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Characterization of turbulent jets from high-aspect-ratio rectangular nozzles

J. Mi,[a] R. C. Deo, and G. J. Nathan
School of Mechanical Engineering, University of Adelaide, South Australia 5005, Australia

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Turbulent free jets issuing from rectangular slots with various high aspect ratios (15–120) are characterized. The centerline mean and rms velocities are measured using hot-wire anemometry over a downstream distance of up to 160 slot heights at a slot-height-based Reynolds number of 10 000. Experimental results suggest that a rectangular jet with sufficiently high aspect ratio (>15) may be distinguished between three flow zones: an initial quasi-plane-jet zone, a transition zone, and a final quasi-axisymmetric-jet zone. In the quasi-plane-jet zone, the turbulent velocity field is statistically similar, but not identical, to those of a plane jet. © *2005 American Institute of Physics*. [DOI: 10.1063/1.1928667]

Rectangular turbulent jets have been investigated more extensively than any other noncircular jets over the past four decades or so (see the work of Gutmark and Grinstein,[1] and references therein). A main reason is believed to be that these jets are most widely utilized in technical applications in aerospace and chemical and mechanical engineering due to their relatively simple and flexible configurations. Previous studies, e.g., those by Quinn[2] and Zaman,[3] found that the aspect ratio of nozzle exit, i.e., $\mathrm{AR}=w/h$, where w and h are the long and short sides of rectangle, influences the jet turbulent mixing (e.g., momentum transfer) process. Based on his velocity measurements for AR=2, 5, 10, and 20, Quinn[2] concluded that the rate of mixing in the near field of the jet increases with increasing AR as long as the mean flow remains three dimensional. However, measurements of the mass flow rate of a rectangular jet at the exit velocity of Mach number of 0.95 by Zaman[3] for $\mathrm{AR}=2-38$ suggest that the entrainment rate only increases significantly with aspect ratio for $\mathrm{AR}\geqslant 8$.

Rectangular jets with high aspect ratios have also been considered to be two-dimensional (2D) plane jets by some investigators, e.g., Hussain and Clark[4] for AR=44 and Namar and Otugen[5] for AR=56, without confirmation of the applicability. A standard plane jet, as well known, is commonly produced by a high-AR rectangular nozzle with two parallel sidewalls attached to the nozzle short sides so that, to ensure the two dimensionality, the jet is forced to entrain/mix the ambient fluid only in the direction normal to the long sides. As shown unambiguously by previous measurements (e.g., Ref. 6) for a plane jet, the centerline decay of mean velocity follows the relation $U_c \sim x^{-1/2}$ and the half-width varies linearly with x in a region not far downstream from the potential core. To our best knowledge, however, no previous work has formally confirmed or denied the existence of a statistically quasi-2D region in a high-AR rectangular jet without sidewalls. Further, should that region exist, the dependence of its axial extent on AR would be of significant interest.

To address the above issues, present measurements are performed of both the mean and rms velocities along the centerlines of rectangular jets with different high aspect ratios. The specific aim is to examine whether these velocity components, and the half velocity width, vary truly in similar manners to those for a plane jet over a certain initial flow region.

A rectangular nozzle with radially contracting long sides was used for the present study. The height of the nozzle was fixed at $h=5$ mm but its width was varied from $w=75$ mm to $w=600$ mm to achieve four aspect ratios of AR=15, 30, 60, and 120. The inner radius of the long-side nozzle is $R=36$ mm so that $R/h=7.2$. Measurements at the exit plane confirm that this nozzle, like a conventional smoothly contracting one, produces a “top-hat” mean velocity profile at the exit, with the turbulence intensity of ≈0.5% in the central core region (not presented here). The jet facility was confined in a room of dimensions 18 m (long) × 7 m (wide) × 2.5 m(high); the nozzle was horizontally located about the midpoint between the floor and ceiling. Based on the model for the effect of confinement on momentum conservation proposed in Ref. 7, the present momentum loss would be only 0.1%–0.5% for $\mathrm{AR}=15-120$ at the maximum downstream distance ($160h$). The present jets should hence well resemble free rectangular jets in an infinite environment. Other details of the jet facility, given in Ref. 8, are not provided here for conciseness. For all the cases tested, the jet exit velocity is $U_j \approx 30$ m/s, which corresponds to a Reynolds number $\mathrm{Re}_h(\equiv U_j h/\nu)$ of $\approx 10^4$.

