \frac{dt}{\sigma^2 t} = \int \frac{dT}{(T - T_i)} + \int \frac{\mathbf{u}\cdot d\mathbf{p}}{(T - T_i)}$$

$$= \int \frac{dT}{(T - T_i)} + (\sqrt{2m}\,|\mathbf{u}|\,\varepsilon) \int \frac{d\sqrt{T}}{(T - T_i)} \quad \ldots\ (4\text{-}20)$$

where $\varepsilon = (\cos\theta/\cos\phi)$, with $\cos\theta = (\mathbf{u}\cdot d\mathbf{p})/(|\mathbf{u}|\,|d\mathbf{p}|)$, $\cos\phi = (\mathbf{p}\cdot d\mathbf{p})/(|\mathbf{p}|\,|d\mathbf{p}|)$,

$|d\mathbf{p}| = (d|\mathbf{p}|)/\cos\phi$ and $|\mathbf{p}| = \sqrt{2mT}$. Using the Gaussian integrals in Appendix III,

equation (4-20) results in the following expression:

$$\log\left|\frac{t^2}{(\hbar^2 t^2/4m^2\sigma_o^{\,4}) + 1}\right| + A = \log|T - T_i| + \varepsilon\,\log\left|\frac{\sqrt{T} - \sqrt{T_i}}{\sqrt{T} + \sqrt{T_i}}\right|$$

where A is a constant of integration. In order to ensure consistency of left and right hand sides of the above equation, the value of A is set to zero. Taking exponentials gives:

$$\frac{C\,t^2}{(\hbar^2 t^2/4m^2\sigma_o^4) + 1} = (T - T_i)\left(\frac{\sqrt{T} - \sqrt{T_i}}{\sqrt{T} + \sqrt{T_i}}\right)^{\varepsilon}$$

where $C = e^A = 1$. The effect of the ε term is to vary the time taken for the transfer of energy from field to particle, depending on the particle's position in the wave packet. The time taken for a complete transfer of energy will be when T equals the final kinetic energy of the particle (T_f). If the particle is in a forward and central region of the packet so that $\varepsilon \approx 1$, then for $T = T_f$, the time for transfer is:[25]

$$t = \frac{2m\sigma_o^2\,(\sqrt{T_f} - \sqrt{T_i})}{\sqrt{4m^2\sigma_o^4 - \hbar^2(T_f + T_i - 2\sqrt{T_f T_i}\,)}}$$

There are very few references to the energy content of the wave field to be found in the literature. Given that the role of energy is essential to the complete description of any physical system, the potential energy stored within the wave field is an important quantity. We have seen that this energy content is given by (H – T). In the case of a classically-free Gaussian wave packet, an expression for the energy stored may be found in terms of the functions R, S and their derivatives (although it will not be in a simple form). Using equations (4-12) and (4-17) we find:

$$(H - T) = (3\hbar^2/8m\sigma_o^2) - (\hbar^2 t/4m\sigma_o^2\sigma^2)\,[\mathbf{u}\cdot(\mathbf{x} - \mathbf{u}t)] - (\hbar^4 t^2/32m^3\sigma_o^4\sigma^4)\,(\mathbf{x} - \mathbf{u}t)^2$$

$$= (3\hbar^2/8m\sigma_o^2) - (\hbar^2 t/4m^2\sigma_o^2\sigma^2)\,[m\mathbf{u} + (\hbar^2 t/8m\sigma_o^2\sigma^2)(\mathbf{x} - \mathbf{u}t)]\cdot(\mathbf{x} - \mathbf{u}t)$$

$$= (3\hbar^2/8m\sigma_o^2) - (\hbar^2 t/4m^2\sigma_o^2\sigma^2)\,[m\mathbf{u} + (\hbar^2 t/4m\sigma_o^2\sigma^2)(\mathbf{x} - \mathbf{u}t)]\cdot(\mathbf{x} - \mathbf{u}t)$$
$$+ (\hbar^4 t^2/32m^3\sigma_o^4\sigma^4)(\mathbf{x} - \mathbf{u}t)^2$$

$$= (3\hbar^2/8m\sigma_o^2) + (\hbar^2 t/2m\sigma_o^2)[(\nabla S)/m]\cdot[(\nabla R)/R] + (1/2m)(\hbar^2 t/2m\sigma_o^2)^2\,[(\nabla R)/R]^2$$

where $(\nabla R)/R = [-(\mathbf{x} - \mathbf{u}t)/2\sigma^2]$ and $(\nabla S) = m\mathbf{u} + (\hbar^2 t/4m\sigma_o^2\sigma^2)\,(\mathbf{x} - \mathbf{u}t)$.

Now we shall make use of the identity:

$$\nabla\cdot\left(\frac{\nabla R}{R}\right) = \frac{R(\nabla^2 R) - (\nabla R)^2}{R^2} = -\frac{3}{2\sigma^2} \quad \ldots.\ (4\text{-}21)$$

so that

$$\frac{\hbar^2 t}{2m\sigma_o^2} = -(\nabla^2 S)/[\nabla\cdot\left(\frac{\nabla R}{R}\right)] = \frac{R^2(\nabla^2 S)}{(\nabla R)^2 - R(\nabla^2 R)}$$

where $(\nabla^2 S) = (3\hbar^2 t/4m\sigma_o^2\sigma^2)$. Using equation (3-15) from Section 3.6, we have:

$$\nabla^2(dS/dt) = (3\hbar^2/4m\sigma_o^2\sigma^2)$$

and with equation (4-21) we find that:

$$\frac{3\hbar^2}{8m\sigma_o^2} = \left(\frac{-3}{4}\right)\nabla^2\left(\frac{dS}{dt}\right)/[\nabla\cdot\left(\frac{\nabla R}{R}\right)] = \left(\frac{3}{4}\right)\frac{R^2\nabla^2(dS/dt)}{(\nabla R)^2 - R(\nabla^2 R)}$$

Then $(H - T) =$

[25] *ibid.*, p.3074.

$$(\frac{3}{4})\frac{R^2\nabla^2(dS/dt)}{(\nabla R)^2 - R(\nabla^2 R)} + \frac{R^2(\nabla^2 S)}{(\nabla R)^2 - R(\nabla^2 R)}(\frac{\nabla S}{m})\cdot(\frac{\nabla R}{R})$$

$$+ (\frac{1}{2m})\left[\frac{R^2(\nabla^2 S)}{(\nabla R)^2 - R(\nabla^2 R)}\right]^2 (\frac{\nabla R}{R})^2 \quad \text{.... (4-22)}$$

We shall reason to consider this expression for the wave field energy content in Section 4.5.

4.4 Quantum Action and Reaction ?

The wave field acts on the quantum particle but the particle does not appear to react back on the wave in the sense that the shape or size of the wave field is not directly affected by the particle. This lack of a classical reaction is viewed as a flaw in the Causal Theory by some commentators.[26] The absence of a classical reaction constitutes a conceptual problem for the Causal Theory for it conflicts with the Principle of Reaction (Newton's Third Law) which may be stated as follows:[27]

♦ *Principle of Reaction*

> Any interaction between two physical entities has a mutual effect on both entities. The forces of interaction are equal and opposite, and act along straight lines joining the locations of the entities.

(The Principle of Reaction is commonly paraphrased as 'for every action there is an equal and opposite reaction'.) In the example of a charged particle accelerated by an external electric field between charged plates used in Section 3.4, there is an obvious action of the external field on the particle but what is the reaction and how is it mediated? Before explicitly answering this question, consider the following description of fields by Noel Doughty in his text *Lagrangian Interaction*:

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26 Anandan, J. and Brown, H.R., 'On the Reality of Space-Time Geometry and the Wavefunction', *Foundations of Physics* **25** (1995) p.359.

27 Doughty, N.A., *Lagrangian Interaction: An Introduction to Relativistic Symmetry in Electrodynamics and Gravitation* (Addison-Wesley, Sydney, 1990), pp.116-117.

> Fields are thus of two forms, those like gravity or electromagnetism which are generated by a source (for example mass or electric charge), and those which are not and represent the sources themselves, such as the non-relativistic Schrödinger wave function … The field equations of a sourced, or mediated field, can be recognised by the presence in them of a term, *the source term*, which does not contain the field itself.[28]

A charged particle is surrounded by its own very small electric field which is independent from any external field. Both the particle's field and an external field (each with its own source) are distorted in shape when they interact. The particle reacts back on the external field via its own electric field. The standard answer to the above question is, of course, that the particle exerts a force (albeit almost totally negligible) on the plates equal and opposite to that which it experiences. This ensures agreement with the Principle of Reaction.

However, the issue of action and reaction is really a more general question of the status of the Principle of Reaction and whether it has universal validity. Indeed, it does not appear that the Principle of Reaction can be universally valid as there are counter-examples in electrodynamics where an action is not accompanied by a corresponding *equal and opposite* reaction.[29] This situation is not the same as for energy conservation since action-reaction is not found to apply in all circumstances. It seems to be the case that classical action-reaction applies in cases of contact phenomena but (as specific counter-examples indicate) not necessarily to all interactions.[30] We are, therefore, justified in holding to the Principle of the

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[28] *ibid.*, p.139 (his italics).

[29] Goldstein, H., *Classical Mechanics* (Addison-Wesley, Reading, M.A., 1980) pp.7-8; Fowles, G.R., *Analytical Mechanics* (Holt, Rinehart and Winston, N.Y., 1977), p.44.

[30] Lange, M., *An Introduction to the Philosophy of Physics: Locality, Fields, Energy, and Mass* (Blackwell, Oxford, 2002) pp.163 n.2 & 234.

Conservation of Total Energy and to denying the applicability of the Principle of Reaction to individual quantum systems.

One suggested approach that has appeared in the literature to 'rectify' the action-reaction problem within the Causal Theory is to add a source term to the Schrödinger equation.[31] This, however, would lead to a non-linear wave equation which would produce predictions in conflict with well-established empirical results. Instead of viewing the absence of a classical reaction as a defect in the Causal Theory, this should be seen as a *new insight* into the quantum domain. Doughty rightly points out that the Schrödinger wave field is not a mediated field. Therefore there is no familiar means to carry a reaction from the quantum particle to the wave field. Indeed, Cushing has suggested that our intuitions about classical action-reaction might not be reliable in the quantum realm.[32] This is a very plausible suggestion for we have seen that the total energy of a quantum system is conserved because energy transformations between particle and wave field are facilitated through the quantum potential, despite the absence of a classical reaction on the wave field.

Part of the difficulty of applying the Principle of Reaction to a quantum system is thinking of the quantum particle and its wave field as if they were on par with say, an external electric field and an introduced charged particle. The electric field example, useful as it is in demonstrating field characteristics, cannot be taken as accurately representing all aspects of quantum entities because the wave field is not a mediated field. The assumption that wave field and particle can be treated as separate but interacting entities that the Principle of Reaction applies equally to is the cause of

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[31] Squires, E.J., 'Some Comments on the de Broglie-Bohm Picture by an Admiring Spectator' in van der Merwe, A. and Garuccio, A. (eds), *Waves and Particles in Light and Matter* (Plenum Press, New York and London, 1994) p.129; Abolhasani, M. and Golshani, M., 'Born's Principle, Action-Reaction Problem, and Arrow of Time', *Foundations of Physics Letters* **12** (1999) p.304.

the problem. The wave field and its quantum particle(s) constitute a single entity, as has been emphasised throughout this dissertation. The Principle of Reaction, however, was formulated to apply to the interaction of separate entities.

We should also ask whether further consideration of energy transfer processes might enlighten this issue. Equations (4-8) and (4-9) together indicate that the energy transfer from particle to wave field is due to a change in the form of the wave field. This transfer of energy is an event that results from changes in the wave field, i.e. one part of a quantum system influencing another part of the system. This implies that the classical ideal of an action accompanied by an equal and opposite reaction need not be realised in the quantum domain. This is also echoed in Holland's response to the action-reaction issue. He writes:

> ... But while it may be reasonable to require reciprocity of actions in classical theory, this cannot be regarded as a logical requirement of all theories that employ the particle and field concepts, especially one involving a nonclassical field.[33]

The Principle of Reaction, as classically formulated, cannot account for all types of field interactions and quantum processes. This conclusion avoids conflict with the Causal Theory. The Principle of Reaction needs revision if it is to be applicable to quantum entities.

4.5 The Wave Field and the Quantum Potential

The mathematical expression of the quantum potential is very different from that found for potential functions associated with classical fields. Why is the quantum potential so dissimilar? In Bohm's original theory, the form of the quantum potential was merely accepted as given by the mathematics and not requiring further

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[32] Cushing, J.T., *Quantum Mechanics: Historical Contingency and the Copenhagen Hegemony* (Chicago University Press, Chicago, 1994) p.46.
[33] Holland, P.R., The Quantum Theory of Motion, *op. cit.*, p.26.

explanation. Others have seen a need to specify an origin for the quantum potential.[34] (A summary of efforts to derive the quantum potential may be found in Carroll's 2004 eprint.[35])

We are now in a position to list relevant features of the wave field and the quantum potential:

(i) The wave field exhibits the usual wave properties (i.e. reflection, transmission, diffraction, interference) and obeys the Principle of Linear Superposition.

(ii) Since the Schrödinger equation is homogeneous, the wave field is not a radiated field and there is no source term for the field.

(iii) The environment surrounding a quantum particle (in part) determines the shape of its wave field.

(iv) The wave field is the repository of potential energy in a quantum system.

(v) The wave field acts on the quantum particle similar to an external field and receives or imparts energy and momentum to the particle.

(vi) The quantum potential represents a portion of the energy contained in the wave field and is the amount of potential energy available to the particle at its specific position in the wave field.

(vii) The quantum potential term is independent of the intensity of the wave field.

(viii) Non-local connections between particles in a many particle quantum system are facilitated through the quantum potential.

Given these features, what might be inferred about the 'origin' of the quantum potential? At a coarse level of description, Q is the potential energy function of the wave field and its 'origin' may be understood as deriving from the potential energy of the wave field. In classical mechanics, the form of a potential energy function is found by integrating the expression for the force between classical particles where the force expression contains a source term for the classical field.

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34 Hiley, B.J. and Peat, F.D., 'The Development of David Bohm's Ideas from the Plasma to the Implicate Order' in Hiley, B.J. and Peat, F.D. (eds), *Quantum Implications: Essays in Honour of David Bohm* (Routledge, London and New York, 1987) p.12.

