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THE ANALYSIS OF COVARIANCE METHOD FOR THE RELATION BETWEEN A PART AND THE WHOLE

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At the suggestion of Dr. C. I. Bliss and by the courtesy of Professor H. G. O. Holck, whose data I shall use, the following note may serve to illustrate the extreme simplicity with which the technique derived from the analysis of covariance may be applied to problems concerned with the relation of a part to the whole, such as are constantly arising in many fields.

The data consist of the body weights in kilograms and the heart weights in grams of 144 cats used in a group of digitalis assays.[1] Of these 47 were females and 97 males. These data are presented in the table at the end of this note. To simplify the calculations only one decimal place was used for each value. Thus we have:

TABLE 1

TOTAL WEIGHTS

	Females	*Males*
Number	47	97
Total body weight	110.9 Kg.	281.3 Kg.
Total heart weight	432.5 g.	1098.2 g.
Heart as fraction of entire body	.3900%	.3904%

The observed variation in these two measurements can, of course, be expressed by means of the sums of squares and products, as in the following tables. The rather intimidating phrase "spurious correlation" used in the earlier literature sometimes prevents workers from taking the simplest course. Obviously it would be easy to derive from

1 Holck, Harald G. O., Kazuo K. Kimura, and Barbara Bartels, "Effect of the Anesthetic and the Rate of Injection of Digitalis upon Its Lethal Dose in Cats," *Journal of the American Pharmaceutical Assn. 35:* 366–370 (1946).

the crude figures the corresponding square and product for the difference between our variates, representing the weight of the body less the weight of the heart.

TABLE 2

SUMS OF SQUARES AND PRODUCTS FOR BODY AND HEART WEIGHTS

Females	*d.f.*	*(Body)*^{2}	*(Body · Heart)*	*(Heart)*^{2}
Total	47	265.13	1029.62	4064.71
Correction for Mean	1	261.677	1020.516	3979.920
Difference	46	3.453	9.104	84.790
Males				
Total	97	836.75	3275.55	13056.17
Correction for Mean	1	815.77	3185.07	12435.700
Difference	96	20.98	90.48	620.470

From the corrected sums of squares and products we may find the regressions of heart weight on body weight, namely .2637% for females and .4313% for males. It will be noticed that these values are in the first case less and in the second case more than the average contribution of the heart to the total weight. The significance of such differences is often of importance, and must often appear to present a rather complex problem.

We may, however, recognize in the above table the requisite data for the simplest form of an analysis of covariance, and test at once the homogeneity of the regressions therein. We may set up the regression equation $H = a + bB$, where H represents the heart weight and B the total body weight. If b is actually less than the average percentage, .39%, then a must be significantly greater than 0 and vice versa for b greater than .39%. a will be significant if the residual variance is significantly reduced by the inclusion of a in the regression equation. The partition of the variation in heart weight after adjustment for body weight by covariance furnishes such a test. Thus for females, since $(1029.62)^2/265.13 = 3998.481$, we have a remainder of 66.229 g.^{2} for 46 degrees of freedom. Consequently, the significance of the observed difference between the regression value for heart weight and the average value can be tested by the following table.

TABLE 3

VARIATION IN HEART WEIGHT ADJUSTED FOR BODY WEIGHT

	Females			*Males*		
	d.f.	*(Heart)*^{2}	*M.S.*	*d.f.*	*(Heart)*^{2}	*M.S.*
From original totals	46	66.229	.....	96	233.669	.....
After correction for mean	45	60.787	1.351	95	230.259	2.424
Reduction due to mean	1	5.442	5.442	1	3.410	3.410

The reduction due to the mean for females is almost significant (the variance ratio is 4.03 while the 5% significance level is 4.06), indicating a strong probability that a is actually different from 0. Hence in

TABLE 4

ORIGINAL DATA ON LIVE BODY WEIGHT IN KILOGRAMS AND FRESH HEART WEIGHT IN GRAMS OF FEMALE AND MALE DOMESTIC CATS USED IN EXPERIMENTS ON DIGITALIS