Velocity measurements were performed over the region $0 \leqslant x/h \leqslant 160$ using hot-wire anemometry. The hot-wire (tungsten) sensor is 5 μm in diameter and ≈0.8 mm in length, aligned in parallel to the long nozzle sides. Signals obtained were low-pass filtered with a high and identical cutoff frequency of $f_c=9.2$ kHz at all the measured locations. They were then digitized at $f_s=18.4$ kHz via a 16 channel, 12-bit A/D converter on a personal computer. The sampling

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[a] Electronic mail: jcmi@mecheng.adelaide.edu.au

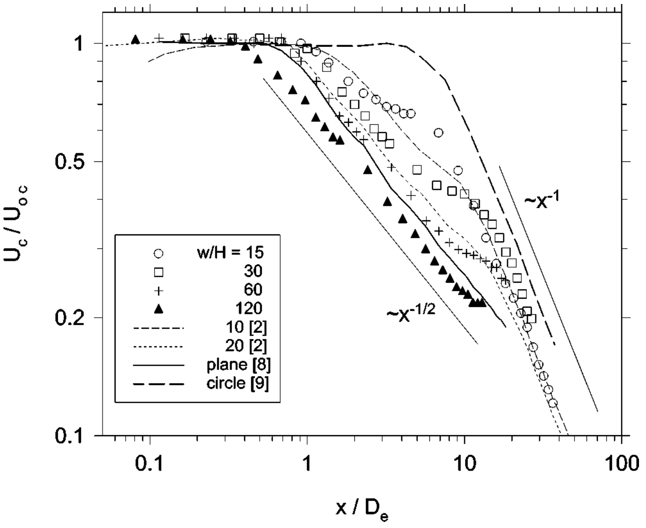

FIG. 1. Centerline variation of the normalized mean velocity U_c/U_{oc} vs x/D_e.

duration was $\approx$22 s. Calibrations of the hot wire were conducted using a standard Pitot tube, located side by side with the hot-wire probe, at the jet's exit, where $\overline{u^2}^{1/2}/U_j \approx 0.5\%$, before and after measurements for each AR. The fourth polynomial curve was used to fit data points for conversion from voltages to velocities. The maximum cumulative errors in the mean and rms velocities were found to be $\approx \pm 1\%$ and $\pm 2.5\%$, respectively.

Figure 1 presents, in log-log form, the streamwise mean velocity decay along the jet centerline for all the aspect ratios of investigation. The plot's vertical and horizontal axes are for U_c/U_{oc} and x/D_e, respectively, where U_c and U_{oc} are the centerline mean velocity and its exit value, and D_e denotes the equivalent diameter defined by $(1/4)\pi D_e^2 = wh$, i.e., $D_e = \sqrt{4wh/\pi} \approx 1.13\text{AR}^{1/2}h$. For comparison, also shown on the plot are those for a plane jet with the same Reynolds number of $\text{Re}_h = 10\,000$ (and AR=60) from the work of Deo[8] and for a circular jet at $\text{Re}_D = 15\,000$ from the work of Mi *et al.*[9]