This is not the case in the Causal Theory for there is no source term and the force expression does not have a general form (such as the inverse square law). Therefore, the form of the quantum potential cannot be derived, in general, by this means. This is sometimes expressed by stating that the quantum potential is not a pre-assigned function of the coordinates.

This identification of Q as the potential energy function is not an entirely satisfactory response to the question of the quantum potential's origin and is, by no means, a complete explanation of its nature. We have not, for example, explained why the quantum potential takes the form $(-\hbar^2/2m)(\nabla^2 R/R)$ rather than one more akin to the familiar classical potential functions which essentially have a (1/r) dependence, where r is the distance from the relevant classical particle. Nor have we explained why the effect of the wave field is independent of its intensity. Clearly, these two issues are not mutually exclusive and deserve some further comment.

Consider first the latter issue. The effect of the wave field on its quantum particles depends on how much of its energy is available to the particles via the quantum potential. This, in turn, is related to the total energy stored in the wave field which cannot depend on field intensity (i.e. amplitude squared) for if it did, the wave field could not have an amplitude large enough to store energy up to macroscopic orders of magnitude (cf. Bohm's comment quoted in Section 4.2). Equation (4-22) shows that the energy content of the wave field is also independent of the field's intensity. This can be seen in the same way as we did for the quantum potential, i.e. multiplication of the amplitude R in equation (4-22) by a constant does not change the value of (H − T). The mechanism of energy storage and transfer cannot be the same as for classical fields.

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[35] Carroll, R., 'On the Quantum Potential', *arXiv:quant-ph/0403156* (16 April 2004) and references therein.

In relation to the former issue, the quantum potential is structured in such a manner so as to facilitate energy exchange between wave field and quantum particles without the presence of a mediated field. Again, the mechanism must be different to the classical case. Clearly, if there is no wave field source term then the quantum potential cannot include such a term. This is an obvious restriction on the form of Q. We have also seen that the shape of the wave field is an important factor in the determination of the wave field's energy-momentum storage and transfer characteristics. A potential energy function that can perform the roles discussed above would need specific constraints on its form. In particular, the form of potential functions found in classical electrodynamics and Newtonian gravitational theory are ruled out since these have a (1/r) dependence (just as classical amplitudes do).

We may now deal with a related conceptual problem involving the quantum potential itself. There is an objection to the Causal Theory occasionally made in the literature which claims that the quantum potential is not a physical potential[36] (where 'physical potential' is presumably used in the same sense as in electrostatics.) A conceptual problem arises because the nature of the quantum potential conflicts with the belief that physical potentials must be due to a source (such as an electric charge).[37] However, as detailed above, the quantum potential should be understood as deriving from the potential energy of the wave field. The 'recycling' of energy by the wave field in an isolated quantum system indicates how the quantum potential operates. Although the operation of the quantum potential would be unexpected on the basis of our understanding of classical potentials, this has not prevented a consistent account of energy conservation being developed. The quantum potential is an integral part of these physical processes which do not require classical sources.

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[36] Rae, A.I.M., *Quantum Mechanics* (IOP Publishing, Bristol, 1992) p.227.

[37] Parmenter, R. H. and DiRienzo, A.L., 'Reappraisal of the causal interpretation of quantum mechanics and of the quantum potential concept', *arXiv:quantum-ph/0305183* (22 May 2004) p.1.

The stipulation that all physical potentials must have (classical) sources does not apply to quantum fields.

On the question of the possible violation of Special Relativity with respect to potential energy – what about the instantaneous changes that occur when a quantum system is 'manipulated'? When such manipulation occurs, there will be an instantaneous change to a wavefunction which means that the quantum potential value for a particle in a spatially distant part of the quantum system may also change. The instantaneous change to the form of the wavefunction is a feature of the model being non-relativistic but does not produce superluminal transfer of energy. The transfer of energy to/from an individual particle (the i^{th} particle, say) in a many-particle quantum system from/to its wave field occurs through the particle's quantum potential Q_i where $Q_i = -[\hbar^2/2m_i R]\, \nabla_i^2 R$. The value of Q_i depends on the position of the particle in the wave field and as such, the energy transfer is from a portion of the wave field at that location, i.e. this process occurs locally. (Recall the apt comment of Hermann Weyl in Section 3.4 that every portion of a field has a definite amount of potential energy.[38]) Therefore, there is no violation of Special Relativity when energy is transferred within a quantum system (see also Section 4.7 below).

These considerations imply that the level of analysis which is appropriate to the issue of origin of the quantum potential is one where the nature of the wave field itself is the subject. An in-depth ontological account of the wave field would provide a more substantial explanation of the origin of the quantum potential (as the potential energy function of the wave field) but such an account would need a relativistic approach. In relation to the wave field, however, a relevant question is why does the Schrödinger equation describe its propagation (in the non-relativistic domain)? Even before quantum theory was given any interpretation, no rigorous derivation of the Schrödinger equation had even been attempted from basic physical assumptions. Instead, the Schrödinger equation is justified by appeal to its predictions and the results of experiments, as the following extract from a leading quantum mechanics text indicates:

> Various assumptions have to be made as regards the structure of the [Schrödinger] wave equation ... These assumptions are given a high degree of plausibility ... by relating them to experimental results ... However, no attempt is made to derive the formalism uniquely from a consideration of the experiments.[39]

Many 'derivations' of the Schrödinger equation have been made, each of which is based on different premises.[40] There is no general agreement on its derivation. Yet, it would seem reasonable that the Schrödinger equation should hold by virtue of some general physical principles subject to the specifics of the appropriate domain.

The mathematical form of the (classically-free) Schrödinger equation is similar to the Heat Flow equation of classical physics rather than the classical wave equation (a similarity that has been noticed since the advent of quantum mechanics). The principal difference between the classically-free Schrödinger equation and the Heat Flow equation is the appearance of the imaginary number $i = \sqrt{-1}$ in the Schrödinger equation. However, this is a mathematical convenience as the Schrödinger equation can be rewritten as two coupled, differential equations involving two real functions. The presence of i in the Schrödinger equation is ultimately traceable to wavefunctions being defined as complex functions. In the Causal Theory, the wavefunction represents the wave field within a mathematical

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[38] Weyl, H., *Space-Time-Matter* (Dover, New York, 1952) p.70.

[39] Schiff, L.I., *Quantum Mechanics* (McGraw-Hill Kogakusha, Tokyo, 1968) p.19.

model (which must be the case as any physically acceptable but otherwise arbitrary wavefunction can be normalised). The definition of the wavefunction as a complex function should not, therefore, be considered problematic (cf. the discussion in Section 2.6 of the distinction between the formal machinery of a model representing an aspect of reality and reality itself).

The Heat Flow equation of classical physics (without any heat sources) is:

$$\nabla^2 u = \frac{1}{\beta}\left(\frac{\partial u}{\partial t}\right) \quad \ldots. \text{(4-23)}$$

where $u = u(\mathbf{x},t)$ is temperature and β is a constant.[41] Does the similarity of the (classically-free) Schrödinger equation to the Heat Flow equation hold any significance? We can answer this in the affirmative for the similarity arises through a common functional role of both equations. If one looks at a standard derivation of the Heat Flow equation without the presence of any sources of heat, the derivation proceeds by specifying the energy flux from one region to another due to temperature variation. The resulting equation then describes the transfer of energy (in this case, heat).

The Schrödinger equation describes the propagation through space of a field which has no sources (in the classical sense) but has a finite energy content. The wave field's time development conserves energy and thereby describes an energy flux from one spatial region to another. Doughty provides an apt description:

> The Schrödinger equation can be considered an example of a classical field equation of Galilean relativity which is local in space and time with a simple well-defined additive and conserved local energy.[42]

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[40] Carroll, R., 'Remarks on the Schrödinger Equation', *eprint arXiv:quant-ph/0401082 v2* (22 Mar 2004) pp.1-2.

[41] Boas, M.L., *Mathematical Methods in the Physical Sciences* (Wiley, New York, 1966) p.630.

[42] Doughty, N.A., Lagrangian Interaction, *op. cit.*, p.122.

The spatial energy flux due to the propagation of the wave field together with the lack of sources places restrictions on the form of the equation describing the wave field's time development (i.e. the Schrödinger equation). This, in part, explains the similarity of the classically-free Schrödinger equation to equation (4-23).

4.6 The Wave Field and Physical Measurement

It was noted in Chapter Two that what is commonly called 'measurement' in Orthodox Quantum Theory is only one type of interaction between different physical systems (albeit a most important interaction from the perspective of experimental physics). In the Causal Theory's account of the measurement process, the wavefunction is split into non-overlapping packets which move off independently. The spatially separated, empty wave packets do not affect the particle and the wavefunction Ψ effective evolves:

$$\Psi \rightarrow c_a \psi_a \phi_o$$

There is no 'collapse' of the wavefunction. The various separated parts of the wave field continue to objectively exist albeit as empty quantum waves. The interaction with a device designed to measure an 'observable' of a quantum system transforms the wavefunction Ψ into what would be called an eigenstate of the observable in Orthodox Quantum Theory. This measurement account is described by means of a multi-dimensional configuration space as the interaction involves a many-particle system (viz. the measurement apparatus). Although the mathematical description is in terms of such a configuration space, the actual measurement interaction occurs in physical three-dimensional space, not in a multi-dimensional configuration space, and not in a mathematical Hilbert space. Recall that in Chapter Two, the view that a multi-dimensional configuration space is a real aspect of nature was rejected.

Therefore, we ought to be able to give a *physical* account of what happens to particle and field in three-dimensional space during a measurement process.

We saw in Chapter Two that measurement processes generally introduce uncontrollable (and unpredictable) disturbances to a quantum system. Instead of a collapse on measurement, there is a physical change caused to the wave field with the following results: (i) the wavefunction (which is the mathematical description of the physical wave field) is altered; and (ii) the particle's position, momentum, etc., which depends on the wave field, will generally be changed as a consequence. Since an 'act of measurement' generally changes the form of the wave field, such acts would be better labelled as 'disturbance measurements', for it is conceivable to have 'non-disturbance measurements', i.e. those that do not alter the form of the wave field. This kind of measurement is also known as "protective".[43]

In order to visualise what occurs during one kind of 'disturbance measurement', consider a quantum particle in cubical box of side length L. If one of the walls is removed suddenly then the particle and its wave field would no longer be contained. The wave field will evolve from a stationary state and will change in form as it propagates out of the box from a standing wave to a travelling wave. There will be a corresponding change to the wavefunction. During this transition process, energy will transfer from the wave field to the particle (as seen in Section 4.2). This change would be interpreted in Orthodox Quantum Theory as an instantaneous collapse of the wavefunction since the removal of the wall would be a necessary part of a 'measurement' on the particle. However, the restoration of the particle's kinetic energy on measurement (or any other process) is *not* and *cannot be* an instantaneous process, otherwise the Special Theory of Relativity would be violated. (We have already discussed that even though the non-relativistic Causal Theory is non-local, this does not lead to violations of Special Relativity.) It was found for the case of a classically free particle, that the rate of change of kinetic energy with respect to time is equal to:

$$(dT/dt) = -(\nabla Q)\cdot(\nabla S/m)$$

from which the time of energy transfer would be given by:

$$t = -m\int_{T_i}^{T_f}\frac{dT}{(\nabla Q)\cdot(\nabla S)}$$

where T_i and T_f are the initial and final kinetic energies of the particle respectively. This integral can be evaluated in specific instances (but not for the infinite well since the integral is not defined in that particular case as $\nabla S = 0$).

There is a change in amplitude that accompanies the change in the shape of the wave field over a short time interval immediately after the wall is removed. From equation (4-9) and equation (4-10), we have:

$$\frac{dU}{dt} = \frac{\hbar^2}{2mR}\nabla^2\left(\frac{\partial R}{\partial t}\right) + \frac{Q}{R}\left(\frac{\partial R}{\partial t}\right)$$

The more pronounced the change in shape is, as indicated by $(\partial R/\partial t)$, the greater will be the amount of energy exchanged between particle and wave field. What form does the wave field take upon leaving the box? It would seem that the *exact* mathematical form of such a wave is not possible to find.[44] Given this, one has to assume a form for the wave field as it emerges from the open part of the box. A physically reasonable approximation for the form of the wave field is represented by the initial, normalised Gaussian wave packet:

$$\Psi_o = (2\pi\sigma_o^2)^{-3/4}\exp\{i\mathbf{k}\cdot\mathbf{x} - (|\mathbf{x}|^2/4\sigma_o^2)\}$$

Then the corresponding quantum potential is:

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[43] Aharonov, Y. and Rohrlich, D., *Quantum Paradoxes: Quantum Theory for the Perplexed* (Wiley-VCH, Weinheim, 2005) pp.214-215.

$$Q_o = (\hbar^2/4m\sigma_o^2)\{3 - (|\mathbf{x}_o|^2/2\sigma_o^2)\}$$

where $\sigma_o = (L/2)$ and the particle's position being at the end of the box, i.e. $|\mathbf{x}_o| = L$. This gives the value: $Q_o = (\hbar^2/4mL^2)$. In the stationary state inside the closed box, the quantum potential had a minimum value of $(3\pi^2\hbar^2/2mL^2)$. The difference between these shows a decrease in the value of the quantum potential of slightly less than $(15\hbar^2/mL^2)$. This loss of energy from the wave field appears, of course, as the kinetic energy of the moving quantum particle. Once clear of the obstruction (i.e. the open end of the box) the wave field will tend to the form of a travelling plane wave of constant amplitude. This latter evolution of the wave field is essentially the same as the account given in Section 4.3.

Belinfante once stated that it is a contradiction-in-terms to claim that *free* quantum particles would accelerate. He further asserted that there is no sense in maintaining such a claim without experimental support.[45] Similar sentiments were more recently expressed by Parmenter and DiRienzo[46] which were motivations for their 'non-interactive' approach to the wave field (as presented in Section 3.3). Belinfante's conclusion is, of course, not warranted in the context of the Causal Theory, for classically-free particles are not necessarily quantum mechanically free. The above case of a particle moving out of an opened box is one such example.

4.7 Tunnelling from a Quantum Well

Tunnelling is a quantum phenomenon with no classical analogue. In the case of a potential well (such as a Coulomb force field) quantum mechanics predicts that there

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44 Main, I.G., *Vibrations and Waves in Physics* (Cambridge University Press, Cambridge, 1978) pp.309-310.

