

## THE ACTUARIAL TREATMENT OF OFFICIAL BIRTH RECORDS

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The study of mortality, primarily for the practical purposes of life insurance, has led to the development of a somewhat extensive apparatus of technical ideas, centred upon the Life Table. The actuary, who understands these things, has thus a clear notion of what is meant by "death rate at any age," "expectation of life," "value of a life annuity," etc., etc. It scarcely seems to have been observed, however, that a similar exact system of ideas may be developed with respect to natality, which in conjunction with the life table are of direct bearing upon the important biological aspects of vital statistics. That such a system does not seem to have been developed is presumably due to the lack of the commercial incentive which exists in the case of mortality; for comparisons of biological interest on the other hand, it is obvious that reproduction must be considered with at least equal care to that given to death, especially as the latter event occurs in a large proportion of civilised mankind at ages at which reproduction has ceased, and at which therefore the moment of expiration of individual life is a matter of indifference in respect of the size and constitution of the future population.

The life table, apart from technical conventions, shows the proportion of those born alive who attain each year of age. Thus one may read off from such a table that 80% reach the age of ten, 77% live to be twenty, 72% to be thirty, 66% to be forty, and so on. Such tables may be and are constructed for separate sections of the population, for men and for women separately, or for special occupations, when the data are available. The source of the information is the death certificate, which records the age and occupation of the deceased. The numbers dying in each year of life must be compared with the numbers similarly occupied living at that age, which are obtained from the Census returns; the comparison, provided the occupational categories employed in the two sources coincide, yields the death rate at each age in the occupation concerned, from which the life table may be constructed.

Strictly speaking a man has an occupation for only part of his

life. The years subsequent to the cessation of active work are, however, unimportant biologically for the reason mentioned above; the deaths of children prior to employment would seem to present a difficulty; if, however, we are concerned with the mortality of any class in relation to its natality, or activity in reproduction, it is obvious that the deaths of children should be included in the life table according to the occupations of their fathers, for their real effect is that of a diminution of the net fertility of this class. The life table may thus be completed for each occupation with the exception of a small gap, representing juvenile occupations, which falling at ages of very low death rates, may be easily bridged.

If we wish for more than the one-sided view of the vital statistics of different classes represented by their mortality, it is necessary to know the number of children already born to men of different ages, or, better, the rate at which men at different ages are becoming fathers. The expectation of offspring of a newly born child could then be evaluated by summing the values obtained by multiplying the number dying at each age by the average number of children born to them. Since paternity is not an instantaneous process we require actually the average number of live births already conceived at each age, which could of course be obtained readily by shifting the ages through three-quarters of a year. This method of calculation is open to the objection—amongst others—that it relies to an important extent on the old age groups in which many are dying, rather than the younger ages at which many births are occurring; it will be better in practice to obtain the expectation by multiplying the number living at each age by the rate at which children are appearing three-quarters of a year later. In this way the older age groups in which the occupational classification deteriorates will scarcely make any appreciable contribution.

The expectation of offspring of a living newly born child subjected to the contemporary conditions of mortality and natality provides the simplest, though not the best, measure of the natural increase or decrease in the class concerned. The occupational classification being normally one of husbands', and not of wives' occupations, the life tables should be those of males only, and as regards the children it will be simplest to regard boys only. A practically equivalent comparison could be based on the total children, but in this case we should have to take into account the mortality both of boys and of girls during childhood, which would involve some additional labour. In either case the expectation shows at once whether an occupational group is producing more or fewer children than will be needed to replace their parents, or, in other words if its mortality and natality are those of an increasing or decreasing population, for in the former case the expectation will exceed 1 boy or 2 children, and in the latter case it will fall short of that figure. For most occupations in England and Scotland the writer believes that, if we knew, we should find the population decreasing, but it is characteristic of our present ignorance that even a point of such prime importance cannot be determined at once from official data. The calculations suggested above, though apparently ignoring women, do not do so in reality; for, such important factors as the age of marriage, fertility and mortality of the wives of the occupational group considered will be com-

pletely represented in the natality ascribed to their husbands. Some illegitimate births will, however, necessarily be ignored, if the occupation of the father is unknown; their total, however, will be available for regional, if not for occupational comparisons.

