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R. A. FISHER

Expansion of "Student's„ integral in powers of n^{-1}.

The exact distribution of z found by « Student » in 1908, may be written

$$df = \frac{\frac{n-1}{2}!}{\frac{n-2}{2}!\sqrt{\pi n}} \left(1 + \frac{t^2}{n}\right)^{-\frac{1}{2}(n+1)} dt \qquad \text{I}$$

Clearly, as n tends to ∞, df will tend to its normal value

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2} dt, \qquad \text{II}$$

and the approach to normality may be expressed by expanding (I) in inverse powers of n.

Now

$$-\log\left(1 + \frac{t^2}{n}\right) = \frac{(-t^2)}{n} + \ldots\ldots\ldots + \frac{(-t^2)^p}{pn^p} + \ldots\ldots$$

hence

$$-\frac{1}{2}\left(n+1\right)\log\left(1 + \frac{t^2}{n}\right) = -\frac{1}{2}t^2 + \ldots\ldots\ldots$$

$$\ldots\ldots + \frac{p(-t^2)^{p+1} + (p+1)(-t^2)^p}{2p(p+1)n^p} + \ldots\ldots,$$

moreover

$$\log\left(\frac{\frac{n-1}{2}!}{\frac{n-2}{2}!} \cdot \sqrt{\frac{2}{n}}\right) = -\frac{1}{4n} + \frac{1}{24n^3} - \frac{1}{20n^5} + \ldots\ldots,$$

Metron, 5: 109-120, (1925).

consequently, the logarithm of the ratio which the exact formula (I) bears to its limiting form (II) may be written as,

$$\frac{t^4-2t^2-1}{4n}-\frac{2t^6-3t^4}{12n^2}+\frac{3t^8-4t^6+1}{24n^3}-\frac{4t^{10}-5t^8}{40n^4}$$
$$+\frac{5t^{12}-6t^{10}-3}{60n^5}-\ldots\ldots \qquad \text{III}$$

The exponential expansion of this series will give the expansion of the actual differences of the ordinates, in powers of n^{-1}. Each term contains a polynomial in t^2, and the factor z, standing for the differential coefficient in II.

The first five terms are

$$\left.\begin{array}{l}
(t^4-2t^2-1)\ \frac{z}{4n}\\
(3t^8-28t^6+80t^4+12t^2+3)\ \frac{z}{96n^2}\\
(t^{12}-22t^{10}+113t^8-92t^6-33t^4-6t^2+15)\ \frac{z}{384n^3}\\
(15t^{16}-600t^{14}+7100t^{12}-26616t^{10}+18330t^8+6860t^6+1980t^4-1800t^2-945)\ \frac{z}{92160n^4}\\
(3t^{20}-190t^{18}+4025t^{16}-38976t^{14}+108702t^{12}-63444t^{10}-21270t^8-7800t^6+4455t^4+1890t^2-17955)\ \frac{z}{368640n^5}
\end{array}\right\}\ \text{IV}$$

These polynomials are of some interest in themselves, but for the present we require the integral of each term from t to ∞; these are as follows,

$$\left.\begin{array}{l}
(t^2+1)\ \frac{tz}{4n}\\
(3t^6-7t^4-5t^2-3)\ \frac{tz}{96n^2}\\
(t^{10}-11t^8+14t^6+6t^4-3t^2-15)\ \frac{tz}{384n^3}\\
(15t^{14}-375t^{12}+2225t^{10}-2141t^8-939t^6-213t^4+915t^2+945)\ \frac{tz}{92160n^4}\\
(3t^{18}-133t^{16}+1764t^{14}-7516t^{12}+5994t^{10}+2490t^8+1140t^6+180t^4+5855t^2+17955)\ \frac{tz}{368640n^5}
\end{array}\right\}\ \text{V}$$

The first four of these will be needed in the practical computation of p for values of n exceeding 20; the coefficients of the negative powers of n are given in Table III. For the fifth correction we need only consider the maximum values. These may be found from the zeros of the polynomial in powers of t^2, given in IV; the polynomial is of the 10*th.* degree in t^2, and has five positive roots. For each of these the following Table shows (*i*) the value of t^2, (*ii*) the value of t, (*iii*) the coefficient of n^5 in V (*iv*) the value of n for which the contribution of this term to p is $\pm$ .00001.

