

# Liquid-gas phase transition and Coulomb instability of asymmetric nuclear systems

P. Wang<sup>a</sup>, D. B. Leinweber<sup>a</sup>, A. W. Thomas<sup>a,b</sup> and A. G. Williams<sup>a</sup>

<sup>a</sup>*Special Research Center for the Subatomic Structure of Matter (CSSM) and Department of Physics, University of Adelaide 5005, Australia*

<sup>b</sup>*Jefferson Laboratory, 12000 Jefferson Ave., Newport News, VA 23606 USA*

We use a chiral  $SU(3)$  quark mean field model to study the properties of nuclear systems at finite temperature. The liquid-gas phase transition of symmetric and asymmetric nuclear matter is discussed. For two formulations of the model the critical temperature,  $T_c$ , for symmetric nuclear matter is found to be 15.8 MeV and 17.9 MeV. These values are consistent with those derived from recent experiments. The limiting temperatures for finite nuclei are in good agreement with the experimental points.

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## I. INTRODUCTION

The determination of the properties of hadronic matter at finite temperature and density is a fundamental problem in nuclear physics. In particular, the study of liquid-gas phase transition in medium energy heavy-ion collisions is of considerable interest. Many intermediate-energy collision experiments have been performed [1] to investigate the unknown features of the highly excited or hot nuclei formed in collisions [2,3]. Theoretically, much effort has been devoted to studying the equation of state for nuclear matter and to discussing the critical temperature,  $T_c$ . The calculated critical temperature of symmetric nuclear matter lies in the range 13 - 24 MeV for various phenomenological models [4]- [12]. Glendenning [13] first discussed the phase transition with more than one conserved charge and applied the method to the possible transition to quark matter in the core of a neutron star. Müller and Serot [14] discussed asymmetric nuclear matter using the stability conditions on the free energy, conservation laws and the Gibbs criterion for the liquid-gas phase transition. The liquid-gas phase transition of asymmetric nuclear matter has also

been discussed in effective chiral models [15,16]. It was found that the critical temperature decreases with increasing asymmetry parameter,  $\alpha$ .

For finite nuclei, there is another temperature which is called the limiting temperature,  $T_{lim}$ , as pointed out by Levit and Bonche [5]. Below the limiting temperature, nuclei can exist in equilibrium with the surrounding vapor. When the temperature is higher than  $T_{lim}$  the nuclei are unstable and will fragment. This is called Coulomb instability. The size effect and Coulomb interaction are important in the determination of the limiting temperature, resulting in a lower limiting temperature compared with that for infinite nuclear matter [17]. Recently, Natowitz *et al.* obtained the limiting temperature by using a number of different experimental measurements [18]. From these observations the authors extracted the critical temperature of infinite nuclear matter  $T_c = 16.6 \pm 0.86$  MeV [19]. Their results show that the limiting temperature is in good agreement with the previous calculations employing either a chiral symmetric model [20] or the Gogny interaction [8].

To study the properties of hadronic matter, we need phenomenological models since QCD cannot yet be used directly. The symmetries of QCD can be used to determine largely how the hadrons interact with each other. On this basis, models based on  $SU(2)_L \times SU(2)_R$  symmetry and scale invariance were proposed. These effective models have been widely used in recent years to investigate nuclear matter and finite nuclei, both at zero temperature and at finite temperature [20]- [22]. Papazoglou *et al.* extended the chiral effective models to  $SU(3)_L \times SU(3)_R$  including the baryon octets [23,24]. As well as the models based on the hadron degrees of freedom, there are additional models based on quark degrees of freedom, such as the quark meson coupling model [25,26], the cloudy bag model [27], the NJL model [28] and the quark mean field model [29]. Recently, we proposed a chiral  $SU(3)$  quark mean field model and applied to investigate hadronic matter and quark matter [30]- [33]. This model is very successful in describing the properties of nuclear matter [30], strange matter [31,32], finite nuclei and hypernuclei [33] at zero temperature. A successful model should describe well the properties of nuclear matter, not only at zero temperature but also at finite temperature. In this paper, we will apply the chiral  $SU(3)$  quark mean field model to study the liquid-gas phase transition and Coulomb instability of asymmetric nuclear system and compare our results with the recent experimental analysis.

The paper is organized as follows. The model is introduced in section II. In section III, we apply it to investigate nuclear matter at finite temperature. The numerical results are discussed in section IV and section V summarises the main results.

## II. THE MODEL

Our considerations are based on the chiral  $SU(3)$  quark mean field model (for details see Refs. [31,33]), which contains quarks and mesons as basic degrees of freedom. Quarks are confined into baryons by an effective potential. The quark meson interaction and meson self-interaction are based on  $SU(3)$  chiral symmetry. Through the mechanism of spontaneous chiral symmetry breaking, the resulting constituent quarks and mesons (except for the pseudoscalars) obtain masses. The introduction of an explicit symmetry breaking term in the meson self-interaction generates the masses of the pseudoscalar mesons which satisfy the partially conserved axial-vector current (PCAC) relation. The explicit symmetry breaking term of the quark meson interaction leads in turn to reasonable hyperon potentials in

hadronic matter. For completeness, we introduce the main concepts of the model in this section.