If the expectations of sons is at or very near to unity, that is if the population is only tending to increase to or decrease to a small extent in each generation, then we only need to consider, in respect of rate of population changes, the average interval of time which separates the generations ; but with rapidly decreasing or rapidly increasing groups the distribution of the ages of fathers at the birth of their sons must be taken more fully into account. If two occupations, for example, were each halving their numbers in each generation the rate of decrease *per annum*, and the corresponding need of recruitment, would be greater in that which had the shortest generation, and the sons born early should count for less than the sons born late; equally if two occupations had each an expectation of 1.5 sons, the rate of increase would be greater in that occupation which produced the sons sooner, and in each occupation the early born sons would count for more than the late born. The scheme of ideas for the exact treatment of this factor is included in Malthus' analogy of population increase with compound interest. If capital is to be repaid at a premium the date of repayment is of importance in the calculation of the rate of interest realised. To find the appropriate rate of interest in the analogous problem of human increase we must reduce the future sons, whose advent we are expecting, to their *present value*; and the particular rate of interest which makes their present value (to the new born child) equal to unity provides an exact measure of the geometrical rate of increase of the population. The fact, which Malthus seems scarcely to have considered, of a "natural decrease" of considerable sections of mankind must be represented by negative values of the rate of interest, and these will necessarily occur whenever the expectation of sons is less than unity.

Some rough calculations of the author on the excellent data available for the Commonwealth of Australia, showed that in 1911 that young and virile population was still increasing, but at a rate of less than 1% per annum. The tabulation of births by ages of parents, upon which the calculation was based, was made possible by the inclusion in the birth registrations of the ages of the parents. Unfortunately this is not at present recorded in our own country, though the query is not an objectionable one, while the occupation of the father in combination with which the age data would be particularly valuable, is already recorded.

The exact calculation of rates of population increase or decrease, freed from the temporary and adventitious elements of the momentary age distribution of the population, which tend to govern any indications given by successive census totals, opens the way to the study of a whole new aspect of human vital statistics. For example an epidemic killing largely persons beyond the reproductive age may have little direct influence upon the population of future generations; nevertheless in the crude statistics which depend upon total deaths, it may loom larger than a war which destroys a large proportion of the age groups of early manhood. Simply and solely from the

point of view of the balancing of death by reproduction these lives are not of equal value; and that quite apart from any difference in industrial efficiency or economic value. The same consideration applies to emigration; a shipload of adolescents carries with it, even if their expectation of sons at birth is only unity, a much larger potential population than the number of the passenger list, while the transfer of an equal number of elderly persons should count for little or nothing. In view of these considerations it may be asked, is it not possible to deduce from complete mortality and natality data a consistent system of valuation by which the potential value to the population of future generations, of each age group, may be measured.

There seems no doubt that this can be done merely from the data considered above. For each age group the expectation of future sons may be known, and the intervals of time over which their arrival will be distributed. The present value, at the rate of compound interest previously indicated, of their future progeny, will provide an exact measure of the value of persons of the group considered for the purpose of procreating future generations. It is interesting that such values will give us the proportions of the ancestry of a remote future generation which each age class will represent. In general the value of each individual, which may be conventionally taken to be unity at birth, will increase with age as he escapes the dangers of infantile and child mortality, it will reach its maximum at about 20, and will thereafter decrease as the time for begetting children passes, whether such children are begotten or not. When reproduction has ceased his value as a potential ancestor is obviously zero.

A further property of the value so calculated should be indicated. Knowing the values to assign to each age we may evaluate the whole census population of which the age distribution is, of course, known. The comparison of the *total values* of two census populations, unlike the comparison of the mere numbers, provides, when allowance has been made for migration, a simple measure of population growth or decrease, which may be shown to coincide with the Malthusian rate of interest discussed above, or rather if it is changing, with its value averaged over the intercensal period. In this way can be applied a very simple test as to whether the increase in the number of heads of the population is or is not sufficient to counterbalance the increasing average age.

The convention that unit value is to be ascribed to the newly born is open to no objection so long as we merely wish to compare the values of different age groups; on the other hand it is not suitable for the comparison of different populations. For this purpose a different convention will be more suitable, namely, that in a population in its *steady state* the total value ascribed to the population is equal to the total number of heads living. Actual populations are seldom at or near the steady state appropriate to their birth and death rates, and could scarcely become so unless the frequency at each age of death and reproduction remained constant for nearly a century. The convention of valuation based upon the steady state allows the new born child to count for more among long lived people than among the shortlived, as he obviously ought to do, and in the comparison of the census population with its total value provides a simple index

showing to what extent the actual populations are or are not of ages favourable to reproduction. The total value on this system of the population of this island is probably somewhat above its census total, and it is quite possible that while the total value for reproductive purposes is decreasing, the number of heads is increasing by a kind of transference from potential to actual humanity.