(*i*) t^2	(*ii*) t	(*iii*) C_5	(*iv*) n
1.3133	1.1460	+ .019864	4.567
4.7330	2.1755	— 1.0818	10.158
10.0555	3.1710	+ 10.3695	15.964
17.8547	4.2255	— 19.6603	18.143
29.7308	5.4526	+ 9.1438	15.568

The largest maximum is at $t = 4.2255$; here the value of n, under (*iv*), rises to 18.143. At this point the fifth correction becomes substantial for values of n but little below 20; at $n = 21$, however, the correction is less than 4 in the 6 th. place of decimals. We may conclude that, for values of n exceeding 20, the error involved in neglecting corrections beyond the fourth will not exceed .000005.

Over considerable parts of this region fewer corrections than four will suffice. The diagram shows at a glance how many terms will be needed at each point. The curves pass through those points at which the corresponding correction is $\pm$ ·00001. Only the maxima are shown for the fifth correction. Thus outside the curve II, the first correction alone will suffice, provided we are willing to accept an error of unity in the 5th. place; between the curves II and III, two correction terms should be used, and so on. It will be seen that only in two small regions will the fourth correction be necessary, for values of n exceeding 20, where no more than the minimal accuracy is aimed at; in other regions the expansion is often useful down to the fourth term, if more accurate values of p are required.

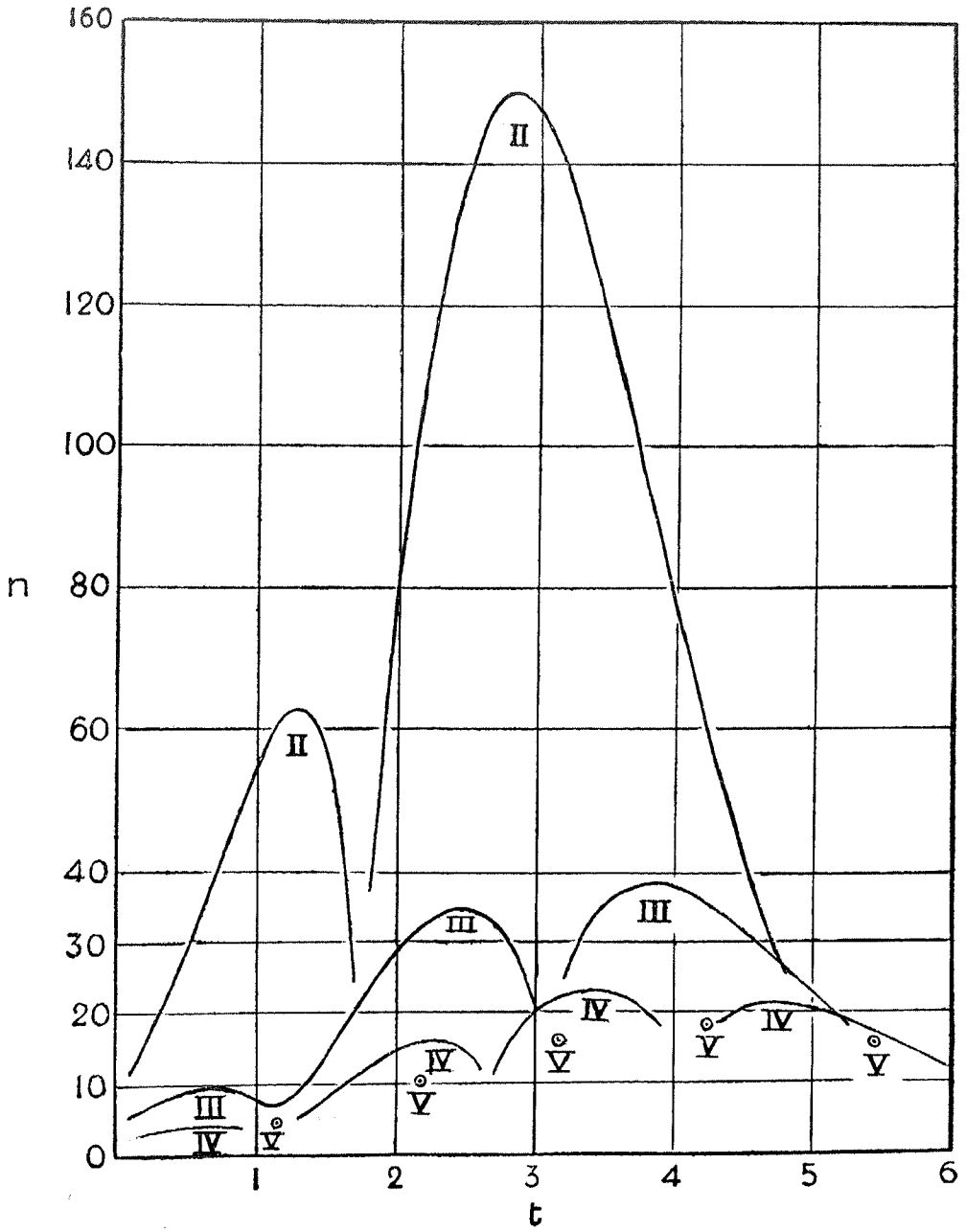
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TABLE I.