In the chiral limit, the quark field  $q$  can be split into left and right-handed parts  $q_L$  and  $q_R$ :  $q = q_L + q_R$ . Under  $SU(3)_L \times SU(3)_R$  they transform as

$$q'_L = L q_L, \quad q'_R = R q_R. \quad (1)$$

The spin-0 mesons are written in the compact form

$$M(M^+) = \Sigma \pm i\Pi = \frac{1}{\sqrt{2}} \sum_{a=0}^8 (\sigma^a \pm i\pi^a) \lambda^a, \quad (2)$$

where  $\sigma^a$  and  $\pi^a$  are the nonets of scalar and pseudoscalar mesons, respectively,  $\lambda^a$  ( $a = 1, \dots, 8$ ) are the Gell-Mann matrices, and  $\lambda^0 = \sqrt{\frac{2}{3}} I$ . The alternatives, plus and minus signs correspond to  $M$  and  $M^+$ . Under chiral  $SU(3)$  transformations,  $M$  and  $M^+$  transform as  $M \rightarrow M' = LMR^+$  and  $M^+ \rightarrow M'^+ = RM^+L^+$ . In a similar way, the spin-1 mesons are introduced through:

$$l_\mu(r_\mu) = \frac{1}{2} (V_\mu \pm A_\mu) = \frac{1}{2\sqrt{2}} \sum_{a=0}^8 (v_\mu^a \pm a_\mu^a) \lambda^a \quad (3)$$

with the transformation properties:  $l_\mu \rightarrow l'_\mu = Ll_\mu L^+$ ,  $r_\mu \rightarrow r'_\mu = Rr_\mu R^+$ . The matrices  $\Sigma$ ,  $\Pi$ ,  $V_\mu$  and  $A_\mu$  can be written in a form where the physical states are explicit. For the scalar and vector nonets, we have the expressions

$$\Sigma = \frac{1}{\sqrt{2}} \sum_{a=0}^8 \sigma^a \lambda^a = \begin{pmatrix} \frac{1}{\sqrt{2}} (\sigma + a_0^0) & a_0^+ & K^{*+} \\ a_0^- & \frac{1}{\sqrt{2}} (\sigma - a_0^0) & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \zeta \end{pmatrix}, \quad (4)$$

$$V_\mu = \frac{1}{\sqrt{2}} \sum_{a=0}^8 v_\mu^a \lambda^a = \begin{pmatrix} \frac{1}{\sqrt{2}} (\omega_\mu + \rho_\mu^0) & \rho_\mu^+ & K_\mu^{*+} \\ \rho_\mu^- & \frac{1}{\sqrt{2}} (\omega_\mu - \rho_\mu^0) & K_\mu^{*0} \\ K_\mu^{*-} & \bar{K}_\mu^{*0} & \phi_\mu \end{pmatrix}. \quad (5)$$

Pseudoscalar and pseudovector nonet mesons can be written in a similar fashion.

The total effective Lagrangian has the form:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{q0} + \mathcal{L}_{qM} + \mathcal{L}_{\Sigma\Sigma} + \mathcal{L}_{VV} + \mathcal{L}_{\chi SB} + \mathcal{L}_{\Delta m_s} + \mathcal{L}_h + \mathcal{L}_c, \quad (6)$$

where  $\mathcal{L}_{q0} = \bar{q} i\gamma^\mu \partial_\mu q$  is the free part for massless quarks. The quark-meson interaction  $\mathcal{L}_{qM}$  can be written in a chiral  $SU(3)$  invariant way as

$$\begin{aligned} \mathcal{L}_{qM} &= g_s \left( \bar{\Psi}_L M \Psi_R + \bar{\Psi}_R M^+ \Psi_L \right) - g_v \left( \bar{\Psi}_L \gamma^\mu l_\mu \Psi_L + \bar{\Psi}_R \gamma^\mu r_\mu \Psi_R \right) \\ &= \frac{g_s}{\sqrt{2}} \bar{\Psi} \left( \sum_{a=0}^8 \sigma_a \lambda_a + i \sum_{a=0}^8 \pi_a \lambda_a \gamma^5 \right) \Psi - \frac{g_v}{2\sqrt{2}} \bar{\Psi} \left( \sum_{a=0}^8 \gamma^\mu v_\mu^a \lambda_a - \sum_{a=0}^8 \gamma^\mu \gamma^5 a_\mu^a \lambda_a \right) \Psi. \end{aligned} \quad (7)$$

In the mean field approximation, the chiral-invariant scalar meson  $\mathcal{L}_{\Sigma\Sigma}$  and vector meson  $\mathcal{L}_{VV}$  self-interaction terms are written as [31,33]

$$\begin{aligned}\mathcal{L}_{\Sigma\Sigma} = & -\frac{1}{2}k_0\chi^2(\sigma^2 + \zeta^2) + k_1(\sigma^2 + \zeta^2)^2 + k_2\left(\frac{\sigma^4}{2} + \zeta^4\right) + k_3\chi\sigma^2\zeta \\ & -k_4\chi^4 - \frac{1}{4}\chi^4\ln\frac{\chi^4}{\chi_0^4} + \frac{\delta}{3}\chi^4\ln\frac{\sigma^2\zeta}{\sigma_0^2\zeta_0},\end{aligned}\quad (8)$$

$$\mathcal{L}_{VV} = \frac{1}{2}\frac{\chi^2}{\chi_0^2}\left(m_\omega^2\omega^2 + m_\rho^2\rho^2 + m_\phi^2\phi^2\right) + g_4\left(\omega^4 + 6\omega^2\rho^2 + \rho^4 + 2\phi^4\right),\quad (9)$$

where  $\delta = 6/33$ ;  $\sigma_0$ ,  $\zeta_0$  and  $\chi_0$  are the vacuum expectation values of the corresponding mean fields  $\sigma$ ,  $\zeta$  and  $\chi$ . The Lagrangian  $\mathcal{L}_{\chi SB}$  generates nonvanishing masses for the pseudoscalar mesons

