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The Expansion of Statistics

[The Inaugural Address of the President, Sir Ronald Fisher, F.R.S., delivered to the Royal Statistical Society on November 19th, 1952]

Thirty or forty years ago it seemed to me a very strange thing that the branch of learning known as Applied Mathematics should fall into two so very unequal parts. On the one side, in the main stream of historical development, were the applications to Astronomy, Physics and Engineering, for which the Universities traditionally made provision by numerous courses in Mechanics, Optics, Electromagnetism, Hydrodynamics, Thermodynamics, and many more specialized subjects, which formed a solidly integrated body of teaching, and the backbone of the education received by the great body of students in Applied Mathematics. On the other side was a slender and tenuous branch of learning, stemming in its origins equally far back to Fermat and Pascal, yet almost neglected in the Universities, and intermittent in its historical development. Its brilliant efflorescence with De Moivre and Bayes, at a period when no one thought gambling to be disreputable, was later almost a handicap, requiring apologies from Laplace and de Morgan in the early nineteenth century. They would rather stress the solid social benefits, and the great commercial success, of the business of Insurance; and sought, rather fruitlessly, to find further social applications for so elegant a mathematical theory, in assessing the probability of correctness of the decisions arrived at by voting, of courts of justice and other committees.

The growth of Actuarial Science was, however, rather a parallel development than an offshoot of the theory of probability. Powerfully influenced by the practical importance of the accurate evaluation of life assurances and annuities, it owed more to the early workers in the Calculus of Finite Differences, especially to Newton, than to the early work in the Theory of Probability; to-day we can recognize that this indebtedness has been discharged with handsome interest, and I have often regretted that courses on the computational techniques of the Calculus of Finite Differences should be so seldom given to others than actuarial students.

The Theory of Errors sprang with equal independence from the practical need to grapple with the problems, at first sight fantastically difficult, confronting astronomers and surveyors, in the combination of observations. The extent to which Statistical Science was broadened and enlarged by this development has not been appreciated by writers on the Theory of Probability. Perhaps only a genius of the magnitude of Gauss could have introduced and consolidated so many new ideas. Unknowns, other than probabilities, were seen to require estimation. Observations had to be discussed that were not merely counts of frequencies, but quantitative measurements on a conceptual continuum. The normal distribution was introduced, not as the necessary consequence of a deductive mathematical argument, but by a choice resting on penetrating understanding of its essential properties. It was amid such extensions, and as part of a much larger whole, that the Theory of Probability began to acquire a genuine importance for human thought.

The diversity of the roots of what we can now recognize as a unitary discipline, and the successive extensions of concept and method as our abstract ideas have been, at one point after another, brought into integral connection with the real world—the world in which it is only in the light of experience, by experiment and observation, that we can guess the rules of our game of chance—are, as it seems to me, worthy of emphasis at the present time, if we are to seize and comprehend the much greater widening of ideas, and diversity of application which characterizes the intensification of statistical activity in the second quarter of the twentieth century.

A few years ago, in a short Presidential address to the International Biometric Society, I ventured to suggest that Statistical Science was the peculiar aspect of human progress which gave

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to the twentieth century its special character ; and indeed members of my present audience will know from their own personal and professional experience that it is to the statistician that the present age turns for what is most essential in all its more important activities. They are the "backroom boys" of every significant enterprise. In the all-important field of scientific research it is they who plan and design the experimental programmes, or the observational surveys, and it is they again who are called in to analyse the results, to assess the evidence, and to separate what is clearly demonstrated from what still requires confirmation. For the business of fact-finding in the fields of economics, sociology or vital statistics, that characteristic and intricately elaborated product of the theory of random sampling, namely, the *Sample Survey*, has shown itself to be economical certainly and of reliable accuracy, but more particularly to be expeditious and adaptable in comparison with older methods. In the field of productive industry that enterprising body known as the Quality Control Engineers, using statistical methods, sometimes of extreme simplicity, but with rapidly increasing refinement and sophistication of statistical techniques, has shown how to reconcile the *efficiency* of mass-production with the accentuated demands for *precision* and *reliability* of, for example, the aircraft industry. Perhaps the production during the last war of such a delicate and potentially dangerous appliance as the proximity fuse, and that by the hundred million, will serve to illustrate how enormously the industrial potentiality of mankind has been increased within a small part of our own lifetimes. The real threat of but a few years back that every aspect of our lives was destined to be harassed by gadget after gadget, constantly out of order and in need of repair, has certainly receded now that every industrial process can be geared to whatever degree of precision is thought to be requisite.