t	$n=1$ $n'=2$	2 3	3 4	4 5	5 6	6 7	7 8	8 9	9 10	10 11
0	.500,0	.500,0	.500,0	.500,0	.500,0	.500,0	.500,0	.500,0	.500,0	.500,0
.1	.531,7	.535,3	.536,7	.537,4	.537,9	.538,2	.538,4	.538,6	.538,7	.538,8
.2	.562,8	.570,0	.572,9	.574,4	.575,3	.576,0	.576,4	.576,8	.577,0	.517,3
.3	.592,8	.603,8	.608,1	.610,4	.611,9	.612,9	.613,6	.614,1	.614,5	.614,8
.4	.621,1	.636,1	.642,0	.645,2	.647,2	.648,5	.649,5	.650,2	.650,8	.651,2
.5	.647,6	.666,7	.674,3	.678,3	.680.9	.682,6	.683,8	.684,7	.685,5	.686,1
.6	.672,0	.695,3	.704,6	.709,6	.712,7	.714,8	.716,3	.717,4	.718,3	.719,1
.7	.694,4	.721,8	.732,8	.738,7	.742,4	.744,9	.746,7	.748,1	.749,2	.750, 1
.8	.714,8	.746,2	.758,9	.765,7	.770,0	.772,9	.775,0	.776,6	.777,8	.778, 8
.9	.733,3	.768,4	.782,8	.790,5	.795,3	.798,6	.801,0	.802,8	.804,2	.805,4
1.0	.750,0	.788,7	.804,5	.813,0	.818,4	.822,0	.824,7	.826,7	.828,3	.829,6
1.1	.765,1	.807,0	.824,2	.833,5	.839,3	.843,3	.846,1	.848,3	.850,1	.851,4
1.2	.778,9	.823,5	.841,9	.851,8	.858,1	.862,3	.865,4	.867,8	.869,6	.871,1
1.3	.791,3	.838,4	.857,8	.868,3	.874,8	.879,3	.882,6	.885,1	.887,0	.888,6
1.4	.802,6	.851,8	.872,0	.882,9	.889,8	.894,5	.897,9	.900,5	.902,5	.904,1
1.5	.812,8	.863,8	.884,7	.896,0	.903,0	.907,9	.911,4	.914,0	.916,1	.917,7
1.6	.822,2	.874,6	.896,0	.907,6	.914,8	.919,6	.923,2	.925,9	.928,0	.929,7
1.7	.830,7	.884,4	.906,2	.917,8	.925,1	.930,0	.933,5	.936,2	.938,3	.940,0
1.8	.838,6	.893,2	.915,2	.926,9	.934,1	.939,0	.942,6	.945,2	.947,3	.949,0
1.9	.845,8	.901,1	.923,2	.934,9	.942,1	.946,9	.950,4	.953,0	.955,1	.956,7
2.0	.852,4	.908,2	.930,3	.941,9	.949,0	.953,8	.957,2	.959,7	.961,7	.963,3
2.1	.858,5	.914,7	.936,7	.948,2	.955,1	.959,8	.963,1	.965,5	.967,4	.969,0
2.2	.864,2	.920,6	.942,4	.953,7	.960,5	.964,9	.968,1	.970,5	.972,3	.973,8
2.3	.869.5	.925,9	.947,5	.958,5	.965,1	.969,4	.972,5	.974,8	.976,5	.977, 9
2.4	.874,3	.930,8	.952,1	.962,8	.969,2	.973.4	.976,3	.978,4	.980,1	.981,3
2.5	.878,9	.935,2	.956,1	.966.6	.972,8	.976,7	.979,5	.981,5	.983,1	.984,3
2.6	.883,1	.939,2	.959,8	.970,0	.975.9	.979,7	.982,3	.984,2	.985,6	.986,8
2.7	.887,1	.942,9	.963,1	.973,0	.978,6	.982,2	.984,7	.986,5	.987,8	.988,8
2.8	.890,8	.946,3	.966,1	.975,6	.981,0	.984,4	.986,7	.988.4	.989,6	.990,6
2.9	.894,3	.949,4	.968,7	.977,9	.983,1	.986,3	.988,5	.990,1	.991,2	.992,1
3.0	.897,6	.952,3	.971,2	.980,0	.985,0	.988,0	.990,0	.991,5	.992,5	.993,3
3.1	.900,7	.954,9	.973,4	.981,9	.986,6	.989,4	.991,3	.992,7	.993,6	.994,4
3.2	.903,6	.957,3	.975.3	.983,5	.988,0	.990,7	.992,5	.993,7	.994,6	.995,3
3.3	.906,3	.959,6	.977,1	.985,0	.989,3	.991,8	.993,4	.994,6	.995,4	.996,0
3.4	.908,9	.961,7	.978,8	.986,4	.990,4	.992,8	.994,3	.995,3	.996,1	.996,6

