Reality of the fundamental topological structure in the QCD vacuum

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(Received 1 July 2005; published 18 August 2005)

Long-range order of a specific kind has recently been found directly in configurations dominating the regularized QCD path integral. In particular, a low-dimensional global structure was identified in typical space-time distributions of topological charge defined via the overlap Dirac matrix. The presence of the order has been concluded from the fact that the structure disappears after random permutation of position coordinates in measured densities. Here we complete the argument for the reality of this structure (namely the conjecture that its existence is a consequence of QCD dynamics and not an artifact of the overlap-based definition of lattice topological field) by showing that the structure ceases to exist after randomizing the space-time coordinates of the underlying gauge field. This implies that the long-range order present in the overlap-based topological density is indeed a manifestation of the QCD vacuum, and that the notion of the *fundamental structure* (structure involving relevant features at all scales) is viable.

DOI: 10.1103/PhysRevD.72.034506 PACS numbers: 11.15.Ha, 11.30.Rd

Numerical lattice gauge theory represents a powerful tool for obtaining quantitative predictions of QCD from first principles. However, apart from extracting physical quantities via numerical simulation, lattice theory is also being used in attempts to decipher the nature of the QCD vacuum. This is quite natural since the latter problem is frequently approached via the *hypothesis* that there exist certain well-defined objects ("structure") that dominate the behavior of typical gauge configurations contributing to Euclidean QCD path integral. Specific properties of such objects are then expected to encode the mechanism QCD uses to induce confinement, spontaneous chiral symmetry breaking, and other effects. If one accepts this logic, then a straightforward approach to the problem of QCD vacuum is to search for a well-defined structure in configurations dominating the evaluation of physical observables in regularized theory, i.e. in equilibrium Monte Carlo configurations of finite lattice systems. Unfortunately, for a long time, such a direct approach has not been fruitful since no obvious structure has been observed either in unmodified equilibrium configurations of the fundamental gauge field nor in the configurations of relevant composite fields derived from it. In fact, the configurations in accessible ensembles appeared to be more or less structureless.

The usual explanation of this fact is rather vague and involves variations on the proposition that a large entropy of fluctuations at the scale of the cutoff obscures any ordered structure that might be present (the *entropy problem*). However, when making this argument, one should (at least mentally) distinguish two possible origins of ultraviolet fluctuations in question. (i) Given a lattice cutoff $\Lambda = 1/a$ there are legitimate *physical* QCD fluctuations at this scale. There is no reason to expect that this part of the fluctuations at the scale of the lattice cutoff should be structureless. Indeed, it would be quite unnatural if the hypothesis of the structure applied only at long distances in the theory with nontrivial ultraviolet behavior. (ii) There are unphysical fluctuations that appear only as artifacts of a field-theoretic description of strong interactions. Such fluctuations are regularization dependent and can be entirely structureless. From this point of view, the entropy problem is not really the problem of large entropy associated with ultraviolet fluctuations. It is rather the problem of a large contribution of artifacts relative to physically relevant fluctuations at the scale of the cutoff. The severity of the issue can thus strongly depend on the lattice action used to define QCD, as well as on the choice of lattice operators for composite fields of interest.

Adopting the above (heuristic) logic as a starting point, the issue of identifying the QCD vacuum structure in unmodified equilibrium configurations has been revisited in Refs. [1,2]. In particular, the configurations of the lattice topological field defined via the Dirac kernel of exactly chiral lattice fermions [3,4] have been computed on unmodified equilibrium gauge backgrounds of pure-glue QCD. The underlying expectation was that the structure, not apparent in the gauge field itself, could become visible in this particular composite field. This was motivated by the fact that the corresponding topological charge density (TChD) operator is constructed in a very different manner than the standard lattice operators, which translates into beautiful continuumlike behavior of this composite field already at the regularized level [5–7]. At the same time, and more importantly for our purposes, it is expected that the lattice operators in this class are necessarily nonultralocal (but still local), similar to nonultralocality of the associated fermionic action [8]. This should soften the impact of ultraviolet gauge fluctuations on the topological field. Moreover, such *chiral smoothing* [9] is expected to be very efficient in eliminating the structureless artifacts-related ultraviolet fluctuations, while still preserving the physical short-distance fluctuations [10], thus providing the window of opportunity for avoiding the entropy problem.

