TABLE IV.

Harvest Year.	a'	b'	c'	d'	e'	f'	Harvest Year.	a'	b'	c'	d'	e'	f'
1854	347	−14	+33	−16	−21	−1	1887	387	−82	−6	+29	−18	+14
1855	398	+90	+41	+8	−35	−22	1888	500	+48	+59	−20	−10	−13
1856	487	−18	+12	+1	−27	+35	1889	498	+72	−1	−11	−36	−10
1857	404	−9	+19	+15	−9	−1	1890	450	+20	+35	−1	−15	−12
1858	441	−69	+69	−38	−12	+15	1891	383	+65	+35	+22	0	+28
1859	412	+64	+3	−14	−4	−5	1892	487	−29	+70	+45	−35	+29
1860	598	+39	+35	−12	−12	+11	1893	398	−38	+37	+26	−16	−4
1861	381	−8	+3	−17	−24	+1	1894	488	−7	+24	+30	−8	+16
1862	451	+22	−13	−13	−21	+18	1895	474	−3	+66	+51	−42	−11
1863	414	−21	+36	+19	−7	+14	1896	399	−27	+21	+32	−4	+13
1864	331	−56	+16	−24	+3	+3	1897	618	−131	+77	−32	+55	−5
1865	454	+43	+61	−30	+29	−11	1898	323	−11	+9	−20	+4	−17
1866	568	−2	+3	+38	−45	+8	1899	405	−20	−42	+27	−17	+5
1867	513	−14	+21	−27	+7	−17	1900	514	−47	+27	+35	+4	+2
1868	346	−11	+14	+26	+42	−11	1901	406	−10	−7	+31	+2	+11
1869	426	−44	−37	+10	−3	−3	1902	382	+30	+18	+12	−5	+2
1870	354	−65	+10	+21	+9	−11	1903	520	+106	+28	−12	−8	−16
1871	451	−7	+16	−23	−32	−16	1904	517	−75	+30	+32	+2	+7
1872	475	+8	+13	−23	+5	−6	1905	428	+35	+58	0	+17	+11
1873	503	−57	+6	+64	−31	+2	1906	389	−37	−13	+11	−9	−15
1874	357	−12	+30	−10	−3	−6	1907	481	−44	+13	+5	−47	+24
1875	526	−11	+42	−33	−26	−42	1908	494	−5	+6	+24	−2	+38
1876	524	−80	+29	+18	−8	+42	1909	419	+52	+26	−26	−5	−7
1877	648	−76	+9	+22	+23	−38	1910	507	−16	+16	+23	−11	+5
1878	532	+38	+10	+20	+1	+35	1911	465	−43	−38	+25	−35	+11
1879	674	+105	+37	+26	−35	−3	1912	672	+18	+33	+109	−9	−2
1880	350	+39	+25	−29	−7	−24	1913	451	−54	−35	+20	−2	+18
1881	603	−78	+110	+35	+41	+16	1914	419	−42	−14	−10	+8	−57
1882	530	−11	−8	+5	−28	+15	1915	604	−14	−36	+65	−29	−55
1883	573	−81	+6	+25	−39	−14	1916	582	−17	−26	+51	+15	+14
1884	422	−53	+35	+1	0	+8	1917	577	+54	+82	+58	−25	−2
1885	434	−20	−37	−11	+9	0	1918	449	−18	+15	−22	−11	+23
1886	508	−83	+33	−30	−13	0							

of the method the numerical values are given in Tables V and VI ; before summation each a' was reduced by 400, the true value of the mean being inserted in the fourth column. The second column gives the successive sums, S_1 to S_6 ; from these the third column is derived by dividing by factors of the form

$$\frac{65 \, . \, 66 \ldots (65+r)}{r!}, \qquad r = 1, \ldots, 5.$$

Thus the third column gives the values of a to f for the series a'. The fourth column gives the corresponding values a' to f' for the series a', obtained by equations (V) ; while the fifth column, obtained by multiplying by factors of the form

$$\sqrt{\frac{(2r+1) \, . \, 65 \ldots (65+r)}{64 \ldots (65-r)}}.$$

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are the actual values of the first five transformed co-ordinates spoken of in Section 4, as x_2' to x_6'.

TABLE V.—Analysis of sequence of values of a'.

—	S_1 to S_6.	a to f.	a' to f'.	x_2' to x_6'.
1	4,521	69·55385	469·55385	
2	128,341	59·832634	+9·72122	+137·85
3	2,603,055	54·3378562	−1·26834	− 23·95
4	40,090,879	49·22841040	+7·79456	+182·35
5	500,901,859	44·570118750	+2·35049	+ 66·31
6	5,313,933,165	40·528492120	−2·62489	− 88·43

In an unchanging series the values x_2', x_3' ... vary about zero in an approximately normal distribution, the standard deviation of which may be obtained from that of the original series ; for

$$\overset{n}{\underset{2}{S}} (x_r'^2)$$

is the sum of the squares of the deviations of the original series from their mean. Slow changes in the original series will be indicated by high positive or negative values in x_2', x_3', ... , and if such slow changes are suspected, it will be better to estimate the variance due to random causes from

$$\overset{n}{\underset{7}{S}} (x_r'^2),$$

from which the first five values have been omitted. From the sums of the squares of the deviations we may thus obtain a series of values each obtained from the last by deducting the square of the corresponding value x_r'; from each such sum may be obtained an estimate of the standard deviation due to random causes, by dividing by the number of squares concerned (degrees of freedom), and taking the square root. Such estimates will be equivalent to those derived from the residuals left after polynomials of the first to the fifth degree have been successively fitted ; but the labour of calculating the polynominal values is avoided.

TABLE VI.

Degrees of freedom.	Sum of squares.	Mean square.	Standard deviation.
64	465,105	7267·3	85·25
63	446,103	7081·0	84·15
62	445,529	7186·0	84·77
61	412,278	6758·7	82·21
60	407,881	6798·0	82·45
59	400,061	6780·7	82·35

The diminution of the estimates of the standard deviation indicates that the first 5 values of x' are on the whole higher than those which follow; for example x_4' is more than double the standard, and suggests strongly that real changes are taking place in the rainfall. To test this more accurately, divide the sum of the squares of the five deviations by the mean square of the remainder, then

$$\chi^2 = \frac{465105 - 400061}{67807} = 9 \cdot 59,$$

whence, entering ELDERTON's table with $n' = 6$, we obtain $P = \cdot 089$. Thus a larger value of χ^2 would be obtained by chance only 8·9 times in a hundred, from a series of values in random order. There is thus some reason to suspect that the distribution of rainfall in successive years is not wholly fortuitous, but that some slowly changing cause is liable to affect in the same direction the rainfall of a number of consecutive years. Another way of putting the same result is that the variance estimated from the residuals of a polynomial of the 5th degree is 93·3 per cent. of the variance of the original series, so that some 6·7 per cent. of the total variance observed in annual rainfall may be ascribed to slow changes, while the remaining 93·3 per cent. of the variance are due to the chance circumstances of each particular year.

We summarise below the results of applying a similar analysis to the rainfall distribution values a' to f' :—

TABLE VII.

——	a'	b'	c'	d'	e'	f'
Mean	469·55	− 11·09	+ 19·22	+ 9·43	− 8·69	+ 0·57
x_2'	+137·85	− 18·62	− 35·01	+ 86·92	+ 8·78	− 2·11
x_3'	− 23·95	+ 56·19	− 33·75	+ 5·45	− 16·94	− 0·84
x_4'	+182·35	− 35·31	− 33·96	+ 13·02	+ 15·72	− 23·80
x_5'	+ 66·31	+ 13·00	+ 45·52	+ 5·76	− 29·78	+ 1·25
x_6'	− 88·43	+ 24·69	+ 33·79	− 20·47	+ 11·50	+ 22·33
Standard Residue	82·35	50·43	30·11	28·19	20·76	19·91
χ^2	9·59	2·17	7·42	10·33	3·78	2·70
P	0·089	0·83	0·19	0·067	0·58	0·74

It is quite clear, from the values of P, that no significant changes are observable in b', c', e' and f'. These fluctuate with large standard deviations about the mean values given in the first line of Table VII, which mean values have not significantly changed during the 65 years examined. With a', and more clearly with d', the mean appears to show gradual changes. In the latter case the change observable is very clear, and of a simple character. The value of x_2', representing a linear increase, exceeds the standard deviation in the ratio 3·083; the probability of such a value occurring by chance is only 0·00205. The remaining values, x_3' to x_6', are individually and collectively insignificant, whence it appears that the slow change in d' is significant, and may be represented by a uniform increase. This is in sharp contrast to the behaviour of a', a contrast which is brought out by plotting successive 10 year means of these two quantities.