$n = 11$ $n' = 12$	12 13	13 14	14 15	15 16	16 17	17 18	18 19	19 20	20 21	∞
.500,0	.500,0	·500,0	.500,0	.500,0	.500,0	.500,0	.500,0	.500,0	.500,0	.500,000,0
.538,9	.539,0	.539,1	.539.1	.539,2	.539,2	.539,2	.539,3	.539,3	.539,3	.539,827,8
.577,4	.577,6	.577,7	.577,8	.577,9	.578,0	.578,1	.578,1	.578,2	.578,2	.579,259,7
.615,1	.615,3	.615,5	.615,7	.615,9	.616,0	·616,1	.616,2	·616,3	.616,4	·617,911,4
.651,6	.651,9	.652,2	.652,4	.652,6	.652,8	.652,9	.653,1	.653,2	·653,3	.655,421,7
.686,5	.686,9	.687,3	.687,6	.687,8	.688,1	.688,3	.688,4	.688,6	.688,7	.691,462,5
.719,7	.720,2	.720,6	.721,0	.721,3	.721,5	.721,8	.722,0	.722,2	.722,4	.725,746,9
.750,8	.751,4	.751,9	.752,3	.752,7	.753,0	.753,3	.753,6	.753,8	.754,0	.758,036,3
.779, 7	.780,4	.781,0	.781,5	.781,9	.782,3	.782,6	.782,9	.783,2	.783,4	.788,144,6
.806,3	.807,1	.807,8	.808,3	.808,8	.809,3	·809,7	.810,0	.810,3	.810,6	.815,939,9
.830,6	.831,5	.832,2	.832,9	.833,4	.833,9	.834,3	.834,7	.835,1	.835,4	.841,344,7
.852,6	.853,5	854,4	.855,1	.855,7	856,2	.856,7	.857,1	.857,5	.857,8	.864,333,9
.872,3	.873,4	874,2	.875,0	.875,6	.876,2	.876,7	.877,2	.877,6	.877,9	.884,930,3
.889,9	.891,0	.891,9	.892,7	.893,4	.894,0	.894,5	895,0	.895,4	.895,8	.903,199,5
.905,5	.906,6	·907,5	.908,4	.909,1	.909,7	.910,3	.910,7	.911,2	.911,6	.919,243,3
.919,1	.920,3	.921,2	.922,1	.922,8	.923,5	.924,0	.924,5	.925,0	.925,4	.938,192,8
.931,0	.932,2	.933,2	.934,0	.934,8	.935,4	.936,0	.936,5	.937,0	.937,4	.945,200,7
.941,4	.942,6	.943,5	.944,4	.945,1	.945,8	.946,3	.946,8	.947,3	.947,7	.955,434,5
.950,3	.951,5	.952,5	.953,3	.954,0	.954,6	.955,2	.955 7	.956,1	.956,5	.964,069,7
.958,0	.959,1	.960,1	.960,9	.961, 6	.962,2	.962,7	.963,2	.963,6	.964,0	.971,283,4
.964,6	.965,7	·966,6	.967,4	.968,0	.968,6	.969,1	.969,6	.970,0	.970,4	.977,249,9
.970,2	.971,2	.972,1	.972,8	,973,5	.974,0	.974,5	.975,0	.975,3	.975,7	.982,135,6
.975.0	.975,9	.976,8	.977 4	.978,1	.978,6	.979,0	.979,4	.979,8	·980,1	.986,096,6
.979, 0	.979,9	.980,7	.981,3	.981,9	.982,4	.982,8	.983,2	.983,5	.983,8	.989,275,9
.982, 4	.983,2	.984,0	.984,6	.985,1	.985,5	.985,9	986,3	.986,6	.986,9	.991,802,5
.985,2	.986,0	.986,7	.987,3	.987,7	.988,2	.988,5	.988,8	.989,1	.989,4	.993,790,3
.987,7	.988,4	.989,0	.989,5	.990,0	.990,3	.990,7	.990, 9	.991,2	.991,4	.995,338,8
.989,7	·990,3	.990,9	.991,4	.991,8	.992,1	.992,4	.992,7	.992,9	.993,1	.996,533,0
.991,4	.992,0	.992,5	.992,9	.993,3	.993,6	.993,8	.994,1	.994,3	.994,5	.997,444,9
.992,8	.993,3	.993,8	.994,2	.994,5	.994, 8	.995,0	.995,2	.995,4	.995,6	.998,134,2
.994,0	.994,5	.994,9	.995,2	.995,5	.995,8	.996,0	.996,2	.996,3	.996,5	.998,650,1
,994,9	.995,4	.995,8	.996,1	.996,3	.996,6	.996, 8	.996,9	.997,1	.997,2	.999,032,4
.995,8	.996,2	.996,5	.996,8	.997,0	.997,2	.997,4	.997,5	.997,6	.997,8	.999,312,9
.996,5	.996,8	.997,1	.997,4	.997,6	.997,7	.997,9	.998,0	.998,1	.998,2	.999,516,6
.997,0	.997,4	.997,6	.997,8	.998,0	.998,2	.998,3	.998,4	.998,5	.998,6	.999,663,1

