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HOW SHOULD I VOTE?

A study of various aspects of voting systems used in Parliamentary elections, particularly in Australia.

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Written to fulfil the requirements of the degree of Ph.D.

NAME: COURSE: Ph. D.

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This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

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FOREWORD

I became interested in practical aspects of electoral systems in the late 1960's, when confronted with a wide choice of candidates in the STV election of the ACT Advisory Council in about 1968. I remember trying hard to work out a voting strategy which would maximise the effectiveness of my vote. From this I became interested in the problem of finding the sampling error for elections to the Australian Senate, which I began in 1970, and worked on sporadically until 1972. For all practical purposes, I had solved the problem at this time, but I was not satisfied with either the theoretical framework or the extent of my assumptions. I left the problem and did not look at it again until 1977, when I developed the framework of the article which eventually appeared in 1980. The answers to the problem of the sampling error, which I obtained for that paper, differed by only one or two percent from those I had originally calculated in 1972.

By then I knew the voting system for the Australian Senate well enough to write a further paper exposing some of its shortcomings and suggesting improvements. I was also asked to act on a Committee of my University on electoral machinery. Out of this work arose a paper on the simultaneous election of representatives, primarily to show how the system for electing the University Council was likely to fail when most needed.

This research was, however, peripheral to my other work in economics. Together with a colleague, I received a large research grant to examine the diffusion of innovations in the wheat industry in South Australia, and I worked primarily in this area for several years. In 1983, however, I made a short submission to the Australian Parliament's Joint Parliamentary Committee on Electoral Reform, highlighting the existence of my two articles on the Senate.

It was not until John Taplin, organiser of Symposia on Electoral Reform at two successive ANZAAS Conferences in 1986 and 1987, contacted me early in 1986 that I again began to focus on further work. In 1986 came the Conference Paper comparing the electoral system of Australia with that of New Zealand and the UK on the one hand and proportional representation systems on the other. In 1987 I followed this with applied work on the game-theory concept of Power in politics for the ANZAAS Symposium that year. I tried to get this work published in *Political Geography*, and received my only outright rejection of any submissions I have made in this area.

By now my dabbling was somewhat more serious, although it has remained a secondary research interest of mine. The paper on Political Power turned out to be pivotal. Although it was a "fizzer" in itself, it inspired work on looking at the variability of the swing at a particular election. The introduction of a stochastic element in this area proved more fruitful. It has enabled a quantification of the variation in the number of seats gained by a party with a particular proportion of the overall vote. This has allowed an estimate of the probability of a tied Lower House, and more importantly, opened the way for an analysis of electoral bias, or more loosely, gerrymandering. More recently, it has provided a basis for work on decisiveness.

Having believed that I had hit on something important, I therefore decided to enrol in a Ph.D. program in this area. I enrolled in September 1992, after a long and for many months inconclusive debate with the University's administration about whether they would accept the kind of thesis I proposed, the possibility of early submission, and whether I had to pay fees.

In the first instance, then, this thesis has cobbled together bits and pieces of my "hobby" research. However, in the last two years, I have done much more. I had intended to make my work on electoral boundaries a central part of the thesis. I decided, after enrolling, that I could not do this. Gelman and King had independently used essentially the same method as mine, and their work had primacy by some months. They then extended their work in exactly the way I had thought I would extend my work in the area, to distinguish it from theirs! While I have included some of my work in this area, I have also concentrated on two other areas, which have enabled me to combine my work in Experimental Economics with my thesis. I have done two sets of experiments, one on the determinants of strategic voting, which breaks new ground in this area, and the other on instrumental and expressive voting. The second set of experiments is just one aspect of recent work I have done in the area of decisiveness, which my supervisor (Professor Jonathan Pincus) suggested I examine more closely. Rather reluctantly, I did so. The newer area has borne rather more fruit than I had imagined it would (though I will only grudgingly admit as much to my supervisor). One of these pieces of fruit is also one of the smallest contributions (in terms of size) to the thesis. In the whole area of Public Choice and electoral theory, the most important single notion is perhaps that of decisiveness. So it is important that the probability that a voter is decisive is calculated using a correct and appropriate method. For the last twenty or so years, it has not been: mistakes of the order of magnitude of 10^{100}, and more, have been made. Consequently, the importance of expressive voting has been overemphasised, and there has been a lack of correspondence between the theoretical probability of being decisive and observed behaviour on turnout. My revision of the calculation of the probability has crowded out a paper I have been intending to write, which would extend the notion of power and the probability of decisiveness to an index of voter inequality based on the Lorenz Curve and Gini coefficient.

It is usual to thank people who have helped and encouraged me in this work. The thesis being done the way it has been, I have a rather short list, as I have worked predominantly alone on it for most of the time.

Kevin Davis and Ken Brewer have both found time to read and comment on my early work while Colin Hughes and Simon Jackman provided useful comments on the papers I sent to them. John Robbins provided useful guidance with the literature on Duverger. My supervisor Jonathan Pincus has been enthusiastic about supervising me, particularly in Public Choice theory, and thorough, to the extent that I have let him be so. Gretel Dunstan has read those chapters I have sent her, and suggested many changes to improve the style. At her prompting, I have also given the notion of subjective probability more prominence. Malcolm Mackerras has been very helpful to me in providing data. I have badgered him in the past about doing work jointly with him, as he has so much background knowledge (which I lack) and I have greater technical expertise. Together we could, I am sure, do more together than the sum of the two parts. Former Deputy Commonwealth Electoral Officer for South Australia, Mr. A.J. Walsh and South Australian Electoral Commissioner, Mr. Andy Becker, and his staff, particularly Mike Duff, have also been helpful with data and suggestions. Ian McAllister and David Butler have been considerate to me as journal editors. Debbie Beckman, Chris Knight and Cathy Pascoe have toiled hard to type this.

I owe the biggest debt of gratitude to my wife, Judith, who, when I told her that I was applying to do a Ph.D., said that I had to work every Sunday on my thesis, free of family responsibilities. I thought I would not need to do so, but she is a far better time manager than I am, and she was right. I dedicate this thesis to her, with the wish that she undertake a Doctorate in her own right, and that I can be as understanding of her as she has been to me.

Part I

Introduction

1. *Background*

This thesis began as a study of the properties and performance of electoral systems in use in Australia. For the most part, it still does that, but some of the later work has more general applicability. The thesis is entitled "How Should I Vote?" because it addresses the dilemma of a voter who wishes to make his or her vote maximise its effect. Does it matter if I vote for a minor party rather than a major one? If I have to vote for more than one candidate, does the order of my voting matter? Is my vote worth more in a marginal seat than a safe seat? Or, for a Federal election, more in one State than another? If the party that I vote for wins over 50% of the vote, will it gain enough seats to win government? Will my vote be selected in a sample to be counted, or in a sample not to be counted?

More recently, I have broadened the thesis to include the topic "Should I Vote?" This makes less sense in Australia, where voting is compulsory, than in places where it is not. Thus, in including this topic, the thesis naturally broadens to encompass voting systems wider than just Australian ones. The question of "Should I Vote?" at first sight appears to be one which should be asked prior to asking "How Should I Vote?" However, there is a degree of simultaneity in these decisions, because it should be necessary to determine the costs and benefits of voting for A rather than B before deciding whether it is worthwhile voting at all. That is, the question "How Should I Vote?" is more a precursor of the question "Should I Vote?" than the reverse.

There is a second sense in which the "Should I Vote?" question is a natural continuation of the themes of the thesis. Most of the thesis topics are concerned primarily with three things: decisiveness, fairness and efficiency. Decisiveness has to do with the boundary between winning and losing, and the question of "Should I Vote?" involves the voter in what amounts to a calculation of the probability of being decisive. This notion also pervades work in the thesis on whether the Parliament is tied, whether the system of electoral boundaries is fair or not (the system is judged at the point where both sides have exactly the same number of votes), and whether or not to vote strategically (it will alter as the probability of being decisive changes). Thus, notions of decisiveness are a unifying force for many of the topics in this thesis.

So too is the notion of fairness. How many Senators should the A.C.T. (which currently has two Senators) be entitled to elect when its population exceeds that of Tasmania, which has 12 Senators? The placing of electoral boundaries is totally a question of fairness. The question of ending the system of sampling the votes, undertaken in the Senate prior to 1984, is one of both fairness and the prevention of corruption.

The question of efficiency in its broadest sense, that is: "What is the best system?" is all-encompassing, and therefore not of value as a classification device.

2 *Papers about electoral systems involving Australia*

The electoral systems used for the Australian Federal Parliament, and for the Australian States and Territories, have a number of relatively rare features. First, Australia is one of only a few countries in the world where the system known as the alternative vote (or as it is called in Australia, preferential voting) is used.

This form of voting is used in single-member constituencies in the Lower Houses of the Federal Parliament (the House of Representatives), five of the six States and one of the two Territories, and in several of the Upper Houses of the States. It is also used in the form of the Single Transferable Vote (STV) for multimember constituencies in the Tasmanian and ACT Lower Houses, as well as in the Upper Houses in the Australian Parliament (the Senate), and in the States of South Australia and NSW. Elsewhere in the world, STV is used to elect Lower Houses in Ireland and Malta.

Second, voting is compulsory - at least, attendance at a polling booth, and the placing of a folded ballot paper or papers in the ballot-box, are compulsory. Third, to complete a valid or "formal" vote, it is necessary to indicate a strict order of preference for all candidates, though there are several exceptions to this rule (for example, in Queensland State elections, the expression of any number of preferences constitutes a formal vote; in the Senate, where there have been large numbers of candidates, it is no longer necessary to complete the preference ordering, although sometimes at least the first twelve preferences must be given).

There has been very little formal study of the properties and performance of electoral systems such as these. The first part of the thesis brings together a number of pieces of analysis of the electoral systems in Australia, but primarily the Federal system, conducted by the author over the past 10 to 15 years.

The first paper is an attempt to justify the present Federal electoral system. There are basically two forms of justification that can be made of a particular electoral system. First, one looks internally at the system, to see whether it is fair to those voting: Is the vote of one person worth more than that of another? Does the composition of Parliament reflect the proportion of electors voting for that party? Is the power of the individual voter to change the outcome of the election equally distributed?

Second, one compares *outcomes* of having one form of electoral system with those of another. This is a much more difficult and subjective comparison than comparing systems internally. The main reason for this is that changing an electoral system is likely to change the nature of the society. However, the extent and speed of the change are not likely to be known, so it is difficult to be definitive about the efficacy of a particular system.

Nevertheless, the first paper attempts to compare the Federal Australian system[1] with those of plurality voting in single-member electorates (as carried out, for example, in the US, UK, Canada, New Zealand and India) and with proportional representation systems. The other papers take the broad shape of the electoral system in Australia for granted. They examine various properties of the system in turn, see how well a particular property performs, and if it is deficient, how it may be changed within the present system. The second and third papers are concerned with aspects of the size and composition of the two Federal Houses.

The second paper looks at the overall representation of States and Territories in the Federal Parliament. The representational requirements for each of the six original States were laid down in the Australian Constitution. However, those of new States have only recently been elucidated. While rules governing representation of the House of Representatives for elections in new States are straight-forward, following on a population

[1]Consisting of 147 single-member electorates of roughly equal members of voters elected by the alternative vote, for the Lower House, and 76 Senators, elected by STV using each State and Territory as a single electorate.

basis, those for the Senate are not. The paper attempts to define rules for determining such representation.

The third paper looks at the size of the House of Representatives, and in particular at an anomaly caused by poor drafting of the Australian Constitution. About half the time, the House of Representatives will have an even number of seats. Given that there are effectively only two parties and only occasionally an Independent, there is a real possibility of impasse if both parties gain the same number of seats at a general election. The likelihood of this occurrence is assessed. Unfortunately, the remedy for this situation - that is, ensuring an odd number of seats - would require a change in the way the Constitution has been interpreted by the High Court.

The fourth, fifth and sixth papers deal with internal aspects of the fairness of the system of voting for the Lower House. Paper four examines the notion of fairness propounded by those interested in proportional representation. They argue that the vote of an individual in a single-member electorate is often "wasted", in that all those voting for the non-winner are not represented in Parliament (at least, not in that seat). Furthermore, for a winner with say 55% of the two-party preferred vote, there is also a sense in which the last 5% (that is, any vote above 50%) is also wasted. This paper links the notion of a wasted vote to the idea of political power, a rather specific concept derived from game theory. Paper four provides the starting-point for about half of the remaining thesis. The fifth paper delineates a method for determining whether a particular set of electoral boundaries is fair in the sense that it would on average give a party gaining over 50% of the two party preferred vote in an election over 50% of the seats, if voting patterns were similar to those of the previous election. In broad terms, therefore, it gives Electoral Boundary Commissioners a means of detecting potential gerrymanders and correcting for them. The sampling error. As an outcome of the paper and a 1983 Joint Parliamentary Select Committee, the Commonwealth Electoral Act was amended to remove sampling altogether. It would seem that the probability of having elected the wrong candidate because of sampling was actually very low, and because of large electorates and the adherence of most electors to party how-to-vote cards, it was not a particularly great risk at any given future election. However, since it was in fact very easy to bypass the problem, Parliament deemed the risk to be not worth taking.

In Ireland, however, it is reasonably likely that since 1922, sampling of votes using STV has resulted in the election of the wrong candidate. Although the best estimate of the number of wrong candidates ever elected up to 1987 was two, the Irish had not at that time moved to amend their system of sampling the votes. Paper eight deals with this matter.

The ninth paper, written soon after the seventh, looked at other anomalies, as well as the sampling problem in the voting procedures for the Australian Senate. The most important of these (and more important than any other technical concern in terms of deciding the composition of the Senate) was a discussion of the way that Senators were chosen either for a three-year term or a six-year term after a double dissolution of the Senate. According to the Australian Constitution, the Senate itself decides which of its newly-elected members should be short-term or long-term. This appears to be a major blunder by the framers of the Constitution. In the limiting case, any bare majority of Senators may give each of themselves three additional years in the Senate. The method which the Senate has actually used to determine who would sit for six years was that they should be the first six elected out of twelve Senators elected from each State. While this method has some simplistic appeal, it is likely to over-represent smaller parties because of the way in which "first six elected" is interpreted, but just as seriously, could give a major and undeserved advantage to a party willing to create two lists of candidates for the election. A method of determining the six year Senators to prevent such distortions was devised in this paper, and as a result was enacted in 1984 in legislation as section 283 of the Commonwealth Electoral Act.

There was only one problem, however. The Senate itself has the power to determine who should be the six year Senators after a double dissolution. So when in 1989 the whole Senate was dissolved, the newly elected Senators realised that if the new provision in the Electoral Act were used, there would be one less ALP and one less Australian Democrat elected for six years (and one more Liberal, and one more National) compared with the old provision of "the first six elected". The ALP and Democrats being in the majority, they were able to vote in favour of the old provision, and thus give themselves each one additional seat for three years. This has been a cynical abuse of power. Because of the technical nature of the problem, it has gone unheralded in the media, apart from some exposure by Malcolm Mackerras, which has not been followed up. The issue of Paper nine has therefore still not been satisfactorily resolved.

3 *Papers on experiments and on decisiveness*

The remaining papers have been written for this thesis, and have not been published, although by the time the thesis has been completed, at least three of them will have been submitted for publication. Paper ten describes an experiment with the alternative vote, to investigate the extent to which strategic (or sophisticated, or insincere) voting occurs, and to examine its determinants. To my knowledge, this is the first such experiment to be undertaken on investigating the properties of the alternative vote, and one of few experiments on strategic voting.

Paper eleven is a reconsideration of the probability that a voter will be decisive. This is probably the most important theoretical paper in the thesis, because the orthodox formula, in use for 20 years, appears to be inappropriate, if not simply wrong. Paper twelve is a calculation of the probability of a voter in the U.S. Presidential election being decisive, calculated on the orthodox formula. Paper thirteen recalculates the probability using the new formulation of Paper eleven. Paper fourteen is an extended review of Brennan and Lomasky's book *Democracy and Decision*, in which the notion of decisiveness is used to contrast two reasons for voting: "expressive" and "instrumental". Paper fifteen describes an experiment to determine to what extent (if any) expressive voting occurs in an experimental setting.

4 *Other*

Paper sixteen is in the nature of an appendix. Its genesis was a consideration of the voting system to elect to Council of the University of Adelaide in South Australia. It examines a system of Borda scores to elect more than one candidate at a time. The literature does not appear to consider this case (or at least, did not appear to do so in 1980 when the paper was written). It compares the multiple-seat Borda election with that of STV and intermediate cases. (The election procedure was devised by a former Registrar at the University of Adelaide, Mr. V. Edgelow. It purports to be a system devised by Professor Nanson of Melbourne in 1884. However, Nanson's system is a "reverse-Borda", in which the candidate with the lowest Borda score is removed and the scores recalculated, the lowest score candidate again removed, and so on, until the last one remaining is the winner. It is clear from the original sources that Nanson did not envisage that his system would ever be used to elect more than one person at a time.)

This thesis consists of the sixteen papers described above, together with a short commentary on each, and a conclusion. Four of the papers have been published.

Paper one is a revised version of some ideas originally put forward to the 1983 Joint Parliamentary Select Committee on the Electoral System and reworked for a paper at the ANZAAS Conference in Palmerston North, New Zealand in 1987.

Paper two is a reworking of a proposal put to the Joint Parliamentary Select Committee on the Electoral System, later in its deliberations, in 1985, on the way in which new States and Territories should be represented.

Paper three was Working Paper 89/8 of the Economics Department, University of Adelaide.

Paper four, entitled "Political Power of Voters in Australian Elections", was given at a Symposium on Electoral Reform, ANZAAS, Sydney, 1988.

Paper five on the detection of electoral bias was published as "Swings and Gerrymanders" in *Electoral Studies*, Vol. 10, no. 4, pp. 299-312, 1991.

Paper six, extending the notion of electoral bias, has been submitted for publication to the *Australian Journal of Political Science* and was returned for revision. It has since been very substantially revised and not yet re-submitted.

Paper seven was published in 1980 by the *Australian Journal of Statistics*, Vol. 22(1), pp. 24-39.

Paper eight was published in the *British Journal of Political Science,* vol 18, number 1, 1988, as a Comment to a paper by Gallagher and Unwin in the same Journal.

Paper nine was published by *Politics* (the forerunner of the *Australian Journal of Political Science*) as "Some Aspects of the Voting System for the Senate" in Vol. 16, pp. 57-62, 1981.

Papers ten to fifteen are new.

Paper sixteen was presented to the Economists Conference, Canberra 1980.

I wish to thank editors of the above journals, and the publishers of *Electoral Studies* (Heineman and Butterworth), for permission to include the published papers in this thesis.

5. *Conclusion*

Because of its applied nature, the work presented for this thesis falls between the four stools of mathematics, statistics, public choice economics and political science. It is hoped, therefore, that in bringing these papers together in this way, it may help to provide a focus for future work.

6. *Postscript: September 1995*

After submission of this thesis, I became aware that the central idea of Paper eleven "The Probability of Being Decisive" had already been expressed in an article "Estimating the Efficacy of a Vote", by I.J. Good and Lawrence S. Mayer, *Behavioral Science*, vol. 20, no. 1, January 1975. This article has been lost sight of in the literature since then, and I am not aware of its having been cited. I was not aware of the article at the time of my submission.

As a result of the examination process, and at the suggestion of one examiner, some changes to the Foreword (pages (iv) and (v)) and to the Introduction (pages 6, 7 and 11) of this thesis have been made.

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Part II

Justification of the Australian electoral system, and the size and composition of the Houses

These papers are concerned with broad aspects of the Australian electoral system. Later parts of the thesis examine more technical aspects of voting.

Paper 1 "Stability and the Voting System in Australia" was written just after a Royal Commission Report on the voting system was presented in New Zealand. That Report suggested a change in the voting system in that country from the present system to one similar to the German mixed multimember constituency system of proportional representation. The aim of the paper was to add to that debate, by suggesting the system common in Australia, of preferential voting, as a further alternative.

It is interesting to note that since that time, New Zealand, as the result of two referendums, has moved to adopt the German proportional representation system, with minor variations. The movement in this direction occurred because minor parties, at election after election in New Zealand, gained substantial proportions of the vote, yet never won more than two seats in Parliament. Meanwhile, in Italy, the reverse has substantially occurred: a proportional representation system prone to short-run instability has been replaced by a system where most of the representation is through single-member constituencies.

In the Australian system, as shown in the paper, there is much greater incentive for the main parties to accommodate to the policies of minor parties, when those policies are thought to be sensible, thus stunting the growth of these minor parties at every turn. It is only the existence of

Upper Houses elected by proportional representation in the Federal system and in some states which allows such minor parties a Parliamentary presence and indeed, their continued survival.

A comparison of the Australian system with that of the USA has not been made. It may be suggested, however, that the lack of party discipline in USA Parliaments, leading to many crossings of the floor of Parliament, plays the same role of incorporating minority views, and holds the number of main parties to two.

Paper 2, "On Even Numbers of Seats in Parliament", is concerned with the possibility of a Lower House which is tied: the two main party groupings obtain exactly half of all the seats. It is a plea for a more flexible interpretation of the Australian Constitution, one which would allow the House always to have an odd number of seats. The problem is again topical, because at the next election (after December 1994) the House will once more have an even number of seats (148).

I think I have severely under-estimated the cost of having a tied House, by taking only the direct costs of another election, and ignoring the substantial indirect costs. These costs include the diversion of activity and thought towards the political process, and the uncertainty generated in many markets by having a Parliament in limbo.

If the High Court were to decide that the Constitution deserves to be interpreted more flexibly in this area, then I would be concerned to allow a gradual increase in the size of the Lower House over time. When population grows unevenly geographically, and the House size is fixed, some areas (states) have to lose representation, even in cases where the absolute size of the population has increased. One would imagine that an increase in population should lead to an increase in representation, *ceteris paribus*. One should therefore attempt, as far as possible, to maintain the numerical representation of slower-growing states, while rewarding *above-average* increases in population with increased representation. This would lead to fewer forced redistributions of boundaries. Where redistributions occurred within states, they would in general not need to be such drastic ones. The population tends to identify with an electorate — after all, why bother to have single-member geographically-based electorates if it were not so? It is likely that there will be a loss in community amenity whenever boundary alterations occur, which are over and above minimal requirements due to population changes.

If changes to the size of Parliament in this way were possible, the size of the Lower House would grow in size to reflect, not the increase in population, but its *differential* increase between states. For all that, let us not lose sight of the paper's main point: an even number of seats in the Australian Parliament could lead to a senseless and expensive deadlock, and one which could be so easily avoided.

Paper 3 "The Representation of Territories in Federal Parliament" is again topical. The Leader of the Opposition, as recently as November 1994, has stated that he wishes to establish ground-rules for the representation of new states (specifically the Northern Territory) by 1997.

PAPER 1

Fischer, A.J. (1987) Stability and the voting system in Australia.
Presented at: ANZAAS Conference, 26-30 January, Palmerston North, New Zealand

NOTE:
This publication is included on pages 15-21 in the print copy
of the thesis held in the University of Adelaide Library.

PAPER 2

ON EVEN NUMBERS OF SEATS IN PARLIAMENT

A.J. Fischer

Abstract

Democracies will sometimes fall into disrepute as a result of inappropriate electoral rules. It is incumbent of Parliaments in these countries to ensure that the mere mechanisms of the electoral process do not interupt the smooth-running of the nation. Suggestions for finding and correcting possible malfunctions of the electoral system should be carefully considered. This paper looks at the potential problem of having an even-number of seats in the Australian Lower House (the House of Representatives), estimates some of the costs that this may involve the nation in, and suggests changes to the system.

ON EVEN NUMBERS OF SEATS IN PARLIAMENT

I INTRODUCTION

This is a plea for an additional criterion to be used by to decide the number of seats for the Australian House of Representatives every so often: never construct the total number of seats in the Australian Lower House to be even.

Currently there are 148 seats. Should the house be evenly divided at 74-74, the party attempting to govern would have to provide a Speaker (unless it could poach a Speaker from the other Party) and could therefore be expected to be in a minority. This would almost certainly require another immediate election, as the Opposition, in a majority on the floor of the House, would more a vote of no confidence in the Government. There is no guarantee that the result of a second general election would be different from the first. Whether or not the same impasse occurred once or more than once, the Governor-General may be required to make some invidious decisions, such as the appointment of a caretaker Prime Minister. Shades of 1975......

This possibility could be avoided by the simple expedient of ensuring that there are always an odd number of seats in the Lower House. Even a majority of one could allow the larger party to govern for what might turn out to be a considerable time. Deaths of Parliamentarians in office are not common, and if a death occurred in the ranks of the Government with a majority of one, it would not be likely to be in a marginal seat; even if it were, there is no guarantee of a by-election loss in such circumstances.

The main reasons for not ensuring an odd number of seats appear to be, first, that no provision has been made for it in the Australian Constitution, and second, the belief that a tied House would be unlikely to occur. In one sense this is correct: the probability of its occurrence is probably about once per 100 years, which is rare enough in relation to the length of time that democracies have existed. However, it is rare in the same way that motor vehicle accidents are rare as a proportion of the number of accident-free journeys. It still pays people to use safety devices. That some do not may be attributable to the belief "It will never happen to me."

II OCCURRENCE AND COST OF A TIED HOUSE

How then could we calculate the occurrence of a tied Lower House? One way is to look at the size of the majorities of the parties over the last few elections. The largest Liberal National Coalition majority was 47; the largest ALP majority since World War II was 24. That is, in the first case, 24 seats changing hands would have just changed the government; in the second case, 12 seats changing in the other direction would have tied the parties. Thus the range of possible wins and losses has been about 36 seats. Assuming that all majorities between +47 and -24 for the Liberal-National coalition are equally likely (and an inspection of the meagre number of data points suggests that this is not an unreasonable hypothesis), there is about a 3% chance of any one of them. That is, there is about a 3% chance of a tied House if there are an even number of seats. Alternatively, we could work out the average state of the House over the last 45 years (this isn't as easy as it looks, because of the difficulty of dealing with a House expanding in size over time): the average is a small Liberal National majority of about 10. The standard deviation is about 10 also. If this set of observations is generated by a normal distribution with those parameters, we can work out the probability of a tied House: by this alternative calculation it is between 4 and 5%. Either way, it is not a negligible probability, unless one believes, magically, that the electorate is never, in aggregate, neutral.

We may now attempt to calculate the cost of allowing a tied House to eventuate, assuming that it would only ever happen once. Assume that the Lower

House always has an even number of electorates. (This assumption will be relaxed later.) Let the cost of its happening at an election held tomorrow be $C. Then since there is only a 3% chance of that (we choose the lower figure), the cost will be 0.03C dollars. If elections on average are 2½ years apart, the probability of its occurrence at the following election will be (0.97) (.03), and if the rate of time discount is a relatively high 5%, then the present value of the cost of that is $\frac{(0.97(0.03)}{(1.05)^{2.5}}C$ dollars. Working out all future costs in the same way, we arrive at a net present value of 0.21C dollars, and since the next election is not always tomorrow but on average is 1¼ years away, this reduces to 0.20C dollars. If the direct cost (to the Electoral Office and to the parties) of running an election is all-up calculated at $100 million, it would be worth up to $20 million to avoid an even number of electorates. If the probability of tying is 5% and not 3% the cost rises to $30 million. This does not allow any cost attributable to the disruption to the country caused by the uncertainties of being in a Parliamentary limbo with a tied House. Presumably this would be considerable.

The rules governing the number of electorates in the Lower House are determined by Section 24 of the Australian Constitution [1], which states that the number should "as nearly as practicable" be double that of the Senate (the Upper House). Currently there are 76 Senators but only 148 MHR's, rather than the 152 suggested by the Constitution. It appears that the requirement "as nearly as practicable" is not adhered to in a strict sense at the moment, so there should be no difficulty in adding or subtracting a seat in future. This Section of the Constitution is, however, rather much of a mess at present. The reason that "as nearly as practicable" was inserted appears to be as follows: ignoring the Territories (which were unrepresented in 1901), the seats of the six States are determined in proportion to their population: there are 72 Senators from the six States, so there should be 144 MHR's. Apportioning these seats to the six States will result in non-integer values which must be converted to integers. This is achieved by taking as the number of MHR's in a State the nearest integer, which means that a State with an entitlement of 13.4 seats will finish with 13, while with 13.6 seats will be granted 14.

If 3 entitlements are rounded up and 3 rounded down among the 6 States, the aggregate of 144 will be achieved exactly. But if more than 3 are rounded up, the aggregate of 144 will increase to 145, 146 or 147, while if less than 3 are rounded up, the aggregate will decrease to 143, 142 or 141. A calculation may determine approximately the probabilities of each of these possibilities for the numbers 141 to 147 as being, approximately, the binomial probabilities 1, 6, 15, 20, 15, 6, 1 out of 64 respectively. That is, there is a 31% chance of 144 seats, a 24% chance of each of 143 and 145, a 9% chance of each of 142 and 146, and a 1% chance of each of 141 and 147.

To the extent that the aggregate of 144 is not always attained exactly by this process, the phrase "as nearly as practicable" has been used. The combined binomial probabilities of an even numbered aggregate of 142, 144 or 146 is exactly 0.5. When an adjustment of one extra seat for Tasmania is made to bring it to the mandatory minimum of 5 seats for an original state, and the 3 seats of the two Territories are added in, the final number of seats is determined, and there will still be a 50% chance that the grand aggregate will be even. The point being made, however, is that the final seat number of 148 is in no way "as nearly as practicable" equal to 152, the number which is twice the number of Senators. The reason for the disparity appears to be that when representation for the Territories was decided, the formula used for the six States was held at the 2:1 ratio of MHR's to Senators. Then the 4 Territory Senators and 3 MHRs were added in. If the 2:1 ratio were adhered to for these seats, there should have been an extra 8 MHR's created for the 4 Territories' Senators. These 8 MHRs, it appears from the Constitution, need not all represent the Territories - 5 of them could represent additional State-seats if only 3 are the appropriate number to be apportioned on a population basis to the Territories. (In fact, a situation like this happened at Federation. When the Constitution was framed, it was not known whether Western Australia would join as an original state. If it were not to join, there were to be 30 Senators, (6 for each of 5 states) and so 60 MHR's. However, if W.A. were to join, there were to be 36 Senators (6 for each of 6 States) and 72 MHR's. But W.A. was entitled to only 5 of these Lower House seats on a population basis, and so the extra 7 seats were apportioned to the other five states.)

We are now in a position to revise the present value of the cost of allowing the possibility of a tied House. It seems from the calculations just made that the probability that there will be an even number of seats in the House is one-half. That being so, the present value of the cost falls, on average, to half of the previously-calculated value of $20 or $30 million - that is, it falls to $10 or $15 million. (Actually it will be somewhat more than this if a new apportionment has just given an even number of seats, because the possibilities of a tie are closer to the present. Conversely it will be somewhat less than this if a new apportionment has just given an odd number of seats. This makes intuitive sense: there is more need to be concerned about a tied House if there is currently an even number of seats than if there is currently an odd number.)

There is one other defence for having 148 MHRs and 76 Senators, and still be in accord with the 2:1 nexus and the "as nearly as practicable" proviso. Start with the House of Representatives number of 148. This must be twice the size of the Senate by the 2:1 ratio rule, so the Senate must have 74 seats. Allot 4 Senators to the Territories, and divide the remaining 70 by 6 to get the allotment per State. To the nearest integer this is 12, so there must be 6 lots of 12, or 72, State Senators, and adding the 4 from the Territories back in gives 76. However, this is a twist of logic. The Constitution says that the House of Representatives should be as nearly as practicable twice the size of the Senate, not that the Senate should be as nearly as practicable half the size of the Lower House.

Thus with 148 members, the size of the House of Representatives is not "as nearly as practicable" twice the size of the 76-member Senate as envisaged by the framers of the Constitution. To justify the situation, one would have to argue that "as nearly as practicable" has now taken on a rather looser interpretation than was intended (or that of course a High Court challenge would find the present interpretation unconstitutional). Given the looser-interpretation explanation, it would not be difficult to justify a further adjustment of one seat whenever the Lower House number turned out to be even.

III MECHANISMS

By what mechanism could the total number of Lower House seats be ensured to be an odd number? Section 24 of the Constitution gives a mechanism to apportion the seats between the six States, but says that the Parliament can make alternative rules for this apportionment if it wishes. It helps to engage in a little algebra at this stage. Suppose there are N Senators in aggregate. Then the "Expected" size for the House of Representatives would be 2N. If the rule for each State's entitlement were to remain the "nearest-integer" rule, then the total number of seats could finish as an even number, either (2N-2), (2N) or (2N+2). If this happened, we could regard this as a first iteration only, and go back to the "Expected" figure of 2N and add one to it, and do the State's apportionment again. The problem with this approach is that the new apportionment might once more give an even number of seats! So might <u>all</u> other apportionments for starting values of 2N - 1, 2N-2, 2N+2, etc. The likelihood of this is small, of course, but the method needs to be foolproof. One way of overcoming the problem within a nearest-integer rule for the States would be to start with the Expected 2N seats for the Lower House, and if the "nearest-integer" aggregate turns out to be even, to iterate using $(2N+\epsilon)$ as the "Expected" starting point, where ϵ is a small positive fraction (such as .001). If this did not work, the starting point would be increased by an extra ϵ each time to $(2N+2\epsilon)$, $(2N+3\epsilon)$, etc, until an odd aggregate ensued. With a computer, the allocation could be done instantaneously. Of course, it could also be arranged that ϵ be negative but this would sometimes have the following drawback. For example, suppose that 152 seats were "Expected", and that 154 arose from the first iteration, with South Australia gaining 13.51 seats and therefore a 14-seat entitlement. If ϵ were negative, then 151.88 as the newly-iterated "Expected" (with $\epsilon = -0.12$) would drop South Australia's entitlement to 13 on account of its raw score of 13.4993, possibly some cause for outrage because of the closeness of 13.4993 to 13.5. Perhaps it would be better to add a seat (bringing the size of the House to 155 than subtract it (to reach 153) in such circumstances.

Another desirable feature if the laws were revamped might be to avoid the loss of a seat in a State which loses its share of the population at a redistribution. If the Lower House remains approximately the same size (that is, the Senate's size is unchanged) then it might be psychologically better and administratively easier to keep that State's entitlement unchanged and to increase the entitlements of other States. This would be possible if the size of the Lower House started off a little less than 2N and over the years gradually increased. Given this additional criterion, it would still be possible to argue for modest increases in the size of the House as being "as nearly as practicable" twice the size of the Senate, where the sense of what is practicable would be in relation to this criterion as well as allowing for the nearest-integer rule and the odd-numbered aggregate.

The question arises as to how flexible "as nearly as practicable" can be. Because of the rotation of Senators, it has always been decided that the number of Senators (from each of the six States, at least) should be an even number. While rotation occurs, therefore, the size of the Senate, when it changes, must increase or decrease by 12 seats. (This assumes a constant representation of two seats for each Territory.) That is, the Senate must have 52, 64, 76, 88 or 100 members, if it is to have between 50 and 100 members inclusive. The Lower House must correspondingly be, as nearly as practicable, 104, 128, 152, 176 or 200. The most flexible that the Lower House can be around the current figure of 152 is from 140 to 164 seats; less than 140 would require a reduction in Senators per State from 12 to 10, while more than 164 would require an increase from 12 to 14. Thus in terms of odd numbers, the Lower House could currently range from 141 to 163 seats and still be within the 2:1 ratio "as nearly as practicable" allowed by the Constitution.

It would also be possible to scrap the nearest-integer rule and to allocate exactly 2N seats (or, more desirably, either (2N+1) or (2N-1), the two nearest odd numbers to 2N) by rounding up as many of the fractions as necessary to get to the required number in aggregate, starting with the largest fraction and working down. For example, if the fractional leftovers for the six States were .9, .8, .7, .6, .5 and .5 (adding to 4.0) then there are four seats still to be allocated, and the allocation would give the seats one each to the States with the 4 highest left-overs (viz .9, .8, .7 and .6). Note one drawback of this procedure: it suffers potentially from the so-called "Alabama paradox", whereby it would be possible for a State to have its representation decreased by one seat when the size of the House increased by one seat. This need not be a problem if the size of the House increased rarely and then only in quantum leaps when the size of the Senate increased, but it could be a problem if the Lower House were to increase in size gradually, as mooted above.