45 Belinfante, F.J., *A Survey of Hidden-Variable Theories* (Pergamon, Oxford, 1973) p.121.

46 Parmenter, R. H. and DiRienzo, A.L., 'Reappraisal of the causal interpretation of quantum mechanics and of the quantum potential concept', *arXiv:quantum-ph/0305183* (22 May 2004) p.5.

is a small but finite probability that particles can be found in a classically forbidden region. Tunnelling arises formally as a consequence of the mathematics (i.e. by the constraints of continuity for the wavefunction and its first derivative at boundaries) but has no other explanation in Orthodox Quantum Theory. Since tunnelling has been experimentally confirmed, it is clearly a real, physical process rather than an artefact of the mathematics. A number of papers have been devoted to aspects of tunnelling within the Causal Theory.[47]

How can particles free themselves when bound in a potential well, i.e. where the magnitude of the particle kinetic energy is less than the potential energy of the well? Consider such a situation where quantum particles are trapped in a well (such as may be produced by an electric field) with insufficient kinetic energy to escape. Quantum mechanics predicts that there is a small probability that some particles can be found outside the well. If we have an N-particle system (N > 1), classically one would expect the particles to be held in a well with a (finite) potential V if

$$[(\nabla_i \mathrm{S})^2/2m_i] < |\mathrm{V}|$$

for all i, $1 \leq i \leq \mathrm{N}$. (It is usual for a well exerting an attractive force to have its potential energy defined to be zero at the 'top' of the well which then requires V to be negative inside the well.) How can any particles become free if bound by an attractive force field? The solution becomes evident when the role of the quantum potential is recognised as facilitating the exchange of energy between the wave field and particles. In the many particle case, individual particles can gain energy from, or lose energy to, the wave field through their associated quantum potential Q_i depending on their position in the wave field, where $\mathrm{Q}_i = -[\hbar^2/2m_i\mathrm{R}]\,(\nabla_i^2\,\mathrm{R})$.

[47] E.g. Dewdney, C. and Hiley, B.J., 'A Quantum Potential Description of One-Dimensional Time-Dependent Scattering From Square Barriers and Square Wells', *Foundations of Physics* **12** (1982): 27-48; Cushing, J.T., 'Quantum Tunneling Times: A Crucial Test for the Causal Program?', *Foundations of Physics* **25** (1995): 269-280.

This transfer of energy occurs locally so that there is no violation of Special Relativity.

Therefore the condition for the *i*-th particle to escape from the well is:

$$(\frac{1}{2m_i})(\nabla_i S)^2 > \left| V + (\frac{\hbar^2}{2m_i R})\nabla_i^2 R \right| \quad .... (4\text{-}24)$$

This condition can be satisfied in two ways depending on the nature of the potential well, the form that the wave field takes within the well, and the positions of the particles. First, an individual particle might gain sufficient energy from the wave field that its kinetic energy becomes large enough to satisfy the inequality (4-24). Second, a small part of the wave field might increase its energy content (and thereby increase the magnitude of quantum potentials Q_i associated with individual particles situated in that part of the wave field) so that the absolute value of the net potential energy in this region of the wave field (i.e. $|V + Q_i|$) is less than an individual particle's kinetic energy. The additional energy in both cases is gained at the expense of a portion of the kinetic energies of other (non-tunnelling) particles in the system. Either way, this would allow a small fraction of the total number of particles to break free of the binding force of the well.

CHAPTER 5
THE EXCLUSION PRINCIPLE

THE EXCLUSION PRINCIPLE plays an important role in quantum physics and has effects that are almost as profound and as far-reaching as those of the principle of relativity … [the Exclusion Principle] enacts vetoes on a very basic level of physical description.

— Henry Margenau [1]

5.1 What is the Exclusion Principle?

Why is it that electrons within an atom do not all collect in the lowest orbital? This is a long-standing, unsolved problem of atomic physics. In 1925, Wolfgang Pauli published a limited version of the Exclusion Principle from his studies of the fine structure of atomic energy levels and the earlier suggestions of E.C. Stoner.[2] This limited version is known as Pauli's Principle and may be stated as follows:[3]

♦ *Pauli's Principle*

In an atom there cannot be two or more electrons with the same quantum numbers.

Since a set of quantum numbers specifies a unique state of a particular physical system, Pauli's Principle provides a *simple rationale* for the existence of the observed atomic electron 'shells'. A complete and consistent explanation of why Pauli's Principle holds has never been advanced. Indeed, since its inception, the status of Pauli's Principle has been axiomatic. This is well summarised in the following statement by Lindsay and Margenau:

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[1] Margenau, H., *The Nature of Physical Reality* (McGraw-Hill, New York, 1950) p.427.

[2] Jammer, M., *The Conceptual Development of Quantum Mechanics* (McGraw-Hill, New York, 1966) p.143; Duck, I. and Sudarshan, E. C. G., *Pauli and the Spin-Statistics Theorem* (World Scientific, River Edge, 1997) pp.21-22.

[3] Pauli, W., 'Uber den Zusammenhang des Abschlußes der Elektronengruppen im Atom mit der Komplexstruktur der Spektren', *Zeitschrift für Physik* **31** (1925) p.776.

> There is *no way of deducing* Pauli's principle; its validity has to be inferred from its results ...[4]

Pauli's Principle was generalised when it was realised that the Principle applies not just to electrons but to all fermions of the same type. Quantum particles are identical if they have the same mass, electric charge, etc. Fermions are sometimes defined to be those identical quantum particles that when part of a quantum system, the system has a wavefunction that is antisymmetrical in its form (see below). The antisymmetrical form of the wavefunction is taken as a 'brute fact', i.e. as a defining characteristic of fermions. However, this definition cannot be used for a single particle system. A better definition is that a fermion is a quantum particle whose system exhibits a half-odd integer value for its intrinsic angular momentum (see Section 5.2). The latter definition is more basic than the former as it can be given for just one fermion. The generalisation of Pauli's Principle is called the Exclusion Principle:

- *Exclusion Principle*

  In a quantum system, two or more fermions of the same kind cannot be in the same (pure) state.[5]

The Exclusion Principle acts primarily as a selection rule for non-allowed quantum states and cannot be deduced as a theorem from the axioms of Orthodox Quantum Theory. Pauli himself, admitted (with some frustration) that the Exclusion Principle could not be deduced. He wrote:

> ... I was unable to give a logical reason for the exclusion principle or to deduce it from more general assumptions. ...

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[4] Lindsay, R.B. and Margenau, H., *Foundations of Physics* (Dover, New York, 1957) p.491 (italics mine).

[5] Sudbery, A., *Quantum Mechanics and the Particles of Nature* (Cambridge University Press, Cambridge, 1986) p.72; Ballentine, L.E., *Quantum Mechanics: A Modern Development* (World Scientific, Singapore, 1998) p.476.

> in the beginning I hoped that the new quantum mechanics … [would] also rigorously deduce the exclusion principle.[6]

This is a serious admission of incompleteness given that the Exclusion Principle is acknowledged as acting at a very basic physical level and having ramifications for all phenomena.

The standard approach in introductory textbooks on quantum mechanics is to justify the Exclusion Principle by appealing to the 'indistinguishability' of identical particles. In Orthodox Quantum Theory, it is assumed that identical particles cannot be distinguished from each other. Suppose we have two non-interacting, identical quantum particles. Let the first particle have coordinates $\mathbf{x}_1$ and its wavefunction be denoted $\psi_A(\mathbf{x}_1)$. Similarly, let the second particle's coordinates be $\mathbf{x}_2$ with wavefunction denoted $\psi_B(\mathbf{x}_2)$. The composite system consisting of these particles is represented by a single wavefunction, denoted $\psi(\mathbf{x}_1, \mathbf{x}_2)$. This wavefunction is a solution of the two-particle Schrödinger equation and is equal to the product of the individual wavefunctions, i.e.

$$\psi(\mathbf{x}_1, \mathbf{x}_2) = \psi_A(\mathbf{x}_1)\psi_B(\mathbf{x}_2) \quad \ldots.\ (5\text{-}1)$$

The 'indistinguishability' argument proceeds by claiming that since the identical particles are indistinguishable, their coordinates merely serve to label the particles and an exchange of such 'labels' cannot be empirically meaningful. This would require the two-particle wavefunction to yield the same probability density regardless of whether the particle 'labels' are exchanged or not. If an exchange of particle 'labels' has no empirical import then the following equalities should hold:

$$|\psi(\mathbf{x}_1, \mathbf{x}_2)|^2 = |\psi_A(\mathbf{x}_1)\ \psi_B(\mathbf{x}_2)|^2 = |\psi_A(\mathbf{x}_2)\ \psi_B(\mathbf{x}_1)|^2 = |\psi(\mathbf{x}_2, \mathbf{x}_1)|^2$$

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6 Pauli, W., *Exclusion Principle and Quantum Mechanics* (Editions du Griffon, Neuchatel, Switzerland., 1947) p.136.

Using equation (5-1), it is easily shown that the probability densities found before and after exchange of the particle 'labels' are *not* equal.[7] However, this may be corrected by the technique of linearly combining the wavefunctions. Since $\psi_A(\mathbf{x}_1)\psi_B(\mathbf{x}_2)$ and $\psi_A(\mathbf{x}_2)\psi_B(\mathbf{x}_1)$ are both solutions of the (two-particle) Schrödinger equation, so is any linear combination of them (as the Schrödinger equation is linear). Using this method, the composite system's wavefunction may be expressed as the following two kinds of linear combinations:

$$\psi(\mathbf{x}_1, \mathbf{x}_2) = (1/\sqrt{2})\,[\psi_A(\mathbf{x}_1)\psi_B(\mathbf{x}_2) \pm \psi_A(\mathbf{x}_2)\psi_B(\mathbf{x}_1)] \quad \ldots. (5\text{-}2)$$

where the factor $(1/\sqrt{2})$ is required for normalisation.

If the sign between the two terms in equation (5-2) is positive then ψ is said to be symmetric with respect to the exchange of coordinates as $\psi(\mathbf{x}_1, \mathbf{x}_2) = \psi(\mathbf{x}_2, \mathbf{x}_1)$. If the sign is negative then ψ is said to be antisymmetric with respect to the exchange because $\psi(\mathbf{x}_1, \mathbf{x}_2) = -\psi(\mathbf{x}_2, \mathbf{x}_1)$. It is the case that only symmetrical and antisymmetrical wavefunctions are 'found' in nature.[8] Both these types of wavefunction satisfy the required probability density equality, but only antisymmetrical wavefunctions entail the Exclusion Principle. This is seen if, in equation (5-2), we take the negative sign and make $\mathbf{x}_1 = \mathbf{x}_2$ then $\psi = 0$, i.e. there is no corresponding quantum state. Note that the presence of the two terms inside the brackets of equation (5-2) indicates that the particles do not act independently unlike those represented by equation (5-1).

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[7] See, for example, the treatment in: French, A.P. and Taylor, E.F., *An Introduction to Quantum Physics* (Nelson, Middlesex, 1978) p.561.

[8] Greenhow, R.C., *Introductory Quantum Mechanics* (Hilger, Bristol and New York, 1990) p.213; Zettili, N., *Quantum Mechanics: Concepts and Applications* (Wiley, Chichester, 2001) p.444; Omar, Y., 'Indistinguishable Particles in Quantum Mechanics: An Introduction', *Contemporary Physics* **46** (2005) p.443.

A full treatment of the antisymmetry will also take into account a system's intrinsic angular momentum. The Exclusion Principle arises from the wavefunction of a system of fermions being antisymmetric in its form, as was grasped initially (and independently) by both Heisenberg and Dirac in 1926.[9] It should be noted, however, that the Exclusion Principle is not equivalent to the condition that fermionic systems have antisymmetrical wavefunctions (as asserted in many quantum mechanics texts) but follows from this condition.

The standard textbook argument is not valid for it has been shown elsewhere that the conclusion that the wavefunction of a fermionic system is antisymmetric in form does not follow from the indistinguishability criterion alone. Assumptions in addition to the indistinguishability of identical particles are needed to arrive at the result of antisymmetrical wavefunctions[10] (as will also be discussed below). Nor does the antisymmetric form of fermionic wavefunctions arise from the requirements of relativistic invariance. This is a justification for the antisymmetric form that is erroneously claimed in the literature as having been conclusively established by Pauli.[11] It is only the case that relativistic invariance is merely *consistent* with antisymmetric wavefunctions.[12] The fact that the total wavefunction for a system of fermionic particles takes an antisymmetric form rather than a symmetric one, or a form exhibiting another symmetry, or a form that exhibits no symmetry, has not been satisfactorily explained. The assumption that fermionic

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[9] Dirac, P.A.M., 'On the Theory of Quantum Mechanics', *Proceedings of the Royal Society A* **112** (1926) pp.669-670; Heisenberg, W., 'Mehrkörperprobleme und Resonanz in der Quantenmechanik', *Zeitschrift für Physik* **38** (1926): 411-426.

[10] Harris, L. and Loeb, A.L., *Introduction to Wave Mechanics* (McGraw Hill, New York, 1963) p.244; Messiah, A.L.M. and Greenberg, O.W., 'Symmetrization Postulate and Its Experimental Foundation', *Physical Review* **136** (1964) pp.B248- B249; De Muynck, W. and van Liempd, G., 'On the Relation between Indistinguishability of Identical Particles and (Anti)symmetry of the Wave Function in Quantum Mechanics', *Synthèse* **67** (1986) p.478.

[11] Reif, F., *Fundamentals of Thermal and Statistical Physics* (McGraw-Hill-Kosahusha, Tokyo, 1981) p.332; Itzykson, C. and Zuber, J.-B., *Quantum Field Theory* (McGraw-Hill, Singapore, 1987) pp.149-150.

wavefunctions are antisymmetric is added to Orthodox Quantum Theory as an additional postulate.[13] Further, this antisymmetry cannot be given a *physical* explanation within the confines of Orthodox Quantum Theory because the wavefunction is only considered to be an abstract entity that does not represent anything physically real.

Discussion on the indistinguishability and identity of quantum particles has a long history in the literature.[14] It remains an area of intense philosophical debate and also of disagreement between physicists and some philosophers of physics. Although many accounts of identical quantum particles assume that these particles are *always* indistinguishable, it can be well argued that this is not the case (see below). In this thesis, the criterion for identical particles to be indistinguishable is that the particles' individual wavefunctions have (spatially) overlapped at some particular time in the past. This means that identical quantum particles can be distinguished if they are sufficiently apart (such as when each is in different and well separated atoms) and have remained so for then the overlap of their individual wavefunctions is zero.[15]

If it is assumed that a composite system's state is given by the simple product of the individual wavefunctions, i.e. as given by equation (5-1), then this is also assuming that there is no overlap of these wavefunctions. This can be seen by

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12 Dieks, D., 'Quantum Statistics, Identical Particles and Correlations', *Synthèse* **82** (1990) pp.134-135; van Fraassen, B.C., *Quantum Mechanics: An Empiricist View* (Clarendon, Oxford, 1991) p.384.

