

THE UNIVERSITY OF ADELAIDE

**Department of Mechanical Engineering** 

# DESIGN AND STRUCTURAL MODIFICATIONS OF VIBRATORY SYSTEMS TO ACHIEVE PRESCRIBED MODAL SPECTRA

PhD Dissertation

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# Abstract

This thesis concerns with problems associated with design and structural modification of vibratory systems. The aim is to meet prescribed modal and spectral requirement. Several common problems encountered in practical engineering applications are described, and novel strategies for solving these problems are then proposed. The mathematical formulations of these problems have been generated, and solution methods are developed.

The first problem concerns with developing a systematic approach for design of conservative vibratory systems with prescribed natural frequencies. Since, in general, this problem has more design parameters (namely the independent elements of the mass and stiffness matrices) than number of design constraints (i.e. the number of natural frequencies to be assigned), it has a family of solutions. We are only interested in these solutions which are physically realisable, i.e. solutions which can be physically constructed. We thus assume that a physically realisable stiffness matrix of a system is known, and then calculate a realisable mass matrix, so that the desired natural frequencies are obtained.

A second problem concerns with a case where in addition to the prescribed natural frequencies, corresponding mass-normalised mode shapes are also specified. This problem is analysed for situations where all of the system's natural frequencies and mode shapes are specified, and also for the case when these frequencies and their associated mode shapes are only partially prescribed. When all of the natural frequencies and mass-normalised mode shapes are shapes are prescribed, the problem is overdetermined, i.e there are more constraints than

there are independent design parameters. In general, there are no physically realisable solutions for this case. Therefore, we formulate and solve an optimisation problem leading to an approximate solution which is optimal in a specified sense. A partial specification of natural frequencies and mode shapes may result in a problem which has no realisable solutions, a unique solution, or a family of solution. This depends on the ratio between the number of prescribed and a total number of natural modes for the system. We present a solution method which can cope with any of these cases.

The two remaining problems concern with determining the necessary structural modifications to an existing system, based on measured modal analysis data. Problem 3 deals with assignment of natural frequencies only, while in Problem 4 we assume that both the natural frequencies and the corresponding mass-normalised mode shapes are specified. In both of these problems, our aim is to determine the necessary modifications based on the measured test data only. Because modal analysis data only partially describe the system, we are unable to obtain exact solutions to either problem. We overcome the difficulties of the inherent truncation of the modal data by formulating and solving optimisation problems giving approximate solutions which are optimal in a specified sense.

The developed algorithms were numerically tested on arbitrarily chosen examples, and a simple experiment was designed and carried out to test the suitability of the generated theory in a practical application.

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D.D.Sivan

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# **Glossary of Principal Symbols**

п	-	number of degrees-of-freedom
i	2	index
j		index
ω <sub>i</sub>	-	i <sup>th</sup> natural frequency
Μ	-	mass matrix of a system
K	<del></del>	stiffness matrix of a system
Λ	-	eigenvalues matrix of a system
Φ	-	corresponding mass-normalised eigenvectors matrix
$\lambda_{i}$		ii <sup>th</sup> diagonal element of $\Lambda$ , $\lambda_i = \omega_i^2$
$\phi_{i}$	-	corresponding $i^{th}$ column vector of $\Phi$
$\Lambda^{\star}$	-	desired eigenvalues matrix
$\Phi^*$	-	desired mass-normalised eigenvectors matrix
λ <sub>i</sub> *	-	ii <sup>th</sup> diagonal element of $\Lambda^*$
$\phi_{i}^{*}$	-	corresponding $i^{th}$ column vector of $\Phi^*$
$\mathbf{m}_{i}$	-	mass of i <sup>th</sup> element
$\mathbf{k}_{ij}$	-	ij <sup>th</sup> element of K
$\mathbf{s}_{ij}$	-	the stiffness constant of a spring connecting mass i to mass j
т	-	number of truncated modes ( $m \le n$ always)
$\Lambda_1$	-	eigenvalue matrix containing first $m$ eigenvalues of $\Lambda$
$\Phi_1$		matrix containing first $m$ corresponding column vectors of $\Phi$
$\Lambda_1^*$	-	eigenvalue matrix containing first <i>m</i> eigenvalues of $\Lambda^*$
$\Phi_1^*$	-	matrix containing first <i>m</i> corresponding column vectors of $\Phi^*$
I <sub>n</sub>	-	Identity matrix of size n (suffix n can be any value)
$\Lambda_2$	-	eigenvalue matrix containing last $n$ - $m$ eigenvalues of $\Lambda$
$\Phi_2$	2	matrix containing corresponding <i>n</i> - <i>m</i> column vectors of $\Phi$
$\Delta M$	-	mass modification matrix
ΔK	-	stiffness modification matrix

Х

δm <sub>i</sub>	H.	modification to an i <sup>th</sup> mass element		
$\delta s_i$	<u>=</u> >	modification to an i <sup>th</sup> stiffness element		
$\mathbf{M}_{mod}$	Ξ.	a mass matrix of a modified system		
$\mathbf{K}_{mod}$	=:	a stiffness matrix of a modified system		
<b>B</b> <sub>ij</sub> <sup>(K)</sup>	¥?	a connectivity matrix describing the connections of spring s <sub>ii</sub>		
Уĸ	Ξ.	an augmented vector for stiffness sensitivity		
$\mathbf{F}_{\mathbf{K}}$	-	a stiffness sensitivity matrix		
S	-	a vector consisting of all independent spring constants $s_{ij}$		
$\mathbf{B}_{i}^{(M)}$	-	a mapping matrix for m <sub>i</sub> onto M		
У <sub>М</sub>	-	an augmented matrix for mass sensitivity		
m	-	a vector consisting of all independent mass elements		
$\mathbf{F}_{\mathbf{M}}$	ŭ.	a mass sensitivity matrix		
$\mathbf{k}_{i}$	i.	i <sup>th</sup> column vector of <b>K</b>		
X	-	an arbitrary symmetric matrix		
Y	-	an arbitrary symmetric matrix		
D		a diagonal matrix		
R	-	in sections 2, 8 and 9, a residual matrix defined by (2.15)		
	-	in section 4, a matrix defined by (4.27)		
		in section 5, a matrix defined by (5.6)		
		in section 7, a matrix defined by (7.19)		
Ψ	-	in sections 2, 8 and 9 is defined in (2.18) and (2.19)		
	Ξ	in section 3, a modal matrix of a modified system		
	inter Ter Ann	in section 5, defined by (5.39)		
$ar{f \Lambda}$		eigenvalues matrix which is an approximation to $\Lambda^{\star}$		
$ar{\Phi}$	-	in section 8, a corresponding mass-normalised eigenvectors matrix		
	÷	in section 9, an approximation to $\Phi^*$		
Α		in section 3 and 5, any real symmetric matrix		
	-	in section 5, a matrix defined by (5.23)		
	-	in section 6, a matrix defined by (6.23)		
$a_i$	-	$i^{th}$ column vector of <b>A</b>		
Ω	-	in section 3, an eigenvalue matrix of a modified system		
	-	in section 8, an eigenvalue matrix of (8.3)		
α		in section 4, a scalar constant defined by (4.3)		

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G <sub>i,j</sub>	-	scalar constants defined by (4.23)
F <sub>i</sub>	-	scalar constants defined by (4.24)
t	-	an iteration index
U	-	an orthonormal matrix
З	-	a residual error function
E	-	a residual error function defined by (4.35)
Q	-	an orthonormal matrix
$\hat{v_1}, \hat{v_2}$	-	two arbitrary vectors in example of Figure 5.1
<i>ΰ</i> ,*, ύ	2 =	two orthogonal vectors in example of Figure 5.1
$\bar{A}_1, \bar{A}$	Ā <sub>2</sub> -	the square of the lengths of the projection vectors in Figure 5.1
$\mathbf{G}_{(i)}$	-	a matrix defined by (5.30) and (5.43)
$\mathbf{z}_{i}$	-	a vector defined by (5.31) and (5.44)
E <sub>(i)</sub>	٠	a matrix defined by (5.32) and (5.45)
$\Phi_{(i)}$	-	a modal matrix defined by (5.39)
$\mathbf{E}_{\mathbf{X}}$	1). <del></del>	a known component of the IP Decomposition of $\mathbf{X}$
D <sub>x</sub>		a diagonal component of the IP Decomposition of $\mathbf{X}$
$\beta_{ij}$	-	known coefficients in $D_X$ defined in (7.1)
$\mathbf{F}$	Э.	in section 7, a matrix defined by (7.23)
	-	in section 8, a matrix defined by (8.4)
Ε	-	a matrix defined by (7.22)
G	-	in section 7,a matrix defined by (7.33)
	Ξ	in section 8, a matrix defined by (8.5)
$\mathbf{H}_{\mathbf{i}}$	-	a mapping matrix defined by (7.36)
$\mathbf{g}_{i}$	-	an $i^{th}$ column vector of G
Р	Ē	in section 7, a sensitivity matrix defined by (7.42)
	-	in section 9, a matrix defined by (9.3)
Т	-	a matrix defined by (9.4)
Η		a matrix defined by (9.5)
$\sigma_{i}$		i <sup>th</sup> singular value of any matrix
S	-	a diagonal matrix of singular values

In addition to above principal symbols, some symbols for local applications are introduced and defined in the appropriate sections of the text.



# Section 1

# **INTRODUCTION**

Design of mechanical systems for a specified range of static and dynamic requirements is a basic problem in Mechanical Engineering. However, while design of static systems is well established, design to meet the dynamic requirements is not yet fully developed. This thesis deals with several problems encountered in the design and analysis of dynamic vibratory systems. The common aim among these problems is to develop algorithms which would allow a systematic approach for the design of vibratory systems with prescribed natural frequencies and mode shapes.

A classical problem in vibration analysis is to determine spectral properties (namely the natural frequencies and mode shapes) of a system with known physical parameters (i.e. mass, stiffness and damping space functions), under specified excitation forces and for a given set of the initial and boundary conditions. Solution technique requires precise knowledge of all the above data as well as a set of the partial differential equations which

describe the motion of the system. In practice, however, these precise data and the description of motion is only available for a limited range of relatively simple systems.

For complex systems, where this information is not easily available, it is customary to develop a discrete analytical model which approximates the behaviour of a real system. In problems analysed in this thesis we only consider conservative analytical models (i.e Mass-Spring and Finite Element with no damping). This was done in order to simplify the analysis. We realise that some amount of damping is always present in physical systems, however for many mechanical systems its magnitude is relatively small and may be ignored.

Once a suitable model is selected, its physical parameters can be determined, and then the spectral properties can be evaluated. The requirements that the system has prescribed natural frequencies and mode shapes is ensured by the adjustment of the physical parameters in the model until the desired spectral properties are obtained. This is a "trial-and-error" process and it requires repeated computations of the spectral properties for each modification of the model. Furthermore, if the spectral requirements consist of multiple constraints, then currently there is no systematic method for determining the necessary modifications to physical parameters which improve one or more dynamic property without detrimental effects on any of the others. In this thesis we have developed systematic methods for obtaining the physical parameters of the system from the prescribed spectral data. Thus we have solved the classical vibration problem in "reverse", and consequently the class of problems that we have analysed is known as *inverse vibration problems*.

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When the necessary physical parameters for the selected model are determined, the design stage is completed. The next stage is the development process where a prototype of the system is built and is then tested experimentally. The measured spectral properties are then checked for compliance with the design specifications. A common method for measuring the dynamic behaviour of the mechanical systems is the modal analysis testing. If the results of modal analysis tests show that the system does not meet the design requirements, then some structural modifications are needed. At present the most common method for determining these modifications is a "trial-and-error" experimental process. The limitation of this approach is that it is costly, time consuming and does not cope well if multiple constraints are placed on the spectral properties. In this thesis we have developed a systematic methods for determining the necessary structural modifications based on the results of modal analysis tests. In these problems we determine the necessary structural modifications using only the physically measured spectral properties, and assume that the analytical model is not available. Therefore the error due to discrepancy between the dynamic behaviour of the selected analytical model and the actual physical system is removed from the calculations.

Using the measured modal analysis data presents several problems. Firstly, since some amount of damping is always present in the physical system, the measured modes will be complex. However, the survey of the relevant literature have shown that there have been several published papers dealing with the methods of extracting the real modes from the measured complex ones. Therefore, we assume that any one of the accepted methods (which are described in the section 3 of this thesis) may be applied prior to the application of our algorithms. Secondly, since modal analysis data do not contain a complete description of the system, there is insufficient information to find the exact modified parameters which yield the desired spectrum. The difficulties arising from the inherently truncated data provided by modal analysis were overcome by formulating suitable optimisation problems, which could then be solved.

A common requirement imposed on the solution to all problems studied in this thesis was that all determined parameters of the system should be *physically realisable*. This requirement resulted in the following two *physical realisability constraints*:

- 1) all determined masses and stiffnesses must be real and non-negative,
- 2) the shape of the obtained mass and stiffness matrices must comply with the requirements of the selected analytical model.

Failure to comply with the physical realisability constraint (1) would result in a system which can not be physically reconstructed. And failure to meet the constraint (2) would prevent the translation of the obtained mathematical solution into the real physical system. These constraints were dealt with separately for each problem studied, depending on the chosen analytical models and assumed known data.

Two different kinds of spectral requirements were examined. Initially we have examined cases where only natural frequencies assignments were sought. Then assignment of both the natural frequencies and the corresponding mode shapes was investigated. Application

of the two types of spectral constraints to both the design and development stages as described above resulted in the formulation of four distinct problems. Description and definition of these four problems is presented in section 2. Section 3 contains a review of background knowledge and literature survey. Analysis of these four problems is described in sections 4, 5, 8 and 9. In sections 6 and 7 we present solutions to some different variations of the problem described in section 5. Developed algorithms for the solution to the above problems were extensively tested on numerical examples, and the results from some selected few of these examples are presented in appropriate subsections for each problem. The practical application of the developed theory to a real physical structure was confirmed by designing and carrying out a simple experimental program, results of which are presented in section 10. Conclusions and summary of this work is given section 11. The relevant references are listed in section 12, and the raw measured data from the experiments are shown in the Appendices.

# Section 2

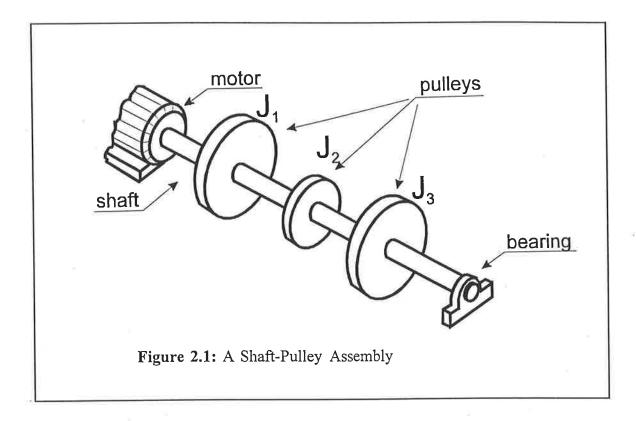
# PROBLEM

# DESCRIPTIONS AND DEFINITIONS

In this section we describe some common engineering design problems where spectral requirements are included in the design specifications. The current standard design procedures are discussed and some changes to these procedures are suggested. These changes result in the formulation of four distinct inverse vibration problems.

### 2.1 Problem 1: Design to Achieve Desired Natural Frequencies

Consider the shaft-pulley system shown in Figure 1. Design of such systems for a specified range of loads and power transmission requirements is one of the basic tasks in mechanical engineering.



Suppose that design specifications state the range of expected rotary speeds of the shaft, and hence restrict the allowed natural frequencies of the system. The current standard design approach for such a system is as follows:

## Current Design Process

*Input*: Total number of pulleys and their position on the shaft; Loads on the system and power transmission requirements; Bands of restricted frequencies.

## Design Procedure:

- Select materials and sizes for all pulleys (Note: this may be predetermined), and hence obtain an estimate of each pulley's mass.
- Select material for the shaft, and determine the required size(s) of the shaft to operate under the required loading conditions.
- 3. Select a suitable analytical model for the vibrational analysis of the system, and using information from steps 1 and 2 construct the mass and stiffness matrices of the system.
- 4. Using the mass and stiffness matrices obtained in step 3 calculate the natural frequencies for the torsional and transverse modes of vibration, and check that these frequencies do not fall within the restricted frequency ranges.
- 5. If natural frequencies do not fall into the restricted ranges, then design process is finished. If they do, then modify the mass and stiffness matrices and repeat from step 4.

Output (In theory): A shaft-pulley system which meets all design specifications.

It is clear that step 5 constitutes an open-ended iterative loop in the above procedure. If the natural frequency restrictions are relatively simple (e.g. that all natural frequencies must be above some specified value), then the above procedure would perform satisfactorily. However, if the design constraints are more complex (e.g. if it is necessary to interlace some or all of the natural frequencies in-between the restricted frequency bands), then the above procedure would not be adequate. This is because there exists no well established method for modifying the mass and stiffness matrices to produce the exactly desired

adjustments in the natural frequencies. Therefore, obtaining the desired natural frequencies by mass and stiffness modification would resort to a "trial-and-error" process with no guarantee of ultimate success. We note that the determination of the required mass and stiffness modifications to produce prescribed changes in the natural frequencies constitutes an inverse vibration problem. Thus we propose to formulate a suitable inverse vibration problem, which could be solved, and include the solution for this problem in the current design procedure. This modification to the current design procedure would remove the open-ended loop described in step 5, and ensure availability of solution which meets all design specifications.