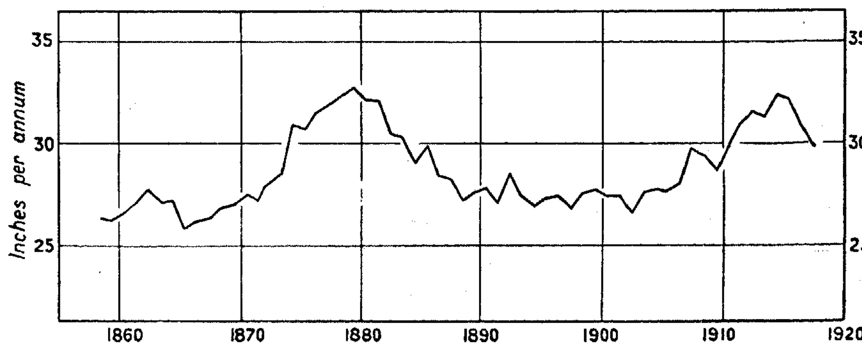
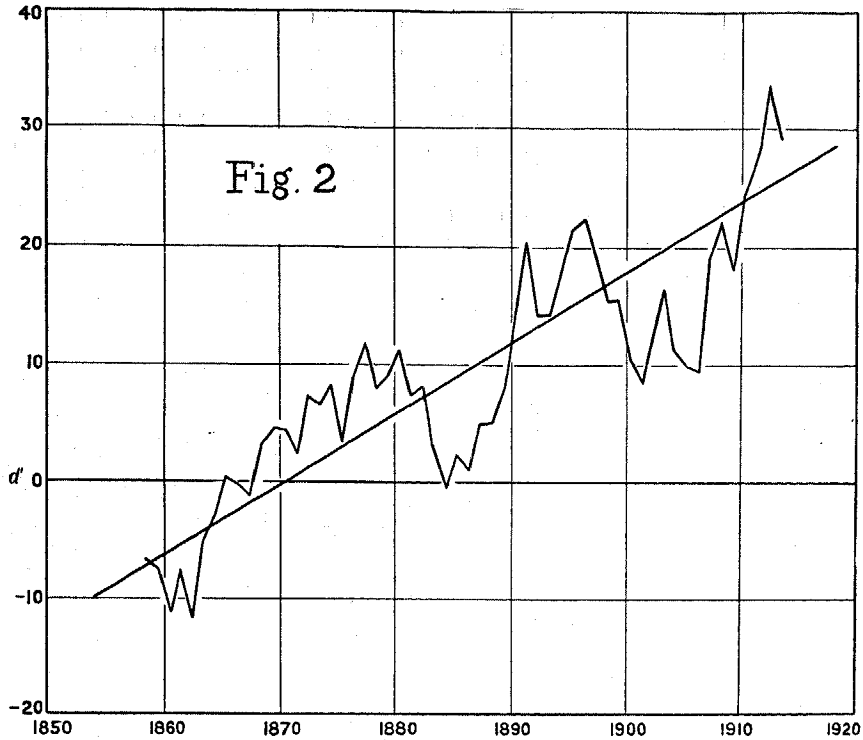


Fig. 1.

In the case of the total rainfall (fig. 1) measured by a', there would seem to have been an excess of rain for a series of years about 1879, and a second spell of about equal intensity about 1914; the interval being 35 years. At these periods the annual rainfall average rises to about 32 inches. Prior to the first wet spell, and in the intermediate period, the average annual rainfall is about 27 inches. Such a change is small compared to the annual fluctuations; we have seen that it accounts for only about 6·7 per cent. of the variance, and that the whole effect is scarcely significant in our data. The distribution value d', on the other hand, which measures not the total rainfall but its distribution over the year, shows a distinct and apparently uniform increase. Judging from the straight line (fig. 2), its mean value has changed from -10 in 1854, to nearly $+29$ in 1918; its mean value $+9·43$, given above, is thus not a permanent feature of the climate of the district. As in all rainfall features the annual fluctuation is very great; of the variation observed in d' in the 65 years only 12·3 per cent. is ascribable to the linear change, the remaining 87·7 per cent. being apparently due to random fluctuations. In the absence of a similar analysis of rainfall at other stations, it would be premature to discuss the possible causes of this remarkable and progressive change in the climate. It may be remarked that such little additional information as is to be obtained from the monthly records indicates that the most marked feature of the change in progress is an



Ten year means of the distribution value d'.

increase in the December rainfall with perhaps some relative reduction of rain in Spring and Autumn.

The mean values of c', d', and e', differ significantly from zero, owing to the seasonal variations of rainfall. The series of mean values, neglecting the mean value of f' which is altogether negligible, represent the mean course of the rainfall sequence during the year. This is represented in fig. 3, where the mean monthly values have been inserted for comparison; these values have been reckoned *per day* to eliminate the effect of the varying lengths of the months. The monthly means are of course subject to large probable errors, and the smooth curve gives a slightly better estimate of the average sequence; this sequence is necessarily in process of modification owing to the progressive increase of d'.

The standard deviations of the six distribution values are in good mutual agreement and evidently arise from a common cause, namely the random fluctuation of rain in a six-day period. This may be seen by multiplying them by the factors necessary to transform a', ..., f' into $\rho_1, \rho_2, \ldots, \rho_6$ (Section 5), that is by

$$\sqrt{61}, \quad \sqrt{\frac{3.61.62}{60}}, \text{ etc.};$$

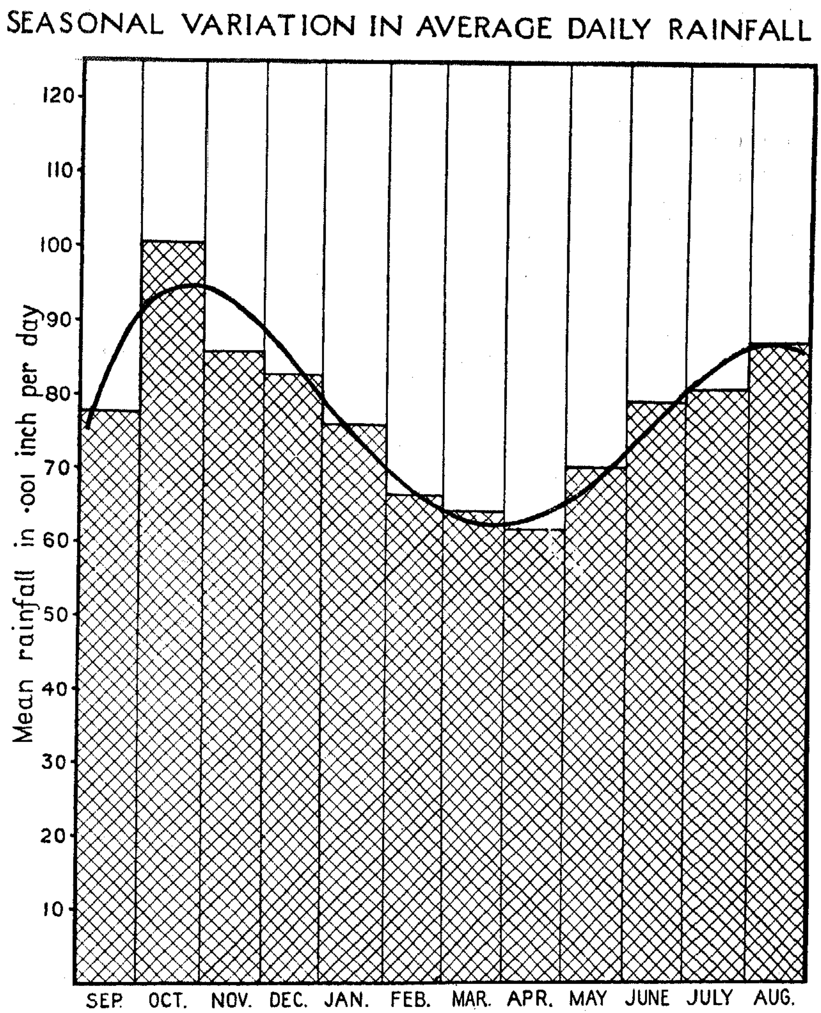


Fig. 3.

the standard errors of $\rho_1, \ldots, \rho_6$ are then

σ_{ρ_1}.	σ_{ρ_2}.	σ_{ρ_3}.	σ_{ρ_4}.	σ_{ρ_5}.	σ_{ρ_6}.	Mean.
643	693	552	643	573	660	627

These may be regarded as independent estimates of the standard deviation of the rainfall (in thousandths of an inch) in a six-day period, each derived from 59 independent squares, which is equivalent to a random sample of 60 values. The variation of these estimates is just of the order to be expected on the basis of random sampling. The variation of the rainfall distribution values is thus nearly what we should deduce from the supposition that in addition to the small contributions made by secular and seasonal variation, each 6 day period varied independently with a standard deviation 0·627 inch, or a coefficient of variation of 134 per cent.

The distribution of the distribution values are not materially skew as far as can be judged from a series of 65 values. The following values obtained from the moments of the sample show the observed deviations from normality, with the standard errors appropriate to a normal population

	a'.	b'.	c'.	d'.	e'.	f'.	Standard error.
$\gamma_1 = \sqrt{\beta_1}$	$+0.493$	$+0.336$	$+0.232$	$+0.566$	$+0.614$	-0.546	± 0.306
$\gamma_2 = \beta_2 - 3$	-0.290	$+0.066$	$+0.354$	$+0.510$	$+0.920$	$+1.006$	± 0.612

These figures suffice to show that the observed values hardly differ significantly from samples of normal distributions, although there are indications that, if the values for a considerably longer series of years were available, real departures from the normal distribution would become apparent.

