Landau gauge Jacobian and BRST symmetry

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Abstract

We propose a generalisation of the Faddeev-Popov trick for Yang-Mills fields in the Landau gauge. The gauge-fixing is achieved as a genuine change of variables. In particular the Jacobian that appears is the modulus of the standard Faddeev-Popov determinant. We give a path integral representation of this in terms of auxiliary bosonic and Grassman fields extended beyond the usual set for standard Landau gauge BRST. The gauge-fixing Lagrangian density appearing in this context is local and enjoys a new extended BRST and anti-BRST symmetry though the gauge-fixing Lagrangian density in this case is not BRST exact.

Key words: BRST, gauge-fixing, ghosts, determinant
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1 Introduction

The elevation of Faddeev-Popov (FP) gauge-fixing of Yang-Mills theory beyond the realm of perturbation theory has been intensely pursued in recent years for many reasons. Nonperturbative gauge-fixed calculations on the lattice are being compared to analogous solutions of Schwinger-Dyson equations [1,2]. As well, the long-term goal of simulating the full Standard Model using lattice Monte Carlo requires the Ward-Takahashi identities associated with BRST symmetry [3] in order to control the lattice renormalisation. The main impediment to nonperturbative gauge-fixing is the famous Gribov ambiguity [4]: gauges such as Landau and Coulomb gauge do not yield unique representatives on gauge-orbits once large scale field fluctuations are permitted. To some extent one could live with such non-uniqueness if one could incorporate all Gribov copies in a computation. However the no-go theorem of Neuberger

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[5] obstructs even this: (a naive generalisation of) BRST symmetry forces a complete cancellation of all Gribov copies in BRST invariant observables giving 0/0 for expectation values. In particular, Gribov regions contribute with alternating sign of the FP determinant.

Here we shall propose an approach which takes seriously that gauge-fixing when seen as a change of variables involves a Jacobian being the absolute value of the Faddeev-Popov determinant. Usually the absolute value is dropped either because of an *a priori* restriction to perturbation theory or because of the identification of the determinant in terms of an invariant of a topological quantum field theory [6] such as the Euler character [7,8]. In the latter case the Neuberger problem is encountered.

The approach we describe in the following is not restricted to perturbation theory. Moreover, because it will be seen to involve a gauge-fixing Lagrangian density that is not BRST exact it falls outside the scope of the preconditions for the Neuberger problem. In the next section we shall derive the Jacobian associated with gauge-fixing in the presence of Gribov copies. We shall give a representation of the "insertion of the identity" in this case in terms of a functional integral over an enlarged set of scalar and ghost fields. The extended BRST symmetry of this new gauge-fixing Lagrangian density will be described though we will see that the final form of the gauge-fixing Lagrangian is not BRST exact.

2 Field theoretic representation for the Jacobian of FP gauge fixing

In the following we shall formulate the problem in the continuum approach to gauge theory.

Our aim is to generalise the standard formula from calculus for a change of variable:

$$\left|\det\left(\frac{\partial f_i}{\partial x_j}\right)\right|^{-1}_{\vec{f}=0} = \int dx_1 \ldots dx_n \delta^{(n)}(\vec{f}(\vec{x})). \tag{1}$$

Here one is changing from integration variables $\vec{x}$ to those satisfying the condition $\vec{f}(\vec{x}) = 0$ and where, for Eq.(1) to be valid, in the domain of integration of $\vec{x}$ there must be only one such solution. In the context of gauge-fixing of Yang-Mills theory the generalisation of Eq. (1) is

$$\left|\det\left(\frac{\delta F[{}^g A]}{\delta g}\right)\right|^{-1}_{F=0} = \int \mathcal{D}g\, \delta[F[{}^g A]] \tag{2}$$

where A_μ represents the gauge field, g is an element of the $SU(N)$ gauge group, $\mathcal{D}g$ is the functional integration measure in the group and