Though perhaps it yields its greatest dividends in the improvement of quality at the level of production, sampling theory is equally happily adaptable to commercial guarantees and acceptance procedures, where the types of tests known as Sequential have been especially successful. In the commercial field also, the possibility of gauging the intensity and tenacity of Consumer Preference promises to lead to a more beneficent or at least less reckless and wasteful use of the medium of advertisement. Already the machinery exists, though it can scarcely be said yet to be organized or unified, by which the actual requirements of that statistical aggregate, the consumer, can be competently ascertained, and transmitted along the whole chain, to the designer and the production engineer.

This rather tumultuous overflow of statistical techniques from the quiet backwaters of theoretical methodology, to which they belonged only a few years ago, into the working parts of going concerns of the largest size, suggests that hidden causes have been at work for much longer than the period of manifest efflorescence, preparing men's minds, and shaping the institutions through which they work, so that, quite suddenly when the academic tools had become sufficiently sharp and accurate, or, perhaps equally important, sufficiently realistic, there was no end to the number of applications impatiently awaiting methods which could, really, deliver the goods. With the aid of a few examples, drawn only from the story of "the improvement of natural knowledge", I propose indeed to suggest that the period of inconspicuous gestation extended through most of the nineteenth century, and was responsible for a goodly number of fundamental advances in subjects the students of which are quite unaware of their statistical origin.

In my first example the facts are, I believe, indisputable. Charles Lyell, the geologist, was born in 1797, and in 1830 there appeared the first of the three volumes of his celebrated *Principles of Geology*. The second volume was received by Darwin at Monte Video in 1832 during the outward voyage of the "Beagle", and no book could have been more stimulating for the theories which Darwin later formed. The third volume is dated 1833, three years after the first. The work is, of course, a scientific classic, a masterpiece of lucid and effective literary style, embodying proposals by the author of revolutionary importance, and preserving for readers of our own age the memorials of a scientific environment with its ideas and arguments so remarkably different from our own.

Geologists prior to Lyell had recognized the sequences of strata which we know as Primary and Secondary, using in the first place the regularity of order of superposition in the same locality. They observed, too, that particular components of these formations could be recognized, though far apart, by their characteristic fossils. They could not by these means recognize or establish the order among the Tertiary rocks, for, in the part of the world then accessible, these occur in patches, and not over wide areas overlying one another. Lyell determined the order and assigned to the successive rock masses the names they now bear by a purely statistical argument. A rich group of strata might yield so many as 1,000 recognizable fossil species, mostly marine molluscs. A certain number of these might be still living in the seas of some part of the world, or at least be morphologically indistinguishable from such a living species. It was as though a statistician had a recent census record without recorded ages, and a series of undated records of previous censuses in which some of the same individuals could be recognized. A knowledge of the Life Table would then give him estimates of the dates, and, even without the Life Table, he could set the series in chronological order,merely by comparing the proportion in each record of those who were still living.

With the aid of the eminent French conchologist M. Deshayes, Lyell proceeded to list the identified fossils occurring in one or more strata, and to ascertain the proportions now living. To a Sicilian group with 96 per cent. surviving he gave, later, the name of Pleistocene (mostly recent). Some sub-appenine Italian rocks, and the English Crag with about 40 per cent. of survivors, were called Pliocene (majority recent). Forty per cent. may seem to be a poor sort of majority, but no doubt scrutiny of the identifications continued after the name was first bestowed, and the separation of the Pleistocene must have further lowered the proportion of the remainder. The Miocene, meaning "minority recent", had 18 per cent., and the Eocene, "the dawn of the recent", only 3 or 4 per cent. of living species. Not only did Lyell immortalize these statistical estimates in the names still used for the great divisions of the Tertiary Series, but in an Appendix in his third volume he occupies no less than 56 pages with details of the classification of each particular form, and of the calculations based on the numbers counted. There can be no doubt that, at the time, the whole process, and its results, gave to Lyell the keenest intellectual satisfaction.