7. *Correlations of Rainfall Distribution Values.*

The rainfall distribution values have been calculated from uncorrelated functions of the time, and if the rainfall at different periods of the year were uncorrelated the rainfall distribution values should also be uncorrelated. If, on the other hand, rainfall at one time of the year were associated with rainfall or lack of rainfall at another time, then correlations would appear in the rainfall distribution values, and such correlations would indicate the nature and extent of such mutual correlations between different parts of the season. HOOKER observed, in the short period of 21 years at his disposal, a correlation as high as + 0·6 between the winter rainfall (1st to 8th weeks) and that of the previous autumn (37th to 44th weeks), and concluded that so large a correlation indicated a real interdependence between winter and autumn rainfall. More recently W. T. RUSSELL (13, 1922) has calculated the correlation coefficients for monthly rainfall between consecutive and between alternate months. For Greenwich (65 years) no appreciable correlations are found between successive months, but in the alternate months it is found that June and August have a positive correlation + 0·55. Such a value should occur by chance from uncorrelated material once only in 900,000 times, so that even allowing for the fact that it is the highest of the 24 values obtained, it would appear to be significant of a real association. It should, however, be borne in mind that any secular changes either in total rainfall, or in its distribution through the year, as may exist at Greenwich, have not been eliminated in this coefficient, and, in the second place, the distribution of monthly rainfall is far from normal, and in consequence the probable error of such a determination may be distinctly higher than that calculated for normal distributions.

The correlations obtained between the 65 values $a', \dots, f'$ are not, in any case, high. The transformed values ($z = \tanh^{-1} r$) are set out in Table VIII, since these values are distributed in random samples in approximately normal distributions, with the same standard error $\pm 0\cdot127$.

TABLE VIII.—Values of $z = \tanh^{-1} r$.

—	a'.	b'.	c'.	d'.	e'.
b'	−0·01	—	—	—	—
c'	+0·19	+0·09	—	—	—
d'	+0·38	−0·07	−0·09	—	—
e'	−0·06	−0·23	+0·18	−0·16	—
f'	−0·03	−0·08	+0·08	+0·16	−0·06

None of these correlations excite remark except that between a' and d'; these two variates are the two which show secular variation, and it is therefore of more interest to obtain the corresponding values after the secular variation has been removed. These are found by treating the sum of the products of deviations as the sum of squares has been treated in Table VI, deducting successively the products of the values $x_2', \dots, x_6'$, for the two variates concerned, calculating r from the sums of squares similarly treated, and transforming to the z scale as before. The values so obtained are shown in Table IX, with standard error $\pm 0\cdot1325$.

TABLE IX.—Values of z after Eliminating Secular Change.

—	a'.	b'	c'.	d'.	e'.
b'	+0·0342	—	—	—	—
c'	+0·2880	+0·0800	—	—	—
d'	+0·3103	−0·0526	−0·0228	—	—
e'	−0·0784	−0·2100	+0·2380	−0·1955	—
f'	+0·0356	−0·1119	+0·0381	+0·2092	−0·0525

The correlation of a' with d' now exceeds its standard error in the ratio 2·34; the probability of obtaining so large a value by chance from uncorrelated material is 0·019, and since this is the largest value of 15, it can scarcely be regarded as proof of association. To test the values collectively from the series of their squares, we obtain

$$\chi^2 = 22\cdot63, \qquad n' = 16, \qquad P = 0\cdot093,$$

from ELDERTON'S Table, showing that although there are signs of association among the rainfall distribution values, such association, if it exist, is not strong enough to show up significantly in a series of about 60 values.

8. *The Regression of Yield on the Distribution Values, and on the Rainfall at Different Seasons.*

The sums of squares and products of the distribution values provide a basis for the calculation of the partial regression coefficients of the yields upon them, but before applying them in this way it is necessary to allow for the fact that 5 years' yields, namely, 1890, 1891, 1905, 1906 and 1915, have been omitted; we must therefore deduct from the sums the contributions of these years. Table X shows the polynomial values of a', b', ... , f' for these years :—

TABLE X.

Year.	t.	a'.	b'.	c'.	d'.	e'.	f'.
1890	4	465·64	−13·21	33·17	9·11	− 9·73	4·16
1891	5	461·66	−12·36	33·36	9·10	− 9·42	4·84
1905	19	458·63	−11·96	10·57	18·88	− 6·03	2·00
1906	20	463·62	−12·52	7·98	20·29	− 6·17	0·97
1915	29	524·83	− 8·98	2·73	30·09	−10·66	− 4·13

calculated from the polynomial coefficients of Table VII. These values were subtracted from the observed values of Table IV, and the squares and products of the deviations deducted from the corresponding sums. The corrected sums, answering now to 54 degrees of freedom, were as follows :—

TABLE XI.

—	a'.	b'.	c'.	d'.	e'.	f'.
a'	+380,853	+ 14,984	+44,122	+38,013	− 5,278	+ 8,121
b'	+ 14,984	+140,110	+ 4,031	− 4,241	−14,577	− 8,921
c'	+ 44,122	+ 4,031	+49,302	+ 906	+ 6,746	− 1,392
d'	+ 38,013	− 4,241	+ 906	+44,944	− 5,790	+ 8,163
e'	− 5,278	− 14,577	+ 6,746	− 5,790	+24,429	− 2,769
f'	+ 8,121	− 8,921	− 1,392	+ 8,163	− 2,769	+19,670

Since we require to find the partial regressions for 13 separate plots, it is worth while to invert the determinant formed of these numbers, so as to obtain a matrix of multipliers each of which is the co-factor of the corresponding number above, divided by the determinant. Table XII shows these multipliers in millionths.

TABLE XII.

—	a'.	b'.	c'.	d'.	e'.	f'.
a	+3·235054	−0·304445	− 2·947149	− 2·535496	+ 0·672526	− 0·542793
b	−0·304445	+8·094433	− 1·082929	+ 1·042372	+ 5·775040	+ 4·100483
c	−2·947149	−1·082929	+24·070588	+ 0·730487	− 7·637592	+ 1·050675
d	−2·535496	+1·042372	+ 0·730487	+26·747433	+ 5·214984	− 8·794578
e	+0·672526	+5·775040	− 7·637592	+ 5·214984	+48·604978	+ 6·478923
f	−0·542793	+4·100483	+ 1·050675	− 8·794578	+ 6·478923	+57·557904

For each plot six correlation tables were constructed with the six rainfall distributions values, using the values in Tables III and IV, corresponding to the 60 years available. The six sums of products obtained from these tables, multiplied by the values of any column of Table XII and added, give the regression of yield on the corresponding rain variate. Table XIII gives the values obtained for these regression coefficients, the crop being measured in bushels per acre.

TABLE XIII.

Plot.	Regression on—					
	a'.	b'.	c'.	d'.	e'.	f'.
2B	− 40·9895	+ 2·1969	− 8·0261	− 46·2368	− 15·8457	+ 24·0548
3+4	− 20·2933	− 0·0582	+ 0·1506	+ 13·7263	+ 14·2433	+ 27·0821
5	− 20·7766	− 3·1878	− 0·0575	+ 13·9806	+ 7·2414	+ 18·8043
6	− 39·7077	− 0·6177	+ 34·1075	− 5·7556	− 8·3922	+ 51·2116
7	− 42·4430	− 3·1822	+ 36·3066	− 31·9965	− 16·6956	+ 83·5923
8	− 43·8350	− 11·7970	+ 35·9850	− 31·6292	+ 8·1368	+110·1433
10	− 25·7070	− 20·2730	− 15·3172	− 15·6733	+ 35·8060	+140·2458
11	− 30·4546	− 14·8637	− 9·8165	+ 1·6230	+ 17·7579	+113·9780
12	− 55·9573	− 13·9676	+ 20·9149	− 22·4509	− 23·0060	+113·0384
13	− 45·1447	− 3·9169	+ 63·5960	− 48·0067	− 24·4347	+ 92·2415
14	− 42·6026	− 12·9980	+ 25·9731	− 30·7156	− 24·8636	+115·5578
17 & 18 miner'ls	− 24·0377	+ 0·5955	+ 17·2943	+ 5·6818	− 3·8642	+ 33·4268
17 & 18 am'onia	− 42·6939	+ 3·8539	+ 60·9060	− 44·3881	+ 2·7304	+120·5257

These coefficients give directly linear regression formulæ expressing the deviation of the wheat crop on each plot in terms of the rainfall distribution values. Equally, as explained in Section 3, they enable us to estimate the average benefit or loss in bushels per acre ascribable to an additional inch of rain at any time during the year. For this purpose we divide the six regressions corresponding to any plot by

$$\frac{(r\,!)^2}{(2r)\,!} \cdot 61 \,.\, 62 \ldots (61+r),$$

and we have the six coefficients of t^0, t, t^2 ... in

$$a = \mathrm{A} + \mathrm{B}t + \mathrm{C}\,(t^2 - n_2) + \ldots,$$

expressing a in terms of the polynomials of Section 3, wherein t is the time in 6-day intervals measured from the central period of the year. Figs. 4 to 8 show the course of the function a throughout the year.

9. *Discussion of Figs.* 4 *to* 8.

It should be emphasised that the information provided by a comparison of the rain record with the subsequent yields tells us the effect, not of so much rain, as such, but of the total meteorological phenomena in fact associated with rain, at the time of the year considered. Thus in our records rain is associated with lower temperatures in summer, in winter with higher temperatures; and generally with diminished sunshine. The effects of these, to the extent in which they are associated, will be incorporated in the total effects shown in the charts, and are in fact an integral part of the value of a rain record as a means of foreseeing the prospects of the crop.

Even in the case of the rainfall itself, however, a detailed consideration of the ways in which it may affect the crop would lead to a most intricate discussion. It would be of the greatest value to know how important to the final crop, and how frequently influential, are not only the actual moisture available in the soil, but also the degree of soil aeration; how frequently, and to what extent, root development is hindered by soil saturation, or by an accumulation in toxic concentration of carbon dioxide. We should expect these factors to be intimately connected with rainfall, both in its direct effect in supplying fully aerated water, and in its indirect effects upon the soil texture.