Figure 1 demonstrates that, for a rectangular jet with $\text{AR} \geqslant 30$, there are four distinct decay regions of U_c. The first is the nondecaying region where U_c is constant, the second is the half-power-law decaying region where $U_c \sim x^{-1/2}$, then followed by a transition, and the fourth is the inversely linear decaying region where $U_c \sim x^{-1}$. For AR=15, there is no obvious region in U_c where $U_c \sim x^{-1/2}$. Evidently, as AR increases, the extent of the half-power-law region increases while that of the other regions appears to reduce. It is interesting to compare the centerline variations of U_c/U_{oc} for the present rectangular and previous plane jet for the case of AR=60. These nozzles are identical except for the presence (planar) and absence (rectangular) of two parallel sidewalls attached. The two data sets are in a good agreement for $x/D_e \leqslant 6$ (or $x/h \leqslant 52$). However, beyond that location, U_c continues to decay in the manner of $U_c \sim x^{-1/2}$ in the plane jet but, in the rectangular jet, it undergoes a transition to the final inversely linear region, like the case for an axisymmetric jet. Apparently the curves of U_c/U_{oc} for all AR tend to merge and follow $U_c \sim x^{-1/2}$ at $x/D_e > 20$. On the other hand, it is interesting to note that the far-field velocity decay is significantly greater in the rectangular than in the circular jet, although both scale with $U_c \sim x^{-1}$, presumably due to stronger upstream three dimensionality in the former.

Trentacoste and Sforza[10] claimed, based on their pressure-probe measurements, the validity of $U_c \sim x^{-n}$ in a characteristic region following the potential core for any aspect ratios and reported that the exponent n varies between 0.3 and 1 for AR=1–40. Our measurements, Fig. 1, do not support their claim nor do the measurements of Quinn[2] for AR=2, 5, 10, and 20 (those for the latter two are reproduced on the plot), although this was not noted by Quinn. Apparently, Quinn's results support our finding. In particular, in his case a distinct half-power-law region can be identified even for AR=10; but the relation $U_c \sim x^{-n}$ $(n < 1)$ is not valid for AR=5 and 2. Nevertheless, there is a discernible difference between the present results and those of Quinn, as seen in Fig. 1. This can be explained because he used sharp-edged orifice plates while our nozzle-exit profiles are smoothly (radial) contracting. Mi *et al.*[11] have observed significant differences between mixing characteristics of two circular jets issuing, respectively, from a smooth contraction and a sharp-edged orifice plate. Similar observations are also made in the work of Deo[8] for plane jets.

It is proposed from the above results that a high-AR rectangular jet may be distinguished between the three flow zones: i.e., (1) an initial quasi-plane-jet zone including a potential core, (2) a transition zone, and (3) a final quasi-axisymmetric-jet zone. However, to verify this unambiguously, it is necessary to further check the centerline evolutions of the rms velocity normalized by the local mean u_c'/U_c and the mean velocity half-width $Y_{1/2}$ in the minor axis plane. Previous studies for the plane jet (e.g., Refs. 6 and 12) and the circular jet (e.g., Ref. 13) have consistently identified the following self-similar relations in their respective far field:

$$u_c'/U_c \approx C(\text{const}), \quad Y_{1/2} \sim x.$$

These relations should also be evident in both the initial quasi-plane-jet and final quasi-axisymmetric-jet zones of the rectangular jet, if they exist.

Figure 2 presents the axial variation of u_c'/U_c in the rectangular jets, along with those of Deo[8] for a plane jet (AR =60) and of Mi *et al.*[9] for a circular jet. The log-log presentation is used to highlight the initial-field variation. A plateau is evident in u_c'/U_c over the range $0.7 < x/D_e < 7$ for all jets of $\text{AR} \geqslant 30$, although more scattering than U_c/U_{oc} as expected, and the plateau widens with increasing AR. This supports the existence of the initial quasi-plane-jet zone as suggested by the mean velocity decay. Figure 2 also demonstrates that, for AR=15 and 30, u_c'/U_c approaches the far-field asymptotic value at $x/D_e \geqslant 20$ which is nearly the same as that for a circular jet. Our measurements for higher values of AR (40–120) were made over a downstream distance of $13D_e$–$18D_e$ (equivalent to $160h$), which is not sufficiently far downstream into the final quasi-axisymmetric-jet zone. Nevertheless, the trends in both u_c'/U_c and U_c/U_{oc} suggest that all the rectangular jets asymptote to an axisymmetric-jet-like flow at $x/D_e \geqslant 25$, regardless of the