IV RECOMMENDATIONS

The size of the House of Representatives is currently 148. The following rules should apply for future numbers of MHRs.

(1) Whenever a change is required in the number of MHR's, there should be the smallest possible increase, rather than decrease, but consistent with the following rules

(2) The number must be odd

(3) The number cannot exceed 163 while the Senate remains at 76

(4) The representation from each State should not decrease, unless the number of a State's MHRs calculated on a population basis is at least one whole number below its current representation. (For example, a State with a representation of 30 seats which now is entitled to only 29.3 seats on the new population figures should still receive 30 seats. However, if it is now entitled to only 28.9 seats, it would lose a seat. To ensure the State's representation remains at 30 seats when it is entitled only to 29.3 on a population basis, the size of the House should be increased by enough seats to bring the 29.3 above 29.5, where that is possible. If the House is already at 163 seats, however, the State would continue with 30 seats unless or until the State's entitlement dropped to 28.99 or less.)

(5) The representation of those States whose entitlement remains the same or increases should be rounded off to the nearest integer, as at present. Should this result in aggregate in the House having an even numer of representatives, the starting size of the House should be successively increased by a small fraction, enough times to allow one State to increase its representation by one seat and to simultaneously increase the aggregate by one seat to an odd number. If the even "Expected" number of representatives were 164 before this adjustment, however, an equivalent process would have to be used to round down to 163 seats. (An alternative rule to (5) would be to round off each State's entitlement to the nearest integer, and if the House had an even number of seats as a result, to take the State which had not been rounded up and which had the largest remainder, and round it up rather than down.)

V DISCUSSION

The cost-benefit analysis of Section II applies to any single-member Parliament with an even number of seats where the Speaker has only a casting vote. A similar analysis could therefore be done for each ~~and~~ legislature, although where there are more than 2 parties in the Parliament, a tied House may untie itself by one of the parties changing sides. So this analysis will usually only apply to strict two-party legislatures with no changing of party allegiance once Parliament sits. It could apply, of course, if a small third party could only ever side with the smaller of the two main parties, so tying the numbers (for example a small Communist Party siding with a larger Social Democrat party, and between them capturing exactly half of the seats).

Sections III and IV apply only to the peculiar case of the Australian House of Representatives, whose size at twice that of the Senate (and with States' entitlements of seats rounded off to the nearest integer) will often be even. Democracies ~~Discussions~~ will occasionally fall into disrepute or even become unstuck as a result of inappropriate electoral rules. It is incumbent of Parliamentary democracies to ensure that the mere mechanics of the electoral process do not interrupt the smooth-running of the nation. Therefore, suggestions for debugging the electoral system of possible malfunctioning should be considered carefully.

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PAPER 3

Representation of Territories in Federal Parliament

1. Introduction

Representation of Territories in Federal Parliament should be based on formulae which are valid for a wide variety of circumstances, and which therefore should be better able to accommodate population and other changes over time.

In order to determine principles, I shall consider not just the two main Territories for which the formulae would be immediately applicable, but look at the question quite broadly, and to consider the A.C.T. and N.T. as two cases within a general framework.

Let us therefore consider the general question of the admission of any new State into Australia. Such a State may be currently a separate country (e.g., New Zealand or a smaller Pacific country), part of a separate country (e.g., North Island of New Zealand and South Island of New Zealand), a splitting of a current State (e.g., Northern N.S.W.) or admission to Statehood of an existing Territory. All of these cases, except the last, are at the moment unlikely candidates to become States. Furthermore, a union with New Zealand, should it ever occur, would presumably not follow the path of New Zealand applying to become one or two States, but we shall not be deterred by the low probability of this happening, and shall formulate general rules which could cover the eventuality.

2. House of Representatives

For the House of Representatives, the situation is clear. With a minor exception in the case of Tasmania, representation currently follows the rule that all electorates have the same number of voters, with a small tolerance of error. The requirement that Tasmania have five electorates as a minimum is a small distortion in that it should have four on the basis of population. This distortion would be magnified if Tasmania were to grow more slowly than the rest of Australia and the Parliament were not increased in size from time to time. However, although Tasmania has grown more slowly, this has been more than offset by the increased size of Parliament, and so is never likely to be a great distortion.

However, suppose that we took the view that any new State should receive a minimum of 5 seats in the House of Representatives. Would we be prepared to give 5 seats to Norfolk Island or Nauru or another island community with a population of 10,000? Presumably not. One suggestion by H. Theil and L. Schrage (1977) is that to determine the representation of each state in the European Parliament, the *square root* of the population be used as the factor of proportionality rather than population itself. However, that was for a single-chamber system, and not for a bicameral system in which rights of existing states have already been established.

The aberration in the case of Tasmania is so small that it seems that the same general rule that applies elsewhere should also apply for any new State, viz all electorates in the new State should have the same number of voters as in all other States, with a similar tolerance. The exceptions are for States with huge or tiny populations. If for example Indonesia sought Statehood on this basis, its representatives would swamp the Australian Parliament, which would not be countenanced by the Australian people: any union with large countries would not be by the method of Statehood. For States with a population of less than 50% of an ordinary electorate, a problem exists in determining their representation. (In the region of 50% to 150% of an ordinary-sized electorate (in terms of voting population), the number of representatives may be rounded off to 1). It is not clear to me that States with population less than 50% of an ordinary electorate should have separate representation in the Lower House: the population of small island communities would have to seek some other form of relationship with Australia, other than getting a vote in referenda and possibly Senate representation. For very small populations, incorporation into an electorate of an existing State would allow that population to vote in House of Representatives elections, and to be represented by a member whose responsibilities included these voters.

If the distortion caused by Tasmania is still thought to be a problem in principle, we may be guided by a theorem in the economics discipline, called the Theory of Second-Best, which deals with such cases. For example, if there are "first-best" conditions of perfect competition everywhere in the economy, and if the government is to begin a new monopoly enterprise in a new industry, the socially optimal pricing policy for the government monopoly is for it to act as if it were also perfectly competitive. However, if in the economy there is already a monopoly among an otherwise competitive set of markets, and the government sets up its new monopoly in the new industry, the socially optimal pricing policy for it is no longer the competitive price, but one which is between the competitive price and the monopoly price. (This is known as the second-best solution). Applied in this case, it says that voters in a new State should be allowed electorate sizes lower than the average for mainland Australia, but not as low as those operating in Tasmania.

In all, therefore, the immediate consequence of the above argument is that representation for the House of Representatives for the Territories of A.C.T. and N.T. should remain as it currently stands, at 2 and 1 seats respectively. In the longer term, representation should be on the same basis as exists in the rest of Australia. If any concession is made in terms of the size of the population in an electorate, it should not make voters in either Territory better-off than those in Tasmania.

3. **The Senate**

It is harder to formulate formulae for the representation of new States in the Senate. A widely accepted principle on which to base decisions about the distribution of entitlements is Rawls' Principle, which states that the distribution is made less unequal (and is therefore desirable) if the entity with the least entitlement is granted more from those who have more. But in the Senate, it depends on what the entity should be. Until now, the entity has been the State, and all States have equal entitlement. Accordingly to Rawls, therefore, to be fair, any new State should receive the same entitlement, currently 12 Senators.

This is essentially what happened in the U.S.A. when it opened up new States in the West in the 1800's and into this century. Giving these States equal Senate representation effectively diluted the voting strength of the existing States, as the new States had relatively small populations in most cases. Over the years, westward movement of the population has tended to reduce this distortion to some degree, and to give representation somewhat more equally with population. However, it is not clear that this is the best formula for Australia, partly because of the relatively greater diminution of power to the existing States by increasing the size of the Senate by 20 (a 26% increase) on admission of the A.C.T. and the N.T. as full States. (It would, however, be a means of "loosening" the nexus between the two Houses, in that the creation of 40 extra House of Representative seats would be possible in consequence).

However, the overriding reason that new States should not get 12 Senators is that the Senate is no longer a States house, if it ever were. It is based mainly on party lines, and because of proportional representation, is able to encompass some minority views (that is, other than those of the A.L.P. or the Liberal-National coalition). People do not vote for Senators from South Australia because they represent South Australia first and foremost, but because they are Labor, Liberal or Democrat. And the same thing occurs in other States.

Therefore, history and our Constitution apart, there is no real justification for having representation based on States to any extent.

To grant 12 Senators to new small-population States would be to increase the inequalities in voting power of people in Australia. *Individual voters* in N.S.W. have a smaller proportionate representation in the Senate than those in other States, and this inequality would not only be perpetuated, but would be made much worse if new small States were to be given a full complement of Senators.

It would be a different story if the Senate were truly a States' House (more like the meetings of Premiers), but as it does not act this way, there is no point in trying to perpetuate a system of representation for new States which enshrines this outdated principle. (I would wish to argue a similar case - that the basis of representation for the Senate should not necessarily be equal numbers in each State - if this were not part of the Constitution, and one which would not be possible in practice to alter).

A different set of possibilities exists if the entity to receive the entitlement of representation were the individual. The first is to give new States the same representation ratio as is given to the best-off State, Tasmania, with 12 representatives for about 420,000 people, or about 1 per 35,000. At the other end of the scale, new States could be given the same representation ratio as is given to the worst-off State, N.S.W., with 1 per 450,000 (approx.). In between, new States could be given the same representation ratio as the average for all States of about 1 per 200,000 people.

If population is to be the basis of representation, therefore, inequality is definitely increased if representation of a new State is less than one Senator per 450,000 people or more than one per 35,000, at present population levels. (To be more precise, one should talk of electors rather than persons, but it is easier to talk of people. If these principles were ever legislated upon, they would need to be fine-tuned to talk of representation per elector rather than per person).

Thus the *minimum* representation for a new State is one Senator per 450,000, the current representation in N.S.W. However, this may not be optimal, as it would make people in Territories equal to the worst-off Australians (those who live in N.S.W.) in terms of representation.

If we decide that we have to live with *existing* inequalities, but that for any new States, we say that the representation *for each member of the population* should be the same as that of other Australians, the fairest method is to give representation on the basis of 1 per 200,000, because that is the average representation in the existing six States of Australia. This would probably be close to the Second-best optimum.

The most generous scheme would be to give representation on the basis of that of the best-off State, Tasmania. That is, one per 35,000. However, this would be to the disadvantage of everyone in the more populous States, who receive less representation than this. This reasoning cannot therefore be sustained if population is the basis for representation of new States in the Senate. Nevertheless, it may well be appealing to some, and its ramifications should be spelt out. On current population, it would give the A.C.T. about 6 or 7 Senators and the Northern Territory 3. At such time that the population of either of these two territories equalled that of Tasmania (i.e., the least populous original State), representation should equal that of Tasmania, and not go beyond it. That is, while Tasmania and all other original States have 12 Senators, that would be the maximum for any new State also.

I would argue, however, for a less-generous scheme (for less-populated new States) on the basis of average representation, i.e., one per 200,000 at present, but would equally argue that a new State with more than 2.4 million people should receive more than 12 Senators. In the unlikely event of New Zealand becoming a State of Australia, it would give New Zealand something like 15 to 20 Senators, and this would not alter if New Zealand decided to become two States (North and South Islands), as the number of Senators would be shared between the two islands. (Of course, for many other formulae put forward, becoming two States of Australia would give New Zealand double the Senate representation compared with its becoming a single State).

On this basis, the A.C.T. and the N.T. should receive only one Senator each, which is less than their current entitlement. There is good reason why both places should continue to have a minimum of two Senators. Firstly, they already have two Senators, and it would be difficult to take them away. Secondly, if Senate representation is based on proportional representation, it would allow both of the main parties to be represented if the two Senators are elected simultaneously, as at present. This does not alter the balance of power in the Senate. Neither main party is advantaged or disadvantaged by such a scheme. Nor does it really affect the Democrats or other smaller party, as the *difference* on the floor of the Senate between the voting strengths of the main parties remains constant by adding one to each side. (In the event of the support of one of the main parties in a Territory falling below 33% of first preference votes, however, this may not hold. So be it, if one party loses popularity to such a degree). With larger numbers of Senators per new State, however, such a chummy scheme (in effect, of "pairing") is much more likely to be upset.

Thirdly, an argument based on community of interest says that both main parties need to be represented in a new State in the Senate, because otherwise a main party may have no Federal representation at all in that State. (The argument has held for some years in Tasmania, also, as all five MHR's belonged first to the ALP in the early 1970's and then to the Liberals in the late 1970's).

Therefore, the rule should be that if a new State is admitted into the Commonwealth, it should have a minimum of two Senators, however small its population. When there are two Senators, these Senators should be elected simultaneously. The period of their election appears to be immaterial - either 6 years, 3 years, at every first or second election of the Lower House, or at a Double Dissolution if it comes sooner. (Given that the basis of the representation is not based on equality between States, there is no reason to prefer any of these terms over the others).

As the population of the new State increases, its Senate representation should be on the basis of one Senator per average number of electors in Australia. The practical effect of this is that up to current populations of about half a million, the representation should remain at two Senators. There is an argument to suggest that the number of Senators should be an even number, but this argument has most effect if the Senators are elected in rotation, which they need not be. If they were rotated, it would be sensible that, on becoming eligible for 3.0 Senators, a new State became entitled to four Senators, two at each rotation. For the scheme which I prefer less (one Senator per average number of voters in Tasmania i.e., one per 35,000 population approximately) together with rotation, the rule should be that there be an even number of Senators, the calculated number being rounded to the nearest even number. For example, if the A.C.T. were entitled to 6.9 Senators by formula, it would get six, but if it were entitled to 7.1 Senators, then it would get eight. With six Senators, there would be three to be elected at each rotation; with eight, four would be elected at any one time.

4. Concluding Remarks

This paper has considered new States, but obviously new State status need not be a concomitant of existing Australian territories receiving representation along the lines suggested. Equally obviously, however, the proposals need not mandatorily apply to Territories. For example, it would be ludicrous for two lighthouse keepers to be the two electors and Senators for a Territory.

References

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Mathematical Formulae Summarising Proposals:

1. House of Representatives

Let N = the number of electors in Australia (including new States)

E = the number of electorates in Australia (including new States)

N_T = the number of electors in Tasmania

N_S = the number of electors in a new State

While Tasmania has a minimum of 5 electorates, the average number of electors per electorate in the rest of Australia, (excluding Tasmania but including new States) is given by

$$\frac{N - N_T}{E - 5}$$

This is greater than $\frac{N_T}{5}$, the number of electors per electorate in Tasmania.

(a) Preferred Proposal

The number of electorates per new State is given by E_S where $E_S = \text{Int}\left(\dfrac{N_s}{\left(\dfrac{N - N_T}{E - 5}\right)}\right)$ (1)

(b) Second-Preference Proposal

The number of electorates per new State is given by a number which allows the number of electors per electorate to lie between $\frac{N - N_T}{E-5}$ and $\frac{N_T}{5}$, that is

$$\left(\frac{N_S}{\left(\frac{N-N_T}{E-5}\right)}\right) > E_S > Int\left(\frac{N_S}{\left(\frac{N_T}{5}\right)}\right) \qquad (2)$$

One particular value of E_S which satisfies this range is $E_S = \text{Int}\left(\frac{N_S}{N/E}\right)$ (3)

That is, the average size of electorate in the rest of Australia (including Tasmania), N/E, is used to divide the number of electors in the new State.

Note that more than one value of E_S may satisfy (2). This second-preference proposal is relatively indifferent between these values of E_S, apart from suggesting that the value of E_S as given by (3) is the most natural value of E_S within the range given by (2). In most cases (1) and (3) will give the same value of E_S in practice. Whichever of the above proposals is used, the value of E_S should be determined after Tasmania's 5 seats but before the allocation of seats to the other States, because of rounding error.

(c) Non-Preferred Proposal

$$E_S = 5 \qquad (4)$$

2. Senate

Let S^* = the total number of Senate places in the six States of Australia (= 72 at present)

N_R = number of electors in these six States

S_T = number of senators for Tasmania.

(a) Preferred Proposal

The average number of electors per Senator in original States

$$= \frac{N_R}{S^*} = \frac{N_R}{72}$$

The number of Senators per new State, given by S_S is given by

$$S_S = \max\left(2, \mathrm{Int}\left(\frac{N_S}{N_R / S^*}\right)\right) = \max\left(2, Int\left(\frac{72\,N_S}{N_R}\right)\right) \qquad (5)$$

where max (a, b) denotes the larger of a and b.

(b) Second-Preference Proposal

Average number of electors per Senator in Tasmania (currently the smallest-population original state) $= \frac{N_T}{S_T}$ ($= \frac{N_T}{12}$ at present)

$$S_S = \min\left(12, \max\left(2, \mathrm{Int}\left(\frac{N_S}{N_T / 12}\right)\right)\right)$$

$$= \min\left(12, \max\left(2, \mathrm{Int}\left(\frac{12\,N_S}{N_T}\right)\right)\right)$$

where min (a, b) denotes the smaller of a and b.

(c) Non-Preferred Proposal

$S_S = 12$

Part III

Fairness of Voting: Single-Member Constituencies

Part III of the thesis begins my work on "decisiveness", which pervades much of the thesis, and is its major theme. The first paper in this Part is paper 4, "Measures of Power in Australia Politics". The power indexes developed by Shapley and Shubik, and by Banzhaf, are examined, but found to be only of limited application in those cases where we have some knowledge of the preferences of individuals and groups within society. These formal indexes suppose that all outcomes are equally likely. Of course, they are not: the Australian Democrats and the Greens are more likely to vote with the Labor government than against it, in the Senate. Measures of power should reflect these realities, and this paper attempts to make a start on solving that problem.

It is easy to see the progression from this paper in three directions: (1) into papers 5 and 6 on electoral boundaries (2) into the work in Part VI on voter decisiveness (3) back to paper 2 on the probabilities of a tied Parliament.

Paper 4 estimates how frequently a given electorate is likely to be pivotal, given that the two parties are within one seat of each other on the floor of Parliament. Should this state of affairs occur, would one party have a larger popular vote than the other, and if so, by how much? This is the work of Papers 5 and 6, on electoral boundaries in Australia and South Australia respectively. Paper 6 extends the work of Paper 5 to consider how to take incumbency into account in attempting to define fair electoral boundaries. This work differs from that in the literature, because the election for the Upper House in South Australia is conducted at the same time is that of the Lower House, and can be used as a benchmark to identify incumbency effects. The way in which Paper 4 affects Part VI of the thesis will be left to the introductory remarks for that Part.

PAPER 4

Fischer, A.J. (1988) Measures of power in Australian politics: an initial investigation.
Presented at: The Centenary Congress of ANZAAS, 16-20 May, Sydney, Australia

NOTE:
This publication is included on pages 49-62A in the print copy
of the thesis held in the University of Adelaide Library.

PAPER 5

Fischer, A.J. (1991) Swings and Gerrymanders.
Electoral Studies, v. 10(4), pp. 299-312

NOTE:
This publication is included on pages 63-76 in the print copy
of the thesis held in the University of Adelaide Library.

It is also available online to authorised users at:

http://doi.org/10.1016/0261-3794(91)90022-K

PAPER 6

Boundary Changes in South Australia

I Introduction

Over the last few years, theoretical work carried out by Gelman and King (1987)[1], (1990a), (1990b), (1991)[2] using United States data, and by Simon Jackman (1993) and Fischer (1992) using Australian data, has made it possible to determine in advance, (as far as *can* be determined in advance) whether a set of electoral boundaries is likely to be biased in favour of a particular political party. The analysis goes beyond the "pendulum" analysis popularised in Australia by Mackerras, insofar as it takes account of the between-seat variability of the swing, and also makes allowance for both the personal vote and the incumbent effect. This paper begins by discussing the South Australian case and the recent history of boundary setting in South Australia. It looks at how the 1992 boundaries performed at the most recent (1993) election, and shows how the results should be modified to take into account incumbent effects. The analysis has two novel features: the use of Upper House votes to act as a benchmark for measuring incumbency and the use of an incumbent-party effect rather than an incumbent-member effect to measure the effect of incumbency, and its effect on boundary changes.

II Background and Summary

Following a referendum passed in 1991, the S.A. Government enacted legislation[3] requiring that, as far as possible, electoral boundaries be set so that a party receiving 50 percent or

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[1] King and Browning.
[2] King and Gelman.
[3] Section 77 of the Constitution Act requires that the number of electors within an electorate should not differ by more than ten percent from the state average. Section 83 of the same Act says that in making a redistribution, if candidates from a party or coalition of parties receive more than 50 per cent of the two-party preferred vote, the boundaries must be set so that as far as practicable they will gain over 50 per cent of the seats.

more of the two-party preferred (2-PP) vote should get 50 percent or more of the seats in the Lower House of Parliament. This followed an election in which the incumbent Labor Party had won government with only 48% of the 2-PP vote, and this in turn followed a period of about 30 years ending in 1970 of a notorious gerrymander by the Liberal government, in which the ALP could not win office despite winning, on occasion, over 55% of the 2-PP vote -see Stock (1991).

When a party gets well over 50% of the 2-PP vote, the parliamentary majority is magnified, so that the percentage of seats held by the party exceeds the proportion of the popular vote. Thus in such cases there is no problem in satisfying the "50% vote, 50% seat" rule. The problem arises when the 2-PP vote is close to 50%, when the rule may not be satisfied. That is, slightly over 50% of the 2-PP vote may lead to slightly lower than 50% of the seats.

But now suppose that the party which scores $(50 + x)\%$ of the 2-PP vote $(x > 0)$ gets $(50 + y)\ \%$ of the seats. Because the marginal vote/seat ratio is not one, and is often unknown, it is difficult to know what percentage of seats the party *would* have obtained *if* it had received only 50% of the 2-PP vote. The pendulum gives us a reasonable idea, but it is unable to tell us how accurate our answer will be. For example, if a party gets 54% 2-PP vote and 57% of the seats, what would be the *expected* number of seats if it obtained only 50% of the vote? The pendulum can answer this after a fashion, but if it came up with an answer, say, that a party which won 50% of the 2-PP vote would finish with an expected 48.7% of the seats, it would not be able to predict the proportion of times the percentage of seats would be 50% or more and the proportion of times it would be less than 50%. In addition, the 48.7% figure would not be a very accurate one, as we shall see.

There are three reasons why a Boundaries Commission, acting without intent to bias, will have difficulty achieving the "50% vote, 50% seats" rule. The first is the natural tendency for votes to cluster. Given the geography of the State, this may give rise to what I shall call a "natural gerrymander". The second is the incumbent effect, which also gives rise to a natural gerrymander, and the third is chance.

The clustering of votes operates to the detriment of the Liberal Party in South Australia. For the most part, large Liberal majorities are locked up in country seats, and are adjacent to other Liberal seats also with large majorities. The ALP, on the other hand, tends to win its seats with smaller majorities.[4]

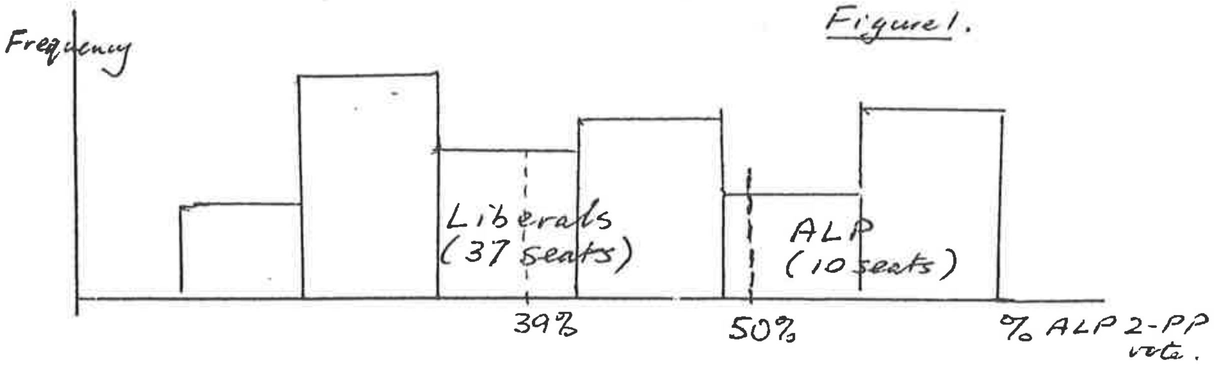
It has not always been so. In times past, areas of high concentrations of blue collar workers such as Port Adelaide locked up huge ALP majorities in a few seats. The country, with a higher proportion of farm labourers and railway workers in the past, returned predominently non-Labor representatives, but with smaller majorities than those of today. Thus the natural gerrymander was against the ALP. Today, with the decline in blue collar employment, the effect is reversed.

The job of present-day electoral commissioners is not made easy by the presence of such clustering in much of the State. There are several ways around the dilemma, none of them easy. The first would be to mix up industrial towns with their hinterlands, rather than maintain the tradition of separating them to allow communities of common interest each to have its own representative. Taken to extremes, the ALP might lose Parliamentary representation from rural South Australia completely. In the short run, this situation may serve the interests of the Liberal Party, but in the longer term, it would not be in the best interests of either side of politics in South Australia. The second solution would be for the electoral commissioners to create corridors to the edge of the metropolitan area, but there are limits to this, and the end result could be laughable. The third would be to work harder

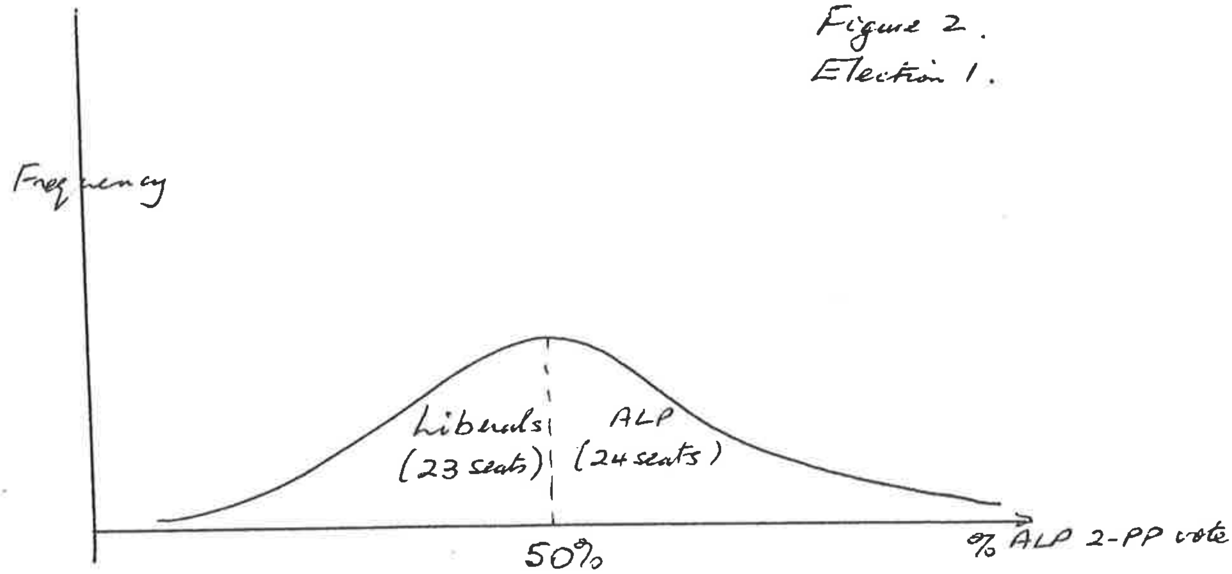
[4] This was the case even before the most recent landslide win to the Liberals. At the 1985 and 1989 elections, Liberal majorities in their safest seats were larger than Labor majorities in theirs.

elsewhere to address the balance, but again, where to find the pockets of heavy Labor concentration?

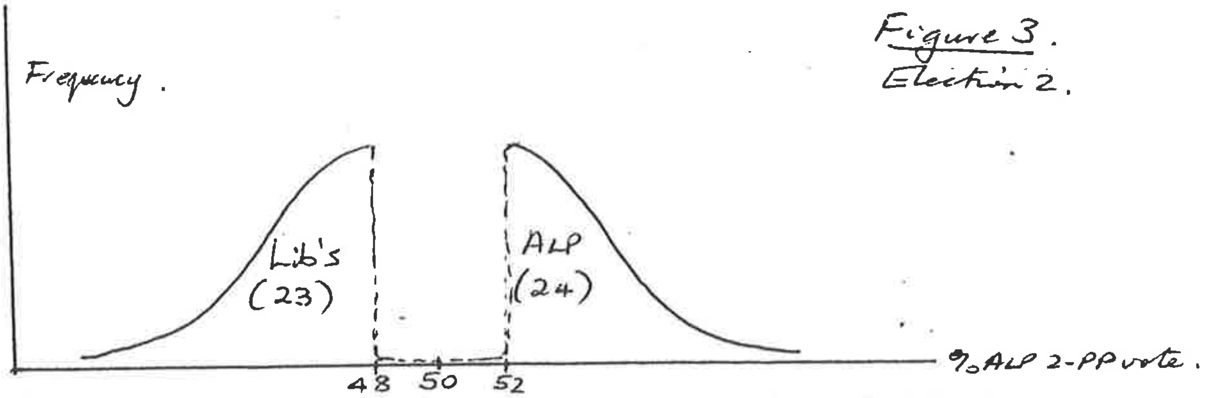
The problem is not so bad when the Liberals are in power, because the second cause of a natural gerrymander, the incumbent effect, works to favour the party in office. Therefore, the bias it affords the Liberal Party in the current circumstances offsets (in fact, more than offsets, as we shall see later) the clustering effect. To see how this works, suppose we graph the frequency of the ALP 2-PP vote in each seat. In stylised form, the resulting histogram is shown in Figure 1.



Now let us make the frequency distribution continuous and look at the case where the ALP gets 50% of the 2-PP vote, not 39% as in Figure 1. Kendall and Stuart (1950) showed that, on average, the frequency distribution will be symmetrical when the boundaries are drawn in a random fashion. This is shown in Figure 2, which we call Election 1.



Suppose that the ALP has 24 seats, having won the 24th seat by just a handful of votes. The Liberals have 23 seats. If the boundaries remain fixed as at Election 2, if each sitting member stands again and gets a 2% advantage for incumbency, and if there is no other change, the frequency distribution will take on the appearance shown in Figure 3.



In this polar example, there can be up to 2% uniform swing to the Liberals and yet the ALP will retain office 24-23, with only as little as 48% of the 2-PP vote. (A uniform 2% swing to the ALP, on the other hand, would also not win it any additional seats).

While the effect may not be so extreme as in the example, there is ample evidence that this phenomenon exists. It happens in all democracies with single member electorates, and it is particularly prevalent in the USA, where district boundaries have changed less frequently, and the personal following of Congress men and women is very high.[5] The effect for the Australian parliament in recent times is illustrated on pages 68 and 69 of this thesis, and a similar pattern for the South Australian parliament was apparent after the 1985 and 1989 elections.[6]

[5] See Gelman and King (1990b). Since the mid-1960's, the incumbency advantage has been of the order of ten percentage points in the U.S. Congress.

[6] Note that the effect in SA intensified after the 1989 election, as there was no redistribution after the 1985 election. In 1992, the Electoral Boundaries Commissioners not only changed many boundaries and the names of many seats, but also removed some of the effect of incumbency by making seats more marginal than had been the case.

The effect in the example above favoured the ALP simply because it was the sitting party. The effect allows sitting parties to survive close contests against the odds. Now that the Liberal Party is in power, the effect will favour it, and this effect more or less counters the clustering effect mentioned earlier.

Chance also plays a role. A party can win more (or fewer) than its share of close contests. In 1989, chance, the incumbent effect and the cluster effect all favoured the ALP, so that by my reckoning[7] the ALP would have had a 50% chance of retaining office with a 2-PP vote of 47.3%. In broad terms, each of the three effects was worth about 0.9% of the vote to the ALP.

However, in 1993, the clustering effect was reduced a little by the new boundaries, and this time, chance favoured the Liberals. The net effect was that, by my calculations, there was virtually no bias in the boundaries. The incumbent effect was insignificant in relation to the size of the swing, but still appears to have kept two or three ALP seats from swinging to the Liberals. Nevertheless, the Liberals will now gain the benefit of the incumbent effect in the additional 14 seats they have won, and, other things unchanged, it is likely to make those seats some 2 to 5 percentage points safer than they were in 1993.[8] The effect will be to favour the Liberal Party by some 0.5 to 1.0 per cent overall. That is, if the election of 1997 were held on the 1993 boundaries, it would be expected that the Liberal Party would have a better than even chance of election if it polled as little as 49.0 to 49.5% of the 2-PP vote.

To complete this section, a word is necessary about what I have called "chance". Some of it is simply luck. But essentially it occurs because the swing in the marginal seats — those required to gain or lose office — differs from the swing in the safely held seats. Federally, the ALP won office in 1987 because it gained seats (it won the marginals) despite an

[7] The method of calculation follows Fischer (1992). See also section III of this paper.
[8] The method of calculation will be described in the next section.

overall swing of 1.0% away from the party. There is some evidence that the marginals swung more than average in 1993 in South Australia, with the Liberals being the recipients of the bonus. In both circumstances, the winning parties essentially made their own luck.

The rest of this paper gives rather more detail about how these results were obtained.

III Method

1. Variability of swing

This study firstly considers what would happen if a further election took place on 1993 boundaries. How much of a swing would be necessary before the seats were distributed 50:50 to the two main parties? What would the state of the parties be (in terms of seats) if both parties had exactly 50% of the 2-PP vote? In undertaking this estimation procedure, it is assumed that the swing is not uniform. Introducing variability into the swing enables a much richer prediction picture to be drawn. The method follows Fischer (1992) and is close to that of Gelman and King (1990a) and of Jackman (1994). In 1989, the swing away from the ALP of 5.2 percentage points had a standard deviation of about 2.5 percentage points, and in 1993, the further swing away from the ALP of 9% had a standard deviation of some 4.3 or 4.4 percentage points. That is, there was a good deal of variability in the swing in 1993: some seats hardly swung at all, while others swung up to about 17%. The larger the variability in the swing, the less accurately can results be predicted. The "pendulum", which is a special case of the analysis, with a swing of zero variability, becomes less and less useful as the swing becomes less and less uniform. However, it turns out that there is little difference between the results of a calculation using a 2.5 percentage point standard deviation or a 4.5 percentage point standard deviation. Since we are most interested in what would happen if the ALP and the Liberal parties both received 50% of the 2-PP we are considering what would happen if there were an 11% swing to the ALP. This is a very large swing: the larger the swing, the more likely it is that there will be some marked variability in the swing, and therefore the more likely it is that the larger swing standard deviation (4.5 percentage points) is the appropriate one to use.

This study also considers what would happen if a further election on 1993 boundaries had a zero overall swing (but a swing standard deviation of either 2.5 or 4.5 percentage points). This is to study the effect of chance on the 1993 result.

2. The personal vote and the incumbent effect: introduction

An important extension of the work is to look at the effect of the personal vote, and that part of the personal vote which is due to the incumbent effect. To do this, we look at the Legislative Council vote at the same election. The Legislative Council, or Upper House, is elected at the same time as the House of Assembly (Lower House). Its term is two Lower House terms. Half the 22 members of the Upper House are elected at each election by the method of STV (Single Transferable Vote) using the whole State as a single electorate. We shall define both an incumbent party effect and an incumbent member effect, as follows. The reasons I do not follow Gelman and King (1990b) or King and Gelman (1991) are firstly that I have very little data. There are only 47 seats in the South Australian Lower House. Secondly, in S.A. there exists a "benchmark" in the form of the Upper House, which Gelman and King lack.

I define a vote in the Lower House as a personal vote for A if a voter's two-party preferred vote for the Lower House is for A, but the same voter's two-party preferred vote for the Upper House is for B, where A and B are candidates for the two main parties of the same respective names. This definition is a fairly restricted one. If a voter votes for A, rather than the usual B, as a result of personal reasons, but then as a result also votes for A's team in the Upper House, it is no longer defined as a personal vote. If a voter votes in opposite directions in the two Houses "to keep the bastards honest", on the other hand, this counts as a personal vote for the candidate in the Lower House. The net effect is probably to underestimate the effect of incumbency. A rather longer project, taking observations over a number of elections, could be performed in order to seek a correspondence between this measure and Gelman and King's (1991b) measure.