The mutual dependence of the two sexes for reproduction ensures that each sex shall play an equal part in the production of future generations, and allows us to base our calculations with a high degree of confidence upon either males or females. For occupational groups the comparisons should be based on men and their sons; for regional comparisons on the other hand either sex may be used, and it is probable that the calculation based on women and their daughters will be the more precise, since their reproductive period is the more sharply defined. Interesting though probably small discrepancies between the results obtained from the two series might perhaps be introduced by aberrations in the sex-ratio in the reproductive period, such as a deficiency of men of these ages following a war, or an excess of men in a newly developed country. Presumably, for example, the women of reproductive age in this country would be producing somewhat more children, if the male age groups from 80 to 45 had not been depleted in the German war; it is also to be presumed that the men now living at these ages would be producing somewhat fewer children. Such effects may be small, but it would be none the less of considerable interest to evaluate them in making regional comparisons.

The primary data required are the ages in addition to the occupations of fathers in birth registration, and, for comparisons other than occupational, the ages of mothers. If these could be obtained forthwith, using the same occupational classification as will be used in the Census of 1931, 5 or 6 years data centred on this Census would be made available; and this would provide not only occupational comparisons of great biological interest, but would have the exceptional additional advantage of referring to a recent period at the time when the results should be ready for publication. While this would be in every way the best plan for ascertaining, in the future, contemporary rates of reproduction, it throws no direct light upon the important changes which have been taking place during the last two generations, and which it is important to evaluate while persons are still living whose reproductive history makes this possible. On this point a most important step was taken in the Census of 1911 by including a question as to the number of children already born to each person included in the Census. Had the dates of these births also been recorded, such a return would have made it possible to reconstruct the reproductive history of any occupational or regional group, of any age, surviving to the date of the Census, and so to extend our knowledge of birth rates back to the reproductive period of the oldest age groups. Apart from the difficulty, previously mentioned, of assigning old persons to correct occupational categories, the information obtained in the 1911 census requires supplementing in one important particular; it is not obvious that the natality of the survivors of a given age class is accurately representative of the natality of men of the same age class who died before the census was taken.

A positive correlation between fertility and longevity has been reported in certain middle class family histories, and it is by no means obvious that such correlations of this kind as may exist should be the same in all classes. A direct examination, and approximate elimination, of this factor should be obtainable from a full return from the same population at a later census; for we could then compare two records of the total children of the same age class up to the date of the first census, based respectively upon the survivors to two different dates. From this point of view it is particularly unfortunate that the question asked in 1911 was not expanded, nor even repeated, in 1921, and exceedingly important that full information should be obtained in 1981.

Unfortunately, owing to changes in the method of occupational classification, in other respects highly desirable, the comparisons with the 1911 census will only be possible for a few well defined occupations, in addition to the country as a whole; however, those occupations for which accurate comparisons seem possible, appear to be groups of very great biological importance.

A possible source for reconstructing the reproductive history of the immediate past, which may be worthy of consideration, would be the inclusion of the census information referred to above,—number of children and dates of their births—in the registration of deaths. Upon the advantages of such a course compared to the repetition of the question in the census, only the experts of the General Registry Office are qualified to judge.

Finally it may be of service to give an algebraical synopsis of the calculations discussed. If  $l_x$  is the probability of a boy living to age  $x$ , and  $b_x$  is the birth rate (of sons) of men of age  $x+\frac{3}{4}$ , reckoned on the number living at age  $x$ , the expectation of sons of a newly born child is

$$\int_0^{\infty} l_x b_x dx$$

The Malthusian rate of geometrical increase will be measured by  $m$ , which is defined so as to satisfy the equation

$$\int_0^{\infty} e^{-mx} l_x b_x dx = 1$$

If the expectation exceeds unity  $m$  will be positive, if it is less than unity  $m$  will be negative. The reproductive value of a man of age  $x$  is  $v_x$ , where

$$l_x e^{-mx} v_x = v_0 \int_x^{\infty} e^{-mt} l_t b_t dt.$$

where  $v_0$  is taken equal to unity, if we wish to give unit value to a newly born boy. If  $v_0$  is to be chosen so that in the steady state the total value of the population is equal to the number of heads, then

$$v_0 \int_0^{\infty} x e^{-mx} l_x b_x dx = \int_0^{\infty} e^{-mx} l_x dx.$$

and the value at all ages will be two or three times greater than on the previous convention.