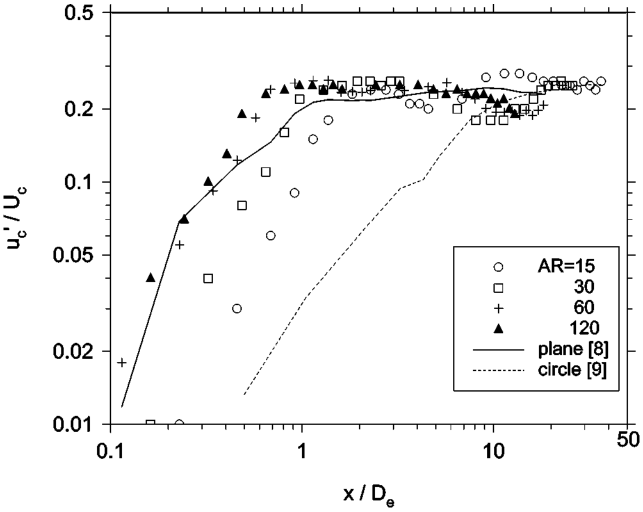

FIG. 2. Centerline variation of the normalized rms velocity u_c'/U_c vs x/D_e.

magnitude of AR. In the transition zone, u_c'/U_c dips with the deceleration of decay of the mean velocity as clearly seen in Fig. 1. Comparison of Fig. 2 with Fig. 1 reveals that the region in which $u_c'/U_c \approx C$ in the near or far field coincides reasonably well with that of $U_c \sim x^{-1/2}$ or $U_c \sim x^{-1}$.

Figure 1 and 2 also suggest that the rate of entrainment of the high-AR jet increases with increasing AR, at least, in the first two zones, i.e., in the quasi-plane-jet and transition zones. This point is reflected consistently by shorter potential core, higher decay rate of U_c/U_{oc}, and greater growth rate of u_c'/U_c, in the quasi-plane-jet zone, with higher AR. It is consistent too with Zaman's measurements[3] of the mass flow rate for AR=2–38, which suggests that the entrainment rate increases significantly with aspect ratio as long as $\text{AR} \geqslant 8$.

The spreading behavior of the mean velocity field also provides a support for an initial quasi-plane-jet zone in the high-AR jet. Figure 3 shows the normalized half-widths $Y_{1/2}/D_e$ and $Z_{1/2}/D_e$ obtained by Quinn[2] for AR=10 and 20 and by Trentacoste and Sforza[10] for AR=40. Here, the half-widths $Y_{1/2}$ and $Z_{1/2}$ are, respectively, lateral distances from the jet axis to the y and z locations at which the axial mean velocity is $0.5U_c$. We also plot the linear relation of $Y_{1/2} \approx 0.104x$ obtained by Deo[8] for a plane jet ($\text{Re}_h = 10\,000$ and AR=60) and that of the half radius, $R_{1/2} \approx 0.094(x-1.42)$, by Quinn[13] for a circular jet ($\text{Re}_D = 204\,000$). Generally for rectangular jets, $Y_{1/2}$ initially grows at a much greater rate than does $Z_{1/2}$. This is due mainly to the near-field large-scale structure, if any, being more coherent along the long than the short sides of the nozzle.