Let the House of Assembly (Lower House) vote for the incumbent party for a particular seat be given by

$$A = X + I + P_I - P_C \qquad (1)$$

where X is a pure party vote

I is the vote that is received by the party holding that seat, whatever that party is, merely because it is the incumbent party. (Such people are assumed to vote along party lines for the Upper House.) I is called the *incumbent party* effect.

P_I is the personal vote of the candidate from the incumbent party (over and above I).

P_C is the personal vote for the challenger (from the non-incumbent party).

Let the Legislative Council (Upper House) vote for that House of Assembly seat be given by

$$L = X + P_{IL} - P_{CL} \qquad (2)$$

where X is as above

P_{IL} is the personal vote of those of the Legislative Council team for the party which holds the Lower House seat.

P_{IC} is the personal vote of those of the Legislative Council team for the party which does not hold the Lower House seat.

Note that, since the electorate for the Legislative Council is the whole State, and since there is a team standing for each main party under a proportional representation system of voting, there cannot sensibly be an incumbent effect for the Upper House. If, however, one does exist, it is likely that it will be small, that is, both P_{IL} and P_{CL} will be very small, because while people may vote for a particular person (such as a well-known footballer, a football coach, or a local mayor) in the Lower House, they are less likely to do so when that person is simply one of a team in the Upper House. They also are less likely to vote personally for someone for the Upper House because the probability of any one candidate being, say, the local mayor, when there are 47 Lower House electorates and one Upper House electorate, is only $\frac{1}{47}$ as great in the Upper House as in the Lower.

Thus the difference between the Lower House vote for some electorate and the Upper House vote within that electorate will be given by

$$A - L = X + I + P_I - P_C - (X + P_{IL} - P_{IC})$$

$$= I + P_I - P_C, \text{ ignoring } P_{IL} - P_{IC} \qquad (3)$$

If *on average* over all electorates, P_I and P_C are of about the same size, $\overline{A} - \overline{L} = I$ (4)

That is, *on average*, the difference between the vote for the Lower House and the Upper House for a particular seat will be equal to an incumbent party effect, due to the fact that a particular party already holds that seat in the Lower House. The incumbent effect in the literature is somewhat different. It is concerned with P_I. We split P_I into two parts, as follows:

$$P_I = P_{I1} + P_{I2}, \qquad (5)$$

where P_{I1} is the *incumbent member* effect and P_{I2} is the personal vote that the incumbent member would have obtained when he or she first stood (i.e. there was no incumbent, at least, not from that party).

We can suppose that *on average* P_{I2} and P_C are of similar magnitudes.[9]

Thus we now have

$$A - L = X + I + P_{I1} + P_{I2} - P_C - (X + P_{IL} - P_{CL}) \tag{6}$$

Given that P_{I2} and P_C, on average, are likely to cancel, and that P_{IL} and P_{CL} are also on average likely to cancel (and will also be small and ignorable), then

$$A- L = I + P_{I1} \tag{7}$$

Adding a stochastic element, ε, to allow for variability, we obtain

$$A- L = I + P_{I1} +\varepsilon \tag{8}$$

where ε is distributed with mean zero and variance σ^2.Thus, over all electorates, the ε cancel out, and we get $\overline{A}-\overline{L} = I + P_{I1}$ (9)

In seats where there is an incumbent party (all seats) but no incumbent member,

$$A - L = I + \varepsilon \tag{10}$$

and where there is also an incumbent member (8) pertains.

Models of the incumbency effect in the existing literature have no "benchmark" such as L. It is rare to have an institutional background in which an experiment can take place which decomposes votes into a pure party effect and an incumbent party and incumbent member effects, so to my knowledge it has never been analysed. The literature looks solely at the value of P_{I1}.

In this part of the analysis, I do not attempt to separate out the component P_{I1}. To do so, it is necessary to look at the vote of a sitting member at the time he or she first entered Parliament (election 1), and to subtract it from the vote from that member's second election (election 2). This is compared with the *average* difference over all its seats between the relevant party's vote from election 1 to election 2. This measures P_{I1}, but obviously with

[9] It may be thought that, on average P_{I2} should exceed P_C: that is, on average, the quality of the incumbent is higher than that of the challenger. However, Gelman and King (1990b) do not detect such an effect.

some error, because there would have been new electors at election 2, death or movement of election 1 electors, and there might have been boundary changes. However, P_{I1}, averaged out over a number of electorates, has been found to be positive. In a recent article, Upton (1994) shows that in the U.K., the incumbent member effect averages 450 votes between elections 1 and 2, and some 600 votes subsequently, in a turnout of some 45,000 to 50,000 per electorate. That is, the effect is of the order of 0.9% to 1.0%, rising to 1.2% to 1.3% of the vote. Curtice and Steed (1980) measure the effect as about 1.5% to 1.6% of the vote. As Upton states:

> There is little doubt that the member of Parliament (MP) accrues an advantage over his or her opponents, possibly by virtue of the exposure to publicity that is inherent in the office. Some individuals will also generate a genuine personal following.

Gelman and King (1991b) estimate the incumbent member effect since the mid 1960's for the U.S. Congress at almost 10%. In small electorates in South Australia, the effect is likely to be a little greater than in the U.K. but not as great as in the U.S.A.
Rather than look at P_{I1}, I look at:

$$I^* = \begin{cases} I + P_{I1}, & \text{where there is an incumbent member} \\ I, & \text{where there is no incumbent member} \end{cases}$$

That is, the size of I^* is, on average, between I and $I + P_{I1}$.

For the 1989 election, the incumbent-member effect was 2.6 percentage points, while for the 1993 election, it was 1.8 percentage points. In each case, the effect is about four standard deviations from zero, so it is almost certainly a real effect.

We must now take this effect into account to determine what is likely to happen when seats change hands from one party to the other. Those seats which have been newly won by the Liberal Party will become much safer Liberal Party seats, because there will be a 2 x 1.8 =

3.6 percentage point incumbent advantage to the sitting Liberals. (The multiplication by two is necessary because an ALP advantage of 1.8% has now changed into a 1.8% disadvantage to it). If this is not taken into account in the redistribution of seats, I estimate that the Liberal Party would be able to win the next election even if it only obtained some 49% to 49.5% of the 2-PP vote.

3. Incumbent effects: an example showing systematic differences between Upper House and Lower House votes between parties.

(a) If we look at incumbent effects for the 1993 election results on a seat-by-seat basis, the effect appears to be greater for ALP members, and smaller for Liberals. However, these figures cannot be used in isolation, as the following example shows. We consider first a situation in which there is no incumbent effect either way. We assume that there are 100 voters in seat 1. In the Lower House, there are 3 candidates: ALP, Liberal and Democrat. In the Upper House, there is in addition a Grey Power candidate.

In the Lower House, the ALP gets 46 votes, the Liberals get 49, and the Democrats get 5. When the Democrat preferences are distributed, 3 of them go to the ALP and 2 to the Liberals, so that the ALP 2-PP vote is 46 + 3 = 49 and the Liberal 2-PP vote is 49 + 2 = 51. The Liberal wins. (See Table below)

We now look at the Upper House vote in the Lower House electorate. We assume that in the Upper House election everyone votes above the line.[10] In the Upper House, the ALP gets 43 votes, the Liberals get 47, the Democrats get 8 and Grey Power gets 2. Both Grey Power preferences go to the Liberals, and of the Democrat preferences, 4 go to the ALP and 4 to the Liberals, so that the ALP 2-PP vote is 43 + 4 = 47 and the Liberal 2-PP vote is 47 + 2 + 4 = 53.

[10] Voters in Upper House elections have the option of voting either "below the line" in which case they vote for all candidates in descending order of preference 1, 2, 3, ..., or "above the line", in which case they vote simply "1" for the party of their choice. The preferences on these votes are determined as if they followed the party's how-to-vote card.

Let us also assume that the difference between the Lower and the Upper House occurred because 2 ALP voters in the Lower House voted Democrat in the Upper House and one voted Grey Power and that one Liberal voter in the Lower House voted Democrat in the Upper House and one voted Grey Power. Had the minor-party voters given their second preference for the Upper House as they did for the Lower House (or in the case of those who voted for different parties, had they given their second preferences in the Upper House to the major party they voted for in the Lower House) the Upper House vote after preference distribution would have been exactly the same as the Lower House vote, namely ALP 49, Liberal 51.

For the final distribution, note that the ALP has a 2% advantage in the Lower House over its Upper House vote and the Liberals have a 2% disadvantage.

It would appear that in the 1993 election, this effect was responsible for an aggregate vote a little over 1% higher for the Liberal candidates in the Upper House than the Lower House.

Seat 1

	First Preference Votes		***Final Distribution***	
	Lower House	**Upper House**	**Lower House**	**Upper House**
ALP	46	43	49	47
Lib	49	47	51	53
Dem	5	8	—	—
Grey Power	—	2	—	—

(b) Now let us add in an incumbency effect, +2 for the ALP in seat 2 and +2 for the Liberals in seat 3, both for the Lower House only. Then the figures become:

Seat 2 (ALP incumbent)

	First Preference Votes		***Final Distribution***	
	Lower House	**Upper House**	**Lower House**	**Upper House**
ALP	48	43	51	47
Lib	47	47	49	53
Dem	5	8	—	—
Grey Power	—	2	—	—

Seat 3 (Liberal incumbent)

	First Preference Votes		*Final Distribution*	
	Lower House	**Upper House**	**Lower House**	**Upper House**
ALP	44	43	47	47
Lib	51	47	53	53
Dem	5	8	—	—
Grey Power	—	2	—	—

In seat 2, the incumbent member effect can be seen to be ALP (Lower House) - ALP (Upper House) = 4 percentage points, while in seat 3 the incumbent-member effect can be seen to be Liberal (Lower House) - Liberal (Upper House) = 0.

In seat 2, the ALP now wins, whereas in seat 3, the Liberals win more comfortably than in seat 1. It is only by averaging over seat 2 and seat 3 that the true incumbency effects are determined. Since there were approximately equal numbers of ALP and Liberal incumbents before the 1993 election, in the analysis done on the actual 1993 election to determine the incumbent effect, a straight (i.e. equally-weighted) average was performed, without making much difference to the analysis.

Thus it is not possible to look at ALP incumbency effects separately from those of the Liberals: only an average effect over both parties can be isolated using this method.

IV The effect of incumbent party on electoral boundaries.

1. How the theory applies to boundaries.

Figure 4 shows in schematic form the percentage of the two-party-preferred vote received by the Liberal Party in 1993. Seats to the right of 50% are held by that Party; seats to the left are held by the ALP.

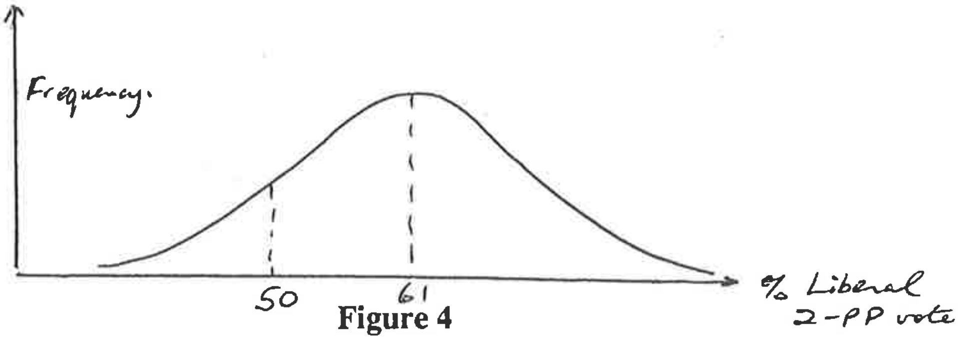


Figure 4

The distribution is approximately symmetrical about the mean vote of 61%.

We continue to make use of a sitting-party effect because it is much simpler to deal with than an incumbent-member effect. The incumbent-party effect is similar to the incumbent-member effect to the extent that when an incumbent retires, the "halo" effect is inherited by the successor from that party.

For electoral boundary considerations, we only really need to look at the seats that changed hands at the last election. This is because the incumbent-party effect will already have been taken into account by the results of the last election, in those seats which did not change sides. For the seats that changed over from ALP to the Liberals at the last election, the incumbent-party effect will reverse its direction at the *next* election.

If other things remained unchanged, the set of winning margins based on the 1993 election results, and altered only to the extent that sitting-party votes will change sides, would shift the distribution of votes (as seen in Figure 4) to the right, in those seats which changed sides. This is shown in Figure 5.

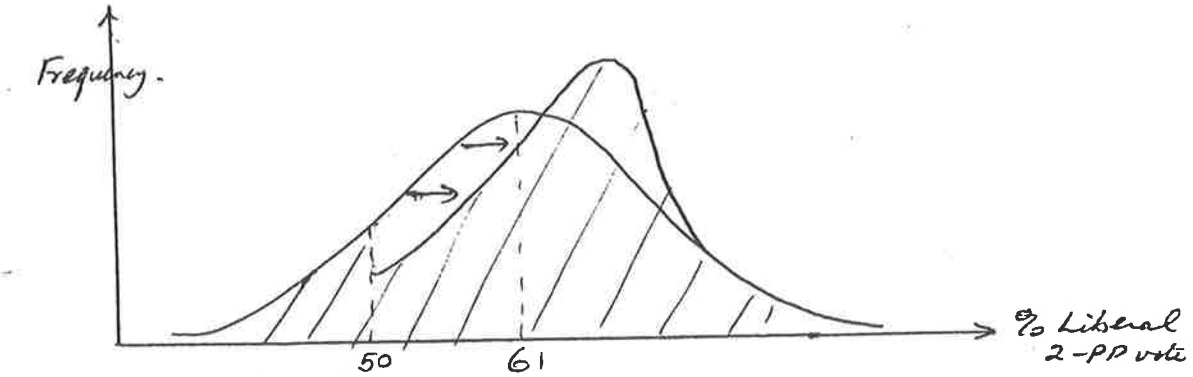


Figure 5: 1993 results, adjusted for sitting-party effect for seats changing hands.

In making this change to the distribution, we have altered its mean. The new mean implies that the Liberal vote would average about 62.5%, not 61%, after this change has been made. But since the swing to reach 50% of the vote is still 11%, not 12.5%, we now adjust this distribution by 1½% to the left, to bring its mean to 61%. This is shown in Figure 6, in which we equivalently shift the horizontal axis 1½% to the right.

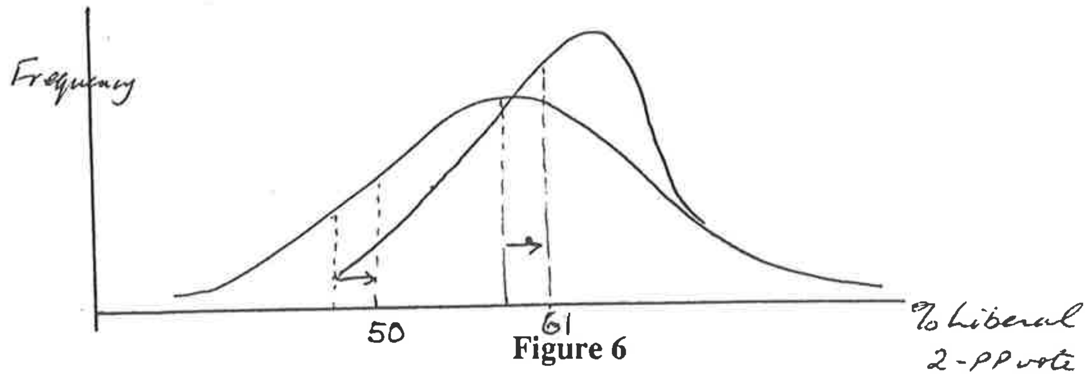


Figure 6

What we have done is shift what are mostly *marginal* seats 5 percentage points to the Liberals, and then shift *all* the seats back by 1½%. The effect overall is to make it relatively hard for the Opposition to win back the seats they lost.

When the calculations are performed, it is clear that a swing back to the ALP to bring the parties to 50:50 of the overall 2PP will not elect 23½ ALP members on average.

Something like an extra 0.5% to 1.0% of the vote would be required to do so. If this factor is not taken into account, the boundaries, technically, are not unbiased.

Ironically, the two main parties are probably taking positions which are opposed to their narrowly-defined best interests in this matter. For suppose the boundaries remained unchanged, and at the *next* election the only change was the switch of incumbent-party votes (from the ALP in 1993 to the Liberals in, say 1997), counter-balanced by a uniform 1½% swing to the ALP, to keep the margin at 11%. By then the distribution of votes *would* look like Figure 6, and the electoral boundary adjustment which I believe should be made in anticipation of this event would occur naturally *after* the next election. However, suppose there was 6% swing to the ALP in 1997, on top of the changes, already suggested, in the seats which changed parties. If the adjustment I suggest is made in *anticipation* of the incumbent-party effect changing sides, it is the Liberal Party which in 1998 or so will gain from the *anticipated* return of the sitting-party effect in those seats which are regained by the ALP.

Let us examine the position diagramatically. As already stated, Figure 6 shows the "1997" election results on unchanged 1993 boundaries and unchanged 1993 voting patterns, except that (a) the newly won Liberal seats become 3.5% safer (5% change in incumbent-party effect, less 1.5% swing to the ALP), and (b) there is a 1.5% uniform swing to the ALP in all other seats, to preserve the mean of 61% Liberal.[11]

[11] There were no adjustments for sitting-party effect in the 1994 redistribution in this example.

Figure 7 shows what would happen in "1997" if there were a *net* 6% swing to the ALP, other things unchanged, given that no allowance is made for changes in sitting party in the 1994 redistribution.

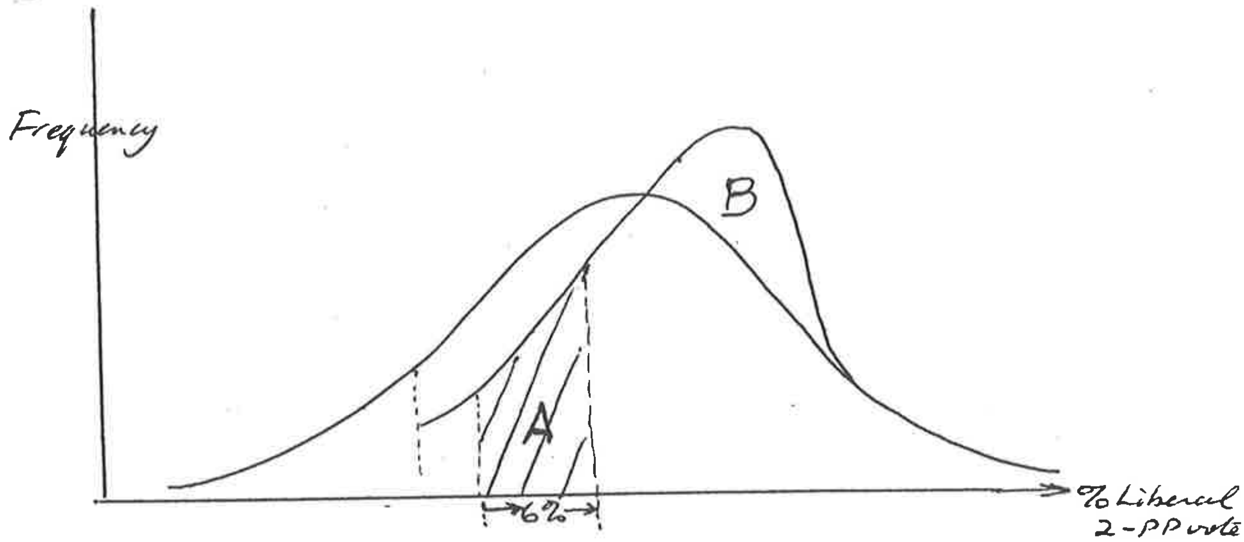


Figure 7: The ALP gains seats marked "A"

In Figure 7, had allowance been made for the incumbent-party effect in the 1994 redistribution, the ALP might have-won, on average, about one more seat.

In 1998, however, the changes due to seats changing parties in 1993 (the "1993 incumbent-party effect") *have* to be taken in to account, if they have not been anticipated in 1994. The Commission in 1998 now takes the bulge B into account for the "2001" election. (It has to so, to the extent that the incumbent-party effect will have materialised.) Operating with these rules (that it does not *anticipate* the changes in incumbent-party effects), it does *not* take the expected return of the incumbent-party effect (caused by the return of A to the ALP) into account in the redistribution of 1998. Thus, Figure 7 also shows what the position will be after the 1998 redistribution if no account is taken in advance of the incumbent-member effect of those marked "A".

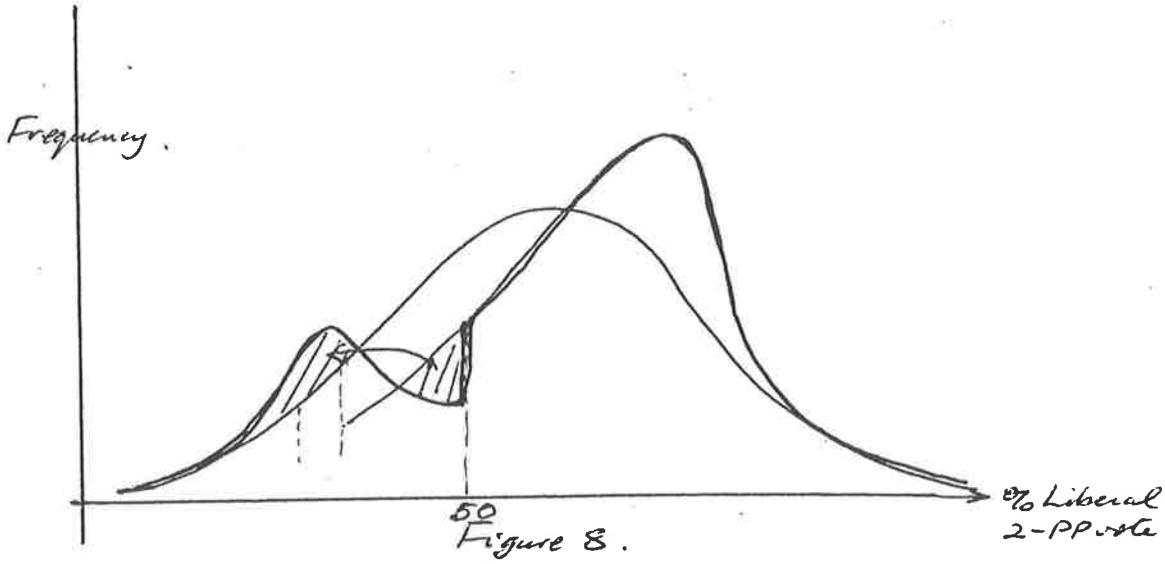


Figure 8 .

Figure 8 shows what would happen if the incumbent-party effect for 2001 is anticipated in 1998. The "A" seats are now safer for the ALP, and so a slightly smaller swing is required for the ALP to gain government, in Figure 8, than in Figure 7, because "A" seats are less likely to swing back to the Liberals than would be suggested by their position in Figure 7.

That is, if the Boundary Commissioners were to take into account the change in incumbent-party effect in advance; they would devise a set of boundaries which, on the previous election's results, would be tilted slightly against the party which has just won extra seats at the previous election. This tilting anticipates the change in voter allegiance at the next election, and therefore makes the next election "fair", so far as that is possible.

If the Liberal Party expects to lose the next election, or expects it to be very close, it is likely to deny strenuously that the incumbent-party effect exists or has any importance, but the Labor Party is likely to insist that it does. However, after the next election, if the Liberal Party loses some seats but retains office comfortably, we may anticipate a reversal of roles. Given that this situation is likely to occur, one might expect a rather more muted response from both parties now.

2. *Taking the incumbent-member effect into account: incumbent member and incumbent party*

There are problems in looking at seats with new incumbent-members, because it is possible that these members will die, lose preselection, decide not to stand, or have their seat disappear in a redistribution. That is a good practical reason for looking at the incumbent-party effect. It is likely to be more stable than the incumbent-member effect. The party is unlikely to die (ie go out of existence), it cannot lose preselection, in practical terms it will always or almost always contest the election, but its degree of incumbency in a particular seat can be muddied by a redistribution.

Redistribution of boundaries aside, therefore, one of the main objections to taking incumbency into account disappears when looking at the incumbent-party.

Another objection to using a incumbent-party effect is that its magnitude is likely to be lower than that of an incumbent-member effect. It is therefore a conservative measure. It should be used as the basis of adjustments on the grounds that a small adjustment which it is possible to implement is better than no adjustment on the one hand, and better than the full adjustment (which cannot easily be undertaken) on the other.

3. *Construction of boundaries using the Incumbent-Party effect*

It is now relatively simple to construct boundaries to take the incumbent-party effect into account. Take a potential new set of boundaries. Where a seat has changed to the Liberal Party at the last election, add the appropriate amount to the Liberal Party vote for that seat, equal to twice the average incumbent-party effect for a single party. This will be of the order of three to five percentage points. (The effect was 3.6 percentage points for the 1993 election.) Subtract enough from all Liberal seats to restore the existing mean Liberal party 2-PP vote over all seats. Now add up the probability that the Liberal Party will win each seat when the swing is 11%, with a standard deviation of 4.5 percentage points. If the Liberal party is expected to win 23.5 seats under these circumstances the method is unbiased.

If this procedure is carried out without making an adjustment for the incumbent-party effect, of say 3%, in the seats which changed hands in 1993, and the ALP does achieve a swing of, say, 11.3%, (ie a little over 11%) it is unlikely that the ALP will win office, even with slightly over 50% of the vote.

One objection to an incumbency effect being included in the analysis is that one does not know its size in any one seat. However, that is also true of other conjectures, such as population projections, yet these are taken into account, despite there being some estimation about the size of the expected population at the next election. A second and more telling objection is that the incumbent-party effect is not present, or is not as great, in the seats which change parties. It may be suggested that when a seat changes parties, it takes more than one election before a new incumbent-party effect fully establishes itself. From the small amount of evidence available, it would appear that this argument does have some force, and that only about half of the effect occurs after one election.

The Commission argues that it would be patently unfair to at least some sitting members if their "swing-to-lose" majorities were notionally changed by even a small uniform percentage figure. (1994 Draft Order, section 7.7). I agree with this view, but I question its relevance. Fairness to individuals who have built up a strong personal vote is not properly a criterion for boundary changes, or the lack of them, (other than "[having] regard to any other matters it thinks relevant"), but obtaining the 50:50 ratio of seats when the vote ratio is also 50:50 is a criterion. If fairness to individuals is jeopardised by the application of this I do not believe it is the Commissioners' prerogative to do anything about that.

However, the "patently unfair" remark misses the point. There would be no more reason to pick on a member with a large personal vote for the adjustment than there would be to pick on a member with a small personal vote.

IV Results

For the 1989 election in S.A., I estimated that the ALP would have received 55.64% of the seats (on average), had the Party finished with 50% of the 2-PP vote. This compares with Jackman's (1994) estimate of 55.79% of the seats, using a similar method.[12]

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[12] It is clear that Jackman's and my methods give very close results in this instance, and each serves as a check on the accuracy of the other.

Given the 1985 results, a swing against the ALP (with a standard deviation 2.5 percentage points) to bring the ALP to 48.1% of the 2-PP vote (its actual vote in 1989) was predicted by the method to give the ALP 22.85 seats in 1989. The ALP gained 24 seats (including pro-ALP independents), so there was a chance factor of 1.15 seats towards the ALP. The ALP would have been expected after 1985 to gain 23.50 seats (half of the total) with 48.75% of the 2-PP vote, so there was some bias in the system in 1985. This was increased, as a result of the 1989 election, so that had new boundaries not been introduced, the ALP would have received, on average, 50% of the seats with 47.3% of the 2-PP vote, or alternatively, 55.64% of the seats (26.50 seats) with 50.00% of the 2-PP vote. (This prediction of 26.50 seats has a standard deviation of 1.14 seats). The new boundaries in 1991 altered this bias in an interesting fashion. Had the 1989 election been re-run with the 1991 boundaries, *ceteris paribus*, I estimate that the ALP on average could have won 50.00% of the seats with 47.8% of the vote, an improvement of 0.5 percentage points. However, because a greater number of *marginal* seats had been created, if the ALP had won 50% of the votes, it would have picked up even more seats, from an expected 26.50 before the redistribution to an expected 26.87 seats after it!

If another election were held on the same boundaries, as applied for the 1993 election, to gain 50% of the seats, the ALP vote would need to increase to 50.0% (swing standard deviation of 4.5), or 49.8% (swing standard deviation of 2.5). Since the swing standard deviation would be likely to be close to 4.5 percentage points for a swing of about 11%, the current system is (miraculously!) unbiased. The probability of such an exact result by chance is actually quite low, so the 1989 Electoral Commissioners really hit the jackpot! (However, when the changed incumbency is taken into account, the situation will change. What was unbiased for 1993, will not be unbiased for the next election, given the changed incumbency.)

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If the 1993 results had been based on the 2-PP votes for the Legislative Council in each electorate, the ALP would have won only 7 seats with 37.7% of the 2-PP vote. To reach 50% of the seats, a swing of 12.4 to 12.5% is required, to 50.1% or 50.2% of the 2-PP vote, so these boundaries are more or less unbiased for this Upper House simulation as well. The implication is that the incumbent effects were not responsible for any distortion in this election, as they seemed to have been in 1989.

Now we add in the incumbency effect, which (other things equal) will make many newly-won Liberal seats safer. Since the ALP vote for these seats included a fairly large incumbency effect, we estimate the incumbent advantage at 2% (c.f. 2.6% in 1989, and 1.8% in 1993) and consequently subtract 2 x 2% from the ALP 2-PP vote for 1993 for those seats which changed hands. This yields an ALP 2-PP vote of 38.0%. To reach an expected 50% of seats, the ALP vote would have to increase to 51.04% (with a swing standard deviation of 2.5% pts) or 51.16% (with a swing standard deviation of 4.5 percentage points). That is, the current position is biased toward the Liberals by 1% to 1.2% after incumbent effects are allowed to work through. With 50% of the vote, the ALP would gain 22.1 seats, or 47.0% of the seats, a 3% bias against Labor.[13]

If the incumbency effect is 1.3% rather than 2%, the ALP will need to poll 50.6% to 50.8% of the vote to gain 50.0% of the seats under these conditions, so with no changes of boundaries, there will be a small bias favouring the Liberal Party at the next election.

[13] The equivalent calculation for the 1989 election yields the figure of a bias of 5.64% (Fischer) or 5.79% (Jackman) towards Labor, as noted at the beginning of Section IV of this chapter.

V Conclusions

The investigation has found that the boundaries for the 1993 election were to all intents and purposes unbiased. It has also found that the same boundaries, if used for a future election, would be likely to result in a smallish bias in favour of the incumbent party, which is the Liberal Party.

What is perhaps more important, however, is that the method used in this investigation can be used to test proposed sets of boundaries in advance. The *modus operandi* would be as follows:

(1) Estimate the 1993 2-PP vote for each proposed new seat for the proposed new boundaries. This is of course subject to error, though it is unlikely that the error will be of any great consequence.[14]

(2) Estimate the extent of the incumbent effect in the seats which changed parties in 1993, if not already taken into account in (1). If the Commission wishes to anticipate this effect, it should follow the directions of what to do from Section IV of this paper.

(3) Work out the expected number of seats for the two main parties when the vote is split 50: 50, and the expected number of 2-PP votes required to split the seats 50: 50 (ie 23.50 seats for each party). This can be done using the method described in Section II.[15]

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[14] The four different sets of 2-PP estimates (constructed by Mackerras, Jaensch, Newton and the S.A. Electoral Department) following the substantial boundary changes after 1989 led to substantially similar predictions.

[15] Gelman and King have a program called "Judgeit" available to do this.

(4) Of the proposals under consideration, and all other things being equal, the Commissioners should probably choose the set of plans for which a 2-PP of 50: 50 goes closest to a 50: 50 seat split.

This paper uses the votes obtained in the Upper House to act as a benchmark in determining the magnitude of an incumbent effect. It also uses the idea of a incumbent-party effect rather than a incumbent-member effect to make operational the boundary changes necessary to anticipate changes in seats won at the previous elections.[16]

The ideas in this paper could be further refined in the following ways:

(1) As shown in the 1994 Draft Order of the S.A. Electoral Districts Boundary Commission, it is likely that the swing differs between metropolitan and country electorates. This has not been taken into account.

(2) An investigation over the elections of the past 20 or so years could be undertaken to determine the extent of the incumbent effect, similar to Gelman and King's (1990b) analysis. However, as the analysis in this paper shows, the incumbent party advantage demonstrated does not require such Gelman and King style analysis to confirm its existence.

(3) Seats which already have a very high proportion of the 2-PP vote for a party which then enjoys a swing in its favour cannot swing much more, and this could be taken into consideration. Again, this factor is unlikely to alter any of the conclusions of this paper.

[16] Calculations of both the incumbent-member and incumbent-party effects in the 1993 election are available on request from the author.

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Part IV

The Australian Senate, and the Single Transferable Vote (STV)

My interest in electoral matters began with work related to the sampling of votes for the STV system of voting as used in the Australian Senate. As mentioned in the Introduction, the Electoral Act of the Commonwealth of Australia was amended as a result of Paper seven "Sampling Errors in the Electoral Process for the Australian Senate" to eliminate the sampling of votes.

When I originally submitted this paper to the Australian Journal of Statistics, I offered its editor a "short" and a "long" version. The "long" version contained an extra four pages of algebra which established equation (2) of the short version. At first I thought I should incorporate the extra pages as an appendix to this Paper in this thesis but when I re-read it, I realised how turgid it was, and have decided to omit it. (It is still available from me, but after 15 years, no-one has yet asked for it).

In 1986, in a paper to the *British Journal of Political Science*, Gallagher and Unwin derived (for a much simpler case) virtually the same results as I had obtained in Paper seven. Not only had my Paper seven been published (in a statistics journal), but also Paper nine in a politics journal. As they failed to acknowledge this work, I wrote a sharp retort to the Journal, but also showed how Gallagher and Unwin's work could be extended in several ways. I had not expected it to be accepted for publication in that form, injured tones and all. But it was, along with a *Reply* from Gallagher and Unwin. They said that their work was exact but mine was an approximation. It was not worth a further reply. Paper eight is my *Comment*. To the extent that, to my knowledge, the sanmpling problem has not been addressed in Ireland, Papers seven and eight are still relevant to that situation.

As mentioned in the Introduction in some detail, Paper nine "Aspects of the Voting System for the Senate" examines a number of anomalies in the voting procedures for the Australian Senate. These problems have been addressed by Parliament but not solved, as the Introduction to this thesis indicates.

PAPER 7

Fischer, A.J. (1980) Sampling errors in the electoral process for the Australian Senate.
Australian Journal of Statistics, v. 22(1), pp. 24-39

NOTE:
This publication is included on pages 108-123 in the print copy
of the thesis held in the University of Adelaide Library.

It is also available online to authorised users at:

http://doi.org/10.1111/j.1467-842X.1980.tb01151.x

PAPER 8

Electoral Distortion under STV Random Sampling Procedures: A Comment

A. J. FISCHER

In a recent Note in this *Journal* by Gallagher and Unwin, it was stated that the element of randomness due to sampling surplus votes in single transferable vote (STV) elections 'has long been recognized, but no previous attempt has been made to assess its impact'.[1] This is incorrect. Work done in Australia (and reported in the leading Australian journals in their respective fields) has comprehensively dealt with this problem both in theory and practice.[2] Since STV is practised in national elections in only three countries (Australia, Ireland and Malta, the most populous being Australia) it is surprising that contributors to, and referees of, this *Journal* should be unaware of such material. It was largely because of the evidence contained in these articles that the Joint Committee on Electoral Reform recommended to the Australian Parliament an amendment to the Electoral Act, to avoid the problems caused by sampling votes, by using the Gregory method of counting them. The Act was amended in this way in 1983.