In a single curve showing the average effect of rainfall (and of the average weather associated with such rainfall) upon the crop ultimately produced, all such contributory causes are included; by the comparison of the curves obtained from different plots, representing different manurial conditions of the soil, we may infer the effect of rainfall upon the availability of the manurial constituents of the soil. As will appear more fully, the predominant feature of such a comparison is the influence of excessive rain in removing soluble nitrates; this effect masks and overshadows all others, partly, I would suggest, by reason of its intrinsic importance to the wheat crop in our wet climate, partly because the plots on Broadbalk show great extremes in the relative abundance of available nitrogen.

The predominance in these curves of the effects which appear to be ascribable simply to the removal of soluble nitrate may perhaps explain two features which otherwise might be unexpected. In the first place the greater part of the effect of rainfall is expressible by means of the linear relations in the manure here represented * ; the quadratic terms, though of great interest and well worth further study, are of much less quantitative importance. Considering merely the effects of rain on the general environment of the growing plant we might expect definite optimum conditions throughout the year to be well marked, with correspondingly important quadratic terms; but the effect of nitrogenous fertilisers is not only approximately linear over the range concerned, but since increasingly heavy drainage generally removes decreasing quantities of nitrates, the tendency of its effect is to reverse the curvature of the regression. In the second place it might have been expected that the effects of rainfall would be closely related to the total crop on the different plots, and that resemblance would be more clearly apparent in the proportionate than in the absolute effects; but since for a wide range of manurial condition the absolute effects of additional nitrogen are not very unequal, and tend to decrease with increasing crop, it is apparent that the similarities in the absolute effect, which we observe for example in plots 8 and 14, are more probably due to quantitative

* For manure, read manner.

changes in nutrition than to indirect modifications of the physiological condition of the crop.

As will be seen from Table II, the 13 plots studied may be arranged in the following classes :—

(i) Low fertility, no added nitrogen. Plots (3 and 4), 5 and (17 and 18) mineral series.

(ii) Progressively higher yields due to progressive increase in nitrogenous dressings, from plot 6, through plots 7, 13 and (17 and 18) ammonium series, to plot 8.

(iii) Dressings which have produced an increasingly unbalanced nutrition with diminution of yield. Plots 12 and 14.

(iv) Unbalanced nutrition produced earlier and more intensely. Plots 10 and 11.

(v) Farmyard manure containing the very heavy contribution of about 200 lb. of nitrogen, with organic matter which has materially lightened the soil.

An examination of these diagrams shows how intimately the response of the crop to weather is connected with the manurial condition of the soil. Classing the plots solely by inspection of the curves of response to rainfall we shall put together every case in which the manurial treatment is alike, and indeed the whole series of curves arrange themselves in sequence of order of increasing abundance of nitrogenous fertilisers. As preliminary to a fuller discussion we may note (i) that in all the plots the average effect of additional rain is harmful. This agrees with HOOKER's finding for Eastern England. (ii) In all save the dunged plot 2B, the average loss is rapidly reduced during August, an observation which finds a simple explanation in the fact that the average date of carting the crop is August 24th, so that as August advances an increasing proportion of years occur in which the rain is too late to affect the crop. The exceptional behaviour of plot 2B indicates that the average loss per inch of rain in the month preceding harvest is even heavier than that shown in the diagram. (iii) In all the plots October is a month in which the average loss per inch of rain is small, or in which rain above the average is positively beneficial. This is the reverse of the condition found by HOOKER, who finds the greatest negative correlation with rain early in October. (iv) In all plots save two, the unmanured plot (3 and 4) and plot 5 which receives mineral (non-nitrogenous) manures only, the autumn period of benefit, or but little loss, from rain, is followed by a period centred in January in which dry conditions appear to be particularly desirable.

At this time of the year each additional inch of rain costs from one to two bushels in the crop. That this effect is scarcely visible on the unmanured plot, and still less so on plot 5, speaks strongly in favour of Sir JOHN LAWES's view that the damage done by winter rain was principally occasioned by the washing out of nitrates from the soil. Plot (3 and 4) and especially plot 5 can have little to lose in this way, and possibly rely for the greater part of their growth upon nitrates produced by bacterial activity as the soil grows warmer in March, April and May. The same view is confirmed by the fact that among the remaining plots the loss due to winter rain is least in plots 2B, 10 and 11, all of which are characterised by the low proportion utilised of the nitrogen supplied.

If these plots seldom suffer from lack of nitrogen, its loss during the winter must be expected to affect the crop relatively little; nevertheless the actual average loss of about one bushel to the inch of rain shows that at this time (*i.e.*, before the spring dressings are applied to plots 10 and 11, and perhaps before bacterial activity can fully tap the resources of the farmyard manure), a temporary shortage sometimes injures the crop. With these preliminary observations we may pass to the consideration of the types of rainfall response curve actually found.

Type I.—Plot 5, mineral manures only, and plot (3 and 4), unmanured. Fig. 4 shows plot 5.

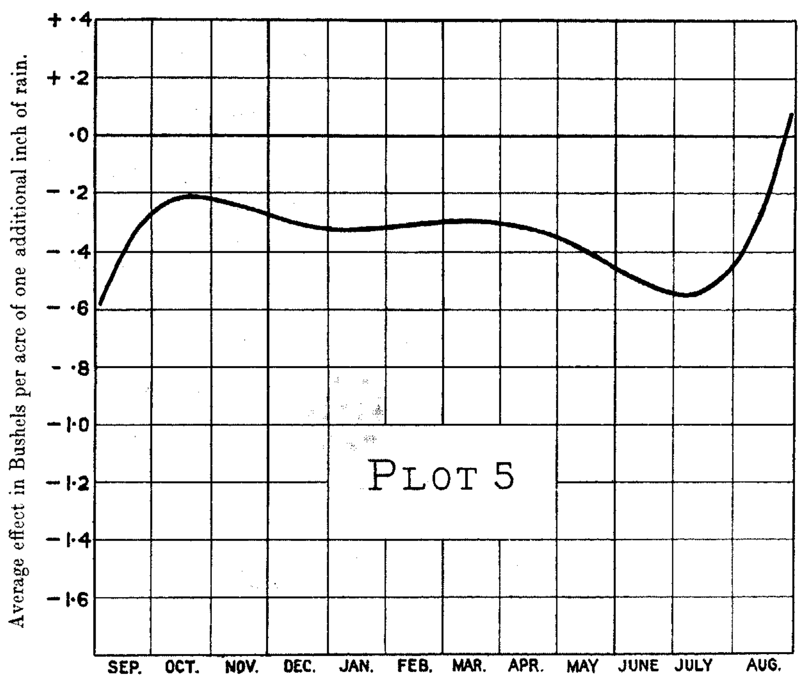


Fig. 4.

In these two plots the crop is severely limited by the lack of available nitrogen. Plot 5 has a mean yield of 14·18 bushels per acre, while plot (3 and 4) yields 12·27. These are averages (1852–1918) covering the whole period over which the rainfall regressions have been calculated. Both plots show heavy deterioration over this period. The small difference in the mean yields, which is nearly constant over the whole period, shows that lack of nitrogen is the dominant limitation in plot (3 and 4) as in plot 5 where the limitation due to lack of nitrogen must be most stringent.

In plot 5 the rain experienced is in excess of the optimum probably at all times of the year. The average loss for additional rain falls from about 0·4, in the September before sowing, to a minimum of 0·22 (1·5 per cent.) towards the end of October, and remains steadily near to 0·30 till the end of May. It falls to a maximum loss of 0·55 (3·9 per cent.) early in July and rapidly diminishes through August. Since the proportion of drainage through the twenty-inch gauge decreases from nearly 80 per cent. in the winter to about 25 per cent. from April onwards, it follows that if the main loss on this plot due to rainfall is ascribed to the washing out of nitrates, the loss due to an equal amount of drainage is increasingly serious as the year advances. This accords with the belief that nitrification becomes more active in the Spring, and that this plot relies largely for its nitrogen supply upon bacterial activity at this season. The curve even suggests the possibility that under the intense nitrogen starvation of this plot the supply of nitrates is of importance as late as the beginning of July, though it must be borne in mind that other effects of rain may become important during the summer.

The curve for plot (3 and 4) is closely similar. September damage of about 0·4 is followed by a minimum of 0·23 (1·9 per cent.) in October. There is slight evidence of a maximum of damage in the winter of 0·37 (3·0 per cent.) followed by a minimum damage of 0·24 (2·0 per cent.) at the end of March; this feature becomes marked in plots in which the nitrogen limitation is less acute, and is perhaps an indication that this condition is less extreme in the unmanured plot than in plot 5. The maximum damage in early summer is 0·58 (4·7 per cent.); little importance can be attached to the positive values of the last 12 days of August, save as in indication that August rain is inoperative after the first week of the month.

Type II.—Plot 6, complete minerals with single dressing of ammonium salts applied half in Autumn and half in Spring; plot 17 and 18 mineral series, receiving mineral manures only, but alternating with the ammonium series which receives a double dressing of ammonium salts. Fig. 5 shows plot 6.

The similarity of the curves for these two plots is striking, and requires that the interpretation put on the yields of the alternating plot should be reconsidered. Plot 6 has a mean yield of 22·58 bushels to the acre, and shows a relatively rapid deterioration; 17 and 18 minerals has a mean yield 14·51, scarcely more than that of plot 5 which receives the same dressing; its average deterioration also resembles plot 5. Hence it has been thought that no appreciable benefit accrued to the alternating series from the previous year's dressing of ammonium salts. It has however been observed (5, 1921) that the alternating series is much more variable in yield than is plot 5, and this suggests that the additional variation is due to variable residue of nitrogenous material. This suggestion is strongly confirmed by the resemblance to plot 6, especially in the effects of winter rainfall.