We begin the formulation of our problem by selecting a discrete mass-spring system as a model for the shaft-pulley assembly. This choice is made because the mass-spring system is a simplest system to analyse, but which also provides a good model for the dynamic behaviour of such assemblies (especially in the torsional modes of vibration). In this model pulleys will be represented as discrete masses and shaft segments between the pulleys as springs. Therefore, modifications to the mass matrix will only affect the pulleys, while changes to the stiffness matrix will only affect the shaft.

The chosen mass-spring analytical model for a shaft-pulley assembly dictates that the mass and stiffness matrices must comply with the following physical realisability requirements: 1) The mass matrix must be real, positive and diagonal.

2) The stiffness matrix must be real and symmetric, with positive dominant main diagonal and negative elsewhere. (Note:" dominant main diagonal" implies that the sum of all elements in each row (or column) is greater than or equal to zero.)

Also, the choice of the mass-spring analytical model restricts the maximum number of natural frequencies that can be assigned to n, where n is the number of pulleys on the shaft. The natural frequencies of a mass-spring system are then the solutions to the following equation:

**K** is a real, symmetric stiffness matrix (nxn),

det (**K** - 
$$\varpi_i^2$$
 **M**) = 0, i = 1,2,...,n. (2.1)

where

**M** is a real, positive and diagonal mass matrix (nxn),  $\varpi_i$  is an i-th natural frequency of the system.

To obtain the desired adjustments in the natural frequencies, a designer has a choice of modifying either the shaft, or the pulleys, or both. Since there are more design parameters (i.e non-zero elements in  $\mathbf{K}$  and  $\mathbf{M}$ ) than constraints (at most *n* desired natural frequencies), the equation (2.1) has a family of solutions. We restrict our interest to only those solutions which are physically realisable, i.e solutions which correspond to realistic physical systems. We are also aware that as well as meeting the spectral specifications, an adequate solution must also satisfy other constraints. Those constraints are that the shaft should be able to operate under the specified loads and that it retains a practical geometry and composition. The practical geometry and composition considerations include the following requirements:

- a) The manufacture of the shaft must be from a single bar stock, i.e uniform material and "in one piece".
- b) The shaft should preferably be of a circular cross-section and its diameter should be kept as uniform as possible, i.e avoid unnecessary "stepping" to minimise manufacture costs and reduce stress concentrations.
- c) There should be no obstructions for the assembly of pulleys onto the shaft, i.e no "troughs" in the middle, etc.
- d) The shaft-pulley assembly must resemble a simply connected mass-spring system,
   i.e no cross-connection between non-adjacent pulleys. This imposes the restriction
   that the stiffness matrix must be tri-diagonal.
- e) The shaft-pulley assembly must operate satisfactorily under the specified loads, i.e minimum strength requirements must be met which dictate minimum size limit.
- f) To maintain a good correlation with the chosen analytical mass-spring model, the mass of the shaft should be considerably less than the mass of the pulleys, i.e maximum size limit.

Clearly, the majority of the above requirements concern only design of a shaft. The only constraint on the design of the pulleys is that they must meet minimum strength requirements. Since current design procedures ensure that the shaft satisfy all of the above requirements, we propose to restrict the modifications for natural frequency adjustment to pulleys only.

We also note that currently the spectral requirements of the design specifications are only used in checking the obtained solution. They are not used directly in any of the design calculations. Thus we believe that a vital piece of design information is not being fully utilised. An alternative approach that we propose is as follows:

- 1) Select a desired set of natural frequencies which are well separated from the restricted frequency bands.
- 2) Determine the required masses of the pulleys which in combination with the stiffness properties of the shaft produce the desired natural frequency spectrum.

The proposed design procedure is then as follows:

## Proposed Design Procedure

*Input*: Total number of pulleys and their position on the shaft; Loads on the system and power transmission requirements; Bands of restricted frequencies.

### New Design Procedure:

- 1. Estimate mass limits for each pulley based on strength requirements.
- Select material for the shaft, and determine the required size(s) of the shaft to operate under the required loading conditions.
- 3. Using the information from step 2 construct the stiffness matrix under the assumption of a mass-spring model.

- Select a set of desired natural frequencies which are well separated from the restricted frequency bands.
- 5. Using the information from steps 3 and 4 calculate the necessary mass matrix for the system to have the desired natural frequencies.
- 6. Check that the obtained masses are within the acceptable limits.

Output: A shaft-pulley system which meets all design specifications.

We note that step 5 in the new procedure involves solving an inverse vibration problem. The mathematical formulation of the inverse problem to be solved is then as follows:

### Problem 1: Mass matrix evaluation to achieve desired natural frequencies.

Given the real, symmetric stiffness matrix **K** and a set of desired natural frequencies  $\{\varpi_1^*, \varpi_2^*, ..., \varpi_n^*\}$ , find a real, positive and diagonal mass matrix **M**, such that the roots of the characteristic polynomial equation

det (**K** -  $\lambda$  **M**)= 0

are  $\lambda = \varpi_i^{*2}$  ( i=1,2,..,n).

The analysis of this problem is presented in section 4. Although the above problem was formulated specifically in relation to the design of shaft-pulley assemblies, the solution may be applied to the assignment of natural frequencies of any vibratory system which can be represented by a mass-spring model. This fact is demonstrated by the results of the experimental tests on the model of a building which are presented in section 10.

## 2.2 Problem 2: Design to Achieve Desired Natural Frequencies and Mode Shapes.

Inverse vibration problems associated with the construction of vibratory systems from the known set of desired natural frequencies and mode shapes have many applications in engineering. These include system reconstruction, modification and design. In this thesis we are concerned with the design problem of constructing a physically realisable mass-spring system with prescribed natural frequencies and mode shapes. This problem arises when controlling the maximal deflection of a harmonically excited system.

The natural frequencies and mode shapes of an undamped vibratory system are characterised by the solutions to the following equation:

$$\mathbf{K}\boldsymbol{\Phi} = \mathbf{M}\boldsymbol{\Phi}\boldsymbol{\Lambda} \,, \tag{2.2}$$

where K is a positive semi-definite symmetric stiffness matrix,
M is a positive definite symmetric mass matrix,
Φ is a modal matrix (which describes the mode shapes of the system),
Λ = diag(λ<sub>1</sub>, ..., λ<sub>n</sub>), is a spectral matrix (which describes the natural frequencies w<sub>i</sub> of the system by relation λ<sub>i</sub> = w<sub>i</sub><sup>2</sup>),

n is the number of degrees of freedom.

If we stipulate that the modal matrix  $\Phi$  must be *mass-normalised*, then it is well known that the following bi-orthogonality relations hold:

$$\Phi^{\mathrm{T}}\mathbf{M}\Phi = \mathbf{I}_{\mathrm{n}} \tag{2.3}$$

$$\Phi^{\mathrm{T}}\mathbf{K}\Phi = \Lambda. \tag{2.4}$$

1 1 1 E &

For a multiple connected mass-spring system, the mass matrix M is real, positive and diagonal. Denote

$$\mathbf{M} = \text{diag}(\mathbf{m}_1, \mathbf{m}_2, ..., \mathbf{m}_n), \mathbf{m}_i > 0, \mathbf{m}_i \in \mathbb{R}; i=1,2,...,n.$$
 (2.5)

The stiffness matrix  $\mathbf{K} = [\mathbf{k}_{ij}]$  is symmetric, and has the following properties:

j=1

(a) 
$$k_{ii} > 0, i=1,2,...n$$
  
(b)  $k_{ij} \le 0, i \ne j; i=1,2,...,n; j=2,3,...,n;$  (2.6)  
(c)  $\sum_{i=1}^{n} k_{ij} \ge 0, i=1,2,...,n$ 

In words, K has positive diagonal elements, non-positive off-diagonal elements, and it is weakly diagonally dominant.

Suppose we want to determine a physically realisable mass-spring system which has a prescribed eigenvalue matrix  $\Lambda$  with corresponding mode shape matrix  $\Phi$ . If we use the orthogonality equations (2.3) and (2.4), we have

$$\mathbf{M} = \mathbf{\Phi}^{-\mathbf{T}} \mathbf{\Phi}^{-1} \tag{2.7}$$

$$\mathbf{K} = \boldsymbol{\Phi}^{-\mathrm{T}} \boldsymbol{\Lambda} \boldsymbol{\Phi}^{-1} \ . \tag{2.8}$$

However, this solution in general would not be physically realisable. This is demonstrated by the following example:

## Example 2.2.1:

Suppose the desired dynamic properties,  $\Lambda^*$  and  $\Phi^*$ , for a five degrees-of-freedom mass-spring system are :

 $\Lambda^* = \text{diag} \{ 50, 100, 200, 400, 800 \}$ 

and

 $\boldsymbol{\Phi}^* = \begin{bmatrix} 0.1 & -0.1 & 0.2 & -0.4 & 0.1 \\ 0.1 & 0.1 & 0.2 & 0.1 & 0.3 \\ 0.1 & -0.1 & 0.3 & 0.2 & -0.4 \\ 0.1 & -0.3 & -0.1 & -0.1 & -0.1 \\ 0.3 & 0.2 & -0.1 & 0.1 & 0.1 \end{bmatrix}$ 

We wish to determine physically realisable M and K which have dynamic characteristics as close as possible to the above desired properties.

There is no exact solution for these data since if we use equations (2.7) and (2.8), we obtain

				-4.7646	
	-4.5515	13.5005	-0.3195	8.6019	-4.9737
$M = \Phi^{*-T}\Phi^{*-1} =$	1.0830	-0.3195	3.5646	-1.1215	1.0213
	-4.7646	8.6019	-1.1215	14.5886	-1.5877
	2.7310	-4.9737	1.0213	~1.5877	8.9672
				-1446.8	
	-2100.0	6358.5	-1.6181	2763.1	-1762.7
$K = \Phi^{*-T} \Lambda^* \Phi^{*-1} =$	230.0	-1618.1	1559.2	-915.4	352.6
				2324.0	
	476.9	-1762.7	352.6	-699.3	945.7

Clearly, both **M** and **K** do not have the form required for a mass-spring system, and therefore are not physically realisable.

Since equations (2.7) and (2.8) represent the unique solution to equations (2.3) and (2.4) we conclude that generally there is no exact physically realisable solution to this problem. However, we may obtain a physically realisable system with spectral properties that are close to the required data, by solving the following optimisation problem:

#### Problem 2: Determination of a Physically Realisable System

Given sets of desired eigenvalues  $\{\lambda_1^*, \lambda_2^*, ..., \lambda_n^*\}$  with corresponding mass-normalised eigenvectors  $\{\phi_1^*, \phi_2^*, ..., \phi_n^*\}$ . Denote by

$$\Phi^* = \left[ \phi_1^* | \phi_2^* | \dots | \phi_n^* \right]$$
(2.9)

the column partitioning of  $\Phi^*$ , and let

$$\Lambda^{*} = \text{diag}(\lambda_{1}^{*}, \lambda_{2}^{*}, ..., \lambda_{n}^{*}). \qquad (2.10)$$

Determine physically realisable **K** and **M** corresponding to a discrete mass-spring system, with spectral properties  $\Phi$  and  $\Lambda$  satisfying eq. (2.2), such that the norms  $\|\Phi^* - \Phi\|$  and  $\|\Lambda^* - \Lambda\|$  are minimised.

The analysis of this problem is given in section 5. A related problem associated with the reconstruction of physically realisable systems from the incomplete prescribed modal and spectral data is considered in section 6. In section 7 we define a special form for the mass and stiffness matrices, and then show how the method developed in section 5 can be extended to reconstructing matrices of this form.

## 2.3 Modifications of the Existing Structures to Obtain Desired Spectral Properties.

Analytical models simulate the behaviour of real physical structures. Application of such models is necessary because we are unable to define the differential equations governing the motion for most practical mechanical systems. In order to define these equations some assumptions about the properties of the real structure, which simplify the analysis, are usually made. The most common technique used in simplifying the analysis of a complex structure is to model this structure as a *lumped parameter* or *discrete* system. In a lumped parameter system the structure is divided into a finite number of discrete elements of known mass, and which are connected to each other by springs and dampers with known stiffness and damping constants. A designer is then able to estimate the forces acting on each element and, thus, obtain the differential equations of motion, which then may be solved.

However, the equations derived in this way may still be too complicated to solve in many cases, and therefore to obtain a solution some other simplifications may have to be made. A commonly used assumption is absence of damping in the structure. Systems without damping are called *conservative* systems, because the energy of the system is not dissipated through damping and thus conserved. Assumption of conservative system greatly simplifies the dynamic response analysis of the system, and in many cases allows a solution to be obtained which would not be possible otherwise. This is especially true in applications with inverse problems.

The number and severity of these assumptions vary greatly depending on the choice of a particular model, and also on the level of sophistication to which this model is developed. However, all assumptions which simplify the analysis of the system also introduce some uncertainty in how accurately does chosen model would simulate the behaviour of a real physical structure. It is well known that the behaviour of all analytical models will differ to some degree from the behaviour of the actual structure. Because of this difference, once the analytical stage is completed, the design process usually requires to build a prototype of the structure and then test this prototype experimentally.

A common method for measuring the dynamic behaviour of vibratory systems is modal analysis testing. The natural frequencies and mode shapes which are measured by modal analysis represent the actual physical dynamic behaviour of the system. Therefore the modal analysis data is free from the inaccuracies due to analytical assumptions. If modal analysis tests show that the dynamic behaviour does not meet design specifications, then some modifications to the system would be necessary. The usual approach at present is to adjust the dynamic behaviour by an experimental trial-and-error process. This process has several disadvantages. The main drawback is that currently there is no systematic method of obtaining modifications which produce exactly the desired changes in the spectral properties. Consequently, if the sought adjustments are relatively complicated, the above process is ill-suited for the task. We want to develop a systematic approach which would allow a designer to calculate the necessary modifications to the structure so that the desired adjustments in the spectral properties are achieved. In developing such systematic approach, there are two choices. The first approach is to use what is known as a *model updating* technique. In this approach the original analytical model is modified in such a way that its spectral properties closely correlate with the measured modal behaviour. This process is carried out by mathematical manipulations using all of the known data, including the physical and spectral properties of the original analytical model, measured modal analysis data and the desired spectral characteristics. Then based on the assumption that the new updated model is now representing the "true" model of the structure, a designer determines the "corrections" which must be applied to the physical parameters of the original model. The original system is then redesigned for the desired dynamic spectrum incorporating these "corrections". The necessary structural modifications to the prototype are then determined by comparing the redesigned system with the original one. The main weakness of this approach is that it assumes that the calculated "corrections" to the physical properties of the analytical model are constant parameters that can be superimposed from one system to another. This may or may not be true, and can vary from one design problem to another. If the modifications to the prototype determined this way still do not produce the desired spectral characteristics when implemented, then it is difficult to see what should be the next step. Repeating the above model updating process would not guarantee any better results, and due to cost and time limitations the procedure can not be carried out indefinitely.

An alternative approach is to determine the necessary modifications to the structure directly from the modal analysis data only, without using any of the data from the analytical model. In this way any inaccuracies due to the analytical model assumptions are eliminated, since only the measured data is used. The modification matrices must still have the form demanded by the analytical model so that they can be translated into actual physical changes in the structure, but the overall effect of analytical assumptions is minimised.

However, using only modal analysis data creates an additional problem. This problem arises from the difficulty in measuring a "complete" set of modal data for many practical structures. Real physical structures have infinite number of natural frequencies, but due to time and equipment limitations only a finite number of these frequencies can be measured. For example, the maximum sampling rate of the available equipment determines the maximum natural frequency that can be measured. Also, in general, the mode shapes can only be measured at a finite number of points and not continuously along the structure.

Fortunately, in most practical engineering problems the design requirements for spectral properties are restricted to a specified frequency range. This frequency range is usually from zero to some maximum stated value. The spectral properties outside this frequency range are of no interest to the designer, and therefore no restrictions on them are imposed. For the problems considered in this thesis we assume that the number of spectral pairs (i.e natural frequencies and corresponding mode shapes) that are measured is equal to the number of spectral pairs in the design constraints. The difficulties arising from the inherently truncated data provided by modal analysis are overcome by formulating optimal modification problems, which are then solved.

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This thesis deals with conservative systems which may be modelled analytically by the generalised eigenvalue problem

$$\mathbf{K}\boldsymbol{\Phi} = \mathbf{M}\boldsymbol{\Phi}\boldsymbol{\Lambda} \ , \ \boldsymbol{\Phi}^{\mathrm{T}}\mathbf{M}\boldsymbol{\Phi} = \mathbf{I}_{\mathrm{n}} \tag{2.11}$$

where K, M,  $\Phi$ ,  $\Lambda$  and *n* are as defined in equation (2.2). We also stipulate an additional requirement for the diagonal elements of  $\Lambda$ , which is that  $\lambda_1 < \lambda_2 < ... < \lambda_n$ .

Partitioning  $\Phi$  and  $\Lambda$  in the form :

 $\boldsymbol{\Phi} = [\boldsymbol{\Phi}_1 | \boldsymbol{\Phi}_2], \quad \boldsymbol{\Phi}_1 n x m \text{ real matrix}, \quad m < n$ (2.12)

and

$$\Lambda = \left[ \frac{\Lambda_1}{0} | \frac{0}{\Lambda_2} \right], \Lambda_1 = \operatorname{diag}(\lambda_1, ..., \lambda_m), \qquad (2.13)$$

we assume that  $\Phi_1$ , measured at *n* points, and  $\Lambda_1$  are known from modal analysis tests, while the submatrices  $\Phi_2$  and  $\Lambda_2$  cannot be obtained by measurements and remain unknown. Any actual structure will be damped and the measured modes will therefore be complex. We assume that the real modes  $\Phi_1$  have been extracted from these complex modes by one of the accepted procedures (see section 3). We also assume that for the low frequency range, the behaviour of the system is adequately modelled by equation (2.11).