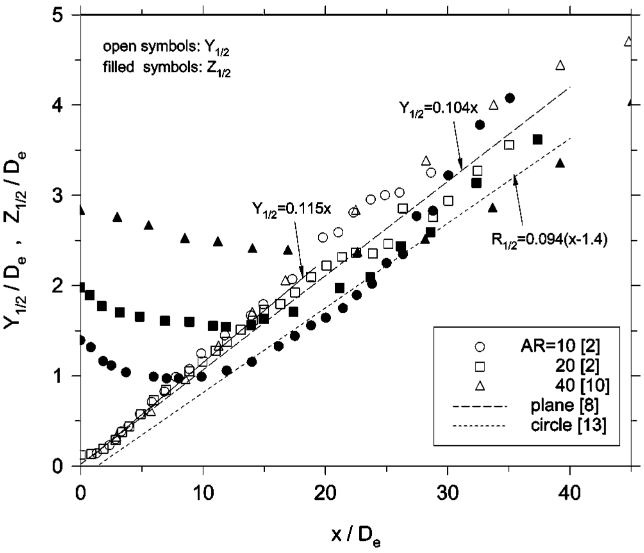

FIG. 3. Streamwise variations of the half-widths $Y_{1/2}$ and $Z_{1/2}$ vs x/D_e.

The growth of $Y_{1/2}$ is approximately linear until it crosses over $Z_{1/2}$, where the so-called "axis switch" occurs. This linear growth appears to be approximately the same for all AR and has a greater slope (≈ 0.115) than that (≈ 0.104) of the planar counterpart, presumably owing to the large-scale structure being more three dimensional in the rectangular jet. As expected, the axial extent of the linear region increases with AR. It is also clear that $Z_{1/2}$ decreases slightly before being taken over by $Y_{1/2}$. This decrease is less significant when AR is increased. Further downstream, the growth in $Y_{1/2}$ decelerates while $Z_{1/2}$ turns to grow. Both finally merge in the final quasi-axisymmetric-jet zone. The initial slight fall of $Z_{1/2}$ is also expected to occur in the plane jet from a nozzle with two parallel sidewalls, because each wall produces a growing boundary layer so that $Z_{1/2}$ should be smaller than the long-side length w and also decrease slowly with x. Therefore, the above observation of the linear growth of $Y_{1/2}$ in the initial flow region provides further evidence to support our deduction of an initial quasi-plane-jet zone in high-AR rectangular jets.

At this point in the discussion, one may ask how distinct the initial quasi-plane-jet is for different values of AR. We address this by using the streamwise evolutions of U_c/U_{oc} and u_c'/U_c against x/h, just as for the plane jet. Figures 4(a) and 4(b) show the results. It appears that the centerline variation of the mean velocity virtually exhibits no difference for different AR as long as the jet is within the quasi-plane-jet zone and so does the rms, despite more scattering. Interestingly for AR=30, 40, 60, or 120, the flow may be considered as a quasi-plane-jet approximately at $x/h \leqslant \text{AR}$ (30, 40, 60, or 120). Even for AR=15, both U_c/U_{oc} and u_c'/U_c differ very slightly from those for other values of AR over the region $x/h \leqslant 12$. Nevertheless, Fig. 4(b) shows a discernible difference in u_c'/U_c between the rectangular and plane jets, particularly, in the near field at $x/h < 20$. The turbulent intensity is significantly lower in the region $5 \leqslant x/h \leqslant 15$ for the plane jet. Correspondingly, the plane jet has a smaller value of $Y_{1/2}$ over the similar region (see Fig. 3). These discrepancies can be explained by a higher degree of three dimensionality of the underlying flow structure in the rectangular than plane jet, since primary ring-like three-dimensional (3D) vortices occur in the former while 2D roller-like vortices exist in the latter.

Some interesting points can be made by comparing Fig. 4 with Figs. 1 and 2. Although scaling the plots with h results in almost identical data of either U_c/U_{oc} or u_c'/U_c for all values of AR at $x/h \leqslant 12$, these data become quite "messy" further downstream (Fig. 4) due to the improper use of h as a length scale beyond the initial quasi-plane-jet zone. By