The numerical experiments of Refs. [1,2] indeed revealed the existence of a nontrivial space-time structure in the overlap-based topological field, contrary to the absence of an observable order when standard naive operators are used.[1] One particular manifestation of the structure is that the topological charge in typical configurations organizes into two sign-coherent locally low-dimensional *sheets*.[2] The sheets can be ''tiled'' with 3D sign-coherent elementary cubes connected via 2D faces but not with 4D coherent cubes. In fact, the fraction of space-time occupied by connected sign-coherent regions built of 4D hypercubes scales to zero in the continuum limit, thus excluding the possibility of the coherence on smooth 4D manifolds. The double-sheet structure is global in the sense that each sheet spreads over the largest possible distances. It contains a global connected substructure—the *skeleton*—consisting of approximately 1D filaments of strong fields. Both the sheets and the skeleton fill a macroscopic (nonzero and, in fact, large) fraction of space-time and their geometric nature is analogous to that of the Peano's curve, i.e. a structure with local attributes of a low-dimensional object but still filling the underlying higher-dimensional space. An important aspect of the order in the topological field is that the two sheets, as well as the oppositely charged parts of the skeleton, are embedded in space-time in a mutually correlated manner so as to yield a negative two-point function of TChD [1,13,14].

The details of the vacuum structure described above, while expected to be relevant for the future understanding of the role of the vacuum in strong-interaction physics, mainly serve here as an unbiased evidence for the basic conceptual point put forward in Ref. [1]. In particular, they support the proposition that there exists a *fundamental structure* (structure involving features at all scales) in gauge configurations dominating the QCD path integral. This structure can be identified and studied directly in lattice-regularized ensembles, and its existence is a direct consequence of QCD dynamics. The above conclusion, if established beyond doubt, is quite far reaching since, in both a conceptual and a practical sense, it forms a basis for the possible understanding of the QCD vacuum in the Euclidean path-integral formalism. To demonstrate the existence of the fundamental structure one has to show that there exists a measurable excess of order in any configuration typical of the QCD ensemble relative to a configuration constructed randomly. In Ref. [1] this has been addressed at the level of the topological field itself. More specifically, it was shown that after randomly permuting the space-time coordinates of topological densities measured in typical gauge backgrounds, the ordered structure described in the previous paragraph ceases to exist. In particular, the sheets built from 3D coherent lattice hypercubes disappear after such random reshuffling of TChD in a given configuration. While indirect, this represents a strong argument supporting the reality of the fundamental structure in the QCD vacuum.

The reason why the above argument is indirect is that it compares the degree of order in the composite field evaluated on the equilibrium gauge background relative to the situation in a disordered composite field. A direct approach should compare the former relative to the situation in the composite field evaluated on a disordered gauge background. To see the difference between the two procedures at the technical level more precisely, let us denote collectively $U^{\mathrm{QCD}} \equiv \{U(x, \mu)\}$ an equilibrium QCD gauge configuration and $q^{\mathrm{QCD}} \equiv \{q(x)\}$ the associated configuration of TChD. If $x \to p_S(x)$ represents a random permutation of the (scalar) space-time coordinates then the operation of randomizing the configuration of TChD corresponds to

$$q^{\mathrm{QCD}} \equiv \{q(x) = q(x, U^{\mathrm{QCD}})\} \to q^R \equiv \{q^R(x) = q(p_S(x), U^{\mathrm{QCD}})\}. \quad (1)$$

At the same time, if $(x, \mu) \to p_V(x, \mu)$ represents a random permutation of the link (vector) space-time coordinates, then the randomization of gauge configuration proceeds via

$$U^{\mathrm{QCD}} \equiv \{U(x, \mu)\} \to U^R \equiv \{U^R(x, \mu) = U(p_V(x, \mu))\} \quad (2)$$

and the associated configuration of TChD is then

$$q^{\mathrm{QCD}} \equiv \{q(x) = q(x, U^{\mathrm{QCD}})\} \to q^{RU} \equiv \{q^{RU}(x) = q(x, U^R)\}. \quad (3)$$

If there is an excess of structure in q^{QCD} relative to q^{RU}, then it can be directly ascribed to the underlying order in the gauge field induced by QCD dynamics. On the other hand, if this is not the case then the structure observed in TChD is mainly induced by the nonultralocal nature of the overlap-based TChD operator. In other words, the structure in q^{QCD} would arise due to artificial amplification of random seeds of coherence and would not be a manifestation of QCD dynamics.