Suppose that a design specification states that the smallest *m* eigenvalues should be  $\lambda_1^*, \lambda_2^*$ , ...,  $\lambda_m^*$ . Then we may ask the following question: If  $\Phi_1$  and  $\Lambda_1$  are known, is it possible to determine physically realisable matrices  $\Delta M$  and  $\Delta K$ , such that  $\Lambda^*$ =diag ( $\lambda_1^*, \lambda_2^*, ..., \lambda_m^*$ ) together with the corresponding modal matrix  $\Phi^*$  satisfy the following equations

$$(\mathbf{K} + \Delta \mathbf{K}) \Phi^* = (\mathbf{M} + \Delta \mathbf{M}) \Phi^* \Lambda^* \text{ and } \Phi^{*T} (\mathbf{M} + \Delta \mathbf{M}) \Phi^* = \mathbf{I}_m ? \qquad (2.14)$$

Since the truncated modal data  $\Phi_1$  and  $\Lambda_1$  do not determine K and M uniquely, it is clear that (2.14) cannot be satisfied generally. We overcome this difficulty by formulating a following related optimisation problem, which gives an optimal approximation, in some particular sense, to the solution of (2.14).

Similar to Ram and Braun [46], let us denote the residual matrix **R** by:

$$R = [(K + \Delta K)\overline{\Phi} - (M + \Delta M)\overline{\Phi}\overline{\Lambda}]$$
(2.15)

where  $\overline{\Lambda}$  and  $\overline{\Phi}$  are some approximations to the desired  $\Lambda^*$  and the corresponding  $\Phi^*$  respectively. Then we formulate the following problem:

## Problem 3: Optimal Modification for Natural Frequencies

Given  $\Phi_1$ ,  $\Lambda_1$  and  $\Lambda^*$ . Find physically realisable incremental matrices  $\Delta K$  and  $\Delta M$  such that:

$$\| (\mathbf{M} + \mathbf{\Delta M})^{-\kappa} \mathbf{R} \|_{\mathsf{F}} \text{ is minimised}$$
(2.16)  
Subject to  $\overline{\Phi} \in \text{span} (\Phi_1).$ 

The requirement that  $\overline{\Phi}$  belongs to the column range of  $\Phi_1$ , is needed to make the problem solvable.

If  $\| \mathbf{R} \| = 0$ , then so is  $\| (\mathbf{M} + \Delta \mathbf{M})^{-1/2} \mathbf{R} \|$ . In this case  $\overline{\Phi} = \Phi^*$  and  $\overline{\Lambda} = \Lambda^*$ , i.e the solution of Problem 3 is also the solution of (2.14). When  $\| \mathbf{R} \|$  is small, the solution of Problem 3 approximates the solution of (2.14). Hence by minimising the norm (2.16) we obtain an *optimal approximation*. In fact, the residual matrix **R** is weighted by  $(\mathbf{M} + \Delta \mathbf{M})^{-1/2}$ , this is for convenience purposes only. The analysis of this problem is presented in section 8. The solution methods are discussed for several commonly used analytical models (including mass-spring and finite elements). The important case where the mass and the stiffness matrices are interrelated is also studied. This analysis may be applicable to the problems of modifying the axially vibrating rod and the transversely vibrating beam.

When modal matrix  $\Phi^*$  is also included in the design specifications, the corresponding optimisation problem becomes:

### Problem 4: Optimal Modification for Natural Frequencies and Mode Shapes

Given  $\Phi_1$ ,  $\Lambda_1$ ,  $\Phi^*$  and  $\Lambda^*$ . Find physically realisable incremental matrices  $\Delta K$  and  $\Delta M$  such that:

 $\| (\mathbf{M} + \mathbf{\Delta M})^{-\mu} \mathbf{R} \|_{\mathsf{F}} \text{ is minimised}$ (2.17) Subject to  $\overline{\Phi} \in \text{span} (\Phi_1).$ 

Problem 4 was studied by Ram and Braun in [46]. The authors have shown that there is a family of solution to this problem, and that it is characterised by the following equations:

$$\Delta M = \Phi_{1}^{T\dagger} \left( \Psi^{-T} \Psi^{-1} - I_{m} \right) \Phi_{1}^{\dagger} + Y - \Phi_{1}^{T\dagger} \Phi_{1}^{T} Y \Phi_{1} \Phi_{1}^{\dagger}$$
(2.18)

$$\Delta K = \Phi_{1}^{T\dagger} \left( \Psi^{-T} \Lambda^{*} \Psi^{-1} - \Lambda_{1} \right) \Phi_{1}^{\dagger} + X - \Phi_{1}^{T\dagger} \Phi_{1}^{T} X \Phi_{1} \Phi_{1}^{\dagger}$$
(2.19)

where  $\Phi_1^{\dagger}$  denotes the Moore-Penrose pseudoinverse of  $\Phi_1$ ,  $\Psi = \Phi_1^{\dagger} \Phi^*$ , and X and Y are arbitrary mxm real symmetric matrices.

The family of solutions described by equations (2.18) and (2.19) contains all possible solutions to Problem 4. However, we are only interested in these solutions which are physically realisable. In [46] Ram and Braun have discussed the requirements for physical realisability, but no method was developed for extracting realisable solutions from the general family described by (2.18) and (2.19). In this thesis our aim was to develop such method for extracting realisable solutions, and the its derivation is given in section 9. It was found that the physically realisable solutions for  $\Delta M$  and  $\Delta K$  are, in fact, independent of the arbitrary matrices **X** and **Y**.

# **LITERATURE**

# **SURVEY**

The scope of this thesis falls within the field of *inverse vibration problems*, and a closely related field in linear algebra known as *inverse eigenvalue problems*. The inverse vibration problems have applications, among others, in the areas of *design*, *model reconstruction* (also referred to as *system identification* problems), *structural modifications* and *model updating* of vibratory systems. The *design* and *structural modifications* problems aim to control the vibrations of the systems in a desired fashion. The *model reconstruction* and *model updating* problems aim at obtaining an optimal analytical model, which approximates as close as possible the vibrations of the actual system. The book of Gladwell [1] and two review papers by the same author [2,3] give an excellent introduction and overview of the current state of knowledge in the general field of inverse vibration problems, and in the area of *model reconstruction* in particular. We draw extensively on the comprehensive material from these sources.

## **3.1 Inverse Problems**

The term *inverse* is used to distinguish these problems from the classical problems in vibration and in linear algebra, which are commonly known as *direct* problems. In the classical direct problems the aim is to determine the *behaviour* of a system (such as natural frequencies and/or mode shapes) from known *physical properties* of the system (e.g. mass, density, elastic constants, size etc). Inverse problems are concerned with the determination or estimation of such physical properties from the desired and/or experimentally measured behaviour. The *design* problems aim to develop methodical algorithms for determining the physical properties of a system from the prescribed desired behaviour. In the *structural modifications* problems the aim is to develop algorithms which use measured modal data (usually from experimental modal analysis testing of a prototype) to determine the necessary modifications to the physical properties which produce the prescribed or desired behaviour. The *model reconstruction* problems concern with the methods of obtaining the physical properties of the system from the measured experimental data, while in the *model updating* problems the aim is to minimise the difference between the measured behaviour and the theoretical behaviour of the existing analytical model.

In the above definition of the inverse problems, the term *determination* refers to problems where the sought properties can be computed exactly from the given behaviour. In these idealised, essentially mathematical problems the fundamental assumption is that all of the required data is *exact* and *complete*, meaning that it is sufficient to determine the system uniquely. The vibration analysis of most engineering problems usually contains a significant

amount of uncertainty. This uncertainty arises from a lack of detailed knowledge about the vibratory system itself (namely the shape of its mass, stiffness and damping matrices), the desired behaviour (which is usually specified only approximately in terms of permitted or restricted ranges), and the experimentally measured data (which always contains some errors due to noise, equipment limits etc). Therefore, in most engineering problems the necessary data is inaccurate and incomplete, thus, at best, permitting only an *estimation* of the sought properties.

Most published work in the field of inverse vibration problems (including the material presented in this thesis) may be classified as a combination of mainly the *determination* approach, with some elements of the *estimation* approach. Thus, in order to make the problems solvable and depending on a type of a particular problem, most of the necessary data (e.g. the shapes of the mass and stiffness matrices, the prescribed natural frequencies and/or mode shapes, etc) are assumed to be known, exact and complete. At the same time, the essential physical limitations (such as a requirement that the values of the mass and stiffness elements must be real and positive, and the inherent perturbations and truncations in the measured modal analysis data) are taken into consideration.

In [1-3] the inverse vibration problems are categorised according to the type of the mechanical systems (namely *continuous* or *discrete*, *damped* or *undamped*), and the type of the prescribed behaviour (namely *spectral* or *modal* (or both), *complete* or *truncated*, *nodal* or *isospectral*). *Nodal* inverse problems concern with the reconstruction of physical properties of the system from data relating to the position of the nodes. The term

*isospectral* refers to the studies of distinct vibratory systems that have the same eigenvalues. Some discussion was also given of applications of the developed system identification techniques to *fault detection* problems.

The paper by Chu [4] gives a very good, up-to-date review of the closely related field of inverse eigenvalue problems. Although this paper is not specifically directed towards the subject of vibration analysis, it contains a substantial amount of information on the latest developments in this field, especially in the area of *design* and control.

# 3.2 Inverse Multiplicative Eigenvalue Problem

Of particular importance to the scope of this thesis, is the so called *inverse multiplicative eigenvalue problem (IMEP)*. This problem was first formulated by Downing and Householder [5] in 1956. Their definition of the IMEP was as follows:

Given a real and symmetric matrix **A**, determine a real, positive and diagonal matrix **D**, such that the equation

$$det \left( \mathbf{A} - \mathbf{\lambda} \mathbf{D} \right) = 0 \tag{3.1}$$

has prescribed roots.

Comparison of the above definition of IMEP with the definition of Problem 1 given in section 2, shows that the two problems are identical. In [5], an iterative algorithm for a solution to IMEP was presented, and this algorithm was shown to have a local quadratic

convergence. When tested on the numerical examples, this local convergence characteristics of the algorithm was found to be inadequate for a robust solution to Problem 1. A further discussion of this algorithm and its performance is given in section 4, where we develop a solution to Problem 1.

Over the following years since its first formulation, the IMEP has attracted interest from several researchers, and a number of papers were published dealing with various aspects of the problem. The formulation of IMEP was expanded and generalised to include complex and non-symmetric matrices **A** and complex diagonal matrices **D**. Several papers were also published dealing with the necessary and sufficient conditions criteria for the existence of a solution to a given IMEP.

Hadeler [6] has defined and proved some sufficient conditions for existence of a solution to an IMEP. An alternative algorithm for the solution was also presented, along with the statement of convergence criterion. From the numerical example given in [6], it appears that the developed algorithm works, but it seems to be more complicated than the method of [5].

Kublanovskaja [7] suggested a general approach to the solution of the so called *generalised inverse eigenvalue problem*, of which IMEP was a specific case. However, due to the more general nature of the problem, the presented algorithm appears to be quite complex. This algorithm is broadly based on the method of Hadeler [6].

De Oliveira [8] expanded the results of Hadeler [6] for the sufficient conditions for a solution to IMEP. The results in [8] allow for the application of the conditions of [8] and the algorithm of [6] to cases where matrix A is non-symmetric. It was shown that the results are also valid for the solution to a so called *inverse multiplicative permanent root problem* (i.e the case when  $\mathbf{D} = \mathbf{I}_n$ ). Although the results of [8] are not directly applicable to Problem 1, this paper provides a good explanation to the results of [6].

A major contribution to the analysis of IMEP was made by Friedland [9,10]. In [9], the author presented a proof of the existence of a solution to an IMEP when both A (which is not necessarily symmetric) and D are complex-valued matrices, and when all principal minors of A are distinct from zero. The paper also contains the mathematical proof that the number of different matrices D (i.e. the maximum number of distinct solutions to an IMEP) is at most n!. This last result is very important for our analysis of Problem 1, and its significance is further discussed in section 4. In [10] the same author suggested a solution for an IMEP when A and D are both complex-valued. Since in our Problem 1 the requirement is for both A and D to be real, this solution is not applicable to our analysis.

Dias da Silva [11] extended the results of Friedland [9,10] for the sufficient conditions to IMEP for complex-valued matrices, and He Xuchu and Dai Hua [12] continued the work of Hadeler [6] and De Oliveira [8] to find better sufficient conditions for the solution to IMEP. The method of proof in [12] is similar to Hadeler's. The presented numerical example illustrates that there exists a solution to IMEP according to the conditions established in [12], even though conditions given in [6] and [8] are not satisfied. Based on

this example, the authors conclude that their conditions are better than those of [6] and [8]. However, in general, the sufficient conditions established in [6] are simpler to calculate.

Biegler-König [13] defined the sufficient conditions for the solution of a generalised inverse eigenvalue problem, of which IMEP is a special case. Formulation and a numerical solution method for this problem was proposed by Kublanovskaja in [7]. While Friedland in [9,10] has proved some conditions for a complex-valued IMEP, in [13] the author is investigating conditions for obtaining a real-valued solution. The method of proof is similar to Hadeler's in [6], and some results of [6] are special cases of the theorems in [13]. The *generalised inverse eigenvalue problem* in this paper is defined as follows:

Let  $\mathbf{A}_i$  be real matrices (i=0,1,2,...,n), and a set of prescribed real eigenvalues  $\{\mathbf{A}_1, \mathbf{A}_2, ..., \mathbf{A}_n\}$ . Find a set of real parameters  $\{\mathbf{c}_1, \mathbf{c}_2, ..., \mathbf{c}_n\}$ , such that a matrix  $A(c) = A_0 + \sum_{i=1}^n c_i A_i$  (3.2)

has eigenvalues  $\{\boldsymbol{\lambda}_1^{\star}, \boldsymbol{\lambda}_2^{\star}, ..., \boldsymbol{\lambda}_n^{\star}\}$ .

The results in [13] were given for general real matrices  $\mathbf{A}_i$  and a special case when matrices  $\mathbf{A}_i$  are both real and symmetric. The IMEP may be expressed in the form required by equation (3.2), by setting  $\mathbf{A}_0 = 0$  and  $\mathbf{A}_i = \mathbf{a}_i \mathbf{e}_i^T$  (i=1,2,...,n), where  $\mathbf{a}_i$  is the i<sup>th</sup> row vector of  $\mathbf{A}_i$  and  $\mathbf{e}_i$  is i<sup>th</sup> element unit vector (i.e the i<sup>th</sup> element in  $\mathbf{e}_i$  is equal to 1, and all other elements are zeros). However, the resultant matrices  $\mathbf{A}_i$  will not, in general, be symmetric. Therefore, it was specifically stated in [13] that a general IMEP described by (3.1) may not be expressed in the form of equation (3.2) if matrices  $\mathbf{A}_i$  are to be symmetric. In the review

paper by Chu [4] (see above) eigenvalue problems of the form (3.2) were classified as *parametrized inverse eigenvalue problems* (in reference to parameters  $\{c_1, c_2, ..., c_n\}$ ), thus dropping the use of the term *generalised*.

Nocedal and Overton [14] gave a summary of the numerical methods for solving eigenvalue problems of the form (3.2), and in a subsequent paper, Friedland, Nocedal and Overton [15] presented a general overview and comparison between the various solution approaches to this problem. A total of four different methods of solution were presented in [15], three of which were published previously and one new. The analysis was given for the problems where the desired eigenvalues were all distinct, and also for the case when there are multiple identical eigenvalues. A general convergence analysis was carried out for both the distinct and multiple eigenvalues. The comparison between the four methods was presented, and a numerical example was given which illustrated the results.

Although the results of [15] were not directly applicable to Problem 1, Joseph [16] has used one of the methods presented in [15] as a basis for an algorithm to solve what he called *inverse eigenvalue problem in structural design*. The author have assumed that the mass and the stiffness matrices of a structure, **M** and **K**, are functions of *n* independent structural parameters  $\{c_1, c_2, ..., c_n\}$ , and comply with the following forms

$$M(c) = M_0 + \sum_{i=1}^{n} c_i M_i$$
(3.3)

$$K(c) = \sum_{i=1}^{n} c_{i} K_{i}$$
(3.4)

Some known results from the *eigenvalue sensitivity* theory were then applied to determine  $\{c_1, c_2, ..., c_n\}$  such that the resulting eigenvalues of the system described by **M** and **K** were equal to the prescribed set of desired eigenvalues  $\{\lambda_1^*, \lambda_2^*, ..., \lambda_m^*\}$   $(m \le n)$ . The method of [16] is directly applicable to solve Problem 1. Numerical testing have shown that the performance of this method has the same problematic local convergence characteristics as the algorithm of Downing and Householder in [5]. However, one major advantage of this method is that it permits assignment of a partial set of desired eigenvalues (i.e when m < n), thus allowing application of the method to problems with truncated modal data. Because of this property, the algorithm was used as a foundation for our solution to Problem 3. Further discussions of the Joseph's algorithm and its application to Problems 1 and 3 are given in the appropriate parts of sections 4 and 8.