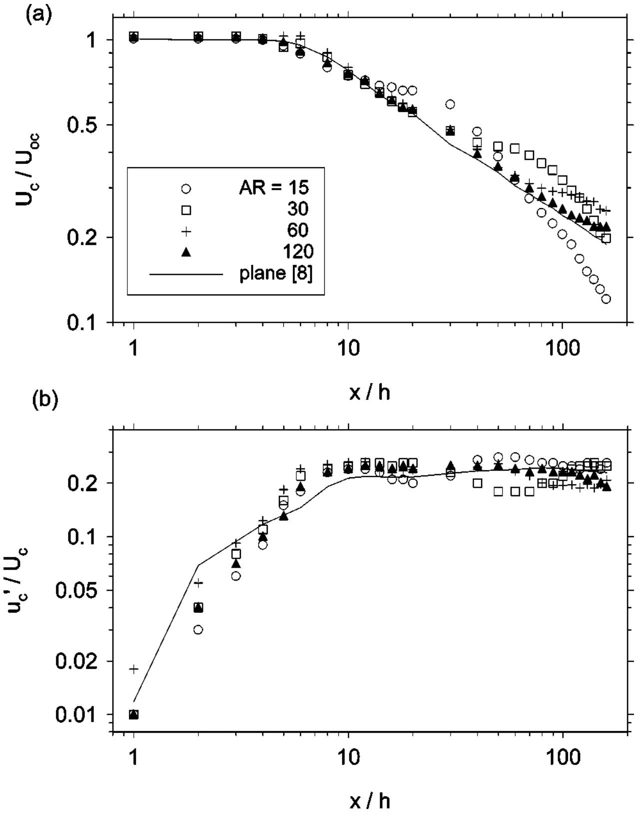

FIG. 4. Centerline variations of (a) U_c/U_{oc} and (b) u_c'/U_c vs x/h.

comparison, the equivalent diameter D_e is more appropriate to act as a length scale to characterize the rectangular jets. This can be seen from Figs. 1 and 2 that show similar variations for all AR$>$15 of both U_c/U_{oc} and u_c'/U_c throughout the measured range of x, which tend to asymptote to single curves as x increases. Even for AR=15, where no quasi-plane-jet zone is present, this observation still can apply for the far field. Further, D_e is more sensible than h as a length scale to describe the jet mixing since the use of D_e (not h) can reflect the true fact that the rate of mixing between the jet and its surroundings increases with AR.[3] In addition, it should be noted that, if the Reynolds number is defined with D_e, its value Re_{D_e} will differ significantly from $\mathrm{Re}_h \approx 10^4$ for each AR and increase with AR since $\mathrm{Re}_{D_e} \approx 1.13\mathrm{AR}^{1/2}\mathrm{Re}_h$. However, the corresponding values of $\mathrm{Re}_{D_e}(\approx 4.4 \times 10^4 - 12.4 \times 10^4)$ are all so high that the Reynolds number effect should be negligible. The analysis of Dimotakis[14] suggests that the critical value of the Reynolds number is 10^4, above which Re_{D_e} will have little influence on jets' mixing characteristics. Otherwise, should the effect of Re_{D_e} remain significant, the present results of Fig. 1 would contradict those of Deo.[8] He has found that, for $\mathrm{Re}_h \leqslant 16\,500$, the plane-jet decay rate of U_c decreases as Re_h increases. On the contrary, the present decay rate increases with AR, and thus Re_{D_e}, throughout the initial quasi-plane-jet zone (Fig. 1).

In summary, a rectangular turbulent jet with sufficiently high aspect ratio ($>$15) is found to undergo in succession the following three distinct zones: (1) initial quasi-plane-jet zone, (2) transition zone, and (3) final quasi-axisymmetric-jet zone. The jet may be treated approximately as a plane jet as long as it is in the first zone. The axial extent of the first zone increases with the aspect ratio. The normalized properties, such as U_c/U_{oc} and u_c'/U_c, in this zone show only slight disparity, based on their evolution against x/h, for different aspect ratios. However, the streamwise evolutions of u_c'/U_c and $Y_{1/2}$ over the first zone differ discernibly from those in comparable plane jets.

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