In this work we perform a numerical experiment to address this issue. In particular, we study the effect of $q^{\mathrm{QCD}} \to q^{RU}$ on the double-sheet structure in the same manner as was done for $q^{\mathrm{QCD}} \to q^R$ in Ref. [1]. We computed both q^{QCD} and q^{RU} for five independent 8^4 equilibrium configurations of Iwasaki gauge action at lattice spacing $a = 0.165$ fm determined from string tension.[3]

[1] Similar results have subsequently been reported also in the case of 2D CP(N-1) models [11].

[2] The notion of a strictly low-dimensional structure has recently been invoked also in the work using projected gauge fields associated with gauge-fixing procedures [12]. It remains to be seen whether there exists a connection to the topological structure on equilibrium backgrounds.

[3] These are actually the first five configurations of ensemble $\mathcal{E}_1$ of Ref. [14].

Topological densities were computed using [3]

$$q(x) = \frac{1}{2\rho}\,\mathrm{tr}\gamma_5 D_{x,x} \equiv -\mathrm{tr}\gamma_5\Big(1 - \frac{1}{2\rho} D_{x,x}\Big), \qquad (4)$$

where D is the overlap Dirac operator [15] based on the Wilson-Dirac kernel with mass $-\rho$. Numerical results presented here were obtained at the value $\rho = 1.368$ ($\kappa = 0.19$). Details of the numerical implementation for the overlap matrix-vector operation needed to evaluate $q(x)$ can be found in Ref. [16]. We wish to point out that while the number of configurations used in this study might seem small, it was found in Refs. [1,2] that the qualitative (and even quantitative) behavior of the observed structure is very robust and changes very little from one configuration to another. This is in fact expected if typical configurations