#### 3.3 Sensitivity Methods

The *eigenvalue sensitivity* theory, referred to above, and the associated subject of *eigenvalue derivatives* was used extensively in this thesis. In its simplest form, the principle of matrix derivatives is demonstrated by a following example. Suppose that we have matrix **A** which is a function of *n* independent parameters  $\{c_1, c_2, ..., c_n\}$  and can be expressed in the form of equation (3.2). Then it is clear that

$$\frac{\partial A(c)}{\partial (c_i)} = A_i \quad , \quad i=1,2,\dots,n \quad . \tag{3.5}$$

Matrices  $A_i$  are commonly called *connectivity* or *mapping* matrices, describing the connections of each individual parameter  $c_i$  or simply defining the grid position of this parameter within the global matrix A(c). Determining matrix derivatives of A(c) with respect to parameters  $c_i$  is a simple process because (3.2) is a linear and uncoupled equation in  $c_i$ . In the eigenvalue sensitivity analysis, the aim is to obtain the relations describing the effect of each of the elements of a matrix on the eigenvalues of that matrix. However, determining the eigenvalues of a matrix is not a linear process. Therefore, there is no exact way of describing the effect of a particular element on the eigenvalues. Consequently, most of the published work on this subject concerned with determining an approximation to these relations, which are optimal in some way. It is clear that obtaining an accurate relations describing the effect of an individual structural parameter (i.e mass, stiffness or damping element) on the natural frequencies and mode shapes (i.e eigenvalues and eigenvectors) is the key to a solution of most problems in inverse vibrations applications to *design* and *structural modifications*.

In a review paper on structural modifications problems, Baldwin and Hutton [17] give a good summary of the literature on eigenvalue sensitivity methods. This paper is primarily concerned with what may be described as *direct structural modification problems*. These are problems in which the aim is to determine the dynamic behaviour of a system based on known changes in the structural parameters and the measured behaviour of the original unmodified structure. The *inverse structural modifications*, where the aim is to obtain the structural modifications necessary to meet specified constraints on natural frequencies and mode shapes, are only briefly discussed. However, most of the theory on sensitivity analysis

is equally applicable to inverse problems. The material presented mainly deals with conservative (i.e undamped) systems, and is classified into three main categories. Namely, the techniques based on the *assumption of small modifications*, these for *localised modifications*, and finally, techniques using the *modal approximation* approach.

The techniques for *localised modifications* deal with determining the dynamic behaviour of the modified system based on the precise knowledge of the location of the structural changes in addition to their magnitude. An interesting characteristic of these problems is that, apparently, the modified behaviour can be determined exactly, although only by an iterative procedure. In all other categories only approximate solutions may be obtained. In applications involving inverse problems a major problematic area is how to satisfy the physical realisability constraints on the necessary modifications based only on the knowledge of the desired and measured behaviour of the system. Thus, specifying additional constraints on the exact location of modifications would add an extra layer of difficulty to what is already a complicated problem. Therefore, in this thesis we have not attempted to specifically formulate or to solve any of the *inverse localised* problems. However, some limited control over the location of structural modifications is possible under some circumstances in the analysis of sections 6 and 9. The reader is referred to these sections for further details.

The category based on the assumption of *small modifications* dealt exclusively with the sensitivity methods. This category was further subdivided into three separate approaches, namely methods based on *Rayleigh's principle*, methods based *on eigenvalue derivatives*,

and finally, methods based on *modal perturbation theory*. All three approaches lead to very similar formulations and are discussed below.

#### 3.3.1 Rayleigh's method

The Rayleigh's approach is based on the equation

$$\mu_i = \frac{\psi_i^T (K + \Delta K) \psi_i}{\psi_i^T (M + \Delta M) \psi_i}$$
(3.6)

where **M** and **K** are the mass and stiffness matrices of the original system,  $\Delta M$  and  $\Delta K$  are the matrices denoting respectively modifications to the mass and stiffness, and  $\mu_i$  and  $\psi_i$  are the i<sup>th</sup> eigenvalue and the corresponding eigenvector of the modified system.

It is assumed that, for small modifications, the mode shapes do not change appreciably and therefore we may substitute  $\psi_i = \phi_i$  into (3.6). Also, applying the orthogonality properties  $\phi_i^T \mathbf{K} \phi_i = \lambda_i$  and  $\phi_i^T \mathbf{M} \phi_i = 1$  (where  $\lambda_i$  and  $\phi_i$  are the i<sup>th</sup> eigenvalue and eigenvector of the original system), equation (3.6) becomes

$$\mu_i = \frac{\lambda_i + \phi_i^T \Delta K \phi_i}{1 + \phi_i^T \Delta M \phi_i}$$
(3.7)

Thus, if  $\Delta M$  and  $\Delta K$ , and  $\lambda_i$  and  $\phi_i$ , are known, then  $\mu_i$  can be calculated. Clearly, if the modifications are not small then the assumption that  $\psi_i = \phi_i$  is not valid, and consequently, the method should not be applied.

# 3.3.2 Eigenvalue derivative method

The *eigenvalue derivative* approach considers the structural modification problem in terms of a rate of change of an eigenvalue with respect to a structural parameter change. It was shown in [17] that

$$\frac{\partial \lambda_i}{\partial c_j} = \phi_i^T \left[ \frac{\partial K}{\partial c_j} - \lambda_i \frac{\partial M}{\partial c_j} \right] \phi_i$$
(3.8)

where  $c_i$  is some j<sup>th</sup> structural parameter.

This relation may then be used in a Taylor series expansion to give a first or second order approximation to the natural frequencies of the modified structure. Early work in this area was conducted by McCalley [18] and Wittrick [19], and, in general, the mathematical foundations have been discussed by Lancaster [20]. A major treatment of the entire problem, including the calculation of the mode shapes of a modified structure, was presented by Fox and Kapoor [21]. In [21] a first-order solution was considered. Two methods were derived to calculate the mode shape derivatives. The first method expressed the eigenvector derivatives in terms of a series expansion in the unmodified eigenvectors and, hence, required the knowledge of the full modal characteristics of the original structure (although truncation is possible). A second method expressed the eigenvector derivative only in terms of the corresponding frequency and eigenvector of the original structure. However, although the second method was potentially more attractive, some numerical difficulties were encountered which prevented its successful implementation. These difficulties were eventually solved by Nelson [22].

Van Belle [23] presented a theory of adjoint structures to calculate the differential sensitivities of mechanical structures. This work has been expanded by Van Honacker [24] to derive expressions for the differential, finite difference and frequency response sensitivities for natural frequencies and mode shapes of a viscously damped vibratory system. Second-order terms of Taylor expansion are included in the analysis to obtain expressions for "large-change" sensitivities.

Second- and higher-order solutions have also been investigated by Rudisil [25, 26], Muira and Schmit [27], Van Belle [28] and Rizai and Bernard [29]. Wang, Heylen and Sas [30] summarised the developed procedures, but found that the methods based on truncated Taylor expansions are limited in their applications to small modifications, and that inclusion of higher-order terms does not always ensure a more accurate solution.

To and Ewins [31] used the closed-form properties of the Rayleigh's method and the theoretical basis of the eigenvalue and eigenvector derivatives analysis, to develop a powerful iterative algorithm, which is not restricted to just small modifications. Instead of assuming that the eigenvectors of a modified system remain unchanged, as in the Rayleigh's method above, the authors express modified modes as linear combinations of the original modes. The coefficients in these linear combinations of original modes were termed *mode participation factors*. The method involves (starting initially from equation of the form (3.7)) obtaining an estimate for the modified eigenvalues of a system, and then using these estimates to calculate the mode participation factors. The method involves. The mode participation factors are then in turn used to calculate the better approximations to the modified eigenvalues. The

procedure is then repeated until convergence is achieved. The authors claim that this procedure has 'superconvergence characteristics', and that the exact modal properties of the modified structure may be determined. The effects of modal truncation and sensitivity to input data perturbations are also presented.

Zimoch [32] used the first-order Taylor expansion to determine the sensitivity matrices for the eigenvalues and mode shapes. The effects of the structural modifications on the dynamic behaviour of the system could then be estimated in a computationally efficient way. The method could be applied to damped as well as conservative systems. A more accurate solution may be achieved by inclusion of a second-order term of the Taylor's expansion. This inclusion does not require any alterations to the procedure, but the penalty is the necessity to carry out much more involved calculations. In [33] Zimoch applied the method of [32] to solve an inverse problem. The formulation of his problem is very similar to our Problem 2, except that no physical realisability constraints were imposed on either mass or stiffness matrices. Without these constraints a solution can be obtained by a trivial process described in section 2.2 (see equations (2.7) and (2.8)). However, the motivation of Zimoch was to develop a more computationally efficient method for determining the changes in the physical parameters of a system to achieve the desired eigenvalues and eigenvectors.

Joseph [16] (see above) used equation (3.8) to develop an iterative algorithm which solves an inverse problem, and, in theory, is not restricted to small modifications. The method required an estimate of the initial values for the structural parameters  $\{c_1, c_2, ..., c_n\}$ . Then the mass and stiffness matrices, **M** and **K**, may be constructed via equations (3.3) and (3.4). Using equation (3.8) and an example of (3.5), the eigenvalue derivatives were calculated for the system described by **M** and **K**. Application of a Newton-Raphson method using these eigenvalue derivatives, allowed calculations of better estimates for  $\{c_1, c_2, ..., c_n\}$ . This procedure could then be repeated indefinitely, until convergence was achieved.

### 3.3.3 Perturbations methods

In the perturbation approach it is assumed that the mass and stiffness matrices of the modified system,  $M_{mod}$  and  $K_{mod}$ , and the corresponding eigenvalue and eigenvector matrices,  $\Omega$  and  $\Psi$ , are related to the properties of the original system by

$$M_{mod} = M + \Delta M \qquad K_{mod} = K + \Delta K$$
$$\Omega = \Lambda + \Delta \Lambda \qquad \Psi = \Phi + \Delta \Phi \qquad (3.9)$$

It is also assumed that for small modifications, all  $\Delta$  terms are sufficiently small. Thus, substituting properties (3.9) into an equation of the form (2.2) governing the motion of the modified system, and neglecting all terms of  $\Delta^2$  and higher, it is possible to obtain an approximate relation between the structural modification  $\Delta M$  and  $\Delta K$  and  $\Omega$  and  $\Psi$ . The precise form of such relationship is dependent on the assumptions and conditions of a particular problem, and thus, are too numerous to be given here.

The first treatment of the structural modification problem was by Rayleigh [34], who used this type of approach and derived an approximate solution in terms of modal coordinates from energy expressions. Jones [35] extended this work to include general perturbations and derived expressions for natural frequency and mode shape changes. Romstad, Hutchinson and Runge [36] investigated a variety of more general perturbation formulations using a power series approach.

Stetson and Harrison [37] have extended the modal perturbation approach to treat the inverse problem of determining the structural modifications necessary to meet the specified constraints on natural frequencies and mode shapes. The method of [37], uses the results from NASTRAN finite element analysis software to determine the analytical model of the original structure. It then processes these data, taking account of the physical realisability constraints, to obtain the necessary changes in the thicknesses of the structural elements. The aim of the method is to minimise the necessary structural changes, while obtaining the desired dynamic properties. Sandstrom and Anderson [38] extended this work by directly relating the physical changes in the natural frequencies and mode shapes to changes in structural parameters.

Sandstrom, Anderson and others [39,40] reported that perturbation approach based on the linear energy formulation gives good accuracy for natural frequency goals, but is often not accurate for significant mode shape changes. In [39] the authors have extended the linear perturbation approach of [37], including all of the non-linear terms in the perturbation equation. This was done because, as was demonstrated by a numerical example of a typical

problem, the second-order terms can be as large as the first-order terms. Thus neglecting second-order terms may lead to large errors, particularly in redesign of mode shapes. In [40] the authors have developed a non-linear, iterative algorithm, which was partitioned into two stages, namely the 'predictor' and the 'corrector' stages. The 'predictor' phase is essentially an improved version of the algorithm in [37], which gives a first-order approximation for the required structural changes. In the 'corrector' phase, these approximations are used to calculate a first-order estimate for the desired eigenvectors. These eigenvectors are then used in general perturbation equations to find the corrections for the structural changes. The process is then repeated as many times as necessary to achieve the acceptable dynamic behaviour. The algorithm of [40] requires precise knowledge of the physical properties of the original structure (i.e its finite element model) as an input to the problem.

Zhang, Wang, Allemang and Brown [41] used the perturbation approach to find an approximate solution to a problem which is identical to our Problem 3. The method uses power expansion of a perturbation equation to find the necessary mass modifications so that the desired natural frequencies are achieved. Results are presented for both the first and the second order approaches. The input to the problem was assumed to consist of only the specified desired natural frequencies, and a truncated set of measured modal analysis data. The method of [41] is applicable to any damped or conservative system, whose mass matrix is diagonal, and where the mass and stiffness matrices are independent of each other (e.g. mass-spring system). The algorithm also allows to control the location of mass modifications, and an optimisation procedure for the best locations and magnitude of mass

modifications is given. However, the method is based on the perturbation approach and its performance is acceptable only if the sought changes in the natural frequencies are relatively small. Numerical simulations have shown that this method performs inadequately in a general case, when desired changes are not sufficiently small. Therefore, our aim was to develop an alternative method which would not have this limitation.

#### **3.4 Modal Approximation Methods**

The *modal approximation* methods are based on the assumption that the eigenvectors of the original unmodified structure form a complete vector basis to describe the motion of any modified structure. Mathematically, this assumption implies that the eigenvectors of the modified system belong to the space spanned by the eigenvectors of the original structure. We have also used this assumption in our formulation of Problems 3 and 4. This approach allows us to obtain an approximate solution to these problems, which is optimal in a Rayleigh-Ritz sense. Our solutions are based on the theory developed by Parlett [42], and which is described in detail at the beginning of section 8.

The results presented in sections 8 and 9 of this thesis, are the extension of the work done by Ram and Braun [43 - 46] in this field. In [43 - 46] the authors dealt with problems arising specifically when the necessary structural modifications are determined based on the modal analysis data, and assuming no knowledge of any other information about the structure. It was shown by Berman [47], that even under the most favourable laboratory conditions, there are severe limitations on obtaining modal analysis data which is complete,

i.e. which completely describes all modes of the system. Therefore, the use of modal analysis results alone (i.e without any additional information from other sources) inevitably introduce the problems of *modal truncation errors*. The presence of *modal truncation errors* imply that there is insufficient information to find the exact values for the physical parameters of a system from the measured modal analysis data. Thus, the difficulties arising from the inherently truncated data provided by modal analysis may only be overcome by formulating problems for approximate solutions which are optimal in some specified sense.

Another problem with using modal analysis data, arise when an assumption is made that a system under consideration is conservative. This assumption of a conservative system was made in [43 - 46] and also in our formulation of Problems 3 and 4. The eigenvalues and eigenvectors of a conservative system are real-valued, whereas eigenvalues and eigenvectors of a damped system are complex-valued. Since any actual structure will always have some degree of damping, the measured modal data is always complex-valued. In the analysis of [43 - 46] and in our analysis of sections 8 and 9, we assume that the real-valued modes may be extracted from these measured complex modes. There are a number of available methods for such extraction, ranging from complicated mathematical procedures to a very simple process of truncating the imaginary part. Zhang and Lallement [48] present a summary of three extraction methods, and describe the comparison of their relative performance. The application of the these methods to a test structure showed that the results are sufficiently accurate. However, due to the requirements placed on the input data in two of the methods, they were judged to be of mathematical interest only. The third method was considered suitable for engineering applications. In a recent paper by Ahmadian, Gladwell

and Ismail [49] it was shown analytically that a real mode most correlated with a complex measured mode is the real part of the same complex mode when it is rotated so that the norm of its real part is maximised. Clearly, an assumption of a conservative system is suitable only for the lightly damped structures. If damping in the structure is not negligible, then the real modes extracted by any of the above methods are completely different from the mode shapes of a conservative analytical model. Consequently, large errors will result from any attempt to correlate this fundamentally different data.

In [43] Ram and Braun used the result of Parlett [42] to formulate and to solve a direct structural modification problem. The developed algorithm yields an approximate solution which is optimal in a Rayleigh-Ritz sense. In [44] same authors derived the upper and lower bounds on eigenvalues (i.e natural frequencies) of a modified structure based on truncated modal testing results. In [43] it was shown that a solution which is optimal in a Rayleigh-Ritz sense provides an upper bound for the predicted natural frequencies of a modified structure. In [44] a method for obtaining the lower bounds for the natural frequencies was developed, and a procedure for predicting a modal truncation error was presented. In [45] the authors obtained bounds on the eigenvectors due to structural modification. In [46] a method developed in [43] was applied to an inverse modification problem. This inverse problem is identical to our Problem 4. The authors were able to characterise all possible family of solutions to this problem (see section 9 for the equations characterising those solutions for  $\Delta M$  and  $\Delta K$ ). However, although constraints for physically realisable solutions problem was not solved and consequently physical

realisability constraints were not enforced. In section 9 we give an alternative formulation for the problem of extracting realisable solutions from the family characterised in [46]. This alternative problem is then solved, thus complementing the results of [46].

Tsuei and Yee [50] presented a solution for a single parameter modification (either a mass or stiffness element) of a conservative system. The method is based on the force response of the original system, and allows to shift one natural frequency to a prescribed value. Since the method does not require iterations, it is computationally efficient. However, due to the coupling between the modes, a prescribed shift in one natural frequency causes other natural frequencies to shift as well, and these "secondary" shifts are not controllable. Thus, this method can not be used for assigning multiple (i.e. more than one) natural frequencies. In [51] same authors extended the results of [50] to applications with damped systems. The new algorithm does require iterations, but it is claimed that it converges very fast. Ram [52] considered the problem of how to enlarge a spectral gap of some vibrating continuous and discrete systems (including taut spring, non-uniform beam and a mass-spring system) by introducing two appropriate oscillators at the proper locations.