$$F[{}^gA] = 0 \tag{3}$$

is the gauge-fixing condition. We shall be interested in Landau gauge $F[A] = \partial_\mu A_\mu$. As in the calculus formula, here Eq.(2) is only valid as long as Eq.(3) has a unique solution. This is known not to be the case for Landau gauge. The FP operator nevertheless is $M_F[A] = (\delta F[{}^gA]/\delta g)|_{F=0}$ and its determinant is $\Delta_F[A] = \det(M_F)$. For the Landau gauge $M_F[A]^{ab} = \partial_\mu D^{ab}_\mu[A]$ with $D^{ab}_\mu[A]$ the covariant derivative with respect to A^a_μ in the adjoint representation. Now the standard FP trick is the insertion of unity in the measure of the generating functional of Yang-Mills theory realised via the identity (which follows from the above definitions):

$$1 = \int \mathcal{D}g \Delta_F[{}^gA] \delta[F[{}^gA]]. \tag{4}$$

By analogy with standard calculus, in the presence of multiple solutions to the gauge-fixing condition Eq.(4) must be replaced by

$$N_F[A] = \int \mathcal{D}g\, \delta(F[{}^gA]) \left| \det M_F[{}^gA] \right|, \tag{5}$$

where $N_F[A]$ is the number of different solutions for the gauge-fixing condition $F[{}^gA] = 0$ on the orbit characterised by A, where A is any configuration on the gauge orbit in question for which $\det M_F \neq 0$.

It is known that Landau gauge has a fundamental modular region (FMR), namely a set of unique representatives of every gauge orbit which is moreover convex and bounded in every direction [9,10]. The following discussion can be found in more detail in [11]. Denoted Λ, the FMR is defined as the set of absolute minima of the functional $V_A[g] = \int d^4x ({}^gA)^2$ with respect to gauge transformations g. The stationary points of $V_A[g]$ are those A_μ satisfying the Landau gauge condition. The boundary of the FMR, $\partial\Lambda$, is the set of degenerate absolute minima of $V_A[g]$. Λ lies within the Gribov region Ω_0 where the FP operator is positive definite. The Gribov region is comprised of all of the local minima of $V_A[g]$. The boundary of Ω_0, the Gribov horizon $\partial\Omega_0$, is where the FP operator M_F (which corresponds to the second order variation of $V_A[g]$ with respect to infinitesimal g) acquires zero modes. When the degenerate absolute minima of $\partial\Lambda$ coalesce, flat directions develop and M_F develops zero modes. Such orbits cross the intersection of $\partial\Lambda$ and $\partial\Omega_0$. The interior of the fundamental modular region is a smooth differentiable and everywhere convex manifold. Orbits crossing the boundary of the FMR on the other hand will cross that boundary again at least once corresponding to the degenerate absolute minima.

Though, at present, there is no practical computational algorithm for constructing the FMR, it exists and we will make use of it for labelling orbits, i.e., $A_{\rm u}$ are defined to be configurations in the FMR, $A_{\rm u} \in \Lambda$. Since every orbit crosses the fundamental modular region once we are guaranteed to have $N_F \geq 1$. In turn the ${}^gA_{\rm u}$ fulfilling the constraint of Eq. (3) would be every other gauge copy of $A_{\rm u}$ along its orbit. Eq.(5) is equal to the number of Gribov copies on a given orbit, $N_{GC} = N_F - 1$, except that copies lying on any of the Gribov horizons ($\Delta_F = 0$) do not contribute to N_F.

The finiteness of N_F in the presence of a regularisation leading to a finite number of degrees of freedom (such as a lattice formulation) can be argued as follows. Consider two neighboring Gribov copies corresponding to a single orbit. If they contribute to N_F they cannot lie on the Gribov horizon. Therefore they do not lie infinitesimally close to each other along a flat direction, namely they have a finite separation. This is true then for all copies on an orbit contributing to N_F: all copies contributing to N_F have a finite separation. But the g which create the copies of $A_{\rm u}$ belong to $SU(N)$ which has a finite group volume. Thus for each space-time point there is a finite number of such g. We conclude then for a regularised formulation that N_F is finite.

Consider then the computation of the expectation value of a gauge-invariant operator $O[A]$ over an ensemble of gauge-field configurations $A_{\rm u}$ which is this set of unique representatives of gauge orbits discussed above.