13 Harris, L. and Loeb, A.L., Introduction to Wave Mechanics, *op. cit.*, p.244; Sudbery, A., Quantum Mechanics and the Particles of Nature, *op. cit.*, p.72.

14 See the references in: French, S. and Rickles, D., 'Understanding Permutation Symmetry' in Brading, K. and Castellani, E. (eds), *Symmetries in Physics: Philosophical Reflections* (Cambridge University Press, Cambridge, 2003).

15 Schiff, L.I., *Quantum Mechanics* (McGraw-Hill Kogakusha, Tokyo, 1968) p.364; Eisberg, R. and Resnick, R., *Quantum Physics of Atoms, Molecules, Solids, Nuclei, and Particles* (Wiley, New York, 1985) p.303; Sakurai, J.J., *Modern Quantum Mechanics* (Addison-Wesley, Redwood City, California, 1985) pp.365-366; Townsend, J.S., *A Modern Approach to Quantum Mechanics* (University Science Books, Sausalito CA., 2000) p.341; Omar, Y., 'Indistinguishable Particles in Quantum Mechanics', *loc. cit.*, p.439; Haroche, S. and Raimond, J.-M., *Exploring the Quantum: Atoms, Cavities and Photons* (Oxford University Press, Oxford, 2006) pp.42-43.

evaluating the expectation value of the square of the distance between two quantum particles (here we shall follow the treatment in Griffiths' 2005 text[16]). Consider two particles with coordinates x_1 and x_2 in a combined state with (normalised) wavefunction $\Psi(x_1, x_2)$. The distance between the particles is $(x_1 - x_2)$ and the expectation value of the square of the distance is:

$$\langle (x_1 - x_2)^2 \rangle = \int\int \Psi^*(x_1, x_2)\, [(x_1 - x_2)^2]\, \Psi(x_1, x_2)\, dx_1\, dx_2$$

$$= \int\int \Psi^*(x_1, x_2)\, [x_1^2 + x_2^2 - 2\, x_1\, x_2]\, \Psi(x_1, x_2)\, dx_1\, dx_2 = \langle x_1^2 \rangle + \langle x_2^2 \rangle - 2\, \langle x_1\, x_2 \rangle$$

Now if we let $\Psi(x_1, x_2) = \psi(x_1, x_2) = \psi_A(x_1)\psi_B(x_2)$, then we find that:

$$\langle x_1^2 \rangle = \int\int \psi^*(x_1, x_2)\, [x_1^2]\, \psi(x_1, x_2)\, dx_1\, dx_2$$

$$= \int \psi^*_A(x_1)\, [x_1^2]\, \psi_A(x_1)\, dx_1 \int \psi^*_B(x_2)\, \psi_B(x_2)\, dx_2 = \langle x^2 \rangle_A$$

where $\langle x^2 \rangle_A$ is the expectation value of x^2 in the (single-particle) state denoted A and ψ_A and ψ_B are taken to be orthonormal wavefunctions (i.e. wavefunctions that are normalised and have a zero inner product). Likewise we find: $\langle x_2^2 \rangle = \langle x^2 \rangle_B$ and $\langle x_1\, x_2 \rangle = \langle x \rangle_A \langle x \rangle_B$, where $\langle x \rangle_A$ is the expectation value of x in the (single-particle) state denoted A, $\langle x \rangle_B$ is the expectation value of x in the (single-particle) state denoted B, etc. Therefore, for $\psi(x_1, x_2) = \psi_A(x_1)\psi_B(x_2)$, we get:

$$\langle (x_1 - x_2)^2 \rangle = \langle x^2 \rangle_A + \langle x^2 \rangle_B - 2\, \langle x \rangle_A \langle x \rangle_B.$$

This is the result for particles that exhibit no symmetry (or antisymmetry) in the form of their wavefunctions i.e. are distinguishable. The wavefunctions have *no overlap.*

Now let's find the expectation value of the square of the distance between two identical fermions by using an antisymmetrical wavefunction, i.e. we let

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16 Griffiths, D.J., *Introduction to Quantum Mechanics* (Pearson Prentice-Hall, New Jersey, 2005) pp.207-208.

$\Psi(x_1, x_2) = \Psi(x_1, x_2) = (1/\surd 2)\ [\psi_A(x_1)\psi_B(x_2) - \psi_A(x_2)\psi_B(x_1)]$. Then, as above, we have:

$$\langle (x_1 - x_2)^2 \rangle = \langle x_1^2 \rangle + \langle x_2^2 \rangle - 2\langle x_1 x_2 \rangle$$

Using $\Psi(x_1, x_2)$, we find that:

$$\langle x_1^2 \rangle = \int\int \Psi^*(x_1, x_2)\,[x_1^2]\,\Psi(x_1, x_2)\ dx_1\,dx_2$$

$$= ½ \int\int [\psi_A^*(x_1)\psi_B^*(x_2) - \psi_A^*(x_2)\psi_B^*(x_1)]\, x_1^2 [\psi_A(x_1)\psi_B(x_2) - \psi_A(x_2)\psi_B(x_1)]\, dx_1\, dx_2$$

$$= ½ \int x_1^2\, |\psi_A(x_1)|^2\, dx_1 \int |\psi_B(x_2)|^2\, dx_2 + ½ \int x_1^2\, |\psi_B(x_1)|^2\, dx_1 \int |\psi_A(x_2)|^2\, dx_2$$

$$- ½ \int x_1^2\, [\psi_A^*(x_1)\ \psi_B(x_1)]\, dx_1 \int [\psi_B^*(x_2)\psi_A(x_2)]\, dx_2$$

$$- ½ \int x_1^2\, [\psi_B^*(x_1)\ \psi_A(x_1)]\, dx_1 \int [\psi_A^*(x_2)\psi_B(x_2)]\, dx_2$$

$$= (1/2)\,[\,\langle x^2 \rangle_A + \langle x^2 \rangle_B\,]$$

where the last two terms are zero due to ψ_A and ψ_B being orthonormal wavefunctions. Likewise we find: $\langle x_2^2 \rangle = (1/2)\,[\,\langle x^2 \rangle_B + \langle x^2 \rangle_A\,]$ and $\langle x_1 x_2 \rangle = \langle x \rangle_A \langle x \rangle_B - |\langle x \rangle_{AB}|^2$ where the quantity $\langle x \rangle_{AB} = \int x\ \psi_A^*(x)\psi_B(x)\ dx$, is a measure of the overlap between the individual wavefunctions ψ_A and ψ_B. Therefore, for $\Psi(x_1, x_2) = (1/\surd 2)\ [\psi_A(x_1)\psi_B(x_2) - \psi_A(x_2)\psi_B(x_1)]$, we get:

$$\langle (x_1 - x_2)^2 \rangle = \langle x^2 \rangle_A + \langle x^2 \rangle_B - 2\langle x \rangle_A \langle x \rangle_B + 2\,|\langle x \rangle_{AB}|^2$$

If the two individual wavefunctions did not overlap then $\langle x \rangle_{AB} = 0$,[17] and the above expression for $\Psi(x_1, x_2)$ would reduce to: $\psi_A(x_1)\psi_B(x_2)$. The fermions would then be distinguishable. In order for this to be the case, the particles must be widely separated and have remained so.

[17] *ibid.*, p.209.

In addition, those that dispute the argument that identical quantum particles can be distinguished by means of their spatial relations do so despite the fact that distinguishability can be demonstrated in experiments and can be shown to be lost when an overlap of wavefunctions occurs.[18]

Since much has been made of the indistinguishability of identical particles and a great deal of attention has been given to the notion, we shall briefly review the main argument. This argument claims that indistinguishability extends to encompass what is called the Permutation Invariance Postulate. This states:

- *Postulate of Permutation Invariance*

  If φ is the state of a composite system whose components are identical particles, then the expectation value of any observable A is the same for all permutations of φ.[19]

Permutation Invariance allows for quantum states that are symmetric, antisymmetric, and of higher symmetry. Permutation Invariance does not restrict states to just symmetric and antisymmetric ones, nor does it assign a type of particle to any particular symmetry class.[20] The restriction to just symmetric and antisymmetric restrict states requires accepting another assumption called the Symmetrization Postulate:

- *Symmetrization Postulate*

  The only possible states of a system of identical particles are described by vectors that are either completely symmetrical or completely antisymmetrical.[21]

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18 Bongs, K. and Sengstock, K., 'Physics with Coherent Matter Waves', *Reports on Progress in Physics* **67** (2004) p.917.
19 van Fraassen, B.C., Quantum Mechanics, *op. cit.*, p.381.
20 Massimi, M., *Pauli's Exclusion Principle: The Origin and Validation of a Scientific Principle* (Cambridge University Press, Cambridge, 2005) p.154.
21 Omar, Y., 'Indistinguishable particles ...', *loc. cit.*, p.439.

The Symmetrization Postulate is *not* implied by the indistinguishability of identical particles[22] but by the results of experiments, i.e. the Symmetrization Postulate is an empirical rule.[23] Some advance in placing the Symmetrization Postulate on a theoretical footing in the context of the Causal Theory have been made by Brown *et al.*,[24] although purely on a mathematical basis. However, this is not the only way to explain the restriction to particular symmetry classes.[25] Indeed, inferences based only on permutation invariance carry little weight in realist versions of quantum mechanics, as has been acknowledged by Huggett:

> … scientific realists cannot accept, as a legitimate argument form, inferences from the unobservability of a distinction to the irreality of the distinction …[26]

It is important to realise that symmetrization is not the only way to deal with the problem. In this thesis, a realist approach will be pursued which attempts to explain the antisymmetrical form of the wavefunction of a fermionic system using physical arguments. This will, therefore, offer a very different account to those that stress the notions of indistinguishability and/or permutation invariance.

The antisymmetry of the wavefunction of a fermionic system constitutes a conceptual problem for the Causal Theory since, if the wave field is a physical field that propagates through space, it should be able to be represented by wavefunctions that do not have any particular symmetry (or antisymmetry). The arguments presented below do not depend on identical particles of the same kind being indistinguishable. In any case, the criterion of 'indistinguishability' itself fails within

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22 Ballentine, L.E., Quantum Mechanics, *op. cit.*, p.475; Massimi, M., Pauli's Exclusion Principle, *op. cit.*, p.155.
23 Ballentine, L.E., *ibid.*, p.475; Omar, Y., 'Indistinguishable particles …', *loc. cit.*, p.444.
24 Brown H.R., Sjoqvist E. and Bacciagaluppi G., 'Remarks on Identical Particles in de Broglie-Bohm Theory', *Physics Letters A* **251** (1999): 229-235.
25 Huggett, N., 'On the Significance of Permutation Symmetry', *British Journal for the Philosophy of Science* **50** (1999) p.326.
26 *ibid.*, p.335.

the context of the Causal Theory. Although the particles are identical in that they have the same mass, charge, etc., they can be distinguished in the Causal Theory by their individual trajectories.[27] What is required though, is that individual wave fields physically overlap for particles that form a single quantum system. This is consistent with the arguments presented above.

5.2 Quantum Mechanical Spin

Intrinsic angular momentum (otherwise known as 'spin') is a characteristic of quantum systems that is very relevant to the Exclusion Principle. We have already noted that the Exclusion Principle prescribes that if the fermions of a particular physical system share the same set of quantum numbers (and this includes the spin quantum number) then they cannot be at the same location. A coherent account of why the Exclusion Principle holds will require a realistic explanation of spin.

The initial concept of spin, as formulated by Uhlenbeck and Goudsmit in 1925, has its origin in the experiments of Stern and Gerlach in which a beam of silver atoms was split in two by passage through a non-uniform magnetic field.[28] Uhlenbeck and Goudsmit proposed that an electron had a magnetic dipole moment which they explained using the classical idea of a extended particle (in this case, an electron) spinning about an axis through its centre.[29] They used this idea to explain the results of the Stern-Gerlach experiments (although the concept of a spinning particle was suggested earlier by R. Kronig[30]). However, it has become clear that

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27 Holland, P.R., *The Quantum Theory of Motion* (Cambridge University Press, Cambridge, 1993) p.284.

28 Stern, O. and Gerlach, W., 'Der experimentelle Nachweis des magnetischen Moments des Silberatoms', *Zeitschrift für Physik* **8** (1922): 110-111; 'Der experimentelle Nachweis der Reichtungsquantelung im Magnetfeld', *Zeitschrift für Physik* **9** (1922): 349-355.

29 Uhlenbeck, G.E. and Goudsmit, S., 'Ersetzung der Hypothese von unmechanischen Zwang durch eine Forderung bezüglich des inneren Verhaltens jedes einzelnen Elektrons', *Die Naturwissenschaften* **13** (1925): 953-954; 'Spinning Electrons and the Structure of Spectra', *Nature* **117** (1926): 264-265.

30 Jammer, M., The Conceptual Development of Quantum Mechanics, *op. cit.*, pp.146-147.

what is called the 'spin of a quantum particle' is *not* the rotational angular momentum of a spinning particle. In other words, spin cannot be due to an extended body rotating about an axis through its centre of mass. The reasons against the axial rotation explanation are readily provided:

- the rotation of an extended particle would not require an additional variable for its specification;
- the spin's vector does not depend on the particle's position and momentum;
- angular momentum due to rotation about the centre of mass cannot take half-odd-integer values;[31] and
- the rate of rotation required to give results in agreement with experiment would need tangential velocities exceeding the speed of light in vacuum![32]

The rotational characterisation has merely assisted in 'picturing' the extra degree of freedom (i.e. spin) required for an accurate description of quantum states. What has also become clear about quantum mechanical spin is that total spin is a conserved quantity and that the square of the spin operator commutes with all other dynamical operators. These two points together with the other characteristics of spin listed above imply that spin must be (in some sense) internal to a quantum system. This has led some theorists to speculate that spin results from an internal structure of the particle.[33] Yet, it does not follow that because spin is internal to a quantum system that it must be due to the particle's structure. We shall return to this issue below.