The model used in the Australian work is more comprehensive than Gallagher and Unwin's in that it recognizes two sources of sampling error, one from the surpluses of candidates already elected (which Gallagher and Unwin do not consider) and the other from eliminated candidates (which they do consider). Besides this, it may well be that the approximations employed in the Australian work are more accurate than Gallagher and Unwin's, the Australian work being based on standard sampling theory from a finite population.

Finally, in the Australian work, a distinction is made on the one hand between the possibility that sampling *might* have affected the result, in a particular electorate, and on the other hand the expected occurrence, over a large number of electorates and elections, of *actually* electing the 'wrong' candidate. (In this instance, a 'wrong' candidate is defined as one who would not have won had all possible samples been chosen and the results from all these samples averaged.) In a particular electorate, if there is a probability of 0.1 per cent that the sampling procedure has affected the result, this is equivalent to saying that the winning margin is 3.09 times the sampling standard deviation. This not unreasonably assumes that the sampling errors for a given electorate are normally distributed. Over a large number of electorates and elections, it is also reasonable to assume that winning margins (each as a proportion of the relevant quota) are rectangularly distributed. From this, it has been shown that the election of a wrong candidate will be associated, on average, with a winning margin equal to 0.399 of the sampling standard deviation.[3] Since Gallagher and Unwin (p. 251) recognize nineteen instances in Ireland where the winning margin is less than 3.09 times the sampling standard deviation, it may be inferred (from the rectangular distribution of winning margins) that on average, $(0.399/3.09) \times 19 = 2.45$ wrong candidates will ever have been elected. This assumes that Gallagher and Unwin's approximations are reasonable ones.

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[1] Michael Gallagher and A. R. Unwin, 'Electoral Distortion under STV Random Sampling Procedures', *British Journal of Political Science*, 16 (1986), 243–53.

[2] A. J. Fischer, 'Sampling Errors in the Electoral Process for the Australian Senate', *Australian Journal of Statistics*, 20 (1980), 24–39 and A. J. Fischer, 'Aspects of the Voting System for the Senate', *Politics*, 16 (1981), 57–62.

[3] Fischer, 'Sampling Errors in the Electoral Process', p. 36.

Another more direct estimate of the number of wrong candidates ever elected in Ireland may be gained simply by adding the probabilities that the wrong candidate might have been elected. From the information given by the authors (p. 251) this adds to about 1.8 wrong candidates ever elected. To the nearest whole number, the best estimate of the number of candidates ever wrongly elected to the Irish Parliament, because of the sampling of votes, is two.

No one can say which two candidates were wrongly elected, or even that there were precisely two (there may have been more or less, but two is the best estimate). But given that the randomness can be entirely prevented by the Gregory method for a relatively small administrative cost, it seems pointless for the Irish system to remain unnecessarily flawed.

Finally, Gallagher and Unwin (p. 243) are unduly harsh on a system they clearly believe to have merit. They show that STV has been given an intermediate rating in an assessment of electoral systems. Unfortunately, the cited criteria used for rating electoral systems are not particularly appropriate ones for the simultaneous election of a number of representatives. For example, an alternative favoured by some is 'approval voting'. Approval voting for *n* places may result in one particular party winning all *n* seats, but with no candidate receiving anything like majority approval. There may well be a very low degree of proportionality in this system between aggregate support for a party and its aggregate representation in Parliament. STV usually achieves a high degree of proportionality and, unlike most list systems of proportional representation, allows freedom of voter choice within a party list, and an expression of preference between lists, as well as allowing the possibility for a voter to jump from candidate to candidate in different parties. STV's lack of monotonicity in these circumstances is unfortunate but a relatively minor flaw, especially in light of Arrow's Theorem[4] and the Gibbard–Satterthwaite Theorem.[5] Lack of short-term stability is still more likely to be regarded as a problem of the electoral system for Ireland (as it is for all proportional representation systems), particularly given the frequency of recent Irish elections.

[4] K. J. Arrow has shown that given only a few, fairly weak anxioms to describe the social choice process, no voting mechanism exists that will satisfy them under all circumstances. See K. J. Arrow, *Social Choice and Individual Values* (New York: Wiley, 1951; revised 1963).

[5] A. Gibbard 'Manipulation of Voting Schemes: A General Result', *Econometrica*, 41 (1973), 587–602 and M. A. Satterthwaite 'Strategy-Proofness and Arrow's Conditions: Existence and Correspondence Theorems for Voting Procedures and Social Welfare Functions', *Journal of Economic Theory*, 10 (1975), 187–217. These theorems state that even in voting systems exhibiting monotonicity, there will always be occasions when some voters will gain more desirable outcomes by misrepresenting their preferences or intensity of preferences. The effect of the Arrow and Gibbard–Satterthwaite theorems is that we cannot hope to find a voting system which will have no undesirable properties. We must therefore make judgements about the relative importance of different violations of desirable properties by different voting systems. Not only is the lack of monotonicity likely to cause relatively minor distortions in STV, it is relatively insignificant compared with the extent to which strategic voting may occur in alternative systems of voting.

PAPER 9

Fischer, A.J. (1981) Aspects of the voting system for the Senate.
Politics, v. 16(4), pp. 57-62

NOTE:
This publication is included on pages 126-131 in the print copy of the thesis held in the University of Adelaide Library.

It is also available online to authorised users at:

http://doi.org/10.1080/00323268108401784

Part V

Strategic Voting and the Alternative Vote

Paper ten is the only paper in this Part. It describes an experiment on strategic voting in the context of preferential voting. It appears that no other experiments have been undertaken to attempt to examine the determinants of strategic voting in this context. The experiment was undertaken under the auspices of the Australian Centre for Experimental Economics (of which I am the Research Director), utilising a Faculty of Economics small grant of $1700. This grant also covered the experiment of Paper 15.

PAPER 10

THE DETERMINANTS OF STRATEGIC VOTING

I INTRODUCTION

In many voting situations where there are three or more candidates, voters may be sure, or fairly sure, that their first preference candidate cannot win the election, but that their second preference candidate has a reasonable chance of doing so. Rather than waste their vote on voting for someone who cannot win, such voters may vote for the second (or later) preference candidate instead. Such behaviour is called strategic voting, and has also been known as insincere, tactical or sophisticated voting.

The purpose of this paper is to investigate the circumstances under which such voting will take place. It does so in a voting scheme known as the alternative vote, or as preferential voting, where each voter has a single vote and there is a single candidate to be elected.

Each voter must express a preference for each candidate or alternative, with "1" denoting the voter's first preference, "2" the voter's second preference, etc. Should one candidate get more than 50% of all first preference voters, he or she is elected. If no candidate gains a simple majority of first preference voters, the candidate with the least such votes is eliminated, and these votes are transferred to the candidate specified by each voter's second preference. (An example of such a scheme is given in Appendix I). It is also a special case of the single transferable vote (STV) where only one candidate is to be elected.

Voting schemes such as this are often used in theoretical discussions of voting systems, but are rarely used in any Parliamentary elections at national or sub national level. For single-member electorates, they are used in Australia in the Lower House of Federal

Parliament, and in the Lower Houses of five of the six Australian States and one of two Territories. They are also used for the election of several State Upper Houses in Australia. (For multi member constituencies, STV is used in the Republic of Ireland, Malta, the Australian Upper House, two Lower Houses in the Australian States and Territories (Tasmania and the ACT) and several State Upper Houses).

The subject of this paper is therefore of general interest to those who wish to evaluate the practical performance of different voting systems, as well as of particular interest to Australians, who use such a system in both national and subnational contexts.

This system, like most others in use, is not free from voter strategy. That is, if some voters believe that the candidate of their first choice will not be elected, there will on occasions be an incentive for them to misrepresent their preferences in an attempt to prevent the election of one of their least-preferred candidates. In majority (ie. "first-past-the-post" or plurality) voting, strategic voting will often act against all but the two parties expected to obtain the largest percentage of the vote. Voters whose most preferred candidate is other than one of the two expected top candidates will tend to vote strategically and desert the small party in order to try to elect one of the top two - that is, "to make his or her vote count". The tendency is likely to be greater wherever the contest between the top two parties is likely to be close. When one party is expected to win well over 50% of the votes cast, however, there is little point in third and fourth party voters voting strategically. In this case, there may be some point in voters for the second party voting strategically for the likely third candidate. They would do this in the hope that, at this or subsequent elections, a good showing for the third party may attract back voters from the first party who had been voting strategically for it, but whose first preference was really for the third party. This phenomenon is more likely to occur at by-elections, where voters can vote against a government without the threat that the loss of the seat will actually change the government.

In the case of preferential voting, only the second of these two mechanisms is likely to operate. The first mechanism (voters whose first preference is for small parties, voting strategically for party one or two) no longer applies, as the voter does not waste the vote by voting for a minor party. In those cases where it may have made a difference in majority voting for a third-party voter to vote strategically, this option occurs automatically in preferential voting, on the elimination of the third candidate and the distribution of preferences. To show how the second mechanism works, it is instructive to work through an example. Suppose candidate C gets 45 first preference votes, D gets 35 and E gets 20 under true revelation of preferences. Suppose also that E's preferences split equally, 10 going to each of C and D, so that on the distribution of preferences, C gets 55 to D's 45, giving the election to C fairly comfortably. But suppose that if D's preferences (which are not counted if D is above E in first preference votes) go entirely or almost entirely to E. If 8 of D's 35 voters vote strategically for E, then E gets 28 votes, to D's 27, and E will be elected if 23 or more of the 27 second preferences of the remaining D votes go to E. From this example, it may be deduced that the following conditions are necessary for strategic voting. The first choice candidate of the voter must be expected not to win, but must be expected to receive more votes than the voter's second choice candidate. In turn, if the second-choice candidate had received more votes than the voter's first-choice candidate, he or she could be expected to have a higher chance of winning than the first-choice candidate. Finally, the candidate expected to win (who must be no higher than the third choice of the strategising voter) must not be expected to gain more than 50% of first-preference votes before the elimination of either of the voter's first choice or second choice candidate.

Very little work has been attempted to understand the determinants of strategic voting in this context, probably because the system has not been in widespread use. Farquarson (1969) and Niemi and Frank (1982 and 1985) began investigations on the plurality system, but only two experimental investigations have been attempted, by Felsenthal et al (1988) and by Rapoport et al (1991). In Felsenthal's study, there were only four participants and in Rapoport's, five participants, in each of ten replicates. The five participants (voters) in Rapoport's experiment had multiple votes (between 1 and 21) in seven different computer-controlled voting scenarios. There was no uncertainty in the sense that all voters knew each other's number of votes and his/her payoffs. (There was uncertainty, of course, in that voters did not know whether other voters would vote strategically or not). With three candidates, there are six different possible preference orderings, and five of these six orderings were used in the experiment, one by each of the five participants. All of the votes of any one participant had to go to the one candidate, and voting was not preferential. This led to the candidate with the most votes, but not necessarily 50% or more of all votes, being elected. Each of the seven different situations was visited 60 times in all, and in each set of 60 outcomes, each of the three candidates won a minimum of three times, which is evidence of a substantial amount of strategic voting.

Experiments in strategic voting have also been carried out by Levine and Plott (1977), Plott and Levine (1978), Herzberg and Wilson (1988), and Eckel and Holt (1989), but these experiments concentrate on voting in committees, and the ways in which strategic voting could manipulate outcomes in a multi-stage agenda.

This paper reports the results of a set of experiments, which in contrast, have conditions much closer to those facing voters in a Parliamentary election. Each voter has one vote, there are usually only three main preference orderings, sometimes four, and there is often uncertainty about the numbers of different sorts of voter. These features change the nature of the conditions facing voters substantially.

In Australian preferential voting situations, the kind of uncertainty modelled by Rapoport et al (1991) basically does not exist. We examine the case of three candidates. Each candidate publishes (and, via supporters, hands out to electors about to enter the polling station) a preferred preference ordering which most voters follow. Since the two main parties (the ALP and the Liberal-National Coalition) usually recommend that voters give their second preferences to the third small party, (the Australian Democrats), two of the six possible preference orderings are ruled out. The Democrats either direct their preferences to one of the two large parties (but not always the same party in different seats or at different elections), or they have a two-sided how-to-vote card, showing how to vote Democrat-ALP-Liberal on one side and Democrat-Liberal-ALP on the other. Therefore, there are effectively either three or at most four possible preference orderings.

We simplify in the first instance by assuming that there are only three preference orderings. What are not known in practical situations are the intensity of preferences, and as mentioned earlier, the exact numbers voting for each alternative. In our experiments, we have altered the relative intensity of preferences and seen how this affects the extent of strategic voting, and perhaps more importantly, we have introduced uncertainty into the number of voters of a particular preference ordering. It appears that some voters with true preference for the main parties will vote strategically if they think their own candidate has no chance of winning, but will not do so (at least not nearly so often) if they think their first true choice has a chance. That is, almost paradoxically, a minor party has a greater chance of winning a "safe" seat of one of the main two main parties than it does when the outcome between the two main parties is in doubt.

II MODELS AND HYPOTHESES

Four models of strategic voting - Niemi and Frank (1982), Farquharson (1969) Felsenthal et al (1988) and Rapoport et al (1991) — which have been proposed in the literature are not really applicable in the real world, because in practice no voter has complete information regarding the preference orderings of all other voters. For very small electorates, say up to 20 or 30 voters, this complete information postulate may hold approximately true, but it becomes more and more difficult to achieve as the electorate expands to thousands, and more. Furthermore, in the real world, a proportion of voters makes mistakes, and uses both dominated and inadmissible strategies.

One of the purposes of this paper is to test whether voters vote differently if they are at least somewhat uncertain as to the preference orderings of other voters, from the case where they know all preference orderings. However, the main purposes of the paper are more practically oriented, as the hypotheses below indicate.

Let us formalise the problem.

Let the sincere primary vote shares for the three candidates C,D and E be v_C, v_D and v_E, and let the final shares of C and D (the Condorcet vote shares), if E is eliminated, be V_C and V_D.

Without loss of generality in what follows[1], let $V_C > V_D$, $v_C > v_E$ and $v_D > v_E$.

It is hypothesised that a voter with sincere preferences C1 E2 D3 (in descending order of preference) is more likely to vote E1 C2 D3 (ie. strategically):

(1) The larger is E's first preference share of the vote

(2) The smaller the difference in the voter's preference payoff between C and E

(3) The *less* certainty there is about the expected vote shares of each candidate

[1] This is the situation of the example given in the introduction. Translating into symbols, $v_C = 45$, $v_D = 35$, $v_E = 20$, $V_C = 55$, $V_D = 45$.

(4) The larger the expected difference between the vote shares of the candidates C and D of the two main parties

(5) If the election is a by-election rather than a general election.

It is further hypothesised that a voter with sincere preferences D1 E2 C3 is more likely to vote E1 D2 C3 (ie. strategically):

(6) The larger is E's first preference share of the vote (same as hypothesis 1)

(7) The smaller the difference in the voter's preference payoff between D and E

(8) The *more* certainty there is about the expected vote shares of each candidate

(9) The larger the expected difference between the vote shares of the candidates C and D of the two main parties (same as hypotheses 4)

(10) If the election is a by-election rather than a general election (same as hypotheses 5)

A further set of hypotheses compares extent of strategic voting occurring between voters with sincere C1 E2 D3 preferences and voters with sincere D1 E2 C3 preferences. The hypothesis is that strategic voting will increase, the lower the probability that one's own first-choice candidate has of winning. When V_C is expected to be greater than V_D, this translates into:

(11) Voters with sincere D1 E2 C3 preferences are more likely to vote strategically than C1 E2 D3 voters.

(12) The less certain are voters about V_C being greater than V_D, the less likely are voters with sincere D1 E2 C3 preferences to vote strategically, and the more likely are voters with sincere C1 E2 D3 preferences to do so.

III EXPERIMENTAL METHOD

The experiment had two replicates, one with 25 undergraduate students from the University of Adelaide and the other with 21. The experiment was done manually and each replicate took two hours to perform. Participants took part in a number of elections and were paid on the outcome of one experiment chosen at random at the end. They were carried out in August 1994.[2]

The results of this experiment are fairly heavily dependent on the sequencing and form of the various elections held within it, so the full set of instructions is included as an appendix. A set of general instructions was placed before participants and read out aloud. To prevent participants looking ahead to see the form of later experiments, specific instructions about each round of voting were screened on an overhead projector. Voting was secret, or at least as secret as possible. At the end of each round, votes were counted and the result of the election announced, in terms of the number of first-preference votes for each candidate. If one candidate received more than 50% of the votes, he/she was declared elected. If not, the candidate with least first-preference votes was eliminated, and his/her second preference added to the first preference votes of one of the other two candidates. If the second and third candidates tied, a coin-toss determined which one would be eliminated. Since there were an odd number of voters in the elections, it was never necessary to use a randomising device to break ties between the remaining two candidates after the distribution of preferences. The result of the election would be known before the start of the next round. There were three exceptions to this, in rounds 16 and 17, 18 and 19, and 20 and 21. In round 16, (as in all rounds up to that point) voters were asked to vote for candidates in this election in isolation from other electorates: they were in fact simply not told about any other electorates. After voting in round 16, with payoff 5, but before knowing the

[2] As well as this, there were four pilot experiments, comprising one short experiment carried out with 20 children from the Gifted Children's group in Adelaide early in 1993, two experiments each of one hour later in 1993, each using 15 undergraduate students from the University of Adelaide, and one experiment of one hour using 11 participants at the Fifth Conference of the Australian Centre for Experimental Economics, October 1993.

result of the election, participants were asked to vote again, in round 17, given the same set of payoffs as in round 16, but now with an additional set of payoffs (payoff 6) if a particular party won the overall election. There was a small chance that the single seat being voted upon could affect the outcome of the overall election. The same logic applied for the other two pairs of rounds (18 and 19, and 20 and 21).

Since learning between rounds was expected, it was decided to make this learning explicit. At the end of round 7, participants were given a short quiz of two questions. The first set of answers was collected. Participants were then told the correct answer before proceeding to the next question. After this question was answered, it too was collected, but the correct answer was not given. The questions related to whether a particular candidate could ever win, given that the preference orderings for all voters were known, and that voters always voted their last preference last. In the particular case chosen, the candidate could never win (so it was rational for those participants whose true preference was for that candidate to vote strategically). The reasons for giving the quiz were to find out who realised this and to see whether that set of participants had been voting strategically. The second reason was to determine whether there was more strategic voting after the correct answer became common knowledge, and the third was simply to alert those participants who had not considered strategic voting, to its possibility.

To overcome the confusion inherent in having true preferences and expressed (sometimes strategic) preferences, participants were designated either blue, red or green voters for a particular round, but did not remain the same colour in all rounds. To avoid confusion, each voter was given either a blue, red or green counter at the beginning of each round. Blue voters would always receive most money if candidate C won, zero if D won, and an intermediate amount if E won. Red voters reversed the amounts received for C and D winning, and green voters received most for E winning, next for C and nothing for D. In later rounds, a fourth set of voters, called yellow voters, were introduced in equal numbers to green voters, receiving most for an E win, nothing for C and an intermediate amount for a D win - that is, swapping C and D amounts compared with a green voter.

The nature of the experiments was such that green (or in later rounds, green and yellow) voters were in a minority. Pilot experiments showed that the small number of such voters played essentially a passive role only: they never had any incentive to vote strategically. In the proper experiments, therefore, green and yellow voters were not played by human participants. Participants were told of the existence of a specified number of green voters. In early rounds there were usually 2, but for several rounds, there were 7. In later rounds there were 2 green and 2 yellow voters. Furthermore, participants were told exactly how each of these green (or green and yellow) voters was going to vote. The effect of this change was to simplify the experiment to one where the participants could be either blue or red voters. It also allowed the number of votes to increase by two: this is important in experiments such as this one which require at least 20 voters.[3,4]

In rounds 1 to 11, the number of red and blue voters in each round was known exactly. Since there was one more red voter than blue, and to start with, two green voters, then

[3] To see why this is so, consider 19 voters. If there are different numbers of blue and red voters, to allow green voters to be decisive, there must be at least two of them, and either 8 or 9 blue, and 9 or 8 red voters respectively. If we wish to see the effect of increasing the number of green voters with only 19 participants, where green voters remain the smallest group, we can only go as far as 4 greens, with 6 and 7 for the main two colours. If there were 5 greens, there could be 6 and 8 of the main two colours, but then the gap between the main two has doubled, thus altering a further condition of the experiment. If there were 6 greens, there would be 6 and 7 of the main two colours, so greens are no longer uniquely the smallest party. The difference between two and four green voters is not very large, and its effect on strategic voting may be small. If we fix the number of blue plus red voters at 19, with a minimum difference (of one) between voters of the two colours, we get 9 and 10 (or 10 and 9) red and blue voters. We can now add in 2 green voters, or any number up to 8 green voters, and still retain green as the smallest group of voters. It also allows all blue and red voters to maintain their same tokens, so that differences in strategic voting as the number of green voters changes will not be due to the changing number and/or composition of red and blue voters.

[4] If an even number of participants had presented for the experiment, of size 2r + 2, they would have been divided into r and (r+2) for the two main colours, and a minimum of 3 green voters would have been added, to allow green to be decisive once again.

if all voters voted sincerely, blue would have won by one vote after the preferences of the two green voters had been accounted for.

The way in which strategic voting may take place is shown by an example taken from the practice round in experiment 1. Payoff 1 from the payoff schedule (see appendices) was used. There were 13 red voters, 12 blue voters, and 2 (non-human) green voters.[5] If all voters voted sincerely, then D would gain one more primary vote than C, but C would then win by one vote from D on the distribution of E's preferences. Thus D cannot win. Payoffs to blue voters would be $20; to red voters would be zero.

If, however, six red voters voted strategically for E in experiment 1 (or five in experiment 2) then E would receive one more primary vote than D, and E would be elected by one vote from C after D's preferences were distributed. Both blue and red voters would receive a payoff of 4. Thus red voters should vote for E. However, if one blue voter votes for D by mistake, D will win, providing that sufficiently few voters vote strategically for E. (In that case, it could even pay risk averse blue voters to vote for E, to go from a payoff of 0 up to 4 instead of 20 down to 4. In experiment 2, in fact, one blue voter voted for D by mistake for several rounds (so that D won). Eventually this voter realised her mistake, and used a more appropriate strategy!)

In round 1, payoff 1 was used, with blue voters getting $20 if C won and $4 if E won, and with red voters getting $20 if D won, and $4 if E won. Round 2 altered payoffs to $10 (sincere first choice wins) and $8 (E wins). In rounds 3 and 4, colours were swapped among participants (one red voter having to remain red), and rounds 1 and 2 were repeated. In round 5, tokens were again swapped (a different participant having to remain red) and payoff 2 was used, to test learning, from round 2.

[5] In experiment 2 there were 11 red, 10 blue and 2 (non-human) green voters.

Then in round 6, the number of green voters was increased to 7 to see if this increased strategic voting. Tokens were again swapped for round 7, and payoff reverted to payoff 1, green voters remaining at 7. (In experiment 1, round 7 was omitted. The reason for increasing the green voters to 7 at round 6 was partly to ensure that participants would be given incentives for learning about how to vote strategically, by ensuring a large green presence. Round 7 was put in so as to ensure that all voters had a chance of voting strategically. In experiment 1, strategic voting was so pervasive that the third candidate, E, won each of the first six rounds, and so there was no need to conduct round 7. However, it was only when the number of green voters increased in round 6 in experiment 2 that strategic voting suddenly became prevalent, so round 7 was not omitted).

At the end of round 7, participants were asked if they thought D could win, given the number of voters of each colour of round 1, and assuming that everyone put their third preference third. They were told the answer to this question, but not to the subsequent question: If D cannot win, can a red voter ever get a payoff greater than zero?

In rounds 8 and 9, with payoff 1, and using the same tokens as in round 7 (or swapping tokens in experiment 1 at the beginning of round 8, given that round 7 and the token swapping at that stage were omitted) voting with two and seven green voters respectively was repeated, to see the extent of learning that had taken place.

Rounds 10 and 11, repeating rounds 8 and 9 with payoff 2, were omitted due to time constraints.

For rounds 12 to 21, there were two yellow and two green voters, with a subsequent neutral effect on candidates C and D in those cases where the preferences of E were to be distributed. The main point about these rounds was that the numbers of red and blue voters were not known, as each participant drew a token (red or blue) from a book bag, with replacement. For rounds 12 and 13, and 16 and 17, there were equal probabilities, and different payoffs. For the other rounds, the colours were in the ratio 2:3. As stated earlier, for rounds 17, 19 and 21, there were two payoffs - one for the election of candidate C, D or E, and one for the victory of party CC or DD, where CC was C's party and DD was D's.

This part of the experiment was completed in two hours, and participants were paid at a later date by rolling an icosahedron whose faces represented the 18 or 19 completed rounds of the experiment. At the time they did this, they were also asked to fill out their preferences for another 12 rounds of a thought experiment, with four rounds for each of three scenarios. In scenario 1, three opinion polls for the election showed C and D each with about 40% of the vote and E with about 20% of the vote, with D having a possible slight advantage of one percentage point, on average, over the three polls. How would the participant vote, for payoff 1 and payoff 2 in turn, if he or she were a red voter? and a blue voter?

The second scenario gave D a seven percentage point advantage, on average over the three polls, over C, with E having the same vote as in scenario 1. (The numbers from this poll were created by adding three to each of D's percentages and subtracting three from C's percentages, and presenting the results in a different order.)

For the third scenario, five percentage points were subtracted from each of C and D and ten added to E, so that the averages were, to the nearest percentage point, C = 33%, D = 40%, and E = 28%.

The reason that three polls were given was to give participants some idea of the variation they could expect, so that in scenario 1, the margin of D over C was in turn 4,

- 1 and 0 percentage points. That is, it was by no means certain that D would win. For scenarios 2 and 3, however, the margin of D over C was in turn 10, 5 and 6, so it was much more probable that D would win over C if E's preferences split 50:50, which they were told was likely. Participants were paid an additional $2 for filling out this set of twelve preference orderings. They did not receive any feedback on who won the election as they did for rounds 1 to 21, and were not paid on the basis of who won.

IV RESULTS

We test the hypotheses in pairs (hypotheses 1 and 6, 2 and 7, etc). The numbers of voters of each kind are pooled over the two experiments.

1(a) Tests of hypothesis 1

(For voters where V_C .> V_D, more C voters will vote for E when E has a higher percentage of the vote).

Table 1: Number of Strategic Voters

(i) Main Experiment

	2 green voters	**7 green voters**
Payoff 1	Round 8 5	Round 9 7
Payoff 2	Round 5 5	Round 6 10
Total:	**10**	**17**

(ii) Thought Experiment*

	E expects 18%	**E expects 28%**
Payoff 1	Round 27 0	Round 31 2
Payoff 2	Round 29 6	Round 33 9
Total:	**6**	**11**

(iii) Aggregate for both types of experiment

	Third party vote low	Third party vote high
Payoff 1	5	9
Payoff 2	11	19
Grand Total:	**16**	**28**

*Red and blue voters swap roles for the thought experiments, so it is red voters who are reported here, whereas it is blue voters reported in the main experiment.

A casual glance at these figures shows that there are more strategic votes in the right hand column than in the left-hand column. (The difference in the grand total is significant at the 10% level). That is, more voters from the leading party vote strategically when the third party is expected to do well. (After paired comparisons (ie. the same student) are taken into account, the difference is significant at the 5% level).

Thus hypothesis 1 is supported.

1(b) Tests of hypothesis 6

(For voters where $V_D < V_C$, more D voters will vote for E when E has a higher percentage of the vote).

Table 2: Number of Strategic Voters

(i) Main Experiment

	2 green voters	**7 green voters**
Payoff 1	Round 8 8	Round 9 15
Payoff 2	Round 5 6	Round 6 13
Total:	**14**	**28**

(ii) Thought Experiment*

	E expects 18%	**E expects 28%**
Payoff 1	Round 26 10	Round 30 9
Payoff 2	Round 28 13	Round 32 18
Total:	**23**	**27**

(iii) Aggregate for both types of experiment.

	Third party vote low	**Third party vote high**
Payoff 1	18	24
Payoff 2	19	31
Grand total:	**37**	**55**

*Blue voters in (ii), red voters in (i).

With one exception, more voters from the candidate expected to come second will vote strategically if the third party is performing at a higher level of support. (This is significant at the 10% level. After paired comparisons have been made, this is significant at the 5% level).

Thus, hypothesis 6 is supported.

1(c) Comparison of strategic voting for the leading party and the second party

Table 3

	Leading Party	**Second Party**
Low support for third party	16	37
High support for third party	28	55
Total:	**44**	**92**

The level of strategic voting is much higher among supporters of the candidate expected to come second compared with the candidate expected to come first. Hypothesis 11 is supported.

2(a) Tests of Hypothesis 2

(Strategic voting is greater among leading party supporters when the differences in payoffs between first and third parties are smaller).

(i) Main Experiment

			Total
Large difference between C and E payoffs	Rounds 1 & 3 (payoff 1) 9	Round 15 (payoff 4) 1	10
Small difference between C and E payoffs	Rounds 2 & 4 (payoff 2) 13	Round 14 (payoff 3) 9	22

(ii) Thought Experiment

Large difference	Rounds 23, 27, 31 (payoff 1) 5
Small difference	Rounds 25, 29, 33 (Payoff 2) 26

(iii) Aggregate for both types of experiments.

Large difference	15
Small difference	48

The phenomenon is more pronounced in the later thought experiment than in the main experiment.

Hypothesis 2 is supported.

2(b) Test of Hypothesis 7

(Strategic voting is greater among second party supporters when payoffs between second and third parties are smaller).

(i) Main Experiment

			Total
Large difference in payoffs (D-E)	Rounds 1 & 3 (payoff 1) 12	Round 15 (payoff 4) 5	17
Small difference in payoffs (D-E)	Rounds 2 & 4 (payoff 2) 18	Round 14 (payoff 3) 9	27

(ii) Thought Experiment

Large difference	Rounds 22, 26, 30 (payoff 1) 23
Small difference	Rounds 24, 38, 32 (payoff 2) 42

(iii) Aggregate for both types of experiments

Large difference	40
Small difference	69

Taken overall, there is a significant relationship: hypothesis 7 is supported.

2(c) Comparison between the leading party and the second party

From 2(a) and 2(b) we obtain the following:

(i) Main Experiment

	Leading Party	Second Party	Total
Large difference in payoffs	10	17	27
Small difference in payoffs	22	27	49

From this, there is weak evidence for the hypothesis that strategic voting is greater among second-party supporters. However, it is less pronounced when the difference in payoffs is small.

(ii) Thought Experiment

	Leading Party	Second Party
Large payoff difference	5	23
Small payoff difference	26	42

Once again, while strategic voting is greater among voters for the second party compared with those of the first party, the difference is much greater when there are large differences in payoffs between the main parties and the third party.

2 (d) Effect of difference in payoffs between main parties and the third party.

There is further evidence from rounds 12 and 13 about the effect of difference in payoffs between the main parties and the third party. It cannot be included under 2(a) or 2(b) because there is no leading party or second party in rounds 12 and 13, as (ex ante) they are the same strength.

Large difference in payoffs	Round 13 (payoff 4) 3
Small difference	Round 12 (payoff 3) 22

From 2(c)(i) and the above table, this gives the total number of strategic votes for large payoff differences in the main experiments of 30, while for small payoff differences it is 71.

If these payoff differences are taken to be proxies for the degree of voter commitment to a party (large differences refer to committed voters; small differences to less-committed voters), then these results have a more practical interpretation. Committed voters for the leading party are unlikely to vote strategically, whereas committed voters for the second party are much more likely to do so - some four or a five times as many have done so in this experiment[6]. Non-committed voters are more likely to vote strategically than committed voters. There is less difference in this case between noncommitted voters of the leading party and those of the second party than for committed voters of these parties.

3(a) **Test of hypothesis 3**

(For the leading party, there will be more strategic voting the less certainty there is about the result).

[6] The magnitude of "four or five times" should not be taken too literally outside this experiment. It is reasonable to suggest, however, that second-party strategic voting is likely to be significantly higher than leading-party strategic voting in the real world.

Payoff Number	Round Number	Number of Strategic Voters
1	1 and 3 (certainty)	9 out of 44 voters
1	23 (less certainty)	3 out of 44 voters
2	2 and 4 (certainty)	13 out of 44 voters
2	25 (less certainty)	11 out of 44 voters

There is no support for hypothesis 3. The test is not a very good one, involving a comparison of the main experiment with the later thought experiment.

It is also likely that in the early rounds of the experiment, not all participants would have been sure that the leading party would win if everyone voted sincerely. Since it is not rational for a leading party voter (under certainty) to vote strategically, it is possible that these results also reflect that learning may have occurred.

3(b) Test of hypothesis 8

(There will be more strategic voting by second party voters the more certainty there is about vote shares).

Payoff	Round Number	Number of Strategic Voters
1	1 and 3 (certainty)	12 out of 46 voters
1	22 (less certainty)	4 out of 44 voters
2	2 and 4 (certainty)	18 out of 46 voters
2	24 (less certainty)	11 out of 44 voters

Hypothesis 8 is supported. However, note that, like hypothesis 3, it compares the main experiment with the thought experiment, where strategic voting does not appear to be as prevalent. Thus the support for this hypothesis may not be robust.

More direct tests between the certainty of the early rounds 1 to 4 with the uncertainty of rounds 12 and 13 cannot be made, because the payoffs differed between these rounds. For what it is worth, the following figures were obtained by combining the results of the tests of hypotheses 3 and 8. It is clear that the size of the payoffs is driving the results.

Main experiment					Thought experiment			
Round	Payoff Number	Number Voting Strategically	Total Voters	Proportion Strategic	Round	Number Voting Strategically	Total Voters	Proportion Strategic
12 (uncertain)	3	22	46	.48				
2 & 4 (certain)	2	31	92	.34	24, 25 (not certain)	22	44	.50
1 & 3 (certain)	1	21	92	.23	22, 23 (not certain)	7	44	.16
13 (uncertain)	4	3	46	.07				

3(c) Comparison of strategic voting between leading and second parties.

For what the figures are worth, 3(a) and 3(b) show that there is a little more strategic voting among voters of the second parties.

4(a) Test of hypotheses 4 and 9: (The larger the difference in the votes of the two main parties, the greater the strategic voting).

(i) Main experiment

Rounds 12 and 14 both used payoff 3. For round 12, there was an equal chance of blue and red tokens. For round 14, the tokens were in a 40:60 relationship.

Payoff 3 (little difference between (C or D) and E payoffs)

Candidate's chances (expected)	Number of Strategic Voters
50:50 (expt 12)	22
60% for leading party (expt 14)	18 (9 for leading party, 9 for second party)

Rounds 13 and 15 used payoff 4, and otherwise were the same as 12 and 14 respectively.

Payoff 4 (big payoff differences)

Candidate's Chances (expected)	Number of Strategic Voters
50:50 (expt 13)	3
60% for leading party (expt 15)	6 (5 for second party, 1 for leading party)

There is no support for the hypothesis when there is little difference between the payoffs of the main parties and the third party. There is weak support when there are big differences in payoffs. Strategic voting will occur much less, as there is not much point in supporting the third party if there is little to be gained by doing so.

However, there is weak evidence to support the belief that strategic voting will be greater among supporters of the second party if it is thought that the second party cannot win, even when the payoff for such voting is low.

Rounds 16, 18, and 20 also provide evidence for hypotheses 4 and 9. All have payoff 5, which is a relatively high payoff for E, the third candidate. Round 16 has 50:50 probabilities of red and blue; the other two rounds have 60:40 or 40:60.

Round Number	Proportion Strategic
16 (50:50)	19 out of 46 = 0.41
18 and 20 (leader)	11 out of 58 = 0.19
18 and 20 (second)	12 out of 34 = 0.35

Strategic voting, where the rewards for voting for the third party are relatively high (or in other words where the voter is not committed to the leading two parties to any extent) does not occur more when the voting strengths of the two main candidates are unequal, contrary to hypotheses 4 and 9. Supporters of the second candidate did not increase their propensity to vote strategically when voting strengths were unequal, and supporters of the leading candidate decreased significantly (in a statistical sense) their propensity to vote strategically, when voting strengths diverged.