Note that for a gauge-invariant observable, it makes no difference whether $A_{\rm u} \in \Lambda$ or if the $A_{\rm u}$'s are any other unique representatives of the orbits.

The expectation value on these configurations

$$\langle O[A] \rangle = \frac{\int \mathcal{D}A_{\rm u} O[A_{\rm u}] e^{-S_{YM}}}{\int \mathcal{D}A_{\rm u} e^{-S_{YM}}} \tag{6}$$

is well-defined. Since in any regularised formulation N_F is a finite positive integer, we can legitimately use Eq.(5) to resolve the identity analogous to the FP trick and insert into the measure of integration for an operator expectation value. We thus have

$$\langle O[A] \rangle = \frac{\int \mathcal{D}A_{\rm u} \frac{1}{N_F[A_{\rm u}]} \int \mathcal{D}g\, \delta(F[{}^gA]) \left| \det M_F[{}^gA] \right| O[A]\, e^{-S_{YM}[A]}}{\int \mathcal{D}A_{\rm u} \frac{1}{N_F[A_{\rm u}]} \int \mathcal{D}g\, \delta(F[{}^gA]) \left| \det M_F[{}^gA] \right| e^{-S_{YM}[A]}}. \tag{7}$$

We can now pass $N_F[A_{\rm u}]$ under the group integration $\mathcal{D}g$ and combine the latter with $\mathcal{D}A_{\rm u}$ to obtain the full measure of all gauge fields $\mathcal{D}({}^gA_{\rm u})$ which we can write now as $\mathcal{D}A$. N_F is certainly gauge-invariant: it is a property of the orbit itself. So $N_F[A_{\rm u}] = N_F[{}^gA_{\rm u}] = N_F[A]$. Thus we can write

$$\langle O[A]\rangle = \frac{\int \mathcal{D}A \, \frac{1}{N_F[A]}\delta(F[{}^gA]) \left|\det M_F[{}^gA]\right| O[A]\, e^{-S_{YM}[A]}}{\int \mathcal{D}A \, \frac{1}{N_F[A]}\delta(F[{}^gA]) \left|\det M_F[{}^gA]\right| e^{-S_{YM}[A]}}. \tag{8}$$

Perturbation theory can be recovered from this of course by observing that only A fields near the trivial orbit, containing $A=0$ and for which $S_{YM}[A]=0$, contribute significantly in the perturbative regime: the curvature of the orbits in this region is small so that the different orbits in the vicinity of $A=0$ intersect the gauge-fixing hypersurface $F=0$ the same number of times. Then the number of Gribov copies is the same for each orbit, N_F is independent of $A_{\rm u}$ and we can cancel N_F out of the expectation value. In that case

$$\langle O[A]\rangle = \frac{\int \mathcal{D}A \, \delta(F[A]) \left|\det M_F[A]\right| O[A]\, e^{-S_{YM}[A]}}{\int \mathcal{D}A \, \delta(F[A]) \left|\det M_F[A]\right| e^{-S_{YM}[A]}}. \tag{9}$$

In turn, observing that fluctuations near the trivial orbit cannot change the sign of the determinant, the modulus can also be dropped and one recovers the usual starting point for a standard BRST invariant formulation of Landau gauge perturbation theory. Note that perturbation theory is built on the gauge-fixing surface in the neighbourhood of $A=0$, which for a gauge-invariant quantity will be equivalent to averaging over the Gribov copies of $A=0$ as in Eq. (9). For the non-perturbative regime, the orbit curvature increases significantly and in general there is no reason to expect that N_F would be the same for each orbit. Moreover the determinant can change sign.

Let us focus on the partition function appearing in Eq. (8)

$$\mathcal{Z}_{\rm gauge-fixed} = \int \mathcal{D}A \, N_F^{-1}[A] \left|\det(M_F[A])\right| \delta(F[A])\, e^{-S_{YM}} \tag{10}$$