Pauli claimed that the quantum mechanical spin has a discreteness that is not describable in classical terms, e.g. spin for electrons has two discrete values. In 1927, he introduced the equation which carries his name, in order to accommodate

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31 Sudbery, A., Quantum Mechanics and the Particles of Nature, *op. cit.*, p.138.
32 Jammer, M., The Conceptual Development of Quantum Mechanics, *op. cit.*, pp.149-150.
33 Saxon, D.S., *Elementary Quantum Mechanics* (Holden-Day, San Francisco, 1968) p.317; Penrose, R., *The Emperor's New Mind* (Vintage, London, 1989) p.341.

the spin variable of the electron in non-relativistic quantum mechanics. Erwin Schrödinger had postulated his scalar wave equation in 1926. The Pauli (or Pauli-Schrödinger) equation for a single spin-½ particle (i.e. with the third component of the spin along an arbitrary axis of value $\hbar/2$), of mass m, electric charge e, and magnetic moment μ, has a two-component wavefunction:

$$\Psi = (\psi_a) = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

and is given by:

$$i\hbar(\partial\Psi/\partial t) = \mathbf{H}\Psi$$

where the Hamiltonian operator is:

$$\mathbf{H} = \frac{-\hbar^2}{2m}\left[\nabla - \frac{ie}{\hbar c}\mathbf{A}\right]^2 + \mu\mathbf{B}\cdot\sigma + eA_o + V$$

with $\mathbf{A}$ and A_0 being the electromagnetic potentials, $\mathbf{B} = \nabla \times \mathbf{A}$ is an external magnetic field, c is the speed of light in vacuum, $i = \sqrt{-1}$, and V is a (classical) scalar potential.[34] The vector quantity σ has Pauli's 'spin matrices' as its components:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

where $\sigma^2 = \sigma_x^2 + \sigma_y^2 + \sigma_z^2$. These spin matrices are operators that represent the spin observables, e.g. the z-component of spin would be given by: $s_z = \frac{1}{2}\hbar\sigma_z = \frac{1}{2}\hbar\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. The eigenfunctions of spin represent the states of 'spin up' and 'spin down' are given respectively by the two spinors: $\chi_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\chi_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. The general expression for a system that is not in an eigenstate of 'spin up' or 'spin down' is the superposition: $\chi = a\chi_1 + b\chi_2$ where a, b are complex numbers. These spinor wavefunctions gave the required measured values of spin, i.e. $\pm(\hbar/2)$ with certainty when the system was in an eigenstate, or when in a superposition, with probability $|a|^2$ for 'spin up' and $|b|^2$ for 'spin down'.

In order to meet the need for incorporating spin into Orthodox Quantum Theory, much attention has been given to developing spinor representation and spin algebra as a way of dealing with an aspect of quantum systems (i.e. spin) that was not properly understood. Although it is the case that spinor methods have been *formally* successful, they are really a technical means of not addressing the underlying nature of the spin phenomenon. Indeed, the Pauli equation does not provide any insight into the origin or characteristics of spin, as noted by Lindsay and Margenau:

> Pauli's theory does not explain the origin of the spin, nor does it give any reason for its magnitude. It merely provides a method for incorporating it into quantum mechanics.[35]

We shall address the underlying nature of quantum mechanical spin at the end of this section.

Pauli's approach does have its uses though, for even inadequate formulations can lead to important insights. In the Causal Theory, a version of the Pauli Equation has also been developed.[36] The approach is to let the wavefunction be represented by a spinor of the form:

$$\psi = Re^{i\chi/2}\begin{pmatrix} \cos(\theta/2)\,e^{i\phi/2} \\ i\sin(\theta/2)\,e^{-i\phi/2} \end{pmatrix}$$

where (ϕ, θ, χ) are the Euler Angles for a rigid body undergoing rotation. This leads to a quantum potential given by:[37]

$$Q = -(\hbar^2/2m)(\nabla^2 R)/R + (\hbar^2/8m)\,[(\nabla\theta)^2 + (\sin^2\theta)\,(\nabla\phi)^2\,] \;\ldots.\; (5\text{-}3)$$

where the first term is the usual quantum potential and the second term is spin-dependent. This approach has proved quite useful and provides a better account of

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34 Davydov, A.S. (trans. D. ter Haar), *Quantum Mechanics* (Pergamon Press, Oxford, 1976) p.258.
35 Lindsay, R.B. and Margenau, H., Foundations of Physics, *op. cit.*, p.487.
36 Bohm, D., Schiller, R. and Tiomno, R., 'A Causal Interpretation of the Pauli Equation (A)', *Supplemento al Nuovo Cimento* **1** (1955): 48-66; Bohm, D. and Schiller, R., 'A Causal Interpretation of the Pauli Equation', *Supplemento al Nuovo Cimento* **1** (1955): 67-91; Dewdney, C., Holland, P.R., Kyprianidis, A. and Vigier, J.-P., 'Spin and Non-locality in Quantum Mechanics', *Nature* **336** (8 December 1988): 536-544.

atomic processes than is possible with Orthodox Quantum Theory. The lowest energy level of hydrogen, for example, in the context of the Causal Theory was originally dealt with by Bohm in the first of his 1952 papers. He argued that an electron in this state is at rest since the Coulomb force was exactly balanced by the quantum mechanical force (but with a statistical distribution of possible positions that would be found on measurement).[38] Indeed, the statement that an electron would be at rest has been used to criticise the Causal Theory.[39] Bohm should have realised in 1955 (after the publication of two articles which incorporated spin into the Causal Theory[40]) that the presence of a spin-dependent term in the quantum potential would produce a different answer. The spin-dependent term gives rise to a situation where the Coulomb and quantum forces do not balance each other. The electron would, therefore, not be at rest.

One might have expected that P.R. Holland would have explicitly dealt with this problem in his detailed text on the Causal Theory by taking account of the spin-dependent term as given in equation (5-3). Alas, Holland's comments were not helpful either and might sit better with an advocate of Orthodox Quantum Theory. Holland writes:

> Readers who … exclaim 'I don't believe it' when confronted with a stationary electron in m = 0-states should … be prepared to put aside expectations based on acquaintance with classical physics …[41]

The solution of the motion of the electron in a hydrogen atom was not worked out in sufficient detail until 2002/2003 when spin-dependent trajectories for several

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[37] Holland, P.R., The Quantum Theory of Motion, *op. cit.*, p.391.
[38] Bohm, D., 'A Suggested Interpretation of the Quantum Theory in Terms of "Hidden" Variables I', *Physical Review* **85** (1952) p.173.
[39] Humphreys, W.C., *Anomalies and Scientific Theories* (Freeman, Cooper & Co., San Francisco, 1968) pp.229-230; Audi, M., *The Interpretation of Quantum Mechanics* (University of Chicago Press, Chicago, 1973) p.74.
[40] See footnote 36.

hydrogen eigenstates and transitions were calculated.[42] Here there is a spin-dependent term in the expression for the momentum of a particle with spin **s**. The total momentum is given by: $\mathbf{p} = \nabla S + \nabla(\log \rho) \times \mathbf{s}$ (where $\rho = R^2$) and yields non-stationary trajectories. Figure 5 (below) is an example of such trajectories.

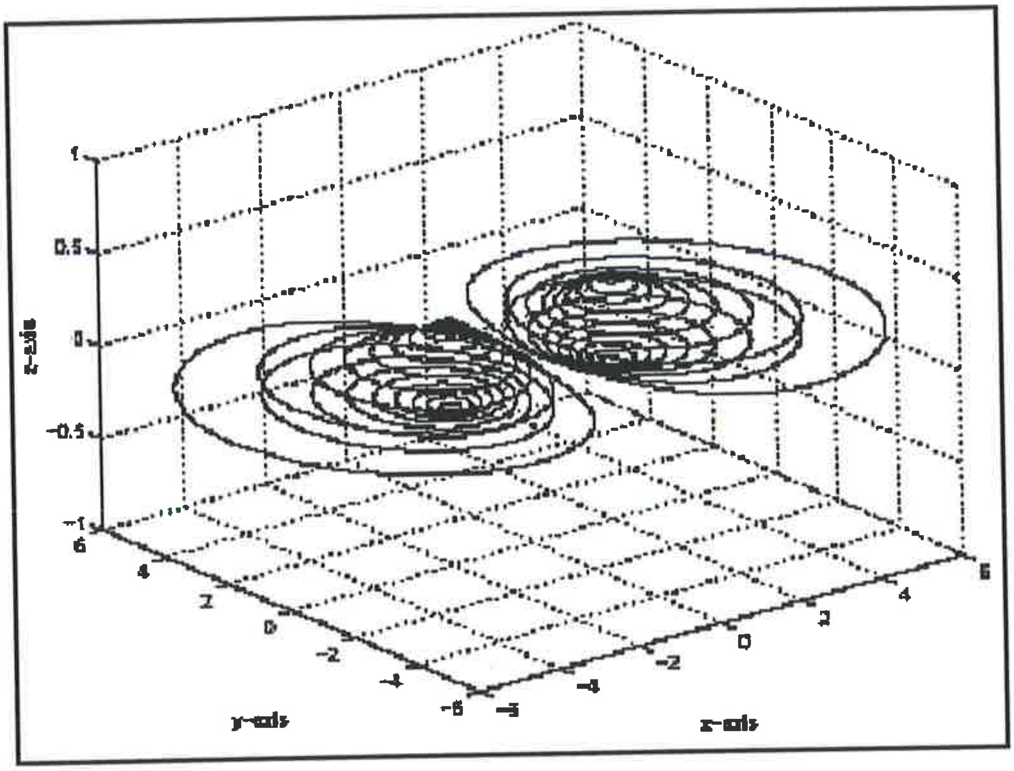

Figure 4: Spin-dependent trajectory for the hydrogen $2p_x$ state [43]

The trajectories in Figure 4 compare well with the calculated probability densities for the $2p_x$ orbital. and imaging of such orbitals (Figure 5).

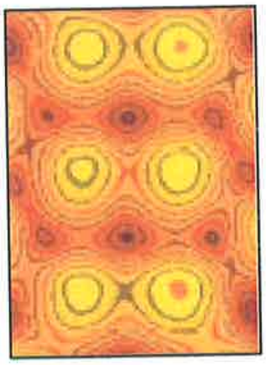

Figure 5: Imaged 2p orbitals [44]

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41 Holland, P.R., The Quantum Theory of Motion, *op. cit.*, p.155.

42 Colijn, C. and Vrscay, E.R., 'Spin-dependent Bohm Trajectories for Hydrogen Eigenstates', *Physics Letters A* **300** (2002): 334-340; 'Spin-dependent Bohm Trajectories associated with an electronic transition in Hydrogen', *Journal of Physics A: Mathematical and General* **36** (2003): 4689–4702.

43 Colijn, C. and Vrscay, E.R., 'Spin-dependent Bohm Trajectories for Hydrogen Eigenstates', *ibid.*, p.338.

This discussion of spin-dependent trajectories disposes of alleged flaw ⑦ in the General Introduction (i.e. that the Causal Theory cannot incorporate spin) which may now be clearly seen as false.

Although the causal version of the Pauli equation is very useful for calculation purposes, it will not be pursued here as its ontology (i.e. a rotating particle) cannot be physically realisable for the reasons cited above. So what is quantum mechanical spin? It was stated above that some theorists speculated that quantum mechanical spin arises from the (presumed) internal structures of quantum particles. However, it has already been noted in Section 3.2 that some quantum particles, such as electrons, do not appear to have internal structures. Most quantum systems (including electrons) have non-zero spins and since their spin cannot originate from internal particle structure (if they do not have any such structure), we have to look elsewhere for the origin of quantum mechanical spin.

We are now in a position to make an important inference about the nature of spin, based on prior findings in this thesis and the existence of a spin-dependent term in the quantum potential. In the Causal Theory, the wavefunction represents an objectively existing field that propagates through space and shares characteristics found with other types of waves, e.g. diffraction, interference, reflection, superposition, etc. Spin can also be seen to be a property of the wave field because the quantum potential (which represents a portion of the wave field's energy) has a spin dependence, as shown by equation (5-3). The conclusion that spin is a property of the wave field furnishes a realistic description of spin and will assist in providing a basis for the Exclusion Principle.

It turns out that the notion that spin is not a property of particles but of fields is not new. In respect to electromagnetic waves, the conclusion that spin is a field

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[44] Herz, M., Giessibl, F.J. and Mannhart, J., 'Probing the shape of atoms in real space', *Physical*

property has been around for years – spin is part of an electromagnetic wave's angular momentum, the part which is dependent on the wave's polarisation.[45] This reveals the connection between spin and polarisation. Consider, for example, a circularly polarised plane electromagnetic wave with a vector potential **A** given by:

$$\mathbf{A} = (\hat{\mathbf{x}} \pm i\,\hat{\mathbf{y}})\,(iE_0/\omega)\,\exp[i\,\omega(t - x/c)]$$

where E_0 is the electric field strength, ω is the angular frequency, c is the speed of light in vacuum, $i = \sqrt{-1}$, and $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$ are the usual Cartesian unit vectors. The polarisation dependent part of the wave's angular momentum (i.e. its spin s) is:

$$s = \pm \frac{1}{\mu_0 c^2} \int \frac{E_0^2}{\omega}\,\hat{\mathbf{z}}\,d^3\mathbf{x}$$

where μ_0 is the (magnetic) permeability of free space constant, $\hat{\mathbf{z}}$ is a unit vector in the z direction, and the ± sign indicates the dependence on polarisation.[46] Further, the explanation of spin as part of a wave's angular momentum has been extended to electrons and other fermions,[47] as Ohanian has commented:

> The lack of a concrete picture of the spin leaves a grievous gap in our understanding of quantum mechanics ... spin could be regarded as due to a circulating flow of energy, or a momentum density in the electron wave field ... this picture of the spin is valid not only for electrons ...[48]

Similar to the case of an electromagnetic wave, wave fields will also have states of polarisation. It is obvious that wavefunctions in non-relativistic quantum mechanics represent scalar waves when describing spinless quantum systems. It might be objected, therefore, that if wave fields have states of polarisation, then

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Review B **68** (2003) p.453015.

45 Belinfante, F.J., 'On the Spin Angular Momentum of Mesons', *Physica* **6** (1939): 887-897; Wallace, P.R., *Mathematical Analysis of Physical Problems* (Holt, Rinehart & Winston, New York, 1972) pp.288-291; Jackson, J.D., *Classical Electrodynamics* (Wiley, New York, 1975) p.333; Ohanian, H.C., 'What is Spin?', *American Journal of Physics* **54** (1986): 500-505.