Now the point of this little history, its point for statisticians, is that the statistical argument by which this revolution in Geological Science was effected was almost immediately forgotten. In later editions of the *Principles* this great Appendix, in which so much labour had been expended, has disappeared; it survived indeed only two years. It had served its purpose; the ladder by which the height had been scaled could be kicked down. Geologists could quickly recognize a fossiliferous stratum by a few characteristic forms with clear morphological peculiarities. There was no need to wait for extensive collection, or statistical tabulation. Nor does it seem to have been thought that future geologists had anything to learn from the example, or the particular method, of Lyell's discovery. His designations of the Tertiary formations remain as records of a forgotten past, like fossils themselves, less intelligible to the geological students than the casts of sea-shells they extract from the rocks.

The obliteration of the Appendix, so decisive for readers of later editions, was presumably actuated by motives stronger than the desire to save space, and these motives have an importance for us in so far as they are characteristic of statistical inquiry. Among thousands of identifications it would be surprising if at least a hundred were not questionable. The state of the evidence on each of these would constantly change as better preserved or more complete specimens came to be scrutinized. The labour of bringing the list up to date would be great, new editions were frequent, and with the Tertiary succession clearly established, and in practical use by all, this labour would serve no immediate purpose. Moreover, Lyell could not say, as might a later geological evolutionist, "These minor differences do not matter, it is only a question of convention at what level of distinctness a new name is required", for he and his associates firmly refused to admit the possibility of the gradual transformation of one form into another. Any "real" difference must mean to them both extinction and creation, and must be given full weight in estimating the proportion of forms "still living".

An example of a not dissimilar character is on record from the early years of the present century. Johannes Schmidt of the Carlsberg Laboratory in Copenhagen was not only an ichthyologist, but also an assiduous biometrician, particularly interested in the numbers of vertebrae and fin-rays of the various species of fish he studied. Usually he was able to establish statistical differences between samples of the same species drawn from different localities ; often even from different parts of the same fjord. With the eel, however, in which the variation in vertebra number is large, Schmidt found sensibly the same mean, and the same standard deviation, in samples drawn from all over Europe, from Iceland, from the Azores and from the Nile. He inferred that the eels of all these different river systems came from a common breeding-ground in the ocean, and it was a major triumph of the research vessel "Dana's" expedition to secure the young eel larvae from a limited region in the Western Atlantic, not far from the breeding-grounds of the different species of eel which inhabits the Eastern rivers of N. America and the Gulf of Mexico. Of course, "every school boy knows", or has been told, about the discovery of the source of the elvers, but a veil has been drawn over the biometrical foundations of the discovery, and my zoological friends have seldom even heard of Schmidt's biometrical studies. One is led to wonder in how many other cases the true sources of important scientific advances have been obscured past recall.

It is easily seen, however, that statistical ideas, often vaguely apprehended, indeed, and sadly lacking in the clarity and facility supplied by a competent statistical technique, formed the silent background of the greatest scientific advances of the nineteenth century. In mid-century the turning-point of Physics was reached with the Kinetic Theory of Gases. The notion that the pressure exerted by a gas was kinetic in origin was, of course, older than Maxwell. It was, however, Maxwell's recognition of a three-dimensional Gaussian distribution, as the form appropriate for the velocity distribution for molecules of equal mass, that made possible calculations of the properties of the aggregate, and so converted an interesting speculation into a working quantitative theory, in terms of which diffusion, conduction of heat, and viscosity could be discussed, and their experimental values used to throw light on the molecular population. With an understanding of the mechanical basis of the gas laws, too, they were quickly seen, as by van der Waals, to be capable of a more accurate restatement. The statistical ideas so injected into physical theory remained, however, somewhat insulated in a specialized application until Willard Gibbs recognized the fundamental nature of Statistical Mechanics, and blazed the trail for the Quantum Mechanics of the present day.

The influence of statistical ideas upon the recognition in Biology of the Theory of Natural Selection is more intricate, and requires probably a fuller examination than I shall be able to give it. The argument was manifestly a statistical one, but put forward so far in advance of the appropriate technical developments that it is only in recent years that writers such as Mather and Lerner have made a start in giving a rational account of the application of deliberate selection to animal and plant improvement. Indeed, Darwin's exposition, as I pointed out some years ago, was very gravely encumbered by the contemporary belief in Blending Inheritance, just as Lyell had been encumbered by the belief in Special Creation. To put across a subtle argument, of unfamiliar form, from among these entanglements certainly required all of Darwin's patience and pertinacity, and was perhaps only possible by reason of his wealth of detailed knowledge. In spite of the widespread consent and conviction which he won, the next two generations saw among theoretical workers in Biology quite incredible confusion of thought on the issues raised by Darwin, echoes of which are still audible; and there were periods, such as the turn of the century in this country, when only a minority would dare to express their belief in his theory. All this can be understood on the view that the force of Darwin's argument lay in statistical considerations with which the biological world had no opportunity for many decades to become familiar.