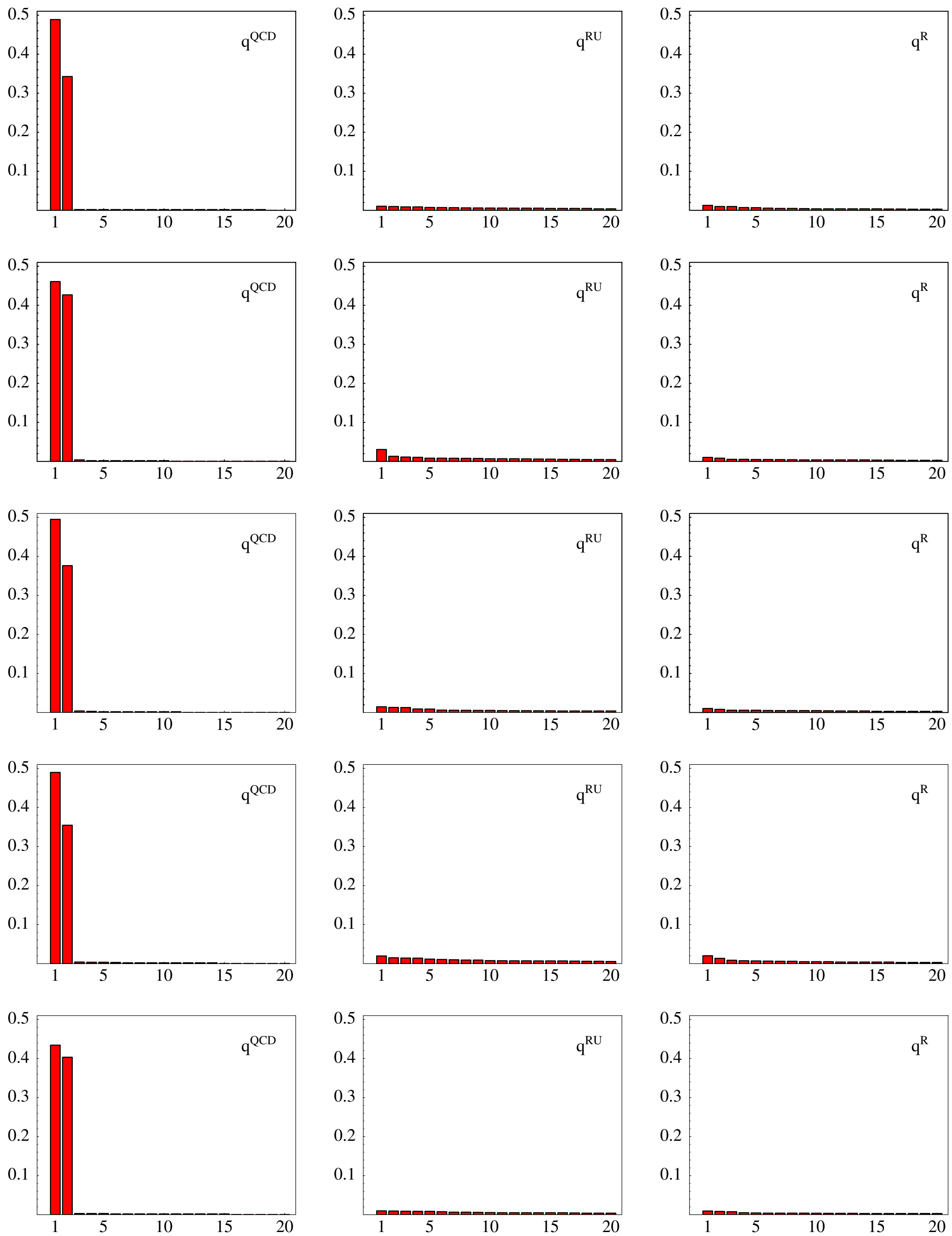

FIG. 1 (color online). Fractions f_k associated with maximal connected structures $\mathcal{R}_k^3$ ordered by size (decreasing f_k) are plotted against k. Each row corresponds to an individual configuration with columns representing q^{QCD}, q^{RU}, and q^R, respectively.

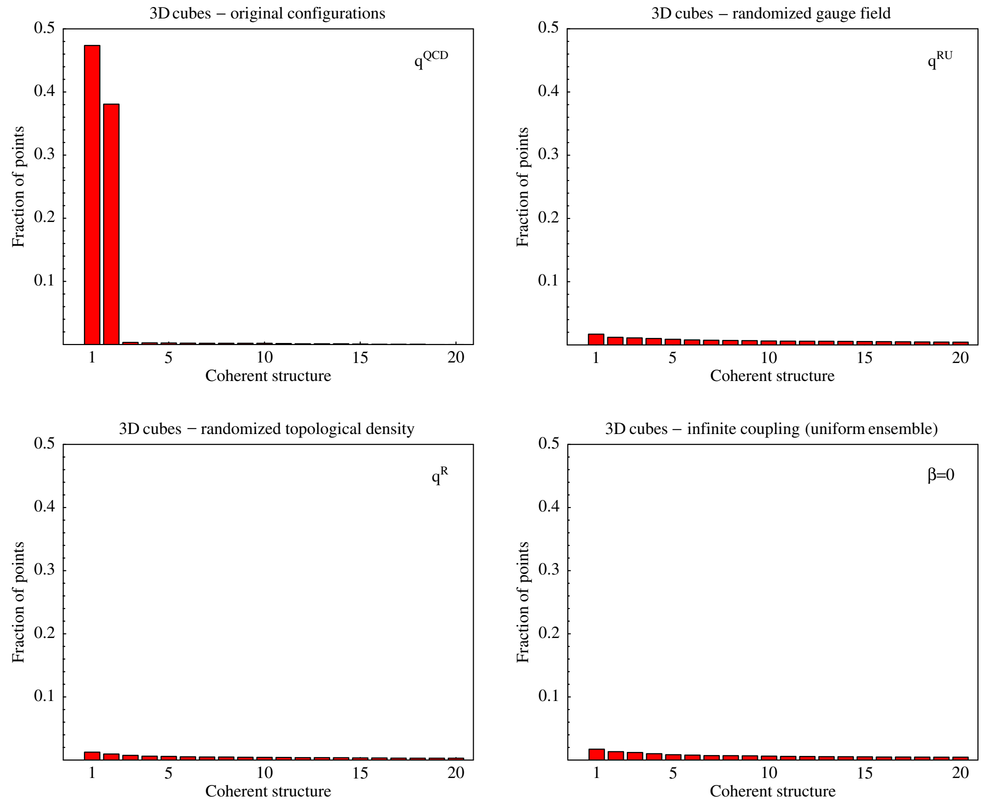

FIG. 2 (color online). The average fractions f_k are plotted against k for $q^{\rm QCD}$ (top left panel), q^{RU} (top right panel), q^R (bottom left panel), and for the uniform ($\beta = 0$) ensemble (bottom right panel).

are indeed dominated by a specific kind of space-time structure. Consequently, a qualitative conclusion (such as one sought here) can be made from just a handful of configurations.