The objective is to generalise the BRST formulation of Eq.(10) such that it is valid beyond perturbation theory taking into account the modulus of the determinant. We thus start with the following representation:

$$\left|\det(M_F[A])\right| = {\rm sgn}(\det(M_F[A]))\, \det(M_F[A]). \tag{11}$$

As mentioned, the factor $\det(M_F[A])$ in Eq.(11) is represented as a functional integral via the usual Lie algebra valued ghost and anti-ghost fields in the adjoint representation of $SU(N)$. Let us label these as $c^a, \bar{c}^a$. It is usual also (see for example [12]) to introduce a Nakanishi-Lautrup auxiliary field b^a. Thus the effective gauge-fixing Lagrangian density

$$\mathcal{L}_{\rm det} = -b^a \partial_\mu A^a_\mu + \frac{\xi}{2} b^a b^a + \bar{c}^a M_F^{ab} c^b \tag{12}$$

yields [12]

$$\lim_{\xi \to 0} \int \mathcal{D}\bar{c}^a \mathcal{D}c^a \mathcal{D}b^a e^{-\int d^4x \mathcal{L}_{\rm det}} = \delta(F[A]) \det(M_F[A]). \tag{13}$$

In order to write the factor $\mathrm{sgn}(\det(M_F[A]))$ in terms of a functional integral weighted by a local action, we consider the following Lagrangian density

$$\mathcal{L}_{\rm sgn} = iB^a M_F^{ab} \varphi^b - i\bar{d}^a M_F^{ab} d^b + \frac{1}{2} B^a B^b \tag{14}$$

with $\bar{d}^a, d^a$ being new Lie algebra valued Grassmann fields and φ^a, B^a being new auxiliary commuting fields. Consider in Euclidean space the path integral

$$\mathcal{Z}_{\rm sgn} = \int \mathcal{D}\bar{d}^a \mathcal{D}d^a \mathcal{D}\varphi^a \mathcal{D}B^a e^{-\int d^4x \mathcal{L}_{\rm sgn}} . \tag{15}$$

Completing the square in the Lagrangian density of Eq.(14), the B field can be integrated out in the partition function leaving an effective Lagrangian density

$$\mathcal{L}'_{\rm sgn} = \frac{1}{2} \varphi^a ((M_F)^T)^{ab} M_F^{bc} \varphi^c - i\bar{d}^a M_F^{ab} d^b \, , \tag{16}$$

where $(M_F)^T$ denotes the transpose of the FP operator. Integrating all remaining fields now it is straightforward to see that the partition function Eq.(15) amounts to just

$$\mathcal{Z}_{\rm sgn} = \frac{\det(M_F)}{\sqrt{\det((M_F)^T M_F)}} = \mathrm{sgn}(\det(M_F)) \, . \tag{17}$$

Thus the representation Eq.(15) can be used for the first factor of Eq.(11). The Lagrangian density of Eq.(14) therefore combines with the standard BRST structures of Eq.(12) coming from the determinant itself in Eq.(11) so that an equivalent representation for the partition function based on Eq.(10) is

$$\mathcal{Z}_{\rm gauge-fixed} = \int \mathcal{D}A^a_\mu \mathcal{D}\bar{c}^a \mathcal{D}c^a \mathcal{D}\bar{d}^a \mathcal{D}d^a \mathcal{D}b^a \mathcal{D}\varphi^a (N_F[A])^{-1} \, e^{-S_{\rm YM} - S_{\rm det} - S_{\rm sgn}} \tag{18}$$

with $S_{\rm det}$ and $S_{\rm sgn}$ the actions corresponding to the above Lagrangian densities Eqs. (12,14).

3 A new extended BRST

The symmetries of the new Lagrangian density, $\mathcal{L}_{\rm sgn}$, are essentially a boson-fermion supersymmetry and can be seen from Eq.(14). In analogy to the standard BRST transformations typically denoted by s, we shall denote them by the Grassmann graded operator t

$$
\begin{aligned}
t\varphi^a &= d^a \\
td^a &= 0 \\
t\bar{d}^a &= B^a \\
tB^a &= 0\,,
\end{aligned}
\tag{19}
$$

such that

$$
t\mathcal{L}_{\rm sgn} = 0 \tag{20}
$$

and trivially $t\mathcal{L}_{\rm YM} = 0$. Eqs.(19) realise the infinitesimal form of shifts in the fields. The operation t is nilpotent: $t^2 = 0$. Using Eqs.(19) we can give the following form for the Lagrangian density $\mathcal{L}_{\rm sgn}$,

$$
\mathcal{L}_{\rm sgn} = t\left(\bar{d}^a(iM_F^{ab}\varphi^b + \frac{1}{2}B^a)\right). \tag{21}
$$