A different approach to a structural modification problem was presented by Coppolino [53]. The author uses the measured truncated modal data and the stiffness matrix from the finite element model of the original structure to determine the so called *residual modal matrix*. The residual modal matrix is obtained by substituting unit load vectors at the location and instead of the required structural modifications. The measured modal matrix, augmented by the residual matrix, then describe an exact static response characterisation of the original

structure due to application of unit loads. This is, in effect, equivalent to obtaining a static response of the modified structure. The numerical example given in [53] for a 1416 degrees-of-freedom system demonstrated the application of the developed method.

An equivalent vector form of equation (2.2) for a conservative system is

$$\mathbf{K}\boldsymbol{\phi}_{i} = \boldsymbol{\varpi}_{i}^{2}\mathbf{M}\boldsymbol{\phi}_{i} \tag{3.10}$$

or, alternatively

$$\mathbf{M}^{-1}\mathbf{K}\boldsymbol{\phi}_{i} = \boldsymbol{\varpi}_{i}^{2}\boldsymbol{\phi}_{i} \tag{3.11}$$

where  $\overline{\omega}_i$  and  $\phi_i$  are the i<sup>th</sup> natural frequency and the corresponding mode shape of a system. There are well established methods for calculating  $\overline{\omega}_i$  and  $\phi_i$  from the measured experimental data. Because both sides of equation (3.11) are multiplied from the right side by  $\phi_i$ ,  $\phi_i$  is sometimes referred to as the *right modal vector*.

Zhang, Allemang and Brown [54] have shown that the same information which is used to extract  $\phi_i$  from the frequency response function of a system, may also be used to extract the so called *left modal vector*,  $\xi_i$ , which is defined by

$$\boldsymbol{\xi}_{i}^{\mathrm{T}}\mathbf{M}^{-1}\mathbf{K} = \boldsymbol{\varpi}_{i}^{2} \boldsymbol{\xi}_{i}^{\mathrm{T}}$$
(3.12)

The left modal vector is related to the right modal vector via

$$\boldsymbol{\xi}_{i} = \mathbf{M}\boldsymbol{\phi}_{i} \tag{3.13}$$

and, assuming mass-normalisation, it can be immediately shown that

$$\boldsymbol{\xi}_{i}^{T}\boldsymbol{\Phi}_{j} = \begin{cases} 1 , i=j \\ 0 , i\neq j \end{cases}$$
(3.14)

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Thus, it appears that more than the usual information can be extracted from the experimental modal analysis results. This additional information can then be used to negate the inherent incompleteness of the measured data, and thus allow to circumvent entirely the effects of the modal truncation errors. However, the extraction of the left modal vectors appears to be very sensitive to noise, and hence it is not clear whether it can be done with sufficient accuracy in practical applications. Also, unlike the well established procedures for extracting right modal vectors, extraction of the left modal vectors requires to solve a set of ill-posed equations of deficient rank, which may lead to additional large errors. If, on the other hand, the left modal vectors are available and accurate, then they can be immediately applied to solve problems in structural modifications.

Based on this assumption of the availability of left modal vectors, Bucher and Braun [55] developed an exact solution to an inverse structural modification problem. Their method allows an exact assignment of the natural frequencies and mode shapes based on incomplete modal analysis data, provided that the prescribed mode shapes belong to the space spanned by the original measured modal vectors. If the prescribed mode shapes do not belong to the space spanned by the measured vectors, a method for approximate assignment is also developed, which gives an optimal solution in a least squares sense.

In two recent papers Bucher and Braun [56,57] have developed a computationally efficient optimisation procedure for minimising the vibratory response of a system by structural modifications. This procedure may be applied in cases where only a truncated set of measured modal data is available, or when a complete set of analytical data is assumed to

be known. In [56] the authors develop the theoretical basis for their method, and in a "companion" paper [57] they provide detailed examples of application of the derived theory.

# 3.5 Model Reconstruction and Model Updating Problems

The work of Gladwell [1-3] (see above) gives a comprehensive introduction to the problems of *model reconstruction*, which are also often referred to as problems of *system identification*. The book [58] and the recent paper [59] by Friswell and Mottershead give a review of the current state of knowledge in the subject of *model updating*. In this section we only present few results which are of most relevance to the scope of this thesis.

Boley and Golub [60] have reviewed various algorithms for reconstruction of Jacobi matrices from the knowledge of their eigenvalues, eigenvalues of their principal submatrices, and/or knowledge of some specified elements of the normalised eigenvectors. Ram and Coldwell [61] found a solution for reconstructing a free-free, multi-connected massspring system (i.e system where none of its springs or masses are attached to the ground, but where each mass may be connected via a spring element to any other masses) from known sets of the natural frequencies of the system. The required sets of natural frequencies included the original system, and the frequencies of the systems when each of the masses, in turn, was pinned to the ground, thus restricting its movement. Gladwell and Movahhedy [62] have obtained the set of the necessary and sufficient conditions to ensure positive mass and stiffness parameters for the three-degree-of-freedom case. Movahhedy, Ismail and Gladwell [63] have examined the problems associated with reconstruction of such systems

from the experimentally measured data. Ram and Gladwell [64] solved the problem of reconstructing the finite element model of an axially vibrating rod from the knowledge of some of its eigenvalues and eigenvectors. The minimum requirement for a closed-form solution is one eigenvalue and two eigenvectors. However, with this minimal data the algorithm is very sensitive to perturbations. This sensitivity of the method is decreased if overdetermined data (i.e more data than is minimally necessary) is used, in which case a solution is obtained by a least squares approach. Ram [65,66] then extended the method of [64] to find the equivalent solutions for reconstruction of a longitudinally vibrating continuous rod and a discrete model of a transversely vibrating beam.

Starek and Inman [67 - 71] have studied the problems associated with the reconstruction of non-conservative systems. In [67] the authors have developed a method of solution which ensured that the mass, stiffness and damping matrices are real-valued, provided that all of the eigenvalues of a system are complex. In [68,69] the method was improved to ensure that the matrices are also symmetric, thus enhancing the physical realisability properties of the solution. In [70] an alternative approach to the method of [69] was presented, which further improved the realisability properties of a solution by ensuring that the obtained matrices, in addition to being real and symmetric, are also positive definite. In [71] the method has been further developed to include systems with real-valued eigenvalues associated with overdamped modes.

# **3.6 General References**

The book of Golub and Van Loan [72] was an invaluable reference for understanding concepts of linear algebra and matrix analysis. In particular, the algorithm for solving an *orthogonal Procrustes problem* [72, p.582] formed a foundation for the solution to Problem 2 described in section 5, and a more general Problem 2(b) presented in section 7. A detailed description of the orthogonal Procrustes problem and the procedure for its solution are given in section 5.

The physical realisability constraints for the mass and stiffness elements were a major focus in this thesis. The principal demand for the realisable mass and stiffness elements is that they must be real and non-negative. Thus, the method for solving a *non-negative least squares problem* given in the book of Lawson and Hanson [73, p. 161] was used as a primary tool in our analysis throughout this thesis. The algorithm for solving a *non-negative least squares problem* is also available as a standard function **nnls** in MATLAB.

Chu [74] has discussed the effect of the rate of convergency of the two methods for an *inverse singular value* problem, which is closely related to the inverse eigenvalue problems. He found that a quadratically converging algorithm converges fast but locally, while a linearly converging algorithm converges globally but at a slower rate. Thus it is possible that the convergence characteristics of an algorithm may be improved by reducing its rate of convergence. We have successfully applied this principle in our solution to Problem 1.

# **Section 4**

# **PROBLEM 1:**

# **DESIGN FOR**

# NATURAL FREQUENCIES<sup>1</sup>

In this section we analyse a solution to Problem 1, which is formulated in section 2.1. In this problem we assume that the vibratory behaviour of a system can be adequately approximated by the behaviour of a conservative mass-spring analytical model. It is also assumed that a physically realisable stiffness matrix **K** and a set of desired natural frequencies  $\{\overline{\omega}_1^*, \overline{\omega}_2^*, ..., \overline{\omega}_n^*\}$  are known.

Denoting:

$$\Lambda^{*} = \text{diag} \ (\lambda_{1}^{*}, \lambda_{2}^{*}, ..., \lambda_{n}^{*}) \ ; \ \lambda_{i}^{*} = \omega_{i}^{*2} \ ; \ 0 > \lambda_{1}^{*} > \lambda_{2}^{*} > ... > \lambda_{n}^{*}$$
(4.1)

we wish to find a real, positive and diagonal mass matrix M such that the roots of the characteristic polynomial

$$\det (\mathbf{K} \cdot \lambda \mathbf{M}) = 0 \tag{4.2}$$

are the prescribed diagonal elements of  $\Lambda^*$ .

<sup>1</sup> Material presented in this section has been published in [75].

This problem is similar to the *inverse multiplicative eigenvalue problem* which was first formulated by Downing and Householder [5]. Many authors (eg. [6 - 15]) have presented alternative methods of solution for this, and similar, problems as well as some partial conditions for the existence of real solutions. Most notable work was done by Friedland [9, 10] and Friedland, Nocedal and Overton in [15]. Recently, Joseph [16] has developed a related method which solves a similar problem to the one studied here. Two of the algorithms ([5] and [16]) can be applied directly to solve the problem under consideration.

It should be noted that the existing methods of solution are based on iterative procedures. The problem, however, can be expressed as a system of n equations with n unknowns. The possibility of finding a closed-form solution is investigated in section 4.1, and a closed-form solutions are obtained there for two and three degrees-of-freedom systems. It appears however that this method cannot be effective for high order systems due to the complexity of the non-linear equations involved. In section 4.2 we describe the requirements for a practical method of solution and discuss the need of a new algorithm. The new algorithm is presented in section 4.3. Some numerical simulations are given in section 4.4, and the conclusions are summarised in section 4.5.

#### 4.1 A Closed-Form Solution

Equation (4.2) can be written in the following form:

$$\det (\mathbf{K} - \lambda \mathbf{M}) = \alpha (\lambda - \lambda_{1}^{*}) (\lambda - \lambda_{2}^{*}) (\lambda - \lambda_{3}^{*}) \dots (\lambda - \lambda_{n}^{*})$$

$$(4.3)$$

where  $\lambda_i^*$  (i=1,2,...,n) are the given eigenvalues, and  $\alpha$  is some constant.

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•

We wish to find positive diagonal **M** such that (4.3) is satisfied. Both sides of (4.3) can be expressed as polynomials in  $\lambda$  of order *n*. Equating the coefficients of the two polynomials will produce *n* equations with *n* unknowns. This is demonstrated by the following examples:

### 4.1.1 A two degree-of-freedom system.

A general two degrees-of-freedom system has the following stiffness and mass matrices:

$$K = \begin{bmatrix} k_{11} & k_{12} \\ k_{12} & k_{22} \end{bmatrix}, M = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}$$
(4.4)

Suppose K and  $\lambda_1^*$ ,  $\lambda_2^*$  are given. The problem under consideration is to find  $m_1$  and  $m_2$  such that (4.3) holds, i.e

$$\det \begin{bmatrix} k_{11} - \lambda m_1 & k_{12} \\ k_{12} & k_{22} - \lambda m_2 \end{bmatrix} = \alpha (\lambda - \lambda_1^*) (\lambda - \lambda_2^*)$$

$$(4.5)$$

Expanding both sides we have

$$m_{1}m_{2}\lambda^{2} - (k_{11}m_{2} + k_{22}m_{1})\lambda + (k_{11}k_{22} - k_{12}^{2}) = \alpha\lambda^{2} - \alpha(\lambda_{1}^{*} + \lambda_{2}^{*})\lambda + \alpha\lambda_{1}^{*}\lambda_{2}^{*}$$
(4.6)

which yields:

$$m_1 m_2 = \alpha \tag{4.7}$$

$$k_{11}m_2 + k_{22}m_1 = \alpha(\lambda_1^* + \lambda_2^*)$$
(4.8)

$$k_{11}k_{22} - k_{12}^2 = \alpha \lambda_1^* \lambda_2^* \tag{4.9}$$

From (4.9)  $\alpha$  can be expressed in terms of the given data as:

$$\alpha = \frac{k_{11}k_{22} - k_{12}^2}{\lambda_1^* \lambda_2^*} \qquad (4.10)$$

and (4.7) and (4.8) can then be used to produce a quadratic equation in  $m_2$ :

$$k_{11}m_2^2 - \alpha (\lambda_1^* + \lambda_2^*)m_2 + k_{22}\alpha = 0$$
(4.11)

The solutions then are

$$m_2 = \frac{\alpha(\lambda_1^* + \lambda_2^*) \pm \sqrt{\alpha^2(\lambda_1^* + \lambda_2^*)^2 - 4k_{11}k_{22}\alpha}}{2k_{11}}$$
(4.12)

$$m_1 = \frac{\alpha}{m_2} \tag{4.13}$$

We note that provided the necessary condition  $k_{11}k_{22} > k_{12}^{2}$  is satisfied, there are two physically realisable solutions if

$$\alpha(\lambda_1^* + \lambda_2^*)^2 > 4k_{11}k_{22} \tag{4.14a}$$

one physically realisable solution if

$$\alpha(\lambda_1^* + \lambda_2^*)^2 = 4k_{11}k_{22} \tag{4.14b}$$

and no real solution otherwise.

## 4.1.2 A three degree-of-freedom system

Suppose

$$\boldsymbol{K} = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{12} & k_{22} & k_{23} \\ k_{13} & k_{23} & k_{33} \end{bmatrix}, \quad \boldsymbol{M} = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}$$
(4.15)

then similar analysis gives:

$$A_1 = m_1 m_2 m_3 \tag{4.16}$$

$$A_2 = B_1 m_2 m_3 + B_2 m_1 m_3 + B_3 m_1 m_2$$
(4.17)

$$A_{2} = C_{1}m_{1} + C_{2}m_{2} + C_{3}m_{3}$$
(4.18)

where

$$A_{1} = \frac{k_{11}k_{22}k_{33} + 2k_{12}k_{13}k_{23} - k_{11}k_{23}^{2} - k_{22}k_{13}^{2} - k_{33}k_{12}^{2}}{\lambda_{1}^{*}\lambda_{2}^{*}\lambda_{3}^{*}};$$

$$A_{2} = A_{1}(\lambda_{1}^{*} + \lambda_{2}^{*} + \lambda_{3}^{*}); A_{3} = -A_{1}(\lambda_{1}^{*}\lambda_{2}^{*} + \lambda_{1}^{*}\lambda_{3}^{*} + \lambda_{2}^{*}\lambda_{3}^{*});$$

$$B_{1} = k_{11}; B_{2} = k_{22}; B_{3} = k_{33}; C_{1} = k_{23}^{2} - k_{22}k_{33};$$

$$C_{2} = k_{13}^{2} - k_{11}k_{33}; C_{3} = k_{12}^{2} - k_{11}k_{22}.$$

$$(4.19)$$

Solving for  $m_2$  and  $m_1$  as functions of  $m_3$ , we obtain:

$$m_2 = \frac{C_1 \left[ A_2 - \frac{B_3 A_1}{m_3} \right] - B_2 m_3 (A_3 - C_3 m_3)}{(C_1 B_1 - C_2 B_2) m_3}$$
(4.20)

and

$$m_1 = \frac{A_1}{m_2 m_3} \tag{4.21}$$

Substitution of (4.20) and (4.21) into (4.18) results in the following  $6^{th}$  order polynomial in  $m_3$ :

$$B_{2}B_{1}C_{3}^{2}m_{3}^{6} - 2B_{2}B_{1}A_{3}C_{3}m_{3}^{5} + (B_{2}A_{2}C_{2}C_{3} + B_{2}B_{1}A_{3}^{2} + A_{2}B_{1}C_{1}C_{3})m_{3}^{4} +$$

$$(B_{2}^{2}A_{1}C_{2}^{2} - 2B_{2}A_{1}B_{1}C_{1}C_{2} + A_{1}B_{1}^{2}C_{1}^{2} - B_{3}B_{2}A_{1}C_{2}C_{3} - B_{2}A_{2}A_{3}C_{2} - B_{3}A_{1}B_{1}C_{1}C_{3} - A_{2}B_{1}A_{3}C_{1})m_{3}^{3} + (B_{3}B_{2}A_{1}A_{3}C_{2} + B_{3}A_{1}B_{1}A_{3}C_{1} + A_{2}^{2}C_{1}C_{2})m_{3}^{2} - 2B_{3}A_{1}A_{2}C_{1}C_{2}m_{3} + B_{3}^{2}A_{1}^{2}C_{1}C_{2} = 0$$

$$(4.22)$$

There are generally six roots to the polynomial (4.22). We may substitute each root into (4.20) and find corresponding  $m_2$ . Then (4.21) gives us  $m_1$ . There are therefore at most six possible solutions in this case. However, in general, the existence of a physically realisable solution is not guaranteed.

### 4.1.3 The general case

Let 
$$\binom{n}{p}_q$$
,  $q = 1, \dots, \frac{n!}{p!(n-p)!}$ 

denote the combinations of elements  $\{m_1, m_2, ..., m_n\}$  taken p elements at a time without repetitions, and let

$$\prod \binom{n}{p}_{q}, \ q=1,\dots,\frac{n!}{p!(n-p)!}$$

denote the products of the elements in these combinations.

Then for a general  $n \times n$  system (4.2) can be written as follows

$$\det (\mathbf{K} - \lambda \mathbf{M}) = \lambda^{n} \left( \prod {n \choose n} \right) + \lambda^{n-1} \left[ \sum_{q=1}^{n} \left[ G_{n-1,q} \prod {n \choose n-1}_{q} \right] + \dots + \lambda \left[ \sum_{q=1}^{n} \left[ G_{1,q} \prod {n \choose 1}_{q} \right] \right] + G_{0} = 0$$

$$(4.23)$$

where  $G_{i,q}$  (i=1,...,n-1) are constants and are explicitly determined from the elements of K.