TABLE I. (*continued*)

	$n=1$ $n'=2$	2 3	3 4	4 5	5 6	6 7	7 8	8 9	9 10	10 11
3.5	.911,4	963,6	980,3	.987,6	.991,4	.993,6	.995,0	.996,0	.996,6	.997,1
3.6	.913,8	.965,4	.981,6	.988,6	.992,2	.994,3	.995,6	.996,5	.997,1	.997,6
3.7	.916,0	.967,0	.982, 9	.989,6	.993,0	.995,0	.996,2	.997,0	.997,5	.997,9
3.8	.918,1	.968,6	.984,0	.990,4	.993.7	.995,5	.996,6	.997,4	.997,9	,998,3
3.9	.920,1	.970,1	.985,0	.991,2	.994,3	.996,0	.997,1	.997,7	.998,2	.998,5
4,0	.922,0	.971,4	.986,0	.991,9	.994,8	.996,4	.997,4	.998,0	.998,4	998,7
4.1	.923,9	.972,7	.986,9	.992,6	.995,3	.996,8	.997, 7	.998,3	.998,7	.998,9
4.2	.925,6	.973,9	.987,7	.993,2	.995,8	.997,2	.998, 0	.998,5	.998,8	.999,1
4.3	.927,3	.975,0	.988.4	.993,7	.996,1	.997,5	.998,2	.998,7	.999,0	.999, 2
4.4	.928,9	·976,0	.989,1	.994,2	.996,5	.997,7	.998,4	.998,9	.999,1	.999, 3
4.5	.930,4	.977,0	.989,8	.994,6	.996.8	.997,9	.998,6	.999,0	.999,3	.999,4
4.6	.931,9	.977,9	.990,3	.995,0	.997,1	.998,2	.998,8	.999,1	.999,4	.999,5
4.7	.933,0	.978,8	.990,9	.995,3	.997,3	.998,3	,998,9	.999,2	.999,4	.999,6
4,8	.934,6	.979,6	.991,4	.995,7	.997, 6	.998,5	.999,0	.999,3	.999,5	.999,6
4.9	.935,9	.980,4	.991,9	.996,0	.997, 8	.998,6	.999,1	.999,4	.999,6	.999,7
5.0	.937.2	.981,1	.992,3	.996,3	.997,9	.998,8	.999,2	.999,5	.999,6	.999.7
5.1	.938,4	.981,8	.992,7	.996,5	.998,1	.998,9	.999,3	.999,5	.999,7	.999,8
5.2	.939,5	.982,5	.993,1	.996,7	.998,3	.999,0	.999,4	.999,6	.999,7	.999,8
5.3	.940,6	.983,1	.993,4	.997,0	.998,4	.999,1	.999,4	.999,6	.999,8	.999,8
5.4	.941,7	.983,7	.993,8	.997,2	.998,5	.999,2	.999,5	.999,7	.999,8	.999,8
5.5	.942,8	.984,2	.994,1	.997,3	.998,6	.999,2	.999,5	.999,7	.999,8	.999,9
5.6	.943,8	.984,8	.994,4	.997,5	.998,7	.999,3	.999,6	.999,7	.999,8	.999,9
5.7	.944,7	.985,3	.994,6	.997,7	.998,8	.999,4	.999,6	.999,8	.999,9	.999,9
5.8	.945,7	985,8	.994,9	.997,8	.998,9	.999,4	.999,7	.999,8	.999,9	.999,9
5.9	.946,6	.986,2	.995,1	.997,9	.999,0	.999,5	.999,7	.999,8	.999,9	.999, 9
6.0	.947,4	.986,7	.995,4	.998,1	.999,1	.999,5	.999,7	.999,8	.999,9	.999,9