Plot 6 has a strongly marked minimum of damage in early October, when the loss is only 0·10 bushel per inch of rain, or 0·4 per cent.; this is followed by a winter period in which rain is on the average particularly harmful to the extent of 1·14 (5·0 per cent.); from April to July the average damage is nearly constant at about 0·64 (2·8 per cent.).

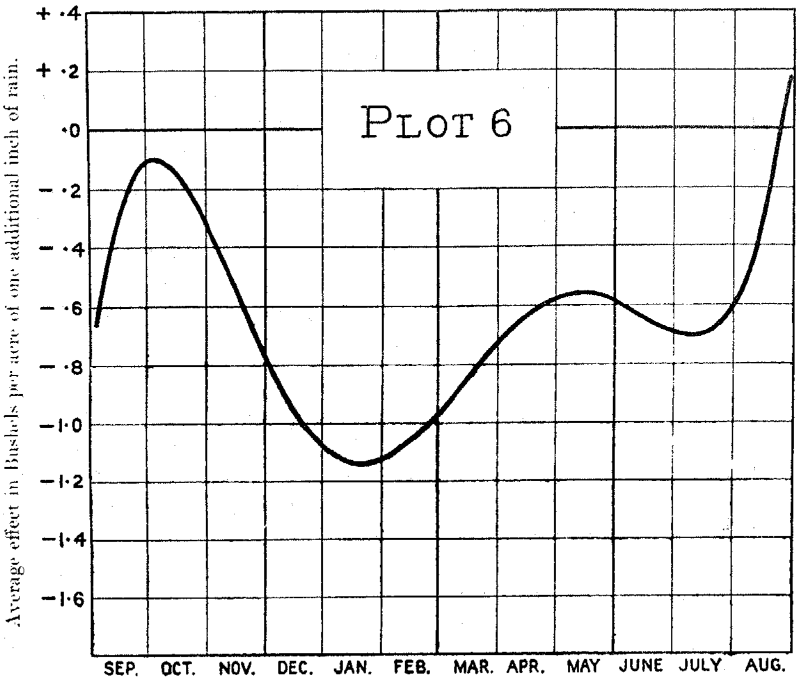


Fig. 5.

The effects on 17 and 18 minerals are, on the whole, smaller, being approximately proportional to the lower yield. A minimum of damage occurs in October 0·10 (0·7 per cent.), followed by a winter maximum 0·64 (4·4 per cent.), and by a nearly stationary period from April to July at 0·42 (2·9 per cent.). The winter effect is smaller both absolutely and relatively than that of plot 6, but if it be admitted that the importance of winter rain lies largely in the washing out of nitrates, the evidence for considerable, though no doubt variable, residues in the alternating series is unmistakable. The fact that the alternating series has not given an appreciable higher yield than that of plot 5, must be ascribed to soil heterogeneity, which has approximately balanced the advantage of the residual nitrogen. That such soil heterogeneity exists in this field may be shown by a comparison of 2*a* with the adjacent plot 2*b*. These have received the same manure since 1885, but during the present century 2*a* has ceased to gain in yield upon 2*b*, but yields very regularly on the average 2 bushels less than the adjacent plot. In view of this fact we cannot deny the possibility that with equal manuring the land of plots 17 and 18 would yield 2 or more bushels less than that of plot 5, and that this circumstance has served to mask the advantage accruing to the mineral series owing to residual nitrogen. It may be mentioned in addition that the mean yield of 17 and 18 ammonium series is 2·36 less than that of plot 7, a difference which may be ascribed to the combined effect of soil inferiority and lack of residual nitrogen; it should be borne in mind that a difference of this amount in a yield of 30 bushels probably would require the addition of considerably more nitrogen than an equal increase in a yield of 14 bushels.

Type III.—Plot 7, complete minerals, with double dressing of ammonium salts, of which one quarter is applied in autumn.

Plot 13, as plot 7 without sulphates of sodium and magnesium.

Plot 17 and 18 ammonium series.

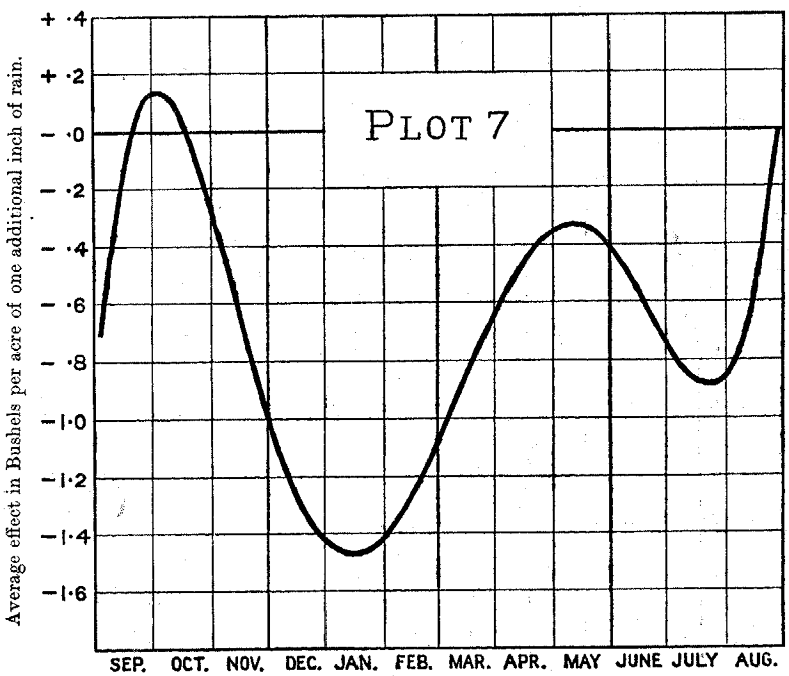


Fig. 6.

These three plots receive similar manurial treatment; the mean yields are 31·37, 30·21 and 29·01 bushels per acre, with mean annual decrements 0·144, 0·123 and 0·114. It might have been anticipated that the decrement should be least on the alternating plot; for the rest it is not obvious to what circumstances the small difference between the plots are to be ascribed.

The three rainfall curves are generally similar, all indicate a slight benefit for something over a month in autumn, the maxima being + 0·14 (0·4 per cent.), + 0·43 (1·4 per cent.), and + 0·41 (1·4 per cent.); the winter damage is strongly marked with values 1·47 (4·7 per cent.), 1·82 (6·0 per cent.) and 1·85 (6·4 per cent.). There is in all cases a clear period of minimum damage in May, the values falling to 0·34 (1·1 per cent.), 0·34 (1·1 per cent.) and 0·20 (0·7 per cent.), followed by a second period of maximum damage in July which is however less severe than the winter maximum, the values are

0·89 (2·8 per cent.), 0·79 (2·6 per cent.) and 0·94 (3·2 per cent.). On the average plot 13 suffers somewhat more heavily from rain than do plots 7 and the ammonium series, but in considering the effects at different times of the year it is seen to agree with the latter closely during the first half of the year, and more nearly with plot 7 during the second half.

Type IV.—Plot 8 as plot 7 with additional ammonium salts applied in the spring.

Plot 12 as plot 13 with substitution of sodium for potassium sulphate.

Plot 14 as plot 13 with substitution of magnesium for potassium sulphate.

Fig. 7 shows plot 8.

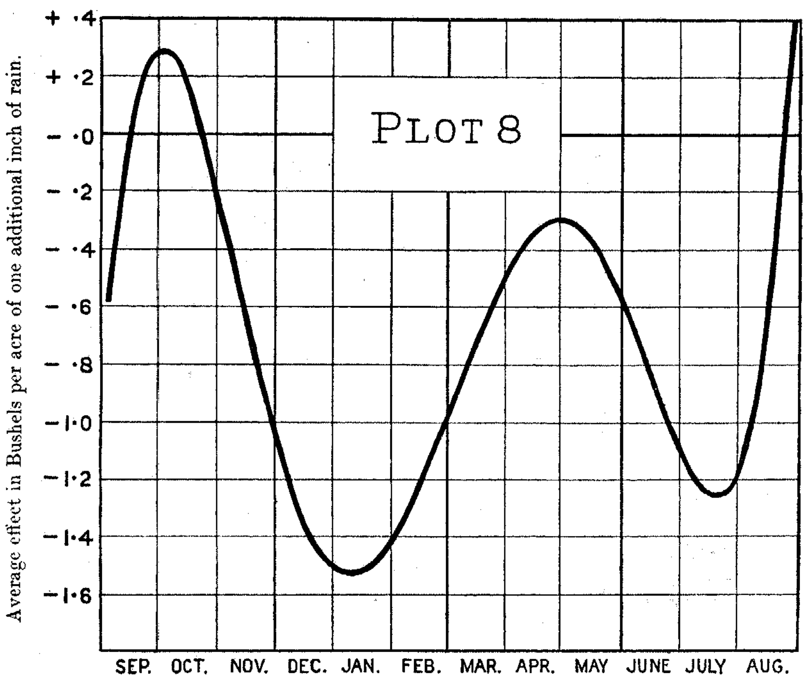


Fig. 7.