Similarly we may write

$$\alpha(\lambda - \lambda_1^*)(\lambda - \lambda_2^*)...(\lambda - \lambda_n^*) = \alpha\lambda^n + \alpha F_1 \lambda^{n-1} + \alpha F_2 \lambda^{n-2} + ...$$
  
... +  $\alpha F_{n-1} \lambda + \alpha F_n = 0$  (4.24)

where the coefficients  $F_i$  (i=1,...,n) are constants and can be determined from  $\lambda_1^*$ , ...,  $\lambda_n^*$ .

Therefore equating the coefficients of (4.23) and (4.24), we have following system of n equations in n unknowns  $(m_1, ..., m_n)$ :

$$\alpha = \frac{G_0}{F_n} = \prod {\binom{n}{n}}$$

$$\alpha F_1 = \sum_{q=1}^n \left[ G_{n-1,q} \prod {\binom{n}{n-1}}_q \right]$$

$$\vdots$$

$$\alpha F_p = \sum_{q=1}^{\frac{n!}{p!(n-p)!}} \left[ G_{n-p,q} \prod {\binom{n}{n-p}}_q \right]$$

$$\vdots$$

$$\alpha F_{n-1} = \sum_{q=1}^n \left[ G_{1,q} \prod {\binom{n}{1}}_q \right]$$
(4.25)

It is difficult to determine a solution to these equations when n is large (we were unable to find a solution for the 4x4 case using a symbolic manipulator!). Friedland in [9] found that there are at most n! different solutions for the general problem. However, the existence of a physically realisable solution is not guaranteed. Also, in practical engineering designs, it is likely that the desired natural frequencies would be permitted to have some finite tolerance ranges. Thus any solution that would fall within these tolerances, would be suitable. The method presented in section 4.2 requires that the values for the desired natural frequencies be specified precisely, and then allows a finite number of possible solutions to be obtained. Approximate solutions cannot be obtained by this method.

### 4.2 Existing Methods

Two of the methods of solution for the inverse multiplicative eigenvalue problem, namely Downing and Householder (from now on D&H) [5] and Joseph [16], can be applied directly to the problem investigated here. Both methods are iterative, and have a local quadratic convergency. Thus, they do converge when the starting point is sufficiently close to a local solution. If they converge, the solution is obtained with a small number of iterations. However, if the starting point is not sufficiently close to a local solution the iterative algorithms may diverge or oscillate about the true solution.

Chu in [74] has discussed the effect of the rate of convergency of the two methods for a closely related *inverse singular value* problem. He found that a quadratically converging algorithm converges fast but locally, the linearly converging algorithm converges globally but at a slower rate. Thus it is possible that the convergence characteristics of an algorithm may be improved by reducing its rate of convergence.

A further consideration about the suitability of the two currently available methods was made based on the physical realisability criteria for the solution. In order to satisfy the physical realisability criteria, any obtained solution (i.e mass matrix) must be real, positive and diagonal. Although both methods always satisfy the diagonality requirement, the algorithm of Joseph may converge into a complex solution. The D&H algorithm, on the other hand, if converge, would always converge into a real and positive solution. Thus, the D&H algorithm was chosen as a basis for a new algorithm, and it is summarised below:

Algorithm 4.1: Downing and Householder Method

*Input:* Stiffness matrix **K** and eigenvalues matrix  $\Lambda^*$ .

#### Algorithm:

- 1) Choose an initial guess for a diagonal, positive-definite mass matrix  $M_0$ .
- 2) Set iteration index t=0.
- 3) Calculate the spectral decomposition:

 $M_{t}^{-\frac{1}{2}}K M_{t}^{-\frac{1}{2}} = U_{t}\Lambda_{t}U_{t}^{T}$ 

where  $\Lambda_t = \text{diag}(\lambda_1, \lambda_2, ..., \lambda_n), \lambda_1 > \lambda_2 > ... > \lambda_n$  is an eigenvalue matrix,  $U_t$  is an nxn orthonormal matrix (i.e  $U_t U_t^{\mathsf{T}} = I_n$ ) and  $I_n$  is the nxn identity matrix.

4) Calculate the real diagonal matrix  $Z_t$ , satisfying:  $\Lambda_t = \Lambda^* (I_n + Z_t)$ 

If the elements of  $\mathbf{Z}_{t}$  are sufficiently small, then stop.

5) Solve a system of linear equations:

$$\sum_{j=1}^{n} |u_{ji}|^2 d_{jj} = z_{ii} \qquad (i = 1, ..., n)$$

where  $u_{ii}$  and  $z_{ii}$  are elements of  $U_t$  and  $Z_t$  respectively.

6) Set  $D_t$ =diag(d<sub>11</sub>, ..., d<sub>nn</sub>).

7) Calculate the next iteration for the mass matrix :

$$M_{t+1} = (I_n - 0.5D_t)^{-2}M_t$$

8) Set iteration index t = t+1, and repeat from step 3.

*Output* (if converges): Real, positive and diagonal mass matrix M.

#### 4.3 The New Algorithm

Suppose that steps 1 to 3 of the D&H algorithm were carried out. Then, the spectral decomposition

$$\mathbf{M}_{t}^{-1/2}\mathbf{K}\mathbf{M}_{t}^{-1/2} = \mathbf{U}_{t} \mathbf{\Lambda}_{t}\mathbf{U}_{t}^{\mathrm{T}}$$
(4.26)

is known.

Define R as follows:

$$\mathbf{R} = \mathbf{U}_{t} \, \boldsymbol{\Lambda}^{* \frac{1}{2}} \boldsymbol{\Lambda}_{t}^{-\frac{1}{2}} \mathbf{U}_{t}^{\mathrm{T}} \,, \tag{4.27}$$

then

$$R \ U_{\ell} \Lambda_{\ell} U_{\ell}^{T} R = U_{\ell} \Lambda^{*} \ U_{\ell}^{T}$$

$$(4.28)$$

which means that the eigenvalues of  $\mathbf{RM}_{t}^{-4}\mathbf{KM}_{t}^{-4}\mathbf{R}$  are the diagonal elements of  $\Lambda^{*}$ . Thus, if **R** is real and diagonal, then an exact solution for the mass matrix  $\mathbf{M}=\mathbf{M}_{t+1}$  is given by:

$$\mathbf{M}_{t+1} = \mathbf{R}^{-1} \ \mathbf{M}_t \ \mathbf{R}^{-1} \ . \tag{4.29}$$

But in general **R** is not diagonal, and consequently  $\mathbf{M}_{t+1}$  would not be diagonal either. However, if we could find a real diagonal matrix, which is close to **R** in some sense, it may obtain an approximation for  $\mathbf{M}_{t+1}$ . Therefore, we define the following optimisation problem.

Given **R** as in (4.27). Find a real diagonal matrix **D**, such that the residual error:

$$\varepsilon = \|\boldsymbol{R} - \boldsymbol{D}\|_{F}^{2} \tag{4.30}$$

is minimised.

Applying the equality:

$$\|\boldsymbol{R} - \boldsymbol{D}\|_{F}^{2} = trace(\boldsymbol{R}^{T}\boldsymbol{R}) + trace(\boldsymbol{D}^{T}\boldsymbol{D}) - 2 trace(\boldsymbol{D}^{T}\boldsymbol{R})$$

$$(4.31)$$

then for any matrix **R** of (4.27) and using the fact that **D** is diagonal,  $\varepsilon$  is given by:

$$\varepsilon = trace(\mathbf{R}^{T}\mathbf{R}) + \sum_{i=1}^{n} \left[ d_{ii}^{2} - 2d_{ii}r_{ii} \right]$$

$$(4.32)$$

$$= trace(\mathbf{R} \ ^{T}\mathbf{R}) + \sum_{i=1}^{n} [d_{ii} - r_{ii}]^{2} - \sum_{i=1}^{n} r_{ii}^{2}$$
(4.33)

where  $d_{ii}$  and  $r_{ii}$  are the diagonal elements of **D** and **R** respectively. Then from (4.33) it is clear that  $\epsilon$  is minimised when

$$d_{\rm ii} = r_{\rm ii} \ . \tag{4.34}$$

Thus, the residual error  $\varepsilon$  is minimised when the diagonal elements of **D** are equal to the diagonal elements of **R**. Applying this result, the approximation for the mass matrix  $\mathbf{M}_{t+1}$  is equal to  $\mathbf{D}^{-2}\mathbf{M}_{t}$ . The following iterative algorithm is then proposed:

#### Algorithm 4.2: The New Method

*Input:* Stiffness matrix **K** and eigenvalues matrix  $\Lambda^*$ .

#### Algorithm:

- 1) Choose an initial guess for a diagonal, positive-definite mass matrix  $M_0$ .
- 2) Set iteration index t = 0.

3) Calculate the spectral decomposition:

 $M_{t}^{-\%}KM_{t}^{-\%} = U_{t}\Lambda_{t}U_{t}^{T}$ 

4) If the norm  $\| \Lambda^* - \Lambda_t \|_F$  is sufficiently small, then stop.

5) Calculate  $\mathbf{R} = \mathbf{U}_{t} \mathbf{\Lambda}^{* \vee} \mathbf{\Lambda}_{t}^{-\vee} \mathbf{U}_{t}^{\top}$ 

- 6) Form a real diagonal matrix **D** from the diagonal elements of **R**.
- 7) Calculate the next iteration of mass  $M_{t+1}=D^{-2}M_{t}$
- 8) Set iteration index t = t+1 and repeat from step 3.

*Output:* Real, positive and diagonal mass matrix **M**.

This algorithm and the two existing methods were tested on some numerical examples, and their performances were compared. It appears that the new algorithm is linearly convergent, and thus, in general, a significantly larger number of iterations is necessary than with the two existing methods. However, this disadvantage is balanced by a better global behaviour, in the sense that it usually converges to a solution even if the initial guess is not close to an actual solution. This relation between the rate of convergence of an algorithm and its convergence characteristics is similar to the one described by Chu in [74].

In all numerical examples tested, the new algorithm has converged into an optimal local solution. Furthermore, the diagonal elements of D are always real, and as a result M is always real and positive.

#### 4.4 Numerical Examples

The algorithms were tested with various combinations of  $\mathbf{K}$  and  $\Lambda^*$ . As expected, the parameters that have the most significant influence on the performance of the algorithms are the initial guess for the mass matrix. We present here the results of the numerical tests for one combination of  $\mathbf{K}$  and  $\Lambda^*$ , for several different initial guesses.

Consider the 10 degree-of-freedom system, with stiffness matrix

54	-										
	200	-10	-20	-5	-5	-10	0	0	-50	-50	
								-10			
	-20	0	300	-40	-30	-60	-10	0	-20	-10	
	-5	0	-40	400	-30	-40	-50	-20	-10	-70	
	-5	0	-30	-30	150	-10	-5	-5	-20	0	
<i>K</i> =	-10	0	-60	-40	-10	250	0	0	0	-80	
	0	-20	-10	-50	-5	0	120	-5	0	-10	
	0	-10	0	-20	-5	0	-5	250	0	-100	
	-50	-20	-20	-10	-20	- 0	0	0	350	-40	
	-50	-10	-10	-70	0	-80	-10	-100	-40	400	

We wish to find a positive-definite and diagonal matrix  $\mathbf{M}$ , such that the eigenvalues of the system are the diagonal elements of :

 $\Lambda^* = \text{diag}$  (500, 450, 400, 350, 300, 250, 200, 150, 100, 50).

Define

$$\mathbf{E}_{t} = \| \boldsymbol{\Lambda}^{*} \cdot \boldsymbol{\Lambda}_{t} \|_{2} \tag{4.35}$$

where t is an iteration index.  $\mathbf{E}_t$  is then equal to the maximum difference between any two corresponding elements of  $\Lambda^*$  and  $\Lambda_t$ , and thus measures the accuracy of the solution. The results of six tests are presented here. The initial conditions are shown in Table 4.1.

Test No.	Diagonal Elements of the Initial Guess, $M_0$	Diagonal Elements of the $\Lambda_0$	Residual E <sub>o</sub>
I	1, 1, 1, 1, 1, 1, 1, 1, 1, 1	507.2, 415.4, 364.6, 335.4, 231.2, 206.3, 160.6, 129.1, 108.4, 61.8	68_8
2	1, 2, 1, 3, 1, 4, 1, 5, 1, 6	369.5, 313.0, 191.1, 165.1, 139.5, 97.5, 83.5, 52.6, 48.0, 22.6	208,9
3	1, 1, 1, 2, 1, 1, 1, 1, 4, 4	345.3, 271.8, 245.6, 211.5, 201.4, 145.2, 116.6, 93.0, 87.8, 39.3	178.2
4	4, 1, 10, 3, 2, 2, 2, 2, 4, 10	151.8, 130.5, 124.1, 107.3, 88.9, 73.6, 52.8, 49.7, 31.4, 15.6	348.2
5	1 , 1 , 1, 1, 1, 1, 1, 1, 4, 4	428.1, 344.2, 267.2, 226.7, 202.7, 145.5, 120.6, 93.2, 88.6, 40.7	132.8
6	1.5, 0.5, 3.5, 1.0, 0.5, 1.0, 0.5, 0.5, 1.5, 3.5	522.9, 443.0, 312.5, 266.4, 245.2, 234.6, 171.9, 138.6, 94.1, 46.5	87,5

#### **Table 4.1: The Initial Conditions**

Figure 4.1 displays the results from the first twelve iterations of each algorithm. The logarithmic scale in this figure shows that the new algorithm converges linearly. Table 4.2 shows the obtained solutions for the various algorithms. The criteria used to terminate iterations was either when  $E_t > 10^4$  (i.e algorithm diverged), or when  $E_{t+1} - E_t < 0.01$  (i.e algorithm converged).

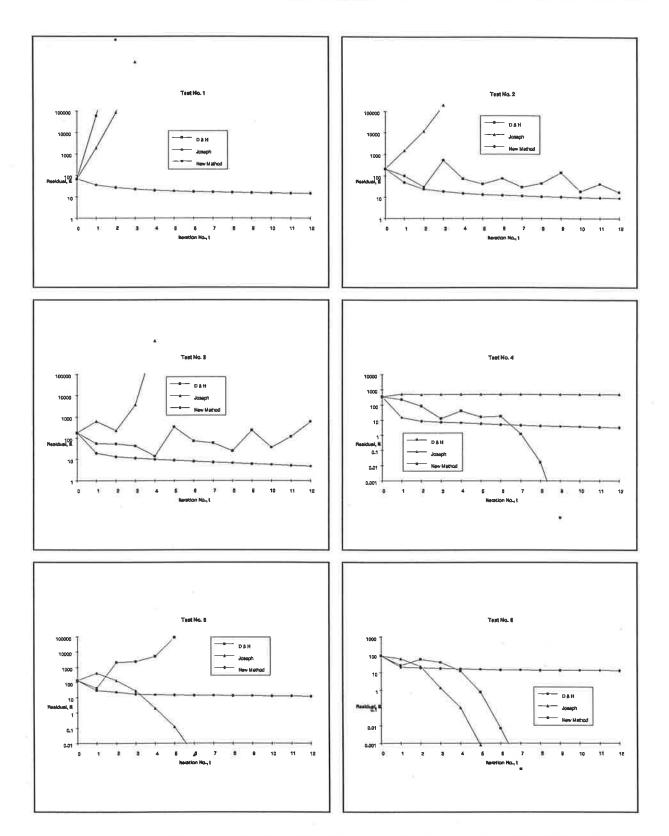


Figure 4.1 : Residual ( $E_t$ ) vs Iteration Number (t) plot for the first 12 iterations

Section	4: Pro	oblem .	1-	Design	for	Natural	Frequencies
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Test No.	Method	Iterations to Convergence	Diagonal Elements of the Obtained Mass Matrix, M t	Diagonal Elements of $\Lambda_t$	Residual E 1
		(t)	(given to 2 decimal places only)	(given to 1 decimal place only)	
	D & H	Diverge			-
1	Joseph	Diverge			
	New Method	425	0.60, 1.60, 0.96, 0.95, 0.75, 1.10, 0.83, 0.68, 1.03, 1.16	504.8, 451.4, 401.0, 353.1, 294.7, 254.9, 201.4, 151.2, 94.8, 50.7	5.3
	D & H	No Convergence	(B)		-
2	Joseph	Diverge	-	3 <b>2</b>	
	New Method	113	0.56, 0.99, 0.74, 1.27, 0.46, 1.18, 0.375, 1.80, 0.77, 3.27	500.4, 450.6, 399.9, 351.6, 297.8, 251.3, 197.5, 149.9, 99.9, 50.5	2.5
	D & H	No Convergence		÷	
3	Joseph	No Convergence		17.	-
	New Method	1050	0.70, 0.54, 0.71, 1.15, 0.50, 0.65, 0.62, 0.585, 3.64, 3.58	500.0, 450.0, 400.0, 350.0, 300.0, 250.0, 200.0, 150.0, 100.0, 50.0	0.0012
	D & H	9	1.36, 0.26, 3.24, 0.90, 0.49, 0.75, 0.51, 0.58, 1.43, 2.97	500.0, 450.0, 400.0, 350.0, 300.0, 250.0, 200.0, 150.0, 100.0, 50.0	3.9 x 10 <sup>-6</sup>
4	Joseph	4	Physically Unrealisable	All zeros	500
	New Method	396	1.36, 0.33, 3.13, 0.93, 0.60, 0.63, 0.49, 0.57, 1.13, 3.16	500.1, 449.9, 400.0, 351.8, 298.3, 249.9, 200.1, 150.0, 100.0, 50.0	1.8
	D & H	Diverge	•	2 <b></b>	*
5	Joseph	6	0.83, 0.34, 1.73, 1.29, 0.31, 0.61, 0.50, 0.65, 3.71, 3.36	500.0, 450.0, 400.0, 350.0, 300.0, 250.0, 200.0, 150.0, 100.0, 50.0	0.0017
	New Method	2356	0.58, 0.62, 0.74, 0.89, 0.52, 0.93, 0.50, 0.65, 3.62, 3.57	500.0, 450.1, 400.0, 350.0, 300.1, 250.1, 199.8, 150.0, 100.0, 50.0	0.2
	D&H	7	1.15, 0.27, 3.36, 0.94, 0.43, 0.86, 0.38, 0.59, 2.16, 2.79	500.0, 450.0, 400.0, 350.0, 300.0, 250.0, 200.0, 150.0, 100.0, 50.0	3.7 x 10 <sup>-5</sup>
6	Joseph	5	1.36, 0.35, 3.23, 1.06, 0.38, 0.71, 0.48, 0.53, 1.44, 2.99	500.0, 450.0, 400.0, 350.0, 300.0, 250.0, 200.0, 150.0, 100.0, 50.0	0.0008
	New Method	2137	1.45, 0.43, 3.62, 1.43, 0.35, 0.67, 0.42, 0.54, 1.2, 2.35	500.0, 450.0, 400.0, 350.0, 300.0, 250.0, 200.0, 150.0, 100.0, 50.0	0.0103