$n = 11$ $n' = 12$	12 13	13 14	14 15	15 16	16 17	17 18	18 19	19 20	20 21	∞
.997,5	.997,8	.998,0	.998,2	.998,4	.998,5	.998,6	.998,7	.998,8	.998,9	.999,767,4
.997,9	.998,2	.998,4	.998,6	.998,7	.998,8	.998,9	.999,0	.999,0	.999,1	.999,840,9
.998.2	.998,5	.998,7	.998,8	.998,9	.999,0	.999,1	.999,2	.999,2	.999,3	.999,892,2
.998,5	.998,7	.998,9	.999,0	.999,1	.999,2	.999,3	.999,3	.999,4	,999.4	.999,927,7
.998,8	.998,9	.999,1	.999,2	.999,3	.999,4	.999,4	.999,5	.999,5	.999,6	.999,951,9
.999,0	.999,1	.999,2	.999,3	.999,4	.999,5	.999,5	.999,6	.999,6	.999,6	.999,968,3
.999,1	.999,3	.999,4	.999,5	.999,5	.999,6	.999,6	.999,7	.999,7	.999,7	.999,979,3
.999,3	.999,4	.999,5	.999,6	.999,6	.999,7	.999,7	.999,7	.999,8	.999,8	.999,986,7
.999,4	.999,5	.999,6	.999,6	.999,7	.999,7	.999,8	.999,8	.999,8	.999,8	.999,991,5
.999,5	.999,6	.999,6	.999,7	.999,7	.999,8	.999,8	.999,8	,999,8	.999,9	.999,994,6
.999,5	.999,6	.999,7	.999,8	.999,8	.999,8	.999,8	.999,9	.999,9	.999,9	.999,996,6
.999,6	.999,7	.999,8	.999,8	.999,8	.999,9	.999,9	.999,9	.999,9	.999,9	.999,997,9
.999,7	.999,7	999,8	.999,8	.999,9	.999,9	.999,9	.999,9	.999,9	.999,9	.999,998,7
.999,7	.999,8	.999,8	.999,9	999,9	.999,9	.999,9	.999,9	.999,9	.999,9	.999,999,2
.999,8	.999,8	.999,9	.999,9	.999,9	.999,9	.999,9	.999,9	1.000,0	1.000,0	.999,999,5
.999.8	.999,8	.999,9	.999,9	.999,9	.999,9	.999,9	1.000,0			.999,999,7
.999,8	.999,9	.999,9	.999,9	.999,9	.999,9	1.000,0				.999,999,8
.999,9	.999,9	.999,9	,999,9	.999,9	1.000,0					.999,999,9
.999,9	.999,9	.999,9	,999,9	1.000,0						.999,999,9
.999,9	.999,9	.999,9	1.000,0							1.000,000,0
,999,9	.999,9	.999,9								
.999,9	.999,9	1.000,0								
.999,9	1.000,0									
.999,9										
.999,9										
1.000,0										

TABLE II. — *Supplementary table for high values of t.*

t	$n=3$ $n'=4$	$n=4$ $n'=5$	$n=5$ $n'=6$	$n=6$ $n'=7$	$n=7$ $n'=8$	$n=8$ $n'=9$	$n=9$ $n'=10$	$n=10$ $n'=11$
6.0	.995,364	.998,059	.999,077	.999,518	.999,729	.999,838	.999,899	.999,934
6.5	.996,303	.998,555	999,357	.999,684	.999,833	.999,906		.999,966
7.0	.997,007	.998,904	.999,542	.999,788	.999,894	.999,944	.999,968	
7.5	.997,544	.999,155				.999,965		
8.0	.997,962	.999,338	.999,754	.999,898	.999,954			
8.5	.998,290							
9.0	.998,552	.999,578	.999,859	.999,947				
10.0	.998,936	.999,719	999,915	.999,971				
11.0	.999,196							
12.0	.999,377	.999,862	.999,965					
14.0	.999,605	.999,924						
16.0	.999,735	.999,955						
20.0	.999,863							
24.0	.999,921							
28.0	.999,950							

Linear interpolation between adjacent entries will give four figure accuracy.