In most respects plot 8 contrasts strongly with plots 12 and 14, which however it resembles in its response to rainfall. The mean yield is 35·69 against 28·32 and 27·77, while it shows but slight deterioration, 0·092 annually, as against the heavy deterioration, 0·181 of plot 12, and 0·231 of plot 14. The resemblance consists in the greater relative abundance of nitrogen supplied to these plots than to those in Type III, and the correspondingly less relative abundance of nitrogen than those in Type V. In the case of plot 8, although the yield is greater than that of plot 7, yet since it receives a triple in place of a double dressing of ammonium salts, it is clear that this plot must less frequently suffer from lack of nitrates. Plots 12 and 14 on the other hand receive the same nitrogenous dressing as plot 7, but owing to lack of potash yield some 3 bushels an acre less ;

consequently with them also crop limitation through lack of nitrates must be less frequent. The same argument applies still more strongly to plots 10 and 11 in the next type.

The benefit of October rain is relatively large in plots 8 and 14, and plot 12 also reaches positive values at this period, the three values are + 0·29 (0·8 per cent.), +0·10 (0·3 per cent.) and + 0·36 (1·3 per cent.); the winter damage is strongly marked, though on the whole less so than in the preceding group, 1·53 (4·3 per cent.), 1·65 (5·8 per cent.), 1·52 (5·5 per cent.). The chief contrast lies in the deepening of the second period of maximum damage; in consequence of this the time of minimum summer damage is shifted back to the beginning of May with values 0·30 (0·8 per cent.), 0·49 (1·7 per cent.) and 0·23 (0·8 per cent.); while the second maximum reaches in late July the values 1·27 (3·5 per cent.), 1·48 (5·2 per cent.), 1·23 (4·4 per cent.), showing that dry weather at this period is to these plots nearly as beneficial as in January.

Type V.—Plot 10 double dressing ammonium salts only.

Plot 11 double dressing ammonium salts with superphosphate. Fig. 8 shows plots 10 and 2B.

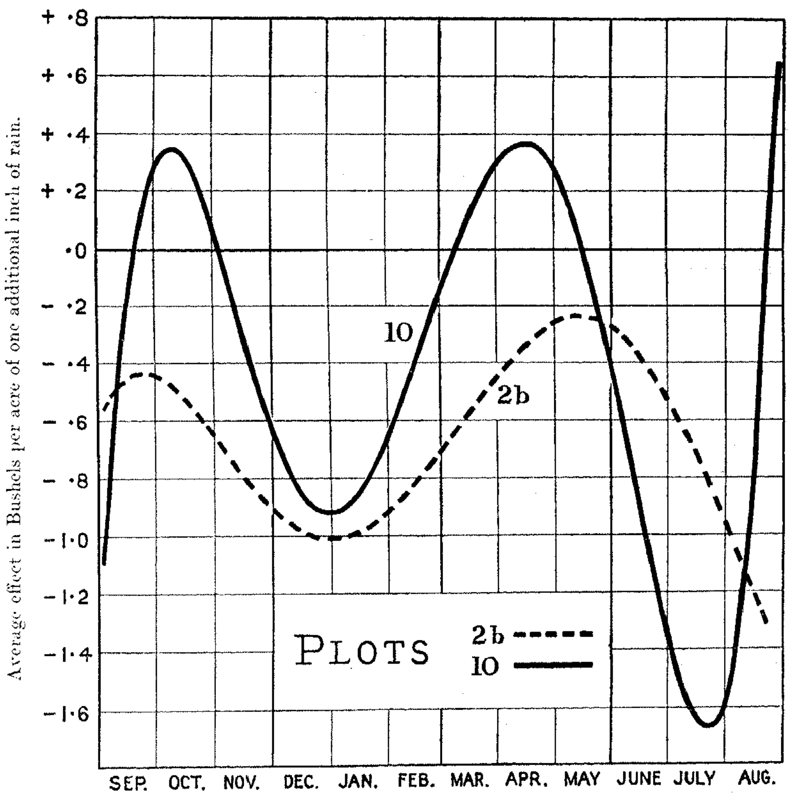


Fig. 8.

In these two plots the yield is very seriously lowered by lack of potash, and in the case of plot 10 by lack of phosphate also. The mean yields are 19·50 and 22·05, with heavy annual deterioration 0·157 and 0·219. Plot 10 with the lower yield is, as might be expected, the more extreme example of the type.

Both curves are positive in October with values + 0·35 (1·8 per cent.) and + 0·16 (0·7 per cent.); the winter damage is less marked in absolute value than in the types III and IV., being only 0·92 (4·7 per cent.) and 0·85 (3·8 per cent.); in April positive values are reached in both cases, those of plot 10 being even higher than in October, + 0·37 (1·9 per cent.) and + 0·03 (0·1 per cent.). The July damage is more extreme than in preceding types being considerably greater than that in January, namely, 1·67 (8·5 per cent.) and 1·43 (6·5 per cent.). In both the April and the July values plot 10 contrasts more strongly than plot 11 with the plots of type IV.

Type VI.—The dunged plot 2B (fig. 8).

This plot differs from all the others in the texture of its soil, and to this perhaps may be ascribed the absence of strong contrasts in the effects of rain at different seasons; the average advantage of dry weather is comparable with the other plots, but the only marked feature of the curve is the severe damage caused by rain immediately preceding harvest. This feature may perhaps be analogous to the heavy damage of July rain in plots 10 and 11, which it should be noted resemble the dunged plot in their relative abundance of nitrogen. Plot 2B differs strikingly from all the other plots, and especially from plots 10 and 11 in the relative constancy of its yield.

The mean yield is 34·55 bushels per acre. The autumn minimum of rain damage is 0·43 (1·3 per cent.), the winter maximum 1·00 (2·9 per cent.), the summer minimum is well marked and prolonged at 0·24 (0·7 per cent.), after which the curve falls to a final value of 1·39 (4·0 per cent.); as we have seen, the mean damage for harvest rain which catches the crop is no doubt greater.

10. *The Value of Rainfall Regressions as Prediction Formulæ.*

The extent to which the variation in crop yield is predicable from a rainfall record may be calculated from the coefficients of multiple regression of the last section. As we have seen in section 2 the coefficient of multiple correlation gives a much exaggerated notion of the prediction value of a regression formula, if calculated from a small sample. In the present instance, although our series of values is a long one, the number of degrees of freedom has been whittled down to 54 by the rejection of unsuitable years, and the elimination of slow changes; six meteorological variates have been used, which represents the utmost economy in view of the complexity of the meteorological sequences. The distribution of the variance, between the 6 degrees of freedom of the regression formula, and the 48 degrees of freedom in which the variates may differ from the regression formula is shown in Table XIV, in which are also shown the multiple correlation R, and the percentage of variance (A_1 of section 2) ascribable to the average effect of rain.

TABLE XIV.

Plot.	Sum of squares. Regression formula 6 degrees.	Sum of squares. Deviations 48 degrees.	Total 54 degrees.	R.	A_1 per cent.
2B	884	996	1,880	0·6859	40·42
3 & 4	149	336	485	0·5548	22·13
5	163	401	564	0·5370	19·93
6	563	1,062	1,625	0·5885	26·46
7	790	2,091	2,881	0·5236	18·34
8	959	2,083	3,042	0·5614	22·96
10	796	1,827	2,623	0·5509	21·63
11	668	2,501	3,169	0·4592	11·23
12	1,395	2,069	3,464	0·6345	32·79
13	985	1,938	2,923	0·5803	25·39
14	938	2,069	3,007	0·5584	22·58
17 & 18 M.	199	686	885	0·4736	12·73
17 & 18 A.	965	1,189	2,154	0·6693	37·90

It will be seen how very inadequate is the value of R to indicate the value of prediction formula; the extreme values of R in the above totals are 0·459 and 0·686, but these values indicate in this case that in one plot 11 per cent., and in the other 40 per cent. of the variance, is expressible in terms of the sequence of rain records.

It is remarkable that so much of the variance as 40 per cent. should be expressible in terms of a single meteorological element such as rainfall, especially when it is remembered that all causes of variation without exception, including casual errors, and the quadratic terms of the rainfall effect, are included in the remaining 60 per cent. This leads us to think that a record of rainfall, in spite of the many disabilities which have been urged against it, is of more value than the record of any other single element, in characterising the season.

The effects ascribed to rain are in most plots clearly significant; the values of P, calculated from the formula

$$(1 - R^2)^{24} (1 + 24R^2 + 300R^4)$$

range from 0·0000186 to 0·0659.

The excessive variation of plot 11 thus masks the rainfall effect in the same manner that it masks the slow changes in this plot; the rainfall effect is somewhat the more important and shows up more clearly. In both cases the similarity of the curves of plot 11 to those of other plots, especially those which it would be expected most nearly to resemble, shows that no serious deviations have been introduced into these by random fluctuations.

The probable errors of random sampling of the regressions of crop on rainfall may be calculated as demonstrated by the author in 1922 (9). The number of years is sufficient to ensure the effective normality of the distributions. In the comparison of the regression, of any two plots, we may anticipate that the random errors are on a substantially smaller scale, since the experience of all the plots is drawn from an identical series of seasons.

No statistical estimate of the accuracy of curves for purposes of comparison can be made in the absence of strictly parallel plots. It may, however, be confidently anticipated that random errors of this kind are, if anything, of a smaller order than the differences between the curves obtained for plots 7, 13 and (17 and 18) ammonium series.

Reality of Slow Changes.

The prediction formulæ which we have obtained are entirely independent of the slow changes which appear to have taken place in yield and in weather. In presenting the evidence for slow changes in yield (5) it was presumed, in the absence of a full investigation of the sequence of rainfall records, that favourable and unfavourable weather conditions fluctuated independently from year to year; in fact, that slow changes were absent from the meteorological series. We now know that a small percentage of the variance of annual rainfall should probably be ascribed to slow changes, and that sequences of wet years did in fact occur about 1879 and 1914. Since these two periods agree with the two main depressions in yield, it becomes at first sight questionable whether the slow changes in yield may not after all be ascribable to meteorological effects.