#### **Table 4.2: Obtained Solutions**

In test no.1 the D&H and Joseph's algorithms both diverged from first iteration onwards. The new algorithm achieved first iteration with  $E_1=35.7$  and converged to an approximate solution with  $E_{425}=5.3$  after 425 iterations. In test no.2 the Joseph's algorithm diverged from first iteration onwards. The D&H algorithm has neither converged nor diverged. It exhibited oscillatory behaviour about the solution. The closest point reached by the algorithm was at  $\mathbf{E}_{12}$  =17.0 after 12 iterations. The new algorithm achieved  $\mathbf{E}_1$ =49.8,  $\mathbf{E}_2$ =23.5 and converged at  $\mathbf{E}_{113}$ =2.5. In test no.3 both D&H and Joseph's algorithms have shown an oscillatory behaviour for the first few iterations and then began to diverge from t=2 and t=10 respectively. The new algorithm achieved  $\mathbf{E}_1$ =19.1,  $\mathbf{E}_2$ =13.3 and converged at  $\mathbf{E}_{1050}$ =0.0012. In test no.4 the D&H algorithm converged quadratically at t=9. The Joseph's algorithm has also converged, but not in the desired direction (refer Table 4.2 and Figure 4.2). The new algorithm achieved  $\mathbf{E}_1$ =14.4,  $\mathbf{E}_2$ =8.5 and converged at  $\mathbf{E}_{396}$ =1.8. In test no.5 the D&H algorithm diverged from the second iteration onwards, the Joseph's algorithm converged quadratically after 6 iterations, and the new algorithm achieved  $\mathbf{E}_1$ =29.1,  $\mathbf{E}_2$ =22.7 and converged at  $\mathbf{E}_{2356}$ =0.2. In test no.6 both the D&H and Joseph's algorithms have converged quadratically after 7 and 5 iterations respectively, and the new algorithm achieved  $\mathbf{E}_1$ =20.1,  $\mathbf{E}_2$ =18.3 and converged at  $\mathbf{E}_{2137}$ =0.01.

Thus we note that in some of the above examples the new method performs better, in terms of global convergency, than the existing algorithms.

#### 4.5 Conclusions

In this section we considered the problem of selecting the masses of a mass-spring system to achieve the desired natural frequencies. A closed-form solution for a two and three degree-of-freedom systems was given, but it appears impractical to obtain similar solutions for high order systems. Two existing iterative methods were then examined numerically, and found to have a local quadratic convergency. A new iterative method was then suggested, and numerical simulations show that it has better global convergency, but at a slower rate. Similar behaviour has been observed by Chu in [74] for a closely related inverse singular value problem. Perhaps the best strategy is to begin iterations with our method, and then switch to a quadratic method once within a proximity of a solution.

Although D&H method was selected as a basis for our algorithm, the method of Joseph is better suitable for problems where the desired natural frequency spectrum is incomplete (i.e less than n desired natural frequencies are specified) or for systems where mass and stiffness matrices are not independent of each other. Therefore, in section 6, where we analyse a problem of structural modifications to achieve prescribed natural frequencies, it was more convenient to base our solution algorithm on the method of Joseph.

### Section 5 PROBLEM 2:

## DESIGN FOR NATURAL FREQUENCIES AND MODE SHAPES<sup>2</sup>

In this section we present a solution to Problem 2, which was formulated in section 2.2. In Problem 2 we wish to determine mass and stiffness matrices **M** and **K** corresponding to a physically realisable mass-spring system, such that its modal and spectral properties, described by the modal matrix  $\Phi$  and the spectral matrix  $\Lambda$ , are as close as possible to the prescribed modal and spectral matrices  $\Phi^*$  and  $\Lambda^*$ .

We realise that the two problems of determining  $\Phi$  and  $\Lambda$  corresponding to a realisable system can be solved separately. Also note that satisfying equations (2.3) and (2.4) is a sufficient condition for equation (2.2) to hold.

 $<sup>^2</sup>$  Material presented in this section has been accepted for publication in the Journal of Sound and Vibration, reference [76].

Consider now the problem of determining the optimal mode shape matrix  $\Phi$ ,

#### Problem 2.1: Determination of Mode Shapes

Given  $\Phi^*$ , determine  $\Phi$  such that  $\mathbf{M} = \Phi^{-T} \Phi^{-1}$  is a diagonal positive definite matrix, and which minimises the norm  $\|\Phi^* - \Phi\|$ .

We analyse this problem in section 5.1. Once the solution  $\Phi$  is found, we solve the following problem

#### Problem 2.2: Determination of Eigenvalue Matrix

Given  $\Lambda^*$  and  $\Phi$ , determine  $\Lambda$  which minimises the norm  $\|\Lambda^* - \Lambda\|$ , such that  $\mathbf{K} = \Phi^{-T} \Lambda \Phi^{-1}$  satisfies the properties given by (2.6).

We present the global optimal solution to this problem in section 5.2. Determining the global optimal solution is computationally expensive. We therefore present another, local optimal approximation in section 5.3. A numerical example demonstrating the algorithms is presented in section 5.4, and conclusions are drawn in section 5.5.

#### 5.1 Mode Shape Optimisation.

Let  $\mathbf{D} = \text{diag} (d_1, d_2, ..., d_n), d_i \neq 0$ , and let  $\mathbf{Q}$  be an orthonormal matrix, that is,  $\mathbf{Q}\mathbf{Q}^T = \mathbf{I}_n$ . If  $\Phi = \mathbf{D}\mathbf{Q}$ , then the mass matrix  $\mathbf{M}$  obtained by equation (6) is physically realisable, since

$$\mathbf{M} = \mathbf{\Phi}^{-T} \mathbf{\Phi}^{-1} = (\mathbf{D}^{-1} \mathbf{Q}) (\mathbf{Q}^{T} \mathbf{D}^{-1}) = \mathbf{D}^{-2}$$
(5.1)

is a positive definite diagonal matrix.

Thus a solution to Problem 2.1 can be obtained by determining a diagonal matrix  $\mathbf{D}$  and an orthonormal matrix  $\mathbf{Q}$ , such that

$$\min_{\mathbf{D},\mathbf{Q}} \| \Phi^* - \mathbf{D}\mathbf{Q} \|$$
 (5.2)

In solving this problem we will make use of the following result. Given two *nxn* matrices **A** and **B**, the well known *orthogonal Procrustes problem*, is to determine an orthonormal matrix **Q**, such that

$$\min_{\mathbf{Q}} \| \mathbf{A} - \mathbf{B}\mathbf{Q} \|_{\mathsf{F}}.$$
 (5.3)

An algorithm for solving this problem is given below (see e.g. Golub and van Loan [72, p.582]).

Algorithm 5.1: Orthogonal Procrustes Problem

Input: Two nxn matrices A and B.

Algorithm:1)Set  $C = B^T A$ .2)Compute the singular value decomposition  $C=U\Sigma V^T$ .3)Evaluate  $Q=UV^T$ .

Output: Orthonormal Q, which solves (5.3).

Thus we may choose a diagonal matrix  $\mathbf{D}_0$  as an initial guess and obtain an orthonormal  $\mathbf{Q}_0$  which minimises  $\| \Phi^* - \mathbf{D}_0 \mathbf{Q}_0 \|_F$ , by using Algorithm 5.1. We now show how to obtain a matrix  $\mathbf{D}_1$  such that

$$\left\| \boldsymbol{\Phi}^{*} - \mathbf{D}_{1} \mathbf{Q}_{0} \right\|_{\mathrm{F}} \leq \left\| \boldsymbol{\Phi}^{*} - \mathbf{D}_{0} \mathbf{Q}_{0} \right\|_{\mathrm{F}}.$$
(5.4)

The Frobenius norm is invariant under orthonormal multiplication. Hence

$$\| \Phi^* - \mathbf{D}_1 \mathbf{Q}_0 \|_{\mathbf{F}} = \| \Phi^* \mathbf{Q}_0^{\mathbf{T}} - \mathbf{D}_1 \|_{\mathbf{F}},$$
 (5.5)

Define 
$$\mathbf{R} = \boldsymbol{\Phi}^* \mathbf{Q}_0^{\mathrm{T}},$$
 (5.6)

and denote  $\epsilon = \| R - D_1 \|_F^2$  (5.7)

Using the equality

$$\|R - D_1\|_F^2 = trace(R^T R) + trace(D_1^T D_1) - 2 trace(D_1^T R)$$
(5.8)

we find that

$$\epsilon = trace (\mathbf{R}^T \mathbf{R}) + \sum_{i=1}^{n} \left[ d_{ii}^2 - 2d_{ii}r_{ii} \right]$$
(5.9)

$$= trace(\mathbf{R}^{T}\mathbf{R}) + \sum_{i=1}^{n} \left[ d_{ii} - r_{ii} \right]^{2} - \sum_{i=1}^{n} r_{ii}^{2}$$
(5.10)

where  $\mathbf{D}_1 = \text{diag}(\mathbf{d}_{ii})$  and  $\mathbf{R} = [\mathbf{r}_{ij}]$ . Then from (5.10) it is clear that  $\epsilon$  is minimised when

$$d_{ii} = r_{ii} agenum{5.11}{}$$

Thus, the residual error  $\epsilon$  is minimised when the diagonal elements of  $\mathbf{D}_1$  are equal to the diagonal elements of  $\mathbf{R}$ . Having determined a diagonal matrix  $\mathbf{D}_1$  satisfying (5.4), we can reapply the Algorithm 5.1 with  $\Phi^*$  and  $\mathbf{D}_1$  as an input and find an orthonormal matrix  $\mathbf{Q}_1$  such that

$$\| \Phi^* - \mathbf{D}_1 \mathbf{Q}_1 \|_{\mathrm{F}} \le \| \Phi^* - \mathbf{D}_1 \mathbf{Q}_0 \|_{\mathrm{F}} .$$
 (5.12)

Continuing in this manner iteratively, we obtain an approximation to the Problem 2.1. The following algorithm summarises this result.

Algorithm 5.2: Approximate Solution to Problem 2.1

*Input*: An nxn modal matrix  $\Phi^*$ .

#### Algorithm: 1) Set initial guess $D_0$ and a tolerance for convergence $\epsilon$ .

- 2) For i=0, 1, 2, ...
  - a) Evaluate  $C = D_i^T \Phi^*$ .
  - b) Compute the singular value decomposition  $C=U\Sigma V^{T}$ .
  - c) Evaluate  $Q_i = UV^T$ .
  - d) Obtain  $\mathbf{R} = \Phi^* \mathbf{Q}_i^T$ .
  - e)  $\mathbf{D}_{i+1} = \text{diag} (\mathbf{r}_{11}, \mathbf{r}_{22}, ..., \mathbf{r}_{nn}).$
  - f) Test convergence
    - (i) Set  $N_1 = \| \Phi^* D_i Q_i \|_F$ ,  $N_2 = \| \Phi^* D_{i+1} Q_i \|_F$ .
    - (ii) If  $(N_1 N_2) \le \epsilon$ , go to 3.

3) 
$$D = D_{i+1}, Q = Q_{i}$$
.

*Output:* A diagonal matrix **D** and an orthonormal matrix **Q** which approximate the solution of (5.2).

It follows from (5.4) and (5.12) that  $\| \Phi^* - \mathbf{D}_i \mathbf{Q}_i \|_F$  is a monotonic non-increasing function of an iteration index i. The Algorithm 5.2 thus necessarily converge.

The geometrical interpretation of the Algorithm 5.2 is also clear. The stage of calculating  $Q_i$  is equivalent to finding an optimal *rotation* of the matrix  $D_i$  into  $\Phi^*$ . The subsequent stage of determining  $D_{i+1}$  is equivalent to *projecting* the column vectors of  $\Phi^*$  onto the axis defined by the column vectors of  $Q_i$ . This *rotation-projection* procedure is then carried out

until an optimal combination of **D** and **Q**, which solves (5.2), is achieved. Note that problem (5.2) itself can be described geometrically for a two- and three-dimensional space. Figure 5.1 shows an equivalent geometrical problem to (5.2) in a two-dimensional space.

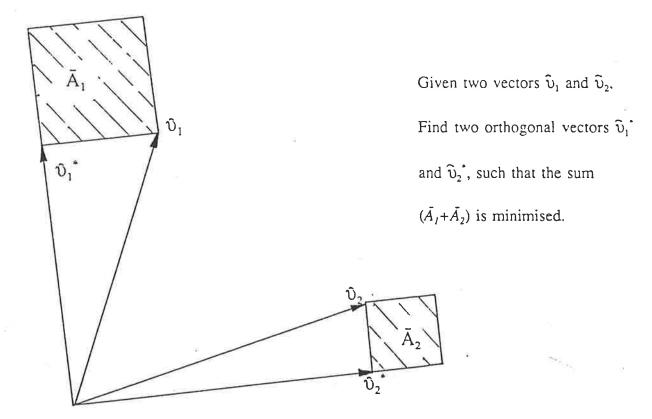


Figure 5.1: Geometric problem equivalent to (5.2) in a two-dimensional space.

#### 5.2 Global Optimisation for Eigenvalues

Using the method described in section 5.1, we obtain a matrix  $\Phi = \mathbf{D}\mathbf{Q}$ , which satisfies the physical realisability criteria for **M** while minimising  $\| \Phi^* - \mathbf{D}\mathbf{Q} \|_{F}$ . In this section we will use this result to obtain a physically realisable **K** which satisfies equation (2.4) while minimising  $\| \Lambda^* - \Lambda \|$ .

The physical realisability criteria for the connectivity of  $\mathbf{K}$ , as described in (2.6), arise from the requirement that the stiffness of all the springs in mass-spring systems must be nonnegative. Thus, if we ensure that all the springs have non-negative stiffness, then we necessarily satisfy the conditions of (2.6).

The stiffness matrix K, may be written in the following form

$$K = \sum_{p=0}^{n-1} \sum_{q=p+1}^{n} s_{pq} \boldsymbol{B}_{pq}^{(K)}$$
(5.13)

where  $s_{pq}$  is the stiffness of the spring connecting mass p to mass q,  $s_{op}$  represents the stiffness of the spring which connects mass p to the ground, and  $\mathbf{B}_{pq}^{(K)}$  is the matrix describing the spring connection between mass p and mass q,

$$B_{pq}^{(K)} = \begin{bmatrix} b_{ij}^{(K)} \end{bmatrix} = \begin{cases} b_{pp}^{(K)} = b_{qq}^{(K)} = 1\\ b_{pq}^{(K)} = b_{qp}^{(K)} = -1\\ b_{ij}^{(K)} = 0 \text{ elsewhere} \end{cases}, \ (p \neq q).$$
(5.14)

Substituting equation (5.13) into equation (2.4), we obtain

$$\mathbf{\Lambda} = \sum_{p=0}^{n-1} \sum_{q=p+1}^{n} s_{pq}(\mathbf{\Phi}^T \mathbf{B}_{pq}^{(K)} \mathbf{\Phi}) \quad .$$
(5.15)

Each of the ij-th element of  $\Lambda$  is thus given by

$$\lambda_{ij} = \sum_{p=0}^{n-1} \sum_{q=p+1}^{n} s_{pq} (\mathbf{\phi}_i^T \mathbf{B}_{pq}^{(K)} \mathbf{\phi}_j) \quad .$$
 (5.16)

Let N=  $\frac{1}{2}(n^2 + n)$  and construct the vectors

$$\mathbf{y}_{\mathbf{K}} = (y_1^{(K)}, y_2^{(K)}, ..., y_N^{(K)})^{\mathrm{T}} = (\lambda_{11}, \lambda_{12}, \lambda_{13}, ..., \lambda_{1n}, \lambda_{22}, \lambda_{23}, ..., \lambda_{2n}, \lambda_{33}, ..., \lambda_{nn})^{\mathrm{T}}$$
(5.17)

and

$$\mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \dots, \mathbf{x}_N)^{\mathrm{T}} = (\mathbf{s}_{01}, \mathbf{s}_{12}, \mathbf{s}_{13}, \dots, \mathbf{s}_{1n}, \mathbf{s}_{02}, \mathbf{s}_{23}, \dots, \mathbf{s}_{2n}, \mathbf{s}_{03}, \dots, \mathbf{s}_{nn})^{\mathrm{T}}.$$
(5.18)

Denote

$$F_{K} = \left[f_{ij}^{(K)}\right] = \frac{\partial y_{i}^{(K)}}{\partial x_{j}}, \qquad (i,j = 1, 2,..., N)$$
(5.19)

then all the elements of  $\mathbf{F}_{\mathbf{K}}$  can be evaluated using equation (5.16). Equation (5.15) can be written in a vector form

$$\mathbf{F}_{\mathbf{K}} \mathbf{x} = \mathbf{y}_{\mathbf{K}}.$$

In order to satisfy the physical realisability criteria we require all the elements of  $\mathbf{x}$  to be non-negative. Setting  $\Lambda = \Lambda^*$  we may determine the vector  $\mathbf{y}_{\mathbf{K}}$  and solve the following *nonnegative least squares problem* 

$$\min_{\mathbf{x}} \| \mathbf{F}_{\mathbf{K}} \mathbf{x} - \mathbf{y}_{\mathbf{K}} \|_{2}, \text{ subject to } \mathbf{x} \ge 0.$$
 (5.21)

An algorithm for the solution of this problem is given in [73, p.161]. (The standard MATLAB function **nnls** solves this problem). Thus the stiffnesses  $s_{pq}$  can be obtained from the solution **x** of (5.21), via equation (5.18), which in turn determines the matrix **K** by (5.13).