It must seem strange in view of Quetelet's early advocacy of Statistics as an educational discipline, that so many leading, and by the standards of their time, well educated, men were quite unaware that they had anything to learn in this field. Perhaps its name was too closely associated with politics and affairs of State, to attract those retiring and philosophic men who were inclined to give their lives to Science. It seems almost certain, however, that Gregor Mendel had never heard of Quetelet during his studies at Vienna (about 1855) or elsewhere. He recognized for himself that one of the important novelties in his approach to the study of inheritance was the use of statistical enumerations, and William Bateson, whom no one could suspect of being prejudiced in favour of statistical methods, called attention to the same circumstance. Yet much that Mendel needed and that was apparently beyond his reach could have been found in Quetelet, or indeed in the earlier eighteenth century authors. Moreover, Mendel was a well-read and studious man. I cannot doubt that although the biological teaching given in Vienna in the mid-nineteenth century was not worse than that given elsewhere—not worse we might say than much that is still given a hundred years later—yet it left Mendel with not an inkling that any of the puzzles he might encounter in the course of Biological Research had ever been helpfully considered and discussed by writers on Statistics. Mendel's use of Statistics in his great discovery was yet another sporadic outburst quite independent of the academic tradition.

I cannot, however, leave the question of Natural Selection without mention of Malthus, whose distinguished part in the foundation of this Society must predispose us in his favour. Both Darwin and Wallace ascribe their first confident appreciation of the potency of Natural Selection, not to personal contact with Malthus, but to the effects of casually reading his Essay on Population.

How easy it would be to say, "Here is the source of that subtle statistical argument which convinced both Darwin and Wallace that they had uncovered the effective cause of Organic Evolution". I do not say it, because I do not believe it. What is there in the Essay to have this effect? There is the doctrine of excessive reproduction; i.e., that in Man, the species tends to produce more off-spring than would be required for a stationary population. A most important fact for human policy; but what is the realistic and philosophic observer of Nature to make of it? He sees species such as thrushes with four eggs in the year, or foxgloves with ten thousand seeds each showing a reproductive activity about equal to the rate of elimination. If the reproduction falls below this level the numbers in the species will fall off, as must on the whole be the case more frequently than not ; but if in more favourable cases fewer die than are reproduced the numbers will go up. If we believe, as I should concede we have a right to do, that generally the density of each species is somehow stabilized, then we shall infer, though we shall not actually observe, that an increase in population will generally make it harder to survive, or to breed, and that a decrease will generally make life easier. We do not, however, see these additional checks to population at work ; they are rather subtle inferences, and the naturalist will not be tempted to label them "misery and vice". These are not the terms which spontaneously come to mind in considering crowded communities of, for example, winkles, or primroses; nor, of course, is "moral restraint" any more appropriate.

Yet something in Malthus undoubtedly influenced both Darwin and Wallace, for they begin at once to sketch pictures of a very unrealistic world in which animals reproduce, but do not die, at least until after they have reproduced abundantly—pictures in which the planet's land area is overrun with elephants, and in which the volume of the oceans is insufficient to accommodate all the herrings. This is certainly due to Malthus, but it is scarcely his logic but his rhetoric, which has gone to their heads. In truth, when I try to explain the Theory of Natural Selection to students, the phrases I find least helpful are those rather journalistic slogans, "The Struggle for Existence" and "The Survival of the Fittest". Each, of course, has a defensible meaning, but it is, I fear, only their dramatic violence that makes them effective.