The question now is how to combine this with the standard BRST transformations

$$
\begin{aligned}
sA^a_\mu &= D^{ab}_\mu c^b \\
sc^a &= -\frac{1}{2}gf^{abc}c^bc^c \\
s\bar{c}^a &= b^a \\
sb^a &= 0.
\end{aligned}
\tag{22}
$$

The transformations due to t and s are completely decoupled except that the latter also act on the gauge field on which the FP operator M_F depends. We propose the following unification of these symmetry operations. Consider an operation $\mathcal{S}$ *block-diagonal* in s and t: $\mathcal{S} = \mathrm{diag}(s,t)$. The operator acts on the following multiplet fields:

$$\mathcal{A}^a = \begin{pmatrix} A^a_\mu \\ \varphi^a \end{pmatrix}, \ \mathcal{C}^a = \begin{pmatrix} c^a \\ d^a \end{pmatrix}, \ \bar{\mathcal{C}}^a = \begin{pmatrix} \bar{c}^a \\ \bar{d}^a \end{pmatrix}, \ \mathcal{B}^a = \begin{pmatrix} b^a \\ B^a \end{pmatrix}. \tag{23}$$

We see that these fields transform under $\mathcal{S}$ completely analogously to the standard BRST operations

$$\begin{aligned} \mathcal{S}\mathcal{A}^a &= \mathcal{D}^{ab}\mathcal{C}^b \\ \mathcal{S}\mathcal{C}^a_i &= \mathcal{F}^{abc}_{ijk}\mathcal{C}^b_j\mathcal{C}^c_k \\ \mathcal{S}\bar{\mathcal{C}}^a &= \mathcal{B}^a \\ \mathcal{S}\mathcal{B}^a &= 0\,, \end{aligned} \tag{24}$$

where $i, j, k = 1, 2$ label the elements of the multiplets, and

$$\begin{aligned} \mathcal{D}^{ab} &= \mathrm{diag}(D^{ab}_\mu, \delta^{ab}) \\ \mathcal{F}^{abc}_{111} &= -\frac{1}{2} g f^{abc}, \quad \mathcal{F}^{abc}_{ijk} = 0 \quad \text{for} \quad ijk \neq 111. \end{aligned} \tag{25}$$

Note that nilpotency is satisfied, $\mathcal{S}^2 = 0$. We shall refer to this type of operation as an *extended* BRST transformation which we distinguish from the BRST–anti-BRST or double BRST algebra of the Curci-Ferrari model [13,14]. We can thus formulate the gauge-fixing Lagrangian density for the Landau gauge as

$$\mathcal{L}_{\mathrm{gf}} = \mathrm{Tr}\ \mathcal{S} \begin{pmatrix} \bar{c}^a F^a & 0 \\ 0 & \bar{d}^a(iM^{ab}_F\varphi^b + \frac{1}{2}B^a) \end{pmatrix}. \tag{26}$$

This approach admits also an extended anti-BRST operation:

$$\begin{aligned} \bar{\mathcal{S}}\mathcal{A}^a &= \mathcal{D}^{ab}\bar{\mathcal{C}}^b \\ \bar{\mathcal{S}}\bar{\mathcal{C}}^a_i &= \mathcal{F}^{abc}_{ijk}\bar{\mathcal{C}}^b_j\bar{\mathcal{C}}^c_k \\ \bar{\mathcal{S}}\mathcal{C}^a &= -\mathcal{B}^a \\ \bar{\mathcal{S}}\mathcal{B}^a &= 0\,. \end{aligned} \tag{27}$$