$$\mathcal{L}_{\chi SB} = \frac{\chi^2}{\chi_0^2}\left[m_\pi^2 F_\pi\sigma + \left(\sqrt{2}m_K^2 F_K - \frac{m_\pi^2}{\sqrt{2}}F_\pi\right)\zeta\right],\quad (10)$$

leading to a nonvanishing divergence of the axial currents which in turn satisfy the relevant PCAC relations for  $\pi$  and  $K$  mesons. Pseudoscalar and scalar mesons as well as the dilaton field,  $\chi$ , obtain mass terms by spontaneous breaking of chiral symmetry in the Lagrangian (8). The masses of the  $u$ ,  $d$  and  $s$  quarks are generated by the vacuum expectation values of the two scalar mesons  $\sigma$  and  $\zeta$ . To obtain the correct constituent mass of the strange quark, an additional mass term has to be added:

$$\mathcal{L}_{\Delta m_s} = -\Delta m_s \bar{q} S q\quad (11)$$

where  $S = \frac{1}{3}(I - \lambda_8\sqrt{3}) = \text{diag}(0, 0, 1)$  is the strangeness quark matrix. Based on these mechanisms, the quark constituent masses are finally given by

$$m_u = m_d = -\frac{g_s}{\sqrt{2}}\sigma_0 \quad \text{and} \quad m_s = -g_s\zeta_0 + \Delta m_s,\quad (12)$$

where  $g_s$  and  $\Delta m_s$  are chosen to yield the constituent quark mass in vacuum – we use  $m_u = m_d = 313$  MeV and  $m_s = 490$  MeV. In order to obtain reasonable hyperon potentials in hadronic matter, it has been found necessary to include an additional coupling between strange quarks and the scalar mesons  $\sigma$  and  $\zeta$  [31]:

$$\mathcal{L}_h = (h_1\sigma + h_2\zeta)\bar{s}s.\quad (13)$$

In the quark mean field model, quarks are confined in baryons by the Lagrangian  $\mathcal{L}_c = -\bar{\Psi}\chi_c\Psi$  (with  $\chi_c$  given in Eq. (14), below). The Dirac equation for a quark field  $\Psi_{ij}$  under the additional influence of the meson mean fields is given by

$$\left[-i\vec{\alpha}\cdot\vec{\nabla} + \chi_c(r) + \beta m_i^*\right]\Psi_{ij} = e_i^*\Psi_{ij},\quad (14)$$

where  $\vec{\alpha} = \gamma^0\vec{\gamma}$ ,  $\beta = \gamma^0$ , the subscripts  $i$  and  $j$  denote the quark  $i$  ( $i = u, d, s$ ) in a baryon of type  $j$  ( $j = N, \Lambda, \Sigma, \Xi$ );  $\chi_c(r)$  is a confinement potential, i.e. a static potential providing confinement of quarks by meson mean-field configurations. The quark effective mass,  $m_i^*$ , and energy,  $e_i^*$ , are defined as

$$m_i^* = -g_\sigma^i\sigma - g_\zeta^i\zeta + m_{i0}\quad (15)$$

and

$$e_i^* = e_i - g_\omega^i \omega - g_\phi^i \phi - g_\rho^i \rho, \quad (16)$$

where  $e_i$  is the energy of the quark under the influence of the meson mean fields. Here  $m_{i0} = 0$  for  $i = u, d$  (nonstrange quark) and  $m_{i0} = \Delta m_s = 29$  MeV for  $i = s$  (strange quark). Using the solution of the Dirac equation (14) for the quark energy  $e_i^*$  it has been common to define the effective mass of the baryon  $j$  through the ansatz:

$$M_j^* = \sqrt{E_j^{*2} - \langle p_{j\,cm}^{*2} \rangle}, \quad (17)$$

where  $E_j^* = \sum_i n_{ij} e_i^* + E_{j\,spin}$  is the baryon energy and  $\langle p_{j\,cm}^{*2} \rangle$  is the subtraction of the contribution to the total energy associated with spurious center of mass motion. In the expression for the baryon energy  $n_{ij}$  is the number of quarks with flavor "i" in a baryon with flavor  $j$ , with  $j = N \{p, n\}, \Sigma \{\Sigma^\pm, \Sigma^0\}, \Xi \{\Xi^0, \Xi^-\}, \Lambda$  and  $E_{j\,spin}$  is the correction to the baryon energy which is determined from a fit to the data for baryon masses.

There is an alternative way to remove the spurious c. m. motion and determine the effective baryon masses. In Ref. [34], the removal of the spurious c. m. motion for three quarks moving in a confining, relativistic oscillator potential was studied in some detail. It was found that when an external scalar potential was applied, the effective mass obtained from the interaction Lagrangian could be written as

$$M_j^* = \sum_i n_{ij} e_i^* - E_j^0, \quad (18)$$

where  $E_j^0$  was calculated to be only very weakly dependent on the external field strength. We therefore use Eq. (18), with  $E_j^0$  a constant, independent of the density, which is adjusted to give a best fit to the free baryon masses.