TABLE III.

t	C_1	C_2	C_3	C_4
.1	.010,023,1	— .001,261	— .001,55	+ .000,4
.2	.020,334,2	— .002,616	— .003,08	.000,8
.3	.031,178,5	— .004,177	— .004,53	.001,3
.4	.042,719,3	— .006,087	— 005,86	.001,7
.5	.055,010,2	— .008,509	— .006,97	.002,2
.6	.067,977,8	— .011,595	— .007,72	.002,6
.7	.081,420,2	— .015,432	— .007,96	.002,8
.8	.095,018,8	— .019,991	— .007,74	.002,6
.9	.108,363,2	— .025,066	— .006,51	.002,1
1.0	.120,985,4	— .030,246	— .005,04	.001,1
1.1	.132,399,7	— .034,907	— .003,76	.000,4
1.2	.142,144,2	— .038,248	— .003,75	.001,1
1.3	.149,819,0	— .039,363	— .006,56	.005,8
1.4	.155,117,7	— .037,344	— .014,10	.018,2
1.5	.157,849,6	— .031,399	— .028,41	.043,0
1.6	.157,951,2	— .020,971	— .051,29	.084,9
1.7	.155,486,7	— .005,832	— .083,84	.161,2
1.8	.150,637,0	+ .013,846	— .126,09	.232,3
1.9	.143,682,2	.037,483	— .176,64	.335,1
2.0	.134,977,5	.064,114	— .232,56	.446,6
2.1	.124,924,4	.092,473	— .289,45	.551,2
2.2	.113,944,4	.121,099	— .341,85	.627,7
2.3	.102,451,8	.148,472	— .383,79	.652,2
2.4	.090,832,2	.173,147	— .409,50	.601,0
2.5	.079,425,1	.193,877	— .414,17	.455,3
2.6	.068,512,4	.209,710	— .394,59	+ .205,6
2.7	.058,312,9	.220,052	— .349,67	— .145,4
2.8	.048,980,8	.224,698	— .280,61	— .580,9
2.9	.040,609,7	.223,749	— .190,82	— 1.070,4
3.0	.033,238,9	.217,715	— .085,59	— 1.572,7
3.1	.026,862,2	.207,289	+ .028,49	— 1.368,5
3.2	.021,437,7	.193,351	.144,23	— 1.610,8
3.3	.016,897,1	.176,859	.254,58	— 1.837,2
3.4	.013,155,2	.158,774	.353,27	— 2.034,8
3.5	.010,117,7	.139,996	.435,35	— 2.732.8

TABLE III (*continued*).

t	C_1	C_2	C_3	C_4
3.6	.007,687,9	.121,313	.497,49	— 2.503,1
3.7	.005,772,2	.103,368	.538,13	— 2.123,7
3.8	.004,282,3	.086,649	.557,33	— 1.625,7
3.9	.003,139,7	.071,486	.556,67	— 1.050,1
4.0	.002,275,1	.058,066	.538,80	— .441,7
4.1	.001,629,5	.046,453	.507,08	+ .155,5
4.2	.001,153,8	.036,613	.465,19	.702,6
4.3	.000,807,4	.028,438	.416,81	1.169,2
4.4	.000,558,6	.021,773	.365,31	1.535,3
4.5	.000,382,1	.016,436	.313,56	1.791,6
4.6	.000,258,4	.012,235	.263,86	1.938,7
4.7	.000,172,8	.008,984	.217,87	1.985,3
4.8	.000,114,3	.006,508	.176,63	1.946,2
4.9	.000,074,6	.004,652	.140,70	1.839,4
5.0	.000,048,3	.003,281	.110,17	1.683,9
5.1	.000,030,9	.002,284	.084,85	1.498,6
5.2	.000,019,5	.001,570	.064,29	1.299,5
5.3	.000,012,2	.001,065	.047,96	1.100,4
5.4	.000,007,6	.000,713	.035,27	.910,7
5.5	.000,004,6	.000,472	.025,47	.379,9
5.6	.000,002,8	.000,308	.018,15	.506,5
5.7	.000,001,7	.000,199	.012,74	.437,0
5.8	.000,001,0	.000,127	.008,82	.349,5
5.9	.000,000,6	.000,080	.006,02	.262,6
6.0	.000,000,3	.000,050	.004,05	.193,9