Now let us look at what happens in rounds 17, 19 and 21, the counterparts of 16, 18 and 20, except that now this is but one seat at a general election which is expected to be fairly close. Strategic voting, when the result in the seat is expected to be close, all but ceases, falling from 19 to 4 out of 46. When the result is no longer expected to be close, the second candidate's voters still largely continue to vote strategically in the same proportions (12 down to 10 out of 34), but the leading candidate's defectors almost all return to the fold (11 down to 2 out of 58). While these absolute-number-changes impinge on hypotheses 5 and 10, it is the relative changes which bear on hypotheses 4 and 9.

Round Number	Proportion Strategic			
17 (50:50)	4/46	=	0.09	(1)
19 and 21 (leading candidate)	2/58	=	0.03	(2)
19 and 21 (second candidate)	10/34	=	0.29	(3)
19 and 21 (total)	12/92	=	0.13	(4)

While (1) and (2), (1) and (3), and (2) and (3) are statistically highly significant (1) and (4) are not significantly different. So while the proportion of strategic voters increased, from 9% to 13%, when the two parties were less-evenly matched, as hypothesised by hypotheses 4 and 9, this experiment was not large enough to distinguish between a real effect and one due to chance alone.

(ii) Thought experiment

The leading candidate increases its probability of winning from slightly better than a 50% chance in round 23, to becoming fairly certain of winning in round 27. The payoff is payoff 1. The same occurs in rounds 25 and 29, but with payoff 2 (small payoff differences).

Strategic Voting

	Uncertain Winner	**Near-Certain Winner**	**Total**
Payoff 1	3	0	3
Payoff 2	11	6	17
Total	14	6	20

Thus, strategic voting decreases (from 14 to 6) among voters for the leading party as it becomes more certain of winning.

Rounds 22 and 26, and rounds 24 and 28, are the counterparts of rounds 23 and 27, and 25 and 29, for the second candidate, who sees its chances slip from slightly less than 50% back to near zero.

Strategic Voting

	Uncertain Loser	**Near-Certain Loser**	**Total**
Payoff 1	4	10	14
Payoff 2	11	13	24
Total	15	23	

In this case, as expected, strategic voting increases as voters abandon the almost-certain loser. However, this increase in strategic voting (15 to 23) for the second candidate only just balances the decrease occurring amongst voters for the leading candidate, the total over both sorts of candidate being 29-29. Notice that there may be a slight increase for payoff 1 (7 to 10) - that is, an increase in strategic voting by voters more committed to a party, when the contest is unequal; however, the opposite occurs among less committed voters, who get on the expected winner's bandwagon when the winner is more certain. However, these are all weak effects. (It does, however, provide a rational explanation for bandwagon effects in politics. While this effect is likely to be small, it is interesting nevertheless that the effect is more marked amongst less-committed voters, which is a characteristic of such a phenomenon).

5(a) Testing of hypothesis 5

(Strategic voting by those whose true first preference is for the leading candidate is greater at a by-election/less at a general election).

<u>Main Experiment</u>

As mentioned already, rounds 18 and 20 represent by-elections, and 19 and 21 the corresponding general election. Strategic voting by voters for the leading candidate reduced from 11 out of 58 to at the by-election 2 out of 58 for the general election, a significant fall.

Hypothesis 5 is thereby supported.

5(b) Testing of hypothesis 10

(As for hypothesis 5, but now for the second candidate).

Main experiment

Strategic voting reduced from 12 to 10 out of 34 votes, which was not significant, although in the predicted direction.

Hypothesis 10 is not supported.

5(c) Testing of hypotheses 5 and 10 together

In rounds 16 and 17, there were no leading and second candidates, as each had an equal chance. Strategic voting declined from 19 out of 46 (round 16 - by-election) to 4 out of 46 (round 17 - general election). Combined with the results in 5(a) and 5(b), the reduction in strategic voting from by-election to general election was from 42 to 16 out of 138, a decrease from 0.30 to 0.12, which is highly significant. Hence the hypothesis that strategic voting will be greater at by-elections is borne out.

6. Testing of hypothesis 11

(Voters for the second candidate are more likely to vote strategically than those of the first candidate).

This has been covered already as an adjunct to the first ten hypotheses, and has generally been confirmed.

7. Testing of hypothesis 12

(Voters for leading candidates are less likely to vote strategically, the more certain they are of winning; voters for second candidates are more likely to vote strategically, the more certain they are of losing).

For the most part, this has been covered already as an adjunct of the discussion of the first ten hypotheses, particularly hypotheses 4 and 9. The conclusions were that voters for leading candidates are less likely to vote strategically, the more certain they are that their candidate will win. However, whether voters for the second candidate vote strategically more often when the cause seems lost depends on their intensity of feeling for the third candidate and on whether the election is a general election or a by-election. The changes observed in these experiments were mostly in the direction hypothesised, but were not very often significant.

V DISCUSSION

Several important results stem from this experiment. One is to do with the act of setting up the experiment. In framing an experiment, it is often necessary to dissect the problem in such a way that its testing becomes quite trite, or trivial. The answers appear so obvious that the experiment itself becomes redundant, or at least appears to be so. To some extent that has happened in this case: *of course*, one might say, it is obvious that the larger is E's (the third candidate's) share of the primary vote, the more likely it is that E will gain further strategic votes from the main two candidates, (particularly, one might think, from the less-preferred of these two candidates, if he or she has little or no chance of winning). And the fact that the experiment bears this out now appears inconsequential. Perhaps the case for greater strategic voting if the payoffs to E are higher rather than lower is even more obvious. Yet until now, none of these things appear to have been comprehensively canvassed or analysed systematically. These things have become apparent in the process of designing the experiment, and in this case the design stage of the experiment has been the most important stage of this project. But now let us look more closely at the first "obvious" proposition, that more voters of the main two parties will vote for E (*ceteris paribus*) if E already has more rather than less votes. Consider a red voter operating under payoff 1, in circumstances where D is unlikely to beat C, the leading candidate. As a decision theory problem, the red voter gets \$20 if D wins and \$4 if E wins. If D has less than a 1 in 5 chance of winning, the expected outcome for the voter is less than \$4. On the other hand, suppose E is certain to win, if only he or she can get above D, so that D's preferences flow on to E and E is thereby elected. Then the expected outcome for the voter is \$4.

To be rational, the voter must vote for E *whatever* is E's primary vote! Implicitly, of course, the voter is likely to be saying: the smaller is E's likely primary vote, the less chance E has of being elected. However, that is not what happened in experiment 2. Very few participants voted strategically in the first rounds in that experiment, despite the fact that a little thought would make it obvious that E was certain to win over C if only E could finish the primary voting above D. Yet as soon as the number of (non-human) green voters increased from 2 to 7, immediately a large number of red voters changed to a strategic E vote, only to desert back to D when the green voters dropped back to 2.

This implies that these participants, and I believe, voters in general, do not regard voting as if it were simply a decision-theory problem. Most voters are either unwilling or unable to consider an alternative voting pattern if their most-preferred candidate stands no chance of election. For reasons that amount to bounded rationality, voting for most people resembles an act of faith.[7]

There are three other pieces of evidence to bolster this conclusion. Let us look at the thought experiments 22 to 25 again. If E is eliminated, then candidates C and D have about an equal chance of winning, with D a slight favourite on the available evidence. Consider payoff 2 for a blue voter, in which a C win delivers \$10 but an E win delivers \$8. Then the break-even point is $10\ P_C = 8\ P_E^C$ where P_C is the probability that C beats D (E eliminated) and P_E^C is the probability that E beats D (C eliminated). So if P_E^C is 1, $P_c = 0.8$. That is, the probability that C will beat D has to be very high, (on the understanding that if E could only gain more votes than C, then E would beat D with certainty). But the same is also true for a red voter voting for D. If $P_E^D = 1$, where P_E^D is the probability that E wins against C (D eliminated), the probability that D will beat

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[7] This also bears on the subject of whether people vote instrumentally (that is, as if their vote will alter the result of the election) or expressively (that is, for any other reason). The former would seem just too difficult a task for most people to comprehend, so they do not vote, it would seem, because they believe their vote will be decisive, but for a range of other reasons.

C then has to be at least 0.8 before a voter should vote for D. It is of course impossible that $P_C = P_D = 0.8$ as $P_C + P_D = 1$. That implies that if a voter, when red, votes for D, then he or she when blue, must vote for E to maximise his or her utility, if $P_E^D = P_E^C = 1$. The objection to this is that P_E^C and/or P_D^C may not be equal to 1. To simplify matters, suppose $P_E^C = P_E^D$ (P_E, say). Then the critical condition in the more general formulation is that $P_C = 0.8P_E = P_D$. Since $P_C + P_D = 1$, then $1.6\ P_E = 1$, so $P_E = 0.625$.

Hence in this example, where a voter has to consider what to do if in turn he or she is a red voter and then a blue voter, it is consistent to vote non-strategically only if it is believed that $P_E \leq 0.625$. The fact that less than a quarter of voters voted strategically in this case suggests that voters were not considering this case as they would a decision problem, as they were encouraged to believe that P_E was close to one.

The second piece of evidence is that, without feedback on the thought experiments, as there were with the main experiments, voters tended to unlearn the lessons gained in the main experiments, and to vote strategically less often than they were prepared to do in the main experiments. (Again, however, this may reflect the real world case where P_E^C and P_E^D are less than one).

Voting in real life is an activity undertaken relatively rarely, with very little explicit information or feedback being given. In the experiment, the consequences of both sincere and strategic voting were made explicit, and the number of voters and the payoffs of the three or four sorts of voters were known to all voters. In the first 8 or 9 rounds the actual numbers of each sort of voter were also known. As well as this, feedback about the results of the election were given out every few minutes. Thus learning was relatively easy to achieve. Such sophistication is not possible in the real world, so perhaps the reversion to less strategic voting in a situation closer to that of the real world is not surprising.

The third piece of evidence is the baleful effect of the general election on strategic voting. The probability that the seat in question would alter the result of the general election was one in ten, by assumption. If a voter's vote in the electorate were decisive, its value in changing the result of the election as a whole would therefore still only be 0.1 times the value of winning the election (which was $16). Thus expected value of the payoff for voting sincerely would be increased by the amount $Z = 1.6p$, where p is the probability of being decisive. At most, therefore, Z = $1.60, and realistically, since p is unlikely to be more than 0.2, Z would be no more than $0.30 in expected value.

The probability of being decisive was, in the event, even smaller than this, so the effect of considering the election in one seat as embedded in a general election was to eliminate quite a lot of strategic voting for what would have been quite a small gain.

Taken singly, none of these pieces of evidence amounts to much. But together, they build a picture of voters who use simple rules to make decisions. In a simpler system (for example, voting with a single cross) where strategic voting is easier to understand, it is still likely that a significant proportion of voters would be unable or unwilling to undertake strategic voting, even when a close examination of the voters' circumstances would indicate it was in their best interests to do so.

As psychologists such as Simon, and more recently Kahneman and Tversky and the German school of experimental economists have shown, many people reach the limit of their cognitive abilities very rapidly when they are faced with quite simple decision problems. It is not surprising that similar phenomena have been demonstrated in this voting experiment.

VI CONCLUSIONS

This experiment has investigated the determinants of strategic voting in a preferential voting situation. It has examined twelve hypotheses and confirmed most of them. Voters from the two main parties will vote strategically more often if the third party is likely to perform well in its own right, giving a bandwagon effect to the third party. However, voters who are committed to one of the two main parties are less likely to vote strategically if their own candidate has a chance of winning. A close contest is less likely to deter less-committed voters from voting strategically. Strategic voting was shown to be much more common in a by-election situation than in a general election.

The question arises as to how this information can best be used by the parties concerned. For small parties and independent candidates, here are some guidelines.

(1) If possible, seek election at a by-election. It should be easier to win a seat this way. (This is borne out in practice.)

(2) The candidate should already be very well-known, to generate enough votes in his or her own right so that main-party voters believe that the candidate has a chance of winning. Only then will significant numbers of the second party begin to vote strategically. Such people in our society are restricted almost exclusively to well known sporting figures, TV personalities, local mayors, or politicians changing from one jurisdiction to another.

(3) The contest between the two main parties must not be too close. If it is close, main-party voters will be less likely to jeopardise the possibility of their own candidate's election.

From personal observation, and not tested in the experiment, a fourth guideline may also be suggested.

(4) It is helpful if the leading candidate is not personally popular.

The defence of the two main parties is to avoid the above circumstances as much as possible. Guidelines include:

(1) Avoid by-elections wherever possible.

(2) Try to ensure that the election is fought on national issues, and that the main parties have clear and distinguishable stands on them. A polarised electorate is less likely to contemplate voting for third parties.

(3) Effective politicians who are unpopular local members should be hidden wherever possible in the midst of the party list for multimember constituency Upper Houses.

(4) Where the seat is "safe" for the other main party (other than for the possibility of a third party winning with the help of strategic voting) to field as poor a candidate as possible and to spend as little campaign time as possible in the seat, consistent with not harming the party's chances in other seats or for other elections in that seat (eg for the Upper House) held at the same time.

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APPENDICES

1. Handout to subjects: instructions.

2. Overhead projector questions and instructions.

3. Handout to subjects: "thought experiments".

4. Results.

APPENDIX I

Alternative Vote Experiment

INSTRUCTIONS TO PARTICIPANTS

I *Introduction*

1. In Australian elections, voters must express a preference for all candidates. If no candidate gets 50% of first preference votes, the candidate with the least first preference votes is eliminated, and his or her second preferences now counts as first preference votes for the candidates indicated

2. An example:

Labor gets 45% of the vote, say	45 votes
Liberal gets	45 votes
Democrat gets	10 votes
	100 votes

Democrat votes: 6 of the 10 gave Labor the second preference, 4 of the 10 gave Liberal the second preference. That is, the configuration of the voting papers was as follows.

	6 votes	4 votes
Labor	2	3
Liberal	3	2
Democrat	1	1

When the Democrat candidate was eliminated, Labor finishes with 45 + 6 = 51 *elected.*
Liberal finishes with 45 + 4 = 49.

II *Method*

1. Today you are doing a voting experiment using the alternative vote.

2. You will be a voter, and you will be told how much you will gain if a particular candidate is elected. In the real world, of course, you don't get real dollars in such a direct way, but you may get tax cuts (and lower Government services!) with one party and so think you are better (or worse) off as a result. In everyday language we even express this with the following sort of question: "Will you be better off under Labor or Liberal?" The task is made easy for you in this experiment - you are told exactly how much better off you will be.

3. There will be quite a lot of voting to do, and you will win real money (although for some that may turn out to be $0!) as a result. After you have voted, *ONE* round will be chosen, and you will be paid according to its result, so you should treat each round seriously, because it could be the round chosen to be paid real money. The payment scheme is as follows: you get $10 whatever happens. On top of that, depending on who wins the chosen election, you could win between $0 and $20 extra.

4. The experimenter will now give you your voter number also called your player number.

5. For various rounds of the experiment, you will be either a Blue, Red, Yellow or Green voter, at random. Note that the colour you are in a particular round does NOT represent a PARTY: it represents a type of VOTER. The parties are represented as party CC, party DD and party EE, and have no significance in terms of actual political parties. The candidates of these parties are called C, D and E respectively. You are assigned a coloured counter each round so that there is no ambiguity about the type of voter you are in that round: it is a means of giving the experimenter greater control of the mechanics of the experiment.

6. You are given several pieces of paper. The first is your PLAYER RECORD. Please put your name, player number and date on the top. The name you put down should enable you to be contacted through the student mail system in case of any query. The second piece of paper [PAYOFF TABLE] shows a table of payoffs for different types of experiment. For round P1 (practice round 1) we play payoff type 1, so if you are a BLUE voter you will get $20 if C is elected, nothing if D is elected and $4 if E is elected. The table shows also what the payoffs are for voters of the other colours.

7. The small slips of paper are voting papers. At the top, in each round, put your player number and round number. At the start of each round, you will be given a counter of a particular colour. Do not show it to anyone. Write its colour on your voting slip.

8. In the first rounds, you will be told how many counters of each colour there are. Not all rounds need to be played.

9. After a certain number of rounds, the number of counters of some colours will be determined by a probability. For example, in some of these rounds, there will be an equal probability of there being an equal number of red and blue counters. In other rounds, there will be unequal probabilities. Details of what will happen will be explained at the beginning of each round.

10. For all these rounds, this is an experiment in isolation. However, for the last rounds, you are told that (for the last rounds only) this is only one seat in a parliament of many seats, and that there are two payoffs - one for winning the seat and the other for winning the election as a whole.

PLAYER RECORD

DATE ....

PLAYER NUMBER ....

NAME.......................................

ROUND	PAYOFF TYPE	COLOUR	WINNER	AMOUNT WON
P1				
P2				
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				
16				
17				
18				
19				
20				
21				

PAYOFF TABLE

PAYOFF TYPE	WINNER	BLUE	RED	GREEN	YELLOW
1	C	20	0	3	-
	D	0	20	0	-
	E	4	4	10	-
2	C	10	0	3	-
	D	0	10	0	-
	E	8	8	10	-
3	C	10	0	3	0
	D	0	10	0	3
	E	9	9	10	10
4	C	20	0	3	0
	D	0	20	0	3
	E	1	1	10	10
5	C	4	0	3	0
	D	0	4	0	3
	E	3	3	10	10
6	CC	16	0	12	0
	DD	0	16	0	12

APPENDIX 2

The following instructions were put on an overhead projector. I did not wish subjects to be able to gain knowledge of any future rounds, so the rounds were only revealed to subjects as they occurred. They are cryptic, because they were supplemented by explanation and the answering of any questions.

Overhead Projector Instructions

You will represent a red or blue voter. The behaviour of green voters is straight-forward, so they are not represented by people in this round.

All green voters vote:	C	2
	D	3
	E	1

Similarly, the behaviour of yellow voters is also straight-forward, and so they are not represented by people in this round.

All yellow voters vote:	C	3
	D	2
	E	1

There are 2 green voters in this round of the experiment.*

[*This referred to both experiments, which had an odd number of human subjects.

Had this number been even, then there would have been three green voters.]

ROUND 1

There are: 10 blue voters*

11 red voters

2 green voters

Payoff 1.

Please vote

*[This referred to experiment 2. For Experiment 1, there were 11, 12 and 2 voters respectively.]

ROUND 2

As for ROUND 1, <u>except</u> Payoff 2 (instead of Payoff 1.)

ROUND 3

Pass your token to the person next to you - ie. swap red↔blue.

Note: One or two must remain same colour.

As for ROUND 1. (Payoff 1)

ROUND 4

As for ROUND 3, <u>except</u> Payoff 2 (instead of Payoff 1.)

ROUND 5

Swap tokens again

(A different one or two to remain same colour)

Payoff 2.

ROUND 6

The number of green voters increases to 7.

Payoff 2.

ROUND 7

Swap tokens.

(A different one or two to remain the same).

Payoff 1.

(Still 7 green voters).

TWO QUESTIONS*

[*At this stage, subjects were asked two questions, one at a time, in order to determine who was alert to the possibility of voting strategically.]

QUESTION 1

PLAYER NUMBER: ....   *NAME:*...................................

1. In the first 5 rounds, there were .... blue voters, ... red voters and .... green voters. If every voter gives last (=3) preference to the candidate which would give him/her a zero payoff, can candidate D ever win?

   Yes/No

   Reasons:

   -

   -

   -

QUESTION 2

PLAYER NUMBER: ....   *NAME:*....................................

If candidate D cannot win, can red voters ever gain a payoff greater than zero?

[Hint: The candidate with the least first-preference votes is omitted]

Yes/No

Reasons:

ROUND 8

The number of green voters is now 2.

Payoff 1.

ROUND 9

The number of green voters is now 7.

Payoff 1.

ROUND 10

Swap tokens.

The number of green voters is *

Payoff 2.

[*This round was omitted in both experiments.]

ROUND 11

The number of green voters is *

Payoff 2.

[*This round was omitted in both experiments.]

FOR SUBSEQUENT ROUNDS, THERE WILL BE 2 GREEN AND 2 YELLOW VOTERS.

IN THE BOOK BAG, THERE ARE <u>EQUAL</u> NUMBER OF <u>RED</u> AND <u>BLUE</u> TOKENS. CHOOSE A TOKEN [from the book bag], LOOK AT IT, <u>AND REPLACE IT</u>.

ROUND 12

Payoff 3.

ROUND 13

Fresh draw from book bag.

Payoff 4.

ROUND 14

There are 2 blue and 3 red tokens in book bag.

Fresh draw.

Payoff 3.

ROUND 15

As for ROUND 14, but payoff 4.

For Round 16, payoff 5 represents the seat Y in Parliament. Payoff 6 represents the payoffs to the voter, depending on which party wins the election.

The probability that CC wins the elections is 70%

The probability that DD wins the election is 20%

The probability that the election is tied, and that your seat decides the election, is 10%.

Seat Y is evenly-balanced, but it is one seat in a Parliament of 147 seats. If the overall result of the election is very close, there is a small chance that the result in your seat will decide the election. The result overall with the exception of seat Y will be determined by chance. There is a 1 in 10 chance of being tied, so that the result in seat Y will determine the whole election.

ROUND 16

Equal red and blue tokens in book bag.

Fresh draw.

Payoff 5.

[Result announced after Round 17].

ROUND 17

As for ROUND 16, but payoffs 5 AND 6.

ROUND 18

2 blue and 3 red tokens in book bag.

Fresh draw.

Payoff 5.

[Result announced after Round 19].

ROUND 19

As for ROUND 18, but payoffs 5 AND 6.

ROUND 20

2 blue and 3 red tokens in book bag.

Fresh draw.

Payoff 5.

[Result announced after Round 21].

ROUND 21

As for ROUND 20, payoffs 5 <u>and</u> 6.

[This was the end of the class experiment. When coming later for payment, students filled out the following "thought experiment".]

PLAYER NUMBER: .... *NAME:*....................................

You are a voter in an electorate of 50,000 people.

Opinion polls give the following results.

	Poll 1	Poll 2	Poll 3
C	38%	41%	43%
D	42%	40%	43%
E	20%	19%	14%
	100	100	100

There may also be a swing of up to 2% either way before the election. E's preferences are thought to be evenly-divided.

<u>Round 22</u> You are a blue voter: please place your preferences below, when your payoff is given by payoff 1.

C
D
E

<u>Round 23</u> You are a red voter in the same circumstances. Payoff 1. How do you vote?

C
D
E

<u>Round 24</u> You are a blue voter, with payoff given by payoff 2. Please vote below

C
D
E

<u>Round 25</u> You are a red voter in the circumstances: Payoff 2. How do you vote?

C
D
E

Opinion polls give the following results

	Poll 1	Poll 2	Poll 3
C	35	40	38
D	45	46	43
E	20	14	19

<u>Round 26</u> You are a blue voter. Payoff 1. Please vote.

C
D
E

<u>Round 27</u> You are a red voter in the same circumstances (Payoff 1). Please vote.

C
D
E

<u>Round 28</u> You are a blue voter. Payoff 2. Please vote.

C
D
E

<u>Round 29</u> You are a red voter. Payoff 2. Please vote.

C
D
E

	Poll 1	Poll 2	Poll 3
C	33	30	35
D	38	40	41
E	29	30	24

<u>Round 30</u> You are a blue voter. Payoff 1. Please vote.

C
D
E

<u>Round 31</u> You are a red voter. Payoff 1. Please vote.

C
D
E

<u>Round 32</u> You are a blue voter. Payoff 2. Please vote.

C
D
E

<u>Round 33</u> You are a red voter. Payoff 2. Please vote.

C
D
E

APPENDIX 4

Experiment 1.

				Number of Voters			Result			After Prefs			Winner	Strategic Voting			Wrong First Choice		Wrong Second Choice
ROUND	PAYOFF	Colours Change	Probability	Blue	Red	Green	C	D	E	C	D	E		Blue	Red	Other	Chose C	Chose D	
Practice	1			12	13	2	8	8	11	-	8	19	E	4*	5				2
										8	-	19†							1
1	1	S		12	13	2	7	8	12	-	8	19	E	5*	5				
2	2	S		12	13	2	6	7	14				E	6*	6				1
3	1	C		12	13	2	8	9	10	-	11	16	E	4*	3	1	2	1	2
4	2	S		12	13	2	6	7	14				E	5*	7			1	1
5	2	C		12	13	2	8	8	11	9	-	18	E	4*	5		2	2	3
6	2	S		12	13	7	7	7	18				E	5*	6		1	1	4
7	Omitted																		
8	1	C		12	13	2	7	8	12	-	8	19	E	5*	5				1
9	1	S		12	13	7	8	5	19				E	5*	7		1		2
10	Omitted																		
11	Omitted																		
12	3	R	50/50	11	14	4	5	8	16				E	6	6				
13	4	R	50/50	10	15	4	10	13	6	13	16	-	D	1	1		1		3
14	3	R	60/40	18	7	4	13	3	13	13	-	16	E	5@	4			1	
15	4	R	60/40	15	10	4	13	8	8	16	13	-	C	1@	3			1	2
16	5	R	50/50	14	11	4	8	8	13	-	8	21	E	6	3		1	1	1
17	5, 6	S		14	11	4	13	11	5	15	14	-	C	-	1		1	2	2

† if D had been eliminated at random.

Experiment 1.

				Number of Voters			Result			After Prefs			Winner	Strategic Voting			Wrong First Choice		Wrong Second Choice
ROUND	PAYOFF	Colours Change	Probability	Blue	Red	Green	C	D	E	C	D	E		Blue	Red	Other	Chose C	Chose D	
18	5	R	60/40	16	9	4	13	6	10	13	-	16	E	3@	3				1
19	5, 6	S		16	9	4	14	9	6	16	13	-	C	-	2			2	1
20	5	R	60/40	17	8	4	16	5	8				C	3@	1		2		3
21	5, 6	S		17	8	4	17	6	6				C	-	2				1
22	1			23			20		3					3					
23	1				23			20	3						3				
24	2			23			16		7					7					
25	2				23			16	7						7				
26	1			23			17	1	5					5				1	1
27	1				23		25	21	-						-		2		1
28	2			23			17	1	5					5				1	2
29	2				23		-	19	4						4				
30	1			23			19	-	4					4					1
31	1				23		2	19	2						2		2		2
32	2			23			15	-	8					8*					2
33	2				23		2	15	6						6		2		1
		S = same												* denotes irrational					
		C = change												@ denotes may be irrational					
		R = random																	

Experiment 2.

				Number of Voters			Result			After Prefs			Winner	Strategic Voting			Wrong First Choice		Wrong Second Choice	Totals Experiment 1 and 2	
ROUND	PAYOFF	Colours Change	Prob.	Blue	Red	Green	C	D	E	C	D	E		Blue	Red	Other	Chose C	Chose D		B	R
P	1			10	11	2	11	8	4	14	9	-	C	1*	1		2		1	5*	6
P	1	S		10	11	2	11	9	3	13	10	-	C		1		1		1	5*	6
2	2	S		10	11	2	10	7	6	13	10		C	1*	3		1		1	7*	9
3	1	C		10	11	2	9	10	4	11	12		D		2			1		4*	5
4	2	S		10	11	2	7	11	5	10	13		D	1@	2			2		6*@	9
5	2	C		10	11	2	10	9	4	13	10		C	1@	1		1			5*@	6
6	2	S		10	11	7	6	3	19				E	5@	7		1			10*@	13
7	1	C		10	11	7	9	8	11	9	-	19	E	1	3		1	1	3	1	3
8	1	S		10	11	2	10	8	5	12	11	-	C	-	3				1	5*	8
9	1	S		10	11	7	7	5	16				E	2	7			1	1	7*	14
10	Omitted																				
11	Omitted																				
12	3	R	50/50	14	7	4	7	4	4				E	6	4			1	2	12	10
13	4	R	50/50	13	8	4	13	7	5				C	-	1				1	1	2
14	3	R	40/60	7	14	4	4	8	13				E	5	4@		2		2	9	9
15	4	R	40/60	7	14	4	5	14	6				D	2	-				3	5	1@
16	5	R	50/50	10	11	4	6	5	14				E	5	5		1			11	8
17	5, 6	S		10	11	4	7	11	7		12	13	E	3	-				2	3	1
										(12)	(13)†		(D)†								

† If E had been eliminated at random

Experiment 2.

				Number of Voters			Result			After Prefs			Winner	Strategic Voting			Wrong First Choice		Wrong Second Choice	Totals Experiment 1 and 2	
ROUND	PAY OFF	Colours Change	Prob.	Blue	Red	Green	C	D	E	C	D	E		Blue	Red	Other	Chose C	Chose D		B	R
18	5	R	40/60	9	12	4	5	8	12	-	8	17	E	4	$4^{@}$				1	8	$6^{@}$
19	5, 6	S		9	12	4	6	11	8	-	11	14	E	3	$1^{@}$				1	5	$1^{@}$
20	5	R	40/60	8	13	4	4	12	9	-	12	13	E	4	$1^{@}$					5	$4^{@}$
21	5, 6	S		8	13	4	5	12	8	-	12	13	E	3	$1^{@}$					5	$1^{@}$
22	1			21			20		1					1						4	
23	1				21			21							-						3
24	2			21			16	1	4					4				1		11	
25	2				21			17	4						4						11
26	1			21			16		5					5						10	
27	1				21			21	-						-						-
28	2			21			13		8					8						13	
29	2				21			19	2						2			-	1		6
30	1			21			16		5					5					1	9	
31	1				21			21	-						-						2
32	2			21			11		10					10					1	18	
33	2				21			18	3						3				1		9

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Part VI

Decisiveness

Papers 11 to 15 continue the work begun in Paper 4 on political power. The notions of power and decisiveness are very close. Both are concerned with the notion of a voter (or legislator, or party of legislators) being pivotal. The Shapley-Shubik and Banzhaf indexes assume that all outcomes are equally-likely, and apply game-theory concepts. The concept of decisiveness of a single voter has used a mathematical approach to a statistical problem. I show that voter power and the probability of a voter being decisive are essentially the same concept, both being outcomes of a *subjective* probability approach. In the process of doing so, the orthodox formulation of the probability of a voter being decisive is rejected. This is done in Paper 11 "The probability of being decisive". Papers 12 and 13 apply the orthodox and new formulations to the problem of finding a value for the probability of a voter being decisive in the election of the U.S. President. The two formulations give quite different results. In the orthodox formulation, if all voters in all states are assumed to have the probability of 0.5001 of voting for candidate A, then A will almost certainly win every state. In the new formulation, if the best estimate of the probability that a voter will vote for A is 0.5001, in each state, A can expect to win just over half the states, and may quite conceivably lose the overall election.

Paper 14 "Strange Bedfellows" is a critical review of Brennan and Lomasky's book *Democracy and Decision*, in which the low value of the probability of being decisive (P_D), calculated in the orthodox way, is used as a reason for suggesting that voters must vote for "expressive" reasons, and not because they believe they will influence the outcome. The review argues that, for a number of reasons, Brennan and Lomasky appear to have overstated the importance of the distinction between "expressive" and "instrumental" voting.

Paper 15 "A Further Experimental Study of Expressive Voting" tests the proposition that as the probability of being decisive declines, voters are more likely to vote expressively.

PAPER 11

THE PROBABILITY OF BEING DECISIVE

I Introduction

One of the most important concepts in the theory of voting and public choice is that of decisiveness.[1] The probability of being decisive (P_D) in an election with more than a handful of voters is always small, usually very small, and sometimes infinitesimally small. However, for over 20 years, it seems to have been calculated incorrectly (or perhaps, given what follows, to have been calculated "inappropriately"). It is important to clear up the matter, not only because it is important to get the methodology correct, but also because the conclusions being drawn from the existing calculations can be quite misleading. For all that, the methodology I propose still gives values that are small, although not infinitesimal, as the orthodoxy would have us believe.

II Background

In 1968, Riker and Ordeshook wrote on what determines voter turnout. They looked at a sample of voters, whose probability of voting was found to be determined by a number of factors, including the subjective beliefs of the voter as to whether he or she was likely to be decisive. They mentioned in passing that this probability for a large

[1] "Being decisive" means either that there is a winning margin of one vote, so that any one voter for the winner changing sides would change the result of the election, or a tie, so that any one voter changing sides would change the result.

electorate might be of the order of 10^{-8}. However, the probability was apparently not calculated with any "precision" until a paper by Beck (1975). Since then, several writers have used Beck's methodology, and a number have used results based on this methodology. The main result, which has been widely recognised, is that P_D is infinitesimal, unless either the electorate is very small, or the numbers voting for the leading two candidates are very close.

Beck's method is to assume two parties, which I relabel as candidates A and B. He assumes that each voter has the same probability P_A of voting for A and $(1-P_A)$ of voting for B. Suppose A obtains X votes. If there are N voters (where N is even), P_D is given by the binomial probability $P(X = N/2)$.[2] For large N, this is approximated, using Stirling's formula, by:

$$P\left(X = N/2\right) = \left[4p(1-p)\right]^{N/2}\sqrt{\frac{2}{\pi N}} \qquad \text{............ (1)}$$

Beck considered the case of p=0.5, 0.49, 0.45 and 0.4 for N =100, 1,000, 10,000, one million and 100 million, and showed that $P\left(X = N/2\right)$ was infinitesimal (less that 10^{-20}) for p=0.45 when N exceeded 10,000, and for p=0.49 when N exceeded a million.

[2] A similar formula exists for odd N, which we omit.

He found that $P\left(X = N/2\right)$ was relatively high when P_A was exactly 0.5 (eg. 8×10^{-5}, or 1 in 12,500 for N =100 million), but dropped off extremely rapidly, when N was very large, as P_A departed even slightly from 0.5.

He recognised that not all voters had the same value of P_A, and considered cases where half the voters had $P_A = \alpha$ and the other half had $P_A = 1-\alpha$, and found substantially similar results.

Beck's work was followed by Kau and Rubin (1976), and Margolis (1977). The analysis in these two cases is similar to Beck's. Thompson (1982) was worried that the huge change in the probability of decisiveness near $P_A = 0.5$ did not lead to a very large change in the percentage turnout to vote. Owen and Grofman (1984) looked at the composite or two-stage voting system for the US President (i.e., electors vote for an electoral college, which in turn elects the President), again using Beck's approach over both stages. Like their predecessors, they showed a very low value of P_D, and used it to justify voting "expressively", as it has come to be known. That is, since P_D is infinitesimal, if one is going to vote, it should not be with the aim of being decisive, or "instrumental". In particular, they suggest that it would be better for voters whose first preference is for third-party candidates to vote sincerely for such candidates, rather than try to avoid their vote being wasted by voting for one of the candidates of the two main parties. The reason for such advice is that, while the third-party candidate may have no prospect of winning, their vote is so unlikely to be decisive that it is, in essence, wasted, however it is cast. Mueller (1987) surveys the area, again using Beck's formula, and Berg (1990) uses the same method, applying it to the case of a caucus.

Brennan and Lomasky (1993) repeat Beck's method, also commenting that the drop-off in P_D, if P_A departs from 0.5, is dramatic. They use this to argue that observed small increases in turnout, when P_A tends towards 0.5, are inconsistent with such large increases in P_D, *unless factors other than self-interest play a predominant role*, (italics in original). Palfrey and Rosenthal (1983) and Ledyard (1984) both establish game - theoretic models of the decision to vote. They recognise the simultaneity of P_D and the turnout decision. The voters in these analyses know the mean of all the other voters' voting positions.

In my re-evaluation of the size of P_D, I do not find that there is a sudden decline in P_D when P_A departs from 0.5, so that conclusions that rest upon this orthodoxy will no longer hold.

The only writers who obtain a different result are Chamberlain and Rothschild (1981). They say that P_A is unknown (that is not the case with any of the other writers, who assume voters know one another's probabilities). However, they still base their analysis on N voters. Although they recognise that P_D is determined by the fact that P_A is unknown, they solve only a very general case. It is apparent that the implications of their results have not been assimilated by the profession (including it seems, its authors!) Their results conflict with Beck's formula, which is still used. The result proved by Chamberlain and Rothschild is that if the size of the electorate increases by a factor of k, then P_D will be k^{-1} times as large as originally.[3] The example utilised in this paper is consistent with Chamberlain and Rothschild's result. However, Chamberlain and Rothschild's result is an isolated one: it has not thrown any light on the processes used by voters in forming decisions, and has been extremely limited in its usefulness.