This question is most readily answered by calculating the actual depression in yield ascribable to the additional rainfall in the two rainy periods. Plot 2B is most suitable for a comparison for the residual variance is, in comparison to the yield, very small in this plot, and the prediction formula correspondingly accurate. The yield in any year is regarded as made of two parts, part (positive or negative) is due to the rainfall sequence under which the crop was grown, the remainder is the yield handicapped for the advantages or disadvantages ascribable to rainfall. The sequence of 10-year means of these two quantities is shown in fig. 9. It will be seen that the yield, after making allowance for rainfall, still shows strongly the slow changes which originally attracted attention. The changes in weather only account directly for a portion of the slow changes observed; in the first half of the sequence this portion is roughly one-half, but in the latter half it would appear not to exceed one-quarter of the total effect.

The circumstance that in two cases the depression of yield, after allowing for rainfall, coincides with a series of years in which the rainfall was on the average unfavourable, is in accordance with the suggestion previously put forward, that in addition to the immediate effect of rainy weather on the growing crop, a sequence of wet years produced an additional and prolonged unfavourable influence by fostering weeds. Two cases of agreement are, however, quite insufficient to prove that the additional depression of yields is causally connected with the rainy periods. The fact of the coincidence, whether causal or fortuitous, does, however, emphasise the importance of eliminating slow changes in the study of annual figures; if slow changes had been ignored in carrying through the correlational work, we should no doubt have arrived at greatly exaggerated estimates of the harmful effect of rain. Higher correlations would have been obtained and the results would have exhibited one more case of high but essentially unreliable correlations in meteorological agriculture. Against the view that the depression in yields is an indirect effect of a rainy period must be set the fact that in our diagrams the yield appears to fall off somewhat *before* the wet sequence has set in ; both series being 10-year means the slight falsification of the sequence incurred by using this form of representation should have a similar effect in both series.

Effect of Fallow.

Certain years were rejected owing to the crops grown having followed a fallow ; it was suspected that the yields obtained in these years were unduly high, and would vitiate

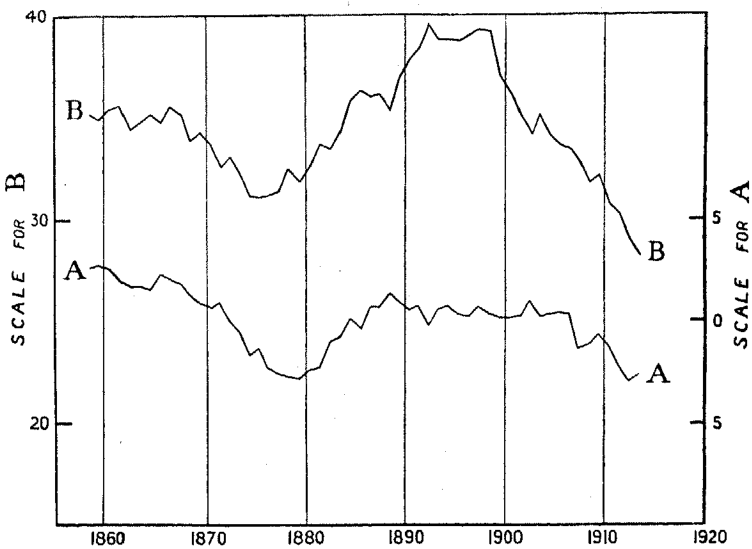


Fig. 9.—Graph A shows the variation in the expectation of wheat yield as judged from the quantity and date of the rain during the year. Graph B shows the additional variation of the actual yields observed after allowance for the expected rainfall effect. Both graphs show 10-year means, in bushels per acre.

the weather correlations if they were retained. A knowledge of the expected effect of weather, together with that of the general fertility of the plot in neighbouring years, makes it possible to estimate the amount by which the fallow benefited the crop. On plot 2B the excess of yield over expectation in 1890, 1891, was 4·48, 6·77 ; in 1905, 1906, the excess values were 6·98, 5·95, while in 1915 the excess was nearly 15·50. In 1916 the two halves of the field were harvested separately, and the half that followed fallow showed an excess of 14·30 bushels over the other. This suggests that the value for 1915 is not so inaccurate as might be thought.

In all these 6 cases the effect of fallow is strongly marked ; the average gain is the very large amount of 9 bushels per acre. It should be remembered that this plot receives a heavy dressing of farmyard manure, and it is doubtful if any part of the gain can be ascribed to the accumulation of plant nutrients. The immediate object of fallowing was in all cases the eradication of weeds, and it may be that the increased yield was mainly due to a greater freedom of the land from weed infestation. If that is so the effect must have been remarkably transient, for the value for 1916 depends on a comparison of the crop immediately following fallow with one which had been fallowed only two years before. The supposition that the benefit was largely a matter of freedom from weeds accords with the higher values of 1915, 1916, when the field, to judge by the average yields on all plots, was much more severely infested than in 1890 or 1905.

The magnitude of the effect of fallow illustrates the complications which arise in the interpretation and statistical reduction of even the best agricultural data. We have attempted to eliminate the large changes which have occurred in the yields of these plots by means of continuous curves, but there is reason to fear that even if the fertility of the soil normally suffers continuous changes from year to year, yet discontinuities were introduced at the periods when fallowing was resorted to. By rejecting the crops immediately concerned the greater part of the errors have no doubt been eliminated, yet the theoretical efficacy of the continuous curve is impaired, however nearly it appears in practice to represent the actual course of events.

11. *Comparison with Previous Results.*

In 1880 LAWES and GILBERT published (12) a long and careful account of the meteorological characteristics of seasons favourable and unfavourable to wheat. The object was to effect a qualitative and preliminary enquiry (p. 174): "As yet, however, the connection between meteorological phenomena and the progress of vegetation is not so clearly comprehended as to enable us to estimate with any accuracy the yield of a crop by studying the statistics of the weather during the period of its growth. . . . . But it is only by a careful comparison of the characters of the seasons on the one hand, and of the quantity and quality of the produce on the other, for many years, that we can hope to acquire sufficient knowledge to enable us to assign to the various agencies, the sum of which constitutes the climate of the year, their respective values in the production of the crop."

In respect of rainfall, LAWES and GILBERT, using principally Rothamsted data, but also some instances of years of exceptional harvests prior to their experiments, concluded that comparative dryness was desirable from Seed-time (November) to harvest, especially in Winter and early Spring. Their table of averages shows no exceptional dryness of October for the favourable years, or exceptional wetness in the unfavourable years. The effect of winter and spring rain is ascribed partly to drainage causing loss of nitrates, and partly to hindering root development.

It appears that in the points upon which they laid most stress the conclusions arrived at by the more exact statistical methods now available and with the aid of 39 more seasons' experience at their station, would have caused LAWES and GILBERT only to reaffirm their conclusions more strongly and with greater precision. The comparison of different plots has emphasised the importance of the drainage of nitrates, and points to further influence of manurial conditions on the response of the crop to July rain. The cause of this is at present obscure, but the effect seems well marked. In addition it has been possible to make some attempt to develop formulæ which shall predict the yield from the weather statistics.

Attention was called to the possibility that autumn rainfall was an important factor in determining the wheat crop by Sir NAPIER SHAW in 1905 (14). SHAW did not use the method of correlation, but pointed out that in the twenty years 1885–1904, with two exceptions, when the yield for Eastern England was above the average the previous autumn rainfall was below the average and *vice versa*. In this set of 20 pairs of values the correlation is actually $-0 \cdot 629$, a value which would only be exceeded by uncorrelated variates in 3 samples of 20 out of a thousand. The particular meteorological variate, autumn rainfall, was picked out of a table giving at least 36 meteorological quantities, and the chance that all of these, if independent and in reality uncorrelated with the crop, should give correlations between $\pm 0 \cdot 629$, was therefore about $0 \cdot 89$. Such a system would therefore be expected to yield so high a correlation only once in 9 trials, and the fact that such a correlation occurred supplied some presumption that autumn rainfall had in reality a perceptible influence on the crop.

In discussing the significance of this result SHAW (14, pp. 318–319) made the interesting suggestion that the association observed might not be due wholly to the effects of autumn weather, but possibly to dry autumns being frequently followed by a favourable succession of weather in Spring and Summer.

> "He would like to say a word with respect of Mr. THOMAS's suggestion that the autumn rainfall was not the dominant factor in determining the subsequent yield of wheat. What surprised him were not the exceptions, but the agreements. Remembering that nine months had to run between the end of autumn and the beginning of harvest, and considering the influence of the intervening rainfall, sunshine and other accidents that might happen to the crop before it was gathered, it was surprising that the connection should be so close as to be expressed possibly numerically. It might be true that two other columns of figures might be tabulated which would show a closer agreement than the two columns put down in the table, but they certainly would not be columns of figures for individual elements. To take two elements out of the whole table, put them side by side, and find them to agree as they did in this case, was astonishing. Of course other influences affected wheat, and it might be that a sequence of influences was required to follow the autumn rainfall in order to bring out the corresponding result. It might be that the relation was a meteorological one, and that a dry autumn itself implied a dry spring or a dry summer, or whatever combination of circumstances was required for a good yield. He could give a certain amount of evidence in favour of the contention that a meteorological relation existed, and that it was not what took place in the autumn alone which might account for the relation, but succeeding events as well, which were associated with a dry autumn. That contingency made the subject one of considerable interest, and one which must be pursued rather more fully than was possible on the present occasion."