Writing $\bar{\mathcal{S}} = \mathrm{diag}(\bar{s}, \bar{t})$ we can extract the standard anti-BRST $\bar{s}$-operations [14,15]

$$\bar{s}A^a_\mu = D^{ab}_\mu \bar{c}^b$$

$$
\begin{aligned}
\bar{s}\bar{c}^a &= -\frac{1}{2} g f^{abc} \bar{c}^b \bar{c}^c \\
\bar{s}c^a &= -b^a \\
sb^a &= 0
\end{aligned} \tag{28}
$$

and those corresponding to $\bar{t}$:

$$
\begin{aligned}
\bar{t}\varphi^a &= \bar{d}^a \\
\bar{t}\bar{d}^a &= 0 \\
\bar{t}d^a &= -B^a \\
\bar{t}B^a &= 0\,.
\end{aligned} \tag{29}
$$

Moreover, the ghosts and anti-ghosts in this extended structure also fulfill the criteria for being Maurer-Cartan one-forms,

$$
\mathcal{S}\bar{\mathcal{C}} + \bar{\mathcal{S}}\mathcal{C} = 0. \tag{30}
$$

However there is no extended BRST–anti-BRST (or double) symmetric form of the gauge-fixing Lagrangian density Eq. (26), unlike the two pieces of which it consists. Such a representation exists in the $s-$sector of Landau gauge:

$$
\mathcal{L}_{\mathrm{gf},s} = \frac{1}{2} s\bar{s} A^a_\mu A^a_\mu\,. \tag{31}
$$

In the $t-$sector, the corresponding structure is

$$
\mathcal{L}_{\mathrm{gf},t} = \frac{1}{2} t\bar{t}\left[\varphi^a M_F^{ab} \varphi^b + \bar{d}^a d^a\right]. \tag{32}
$$

However the complete Landau gauge-fixing Lagrangian density can only be expressed via a trace, namely as

$$
\mathcal{L}_{\mathrm{gf}} = \frac{1}{2} \mathrm{Tr} \mathcal{S}\bar{\mathcal{S}}\mathcal{W} \tag{33}
$$

with

$$
\mathcal{W} = \mathrm{diag}\left(A^a_\mu A^a_\mu, \varphi^a M_F^{ab} \varphi^b + \bar{d}^a d^a\right). \tag{34}
$$

Nevertheless this compact representation formulates the modulus of the determinant in Landau gauge fixing in terms of a local Lagrangian density and follows as closely as possible the standard BRST formulation without the modulus.

4 Discussion and Conclusions

We have thus found a representation for Landau gauge-fixing corresponding to the FP trick being an actual change of variables with appropriate determinant. The resulting gauge-fixing Lagrangian density enjoys a larger extended BRST and anti-BRST symmetry. However it cannot be represented rigorously as a BRST exact object, rather the sum of two such objects corresponding to different BRST operations. This means that some of the BRST machinery is not available to this formulation, such as the Kugo-Ojima criterion for selecting physical states. We discuss cursorily now the perturbative renormalisability of the present formulation of the theory. Note that the procedure leading to Eq. (26) does not introduce any new coupling constants; only the strong coupling constant g is present in $M_F[A]$ coupling the Yang-Mills field to both the new ghosts and scalars. The dimensions of the new fields are

$$[\varphi] = L^0, \quad [d] = [\bar{d}] = L^{-1}, \quad [B] = L^{-2}. \tag{35}$$

Most importantly in this context, the kinetic term for the new boson fields φ^a is *quartic* in derivatives:

$$\mathcal{L}_{\rm kin} = \varphi^a (\partial^2)^2 \varphi^a \,, \tag{36}$$

which is renormalisable, by power counting, since φ^a are dimensionless. Such a contribution is seemingly harmless in the ultraviolet regime: for large momenta propagators will vanish like $1/p^4$. Moreover it should play an important role in guaranteeing the decoupling of such contributions in perturbative diagrams. That such a decoupling should occur is clear from Eq. (11): in the perturbative regime fluctuations about $A_\mu = 0$ will not feel the $\mathrm{sgn}(\det M_F[A])$, so that the field theory constructed in this way must be equivalent to the perturbatively renormalisable Landau gauge fixed theory. For example in the computation of the running coupling constant we expect that this property will lead to a complete decoupling of the t-degrees of freedom so that the known Landau gauge result emerges from just the gluon and standard ghost sectors. Naturally, the new degrees of freedom will be relevant in the infra-red regime, which will be the object of future study.

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