III Probability of Being Decisive: A New Formulation

I shall argue from example. The reason for this will become obvious as the analysis proceeds.

Consider the case of 50,000 electors, and that there are only two candidates, A and B. Assume that we have conducted an opinion poll of a random sample of 300 electors. This is typical of the size of polls taken before an election in marginal seats in many Parliamentary elections. It gives a standard error of about three percentage points for $\hat{P}_A$, the estimated vote-share of A.

Suppose that P_A is estimated to be 0.5. That is, exactly 150 of the 300 say that they intend to vote for A.

We in fact do not know P_A from the knowledge of the proportion who *actually* vote for A, which is essentially what the orthodox (Beck's) method assumes. If we knew that P_A equalled 0.5, the use of Beck's method would give P_D as 1 in 280.

All we have is an estimate of P_A. But we can still use this information to work out the probability that the two candidates will be within a vote of each other. In this problem, $n = 300$, $np = 150$, $npq = 75$ and $\sqrt{npq} = 8.66$. (We ignore the finite population correction). That is, P_A is *estimated* to be 0.5, and likewise, the S.D. is estimated to

[3] In Beck's formula, P_D alters by a factor of $k^{-1/2}$.

be $\frac{8.66}{300}$ = 0.029. This means that we estimate that A will receive 25,000 votes plus or minus a standard deviation of 1,443 votes. Using the standard normal distribution approximation, the probability that the two candidates are within one vote of each other is $\frac{0.399}{1443}$=0.000276, or 1 in 3618. If we took the finite population correction of 0.994 into account, the probability would rise to 0.000277, or 1 in 3607.

If we suppose that the sampled information is the best available and that voters use it to form their subjective probabilities, then 1 in 3618 becomes the voter's estimate of P_D. This is substantially lower than the 1 in 280 of the orthodox method, by a factor of about 13 - in fact the ratio is $\sqrt{\frac{50,000}{300}}$, or about 13.

Now suppose that P_A departs from 0.5. Suppose 153 out of 300 say they will vote for A. We would not expect in real life that this would significantly alter P_D. Nor does it. $\hat{P}_A$ = 0.51, $\sqrt{npq}$ = 8.66 as before, and P_D falls to 0.000271, or 1 in 3689. When $\hat{P}_A$ = 0.52, P_D falls to 0.000255, or 1 in 3917, relatively small changes. Even when $\hat{P}_A$ = 0.55, the probability is still 1 in 6000.

By comparison, P_D, as calculated by the orthodox method, is 0.00357 for P_A = 0.500, 0.000295 for P_A = 0.505 and 0.000098 for P_A = 0.506. These values of P_D decrease very sharply, and the crossover between the two methods in this example is at P_A = 0.5051. That is, the orthodox method gives a much higher value of P_D for A's vote between $P_A = 49.5\%$ and $P_A = 50.5\%$, but a much lower one outside this range. When $P_A = 0.51$, P_D (Beck) = 1.6×10^{-7}, and when $P_A = 0.55$, P_D (Beck) $= 7.6 \times 10^{-113}$.

The orthodox method for an election where the candidates have equal support may now be interpreted as one where $\hat{P}_A = 0.5$, being based on a sample of 50,000 out of an infinite super-population![4] The standard deviation of this is not 1443, but only 112 votes. That is why the orthodox method gives answers which are so peaked. It is also why the value of P_D drops off so spectacularly in the orthodox method.

The one advantage of the orthodox method is that it is "objective". That is, voters who are trying to decide the size of P_D have a known sample size on which to base their subjective beliefs. It is the size of the population!

It may be argued, of course, that the P_A used by the orthodox method is *not* an estimate, but the true proportion who vote for A. But if that is the case, then the outcome of the election will be known precisely, so the voter will know *in advance* whether the final result is within a one vote majority without her/his vote. It cannot therefore be argued that P_A is known exactly. If it is known, P_D will either be exactly 0 or exactly 1.[5]

[4] An "infinite super-population" is an infinite population of individuals from which the actual electorate of N = 50,000 may be drawn as a sample. Of course, other "electorates" very similar to the actual electorate may also be drawn from this population. The orthodox method chooses one of these other electorates, and thereby incurs some sampling error.

[5] Note also that if the outcome is known, the voter may, in light of this knowledge, vote differently. But then, so may other voters. Thus the outcome cannot be known. I am indebted to Gretel Dunstan for pointing this out.

If P_A is not known, it must be estimated. The question which then must be asked is: how do people make their estimates of who will win the election, and how do they decide by how much? In the example, I imply that, by and large, voters cannot do much better than use the result of opinion polls in formulating their subjective beliefs about the likely size of the majority, and by implication, about the probability of a tied vote or majority of one.

Of course they may not do that: the nature of any subjectively-held beliefs is that we are likely never to be able to measure them accurately. To that extent, it is *possible* that people are basing their beliefs as if they were sampling from an infinite population, the sample size being equal to the (finite) number of voters in the electorate. But it is not likely!

A further possible criticism of this revision of the orthodoxy is to say that there must really be an *objective* measure of P_D. The answer is that, after the event, there is: as has already been stated, P_D is zero or unity. Before the event, there is not.[6] One way to proceed would be to *define* the subjective probability as if it were based on the best available estimate of who wins. This is likely to be the opinion poll approach.[7] This approach is the public choice equivalent of a "rational expectation" subjective probability. (See also Ledyard (1981)).

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[6] One could of course look at the frequency of tied votes in different jurisdictions to get a frequentist measure of being decisive, but there are many problems with this, one being the difference in the number of voters in each electorate. Alternatively, one could estimate the frequency of tied votes by examining the distribution of winning margins in that electorate over time. However, the number of data points is very limited, the population of voters, the electoral boundaries, the issues and the candidates all change over time, so there is a real problem in defining the population.

[7] The best estimator of the result is likely to be a Bayesian compilation of all the polls conducted in the electorate, together with those in other electorates, to the extent that the swings in different electorates are likely to be fairly-highly correlated with each other.

Let us summarise the argument so far. It is not possible to calculate the *ex ante* value of P_D by looking at the result of the election (that is, from the *ex post* probability of being decisive). The reason for this is that the *ex post* probability is either zero or one. That is to say: P_D must be based on subjective probabilities and not objective ones. The subjective probabilities will in general not be known, but we can, at least theoretically, decide on what the voter's subjective probability ought to be if it were based on an efficient processing of all the available information or a sort of "rational expectations" subjective probability (RESP). This probability can be associated with the sample size of opinion polls conducted just prior to the election. An upper bound to this "equivalent sample size" would be the sum of the sample sizes of these opinion polls.[8] From the mean and variance of the distribution of the estimated P_A from this sample, P_D can be calculated using standard statistical methods.[9]

There is an interesting and important outcome of the proposed methodology. Since the sample size for opinion polls for a general election is usually about 2,000, whatever the size of the population being sampled, (in order to give an error in each case of about one percentage point) it implies that P_D does not decline any faster for large N than it does for small N, (for N > 10,000, say) as P_A departs from 0.5. To show this, suppose we have a simple random sample of 2,000 out of 50,000, and another of 2,000 out of 100,000,000 and we base our subjective beliefs on these samples. The

[8] If independent polls using different samples have been taken, and the only error of prediction is sampling error, the equivalent sample size would be the sum of the samples of the different polls. This would have to be discounted to the extent that the samples may not be totally independent, or that there will be errors other than sampling error in the prediction process.

[9] These methods, when applied to the orthodox techniques of basing P_A on a sample size of the whole electorate, give the same results as Beck's use of Stirling's formula.

following table shows the values of P_D for several values of P_A.[10] Note that for a given sample size, the standard deviation of the estimated number of votes gained by A will be proportional to the estimated number of votes (ie. will be proportional to N, the number of voters in the electorate). Since P_D is proportional to the reciprocal of this standard deviation, it will be proportional to N^{-1}, the result obtained by Chamberlain and Rothschild (1981).

TABLE 1

Values of the probability of being decisive (P_D)

N $\quad P_A$	0.5	0.51	0.52	0.55
50,000	1.78×10^{-2}	1.20×10^{-2}	3.6×10^{-3}	8.1×10^{-7}
100,000,000	8.9×10^{-6}	6×10^{-6}	1.8×10^{-6}	4×10^{-10}

The following diagrams illustrate the above arguments.

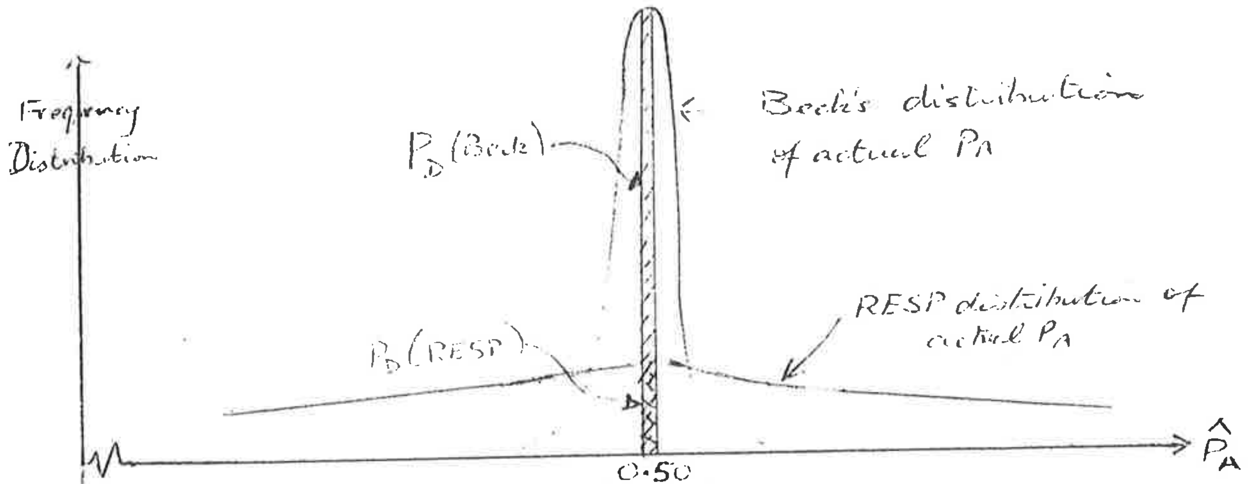

Diagram 1.

[10] We ignore the finite population correction. It makes a small (ie. two percent) difference for N = 50,000, and a non-discernible one for N = 100 million. In practice, the accuracy of the sample will depend on the degree of homogeneity of the population, and the extent to which techniques such as stratification can be used to improve the accuracy of sampling for a given *n*. Almost paradoxically, since a population of 100 million is likely to be less homogeneous than one of 50,000, the relative accuracy of a stratified sample of 2,000 from the larger population is likely to be higher than that of a sample (stratified or unstratified) of 2,000 from the smaller population. This effect is likely to be greater than the opposite effect of including the finite population correction in the calculations, and consequently, both effects are ignored in Table 1.

In Diagram 1: Population = 50,000
Beck's value of $P_A = 0.5$
Opinion poll of 300 yields $\hat{P}_A = 0.5$

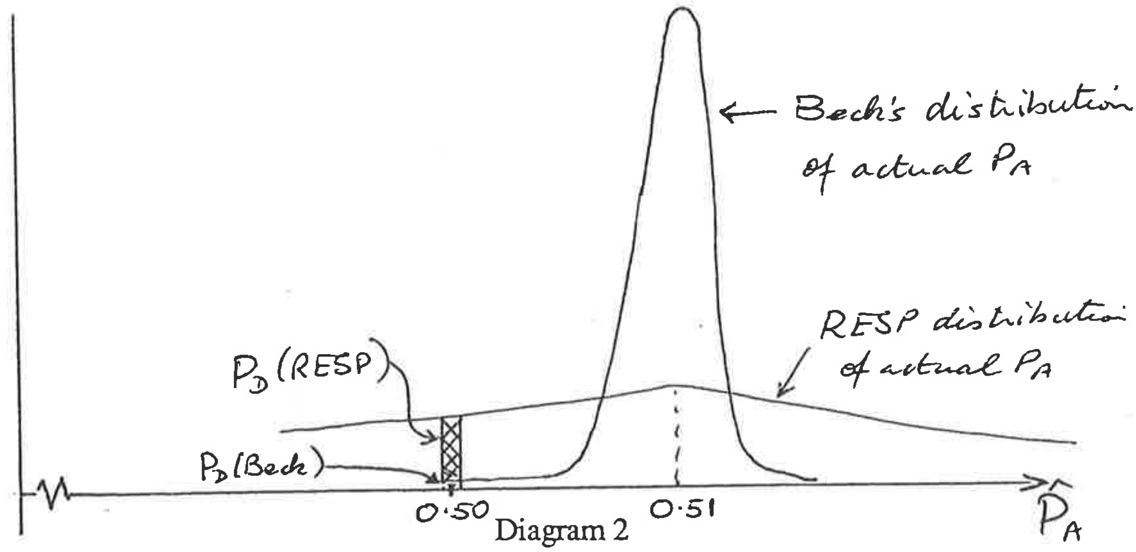

Diagram 2

In Diagram 2: Population = 50,000
Beck's value of $P_A = 0.51$
Opinion poll of 300 yields $\hat{P}_A = 0.51$

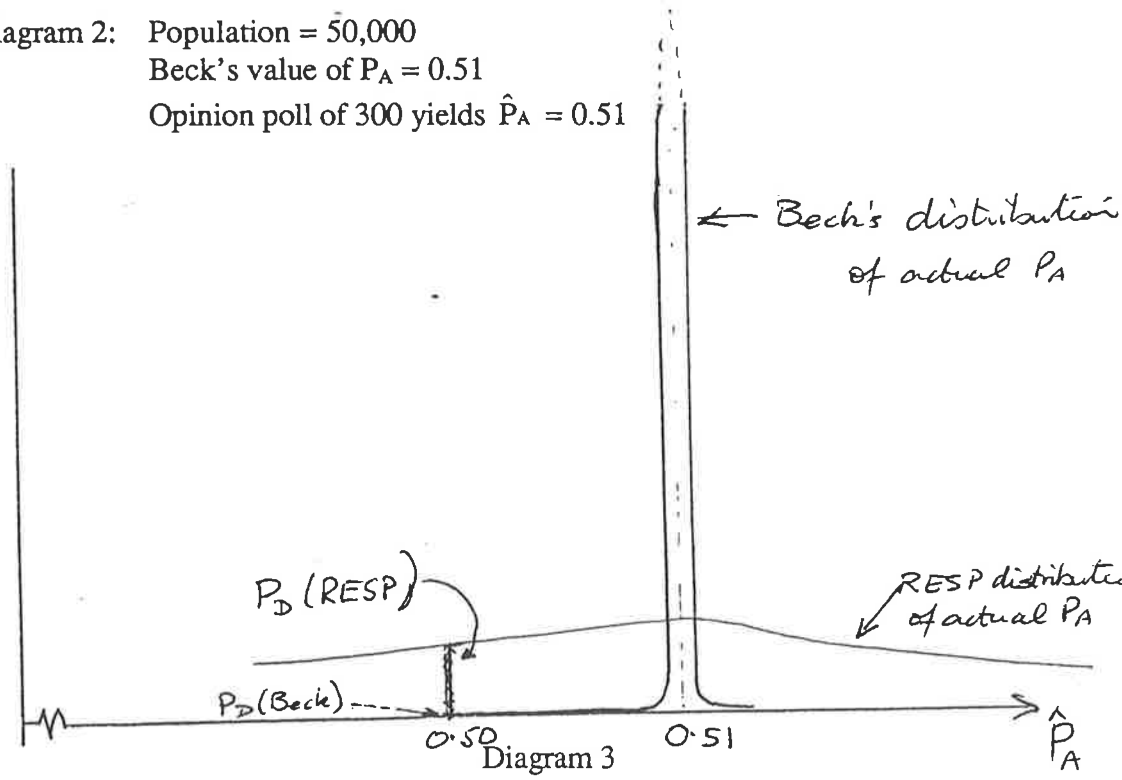

Diagram 3

In Diagram 3: Population = 100,000,000
Beck's value of $P_A = 0.51$
Opinion poll of 300 yields $\hat{P}_A = 0.51$
Beck's distribution is more peaked in Diagram 3 than in Diagram 2; the RESP is not.
P_D (RESP) is smaller in Diagram 3 than in Diagram 2 because the area under the curve representing one vote has a much narrower base. This is the scale effect which drives Chamberlain and Rothschild's result.

IV. Further Fuzziness

The value of P_D has been calculated by the orthodox method as if it were in some sense "objective", and by my revised method, by what amounts to a rational expectations approach to it. Of course in the real world, things are not so simple.

The first problem is that we assume that voters can work out their value of P_D. Given the difficulties which researchers in this area have had, it should not be surprising if voters had subjective probabilities of being decisive which look quite different from the ones researchers, or I, assume may be appropriate.

How does a voter know how to "calculate" the probability of being decisive? As P_D will be small, errors in its calculation are likely to be asymmetric. Much of the evidence from the behavioural decision school suggests that most people cannot comprehend very low probabilities, and therefore tend to exaggerate them (see for example Kahneman and Tversky's (1979) Prospect Theory, or people's willingness to take part in lotteries). It would not be surprising, therefore, if many voters held subjective probabilities of being decisive which were larger than those I have suggested appropriate according to the calculations earlier in this paper. The irony, of course, is that the orthodoxy has a very high value for P_D when P_A is close to 0.5!

The second real-world problem is not just that voters can't calculate the probability of their being decisive, but that they are biased, and often severely so, in their estimation of the likelihood of the outcomes. I refer to Table 2, where Australian voters are asked their voting intentions and who they think will win the next election.

TABLE 2

Which political party do you think will win the next election whenever it is held?

(answers are percentages)

Type of Voter

Winner	All voters	ALP supporters	Coalition supporters
ALP	42	73	19
Coalition	36	15	60
Uncommitted	22	12	21

Source: Newspoll, *The Australian*, 31 May 1994.

A large majority of ALP voters think the ALP will win the next election; a large majority of Liberal National Coalition voters think that the Coalition will win it. Do they have different information, or do they skew the same information for their own purposes? Or are they, in essence, lying to the pollster? Or a combination of these things?

Whatever the truth of the matter, subjective probabilities are being distorted by the voter's environment and/or convictions. Taken over the community as a whole, there may be very little agreement about which party will win, and therefore quite possibly a good deal of uncertainty about whether their own vote matters. If uncertainty were to attach to an otherwise-certain or near-certain event, it would cause its (subjective) probability to decline in the minds of the voter, but if it were to attach to an event with an otherwise low probability of occurrence (ie. being decisive), it is likely to cause its subjectively-held probability to increase.

It is fitting to note, in passing, a further contribution to fuzziness. Aldrich (1993), emphasises that voting is a low-cost, low-benefit activity and any small changes in either costs or benefits can significantly change behaviour.

V Conclusions

There is no objective probability of being decisive. The "rational expectations" probability developed in this paper is not nearly so highly-peaked as the probability calculated by the orthodox methods beginning with Beck (1975). As a consequence, the findings of Barzel and Silberberg (1971) and subsequent writers about the elasticity of voter turnout may no longer be inconsistent with instrumental reasons for voting.

With this new set of calculations, an increase in expected majority from 2% to 4% would alter the probability of being decisive from 1 in 3,900 to 1 in 5,000 (if based on an equivalent sample size of 300), and this, according to Barzel and Silberberg, would decrease turnout by between 1.1% and 1.5%, which is not difficult to believe.

As a consequence of the arguments in this paper, it is likely that a re-evaluation of past empirical work will be undertaken. This work has generally shown a positive relationship between turnout and the closeness of the result, but it has never been possible to tie the empirical findings to the theoretical probabilities, because these declined so rapidly as P_A diverged from = 0.5. The arguments in this paper suggest a much more gradual decline in decisiveness from a peak at $P_A = 0.5$. Furthermore, the ratio of the decline in decisiveness at $P_A = 0.5$ compared with that at $P_A = 0.5 + x_0$ will often be virtually the same (for given x_0) whatever the number of voters in the electorate happens to be, as was shown in Table 1. The orthodox theory, however, has a much steeper decline in decisiveness for electorates with a greater number of voters.

The concerns of Thompson (1982) about the lack of a link between the change in P_D as P_A alters and the change in voter turnout may largely be put to rest, while Owen and Grofman's (1984) arguments in favour of expressive voting must now be reviewed.[11]

One interesting point is that the probability of being decisive does not fall as rapidly for those electors who know very little about who is likely to win, as it does for the cognoscenti: in the extreme case, the complete ignoramus will have as much reason to vote instrumentally when the poll is likely to be a foregone conclusion (for others!) as on those occasions when others believe that the outcome will be very close.

This paper makes five contributions. The first is to emphasise that there is no such thing as a truly objective measure of the probability of being decisive. The second is to re-iterate Ledyard's (1981) point that there may be a "rational expectations" subjective probability of being decisive, based on a proper analysis of the best available information about likely voting behaviour. The third is to recognise that subjective

[11] Mueller's (1987) survey article lists about 10 empirical articles relating turnout to decisiveness which must now be reviewed, and this list is by no means exhaustive.

probabilities are distorted by voters' perceptions and their inability to process information effectively.

New calculations of the probability of being decisive lead to the fourth contribution, namely the appreciation of the fact that the probability of being decisive, logically and efficiently derived from the best available information, will in general neither be sharply-peaked nor fade quickly to the infinitesimal as P_A departs from 0.5. It will be low, but have a relatively uniform distribution over a fairly wide range of values of P_A. For many elections, the probability of being decisive will for many voters be larger than the probability of winning a state lottery.

The final contribution is to resolve the contradiction between the formulation of Beck (1975) of the probability of being decisive and that of Chamberlain and Rothschild (1980), in favour of the latter.

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PAPER 12

The Probability of a Tied Vote for the U.S. Presidency: The Orthodox Formulation

1. *Introduction*

This paper uses Beck's (1975) formulation of the probability of being decisive (P_D) to determine the value of P_D for a two-stage election such as that of the U.S. President. Originally this paper was written to contrast the value of P_D for the two-stage electoral process with that of the one-stage process, where up to 100 million people vote in one huge first-past-the-post electorate. In the two-stage process, electors are assumed to vote within a state as a single electorate. The state has a certain number of electoral college votes. Whichever candidate obtains the most votes in a state is assumed to gain all the electoral college votes for that state. Whoever wins the majority of electoral college votes over the nation wins Presidential office.

Since this paper was drafted, I wrote "The probability of being decisive" (paper 11 of this thesis), from which it is clear that Beck's method (called the "orthodox" method) is severely flawed. This paper has been kept in the thesis for two reasons: first, it shows how the orthodox method grotesquely distorts the value of P_D, and second, it contains methodology which can be used more generally.

Kau and Rubin (1976) and Owen and Grofman (1984) adapt the orthodox method to a two-stage election. Kau and Rubin note that the value of P_D is lower for a two-stage election, while Owen and Grofman use the result to suggest that, if a voter is not going to be decisive, he or she is made better off by voting sincerely (which implies more voters should vote for third parties) because the outcome is decided in any event. Brennan and Lomasky (1993) rework the orthodox formula, but commit what are probably arithmetic errors, so that many of the entries in their table (on page 56) of P_D are incorrect.

Suppose there are only two candidates, A and B, and that all voters in the first instance are regarded as if they will vote for A with the same probability, P_A.Take the case where $P_A = 0.5000$ (exactly), and that there are 100,000,001 votes. Then, as Brennan and Lomasky show, the probability that A wins 50,000,001 to 50,000,000 (ie by one vote exactly) is calculated to be 1 in 12,533.

But suppose that P_A exceeds 0.5000 by a very small margin, say $P_A = 0.5001$. That is, we expect A to win 50.01% of the popular vote. The probability that A will win by exactly one vote is reduced to 1 in 92,608 (not 1 in 1.9×10^6 as given by Brennan and Lomasky) while if $P_A = 0.501$, the probability of being decisive falls to 1 in 9×10^{90} (not 1 in 6×10^{25} as given by Brennan and Lomasky). Such a victory, of an overall 50.1% of the popular vote, is still wafer-thin, and usually too close for public-opinion polls to call accurately in advance. Elections as close as this are quite rare.

Under the assumption that all voters in a state s have the same value of P_A, called P_{As}, we examine a two-stage election in which

(a) P_{As} is the same for each s (i.e. equals P_A)

(b) P_{As} differs for different s.

The orthodox method of calculating the probability of being decisive assumes that P_A is known. We are in effect sampling from an infinite super population, out of which we are drawing a sample of n, equal to the number of voters. All n toss a coin which comes down heads with probability P_A, and if so, vote for A; if not, they vote for B. This gives rise to a very small variance for $\hat{P}_A$ which declines as n increases. Since $\hat{P}_A$ yields an extraordinarily accurate measure of P_A for large n, the distribution of $\hat{P}_A$ is very highly-peaked.

2. *A two-stage election model*

We use a stylised two-stage model of the electoral process. Assume that there are only two candidates. Assume that the candidate who wins the popular vote in a state wins all the electoral college votes in that state. For simplicity, assume that all states except California are of equal size (2 million voters), and that California is four times as big (8 million voters). Without further loss of generality, we can now assume that each state has one electoral college vote and that California has four (all four of whom vote the same way). There are therefore 53 voters, comprising 4 from California and 49 from the other States. This is one of the easiest practical ways of considering states of different sizes. We shall calculate the probability that a voter in California (and in other states) is decisive.

Case 1. $P_A = 0.5$ (exactly) for all voters in each state.

In the first instance, assume that P_A ($=P_B$) $= 0.5000$ (exactly) in all 49 states. For California to be decisive for A, the vote for A prior to California must be 23-26 or 24-25, and California votes 4-0 for A. What is the probability of this occurring?

Using the normal approximation, the probability is calculated to be 0.216.[1]

[1] What is the probability of being between 22½ and 24½ out of 49? In this case n = 49, p = 0.5, q = 0.5, so np = 24.5 and npq = 12.25. Thus $\sqrt{npq}$ = 3.5, so we require the probability of being between z $= -\frac{4}{7}\left(=\frac{22.5-24.5}{3.5}\right)$ and z = 0 for the standard normal distribution, which is 0.216, or about 1 in 5.

The probability of being decisive in 8 million rather than 100 million peaks at p = 0.5, at 1 in about 3545. Thus there is a one in 16,400 probability of being decisive overall. ($1/3545$ times .216). Errors of approximation aside, if p = 0.5 for all voters, there is thus little or no difference in decisiveness between a single stage and a two stage election, which Brennan and Lomasky (Table 4.1, page 57) give as 1 in 12,533, following Beck's method correctly. This is intuitively obvious. If one tosses a coin many million times, all tosses are equivalent if p = 0.5, so it does not matter to decisiveness if the tosses are partially aggregated or not. That is, after any number of votes have been counted, the expected value of the difference between the two candidates will always be zero. It does not matter if there are 50 parcels of votes, one from each of the states, or whether there is one parcel of votes covering all the states, when the last critical vote is added to the pile.

Case 2. P_A is the same for all voters in all states, but $P_A \neq 0.5$

Now assume that P_A varies slightly from 0.5000, and is the same for all voters in all states. Assume that $P_A = 0.5001$ for every voter. The probability of a voter being decisive in California (for the state only) falls only marginally from 1 in 3545 when $P_A = 0.5000$ to 1 in 4160 when $P_A = 0.5001$. However, if $P_A = 0.5001$ for all voters in all states, and if there are 2 million voters in each other states, the probability that B wins any state (other than California) is .00234 or 0.234%.

Then what is the probability that B wins 22½ to 24½ states? This is given by the expression

$\sum_{x=23}^{24} \binom{50}{x} (.0023)^x (.9977)^{50-x}$ and is approximated by the probability that x lies between 22½ and 24½ when np = 50 x .00234 = .117 and $\sqrt{npq} = 0.34$. That is, the expected number of states that B will win is only 0.117, with a standard deviation of 0.34 states. The probability that B could win more than 22 states is essentially zero (since it is more than 65 standard deviations from the mean), so the probability that California could be decisive is also essentially zero. This is counter intuitive. It happens because P_A is estimated with extraordinary high accuracy.

Case 3. P_A is the same for all voters within a state, but varies between states.

In practice, P_{AS} will differ from state to state. In no state is it likely that P_A lies between 0.4999 and 0.5001. We can get a rough order of magnitude of the probability that P_{AS} will be between 0.4999 and 0.5001, by noting that in moderately close contests for the Presidency usually less than 10 out of 50 states have had P_{AS} lie between 0.49 and 0.51. Since results are likely to be uniformly distributed in this range, the probability that any one such state (whose P_{AS} lies between 0.49 and 0.51) lies between 0.4999 and 0.5001 is one in a thousand, so for 10 states the probability that one of them lies between 0.4999 and 0.5001 is one in a hundred. It could therefore be expected that a result as close as this will occur once in a hundred Presidential elections, or once every 400 years - and this is clearly an overestimate, since generally fewer states have a P_{AS} lying between 0.49 and 0.51.

For all practical purposes, therefore, if P_{AS} exceeds 0.5 then A will win the state with certainty, and will lose it otherwise. Suppose then, that the *overall* vote for A for the 49 states excluding California is 50.01%, but that the value of P_{AS} is never really close to 0.50 (and continue to assume that the states all have 2 million who vote). This means that A is likely to win about half the states. It may be, of course, that A wins eight states each with $P_{AS} = 0.6$ and loses 41 states, each with $P_{AS} = 0.481$, to give an overall average vote of just over 50%. The reverse is also true (A loses eight states with $P_{AS} = 0.4$ ($P_{BS}= 0.6$) and wins forty-one states with $P_{AS} = 0.519$ ($P_{BS} = 0.481$). Therefore the probability that California can be decisive in the electoral college in this example is the probability of A having won 23 or 24 out of 49 when P_{AS} lies between about 0.4 and 0.6 for each of the 49 states, never lies so close to 0.5 in any one state that the result in that state is in doubt, and where the overall proportion of votes won by A is 50.01%.

In order to provide orders of magnitude for this occurrence, suppose that P_{AS} (the probability of an *individual* voting for A in the state) is rectangularly - distributed between states and can take values 0.40, 0.41, ..., 0.49, 0.51, 0.52, ..., 0.60. That is, it takes integer values from 40% to 60%, with the exception of 50%. We wish to discover what the distribution of states voting for A will be, on the assumption that all 49 states (except California) are of equal size, and given that the overall vote for A is 50.01%.

We proceed as follows. For a rectangular distribution on (0,1), the variance is $\frac{1}{12}$, so for the numbers 51 to 60, (average 55.5) the variance is ${}^{9^2}\!/_{12}$, and the standard deviation is ${}^{9}\!/_{\sqrt{12}}$. Let U_1 be the rectangularly - distributed winning margins for A (in % terms) 1, 2, ..., 10, and let U_2 be the rectangularly - distributed losing margins for A (in % terms) -1, -2, ...-10. We further suppose that when the overall vote for A is 50.01%, it can be approximated by 50.00%. So if A wins n states and loses m states, we require (to satisfy the overall requirement of A getting 50% of the total vote), that

$$U = \sum_{i=1}^{n} U_{1i} + \sum_{j=1}^{m} U_{2j} = 0 \qquad (1)$$

We also require n + m = 49 (2)

We seek the distribution of (n - m), given (1).

The variance of (1) is $\frac{81}{12}$ x 49, (since there are 49 independent states) and the associated standard deviation is 18.19. If A wins 24 states and loses 25, U has an expected value of - 5.5, and a standard deviation of 18.19.

$P(U = 0|n = 24)$, the probability that A will win just 50% of the popular vote when it wins 24 states, is the probability that U will equal 0, that is, that the probability that a distribution with mean -5.5 and S.D. of 18.19 will equal 0. This probability is proportional to the value of the frequency distribution, which is 0.3811. Similar calculations may be done for A winning 23, 22, ... seats, as shown in the Table. We put the constant of proportionality equal to one, as it will cancel out in equation (3) below.

Value of n	$P(U = 0 \mid n)$
24	.3811
23	.2644
22	.1274
21	.0426
20	.0099
19	.0016
18	.0002

We now use Bayes' Theorem, because what we want is the distribution of the number of A's winning states, given that A wins a tiny majority of the overall popular vote.

That is, we want

$$P(n = 24|U = 0) = \frac{0.5\, P(n=24).\, P(U=0 \mid n=24)}{P(n=24).\, P(U=0 \mid n=24) + P(n=23).\, P(U=0 \mid n=23) + \ldots\ldots} \quad (3)$$

where U = 0 is the occurrence that A wins a tiny majority of the popular vote. The factor 0.5 on the numerator of (3) is to account for the denominator not including any probabilities where A wins.

The probability that A wins exactly 24 states out of 49 when the probability of doing so in any one state is 0.5, is given by the appropriate binomial probability and approximated by the normal, with mean 24.5 and standard deviation of 3.5.

Hence $P(n=24) = P(-\tfrac{2}{7} < z < 0)$ where z is the standard normal variate

$= 0.1124$

We repeat this calculation for other values of n, as given in the Table below.

Value of n	P (n)
24	.1124
23	.1038
22	.0881
21	.0692
20	.0499
19	.0334
Rest (less than 19)	.0432

Combining the above two tables, as specified in equation (3) above, we get the following table.

Value of n	P $(n \mid U=0)$
24	.252
23	.162
22	.066
21	.018
20	.003
Total	.501

The probability sums to 0.500 (apart from rounding error), the other 0.500 coming from the cases where A wins rather than loses, the problem being a symmetrical one. From this, the probability that A will have 23 or 24 States, and that California will be decisive, is $0.252 + 0.162 = 0.414$.

Since this probability is independent of a Californian voter being decisive within California, the probability that a Californian voter will be decisive overall when the popular vote is 50.01% for A is thus $\frac{1}{4160} \times 0.414 = \frac{1}{10{,}048}$ (=.0000995).

This compares with $\frac{1}{92608}$ for the single electorate and 0 for the two-stage process where $P_A = .5001$ for all voters everywhere. The difference occurs because it is a two-stage process, and because P_{AS} is not uniform between states. (Nevertheless, the result is still suspect because in practice the true value of P_A is not known, as assumed here.)

3. *Extensions*

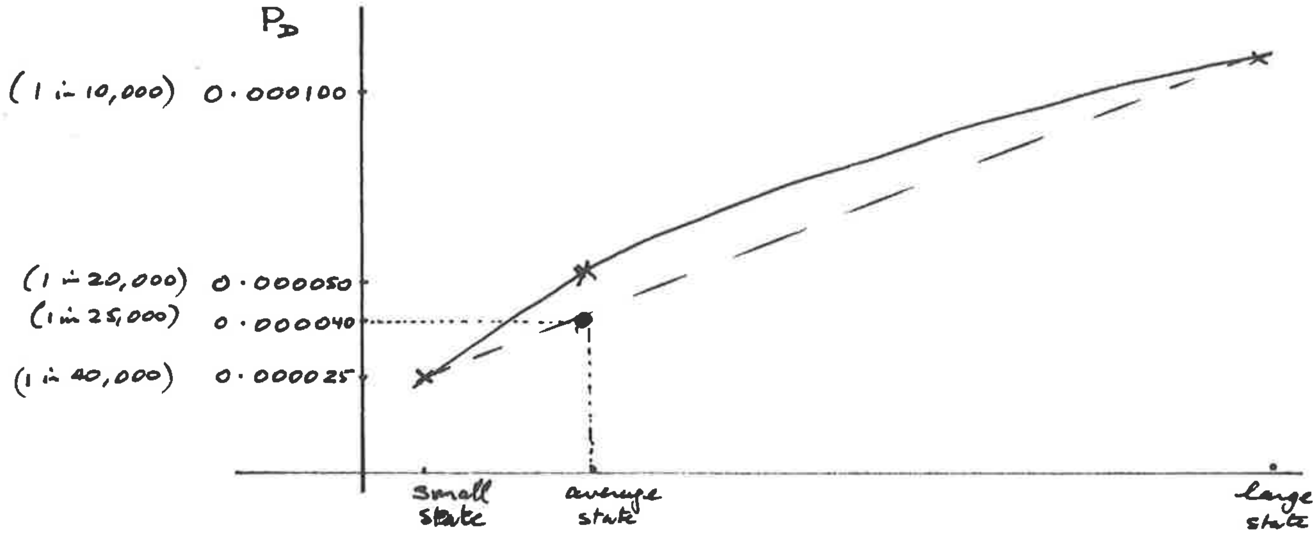
We are now in a position to determine the approximate size of the probability that a voter in one of the 49 smaller states will be decisive. Using Beck's method (see equation 1 of paper 11), the probability of a voter being decisive within a state is inversely proportional to $n^{½}$, where n is the number of voters in the state. The probability of a *voter* being decisive *within* the state is about double that of California (the square root of the ratio of the number of voters in California to the number of voters in another state). When $P_A = 0.5$, it equals 1 in 1845.