A great advance in method is shown by the magnificent paper by HOOKER (10) in 1907. HOOKER systematically correlated rain and accumulated temperature for 8-week periods throughout the year with the yields of a number of farm crops in Eastern England, using a slightly different area from that used by SHAW. HOOKER clearly recognised the two limitations from which the method of correlation suffers, namely that it takes account only of the *linear* relation of the variates, and secondly that the correlations obtained will be much affected by any such meteorological associations between the weather at different times of the year as were suggested by SHAW. It was to remove these two limitations from the treatment of the Rothamsted data that the present method of computing the partial regressions of the yield on the weather at each period of the year was devised, a method which as we have seen can be extended to the discussion of the quadratic terms of the regression function.

HOOKER found high negative correlations between the wheat yield and the rainfall for periods centred in October and January, while between them small positive correlations occurred. In May again positive correlations, this time somewhat larger, made their appearance. It is concluded that a dry September-October ranks first among the wheat's requirements, while the effect of winter rain is ascribed partially to the high correlation (+ 0·6) found between the rainfall of weeks 1–8 and that of the preceding weeks 37–44.

In 1922 HOOKER (11) gave recalculated figures for the same variates ; the results being now based on 35 years, possess considerable significance. Diagram 10 shows the correla-

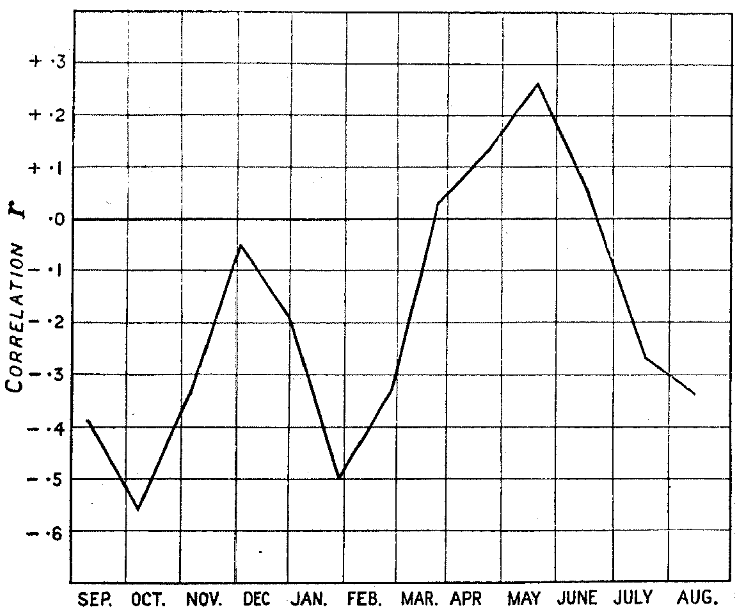


Fig. 10.—Rainfall correlations obtained by HOOKER from official returns of wheat yields for Eastern England, using rainfall for 8-week periods centred at the dates shown.

tions obtained for successive (overlapping) 8-week periods, in respect of which HOOKER draws the following conclusions (p. 120) :—

> "Looking first at the curve showing the connection between rain and wheat, the most striking features are the dips about October and in winter, both very nearly identical. As a matter of fact the winter dip is slightly greater than the other. But no weight can be attached to so slight a difference, and we must apparently regard a dry seed-time and a dry subsequent winter as about of equal importance. Sir NAPIER SHAW appears to have been the first to call attention to the great advantage of a dry autumn, and my calculations fifteen years ago pointed to this period being more important than the winter. It is not very clear from the writings of GILBERT and LAWES how far they realised this feature of the wheat's requirements ; it would seem that they were aware of it, but they lay all stress upon the winter rainfall and the washing of the nitrates out of the soil. Wheat is a deep-rooting plant, and I believe it to be probable that, if experiments could be devised to test the point, it would be found that the real effect of a very wet autumn is mechanical—that it is then more the clogging of the soil that prevents the proper development of the root system than the absence of sufficient nitrates. The effect of the loss of the latter is probably more felt later on, during the winter, after the plant is established, and this is reflected in the heavy negative coefficient with rain during the winter months. GILBERT and LAWES have laid so much emphasis upon the washing out of the nitrates that it is generally overlooked that they did in fact recognise the hindrance to root development caused by saturated soil. It seems reasonable to conclude from the data that the mechanical effect of the autumn is just as important as the chemical effect of the winter ; and it is, moreover, one that is practically irremediable."

The statement that the winter dip is slightly greater refers to the partial correlations obtained after eliminating the effects of temperature ; we are here only concerned with the total effects, and it is apparent that HOOKER's data contain substantial evidence for a real deleterious effect of Autumn rainfall. In other respects the series of correlations is not out of harmony with several of the Broadbalk plots, the characteristic period of winter damage is clearly recognisable in both, and we may identify the positive correlations obtained in late Spring with the minimum of damage observed on several of our plots at the same season. The July values also are suggestive of the more highly nitrogenous plots on Broadbalk. The fact that HOOKER's series appears to incline on the whole to more positive values than are observed in our regressions is not improbably due to the inclusion in his data of yields from lighter lands, more susceptible to summer droughts than is the heavy loam of Broadbalk ; in addition his mean rainfall is some four inches less than in our series.

The earlier half of the series, in which the Broadbalk plots are most alike, is alone in contradiction to HOOKER's series. Of course, if strong meteorological correlations existed between the several parts of the year, no resemblance need be expected between the actual regressions and the correlations which are, in a complicated manner, compounded from them; and even slight meteorological correlations might seriously disturb the numerical values of the correlations with the crop; but our meteorological data show that such correlations between weather at different periods of the year, though probably present, are quite small; and since the agreement, in its general qualitative outline, has not been disturbed from January onwards, it is probable that other factors contribute to the striking difference observable in the effects of October rain. In particular it may be noted that HOOKER's values at this period, allowing for the fact that adjacent points are not independent, but have four identical weeks in common, show somewhat abrupt changes; the values found for November-December agree well with our maximum in October, and the damage ascribed by HOOKER to "seed-time" may be paralleled by the somewhat increased damage shown in many of our curves in September. In this connection an investigation is desirable of the accuracy with which HOOKER's process of correlating with sets of 8-consecutive weeks, is able to indicate the true maxima or minima of the correlation curve; and to what errors such estimates are exposed owing to the capricious incidence of heavy rain.

A consideration which more probably contains a solution of the discrepancy is that under industrial farming conditions rain in late autumn and early winter prevents the sowing of large areas of wheat, the land being sown in spring with oats or barley. This is a factor which must strongly influence HOOKER's results, whereas under experimental farming conditions it is inoperative. HOOKER mentions that the wheat area has a correlation $-0 \cdot 41$ with rain of weeks 37–44, but does not mention the correlation of neighbouring periods; these it would be necessary to know in tracing out the influence of the variable wheat areas on the yield.

It would seem probable that the proportion of land lost to wheat in this way should differ from district to district, and even more probable that such loss should not be proportionately distributed between the two main classes of wheat land in which the wheat follows respectively clover and roots. There are thus many points at which the effect of autumn rainfall upon cropping might give rise to an apparent influence on yield.

It is admittedly by no means necessary that the response of crop to weather should be the same on one particular type of land as it is on the average wheat land of several counties; the curve of regression may be as much influenced by soil type, as we now know it to be by the manurial condition of the soil. In this respect data of crop averages grown under industrial conditions would be of the greatest value in supplementing by shorter series over a more extensive area the long series from a single field which the Broadbalk data provide. It is to be feared, however, that the Ministry of Agriculture's returns are not sufficiently accurate to meet this need, being based principally upon eye estimates of yield before harvest. Such estimates may be closely correlated with the true yields, and yet fail to give the values of the required regressions even with approximate accuracy; for there is reason to fear that the deviations from the known mean yield are systematically, though of course unconsciously, underestimated. In inquiries respecting the weather it would be important to ascertain also that the known meteorological peculiarities of the year under review, the supposed effects of which upon the crops may have been discussed in the press, are without systematic influence upon the judgments of reporters.

12. *Summary.*

The study of the theoretical distributions of statistics emphasises the dangers of applying the methods of multiple correlation to small samples, and the necessity of extensive crop data in the study of meteorological agriculture.

By a special procedure involving the analysis of separate meteorological sequences it is possible to obtain an adequate mathematical expression of the average effects of the meteorological influences indicated by different instrumental observations at different times of the year.

The errors involved in the correlation of residuals of series changing in an unknown manner may be minimised by the method of polynomial fitting; such errors are probably insignificant when this procedure is applied to rain data and wheat yields.

The rain data for Rothamsted have been analysed for 65 years; there are some indications that the wet years tend to occur in spells; a continuous and progressive change is observable in the distribution of rain through the year; in other respects the sequence appears to be fortuitous. Rainfall changes account for only a portion of the slow changes observed in the yields.

Curves showing the average effect on the yield, for each additional inch of rain, throughout the year, have been obtained for 13 plots of Broadbalk wheat field, which have been under uniform experimental treatment since 1852. On all the plots dry weather is generally beneficial. A detailed comparison of the several plots indicates a predominant influence of the effect of rain in removing soil nitrates; the cause of other well-marked features cannot safely be asserted without further research, which it is hoped may be facilitated by the body of facts expressed in these curves.

Previous investigations bearing particularly on the present data are briefly discussed in the last section.

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