Using the square root ansatz for the effective baryon mass, Eq. (17), the confining potential  $\chi_c$  is chosen as a combination of scalar (S) and scalar-vector (SV) potentials as in Ref. [31]:

$$\chi_c(r) = \frac{1}{2} [\chi_c^S(r) + \chi_c^{SV}(r)] \quad (19)$$

with

$$\chi_c^S(r) = \frac{1}{4} k_c r^2, \quad (20)$$

and

$$\chi_c^{SV}(r) = \frac{1}{4} k_c r^2 (1 + \gamma^0). \quad (21)$$

On the other hand, using the linear definition of effective baryon mass, Eq. (18), the confining potential  $\chi_c$  is chosen to be the purely scalar potential  $\chi_c^S(r)$ . The coupling  $k_c$  is taken as  $k_c = 1$  (GeV fm<sup>-2</sup>), which yields root-mean-square baryon charge radii (in the absence of a pion cloud [35]) around 0.6 fm.

### III. NUCLEAR MATTER AT FINITE TEMPERATURE

Based on the previously defined quark mean field model the Lagrangian density for nuclear matter is written as

$$\begin{aligned} \mathcal{L} = & \bar{\psi}(i\gamma^\mu\partial_\mu - M_N^*)\psi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\zeta\partial^\mu\zeta + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}\rho_{\mu\nu}\rho^{\mu\nu} \\ & - g_\omega\bar{\psi}\gamma_\mu\psi\omega^\mu - g_\rho\bar{\psi}\gamma_\mu\tau_3\psi\rho^\mu + \mathcal{L}_M, \end{aligned} \quad (22)$$

where

$$F_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu \quad \text{and} \quad \rho_{\mu\nu} = \partial_\mu\rho_\nu - \partial_\nu\rho_\mu. \quad (23)$$

The term  $\mathcal{L}_M$  represents the interaction between mesons which includes the scalar meson self-interaction  $\mathcal{L}_{\Sigma\Sigma}$ , the vector meson self-interaction  $\mathcal{L}_{VV}$  and the explicit chiral symmetry breaking term  $\mathcal{L}_{\chi SB}$ , all defined previously. The Lagrangian includes the scalar mesons  $\sigma$ ,  $\zeta$  and  $\chi$ , and the vector mesons  $\omega$  and  $\rho$ . The interactions between quarks and scalar mesons result in the effective nucleon mass  $M_N^*$ , The interactions between quarks and vector mesons generate the nucleon-vector meson interaction terms of equation (22). The corresponding vector coupling constants  $g_\omega$  and  $g_\rho$  are baryon dependent and satisfy the  $SU(3)$  relationship:  $g_\rho^p = -g_\rho^n = \frac{1}{3}g_\omega^p = \frac{1}{3}g_\omega^n$ .

At finite temperature and density, the thermodynamic potential is defined as

$$\Omega = -\frac{k_B T}{(2\pi)^3} \sum_{N=p,n} \int_0^\infty d^3\vec{k} \left\{ \ln \left( 1 + e^{-(E_N^*(k) - \nu_N)/k_B T} \right) + \ln \left( 1 + e^{-(E_N^*(k) + \nu_N)/k_B T} \right) \right\} - \mathcal{L}_M, \quad (24)$$

where  $E_N^*(k) = \sqrt{M_N^{*2} + \vec{k}^2}$ . The quantity  $\nu_N$  is related to the usual chemical potential,  $\mu_N$ , by  $\nu_N = \mu_N - g_\omega^N \omega - g_\rho^N \rho$ . The energy per unit volume and the pressure of the system are respectively  $\varepsilon = \Omega - \frac{1}{T} \frac{\partial \Omega}{\partial T} + \nu_N \rho_N$  and  $p = -\Omega$ , where  $\rho_N$  is the baryon density.

The mean field equation for meson  $\phi_i$  is obtained by the formula  $\partial\Omega/\partial\phi_i = 0$ . For example, the equations for  $\sigma$ ,  $\zeta$  are deduced as:

$$\begin{aligned} k_0\chi^2\sigma - 4k_1(\sigma^2 + \zeta^2)\sigma - 2k_2\sigma^3 - 2k_3\chi\sigma\zeta - \frac{2\delta}{3\sigma}\chi^4 + \frac{\chi^2}{\chi_0^2}m_\pi^2F_\pi \\ - \left(\frac{\chi}{\chi_0}\right)^2 m_\omega\omega^2 \frac{\partial m_\omega}{\partial \sigma} - \left(\frac{\chi}{\chi_0}\right)^2 m_\rho\rho^2 \frac{\partial m_\rho}{\partial \sigma} + \frac{\partial M_N^*}{\partial \sigma} \langle \bar{\psi}\psi \rangle = 0, \end{aligned} \quad (25)$$

$$k_0\chi^2\zeta - 4k_1(\sigma^2 + \zeta^2)\zeta - 4k_2\zeta^3 - k_3\chi\sigma^2 - \frac{\delta}{3\zeta}\chi^4 + \frac{\chi^2}{\chi_0^2} \left( \sqrt{2}m_k^2F_k - \frac{1}{\sqrt{2}}m_\pi^2F_\pi \right) = 0 \quad (26)$$

where

$$\langle \bar{\psi}\psi \rangle = \frac{1}{\pi^2} \int_0^\infty dk \frac{k^2 M_N^*}{E^*(k)} [n_n(k) + \bar{n}_n(k) + n_p(k) + \bar{n}_p(k)]. \quad (27)$$

In the above equation,  $n_q(k)$  and  $\bar{n}_q(k)$  are the nucleon and antinucleon distributions, respectively, expressed as

$$n_q(k) = \frac{1}{\exp[(E^*(k) - \nu_q)/k_B T] + 1} \quad (28)$$

and

$$n_q(k) = \frac{1}{\exp[(E^*(k) + \nu_q)/k_B T] + 1} \quad (q = n, p). \quad (29)$$