The probability that a *state* will be decisive is inversely proportional to its population. That is, the probability that one of the 49 *states* will be decisive is approximately ¼ of that of California (the ratio of number of voters in a state other than California to the number of voters in California). Multiply these two probabilities (probability that a voter is decisive within the state times probability that the state will be decisive). This is thus (2 x ¼) times the probability that a Californian voter is decisive, or about 1 in 20,000.

If we now relax the assumption that all states apart from California have an equal number of electoral college votes, and assume that the smallest have about one-quarter of the electoral college votes of the average state, we use similar arguments to show that for these states, the probability of being decisive is about half as much as that for the average state, or about 1 in 40,000.

Overall, therefore, the "average" voter in the USA has a small but not absolutely minuscule probability of being decisive when the overall vote is likely to be close, and where that state itself is also likely to be close. From the sketch diagram below, it is clear that the probability of the average voter (P_{ave}) being decisive will be a little less than that of the voter in the average-sized state, or in round figures 1 in 25,000. (To reiterate, this is when the election is likely to be close overall, and when the election is likely to be close within that state, and when P_A is assumed to be known for each state.)

All the probabilities for a voter being decisive in a two-stage election are in the range 1 in 10,000, to 1 in 40,000 when the election in a State is likely to be close and when the overall election is likely to be close.



Obviously, these are the highest estimates obtainable, given the assumptions, because what is being assumed is that (a) the state vote is extremely close and

(b) the overall popular vote for candidate A is very close to 50%

It is possible for a state to be decisive overall, but for the result within the state to be an easy win for one of the two candidates. If this is the case for all states, then the result within the electoral college may be very close indeed, but without any single voter in any state having any chance of being decisive in that state.

4. *Conclusions*

Using the orthodox method, the value of P_D in a U.S. Presidential election is remote — it is infinitesmal. However, where the prior forecasts of the vote show that it could be close, both within the state and overall, risk averse voters would be entitled to believe that the overall probability of being decisive could lie in the region of 1 in 10,000 to 1 in 40,000, depending on the size of the state. *Ceteris paribus*, voters in large states are more likely to be decisive than those in small ones.

Candidates must put their largest efforts into those states which are marginal, but within this category of states there is therefore good reason for candidates to concentrate to an even greater extent on larger states. States which are not likely to be marginal should not be targeted, whatever their size.

For an individual voter, the implications are that if he or she lives in a state where one candidate is virtually certain to win, there is no prospect that such a voter can influence the result. Furthermore, if the result in a state is likely to be very close but overall one candidate is almost certain to win (whatever happens in the state) again the voter cannot influence the result.

Compared with a one-stage process (i.e. election of the President on the aggregate popular vote), the two stage process gives less voters even a moderate chance of being decisive. Only where both the state result is likely to be close *and* the electoral college result is likely to be close, electors in such states will face somewhat higher values of P_D (1 in 10,000 to 1 in 40,000) than those of the whole electorate faced with single-stage voting (1 in 92,000 for $p = 0.5001$). However, it is possible, even likely, that even when the electoral college is extremely close, none of the results within a state is even remotely close, so no single voter can be regarded as decisive.

When the election is converted into a two-stage election using an electoral college, the value of P_D either increases a little, if both the state and overall results are perceived as being very close, or declines to essentially zero, if either the state *or* overall results are not perceived as being close.

Therefore, when Brennan and Lomasky's work on the mechanics of being decisive is expanded, in the case of the US Presidential election, there are mixed results. For the most part, P_D is essentially zero for most voters under both the single stage and the two stage electoral systems. The exceptions are for voters in states where the result is likely to be close in an election where the overall result is also in doubt. For such voters, P_D is still low, but not negligible.

As will be shown in paper 13, the orthodox method is quite misleading. If it is taken at face value, candidates should put no effort whatsoever in any state except those which are absolute cliff-hangers. Most of the other conclusions based on the orthodox method are also wrong. The next paper hopefully restores some sanity to the arguments.

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PAPER 13

The Probability of a Tied Vote for the U.S. Presidency: A Reformulated Calculation

1. Introduction

The previous paper (paper 12) used Beck's (1975) "orthodox" formulation for the probability of being decisive (P_D) in a two-stage election such as that of the U.S. President. However, paper 11 has shown that the methodology used is flawed. Consequently the conclusions that may be drawn from using the orthodox methodology can often be badly astray. This paper looks at the difficulties a voter has when trying to decide his or her value of P_D, given that P_D is subjective, and will change with changing circumstances.

2. A two-stage election model

This part of the analysis follows that of paper 12: it is assumed that there are 49 states, each with two million voters, in addition to California, with 8 million. All states have one electoral college vote, except California, which has four. We assume that in each state, an opinion poll of 400 voters has been taken. In practice the poll size may be different from this, but that merely adds complication and detail, and does not illuminate the method. For greater realism, and to test the sensitivity of the model, a number of assumptions, including poll size, can be changed. A poll size of 400 in each state yields 20,000 over the whole country, which is quite a large opinion poll. However, 400 in a state is not particularly large, and it may be that in some states a large number of people have been polled. The figure of 400, however, is not meant as an *actual* sample size, but an *equivalent* sample size, to reflect the fact that there are non-sampling errors present in the data. It is the size of sample which with zero non-sampling errors, would have the same accuracy as an opinion poll sample which contains all kinds of other errors as well as sampling errors. In these terms, 400 is probably a very generous size; quite possibly a sample of as few as 100, free of non-sampling errors, could give a prediction of the same accuracy as a poll of some 2,000.

Case 1: $\hat{P}_A$ = 0.5 for all States

Assume that in all states, the opinion poll shows that presidential candidate A gets exactly 200 out of the 400 sampled (and the other candidate, B, the other 200). As in case 1 in paper 12, for California to be decisive for A, the vote for A (in terms of the states for and against) must be 23-26 or 24-25, and the four electoral college votes from California are won by A. The probability of that, as calculated in paper 12, is 0.216.

What is the probability that a voter in California is decisive for California? It is the probability that each candidate in California obtains exactly four million votes. If a voter uses RESP (rational expectations subjective probability) then n = 400, $p = ½$, $np = 200$, $npq = 100$, $\sqrt{npq} = 10$. $\sqrt{npq}$ is $^1/_{20}$ times np, so that the S.D. of our estimate of 4,000,000 votes is 200,000 votes. Thus the probability of the two candidates being within one vote within California is given by $\frac{1}{\sqrt{2\pi}} \frac{1}{200{,}000}$ = 0.19947×10^{-5} , or 1 in 501,000. (The probability in all other states is 0.8×10^{-5}, or 1 in 125,000). The probability of a voter in California being decisive overall is the product of the two probabilities (0.216 and 0.19947 x 10^{-5}) or 0.431 x 10^{-6}, or 1 in 2.3 million.

This is a much smaller probability than is obtained using the orthodox calculation, namely 1 in 16,400.

For all other states, the (reformulated) probability is also about 1 in 2.3 million (each other state has ¼ the probability of California of being decisive, but voters in each state other than California have four times the chance of being decisive within the state).

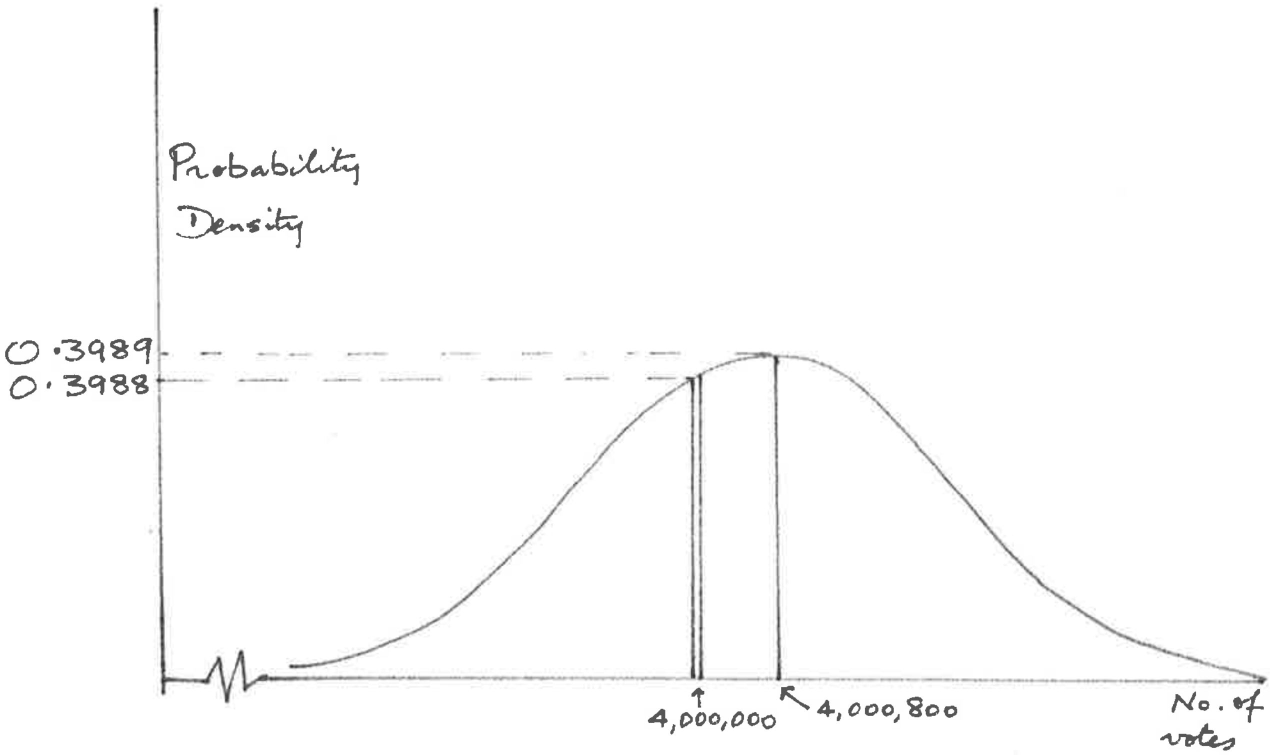
Let us also compare this with the value of P_D from a single stage election, given that the voter's complete information is a sample of 400 in each state. How accurately do we know the overall $\hat{P}_A$? We have a sample stratified by state. For each of 49 states, an estimated 1,000,000 vote for A, with a S.D. of 50,000 ie. a variance of 25 x 10^8. For California, an estimated 4,000,000 vote for A, with a S.D. of 200,000, ie. a variance of 4 x 10^{10}. The estimated total of 53,000,000 votes therefore has a variance of 1625 x 10^8, which gives a S.D. of 403,000. This is 0.76% of 53 million. The probability that a Californian voter will be decisive overall, in a one stage election, *is the same as that of any other state*, viz $\frac{0.399}{403{,}000} = 0.9897 \times 10^{-6}$, or a little less than 1 in 1 million.

Thus P_D for the single stage election is about double that of the two-stage election, given the assumptions made.

Case 2: $\hat{P}_A$ is the same for all voters in all States, but $\hat{P}_A \neq 0.500$

As in paper 12, assume $\hat{P}_A = 0.5001$ for each state. The probability of a voter in California being decisive for California is found by asking the question: if there are 400 voters in sample, and 200.04 of them (!) vote for A, what is the probability that exactly 4,000,000 out of 8,000,000 will vote for A?

If we multiply up the sample and its standard deviation of 10 to the size of the population, we get an estimated 4,000,800 out of 8 million, with a standard deviation of 200,000.



The vote for A is estimated to be $\dfrac{800}{200{,}000}$ = .004 standard deviations from the decisive vote of 4 million.

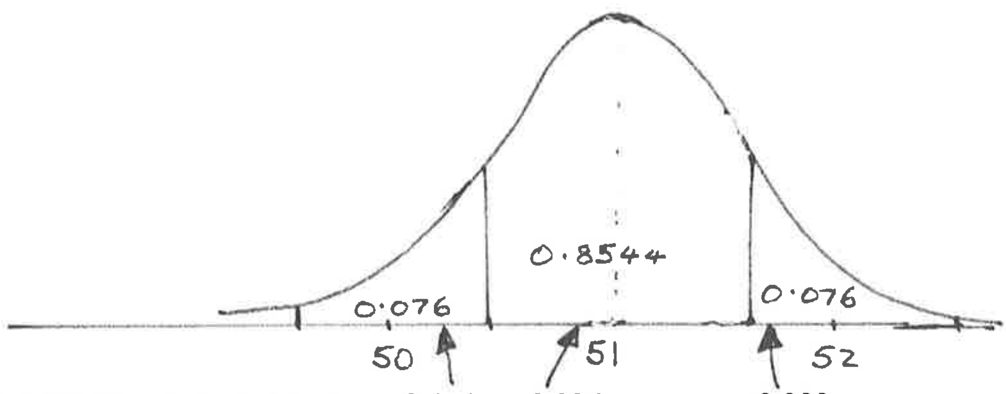
Thus the probability of a voter being decisive within California is $0.3988 \times \frac{1}{200{,}000} =$ 0.1994×10^{-5}, basically the same as that of case 1. The probability that A will win California is 0.5016, the same probability of winning as for any other state. Thus, of all other 49 states, A will win on average 24.58, leaving B with 24.42 on average. The standard deviation of the number of states won by A is still 3.5, and the probability that A will win 23 or 24 states, so that California is decisive, is 0.215.

The overall probability of a Californian voter being decisive is thus 0.428×10^{-6}, or again, 1 in 2.3 million. The figure is insignificantly different from case 1. Note, however, in the orthodox case, the probability goes from 0.6×10^{-4} in case 1 to infinitesimal in case 2.

Case 3: P_A is the same for all voters within a State, but varies between States

We again make use of the probability, calculated for case 3 in paper 12, that either 23 or 24 (of 49) states are won by A, given that the overall vote for A is 50%, and that there are equal probabilities that P_A can take exactly the values 0.40, 0.41, ..., 0.49, 0.51, 0.52, ..., 0.60 for the 49 states being considered. This was calculated to be 0.414, which is therefore the probability that California will be decisive.

We also need to calculate the probability that either 23 or 24 (of 49) states will be won by A, given that the overall vote for A is 51% and given that it is 52%. This is done in the appendix. When the overall vote is 51%, the probability that A gets 23 or 24 seats is 0.096, and when it is 52%, the probability is 0.008. Again, we recognise this as the probability of California being decisive. Note that we do not have to assume any knowledge about the outcome in any particular state to arrive at these probabilities. If the result is based on opinion polls of 400 per state, (for 49 states) the S.D. will be of the order of 350,000 votes, out of a total vote of some 49 to 50 million, or 0.7% error. Thus, when a Californian votes in a Presidential election where the polls suggest that A will have 51% of the overall vote, there is a reasonably high probability that California will be decisive. This could be worked out by weighing the probabilities of being decisive at small intervals in the overall vote by the probability of being within each of these intervals, as given by the polls.



Prob (California is decisive): 0.414 0.096 0.008

We may roughly approximate the answer by considering the three data points available. At $P_A = 0.50$, P (California is decisive) = 0.414, and this is associated with 7.6% of the probability from the distribution N ($\mu = 0.51$, S.D. = 0.0035). From the diagram, note that the amount of probability between 50.5 and 51.5 is 0.8544. The amounts of probability between 49.5 and 50.5, and between 51.5 and 52.5 are 0.076

each. There is thus a 7.6% probability of being associated with P (California decisive) of 0.414, an 85.44% probability of being associated with P (California decisive) of 0.046, and 7.6% probability of being associated with P(California decisive) of 0.008. Multiplying these probabilities gives California an 11.4% probability of being decisive.

If $\hat{P}_A$ = 0.50, this probability will rise to perhaps 25%, while if it is 0.52, it will fall to perhaps some 2%. These are no more than indicative, and it is pointless to be any more precise. The point is that the probability of California being decisive is moderately high for quite a range of values of $\hat{P}_A$.

Now for the voter within California.

(1) If $\hat{P}_A$ (California) = 0.5000 (from the sample of 400), then

P_D (California) = 0.199×10^{-5} (from Case 1)

so we may now multiply this by 0.25, when $\hat{P}_A$ (other states) = 0.5 to get

$\therefore$ P_D (California voter decisive overall | $\hat{P}_A$ (other states) = 0.50)

$$= 0.25 \times 0.199 \times 10^{-5}$$

$$= 0.5 \times 10^{-6}\text{, or 1 in 2 million ...(*)}$$

P_D (California voter decisive overall| $\hat{P}_A$ (other states) = 0.51)

$$= 0.114 \times 0.199 \times 10^{-5}$$

$$= 0.23 \times 10^{-6} \text{ or 1 in 4 million ...(*)}$$

P_D (California voter decisive overall| $\hat{P}_A$ (other states) = 0.52)

$$= 0.02 \times 0.2 \times 10^{-5}$$

$$= 0.4 \times 10^{-7}\text{, or 1 in 25 million ...(*)}$$

(2) If $\hat{P}_A$ (California) = 0.51, then

P_D (within California) $= .923 \times 0.199 \times 10^{-5}$

$= 0.184 \times 10^{-5}$

(Calculation: $\hat{P}_A$ (California) is 0.4 standard deviations from 50%, and the ratio of the heights of the normal curve at 0.4 and 0.0 S.D. is .3683 to .3989, or .923).

Thus P_D (California voter decisive overall) $= 0.46 \times 10^{-6}$, $\hat{P}_A$ (other states) = 0.5

$= 0.21 \times 10^{-6}$, $\hat{P}_A$ (other states) = 0.51

$= 0.04 \times 10^{-6}$, $\hat{P}_A$ (other states) = 0.52

If $\hat{P}_A$ (California) = 0.52, the values of P_D are 0.726 times those of (*), that is, in the range 1 in 2.5 million to 1 in 35 million.

If $\hat{P}_A$ (California) = 0.55, the values of P_D are 0.135 times those of (*), that is, in the range 1 in 15 million to 1 in 200 million.

Realistically, these are probably about as close as anyone will get to a measurement of voter decisiveness for the U.S. President. The figures are order-of-magnitude estimates of the value of P_D for a Californian voter. The results are summarised in the table below.

Table 1

Inverse of the Probability that a voter in California will be decisive in the U.S. Presidential election. *Values are in millions.*

		$\hat{P}_A$ (other states)		
		0.50	0.51	0.52
	0.50	2.0	4.0	25
$\hat{P}_A$ (California)	0.51	2.2	4.4	27
	0.52	2.8	5.5	35
	0.55	15	30	200

3. Conclusions

The table above gives probabilities of a voter in California being decisive, over a range of possibilities in a fairly close presidential election, from 1 in 2 million to 1 in 200 million.

These figures are fairly sensitive to how accurately a RESP voter can judge the election aggregated over other states. The assumption here is that the accumulated knowledge is equivalent to knowing about the votes of a random sample of 20,000 voters. This is probably an overestimate, and if so, the figures in Table 1 should be amended accordingly. A less-accurately known estimate of $\hat{P}_A$ (other states) would lower the value of the largest probability (top left hand corner of the above table) and raise that at the bottom left of the table. The net effect would, I suggest, give values in the table ranging from about 1 in 5 million to 1 in 50 million of being decisive. These are small numbers but not infinitesimal. They are quite different from those calculated by the orthodox method. The orthodox method is completely misleading in its conclusions.

Finally, why California? Why not choose the probability of a voter being decisive in a more "average" state? There are two reasons. I started with a stylised system in which to examine the difference in decisiveness between two different sizes of states. It made sense to take the case of California (the state with the largest population) and the rest, as this was simplest example of two different sizes of state. Second, there's a story about people in California going out to vote when the outcome of the election is already known from the East Coast results. So Californian voters have a special place in the pantheon of American voters.

APPENDIX

In paper 12 ("The Probability of Being Decisive in a U.S. Presidential Election I"), the value of the probability that California would be decisive, given that the overall value of P_A averaged over the other 49 states (the "popular vote") was 0.5000, but where the P_A for each state was different, was worked out. That paper had no need to calculate the value of the same probability in cases where the popular vote for the other 49 states took values other than 0.5000, but this paper does. We calculate the probability that California would be decisive, given that the popular vote for the other 49 states takes in turn the values of 0.51 and 0.52. The method is similar to that of paper 12.

Case 1: The popular vote for A for the other 49 states is 51%. Assume that P_A for these states is distributed with percentage votes for A given with equal probability as 41, 42, ..., 49, and 51, 52, ..., 61. This divides the population into separate losing and winning states each with a rectangular distribution, and omitting the 50% level for convenience. The numbers are shifted up by one percentage point from the case delineated in paper 12. The average of the distribution taken as a whole is 51.05. By subtracting 50 from each observation, we define $U_1 = [-9, -8, ..., -1]$ and $U_2 = [1, 2, ..., 11]$.

Suppose that the 49 states divide so that n= 24 of them are randomly assigned as winners, to the U_2 distribution (51, ..., 61) and m = 25 of them as losers, randomly assigned to the U_1 distribution (41, ..., 49). The expected value of $U = \sum_{i=1}^{25} U_{1i} + \sum_{i=1}^{24} U_{2i}$ is thus 25 x (-5) + 24 x 6 = 19, and its standard deviation is $\sqrt{(\frac{8^2}{12} . 25) + (\frac{10^2}{12} . 24)}$ = 18.26. Since the popular vote is 51%, this implies that the *actual* value of U is 49 (ie, the 49 states, each one percentage point above 50). $P(U = 49| n = 24)$ is thus the probability that a normal distribution with mean of 19 and S.D. of 18.26 equals 49.

The height of the density function is 0.1072 at this point, so $P(U = 49| n = 24)$ is proportional to this probability. Without loss of generality, we put the constant of proportionality equal to one, as this will be cancelled in the analysis which follows.

We now find $P(U = 49| n)$, for other values of *n*, and tabulate the results below. Unlike the symmetrical case dealt with in paper 12, it is necessary to consider values of *n* above 24 as well as below it.

Value of *n*	$P(U = 49\| n)$
32	0.0030
31	0.0159
30	0.0603
29	0.1603
28	0.2996
27	0.3933
26	0.3633
25	0.2355
24	0.1072
23	0.0343
22	0.0077
21	0.0012

What we require, however, is the posterior probability, $P(n = 24 | U = 49)$; that is, we require to find the probability that A wins 24 states when the popular vote for A is 51% over the 49 states, and the winning and losing margins are those described by the rectangular distributions U_2 and U_1 respectively.

As for paper 12, this expression may be derived by Bayes' Theorem as follows:

$$P(n = 24 | U = 49) = \frac{P(n=24).P(U=49| n=24)}{\sum_{n=0}^{49} P(n).P(U=49| n)}$$

To use this formula, we need to find the value of $P(n)$. To illustrate, $P(n = 24)$ is the probability that A wins 24 seats out of 49 when $P_A = 0.51$ and $N = 49$.

$$NP_A = 49 \times 0.51 = 24.99$$

$$\sqrt{NP_A\ q_A} = 3.4993$$

ie. we assume that this is a normal distribution with mean of 25 and a S.D. of 3.5. The probability that n lies between 23.5 and 24.5 is thus the probability that the standard normal deviate lies between $-^3/_7$ and $-^1/_7$, or 0.1092.

The following table gives the requisite values, symmetrical about $n = 25$

Value of n	$P(n)$
26	0.1092
25	0.1134
24	0.1092
23	0.0881
22	0.0692
21	0.0499
20	0.0334

Thus, using Bayes' formula, $P(n = 24 \mid U = 49) = 0.075$

Similarly, $P(n = 23 \mid U = 49) = 0.021$.

The sum of these gives the probability that California is decisive, namely 0.096.

Case 2: The popular vote for A for the other 49 states is 52%. We proceed as for case 1, except that the values of P_A for each state are now assumed to be 42, 43, ..., 49 and 51, ..., 62, with the corresponding $U_1 = (-8, -7, \dots -1)$

and $U_2 = (1, 2, \dots, 12)$.

The expected value of U is now 42.5, with a standard deviation of 19.4, and the actual value of U is 98.

We now find $P(U = 98 \mid n)$; this is tabulated below, together with the values of $P(n)$.

Value of n	$P(U = 98 \mid n)$	$P(n)$
30	0.2047	0.0499
29	0.3346	0.0692
28	0.3984	0.0081
27	0.3443	0.1038
26	0.0982	0.1124
25	0.0325	0.1124
24	0.0077	0.1038
23	0.0013	0.0881
22	0.0002	0.0692

Hence by Bayes' Theorem, $P(n = 24 \mid U = 98) = 0.0065$ and $P(n = 23 \mid U = 98) = 0.0009$.

The probability that California is decisive, when $\hat{P}_A = 0.52$ in other states, is the sum of these two probabilities, namely 0.0074.

PAPER 14

Strange Bedfellows:

A critical review of

Democracy and decision - the pure theory of electoral preference
by G. Brennan and L. Lomasky

Brennan and Lomasky have sought to show that when people vote, their reasons for doing so are primarily to express support for ideology, good government and for the supply of those goods which they think governments ought to provide.

It takes them 225 pages of tightly-argued reasoning to reach a conclusion which many political scientists would regard as obvious, and thoroughly demonstrated. Even within the public choice literature, from which their book arose, there has been a long tradition that the rational voter is unlikely to believe that his or her vote will be the one to decide which of two candidates will be elected. Essentially, therefore, rational voters will cast a vote only if the satisfaction gained from the actual *casting* of the vote exceeds the cost of doing so.

The reason for the book, it seems to me, is that what may be obvious to the overwhelming majority of political scientists, may not be quite so obvious to public choice theorists.

Public choice theory has one overriding hypothesis: political processes can be explained by the self-interest of those involved in political life. This is not to say that other things are of no influence: rather, that (at least for the purpose of public choice argument) they can be ignored, to see how far the main hypothesis will lead. Such a narrow focus is at once both a strength and a weakness. The strength lies in the simplicity of the hypothesis, and the belief that a good deal of observed behaviour can be explained by it. The weakness is that the richness of institutional background, personal interaction, and the myriad of other factors which impinge on the political process are conveniently forgotten. The problem is that what is forgotten for the convenience of simplicity sometimes gets forgotten *altogether* by the same theorists. None, if pressed, would ever say their theories were a complete explanation of the world. Yet it seems that there is a tendency in most of us who indulge in formal modelling to be neglectful of those aspects of reality which are not included in the formal model.

So this book is really addressed to public choice theorists, or at least, the "straw man" among public choice theorists, who is unable to accept that voters may vote for reasons other than those of their narrow self-interest.

The starting-point for the book is the observation that so many people vote, even when the result is beyond doubt. And in large electorates, even if the result is likely to be close, the probability that it will be won by a single vote, so that any individual voter can be decisive as to who is elected, is still quite small. If the only reason that people vote is narrow self-interest, then (according to the more narrow theories) they will do so only if the probability that they will determine who is elected, multiplied by the benefit they gain from this, exceeds the cost of voting. Brennan and Lomasky calculate the probability of an individual affecting the outcome of the vote, and show that in large electorates it is usually infinitesimal. It is therefore usually irrational to vote, if narrow self-interest is all that matters. As Brennan and Lomasky relate (page 35), although considerations other than the expectation of being decisive lead people to the polling station, it has been assumed by public choice theorists that once they are there these considerations play no further part in the voting decision, which thus proceeds in a self-interested way. Brennan and Lomasky's point of departure from such orthodoxy is to dispute this: they assume that such considerations colour the actual voting patterns. By doing so, voters are said to be acting "expressively", that is, giving expressions of support for their preferred ideologies and their favoured causes. Their ballot papers thus reflect a demonstration of voters' desires for good and stable government and other long term considerations, not all of which need be in sympathy with their narrowly-defined short-term interests. Sometimes, their expressive reasons for voting may lead to their vote being different from that based purely on instrumental considerations.[1]

Some interesting corollaries follow: if many voters vote for their longer-term interests, or what they perceive to be in the community's interests rather than their own narrow pecuniary interests, there is likely to be far greater stability in political systems than is implied by public choice theorems such as McKelvey's (1976) result. In this theorem, *any* point in policy space can in theory be reached by a sequence of moves, each of which is approved by a majority of voters. A second corollary is that it is likely that the state, acting paternalistically, will spend more public money on merit goods (such as education) than the median voter would vote for if he or she were decisive. It is suggested that because voters know they are not decisive, they can indulge themselves by voting for those policies and actions which make them feel good about themselves. By voting for a merit good, a voter does not act with the intention of bringing about its provision directly; however, if many voters vote in this way, the effect will be to provide it without any single voter having been *decisive* in the process.

Now while all of this is sensible, it nonetheless flies in the face of narrowly-conceived public choice orthodoxy, and puts Brennan and Lomasky in bed with those more flexible political observers who have already arrived rather more pragmatically at the same conclusions. The

[1] By "instrumental" is meant that the vote will make the difference between losing and winning (or tying), that is, that the voter is "decisive".

authors in essence provide this body of opinion with a more sound theoretical backbone than it has hitherto possessed.

However, it seems to me that Brennan and Lomasky's own framework is also peculiarly narrow. Like many converts to new causes, they are in danger of being excessively enthusiastic about their new position. Their strength and their weakness once again stem from their public choice background, which provides them with a central theme: if people are rational in going out to vote, it must be for expressive reasons, not because they expect to alter the outcome. This theme is pursued relentlessly and narrowly throughout the book: this is their strength. But it is also their weakness, as it acts also as a strait jacket, preventing them from following perhaps more prosaic and sensible directions, as I shall attempt to show.

(1) *The probability of being decisive, P_D, is not as peaked as Brennan and Lomasky suggest.*

Brennan and Lomasky follow the "orthodox" tradition of Beck (1975) and others in calculating P_D[2]. There are two problems with this. The first is that they have calculated many of the orthodox probabilities incorrectly.

I reproduce their Table 4.1 on page 57 of their book, and the correct orthodox inverse probabilities in a table below it for comparison. The figure of 56, in the top left hand corner of their Table 4.1, means that when $P_A = 0.5$, that is, the probability that each voter will vote for

[2] They are aware of Owen and Grofman's (1984) contribution, which notes that if everyone thought it was not worth the effort to go and vote, then very few would. This in turn would raise the value of P_D, so that an equilibrium would be attained at what may be a relatively low turnout. Voters would choose to vote with a certain probability less than one, which, if chosen by all voters, would lead to mutually consistent behaviour. Nevertheless, Owen and Grofman recognise that the value of P_D (calculated by the orthodox method) in most elections, where typically some 40% to 80% of the electorate votes, is very small indeed. Brennan and Lomasky are also aware of Palfrey and Rosenthal's (1984) contribution, where "teams" of voters vie to produce a "public good", which is achieved by the election of a candidate. However, Brennan and Lomasky say that this approach would suggest an equilibrium where *everyone* votes, which would again imply a very low value for P_D.

candidate A with probability 0.5 exactly, then there is one in 56 chance that the winning margin will be exactly one vote, for an electorate of size 2001. The top figure in the next column, also 56, means that if each voter votes for A with probability 0.5001, the probability that the winning margin will be exactly one vote, in an electorate of 2001, will still be 1 in 56.

Table 4.1

(Brennan and Lomasky)*

The orthodox inverse probability of being decisive

	Value of P_A			
Electoral size	**0.5000**	**0.5001**	**0.5010**	**0.5100**
101				
2001	56	56	56	56
20,001	177	177	179	481
200,001	560	566	619	12.3×10^6
10 million	4,000	6,533	60,000	∞
100 million	12,500	1.9×10^6	6×10^{25}	∞

*Brennan and Lomasky refer to $t = P_A - 0.5$ instead of referring to P_A. i.e. when $P_A = 0.5$, $t = 0$; when $P_A = 0.5001$, $t = 0.0001$, etc.

Table 4.1 (corrected)

The orthodox inverse probability of being decisive

	Value of P_A			
Electoral size	**0.5000**	**0.5001**	**0.501**	**0.51**
101	18	18	18	18
2001	56	56	56	84
20,000	177	177	184	9685
200,000	560	562	835	1.33×10^{20}
10 million	3963	4841	1.9×10^{12}	∞
100 million	12,533	92,608	9×10^{90}	∞

Note also that on page 57, the formula for V given in (4.7) is incorrect.

For small electorates, as P_A (the probability with which all voters are assumed to vote for A) diverges slightly from 0.50, say to 0.51, the probability that a single voter will be decisive does not decline appreciably, but there is a very rapid decline in decisiveness as P_A diverges from 0.50 in large electorates.

The second and far more serious problem is that the orthodox probabilities themselves are completely inappropriate. As was shown in paper 11, they are based on regarding the *whole electorate* as a sample from an infinite super-population, where each voter has a *known* probability of voting for A, P_A, where P_A is apparently equal to the proportion of the votes A actually receives. For a start, the result of the election has to be assumed in order to calculate P_A, and if so, then all voters should be considered as capable of knowing who wins when they vote. Thus they will know whether they themselves would be decisive — the probability must either be 0 or 1. It cannot be any number in between.

As was shown in paper 11, if P_D is to mean anything, it must be an *ex ante* concept and must therefore be based on subjective probabilities.

The problem with subjective probabilities is simply that they are subjective, and may vary greatly and without apparent reason, from person to person. To inject some objectivity into the debate, we consider a voter who is in possession of all available information about the likely outcome of the election and can efficiently process that information. This voter is said to have "rational expectations subjective probability" (RESP) for P_A, and hence will form a RESP value of P_D.

The upshot is that when values of P_D are calculated in this way, making some reasonable assumptions abut the accuracy with which the RESP voter will know P_A, then P_D is no longer so peaked near $P_A = 0.5$, nor so small away from $P_A = 0.5$. In papers 12 and 13, the probability of a voter being decisive in elections for the U.S. President is calculated. In the orthodox view, if P_A = 0.5 for all voters in all states, P_D lies between 1 in 10,000 and 1 in 40,000 depending on the voter's state, but if P_A departs for 0.5 only marginally, say to $P_A = 0.5001$ (i.e. the candidate gets 50.01% of the popular vote) there is essentially zero probability that any voter will be decisive in any state.

However, when the reformulated method is used, P_D is almost unchanged at about 1 in 2 million, whether $\hat{P}_A = 0.5000$ or 0.5001. Calculated by this method, P_D declines to about 1 in 200 million if one of the candidates is expected to win 55% of the popular vote. While these numbers are low, they are not of the order of less than 10^{-100}, as calculated by the orthodox method. Brennan and Lomasky, who use the orthodox calculation for P_D, therefore significantly underestimate its value for most values of P_A.

When they turn to the literature on electoral turnout, they would therefore expect to see very few voting except where P_A is expected to be *extremely* close to 0.5. Their argument is that P_D falls off very rapidly away from $P_A = 0.5$, and so, therefore, should turnout. This they do not find, so they are led to conclude that people must be voting expressively. We must suspend judgement on whether their argument is still valid until the turnout figures in a range of published papers are re-worked against the reformulated values of P_D. It seems likely that small decreases in turnout as P_A departs from 0.5 may accord reasonably well with P_D values as newly calculated.

(2) *People are not as rational as Brennan and Lomasky believe.*

Those who have read as far as this will by now no doubt be aghast at the technical turn in the review. The reviewer, it seems, has been seduced into arguing within the same narrow framework as Brennan and Lomasky have employed. Yes, but that has been for the purpose of showing that, *on their own terms*, Brennan and Lomasky's work must be queried.

It is now time to broaden the base of the discussion a little, to argue that people by and large do not rationally form subjective probabilities of who will win. Indeed, it is probably alien for people to form subjective probabilities in any explicit sense at all. Perhaps a handful of the statistically-minded should be excepted from this statement, but that is all. In paper 11 we show that people's subjective probabilities are likely to be systematically warped by their party allegiance, but that is only one tangible manifestation of a whole iceberg of irrationality, apathy and ignorance.