46 Ohanian, H.C., 'What is Spin?', *ibid.*, p.502.

47 Gsponer, A., 'What is Spin?', *arXiv:physics/0308027* (10 Sept 2003) and references therein.

48 Ohanian, H.C., 'What is Spin?', *loc. cit.*, p.501.

quantum mechanical wavefunctions would have to represent vector waves and this might conflict with the representation of quantum systems with spin by spinor waves. The crucial word here is 'represent' for there is more than one formal way to achieve this. In particular, either vector waves or scalar waves plus spinors can be used. Indeed, spinors are used in this way in classical wave theory.[49] Therefore, as previously noted, the representation of spin by the use of spinors is only a method of dealing with the spin phenomenon without needing an understanding of its fundamental nature. (A vector wave approach is also possible, however this will not be developed here.)

The connection between spin and wave field polarisation accounts for the empirical fact that the spin related to protons, electrons and neutrons, i.e. spin ½ fermions, has a two-valued discreteness (commonly called 'spin-up' and 'spin-down'). The observed two-valued discreteness related to spin ½ fermions is determined by the polarisation state of their wave fields. The explanation of spin as the polarisation dependent part of the wave field's angular momentum has not only not been accepted by most physicists who are aware of this explanation, it is almost universally ignored. One principal reason for the non-acceptance is that, in Orthodox Quantum Theory, the wave field is not considered to be a real field.

5.3 The Exclusion Principle in the Causal Theory

We shall delay discussion of why fermionic wavefunctions are antisymmetric in their form until the next section. If we tentatively accept the antisymmetric form (in the context of the Causal Theory) then it is relatively straight forward to provide a causal mechanism to explain the Exclusion Principle.

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[49] Rogalski, M.S. and Palmer, S.B., *Advanced University Physics* (CRC Press, Boca Raton, FL., 2006) pp.401-403.

Consider first the response within Orthodox Quantum Theory to the consequences of assuming an antisymmetrical wavefunction for a system of fermions. This is commonly expressed in textbooks by stating that the electrons in an atom "avoid one another" (or words to that effect)[50] with no further qualification. Occasionally, texts will attempt to gloss over the lack of a proper explanation in Orthodox Quantum Theory for electrons 'avoiding each other' by making some covering statement, such as:

> A system in an antisymmetric state ... exhibits what is called a *statistical repulsion* ...[51]

This strange notion of 'statistical repulsion' comes from dismissing any possibility of a realistic, causal description of quantum phenomena and leaves correlated particle motion completely unexplained.

It was stated in Chapter Two that, in the Causal Theory, the trajectories of quantum particles do not pass through nodes of the wave field. An antisymmetrical wavefunction representing such a wave field obviously will have nodal points. This led Holland to make the following conclusion:

> ... the exclusion principle is incorporated into the [Causal] quantum theory of motion in that particles cannot pass through nodes.[52]

Holland's conclusion may be better appreciated from a dynamical perspective which provides a causal description of particle motion in terms of the effects of the quantum mechanical force. Indeed, Bohm himself was initially very much in favour of such an explanation. He wrote:

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50 Examples include: French, A.P. and Taylor, E.F., An Introduction to Quantum Physics, *op. cit.*, p.569; Sakurai, J.J., Modern Quantum Mechanics, *op. cit.*, p.365; Penrose, R., *The Road to Reality: A Complete Guide to the Laws of the Universe* (Jonathon Cape, London, 2004) p.596.

51 Park, D., *Introduction to the Quantum Theory* (McGraw-Hill, New York, 1974) p.409 (his italics).

52 *ibid.*, p.310.

> ... the [quantum mechanical] force between any two particles may depend significantly on the location of every other particle in the system. An example of such a force is given by the exclusion principle.[53]

Let's see how this is achieved in the Causal Theory. The study of atomic electrons shows that a total antisymmetrical wavefunction can occur in a number of ways (ignoring the interaction between the electrons). In the case of two electrons, we can label them by the numerals 1, 2 and suppose that particle 1 is in state A at position $\mathbf{x}_1$ and particle 2 is in state B at position $\mathbf{x}_2$. The states of interest are those for which the electrons 'avoid each other'. There are three of these which have the collective name of the 'triplet state'. The total 'z-component of spin' for these three states has values of $\hbar$, $-\hbar$, and 0 respectively. Since the Hamiltonian for a system of identical quantum particles does not involve spin operators (in the absence of a magnetic field), their wavefunctions may be given as a product of a spatial component and a spin component.[54] These are, respectively, as follows:

$$\Psi = \{\psi_A(\mathbf{x}_1)\psi_B(\mathbf{x}_2) - \psi_A(\mathbf{x}_2)\psi_B(\mathbf{x}_1)\}\ \alpha(1)\,\alpha(2) \quad .... (5\text{-}4)$$

$$\Psi = \{\psi_A(\mathbf{x}_1)\psi_B(\mathbf{x}_2) - \psi_A(\mathbf{x}_2)\psi_B(\mathbf{x}_1)\}\ \beta(1)\,\beta(2) \quad .... (5\text{-}5)$$

$$\Psi = \{\psi_A(\mathbf{x}_1)\psi_B(\mathbf{x}_2) - \psi_A(\mathbf{x}_2)\psi_B(\mathbf{x}_1)\}\ \{\alpha(1)\,\beta(2) + \alpha(2)\,\beta(1)\} \quad .... (5\text{-}6)$$

where ψ_A, ψ_B are the spatial components of the wavefunctions of particles 1 and 2; α, β denote 'spin up' and 'spin down' respectively; and the normalisation factors have been ignored.[55] The values of the spin components α and β are discrete, independent of position, and each is an eigenfunction of the z-component of spin.

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53 Bohm, D., 'A Suggested Interpretation of the Quantum Theory in Terms of "Hidden" Variables II', *Physical Review* **85** (1952) p.175.

54 Davydov, A.S., Quantum Mechanics, *op. cit.*, pp.297-298; Eisberg, R. and Resnick, R., Quantum Physics of Atoms ..., *op. cit.*, p.303; Greenhow, R.C., Introductory Quantum Mechanics, *op. cit.*, p.212.

55 Enge, H.A., Wehr, M.R. and Richards, J.A., *Introduction to Atomic Physics* (Addison-Wesley, Reading, MA., 1972) p.252; Greenhow, R.C., Introductory Quantum Mechanics, *ibid.*, p.213.

Then, in the above notation, $\psi_A(\mathbf{x}_2)$ is the value of ψ_A at position $\mathbf{x}_2$, $\psi_B(\mathbf{x}_1)$ is the value of ψ_B at position $\mathbf{x}_1$, $\alpha(1)$ is 'spin up' applied at particle 1's position, $\beta(2)$ is 'spin down' applied at particle 2's position, etc. Equations (5-4) and (5-5) describe situations where the spins are the same for both electrons (both 'spin up' or both 'spin down'). Note that the above equations all have antisymmetrical spatial components, so that $\Psi = 0$ if $\mathbf{x}_1 = \mathbf{x}_2$.

We can rewrite equation (5-6) to incorporate generalised spins χ_A, χ_B for particles 1, 2 respectively so that these may include the possibility of general spin states which consist of superpositions of 'spin-up' and 'spin-down', e.g. $\chi = a\alpha + b\beta$, where a, b are numbers with $|a|^2 + |b|^2 = 1$. Then the general two-particle, antisymmetrical wavefunction is:

$$\Psi = \{\psi_A(\mathbf{x}_1)\psi_B(\mathbf{x}_2) - \psi_A(\mathbf{x}_2)\psi_B(\mathbf{x}_1)\}\{\chi_A(1)\,\chi_B(2) + \chi_A(2)\,\chi_B(1)\} \quad \ldots. (5\text{-}7)$$

(again suppressing the normalisation constant). Now let the spatial part of Ψ be given by:

$$\psi = \psi_A(\mathbf{x}_1)\psi_B(\mathbf{x}_2) - \psi_A(\mathbf{x}_2)\psi_B(\mathbf{x}_1) = \mathrm{R}e^{iS/\hbar} \quad \ldots. (5\text{-}8)$$

When $\psi = 0$, the amplitude R in equation (5-8) must be zero (since $e^{iS/\hbar}$ cannot be zero). Thus, as a nodal region of the wave field is approached, the value of R will tend to zero. The (repulsive) quantum mechanical force on each particle is:

$$\mathbf{f}_j = (d\mathbf{p}_j/dt) = -\nabla_j \mathrm{Q} \quad (j = 1, 2) \ldots. (5\text{-}9)$$

where

$$\mathrm{Q} = -\left(\frac{\hbar^2}{2m\mathrm{R}}\right)(\nabla_1^2\mathrm{R} + \nabla_2^2\mathrm{R}) + \text{spin dependent terms} \quad \ldots. (5\text{-}10)$$

Substitution of equation (5-10) into equation (5-9) and ignoring the contribution of the spin dependent terms (since the spatial terms will dominate as R tends to zero) gives:

$$\mathbf{f}_j = \left(\frac{\hbar^2}{2m\mathrm{R}^2}\right) \sum_{k=1}^{2} \left[\mathrm{R}\,\nabla_j(\nabla_k^2\mathrm{R}) - (\nabla_j^2\mathrm{R})(\nabla_k\mathrm{R})\right] \quad \ldots. (5\text{-}11)$$

It can be seen from equation (5-11) that as $\mathrm{R} \rightarrow 0$, $\mathbf{f}_j \rightarrow \infty$. The force $\mathbf{f}_j$ exerted by the wave field on the two fermions prevents them coming into close proximity of each other when their spins are the same (i.e. in cases where the spatial part of the wavefunction is antisymmetric). However, we would not expect literal infinities to occur, only that $\mathbf{f}_j$ can become quite large. The numerator terms in equation (5-11), for example, may serve in some instances to cancel out the 'blowing up' of (1/R).[56] Also, in some cases of motion in external potentials there are instances where there is compensation due to the external potential resulting in a finite value for $\mathbf{f}_j$.[57] More generally, the dynamics as shown by the Causal Theory prevent fermions occupying the same quantum state. This arises as a natural consequence in the Causal Theory.

It is interesting to note that, although the quantum mechanical force is not generally accepted by the physics community, a (so-called) 'Pauli Force' has been postulated/ acknowledged by some researchers because of observed physical effects.[58] These effects are not just restricted to electrons 'avoiding each other' but include the creation of 'cavities' in liquid helium (with radii between 5 and 17 Angström units) which are occupied by electrons or by neutral atoms.[59] In solids, similar 'cavities' are called Fermi Holes:

> ... in terms of the short-range Pauli repulsive force between parallel-spin electrons ... each electron creates a Fermi hole

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56 Holland, P.R., The Quantum Theory of Motion, *op. cit.*, p.227.

57 For specific examples, see Belinfante, F.J., *A Survey of Hidden-Variable Theories* (Pergamon, Oxford, 1973) p.187.

58 Simons, G. and Bloch, A.N., 'Pauli Force Model Potential for Solids', *Physical Review B: Condensed Matter* **7** (1973) p.2755; Apkarian, V.A. and Schwentner, N., 'Molecular Photodynamics in Rare Gas Solids', *Chemical Reviews* **99** (1999) p.1484; Kanorsky, S.I. et al., 'Optical spectroscopy of atoms trapped in solid helium', *Zeitschrift für Physik B: Condensed Matter* **98** (1995) p.3645.

59 Günther, H. et al., 'Lifetime of metastable magnesium atoms in superfluid helium', *Zeitschrift für Physik B: Condensed Matter* **98** (1995) p.395.

> around itself which is due to the repulsion of other electrons with the same spin polarization.[60]

If we consider two electrons moving in an atom's electric field in the context of Orthodox Quantum Theory we find that, in addition to the Coulomb potential, another potential has to be postulated as a pragmatic means of dealing with the circumstances of the electrons 'avoiding each other'. In the literature, it is called the 'exchange potential' and, in the usual quantum mechanical notation, is given by the following integral:[61]

$$\pm \frac{e^2}{4\pi\varepsilon} \int\int \psi^*_A(\mathbf{x}_1)\psi^*_B(\mathbf{x}_2) \left[\frac{1}{|\mathbf{x}_1 - \mathbf{x}_2|}\right] \psi_A(\mathbf{x}_1)\psi_B(\mathbf{x}_2)\ d^3\mathbf{x}_1\ d^3\mathbf{x}_2$$

where e is the electronic charge, ε is the electric permittivity constant, $\mathbf{x}_1$ and $\mathbf{x}_2$ are the particle coordinates, and the sign depends on the 'relative orientation' of the spins. Although this exchange term appears in the expression for total energy, it is not considered in Orthodox Quantum Theory to originate from a real (i.e. physical) potential and is justified as merely a formal expression of the effect of the Exclusion Principle:

> … a term having no classical analogue, called the *exchange potential*. This exchange term has its origin in the Pauli Principle and may be regarded as an expression of an effective repulsion of electrons with the same spin.
>
> … There is no real "potential" in the *N*-electron problem corresponding to this exchange repulsion …[62]

The 'exchange potential' is necessary for the accurate description of an N-electron system in Orthodox Quantum Theory but, like other aspects of the theory, has no physical explanation. In the Causal Theory, the 'effective repulsion' and the

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60 Payami, M., 'Volume change of bulk simple metals and simple metal clusters due to spin polarization', *Journal of Physics: Condensed Matter* **13** (2001) pp.4133-4134.

61 Gasiorowicz, S., *Quantum Physics* (Wiley, New York, 1974) p.289.

62 Rybicki, G.B. and Lightman, A.P., *Radiative Processes in Astrophysics* (Wiley, New York, 1979) p.245 (their italics).

'exchange potential' are explained by the existence of a quantum mechanical force on each particle and the extra energy due to the effects of the quantum potential. Holland has explained such effects of the quantum potential as follows:

> ... we may say that *classical potentials have nonclassical effects in quantum mechanics because their influence is made manifest in the motion of particles via the mediating role of the quantum potential.*[63]

This extra energy is especially large in cases of white dwarf stars and neutron stars. These stellar objects are held from complete gravitational collapse only by the action of the Exclusion Principle.[64] Since the electrons and neutrons in these stars need enormous amounts of energy to resist gravity, the operation of the Exclusion Principle in such stars has been described as "a huge energy storage mechanism",[65] but without any further explanation. The existence of the wave field and the quantum potential provide such an explanation which accounts for stability of these stars and the otherwise mysterious existence of the energy necessary for them to avoid total gravitational collapse.

