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On the Random Sequence.

In an interesting note, E. G. Bilham points out that quite apart from the use of correlation coefficients, the trend of a random sequence of values will change sign on the average twice in three trials. In fact, of three unequal values, one is greater than both the others, one is less than both the others, and the third is neither; one or other of these must come second in order, and if the greatest or the least comes second, the successive differences will be of opposite signs.

Mr. Bilham adds: "By reasoning on generally similar lines, it may be shown that if n successive differences have similar signs, the chances are $n+1$ to one in favour of a change of sign in the next difference." The *caveat* should, perhaps, be added that both rules depend absolutely upon ignoring the changes previous to the sequence used for prediction, as well as on ignoring the actual deviation observed from the mean of the series.

A more general view is to consider the frequency with which a **"run"** of increasing or decreasing values is of 1, 2, 3, . . . differences; in fact, to determine the frequency distribution of length of run. The frequency of a run of n successive increases or decreases is then found to be

$$3\left\{\frac{1}{(n+1)!}-\frac{2}{(n+2)!}+\frac{1}{(n+3)!}\right\}$$

corresponding to the series of fractions

$$\frac{5}{8}, \frac{11}{40}, \frac{19}{240}, \frac{29}{1{,}680}, \frac{41}{13{,}440}, \frac{55}{120{,}263}, \frac{71}{1{,}209{,}600}, \quad . \quad . \quad .$$

If, therefore, a run of one increase only has been observed, the chance that it will be followed by a decrease is $\frac{5}{8}$ and not $\frac{2}{3}$, the latter probability being correct when the last change is known to be an increase, but whether preceded by an increase or a decrease is unknown.

The probability of a run of n or more is evidently

$$3\left\{\frac{1}{(n+1)!}-\frac{1}{(n+2)!}\right\}$$

and consequently when such a run has been observed, the probability of a change is

$$\frac{n^2+3n+1}{(n+1)\,(n+3)} = 1-\frac{n+2}{(n+1)\,(n+3)}$$

The extreme rarity of runs of 5, 6 or 7 differences is of value in the use of such runs as evidence that a sequence is in parts not of a random character; such a test may be refined by counting all the runs of all lengths and comparing the frequency of each class observed with that predicted by the above distribution.

The mean of the distribution is, of course, $1\frac{1}{2}$; but the other moment functions involve e and powers of e; thus the variance is $6\,(e-2\frac{5}{8})=\cdot5597$, and the third moment about the mean is $45\,(41/15-e)=.6773$.

R. A. Fisher.