The equations for the vector mesons,  $\omega$  and  $\rho$ , are:

$$\frac{\chi^2}{\chi_0^2} m_\omega^2 \omega + 4g_4 \omega^3 + 12g_4 \omega \rho^2 = g_\omega^N (\rho_p + \rho_n), \quad (30)$$

$$\frac{\chi^2}{\chi_0^2} m_\rho^2 \rho + 4g_4 \rho^3 + 12g_4 \omega^2 \rho = \frac{1}{3} g_\omega^N (\rho_p - \rho_n), \quad (31)$$

where  $\rho_p$  and  $\rho_n$  are the proton and neutron densities, expressed as

$$\rho_q = \frac{1}{\pi^2} \int_0^\infty dk k^2 [n_q(k) - \bar{n}_q(k)] \quad (q = p, n). \quad (32)$$

In order to describe asymmetric nuclear matter, one can introduce the asymmetry parameter  $\alpha$  which is defined as

$$\alpha = \frac{\rho_n - \rho_p}{\rho_N}, \quad (33)$$

where  $\rho_N = \rho_n + \rho_p$ . For symmetric matter  $\alpha = 0$ , while for neutron matter  $\alpha = 1$ .

Let us now discuss the liquid-gas phase transition. For asymmetric nuclear matter we follow the thermodynamic approach of Refs. [13] and [14]. The system will be stable against separation into two phases if the free energy of a single phase is lower than the free energy in all two-phase configurations. This requirement can be formulated as [14]

$$F(T, \rho) < (1 - \lambda)F(T, \rho') + \lambda F(T, \rho''), \quad (34)$$

with

$$\rho = (1 - \lambda)\rho' + \lambda\rho'', \quad 0 < \lambda < 1, \quad (35)$$

where  $F$  is the Helmholtz free energy per unit volume. The two phases are denoted by a prime and a double prime. The stability condition implies the following set of inequalities:

$$\rho \left( \frac{\partial p}{\partial \rho} \right)_{T, \alpha} > 0, \quad (36)$$

$$\left( \frac{\partial \mu_p}{\partial \alpha} \right)_{T, p} < 0 \quad \text{or} \quad \left( \frac{\partial \mu_n}{\partial \alpha} \right)_{T, p} > 0. \quad (37)$$

If one of the stability conditions is violated, a system with two phases is energetically favored. The phase coexistence is governed by the Gibbs conditions:

$$\mu'_q(T, \rho') = \mu''_q(T, \rho''), \quad (q = n, p), \quad (38)$$

$$p'(T, \rho') = p''(T, \rho''), \quad (39)$$

where the temperature is the same in the two phases.

## IV. NUMERICAL RESULTS AND DISCUSSIONS

### A. Liquid-gas phase transition

The parameters in this model are determined by the meson masses in vacuum and the properties of nuclear matter which were listed in table I of Ref. [36]. We first discuss the liquid-gas phase transition of symmetric nuclear matter. In Fig. 1, we show the pressure of the system versus nucleon density at different temperatures using the square root ansatz for the effective nucleon mass (Eq. (17)). At low temperature, the pressure first increases and then decreases with increasing density. The  $p - \rho_N$  isotherms exhibit the form of two phase coexistence, with an unphysical region for each. At temperature  $T = 15.82$  MeV, there appears a point of inflection, where  $\partial p / \partial \rho_N = 0$ ,  $\partial^2 p / \partial \rho_N^2 = 0$ . This temperature is called the critical temperature. Symmetric nuclear matter can only be in gas phase above this temperature. The pressure versus nucleon density with the linear definition of effective nucleon mass (Eq. (18)) is shown in Fig. 2. In this case, the critical temperature is 17.9 MeV. In both cases, the calculated critical temperature is close to the recent result  $T_c = 16.6 \pm 0.86$  MeV, which was obtained by Natowitz *et al.* [19]. The critical temperature calculated with the Walecka model is larger than 20 MeV. This large  $T_c$  is partially caused by the large incompressibility modulus  $K$  ( $\simeq 540$  MeV). For the same reason the stiff Skyrme interaction SK1 gives a large  $T_c$ , which is close to 20 MeV, while the soft Skyrme interaction SKM\* gives a small  $T_c$  [7]. The QMC model and the effective model based on  $SU(2)$  chiral symmetry provide reasonable values of  $T_c$ , around 15-16 MeV [15,37].

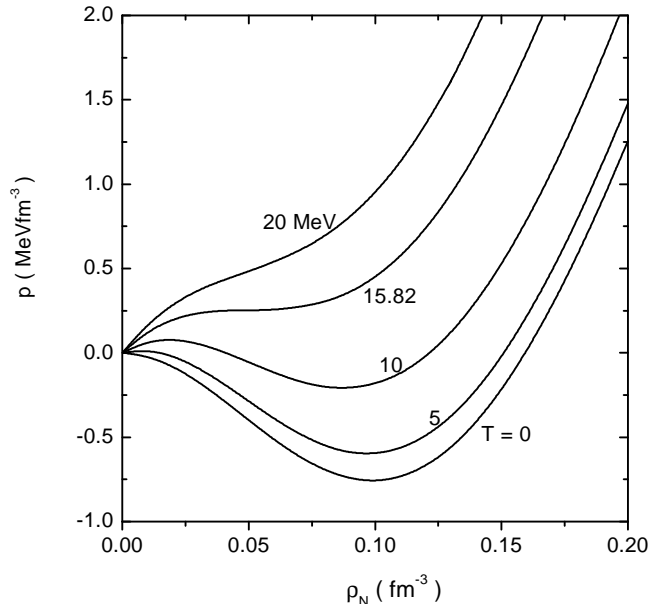


FIG. 1. The pressure of symmetric nuclear matter  $p$  versus nucleon density,  $\rho_N$ , at different temperatures with the square root ansatz of effective nucleon mass.