Now it is very natural to the human race, and very unphilosophical, to imagine that results of towering and majestic importance, such as the evolution of living creatures, must be brought about by equally powerful or violent causes. If we can imagine my winkles, or my primroses, *tense with struggle*, red in their non-existent teeth and claws, it becomes easier to believe that great things are happening to them, that they are being hammered into shape by cosmic forces. Whereas all we know is that each is living its own quiet life, with the life Table and the rates of reproduction characteristic of its genotype, and that for this reason, and for no other, these species are probably changing, imperceptibly slowly, but with a speed sufficient to make them perceptibly different if they keep it up for some millions, or tens of millions, of years. Natural Selection has no need of all this "Sound and Fury, signifying nothing". But Darwin and Wallace, faced with the problem of getting across a doctrine, in many ways obscure, even to themselves, to a rather thick-headed audience by no means predisposed in its favour, must have felt greatly heartened by Malthus' eloquence in depicting the violence of the effects of the reproductive urge in mankind, and by the feeling that here was a natural agency really competent to bring about the great evolutionary effects they had in view.

These at least are my reasons for not tracing the peculiarly statistical nature of the Theory of Natural Selection to the distinguished statistician who in fact inspired both its discoverers. Its sequel through Galton and the mathematical study of Evolution is clearer than its origin. Statistical mathematics is to-day a well-established necessity as much in the understanding of populations and their natural evolution as in that of the technique of animal and plant improvement.

My epilogue and moral lie in the recent history of this Society itself, of which I owe the facts I can give to our Secretary. How has our organization risen to the height of its great opportunity? If we survey the increase in our numbers, over the last fifty years available for comparison, the course of events revealed is very instructive. From 1902 to 1927 the number of Fellows increased from 932 to 1,074, an increase of 142, or, over the twenty-five years, an annual rate of increase of 0·58 per cent. (Speaking to an instructed body I need make no hesitation, or elaborate explanation, if I calculate such a rate of increase as a divided difference of the natural logarithms.) The second twenty-five years, 1927 to 1952, brings the number (taken this year in April instead of December) up to 2,343, and gives an average rate of relative increase, not of 0·58 per cent. but of 3·12 per cent., more than 5 times as fast. Further, the last five years of these twenty-five has a higher rate of increase still, namely of 4·88 per cent. Whereas most organisms, and most societies, grow relatively more slowly as they get older and larger, our own old and large Society shows a relative growth rate characteristic of extreme youth.

Our Society has grown not only in size, but also in variety, by budding, I am glad to say, rather than by fission. Our Study Section dates from 1928; its last five years of growth from 194 to 284 has an annual increase of 7·62 per cent. as compared with 4·88 per cent. for the Society as a whole. The Research Section, founded in 1933, has in the last five years increased from 266 to 580, or at the rate of 15·59 per cent., a figure closely rivalled by the numbers of Fellows of the Society in the Industrial Applications Section, which dates from 1942, and has in the last five years increased at the rate of 14·95 per cent. per annum. The extraordinary performance of the Research Section indicates perhaps where the growth hormones are coming from, and where they are still richest.

These Sections have been founded originally and primarily for the sake of holding additional meetings. To the eight standard meetings of the Society, which still indeed attract the largest numbers, have been added eight or nine or even fifteen meetings of the Study Section. With the Research Section we have four more, about as well attended as those of the main Society, and thirty to forty meetings a year more with the Section of Industrial Applications. Over twenty-five years the total number of meetings has increased about seven-fold, say 7·78 per cent. on an annual average, as compared with a total membership increase of 3·12 per cent. over the same period.

From 1934 the regular publication of our Journal has been duplicated by the formation of Series B (*Methodological*), alongside of Series A, now termed *General.* In 1946 the page size of both series was increased by about 50 per cent., to their great advantage, both at the separate and at the bound stage. Series B is among other things the natural repository for the Proceedings of the Research Section, and has become a journal of the first importance for Mathematical Statistics. This year a third publication has been launched under the name of *Applied Statistics,* for which Mr. Tippett tells me the subscription list is already about 1,250—a figure large enough to make many scientific Journals envious. The needs of the Industrial Applications Section will there be met, I confidently expect, on a self-sustaining basis. Yet, unlike meetings, which are a comparatively economical form of activity, publications are, at the present time, quite inevitably costly. And the only really "mournful number" which I have to offer this evening is that in 1951 our two Journals together cost the Society something approaching £7,000. How this sort of thing is done on our annual subscription is for our very expert Treasurer to explain. The growing-pains of our renascent Society must, however, clearly be distinguished from the concurrent depreciation of the currency.

I can submit, therefore, with this little record before you, that our Society has taken an appropriately vigorous part in this tidal movement of our century, which I have ventured to call the Expansion of Statistics; and that there is every prospect that it will continue to exercise, in developments not yet to be foreseen, a wise and helpful initiative.