5.4 A Basis for the Exclusion Principle

All quantum mechanical textbooks describe the effects of the Exclusion Principle but its explanation is either avoided or put down to symmetry considerations. The importance of the Exclusion Principle as a foundational pillar of modern physics cannot be overstated for atomic structure, the rigidity of matter, stellar evolution, and the whole of chemistry depends on the operation of the Exclusion Principle. Given its absolutely crucial nature to understanding physical processes, an explanation of why

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63 Holland, P.R., The Quantum Theory of Motion, *op. cit.*, p.81 (his italics).
64 Doughty, N.A., *Lagrangian Interaction: An Introduction to Relativistic Symmetry in Electrodynamics and Gravitation* (Addison-Wesley, Sydney, 1990) p.132.
65 Clark, S., 'The bubble that ate the universe', *New Scientist* (12 March 2005) p.30.

the Exclusion Principle holds is long overdue. In order to provide a basis for the Exclusion Principle, it needs to be explained how the wavefunction of a fermionic system takes an antisymmetric form. We have already noted that this antisymmetry does not follow from the indistinguishability of identical particles nor from satisfying relativistic invariance. If a plausible account of the antisymmetry of wavefunctions of fermionic systems was provided, this would lay a much needed basis for the Exclusion Principle as well as resolving the conceptual problem of the existence of the antisymmetric form.

In 1946, Pauli's expressed the following thoughts about the history of the Exclusion Principle to that time and its status:

> … [the exclusion principle] remains an independent principle which excludes a class of mathematically possible solutions of the wave equation. … The history of the exclusion principle is thus already an old one, but its conclusion has not yet been written. … it is not possible to say beforehand where and when one can expect the further development …[66]

The considerable magnitude of the task of 'deducing' the Exclusion Principle should not be underestimated. Even a brief survey of the history of the subject matter[67] indicates that this problem is not going to be *readily* solvable in a mathematically rigorous way and, as Pauli said, it is not possible to predict where further development will occur. Granting this, we shall only attempt in what follows to shed some light on why wavefunctions of simple fermionic systems are antisymmetric and thereby suggest a basis for understanding the Exclusion Principle.

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66 Pauli, W., 'Remarks on the History of the Exclusion Principle', *Science* **103** (22 February 1946) p.215.
67 van der Waerden, B.L., 'Exclusion Principle and Spin' in Fierz, M. and Weisskopf, V.F. (eds), *Theoretical Physics in the Twentieth Century, A Memorial Volume to Wolfgang Pauli* (Interscience, New York, 1960); Jammer, M., The Conceptual Development of Quantum Mechanics, *op. cit.*, chapter 3; Duck, I. and Sudarshan, E. C. G., Pauli and the Spin-Statistics Theorem, *op. cit.*, chapters 1, 2 & 4; and Massimi, M., Pauli's Exclusion Principle …, *op. cit.*, passim.

In the context of the Causal Theory, the antisymmetrical form should be explicable in terms of the well established wave behaviour. We begin with a couple of important observations about the Exclusion Principle. First, if fermions are localised particles (as postulated in the Causal Theory) then the Exclusion Principle cannot operate as a local causal effect. It must be physically manifested as non-local and holistic.[68] This being the case, the Causal Theory is particularly well suited to providing an explanation as it is a non-local theory in which the motion of an individual particle depends on the quantum state as a whole.

Second, the Exclusion Principle is regularly assumed to apply to all physical situations involving fermions of the same kind. However, it has been argued elsewhere that the applicability of the Exclusion Principle should be restricted. Such a case was presented by Toyoki Koga in his *Foundations of Quantum Physics*, where he argued that applying the Exclusion Principle in situations other than stationary states can lead to absurdities (where the term 'stationary state' is understood in its standard meaning). He wrote:

> If we treat such a [non-stationary] state carelessly … we may get solutions which imply unreal states. In order to avoid such mistakes, it is necessary to set forth a criterion by which those solutions that appear to be possible … but [are physically] impossible … are eliminated. Pauli's [Exclusion] principle serves this purpose of elimination. For the same reason, the [Exclusion] principle should not be applied, for instance, to the treatment of nonstationary states.
>
> Pauli's [Exclusion] principle as such cannot be applied thoughtlessly without causing paradoxical results …[69]

Physically, a stationary state results when two travelling waves that are propagating in opposite directions, superimpose on each other.[70] This can be achieved by

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[68] Gibbins, P., *Particles and Paradoxes* (Cambridge University Press, Cambridge, 1987) p.117.

containing a quantum system in an enclosure (e.g. a rigid container) or restricting it to a finite spatial region (e.g. an atomic orbital, the lattice structure within a metal, etc.).[71] Clearly, the Exclusion Principle does not apply to widely separated fermions of the same kind, e.g. an atom of hydrogen on the Moon can be in the same state as one on Mars. In order for the Exclusion Principle to apply, two or more fermions must interact, i.e. there must be substantial overlap of their individual wavefunctions.[72] However, Koga's criterion of applicability to just stationary states is too restrictive, as the Exclusion Principle may be applied to some non-stationary situations. What Koga should have inferred is that the Exclusion Principle applies to a system of identical fermions that has constraints imposed upon it (the most obvious being a stationary state).

The Exclusion Principle is most readily manifested in the case of identical spin-½ fermions in a stationary state and this situation is easiest to explain. We shall proceed by applying the characteristics of the wavefield of a system of two identical spin-½ fermions in order to see what progress this approach can yield. The magnitude of the current task is such that this approach to finding a basis for the Exclusion Principle is, at best, only a promising line of investigation. In what follows, the terms 'wave field' and 'wavefunction' will be used in close association and the reader is reminded of the distinction that 'wave field' refers to the physical quantum field whereas 'wavefunction' refers to its mathematical representation.

Consider a system of fermions, say two neutrons (as this avoids the problem of electrical interaction between the particles) which are in motion but are also well

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69 Koga, T., *Foundations of Quantum Physics* (Wood and Jones, Pasedena, 1980) p.66.
70 Hirose, A. and Lonngren, K.E., *Introduction to Wave Phenomena* (Wiley, New York, 1985) p.97; Rogalski, M.S. and Palmer, S.B., *op. cit.* , p.311.
71 Main, I.G., *Vibrations and Waves in Physics* (Cambridge University Press, Cambridge, 1978) pp.273 & 276; Ingard, K.U., *Fundamentals of Waves and Oscillations* (Cambridge University Press, Cambridge, 1988) p.422.

separated. The individual wave field of each neutron is initially a travelling wave as they are in motion. We can formally be treat the neutrons and their wave fields as a single quantum system. Since the neutrons are well separated, there is no overlap of the individual wave fields and the system is described by a wavefunction which is the product of the individual wavefunctions associated with each neutron (i.e. a solution of the Schrödinger equation for two non-interacting particles).

Let the neutrons move sufficiently close that significant overlap of their individual wave fields occurs. Each neutron will then be subject to the other's wave field. In the case of non-overlapping wave fields it is clear that the wavefunction of the two-neutron system is just the simple product of the individual wavefunctions. However, without invoking the antisymmetry assumption, there is no obvious expression for the form of the two-neutron wavefunction when the individual wave fields first overlap.

Although we do not have an obvious expression for the wavefunction when there is overlap of individual wave fields, we can theorise that this situation may initially be described by a single wavefunction (denoted Ψ_I) which represents a travelling wave, i.e. both neutrons move under the influence of the wave field described by Ψ_I. We noted above that a stationary state is achieved by containing a system in an enclosure or a finite spatial region. The case of atomic electrons is an example of a system contained in a finite region (described by equations (5-4), (5-5) and (5-6)). We shall suppose, for current purposes, that the neutrons move only within a box with rigid walls. The wave field of the two-neutron system (initially represented by Ψ_I) will be successively reflected from one end of the box and then from the other. In the case of a fermionic wave field, reflection at a rigid wall causes

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[72] Herbut, F.and Vujicic, M., 'Irrelevance of the Pauli Principle in Distant Correlations between Identical Fermions', *Journal of Physics A: Mathematical and General* **20** (1987), p.5562.

a change of the wave field's phase of π radians. This is a well-known effect when a physical wave (such as an electromagnetic wave) is reflected from a fixed boundary. However, it is the polarisation of the incident wave field (and not the total spin) that determines whether there is a change of phase on reflection at a fixed boundary.[73]

The wave field reflected from the end of the box (described by a wavefunction denoted Ψ_R) travels in the opposite direction to the wave field that is incident at the box's wall. The wavefunction Ψ_R will therefore differ from Ψ_I in two respects. First, the phase difference between Ψ_R and Ψ_I requires that each term in Ψ_R carry a negative sign. (This is the origin of the minus sign in the antisymmetrical wavefunction for this kind of stationary state.) Second, the wave speed which appears in each term of Ψ_R will be of the opposite sign to the corresponding term in Ψ_I (as incident and reflected wave fields are moving in opposite directions). The spin part of the wavefunction remains the same on reflection of the wave field.[74] The interference between the incident and reflected wave fields produces a resultant wave field that is in a stationary state within the box.[75] This may be described by a total wavefunction (denoted Ψ_T) which has a standing wave pattern given by the sum of the incident wavefunction Ψ_I and the reflected wavefunction Ψ_R. The form of the total wavefunction Ψ_T will be antisymmetric due to the negative signs in the terms of Ψ_R.

This account of forming a total wavefunction that is antisymmetric can also be related to atomic systems and not just 'waves in boxes'. It may be employed, for example, to explain the case of two electrons in the same atomic orbital (such as

73 Cf. the treatment of electromagnetic waves in Jackson, J.D., *Classical Electrodynamics* (Wiley, New York, 1975) pp. 280-282.
74 Greenhow, R.C., Introductory Quantum Mechanics, *op. cit.*, p.215.

found in neutral Helium). Here the account is analogous to the waves generated in a free floating wire (or similar) loop when the wire is twisted and then released. Two waves can be produced which are half a wavelength out of phase that propagate in opposite directions when the wire is released. In an atomic orbital, we would have two wave fields superimposing with the necessary phase difference of π radians, one propagating 'clockwise' and the other 'anticlockwise' around the nucleus. The antisymmetrical form of the total wavefunction for this system would then result from describing the behaviour of these two wave fields.

How might the presented explanation for the behaviour of wave fields be analytically modelled? The main difficulty with modelling this phenomenon is to provide a valid mathematical description of the *initial overlap* of individual wave fields of the two neutrons. What appears in the literature when two (or more) fermions of the same kind are involved is simply to assume that the overall wavefunction for a combined system is antisymmetric without showing how this is achieved. Even quantitative calculations in quantum chemistry assume an antisymmetrical wavefunction for multi-fermion systems or directly import empirical values into their calculations.[76] There are no rigorous mathematical methods to be found in the literature to analytically determine the wavefunctions which describe significant overlap of individual wave fields.

A tentative approach to finding an mathematical description of significantly overlapping wave fields is to model the overlap using superpositions of their individual wavefunctions. Consider again the example of the two neutrons and label them with numerals 1, 2. We can specify, as before, that neutron 1 is in state A at position $\mathbf{x}_1$ and neutron 2 is in state B at position $\mathbf{x}_2$. We shall denote the spatial

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[75] Main, I.G., Vibrations and Waves in Physics, *op. cit.*, p.286.
[76] Atkins, P.W. and Friedman, R.S., *Molecular Quantum Mechanics* (Oxford University Press, Oxford, 1997) p.276.

components of their individual wavefunctions by ψ_A, ψ_B and spin components by χ_A, χ_B. Now when the individual wave fields overlap, they superimpose so that there will be new values for the wave field at each neutron's position. The net field at the position of neutron 1 will be a resultant of neutron 1's own wave field superimposed with the value of neutron 2's wave field at neutron 1's position. Likewise, the net field at the position of neutron 2 will be a resultant of neutron 2's own wave field superimposed with the value of neutron 1's wave field at neutron 2's position. This will be manifest in the values of the wavefunctions at coordinates $\mathbf{x}_1$ and $\mathbf{x}_2$. Let these values be ψ_1 and ψ_2 respectively. Expressions for ψ_1 and ψ_2 can then be formed by superimposing the individual wavefunctions.

The superposition expression with the smallest number of terms leads to the following expressions for wavefunctions ψ_1 and ψ_2:

$$\psi_1 = [\psi_A(\mathbf{x}_1)\chi_A(1) + \psi_B(\mathbf{x}_1)\chi_B(1)] \quad \text{.... (5-12)}$$

$$\psi_2 = [\psi_A(\mathbf{x}_2)\chi_A(2) + \psi_B(\mathbf{x}_2)\chi_B(2)] \quad \text{.... (5-13)}$$

where 1, 2 refer to the values at positions $\mathbf{x}_1$ and $\mathbf{x}_2$. Since the overlap of individual wave fields is explicitly taken into account by equations (5-12) and (5-13), we might try defining Ψ_I by forming the product $\psi_1\psi_2$, by analogy with the case of non-overlapping wave fields. This gives:

$$\begin{aligned}\Psi_I &= [\psi_A(\mathbf{x}_1)\chi_A(1) + \psi_B(\mathbf{x}_1)\chi_B(1)]\,[\psi_A(\mathbf{x}_2)\chi_A(2) + \psi_B(\mathbf{x}_2)\chi_B(2)] \\ &= \psi_A(\mathbf{x}_1)\psi_A(\mathbf{x}_2)\chi_A(1)\chi_A(2) + \psi_A(\mathbf{x}_1)\psi_B(\mathbf{x}_2)\chi_A(1)\chi_B(2) \\ &\quad + \psi_A(\mathbf{x}_2)\psi_B(\mathbf{x}_1)\chi_A(2)\chi_B(1) + \psi_B(\mathbf{x}_1)\psi_B(\mathbf{x}_2)\chi_B(1)\chi_B(2) \quad \text{.... (5-14)}\end{aligned}$$

