



ON THREE-DIMENSIONAL  
HYDRODYNAMIC NUMERICAL MODELLING  
OF WIND INDUCED FLOWS IN STABLY  
STRATIFIED WATERS: A GALERKIN-FINITE  
DIFFERENCE APPROACH

by

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# TABLE OF CONTENTS

SUMMARY . . . . .	i
SIGNED STATEMENT . . . . .	iii
ACKNOWLEDGEMENTS . . . . .	iv
1. INTRODUCTION . . . . .	1.
1.1 A survey of previous work . . . . .	1.
1.2 Introductory remarks on present works . . . . .	10.
2. LINEAR HYDRODYNAMIC EQUATIONS FOR MULTILAYERED SEAS AND THEIR NUMERICAL SOLUTIONS USING GALERKIN-FINITE DIFFERENCE METHODS . . . . .	16.
2.1 Linear hydrodynamic equations . . . . .	16.
2.2 The multilayered formulation . . . . .	18.
2.3 Surface, bottom and interfacial boundary conditions . . . . .	22.
2.4 Transformed equations and boundary conditions . . . . .	24.
2.4.1 Transformation to the $\sigma$ coordinate system . . . . .	24.
2.4.2 Transformed equations . . . . .	25.
2.4.3 Transformed boundary conditions . . . . .	26.
2.5 Solutions via the use of a Galerkin method applied over the vertical space domain: a basis set of <i>B</i> -splines . . . . .	27.
2.5.1 <i>B</i> -spline functions . . . . .	28.
2.5.2 One domain solutions . . . . .	31.

2.5.3 Two domain solutions . . . . .	37.
2.6 Solutions via the use of a Galerkin method applied over the vertical space domain:	
a basis set of eigenfunctions . . . . .	42.
2.6.1 One domain solutions . . . . .	43.
2.6.2 Two domain solutions . . . . .	51.
3. CONSTRUCTION OF EIGENFUNCTIONS . . . . .	56.
3.1 Introductory remarks . . . . .	56.
3.2 A multilayered eigenvalue system . . . . .	57.
3.3 Numerical determination of eigenfunctions . . . . .	59.
3.4 A projection method for estimates of modal composition . . . . .	62.
3.5 Vertical modes in one and two domain systems . . . . .	65.
3.5.1 Description of the system to be modelled . . . . .	65.
3.5.2 Vertical modes . . . . .	68.
4. WIND INDUCED MOTION IN HORIZONTALLY UNBOUNDED SEAS . . . . .	77.
4.1 Introductory remarks . . . . .	77.
4.2 Steady state solutions . . . . .	78.
4.3 Time-dependent wind induced motion . . . . .	91.
4.3.1 Description of the model . . . . .	91.
4.3.2 Integration with respect to time . . . . .	93.
4.3.3 Results . . . . .	99.
5. WIND INDUCED MOTION IN FLAT-BOTTOMED NARROW LAKES . . . . .	134.
5.1 Introductory remarks . . . . .	134.
5.2 Finite difference form of the Galerkin equations . . . . .	135.

5.3 Results . . . . .	140.
6. CONCLUSION . . . . .	165.
BIBLIOGRAPHY . . . . .	169.
APPENDIX I . . . . .	179.
APPENDIX II . . . . .	182.
APPENDIX III . . . . .	183.
APPENDIX IV . . . . .	197.

## SUMMARY

The work presented in this thesis is concerned with the development of a linear three-dimensional hydrodynamic numerical model for wind induced flow in stably stratified flat-bottomed lakes or seas. The perturbed motion is assumed small and deepening of the interfaces under wind action is ignored. The vertical dependence of horizontal currents is determined using Galerkin methods, whereas a finite difference method is used for the integration in time and the horizontal spatial coordinates. Two types of basis functions are taken into consideration: *B*-spline functions (Galerkin-finite element method) and numerically determined eigenfunctions (Galerkin-spectral method).

The proposed Galerkin models can accommodate an arbitrary variation in the vertical eddy viscosity within each layer. Across the interface the eddy viscosity profile can be chosen to be either continuous or discontinuous.

Two types of interfacial conditions are examined. The first is that the horizontal velocities and shear stresses are continuous across the interface and the second is that of zero-stress (used by Heaps, 1966, *Phil. Trans. Roy. Soc., Ser. A*, **259**, 391-416). The condition of continuous horizontal velocities and shear stresses requires only one set of basis functions, whereas the stress-free condition effectively decouples the system into two parts, and hence requires two independent sets of basis functions.

The model performances are demonstrated for the Ekman problems with stratification. Steady state and time dependent responses of an unbounded sea, subject to the impulsive onset of wind stress, are computed using a point model. For the study of inertial motion subject to the local wind stress, a two-layer model composed of the surface layer and the pycnocline is proposed.

The methods are also applied to investigate the transient response of an ide-

alised narrow lake of uniform depth subject to the impulsive onset of wind. Time-dependent behaviour of internal vertical displacements and their convergence rates are compared for the one and two domain systems.