For a given configuration of TChD we follow the procedure of Ref. [1] and determine all maximal connected sign-coherent regions $\mathcal{R}^3_k$, $k = 1, \ldots, K_3$ that can be built from elementary 3D cubes. We emphasize that the cubes are connected via 2D faces so that a consistent lattice 3D hypersurface (individual structure) is defined by any $\mathcal{R}^3_k$. Let $N(\Gamma)$ denote the number of points in arbitrary subset Γ of discretized space-time Ω. Then the fraction f_k of space-time occupied by $\mathcal{R}^3_k$ is given by $f_k \equiv N(\mathcal{R}^3_k)/N(\Omega)$. In what follows we order the structures by decreasing fraction, i.e. we choose the enumeration such that $f_k \geq f_{k+1}$ for all $k \leq K_3 - 1$. The resulting sequences f_k (for $k \leq 20$) are plotted in Fig. 1 with each row representing a situation in a given individual configuration. The first column shows f_k for $q^{\rm QCD}$ with the double-sheet structure appearing in each configuration via dominance of f_1 and f_2.[4] In the second column the corresponding fractions are shown for q^{RU}, i.e. for the randomized gauge field defined in Eq. (3). The key point of this work is that, as clearly seen from the plots, the double-sheet structure disappears when $q^{\rm QCD} \to q^{RU}$. Indeed, only fragmented pieces of coherence survive the random reshuffling of space-time coordinates of the gauge field. In fact, the situation is very similar to the randomization of topological density ($q^{\rm QCD} \to q^R$) as shown in the right column.

In Fig. 2 we plot the configuration averages from the data shown in Fig. 1. In addition, we have included here (lower right plot) the result from the same structure analysis for five configurations of 8^4 lattice generated at infinitely strong coupling ($\beta = 0$). This represents another form of comparison between the structure in QCD and the situation for the disordered gauge field. Indeed, an equilibrium configuration from this ensemble can be generated by independently choosing each link from a uniform distribution. As can be clearly seen from the plots, the result is again completely analogous to that of q^{RU} and q^R. We thus conclude that the double-sheet structure exists due to the underlying order in gauge configurations dominating the QCD path integral and is indeed a manifestation of the QCD vacuum.

To summarize, the goal of this work was to provide additional evidence for the proposition that there exists a fundamental structure (space-time order involving all scales) in configurations dominating the QCD path integral

[4]The results of Ref. [1] indicate that both f_1 and f_2 have a finite continuum limit with the combined double-sheet occupying about 70%–80% of space-time. On the other hand, the fractions f_k for $k \geq 3$ (the "fragments") scale to zero in the continuum limit.

[1]. In particular, it was shown that a particular manifestation of it, namely, the global double-sheet structure observed in the configurations of overlap-based TChD, exists only when appropriate local correlations (local QCD interaction) govern the ensemble. This long-range structure disappears when the space-time correlation among gauge variables is turned off. From this we conclude that the double-sheet structure (as well as the underlying gauge structure inducing it) is *real* in the sense that it exists as a consequence of QCD dynamics. We wish to emphasize two points that we associate with this finding (see also [1]). (i) The observation of the structure directly in the equilibrium configurations via a physically relevant composite field puts the hypothesis of the structure, and with it the approach to QCD vacuum via Euclidean path-integral formalism, on a firmer ground. It also suggests that a direct systematic approach using ensembles of lattice QCD is a viable (and perhaps optimal) avenue to study the problem of the QCD vacuum. (ii) Our approach leads us to the conclusion that the vacuum structure should not be viewed as a purely low-energy concept. Indeed, the structure observed in Refs. [1,2] involves space-time features at all scales (upon taking the continuum limit), and should have manifestations relevant for physics at arbitrary energy. The behavior of the structure at a given fixed scale can be studied in the case of TChD via *effective densities* [17] obtained by the means of the Dirac eigenmode expansion. We emphasize that the qualitative novelty here consists of the view that ‘‘understanding’’ the vacuum structure crucially involves an insight into how the structure changes across all scales.

We are indebted to Terry Draper and Keh-Fei Liu for useful feedback on the manuscript. This work was supported in part by the U.S. Department of Energy under Grants No. DE-FG05-84ER40154 and No. DE-FG02-95ER40907.

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