For the asymmetric case, the situation is more complicated. One cannot get the critical temperature from the  $p - \rho_N$  isotherms. The chemical potentials of the proton and neutron are different. We show the chemical potential versus asymmetric parameter  $\alpha$  at



temperature  $T = 10$  MeV and pressure  $p = 0.12$  MeVfm $^{-3}$  with the square root definition of effective nucleon mass in Fig. 3 (For convenience, we use the reduced chemical potential which is defined as  $\tilde{\mu}_N = \mu_N - M_N$ .) The solid and dashed lines are for proton and neutron respectively. The Gibbs equations (38) and (39) for phase equilibrium demand equal pressure and chemical potentials for two phases with different concentrations. The desired solution can be found by means of the geometrical construction shown in Fig. 3, which guarantees the same pressure and chemical potentials for protons and neutrons in the two phases with different asymmetry parameter  $\alpha$ .

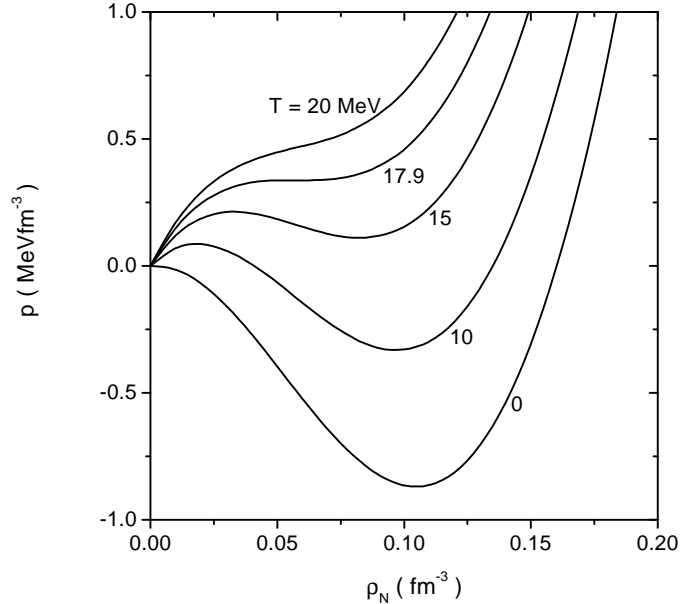


FIG. 2. The pressure of symmetric nuclear matter  $p$  versus nucleon density  $\rho_N$  at different temperatures with the linear definition of effective nucleon mass.

The pairs of solutions found using the method just described, yield a binodal curve which is shown in Fig. 4. There is a critical point where the pressure is about 0.205 MeV-fm $^{-3}$  and the corresponding asymmetry parameter is around 0.7. The two phases have the same  $\alpha$  and therefore the same density at this point. The binodal curve is divided into two branches by the critical point. One branch corresponds to the high density (liquid) phase, the other corresponds to the low density (gas) phase. Assume the system is initially prepared in the low density (gas) phase with  $\alpha = 0.55$ . When the pressure increases to some value, the two-phase region is encountered at point A and a liquid phase at B with a low  $\alpha$  begins to emerge. As the system is compressed, the gas phase evolves from point A to C, while the liquid phase evolves from B to D. If the pressure of the system continues to increase, the system will leave the two-phase region at point D. The gas phase disappears and the system is entirely in the liquid phase. This kind of phase transition is different from the normal first order phase transition where the pressure remains constant during phase transition. If the initial asymmetric nucleon gas is larger than some value, the system enters and leaves the two phase region on the same branch. For example, the system becomes unstable at point A' and a liquid phase with a higher nucleon density begins to emerge at B'. The system is compressed at a fixed total  $\alpha$ , with the gas phase evolving from A' to C' and the liquid phase from B' to D'. The system leaves the two-phase region point C' which is still in the original

gas phase. Therefore, for a given temperature, if the total asymmetry parameter of the system is larger than a critical value, the system cannot change completely into the liquid phase, however large the pressure. In other words, for a system with a fixed asymmetric parameter  $\alpha$ , there exists a critical temperature, above which the system can only be in the gas phase at any pressure.

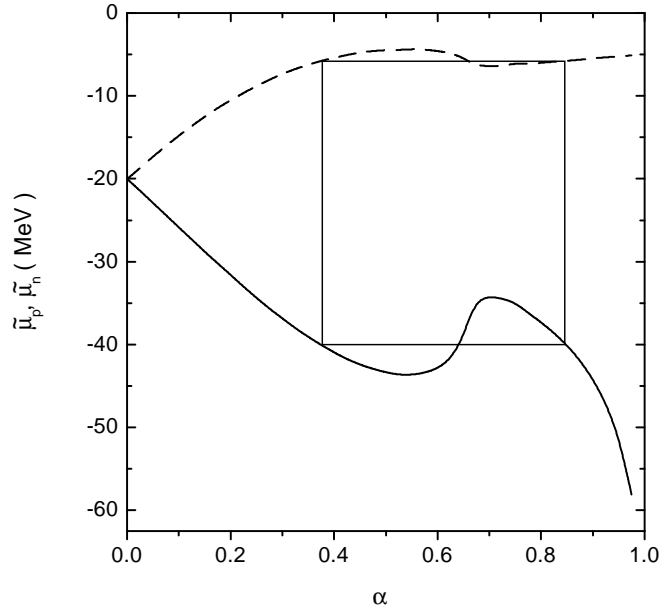


FIG. 3. Geometrical construction used to obtain the chemical potentials and asymmetric parameters in the two-phase coexistence at temperature  $T = 10$  MeV and  $p = 0.12$  MeVfm $^{-3}$ .