Using equation (5-14) and taking account of the differences between Ψ_R and Ψ_I, as stated above, the wavefunction representing the reflected wave field is:

$$\Psi_R = -\,[\psi_A{}^{\ddagger}(\mathbf{x}_1)\,\psi_A{}^{\ddagger}(\mathbf{x}_2)\,\chi_A(1)\,\chi_A(2) + \psi_A{}^{\ddagger}(\mathbf{x}_1)\,\psi_B{}^{\ddagger}(\mathbf{x}_2)\,\chi_A(1)\,\chi_B(2)$$
$$+ \psi_A{}^{\ddagger}(\mathbf{x}_2)\,\psi_B{}^{\ddagger}(\mathbf{x}_1)\,\chi_A(2)\,\chi_B(1) + \psi_B{}^{\ddagger}(\mathbf{x}_1)\,\psi_B{}^{\ddagger}(\mathbf{x}_2)\,\chi_B(1)\,\chi_B(2)] \;\ldots.\; (5\text{-}15)$$

where $\psi_A{}^{\ddagger}$ represents the same function as ψ_A but with the opposite sign of the wave speed appearing in its argument and likewise for $\psi_B{}^{\ddagger}$. Then, using equations (5-14) and (5-15), an expression for the total wavefunction would be:

$$\Psi_T = \Psi_I + \Psi_R$$
$$= [\psi_A(\mathbf{x}_1)\psi_A(\mathbf{x}_2) - \psi_A{}^{\ddagger}(\mathbf{x}_1)\psi_A{}^{\ddagger}(\mathbf{x}_2)]\;\chi_A(1)\,\chi_A(2)$$
$$+ [\psi_A(\mathbf{x}_1)\psi_B(\mathbf{x}_2) - \psi_A{}^{\ddagger}(\mathbf{x}_1)\psi_B{}^{\ddagger}(\mathbf{x}_2)]\;\chi_A(1)\,\chi_B(2)$$
$$+ [\psi_A(\mathbf{x}_2)\psi_B(\mathbf{x}_1) - \psi_A{}^{\ddagger}(\mathbf{x}_2)\psi_B{}^{\ddagger}(\mathbf{x}_1)]\;\chi_A(2)\,\chi_B(1)$$
$$+ [\psi_B(\mathbf{x}_1)\psi_B(\mathbf{x}_2) - \psi_B{}^{\ddagger}(\mathbf{x}_1)\psi_B{}^{\ddagger}(\mathbf{x}_2)]\;\chi_B(1)\,\chi_B(2)$$
$$\ldots.\; (5\text{-}16)$$

Although expected terms (such as $\psi_A(\mathbf{x}_1)\psi_B(\mathbf{x}_2)$) do appear by this process, the required antisymmetrical wavefunction, i.e.

$$\Psi_T = \{\psi_A(\mathbf{x}_1)\psi_B(\mathbf{x}_2) - \psi_A(\mathbf{x}_2)\psi_B(\mathbf{x}_1)\}\,\{\chi_A(1)\,\chi_B(2) + \chi_A(2)\,\chi_B(1)\} \;\ldots.\; (5\text{-}17)$$

will not result from equation (5-16). This implies that the wavefunction Ψ_I as formed from the product of equations (5-12) and (5-13) cannot be a faithful representation of a quantum system where individual wave fields first overlap.

Given that the form of Ψ_T is derived from the form of Ψ_I, the above example indicates that the correct expression for Ψ_I will not be of a simple form. This was not completely unforeseen as mentioned above in relation to the magnitude of the task. In order to find the correct expression for Ψ_I, a better understanding of what occurs when individual wave fields overlap will be required. It should then become apparent how to provide a correct mathematical description of the process. What constitutes such a description of overlapping individual wave fields and a method of deriving the relevant wavefunction remain open questions.

However, a plausible case has been presented which indicates that the antisymmetric form of fermionic wavefunctions in a stationary state arises from the description of the interference between physical wave fields within a bounded region. This explains why the Exclusion Principle is especially pronounced in cases of stationary quantum states. This explanation also resolves the conceptual problem that arises due to the wavefunctions of fermionic systems being antisymmetric in their form. Since the Exclusion Principle is a consequence of the antisymmetric form, this also provides an explanatory basis for the Exclusion Principle which is not duplicated in Orthodox Quantum Theory.

PART IV

CONCLUSIONS

AND

FINAL REMARKS

CONCLUSIONS

In Part I of this dissertation, two principal aims were presented. The topics and problems which relate to these aims have been investigated in the preceding chapters. The results of this analysis are as follows:

Principal Aim (I)

It has been demonstrated that the Causal Theory of Quantum Mechanics is a viable theory and that it provides a realistic account for physical phenomena. This aim was realised by showing that:

- causal explanations of quantum processes (e.g. diffraction, tunnelling, measurement, etc.) can be given in terms of events in spacetime;
- the conceptual problems of the Causal Theory (as stated in Part I) are not insurmountable;
- further theoretical development of the Causal Theory is achievable; and
- a variety of 'myths' and misconceptions about the Causal Theory (as listed in Part I) are all ill-founded and/or unwarranted.

Principal Aim (II)

This aim was realised by presenting solutions to the following conceptual problems of the Causal Theory:

- energy conservation (Section 4.2);
- action and reaction (Section 4.4);
- the quantum potential being a physical potential without sources in the conventional sense (Section 4.5);
- the antisymmetric form of the wavefunction of a fermionic system (Section 5.4).

Some related theoretical problems were also solved such as the energy transfer time from wave field to particle and (in outline) a basis for the Exclusion Principle.

FINAL REMARKS

It has been the contention in this dissertation that the explanatory success of the Causal Theory of Quantum Mechanics is due to it 'mirroring' aspects of an objective (observer independent) existence, i.e. the Causal Theory is a realist theory of physical phenomena. The problems addressed were chosen not just because of their relevance to the aims of this dissertation but also to highlight the 'mirroring' feature of the Causal Theory.

The concept of energy and the processes of energy transformation proved to be important ingredients in explaining quantum phenomena and in gaining a better understanding of quantum systems.

In addition to providing solutions to a number of conceptual and theoretical problems of the Causal Theory, several topics treated in this dissertation offer avenues for further research. Of particular note were the following questions:

- What fundamental properties of the wave field can be identified?
- To what degree will an in-depth account of the wave field require relativistic Causal Theory?
- What is the mechanism by which energy transfers are achieved between a quantum particle and its wave field?
- What further insights would many-particle quantum systems reveal?

  and

- What constitutes a valid mathematical description of the overlap of individual wave fields and how is this to be found?

These questions are indicative of those issues that should be the subject of subsequent studies.

APPENDICES

Appendix I

Axioms of Orthodox Quantum Theory

The axioms appearing below are to be taken as defining the formal aspects of Orthodox Quantum Theory. These axioms refer to a quantum system in a pure state. Such states are uniquely definable by state vectors evolving in time. States other than pure states (i.e. mixed states) do appear in the literature. However, mixed states cannot be represented by wavefunctions but instead are represented by density operators which are used to describe fictitious ensembles of systems of which there is incomplete information. Similar axiomizations are found in the literature* but many tend to combine some of the axioms presented here.

Axiom 1 (State Vectors)

Any (pure) quantum state is described by a state vector which is an element of a Hilbert space. Multiplication of a state vector by a complex number results in a description of the same physical state.

Axiom 2 (Completeness)

A state vector contains all possible information about the quantum state.

Axiom 3 (Linear Superposition)

The addition of two or more state vectors results in a state vector which describes another quantum state.

Axiom 4 (Inner Product)

The inner product of two state vectors ψ_1 and ψ_2, denoted by (ψ_1, ψ_2) on a Hilbert space is defined as a complex number with the following properties:

$$(\psi_1, \psi_2) = (\psi_2, \psi_1)^* \quad (\psi_1, c\psi_2) = c\,(\psi_1, \psi_2)$$

$$(c\psi_1, \psi_2) = c^*\,(\psi_1, \psi_2) \text{ and } (\psi_i, \psi_i) \geq 0,$$

where c is a complex number.

* E.g. Sudbery, A., *Quantum Mechanics and the Particles of Nature* (Cambridge University Press, Cambridge, 1986) chapter 2.

Axiom 5 (Hermitian Operators)

Corresponding to every physical observable is a linear, Hermitian operator on the Hilbert space. These operate on state vectors to give other state vectors.

Axiom 6 (Eigenstate)

A state vector ϕ is in an eigenstate of an operator $\mathbf{A}$ if the equation: $\mathbf{A}\phi = \alpha\phi$ holds, where α is a real number called the eigenvalue.

Axiom 7 (Expansion Postulate)

An arbitrary state vector ψ can be expanded into a complete set of eigenstate vectors (eigenvectors) ϕ_i, where $\psi = \sum_i c_i\phi_i$ is a linear superposition and the coefficients are given by $c_i = (\psi, \phi_i)$.

Axiom 8 (Canonical Commutation Relations)

The canonical commutation relations are defined by the following equations:

$$[\hat{\mathbf{q}}_i , \hat{\mathbf{q}}_i] = 0 = [\hat{\mathbf{p}}_i , \hat{\mathbf{p}}_i] \quad \text{and} \quad [\hat{\mathbf{p}}_i , \hat{\mathbf{q}}_j] = i\hbar\, \delta_{ij}$$

where $\hat{\mathbf{p}}_i$, $\hat{\mathbf{q}}_i$ are operators corresponding to canonical conjugate variables, δ_{ij} is the Kronecker Delta, $\hbar$ is Planck's Constant divided by 2π and $i = \sqrt{-1}$.

Axiom 9 (Time Development of States)

A state vector ψ develops in time according to the equation: $i\hbar(\partial\psi/\partial t) = \mathbf{H}\psi$, where $\mathbf{H}$ is the Hamiltonian operator.

Axiom 10 (Projection Postulate)

A measurement of an observable on a system in a (superposition) state given by $\psi = \sum_i c_i\phi_i$ will project (or reduce) ψ into one of the eigenvectors ϕ_i of the superposition.

Axiom 11 (Quantisation Algorithm)

A measurement of an observable only can yield an eigenvalue of the operator corresponding to that observable.

Axiom 12 (Born Statistical Postulate)

The probability that a measurement on an observable A of a system in a state described by a state vector $\psi = \sum_i c_i\phi_i$ will yield an eigenvalue α_n is $|c_n|^2$, where $\mathbf{A}$ is the operator corresponding to observable A, $\mathbf{A}\phi_n = \alpha_n\phi_n$, with ψ and ϕ_i normalised: $(\psi,\psi) = 1 = (\phi_i, \phi_i)$.

Appendix II

Summary of Defined Principles

Principle of Complementarity

Any application of a classical concept precludes the simultaneous use of other classical concepts which in a different connection are equally necessary for the description of phenomena.

Correspondence Principle

Quantum states and measurements will tend to the corresponding classical case in the limit of large quantum numbers.

Principle of Locality

Elements of reality pertaining to one system cannot be affected by measurements performed at a space-like distance on another system, even if the systems previously interacted.

Principle of Linear Superposition

When several individual states are superimposed, the resultant state is the vector addition of the individuals.

Principle of Stationary Action

The change in the total Action for each infinitesimal variation of the state of a physical system is zero.

Principle of Causality

The same cause always produces the same effect or effects (other things being equal) and the cause temporally precedes, or is simultaneous with, its effects.

Principle of Faithful Measurement

The result of measurement is numerically equal to the value possessed by an observable immediately prior to measurement.

Principle of the Conservation of Total Energy

The energy of a physical system is neither created nor destroyed, but may be transformed from one kind of energy into another, such that it is always theoretically possible to account for the total energy of a system.

Principle of Energy Content

Every physically real entity in the universe contains some finite quantity of energy.

Principle of Reaction.

Any interaction between two physical entities has a mutual effect on both entities. The forces of interaction are equal and opposite, and act along straight lines joining the locations of the entities.

Pauli's Principle

In an atom there cannot be two or more electrons with the same quantum numbers.

Exclusion Principle

In a quantum system, two or more fermions of the same kind cannot be in the same (pure) state.

Appendix III

Identities, Derivatives and Integrals

Gaussian Formulae

$$\Psi(\mathbf{x}, t) = (2\pi s_t^2)^{-3/4} \exp\{i\,\mathbf{k}\cdot(\mathbf{x} - \tfrac{1}{2}\,\mathbf{u}t) - (\mathbf{x} - \mathbf{u}t)^2/4\sigma_o\, s_t\}$$

$$s_t = \sigma_o\,(1 + i\hbar t/2m\sigma_o^2) = |\,s_t\,|\,\exp(i\varphi)$$

$$\sigma = |s_t| = [\sigma_o^2 + (\hbar t/2m\sigma_o)^2]^{1/2} = \sigma_o[1 + (\hbar^2 t^2/4m^2\sigma_o^4)]^{1/2}$$

$$\text{and } \varphi = \arctan(\hbar t/2m\sigma_o^2)$$

Therefore

$$\sigma^2 = \sigma_o^2\ [1 + (\hbar^2 t^2/4m^2\sigma_o^4)]$$

$$\sigma^2/\sigma_o^2 = [1 + (\hbar^2 t^2/4m^2\sigma_o^4)]$$

$$\sigma_o^2/\sigma^2 = [1 - (\hbar^2 t^2/4m^2\sigma_o^2\sigma^2)]$$

$$4m^2\sigma_o^2\sigma^2 = \hbar^2 t^2 + 4m^2\sigma_o^4$$

$$\frac{d\sigma}{dt} = \frac{\hbar^2 t}{4m^2\sigma_o^2\sigma} = \frac{\partial\sigma}{\partial t}$$

$$\frac{\partial^2\sigma}{\partial t^2} = \frac{\hbar^2}{4m^2\sigma^3}$$

$$\frac{d}{dt}\left(\frac{1}{\sigma}\right) = \frac{\partial}{\partial t}\left(\frac{1}{\sigma}\right) = \frac{-\hbar^2 t}{4m^2\sigma_o^2\sigma^3}$$

$$\frac{\partial}{\partial t}\left(\frac{1}{\sigma^2}\right) = \frac{-\hbar^2 t}{2m^2\sigma_o^2\sigma^4}$$

$$\frac{\partial}{\partial t}\arctan\left[\frac{\hbar t}{2m\sigma_o^2}\right] = \frac{\hbar}{2m\sigma^2}$$

$$\int\frac{dt}{\sigma^2} = \left(\frac{2m}{\hbar}\right)\arctan\left[\frac{\hbar t}{2m\sigma_o^2}\right]$$

$$\int\frac{t}{\sigma^2}\,dt = \left(\frac{4m^2\sigma_o^2}{\hbar^2}\right)\log\sigma$$

Standard Definite Integrals‡

$$\int_{-\infty}^{\infty} \exp[-\alpha x^2 - \beta x]\, dx = (\pi/\alpha)^{1/2} \exp[\beta^2/4\alpha]$$

$$\int_{-\infty}^{\infty} x \exp[-\alpha x^2 - \beta x]\, dx = (-\beta/2\alpha)(\pi/\alpha)^{1/2} \exp[\beta^2/4\alpha]$$

$$\int_{-\infty}^{\infty} x^2 \exp[-\alpha x^2 - \beta x]\, dx = [(1/2\alpha) + (\beta/2\alpha)^2](\pi/\alpha)^{1/2} \exp[\beta^2/4\alpha]$$

where α and β are constants.

‡ Dwight, H.B., *Tables of Integrals and Other Mathematical Data* (Macmillan, New York, 1961).

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