Females

Body Weight	*Heart Weight*	*Body Weight*	*Heart Weight*	*Body Weight*	*Heart Weight*	*Body Weight*	*Heart Weight*
2.3	9.6	2.0	7.4	2.1	9.8	2.9	10.1
3.0	10.6	2.3	7.3	2.7	10.8	3.0	13.0
2.9	9.9	2.2	7.1	2.2	9.1	2.2	8.7
2.4	8.7	2.3	9.0	2.3	11.2	2.4	6.3
2.3	10.1	2.1	7.6	2.1	8.1	2.4	8.8
2.0	7.0	2.0	9.5	2.4	10.2	2.5	10.9
2.2	11.0	2.9	10.1	2.7	8.5	2.5	9.0
2.1	8.2	2.7	10.2	2.3	10.1	2.3	9.7
2.3	9.0	2.6	10.1	2.1	8.7	2.3	8.4
2.1	7.3	2.3	9.5	2.2	10.9	2.6	10.1
2.1	8.5	2.6	8.7	2.3	7.9	2.3	10.6
2.2	9.7	2.1	7.2	2.1	8.3	.....	.....

Males

Body Weight	*Heart Weight*	*Body Weight*	*Heart Weight*	*Body Weight*	*Heart Weight*	*Body Weight*	*Heart Weight*
2.9	9.4	2.5	12.7	3.5	15.7	3.5	17.2
2.4	9.3	3.5	15.6	2.8	13.3	3.8	16.8
2.2	7.2	2.4	9.1	2.2	9.1	2.2	8.5
2.9	11.3	2.2	7.6	2.5	7.9	3.3	15.4
2.5	8.8	3.4	12.8	2.4	7.9	2.7	9.8
3.1	9.9	2.6	8.3	3.9	14.4	3.2	11.9
3.0	13.3	3.4	11.2	3.1	12.5	2.9	10.6
2.5	12.7	2.6	9.4	3.7	11.0	3.6	13.3
3.4	14.4	2.7	8.0	3.0	12.4	2.7	12.5
3.0	10.0	3.3	14.9	3.2	13.5	2.9	11.8
2.6	10.5	2.2	10.7	3.3	14.1	3.6	15.0
2.5	8.6	3.2	13.6	3.0	12.7	2.8	10.2
2.8	10.0	2.2	9.6	2.9	10.1	2.5	11.0
3.1	12.1	3.5	11.7	3.0	10.4	2.6	11.5
3.0	13.8	2.5	9.3	2.4	7.9	3.9	20.5
2.7	12.0	3.2	12.3	3.8	14.8	3.0	12.2
2.8	12.0	3.2	13.0	2.0	6.5	2.6	9.4
2.1	10.1	2.7	9.6	3.1	11.5	2.7	9.0
3.3	11.5	2.6	7.7	2.8	9.1	2.5	8.8
3.4	12.2	2.7	9.6	2.3	9.6	2.2	9.6
2.8	13.5	2.0	6.5	3.0	11.6	3.1	13.0
2.7	10.4	3.1	14.3	2.2	7.9	3.3	12.0
3.2	11.6	2.4	7.3	3.4	12.4	2.7	11.1
3.0	10.6	3.6	14.8	3.5	12.9	3.6	11.8
.....	.....	.....	.....	.....	.....	2.8	11.4

females heart weight probably increases less than proportionately to body weight. In the case of males, where the heart weight appears to increase more than proportionately to the body weight, the differ-

ence is not significant with these data. The close agreement between the sexes in the average percentage of the body taken up by the heart seems to mask a real difference in the heart weight to be expected for a given body weight.

It may be noted that the estimated variance of heart weight for given body weight in males, 2.424 g.^{2}, is considerably greater than the value for females, 1.351 g.^{2} The greater residual variance for males possibly was related to their larger size. The heaviest female weighed 3.0 Kg. while nearly 40% of the males exceeded this weight.