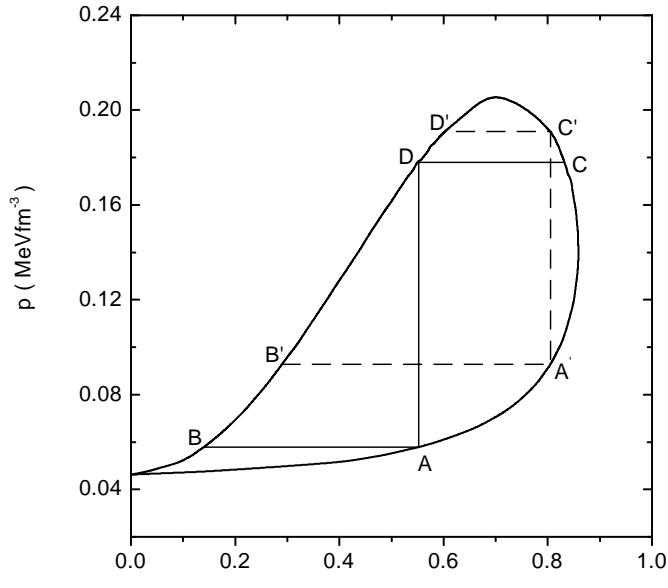


FIG. 4. Binodal curve at temperature  $T = 10$  MeV. The points A through D and A' through D' denote two kinds of phase transition.

The  $\alpha$  dependence of the critical temperature is shown in Fig. 5. The solid and dashed lines correspond to the square root ansatz and the linear definition of effective nucleon mass,

respectively.  $T_c$  decreases with increasing  $\alpha$ . For the square root case, when  $\alpha$  is less than 0.2, the decrease of  $T_c$  is very small. When  $\alpha$  is larger than 0.6,  $T_c$  decreases very fast. If  $\alpha$  is larger than 0.88, the system can only be in the gas phase at any temperature. In the linear case, the critical temperature is 2 MeV larger than that in the square root case. The liquid-gas phase transition can occur for nuclear matter with any  $\alpha$  if the temperature is lower than the critical temperature. The critical temperature of asymmetric nuclear matter was also studied in the Walecka model, the derivative scalar coupling model and the QMC model [10,37] where the authors found different critical temperatures for protons and neutrons. The lower critical temperature was chosen to be the  $T_c$  of the system as an approximation. Their results show that  $T_c$  decreases almost linearly with increasing  $\alpha$ , which is different from the results shown in Fig. 5. We use the stability conditions on the free energy, conservation laws and the Gibbs criterion for the liquid-gas phase transition of asymmetric nuclear matter, following Müller and Serot [14]. This is expected to be better than the approximation used in the earlier discussion.

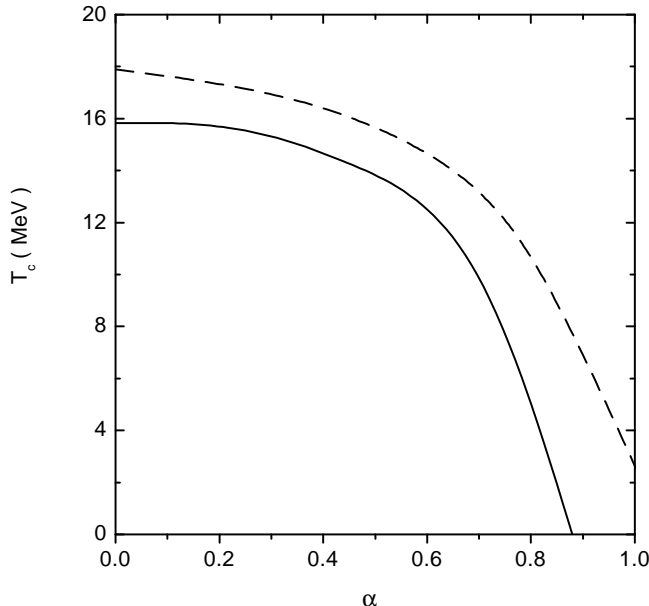


FIG. 5. The critical temperature  $T_c$  versus asymmetric parameter  $\alpha$ . The solid and dashed lines are for the square root and linear cases, respectively.

## B. Coulomb instability

In this subsection, we discuss the liquid-gas phase equilibrium for finite nuclei. Compared with the case of infinite nuclear matter, the size effect and Coulomb interaction are important for finite nuclei. When the Coulomb interaction is considered, the chemical potential for the protons will have an additional term corresponding to a uniformly charged sphere:

$$\mu_{Coul} = \frac{6}{5} \frac{Ze^2}{R}, \quad (40)$$

where  $Z$  and  $R$  are the charge number and radius of finite nuclei. Meanwhile, the pressure also has an extra term:

$$p_{Coul}(\rho) = \frac{Z^2 e^2}{5AR} \rho, \quad (41)$$

where  $A$  is the mass number of the nucleus and  $\rho$  is its density. For a liquid droplet, the surface pressure is

$$p_{surf}(T, \rho) = -2\gamma(T)/R, \quad (42)$$

In the above equation,  $\gamma(T)$  is the surface tension suggested by Goodman *et al.* [38]