Most people, it would seem, have simply not questioned whether their own vote matters, and it is an extravagance to ascribe motives of instrumentality or expression to their behaviour. As Aldrich (1993) points out, voting is a low cost, low benefit activity, and as such may be greatly affected by whim or other trivial circumstance.

From indirect evidence, it would seem that many voters' subjective probability of being decisive could well be much higher than the rational expectation subjective probability. The indirect evidence comes largely from the study of behavioural decision making. Kahneman and Tversky's (1979) Prospect Theory is predicated partly on people not being able to visualise the smallness of very small probabilities, and therefore overestimating them. This is sufficient to explain what might otherwise be regarded as irrational behaviour, such as participating in lotteries, where the sum of the value of the prizes is less than the sum of the value of the tickets, and often substantially less. Other psychologists who have examined how well calibrated people are, when faced with decisions, also have demonstrated persistent cognitive biases in subjects in all walks of life (Lichtenstein et al, 1982). For example, if I predict that there is a 30% chance of rain on the following day, on a number of days over a 1000 day period, there would need to be rain on 30% of such occasions if I were to be well-calibrated. If there is rain on 80%, or on only 8%, of such occasions, I would almost certainly be poorly-calibrated. Studies over widely varying circumstances show that most people are poorly-calibrated. If they think they are "99% certain", they are on average only about 90% certain about a particular occurrence.

The point is that people are poor judges of events with very low or very high probabilities. Generally speaking, they grossly overestimate the probability of low-probability events and they are far more certain about high-probability events than they are entitled to be. Translated to the problem at hand it means that people may believe that they have quite a high probability of being decisive.

(3) *Brennan and Lomasky's use of "instrumental" as synonymous with "self-interest" is inappropriate.*

As I stated earlier, public choice is above all about the pursuit of self interest amongst those involved in politics. This applies to voting as well as other political actions. There are many facets to voter self-interest. At one extreme, all actions can tautologically be said to be based on self-interest, so that even acts of so-called "altruism" give rise to enough inner satisfaction to give positive utility to the donor.

Let us confine ourselves to those acts which make the voter *tangibly* better-off, so as to exclude altruism, at least in its purest form. The act of voting advances this type of self-interest in a number of ways. There is the possibility of being decisive. But clearly there is more to it than that for most voters. The first reason concerns the size of the majority. Brennan and Lomasky take it for granted that the size of the majority is of no concern. Any majority greater than one does not alter the result. While that is strictly true of the candidate, it will not in general be true of the candidate's policies. A candidate who wins one election comfortably, but who then loses some support, to win by only one vote at the next election, is likely to try to jettison those policies or circumstances which have led to this loss of popularity, for fear of a more complete failure at a third election. In safe seats, the size of a candidate's majority is a signal to both winners and losers, and is seen by voters and candidates in this light. I vote for party X, not because it gives me a better deal in this election, but because I believe that in the long-run it will give me a better deal. If I, and those like me, fail to support this party in the short-run, there is less chance it will be in a position to win in the longer-run. We are voting ideologically, which, by Brennan and Lomasky's reasoning, is expressive. Judged from a slightly broader perspective, however, it is no longer clear that such voters are simply expressive voters: they may be instrumental in a wider sense, either about policies, or intertemporally.

It is often important for supporters of likely losers, to vote for them in order to make the loss as small as possible. (Similarly, it is also often important for supporters of candidates expected to win by a large margin, to vote for them in order to make the winning margin as large as possible.) A party which loses a seat by a smaller margin usually will have a greater chance of winning it next time. A very poor result will cause voters for that party to lose hope and enthusiasm, and perhaps to transfer their allegiances to third parties, or not bother to vote next time. Voters for a likely loser may also vote in order to give respectability to the overall aggregate vote of the party. Such voters are in essence signalling to the rest of the electorate or to the state that their party is a force to be reckoned with, and worth voting for at the next, and subsequent, elections. If this process is considered as a whole, a vote for A at this election also represents a discounted vote for A (or for A's party) at subsequent elections. In this very broad sense, therefore, one may regard the vote as having several instrumental components: the main one for the present election, but also other smaller components for future elections.

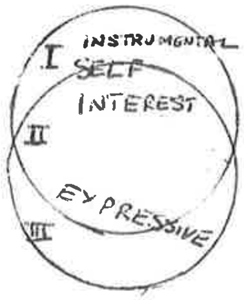
So if we consider the set of all present and future elections, the meaning of instrumentality is a little more blurred. More importantly, however, all the reasons encompassed in this discussion which lead a person to vote are manifestly for the same candidate or party: they all reflect self-interest (in the narrower sense in which I have defined it to exclude altruism). Therefore they all accord with the mainstream of public choice theory, which gives a central role to self-interest. That is, none of these self-interest reasons for voting are likely to be mutually contradictory.

Brennan and Lomasky will have none of this. To them, a vote is instrumental only if it changes the result of the *present* election. Everything else is expressive. Even if the voters' best interests are tied up not only in who wins this election, but also in who wins future elections, their reasons will only count as an instrumental reason for voting in Brennan and Lomasky's calculus, it if relates to the present election alone.

Brennan and Lomasky's propositions make most sense in a world in which there is no time dimension, and in which there is only one election. There is no on-going political party whose future could be considered when voting; there is no sense of voting to uphold the tradition of democracy. In this world, the voter's self-interest (as seen by the public choice theorist) is simply determined by the result of the present election: to be instrumental.

But just as in game theory, where a one-shot Prisoners' Dilemma game may have a different set of optimal strategies from one played an infinite number of times, so too does adding a time dimension to the voting problem alter the conclusions. "Self-interest" is no longer synonymous with instrumentality: now it will include on-going elements. Of course there is more to a voter's life than being instrumental: in a four-dimensional world, both public choice and political science recognise it. It is no wonder that Brennan and Lomasky are in bed with the political scientists: the bed they abandoned contained only a straw man!

The following diagram may help to clarify these matters.



Area 1: Instrumental reasons for voting, (as defined by Brennan and Lomasky). This is the area recognised by public choice theorists in a single election where there are no future elections.

Areas I & II Self-interest reasons for voting.

These include instrumental reasons, but also on-going reasons for supporting a party, and the institution of democracy. They will usually not lead to a different vote from that cast by a simple instrumental voter. These areas are ones which should be recognised by public choice theorists in a set of elections through time.

Areas II & III Expressive reasons for voting.

These are all reasons other than instrumental reasons. Those in area III may or may not lead to a voter casting the same vote as those in area I, or areas I and II.

On top of all this, for many voters the diagram should appear embedded in a mirage, where the boundaries shimmer and are always ill defined. (I am inclined to believe that for other voters the diagram should be embedded in a mirage viewed on a dark night.)

(4) The occasions on which expressive and instrumental reasons for voting lead in opposite directions are likely to be few.

Brennan and Lomasky use a good deal of space demonstrating the possibility that expressive and instrumental reasons for voting may lead to different results. Where the instrumental and expressive reasons for voting coincide, there is no conundrum. It is where they do not that interesting paradoxes may occur. This is most succinctly expressed by Brennan and Lomasky themselves:

> "Individuals who recognise that their own likelihood of being decisive is fairly small, and who stand to accrue large expressive gains by voting against their interests, are strongly induced to vote for the outcome the emergence of which they do not prefer". (Brennan and Lomasky, page 139).

For those who recognise that they are likely to be decisive, expressive reasons which are orthogonal to their interests may well lower the cost of voting to zero or even a negative value. In this case, once they are in the polling-booth, they have no reason to vote other than instrumentally.

Examples of this are people who think they ought to vote, for reasons of civic duty, or for whom voting is a form of entertainment. It is unlikely that people who go to the polling-booth for these reasons have any conflict between their reasons for casting their vote and their reasons for supporting a particular candidate. On the question of voting as civic duty or as entertainment, from my observation of voters on polling day, it appears that voting is a significant event in many people's lives, which are often lived out in front of a T.V. set. To many, the act of participation is important, whether it be participation on a live stage (as opposed to the passive TV stage) or as a civic duty.

A further reason for voting, related to its cost, involves the extent to which someone badgers the potential voter to get him or her to the polling station. A potential voter may find it easier to go to vote than to be pestered by family, friends, neighbours and/or party officials. This vote may well be an instrumental one, given that the coercion reduces the cost of such a vote to zero or below. From the above, it would appear that for many people, there is no positive cost to voting.

For those who do recognise that they are not likely to be decisive[3], and whose expressive reasons for voting are aligned positively with their instrumental reasons for voting, the subsequent vote is *less* likely to differ from a purely instrumental vote than it would be in the case just described, where there is no relationship between their expressive and instrumental reasons for voting. For those who do recognise that they are not likely to be decisive, whose expressive reasons, unrelated to their instrumental reasons, are not large enough on their own to lower the cost of voting sufficiently to get them to vote at all, and whose additional expressive reasons for voting are (a) collectively negatively aligned with their instrumental reasons, and (b) sufficiently strong, that alone or with the unrelated expressive reasons for voting, they will lower the cost to a level which will induce them to vote, *may* (if the negative alignment and the intensity of the reason are sufficiently great), not vote the same as their instrumental vote.

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[3] If readers find this paragraph obscure, they are meant to. It is designed to show that a number of conditions must be met before instrumental and expressive reasons lead to a different voting outcome.

This, then, is the extent of the conundrum. Of course there will be those voters for whom such a conundrum exists: among millions of voters, there will be some for whom expressive and instrumental reasons are negatively related, and who consequently are "induced to vote against their own interests". But given the string of the conditions for such a conundrum, the set of such voters is likely to be at best nearly empty, relative to the size of the whole electorate. The main examples given by Brennan and Lomasky where voters' instrumental and expressive reasons for voting differ are all extended Prisoners' Dilemmas e.g. the failure of any among many bystanders to report a crime. It would seem that this paradox of voting deserves a similar place in the pantheon of voting paradoxes as that accorded to Giffen goods in consumer theory. Giffen goods, which demonstrate the theoretical possibility of an upward-sloping demand curve (and are therefore well known from such theory), are thought to be so rare as not to exist in practice.

In all, Brennan and Lomasky have written a provocative but narrowly-based book. They base much of their argument on values of the probability of being decisive which have since been shown to be inappropriate, they do not consider any of the results of the behavioural decision-making school and few from the political science literature, their definition of "instrumental" is more restricted than what most, including many public choice theorists, would be likely to regard as reasonable, and as a result of all these things, appear to have elevated a minor conundrum into a major theory. This might put them in bed with the political scientists, but the kissing hasn't started yet. They're still in love with the true embodiment of Public Choice, whose straw-man caricature they have left in the other bed.

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POSTSCRIPT

Since this paper was drafted, the results of the experiments in papers 10 and 15 became available.

In paper 15, the results are clearcut: some people do vote expressively. These are the first experimental results to give an unequivocal answer to the question of whether expressive voting has been observed. In paper 10, it was noted that experimental subjects voted strategically when the third-ranked candidate (E) had a high primary vote but did not do so when E had a low primary vote. It was noted that a rational voter should either have voted strategically for E in both cases, or in neither case: it was not rational to change votes. This may be explained in terms of expressive voting, as follows. When E's primary vote is low, the voter believes E has little chance of winning, so does not vote for E. If E does win, however, the rational voter receives the same conditional payoffs as in the case where E's primary vote is higher. Suppose that the voter believes that the probability that E's primary vote (v_E) exceeds D's primary vote (v_D) is β, and that if so, then E will beat C with certainty. On the other hand, if $v_D > v_E$ (which occurs with probability $(1 - \beta)$) then assume that C wins with probability α and D with probability $(1 - \alpha)$. If the payoffs for C, D and E are 10, 0 and 8 respectively, then the expected payoff for voting for E is $8\beta + 10\alpha (1 - \beta) = \beta (8 - 10\alpha) + 10\alpha$.

If $\alpha < 0.8$, it is rational to vote for E, but the smaller is β, the less will be the gain from strategic voting. If β is judged to be negligible, the voter is effectively indifferent between C and E, and so other things (almost) equal, is quite happy to vote for C, the voter's sincere first preference. When this happens, the vote is thereby an expressive one.

These results therefore support Brennan and Lomasky's contention that expressive voting is an important phenomenon. It is really exquisite irony, given the contents of paper 14, that it is my own work that has supplied the experimental evidence to support their case. I imagine paper 14 must therefore undergo revision before it can be submitted for publication.

PAPER 15

A FURTHER EXPERIMENTAL STUDY OF EXPRESSIVE VOTING

1. Introduction

Motives for voting can be divided into two categories: instrumental and expressive. A vote which is instrumental has the capacity for altering the outcome of an election; that is, it is decisive. For a single voter to be instrumental, when all voters have a single vote, the difference between the electorate's two most preferred choices must be at most one vote. Since the probability that a vote will be decisive is usually small in an electorate of more than a handful of voters, it is argued that voters must have non-instrumental reasons for voting. Such reasons are called expressive. The terms instrumental and expressive may also be applied to a voter. Expressive reasons for voting may conceivably change the voter's preferences, as has been argued within the public choice literature by Tullock (1971), Brennan and Buchanan (1984), Brennan (1989), Brennan and Lomasky (1985, 1987, 1992), Brennan and Pincus (1987), Lee (1988) and Lomasky (1985). Outside this literature but within the political science mainstream, a large literature exists on the reasons for voter turnout (which should generally be quite low if voters are rational users of time and the probability of being instrumental is low). Such literature, summarised in a survey article by Aldrich (1993), covers the same arguments as the public choice literature, and more.

In a recent article, Carter and Guerette (1992) (hereafter called CG) report the results of an experiment to test the extent to which expressive voting occurs. Their results are inconclusive: the authors conclude that the results provide at best weak support for the hypothesis that the higher the probability of their being decisive, the more likely people will be to vote instrumentally.

In this paper, I present a somewhat different experiment whose results lend clear support to the expressive voting hypothesis. CG's problems with experimental design have been either sidestepped or attenuated, although other (arguably smaller) problems have occurred in their place. The experimental design links this experiment to free-riding experiments. A comparison of secret and non-secret voting is also made.

2. Experimental Design

Like CG's (1992) experiment, this experiment derives from Tullock's (1971) thought experiment, in which he could either donate \$100 directly to charity, or vote to be taxed \$100 for the same purpose. His cost is \$100 in the first case, but in the second case is \$100 only if he is decisive. Thus, the larger the electorate, the smaller the likelihood of being decisive, and the more likely he would be to vote for the tax.

This is clearly a testable proposition. Giving to charity represents a purely expressive motive for voting, since there is no personal gain it; keeping the money oneself represents a purely instrumental motive, as the only gain is personal. CG designed an experiment in which the probability of being decisive was altered, and the proportion who voted altruistically observed. In that experiment, each took part in a separate election in which theirs was the only vote. However, they were told that there was a pre-assigned probability of that vote being decisive. They were given many examples to ensure that they understood this unusual concept. In the election, two-thirds of all subjects had the choice of voting for \$6 for themselves, or \$2 for charity; the other third had the choice of voting for \$9 for themselves or \$2 for charity. When the probability of being decisive increased, it was hypothesised that voters would more often vote for cash for themselves rather than charity; and when the cash reward increased compared with the fixed amount going to charity, it was also hypothesised that voters would vote for cash more often.

2.1 Problems with Carter and Guerette's experiment

CG's experiment faced several problems. The first is that the subjects received more in cash (either $6 or $9) than was given to charity ($2), so the rational subject who wanted to give to charity would vote for cash and donate $2 (or more) of it to charity. Thus the experiment was more a matter of finding out whether subjects were rational rather than whether they were charitable.

The second problem was that there were very few observations where all the parameters were the same. Although there were 96 students in the whole experiment, they were divided into two groups, with 64 being asked to choose between $6 for self and $2 for charity, and 32 being asked to choose between $9 for self and $2 for charity. However, each of these two groups was subdivided into five further groups, each with a different probability of being decisive (namely 0, 0.01, 0.1, 0.2 and 1.0). There were only 6 observations in four of these groups, 8 in another, 11 or 12 in four other groups, and 18 in the largest group. There would therefore not be much discriminatory power between the alternatives in the hypotheses to be tested.

A third problem was that the difference between $6 cash/$2 charity and $9 cash/$2 charity was also small, so again there was little discrimination possible.

A fourth problem was that different subjects were used for each observation, so the method of paired comparisons could not be used.

2.2 The Design

The experiment I have undertaken is inherently much simpler. It was undertaken using undergraduate students studying intermediate microeconomics at the University of Adelaide. I was able to conduct the first part of the experiment in a lecture class of about 140 students, not all of whom went to every lecture. In the event there were 107 voters, but only 106 put their names on the ballot paper. One of the students, whose name was drawn at random after the experiment was completed, was to receive \$200, either to keep or to give to charity. Students took part in eight different "elections", called voting mechanisms. The voting in all cases was secret (or as nearly secret as is possible in a lecture theatre) but in four of eight mechanisms, the result of the ballot would be known. That is, the identity of the student receiving the money would be known and whether the student accepted the money or gave it to charity would also be known. In the other four mechanisms, the identity of the student whose name was drawn would be known, but what the student did with the money would not be known.

There were two hypotheses to be tested, the main one being hypothesis 1.

Hypothesis 1.

The incidence of expressive voting would increase as the probability of a voter being decisive became smaller.

Hypothesis 2

More people would vote altruistically (expressively) if voting were not secret.

In mechanism 1, if the student's name was chosen at random, the student's own vote (cast before the drawing of the name) would decide what the student did with the money. In mechanism 2, the vote of the whole class would, by simple majority, decide whether the student would keep the money or whether it would go to charity. So for mechanism 1, the probability of being instrumental was unity. For mechanism 2, the probability was clearly less than 1, but depended in each case on the subject's prior beliefs about how other subjects were likely to vote.[1]

[1] Using the orthodox (Beck (1975) formulation) for 106 voters, if each one had a probability of 0.5 voting for self (and therefore 1-0.5 = 0.5 of voting for charity) the probability that the vote would be tied at 53-53 would

For mechanism 3, students were told that, for the student chosen at random, the fate of the $200 would depend on the majority vote of the student's own tutorial class of between 14 and 17 students, with about 10 or 11 of these being expected to vote. The probability of being decisive for such a class should have been higher than that for mechanism 2, but the actual probability would once again be unknown.[2]

Mechanism 4 was similar to that of mechanism 3, but whether the student kept the $200 would depend on the majority vote of all tutorial groups *other than* the student's own. The student could still be decisive, but not for him/herself. If the vote (apart from student A) were tied and if student A voted for the money, it meant that the money would be received by a student *in a different tutorial*; if student A voted for charity, then the randomly-chosen student in *another tutorial* would be required to give the money to charity. So in effect, the student's decision would have the same effect on him or herself as if the probability of being decisive were zero.

For mechanisms 1 to 4, the result was to be non-secret. Mechanisms 5 to 8 were the same respectively as mechanisms 1 to 4, except that now the results were kept secret.

Only one of the eight mechanisms, to be determined at random, would be played for money.

As a result of this experimental design, three of the four problems faced by CG (see section 2.1 above) were avoided, and the fourth partly so. One, money to be paid out to the chosen student in this experiment, either in cash to the student, or to the charity,

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be given by $\binom{106}{53}(\frac{1}{2})^{53}(\frac{1}{2})^{53}$, which by the normal approximation is 0.08. However, as paper 11 shows, that formulation is inappropriate.

[2] According to Chamberlain and Rothschild (1981), if electorate R is k times the size of electorate Q, the probability of being decisive in Q is of the order of k times the probability of being decisive in R. That is, *ceteris paribus*, we would expect the probability of being decisive to be about 10 times as great in one of the ten tutorial classes as in the aggregate of these classes. The Beck (1975) formulation would give a maximum value of the probability of being decisive of about 0.24, but that formulation has been shown to be inappropriate.

would be the same in either case. Two, there were many more observations - 106 for comparison between mechanisms 1 and 2, and between mechanisms 5 and 6. (There were somewhat fewer observations for the other mechanisms as will be explained shortly, but always at least 82). Three, where CG made a distinction in cash payouts (either $6 or $9), no distinction was made in this experiment. Four, each student would vote eight times, so there were many more than 106 observations. (If all 106 students had voted all eight times, there would have been 8 x 106 = 848 observations, but for reasons that follow, the number was only 784, and in the main comparison, we have used 654 votes, based on the eight votes of 82 students. Effectively, this allows us to use paired comparisons with the common pool of 82).

The voting for mechanisms 3 and 4, and 7 and 8, where the tutorial class would decide the fate of the money by majority vote, was conducted in five of the ten tutorials, not in the lecture class. For the students in the remaining five tutorials, voting was conducted at the next lecture, in order to check whether there was a difference caused by students feeling greater bonds of togetherness in a tutorial of about a dozen people (there wasn't). Unfortunately, the pool of students voting for these four mechanisms was different. There were 91 voters, of whom 82 also voted for mechanisms 1, 2, 5 and 6. Thus there were 82 common voters for all eight mechanisms (106 common voters for mechanisms 1, 2, 5 and 6, and 91 common voters for mechanisms 3, 4, 7 and 8).

There was one further problem as a result of carrying out the experiment at two different times. In the intervening time, it was possible that some students decided to share the $200, and if so, to change their votes accordingly. To that end, a further set of questions asking all students if they had agreed to share the $200, in the event of receiving it, was asked. Three students said that they had done so. In only one case, the student went from voting the $200 to charity in mechanisms 2 and 6 to voting it to students in mechanisms 3, 4, 7 and 8.

The one problem faced by CG's experiment which was only partly solved in the experiment reported here, was that it was possible that a student who voted to give the money to charity would be asked to accept the money as a result of the majority vote of his or her lecture or tutorial class. Students were asked to imagine a situation where they would receive the money in front of the class (I did an enactment of handing over $200 in real money with one of the class). They then had to pre-commit themselves to say what they would do if faced with this situation, and to be bound by their pre-commitment. In the event, very few students changed their preferences when asked to precommit themselves.

One further problem introduced by the approach used in my experiment which was not present in CG's was that the probability of being instrumental or decisive was not known precisely by students, unless it was either zero or unity. All that could be inferred was that the probability would be lower in the large lecture class than in the small tutorial class. On the other hand, the scenario of people actually voting is probably a more realistic representation to students of real-world voting than being given a single vote and an exact probability of being decisive.

3. Results

The results refer to the 82 students who filled out responses to all eight mechanisms. Out of 82 x 8 = 656 possible responses, only two were missing, leaving 654 votes in the experiment.

Of the 82 responses:

- 42 voted for themselves or other students all eight times
- 20 voted for charity all eight times
- 20 voted differently on at least one of the eight times.

We shall concentrate on the last 20. Table 1 shows the numbers who voted for themselves and for charity under the eight different mechanisms.

Table 1: 20 swinging voters

	Not Secret		Secret	
	Self	Charity	Self	Charity
Own vote instrumental (mechanisms 1, 5)	18	2	17	3
Vote instrumental in tutorial: P (Instrumental) ≤ 0.24 (mechanisms 3 ,7)	13	7	15	5
Vote instrumental in lecture: P (Instrumental) ≤ 0.08 (mechanisms 2, 6)	12†	8	13† (one	6 missing*)
Vote instrumental in others' tutorials: as if P (instrumental)= 0 (mechanisms 4, 8)	6†	14	10† (one	9 missing*)

* The missing votes were oversights which should almost certainly have been added to the "self" entries.

† If the student who decided, between experiments, to share the $200 (and therefore voted for "self" for mechanisms 3, 4, 7 and 8,) had voted for charity in these cases, this would reduce the figures marked by † by one each, and increase the corresponding "charity" figure by one.

The results are quite clear, and show a monotonic decline in voting for one's own self-interest as the probability of being decisive (instrumental) declines from 1 to 0. The pattern of these votes was not random. Although there were mistakes and inconsistent choices amongst these 20 voters, mostly the individual ballots show a straight progression towards more charity when the probability of decisiveness declines. These results are consistent with hypothesis 1.

Comparing the results of the "charity" columns in Table 1, we see that between two and five voters are more likely to vote for their own interest when the vote is secret than when the result of their vote is known. These results are consistent with hypothesis 2, although the effect is reasonably small. Other things equal, this suggests that more would be given to charity and to public institutions if voting were not secret. Of course, this neglects the role of the secret ballot as a means of safe-guarding people's privacy and in allaying their fears (real or imagined) about coercion with respect to voting when it is not secret.

Those students who voted on only one of the two occasions, and who have not been included in these figures, exhibited very similar patterns of voting. Three tables are provided in the appendix to show the full data in tabular form, including the figures on precommitment, the actual numbers voting, and the percentage voting for their own monetary interests in all eight mechanisms. A second appendix shows instructions to participants and the form of the ballot papers.

4. Conclusion

This experiment clearly shows that the smaller the chance of people's being instrumental, the more they will vote expressively. More will also vote expressively if their vote is not secret. This paper has not attempted a statistical test of this proposition because the figures in Table 1 speak for themselves, and because the probability of being decisive is not known exactly in two of the four categories.

The experiment bears a close relationship to the public goods experiment conducted by Orbell *et al* (1988). In that experiment, some groups were told that their contributions would provide a public good, not for those in their own room, but for a similar group in another room. Although the payoff structures were identical in both treatments, co-operation was almost twice as high (and significantly so) when the public good accrued to subjects in one's own room.

As occurs in many experiments, when the hypothesis to be tested is stripped down to its bare essentials, the result becomes "obvious" and apparently trivial. Nevertheless, given Carter and Guerette's (1992) inconclusive results, this experiment provides the first conclusive direct evidence of the phenomenon of expressive voting. Given the amount that has been written on voter turnout and on expressive voting as a means of explaining turnout, this evidence will go some of the way towards confirming expressive voting theories. These theories can help to explain why ideology and notions of justice are important in democratic politics, and why democratic elections show far greater stability and resilience than some simpler electoral theories would suggest. (Brennan and Lomasky, 1993).

The results show how important is a good experimental design. This experiment was cheaper and simpler than Carter and Guerette's experiment, and gave more conclusive results. Nevertheless, it was not without its own problems, particularly those caused by conducting the experiment over two occasions.[3]

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[3] The simpler expedient of conducting the whole experiment on one occasion would have been less ambitious and, as it turns out, would probably have given even more conclusive results, as the number of observations would have increased from 82 to 106, and the problem that some students might decide to share the $200 if they received it would have been avoided.

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THE MICROECONOMICS II VOTING EXPERIMENT

You are given the following choice:

(1) accept $200, and not give it to charity
(2) accept $200, to give to charity.

One student in the class will be chosen at random, and will be paid the $200.

The decision to keep the money or to hand it to charity will be made by a vote. There are several different voting mechanisms, and you will be asked to vote using each mechanism in turn. ONE of the mechanisms will be used to determine which way the money will be allotted.

Mechanisms 1 to 4 involve non-secret voting, and mechanisms 5 to 8 are the same except that they are secret.

In mechanisms 1 to 4, the voting to keep the money or give it to charity will be decided by

(1) YOU only (mechanism 1)
(2) The whole lecture class (mechanism 2)
(3) Your own tutorial group (mechanism 3)
(4) Other tutorial groups, but not your own (mechanism 4).

Mechanism 1

If your name is selected at random, and if you have voted to keep the money, you keep it. If your name is selected at random and if you have voted to give the money to charity, it will go to the charity of your choice or if you do not specify the charity, it will go to Rwandan refugee relief.

Your identity will be known and so will your choice.

Mechanism 2

(1) You will vote on whether the student, whose name is drawn, will receive the money for him/herself on whether it goes to charity. If a majority of students in the whole class are in favour of the student receiving the money, this will happen. If a majority of students are in favour of the money going to charity, this will happen.

(2) If a majority vote in favour of a student receiving the money, the identity of the student will become known. Since the student may personally have voted to give the money to charity, he or she will then be given a further opportunity of donating the money to charity, and the result of this choice will become known. If you are the student chosen to receive $200 you are asked now to say what you would do when given the choice. (NOTE: This is a PRECOMMITMENT).

Mechanism 3

As for mechanism 2, except that the decision as to whether to give the money to the student or to donate to charity will be made by students in your tutorial by majority vote.

Mechanism 4

As for mechanism 3, except that now you will vote on whether a student in a different tutorial class (ie. a student in all tutorial classes other than your own) will receive the money or it goes to charity.

Mechanism 5

As for mechanism 1, but your choice will not be known, except to the research assistant of the experimenter.

Mechanism 6

As for mechanism 2(1). For mechanism 2(2), the student chosen at random will be given a further opportunity of donating the money to charity, but the result of this choice will not be known. If you are the student chosen to receive $200, you are asked now to say what you would do when given the choice, given that no-one except you and the research assistant will know whether you decide to keep the money or give it to charity.

Mechanism 7

As for mechanism 6, except that the decision will be made by the students in your tutorial.

Mechanism 8

As for mechanism 6, except that the decision will be made by students in other tutorial classes.

For those students in the Monday tutorials, you will answer the questions for mechanisms 3, 4, 7 and 8 in the lecture class; for those who are in Tuesday and Friday tutorials, you will answer questions 3, 4, 7 and 8 in your next tutorial class.

After you have all voted, the name of one student will be drawn at random, and one of mechanisms 1 to 8 will be selected at random. To be eligible for the money, the student must have voted in the election, and if the chosen mechanism involves mechanisms 3, 4, 7 or 8 (ie. involves a tutorial), you must have attended the tutorial in the week in which voting takes place. Otherwise the prize will go to charity.

Your name and student number is required on your voting slips for experimental purposes. It will be known only to the research assistant of the experimenter.

MICROECONOMICS II EXPERIMENT

Name: ....

Student Number: ....

Signature: ....

Mechanism 1. TICK ONE BOX (NOT SECRET) (YOU CHOOSE)

I ACCEPT $200 NOT TO GIVE TO CHARITY ☐
I ACCEPT $200, TO GIVE TO CHARITY ☐

Mechanism 2 (NOT SECRET) (CLASS CHOOSES)

(1) I ACCEPT $200, NOT TO GIVE TO CHARITY ☐
I ACCEPT $200, TO GIVE TO CHARITY ☐

(2) IF YOU RECEIVE THE $200, WILL YOU NOW:

ACCEPT IT FOR YOURSELF ☐
GIVE IT TO CHARITY ☐

Mechanism 5 (SECRET) (YOU CHOOSE)

I ACCEPT $200, NOT TO GIVE TO CHARITY ☐
I ACCEPT $200, TO GIVE TO CHARITY ☐

Mechanism 6 (SECRET) (CLASS CHOOSES)

(1) I ACCEPT $200 NOT TO GIVE TO CHARITY ☐
I ACCEPT $200, TO GIVE TO CHARITY ☐

(2) IF YOU RECEIVE THE $200, WILL YOU NOW:

ACCEPT IT FOR YOURSELF ☐
GIVE IT TO CHARITY ☐

CHARITY OF YOUR CHOICE: ....

MICROECONOMICS II EXPERIMENT(TUESDAY/FRIDAY) TUTES

Name: ....

Student Number: ....

Signature: ....

Mechanism 3 TICK ONE BOX (NOT SECRET) (TUTE CLASS CHOOSES)

(1) I ACCEPT $200 NOT TO GIVE TO CHARITY ☐
I ACCEPT $200, TO GIVE TO CHARITY ☐

(2) IF YOU RECEIVE THE $200, WILL YOU NOW:

ACCEPT IT FOR YOURSELF ☐
GIVE IT TO CHARITY ☐

Mechanism 4 (NOT SECRET) (ALL EXCEPT YOUR TUTE CHOOSES)

(1) I ACCEPT $200, NOT TO GIVE TO CHARITY ☐
I ACCEPT $200, TO GIVE TO CHARITY ☐

(2) IF YOU RECEIVE THE $200, WILL YOU NOW:

ACCEPT IT FOR YOURSELF ☐
GIVE IT TO CHARITY ☐

Mechanism7 (SECRET) (TUTE CLASS CHOOSES)

(1) I ACCEPT $200, NOT TO GIVE TO CHARITY ☐
I ACCEPT $200, TO GIVE TO CHARITY ☐

(2) IF YOU RECEIVE THE $200, WILL YOU NOW:

ACCEPT IT FOR YOURSELF ☐
GIVE IT TO CHARITY ☐

<u>Mechanism 8</u>

(SECRET)
(ALL EXCEPT YOUR TUTE CHOOSES)

(1) I ACCEPT $200 <u>NOT</u> TO GIVE TO CHARITY ☐
I ACCEPT $200, TO GIVE TO CHARITY ☐

(2) IF YOU RECEIVE THE $200, WILL YOU NOW:

ACCEPT IT FOR YOURSELF ☐
GIVE IT TO CHARITY ☐

NOMINATE CHARITY: ....

MICROECONOMICS II EXPERIMENT

Additional Information

This information is needed, because it may influence the results. The answers you give below will in no way influence whether you will receive, or whether a charity of your choice will receive, the $200.

If you receive the $200 and decide to, or are allowed to, keep the money yourself, have you made an agreement with any other student to share the money?

☐ yes

☐ no

If "yes", is the money to be shared

☐ equally?

☐ unequally?

If "equally", how many are sharing? ....
.... and how many of these are in <u>DIFFERENT</u> tutes to you? ....

If "unequally", please explain arrangement, and include in your answer how many are sharing, and if any are in a different tutorial to you ....
....

RESULTS OF EXPERIMENTS IN DETAIL

	Question Numbers																													
	same answers, all Qs		1		2(1)		2(2)		3(1)		3(2)		4(1)		4(2)		5		6(1)		6(2)		7(1)		7(2)		8(1)		8(2)	
PLACE OF EXPERIMENT	S*	C*	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C
L AND T*	41	20	19	2	13	8	16	4	14	7	15	5	7	14	8	11	18	3	14	6	17	2	16	5	16	5	11	9	11	9
L only	13	5	5	1	1	5	4	2	-	-	-	-	-	-	-	-	6	-	4	2	6	-	-	-	-	-	-	-	-	-
T only	3	3	-	-	-	-	-	-	2	1	2	1	1	2	1	2	-	-	-	-	-	-	2	1	2	1	2	1	1	2
Total for question: L	54	25	24	3	14	13	20	6									24	3	18	8	23	2								
Total for question: T	44	23							16	8	17	6	8	16	9	13							18	6	18	6	13	10	12	11
Overall: L AND T			60	22	54	28	57	24	55	27	56	25	48	34	49	31	59	23	55	26	58	22	57	25	57	25	52	29	52	29
Overall: L			18	6	14	10	17	7	-	-	-	-	-	-	-	-	19	5	17	7	19	5	-	-	-	-	-	-	-	-
Overall: T			-	-	-	-	-	-	5	4	5	4	4	5	4	5	-	-	-	-	-	-	5	4	5	4	5	4	4	5
Overall: All			78	28	68	38	74	31	60	31	61	29	52	37	53	36	78	28	72	33	77	27	62	29	62	29	57	33	56	34
Percentages % L AND T			73.2		65.9		70.4		62.1		67.1		69.1		58.5		59.8		72.0		67.9		72.5		69.5		69.5		64.2	
All			73.6		64.2		70.5		65.9		67.8		58.4		59.6		73.6		68.6		74.0		68.1		68.1		63.3		62.2	

* S = Vote to keep money for *Self*

* C = Vote to give money to *Charity*

* L = Lecture class (ie. mechanisms 1, 2, 5, 6)

* T = Tutorial class (ie. mechanisms 3, 4, 7, 8)

Part VII

On the Simultaneous Election of Representatives

As noted in the Introduction, this Part consists of one paper (Paper 16) of the same name. It examines an area of voting which has not had wide exposure. Most papers on multiple-seat elections are concerned with various sorts of proportional representation (PR) schemes. I do not know of any (other than this) which tries to compare other systems with the PR system.

I would be surprised if the section (at the end of the paper) on public good provision is completely correct. At the time the paper was written (1980) it was my first attempt to integrate this aspect into the literature on electoral systems.

The paper has some interesting results.

PAPER 16

Fischer, A.J. (1981) Simultaneous election of representatives.
Presented at: Tenth Conference of Economists, 24-28 August, Canberra, Australia

NOTE:
This publication is included on pages 264-291 in the print copy
of the thesis held in the University of Adelaide Library.