$$\gamma(T) = (1.14 \text{MeVfm}^{-2}) \left[ 1 + \frac{3T}{2T_c} \right] \left[ 1 - \frac{T}{T_c} \right], \quad (43)$$

where  $T_c$  is the critical temperature for infinite symmetric nuclear matter. The Gibbs conditions for two phase equilibrium now becomes

$$p(T, \rho_L, \alpha_L) + p_{Coul}(\rho_L) + p_{surf}(T, \rho_L) = p(T, \rho_V, \alpha_V), \quad (44)$$

$$\mu_n(T, \rho_L, \alpha_L) = \mu_n(T, \rho_V, \alpha_V), \quad (45)$$

$$\mu_p(T, \rho_L, \alpha_L) + \mu_{Coul}(\rho_L) = \mu_p(T, \rho_V, \alpha_V). \quad (46)$$

When the temperature is higher than a temperature (limiting temperature), the above equations have no solution. The finite nuclei cannot exist in equilibrium with the surrounding vapor.

We show in Fig. 6 the mass number dependence of the limiting temperature,  $T_{lim}$ , for nuclei along the line of  $\beta$ -stability:

$$Z = 0.5A - 0.3 \times 10^{-2} A^{5/3}. \quad (47)$$

The solid and dashed lines correspond to the square root ansatz and the linear definition of the effective nucleon mass, respectively. The experimental values obtained recently by Natowitz [18] are also plotted in the figure for comparison. The calculated results in this model are in good agreement with the experimental data. The limiting temperature decreases with increasing mass number. This means that when the temperature is higher than the limiting temperature, the heavy nuclei will fragment to light nuclei. Two useful parameterizations of  $T_{lim}/T_c$ , valid for  $10 \leq A \leq 208$ , are  $(T_{lim}/T_c) = 0.611 - 0.00193A + 3.32 \times 10^{-6} A^2$  (square root case) and  $(T_{lim}/T_c) = 0.591 - 0.00203A + 3.80 \times 10^{-6} A^2$  (linear case), which are comparable with that given in Ref. [19]. Calculations from other models also show that  $T_{lim}$  decreases with increasing mass number. Numerical results show that there is a relationship between  $T_{lim}$  and  $T_c$ . For larger  $T_c$ , the calculated  $T_{lim}$  is also larger. Therefore, the results for  $T_{lim}$  found with the Walecka model and the SK1 interaction are larger, while the results of the SKM\* interaction are smaller compared with the experiments [37]. The values of  $T_{lim}$  calculated for both the QMC model and the effective model suggested by Furnstahl *et al.* [20,37] are close to our results.

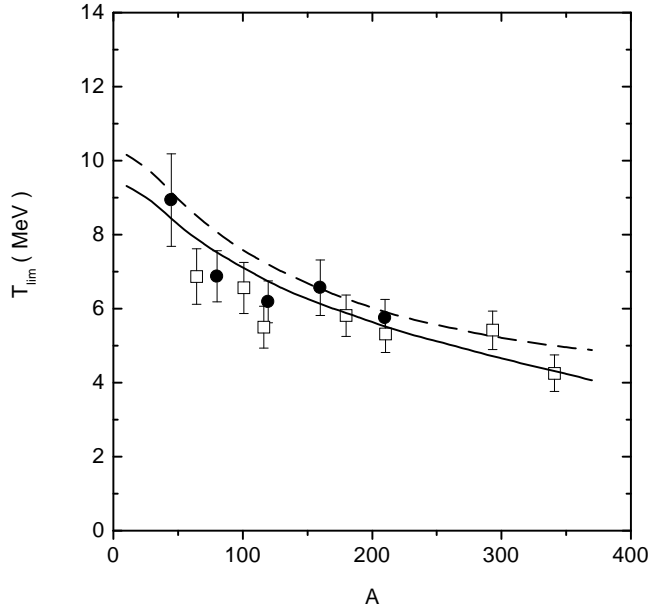


FIG. 6. The limiting temperature  $T_{lim}$  versus mass number  $A$  of finite nuclei. The solid and dashed lines are for the square root and linear cases, respectively. The points with error bars are from Ref. [19]. The data derived from the double isotope yield ratio and thermal bremsstrahlung measurements are represented by the filled circles and open squares, respectively.

## V. SUMMARY

In this paper, we extended the chiral  $SU(3)$  quark mean field model to finite temperature and density. This model describes properties of infinite nuclear matter, finite nuclei and hypernuclei very well at zero temperature. The saturation properties and compression modulus of nuclear matter are reasonable. The hyperon potentials are close to the empirical values for hadronic matter. The results for finite nuclei and hypernuclei are also consistent with experiment. We therefore want to know whether this model can also describe the system at finite temperature. The liquid-gas phase transition of infinite nuclear matter and the Coulomb instability of finite nuclei at finite temperature are discussed in this model. All the parameters have been determined in earlier papers and there is no further parameter to be adjusted. The critical temperatures for two-phase coexistence of symmetric nuclear matter,  $T_c$ , is 15.82 MeV (square root case) and 17.9 MeV (linear case). Both of these values are close to the recent experimental value,  $16.6 \pm 0.86$  MeV.  $T_c$  is found to decrease with either increasing asymmetry parameter,  $\alpha$ , or increasing mass number for finite nuclei. The critical temperature,  $T_c$ , in the linear case is about 2 MeV larger than that in the square root case. The values in both cases are in good agreement with the experimental values found in Refs. [18,19].

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