The above process gives an optimal solution to the eigenvalue matrix optimisation problem, because it is the best positive solution in a least square sense. We note that in order to obtain a solution for the n degrees-of-freedom system, we need to solve an augmented

system (5.21) of dimension N. This is a computational barrier, and an alternative approach is presented in the next section.

#### 5.3 Local Optimisation for Eigenvalues.

Alternatively, the stiffness matrix **K** may be obtained by a local optimisation procedure. Setting  $\Lambda = \Lambda^*$  and multiplying both sides of equation (2.4) by  $\Phi^{-1}$ , we have

$$\mathbf{\Phi}^{\mathrm{T}}\mathbf{K} = \mathbf{\Lambda}^{*}\mathbf{\Phi}^{-1} \ . \tag{5.22}$$

Denote

$$\mathbf{A} = \mathbf{\Lambda}^* \mathbf{\Phi}^{-1} \tag{5.23}$$

and partition A and K as follows

$$\mathbf{A} = [ \ a_1 \ | \ a_2 \ | \ a_3 \ | \ \dots \ | \ a_n \ ], \tag{5.24}$$

$$\mathbf{K} = [\mathbf{k}_1 \mid \mathbf{k}_2 \mid \mathbf{k}_3 \mid .... \mid \mathbf{k}_n].$$
(5.25)

Then from equation (5.22), each column of A is given by

$$\mathbf{\Phi}^{\mathrm{T}}\mathbf{k}_{\mathrm{i}} = a_{\mathrm{i}} \quad (\mathrm{j} = 1, ..., \mathrm{n}) \;. \tag{5.26}$$

We now show how to solve equation (5.26) column by column sequentially. The stiffness matrix  $\mathbf{K}$  for a general mass-spring system of order n has the following form

$$\boldsymbol{K} = \begin{bmatrix} k_{11} & -k_{12} & -k_{13} & -k_{14} & \cdots & -k_{1n} \\ -k_{21} & k_{22} & -k_{23} & -k_{24} & \cdots & -k_{2n} \\ -k_{31} & -k_{32} & k_{33} & -k_{34} & \cdots & -k_{3n} \\ -k_{41} & -k_{42} & -k_{43} & k_{44} & \cdots & -k_{4n} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -k_{n1} & -k_{n2} & -k_{n3} & -k_{n4} & \cdots & k_{nn} \end{bmatrix}$$
(5.27)

and physical realisability requires that

(a) 
$$k_{ij} = k_{ji} \ge 0$$
, for all  $1 \le i, j \le n$ , and  
(5.28)  
(b)  $k_{jj} = \sum_{\substack{i=1 \\ i \ne j}}^{n} k_{ji} \ge 0$   $(j=1,2,...,n)$ 

The physical parameters appearing in the first column of K can be approximated by solving

$$\min_{\mathbf{k}_{1}} \| \Phi^{\mathsf{T}} \mathbf{k}_{1} - a_{I} \| \text{ , subject to } \mathbf{G}_{(1)} \mathbf{k}_{1} \ge 0$$
 (5.29)

where 
$$G_{(1)} = \begin{bmatrix} 1 & 1 & 1 & 1 & \cdots & 1 \\ 0 & -1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & -1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & -1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & -1 \end{bmatrix}$$
;  $G_{(1)} \in \mathbb{R}^{n \times n}$ . (5.30)

Then setting

$$\mathbf{z}_1 = \mathbf{G}_{(1)} \mathbf{k}_1 \tag{5.31}$$

and

$$E_{(1)} = \Phi^T G_{(1)}^{-1} \tag{5.32}$$

we find that (5.29) can be transformed to the standard non-negative least squares form.

$$\min_{\mathbf{Z}_{1}} \| \mathbf{E}_{(1)} \mathbf{z}_{1} - \mathbf{a}_{I} \|, \text{ subject to } \mathbf{z}_{1} \ge 0.$$
(5.33)

The solution  $z_1$  of (5.33) then determines the physical stiffnesses in  $k_1$ , as shown

$$\mathbf{k}_{1} = \mathbf{G}_{(1)}^{-1} \mathbf{z}_{1}. \tag{5.34}$$

1

In a similar manner the physical parameters appearing in  $\mathbf{k}_{j}$ , the j<sup>th</sup> column of  $\mathbf{K}$ , can be approximated. By the symmetry of  $\mathbf{K}$  the first (j-1) elements in the j<sup>th</sup> step have been already determined in the previous steps. Hence denoting

$$\hat{k}_{j} = \left[-k_{1j}, ..., -k_{j-1j}, \sum_{i=1}^{j-1} k_{ij}, 0, 0, ..., 0\right]^{T}$$
(5.35)

$$\overline{k}_{j} = \begin{bmatrix} 0, ..., 0, \overline{k}_{jj}, -k_{j+1j}, ..., -k_{nj} \end{bmatrix}^{T}$$
 (5.36)

we may write

$$k_j = \hat{k}_j + \bar{k}_j \qquad (5.37)$$

. . . .

where  $\hat{\mathbf{k}}_{j}$  is known and  $\mathbf{\bar{k}}_{j}$  is to be determined. Substituting (5.37) into equation (5.26) gives

$$\Phi^T \hat{k}_j + \Phi^T \overline{k}_j = a_j \tag{5.38}$$

Let  $\Phi$  be partitioned in the form

$$\boldsymbol{\Phi} = \begin{bmatrix} \boldsymbol{\Psi} \\ \boldsymbol{\Phi}_{(j)} \end{bmatrix}, \quad \boldsymbol{\Phi}_{(j)} \in \mathbb{R}^{(n-j+1) \times n} \quad . \tag{5.39}$$

Define

$$a_j^* = a_j - \Phi^T \hat{k}_j \quad , \tag{5.40}$$

and by truncating the zero elements of the vector  $\mathbf{\bar{k}}_{j}$  in (5.36), set

$$k_j^* = \left[ \ \bar{k}_{jj}, \ -k_{j+1}, \ ..., \ -k_{nj} \right]$$
 (5.41)

Then a non-negative  $\mathbf{k}_{j}^{*}$  which approximates the solution of equation (5.38) in least square sense, can be obtained by solving

where 
$$G_{(j)} = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 0 & -1 & 0 & \cdots & 0 \\ 0 & 0 & -1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & -1 \end{bmatrix}$$
;  $G_{(j)} \in \mathbb{R}^{(n-j+1) \times (n-j+1)}$  (5.43)

Denote

$$\mathbf{z}_{j} = \mathbf{G}_{(j)} \mathbf{k}_{j}^{\star} \tag{5.44}$$

and

$$E_{(j)} = \Phi_{(j)}^T G_{(j)}^{-1}$$
(5.45)

then the standard non-negative least square form of (5.42) is given by

$$\min_{\mathbf{Z}_{j}} \| \mathbf{E}_{(j)} \mathbf{z}_{j} - \mathbf{a}_{j}^{*} \|, \text{ subject to } \mathbf{z}_{j} \ge 0.$$
(5.46)

Solving (5.46) for  $z_j$ , then  $k_j^*$  can be obtained by

$$\mathbf{k_{j}}^{*} = \mathbf{G_{(j)}}^{-1} \mathbf{z_{j}}$$
 (5.47)

This determines the unknown stiffnesses in the  $j^{th}$  column of **K**. Applying this process for j=2, ..., n evaluates the complete matrix **K** in a physically realisable form. The following algorithm summarises the above process.

#### Algorithm 5.3: Approximate Solution to Problem 2.2

*Input:* A modal matrix  $\Phi$  (obtained in section 5.1), and a desired spectral matrix  $\Lambda^*$ .

Algorithm: 1) Calculate A using equation (5.23) and partition A as in (5.24). This

determines the vectors  $a_j$ , j=1,2,...,n.

- 2) Construct the matrix  $G_{(1)}$  as in (5.30).
- 3) Determine the matrix  $\mathbf{E}_{(1)}$  using (5.32).
- Determine the vector z<sub>1</sub> by solving the non-negative least square problem (5.33).
- Obtain k<sub>1</sub> from (5.34). This determines the first row and column of K=[k<sub>ii</sub>].
- 6) for j = 2, 3, ..., n
  - (a) Set the vector  $\mathbf{\hat{k}}_{j}$  using (5.35).
  - (b) Obtain  $\Phi_{(i)}$  by partitioning  $\Phi$  as in (5.39).
  - (c) Determine  $a_i^*$  from equation (5.40).
  - (d) Construct  $G_{(j)}$  as in (5.43) and calculate  $E_{(j)}$  by (5.45).
  - (e) Determine  $z_j$  by solving the non-negative least square problem (5.46).
  - (f) Calculate  $\mathbf{k_i}^*$  from equation (5.47).
  - (g) Construct vector  $\mathbf{\bar{k}}_{j}$  by augmenting  $\mathbf{k}_{j}^{*}$  with zero elements as shown in (5.36) and (5.41).
  - (h) Obtain k<sub>j</sub> from equation (5.37). This determines the j-th row and column of K, without destroying the symmetry of its first (j-1) rows and columns.

*Output:* A physically realisable stiffness matrix **K** which approximates the solution of Problem 2.2 in the local optimisation sense.

The computational expense of this process is approximately equal to solving n times a nonnegative least square problem of dimensions n, (n-1), ..., 1. This is more efficient then solving an augmented system of dimension N.

#### 5.4 Numerical Example

The local optimisation solution obtained by Algorithm 5.3 is not the optimal solution in the global sense, such as described in section 5.1. It is shown in this section by means of a numerical example that the quality of solution is not greatly affected.

Consider a solution to the problem described in Example 2.2.1. The desired dynamic properties,  $\Lambda^*$  and  $\Phi^*$ , for a five degrees-of-freedom mass-spring system are:

$$\Lambda^* = \text{diag} (50, 100, 200, 400, 800)$$

and

$$\boldsymbol{\Phi}^* = \begin{bmatrix} 0.1 & -0.1 & 0.2 & -0.4 & 0.1 \\ 0.1 & 0.1 & 0.2 & 0.1 & 0.3 \\ 0.1 & -0.1 & 0.3 & 0.2 & -0.4 \\ 0.1 & -0.3 & -0.1 & -0.1 & -0.1 \\ 0.3 & 0.2 & -0.1 & 0.1 & 0.1 \end{bmatrix}$$

and we wish to determine physically realisable M and K which have dynamic characteristics as close as possible to the above data.

It was shown that there is no exact solution for these data since M and K obtained by equations (2.7) and (2.8) are not physically realisable solutions for a mass-spring system. We now show how to determine an optimal solution.

Applying Algorithm 5.2, we obtain a diagonal matrix **D** and an orthonormal matrix **Q**, such that  $\Phi = \mathbf{D}\mathbf{Q}$  is given by

	0.1232	-0.0333	0.2107	-0.3988	0.0414
ľ	0.0578	-0.0363	0.1922	0.1503	0.2680
Φ =	0.1337	-0.0707	0.3292	0.1820	-0.3767
	0.1186	-0.2856	-0.1045	-0.3988 0.1503 0.1820 0.0061 0.0324	0.0073
	0.3233	0.1725	-0.1021	0.0324	0.0088

and  $\| \Phi^* - \Phi \|_F$  is minimised.

Substituting  $\Phi$  in equations (2.7) and (2.8), we obtain

M = diag(4.5152, 7.3516, 3.2650, 9.3757, 6.8583)

	1523.9	-216.2	-392.0 -1356.4 1597.1 -178.5	-146.0	-240.6
	-216.2	4009.4	-1356.4	-48.7	10.5
<b>K</b> =	-392.0	-1356.4	1597.1	-178.5	-135.7
	-146.0	-48.7	-178.5	976.0	-47.9
	-240.6	10.5	-135.7	-47.9	506.5

The mass matrix **M** is now physically realisable, whereas the stiffness matrix **K** is not realisable. Therefore, setting  $\Lambda = \Lambda^*$  and applying the global optimisation procedure described in section 5.2, we obtain the following realisable stiffness matrix

$$\boldsymbol{K'} = \begin{bmatrix} 1512.0 & -227.2 & -337.7 & -144.3 & -245.4 \\ -227.2 & 4012.4 & -1277.9 & -41.7 & 0 \\ -337.7 & -1277.9 & 1690.1 & -37.6 & -36.9 \\ -144.3 & -41.7 & -37.6 & 939.8 & -69.1 \\ -245.4 & 0 & -36.9 & -69.1 & 454.2 \end{bmatrix}$$

and

A mass-spring system corresponding to the mass matrix  $\mathbf{M}$  and the stiffness matrix  $\mathbf{K}'$  is shown in Figure 5.2.

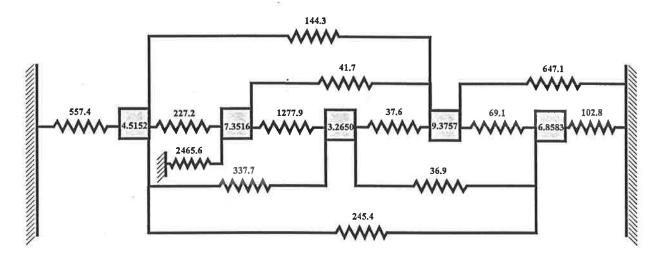


Figure 5.2: A mass-spring system corresponding to M and K'.

This realisable mass-spring system has the following modal data

 $\Lambda' = diag(52.8, 101.2, 214.3, 401.3, 795.2)$ -0.0265 -0.38480.0335 0.2488 0.0982 0.0267 -0.0165 0.1995 0.2639 0.1600 0.0553 -0.0273  $\Phi' =$ 0.3379 0.2010 -0.3846 0.0939 -0.3087 -0.0488 0.0137 -0.00020.3538 0.1203 -0.0691 0.0375 0.0012

This compares reasonably well with the desired properties  $\Lambda^*$  and  $\Phi^*$ .

However, the above solution is computationally expensive. Applying Algorithm 5.3 to the above example, we obtain a physically realisable stiffness matrix

	1523.9	-216.2	-392.0	-146.0	-240.6
				-48.7	0
<b>K</b> <sup>''</sup> =	-392.0	-1356.4	1748.4	0	0
	-146.0	-48.7	0	976.0	-47.9
	-240.6	0	0	-47.9	506.5

Figure 5.3 shows a mass-spring system corresponding to the mass matrix M and the stiffness matrix K''.

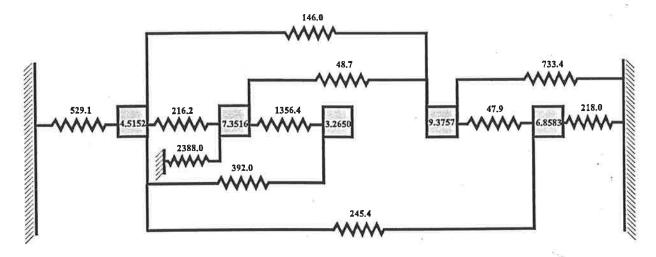


Figure 5.3: A mass-spring system corresponding to M and K''.

This mass-spring system has the following modal properties

$$\Lambda'' = diag(62.6, 104.0, 200.5, 407.6, 821.7)$$

$$\mathbf{\Phi}'' = \begin{bmatrix} 0.0985 & -0.0314 & 0.2479 & -0.3838 & 0.0454 \\ 0.0251 & -0.0174 & 0.2013 & 0.1683 & 0.2574 \\ 0.0471 & -0.0255 & 0.3384 & 0.1863 & -0.3927 \\ 0.0841 & -0.3116 & -0.0474 & 0.0161 & -0.0028 \\ 0.3578 & 0.1087 & -0.0661 & 0.0400 & -0.0021 \end{bmatrix}$$

Section 5: Problem 2 - Design for Natural Frequencies and Mode Shapes

We note that the global optimal solution is slightly better than the local one. They both, however, lead to essentially similar systems. Table 5.1 shows the cosines of the angles between the desired mode shapes and the modes of the physical systems which have been obtained. Let  $\theta$  be the angle between two eigenvectors. Then  $\cos(\theta)=1$  indicates identical eigenvectors.

Mode No., j	1	2	3	4	5
$\cos \measuredangle(\phi^*_{j},\phi_j)$	0.9885	0.9210	0.9988	0.9586	0.9580
$\cos \triangle(\phi^*_{j}, \phi'_{j})$	0.9648	0.9015	0.9852	0.9541	0.9558
$\cos \triangle (\phi_j^*, \phi_j')$	0.9587	0.8948	0.9845	0.9506	0.9571

 Table 5.1: Cosines of angles between the desired mode shapes

 and their approximations

We asked for mass-normalised eigenvectors. Hence the amplitude ratio between the desired mode shapes and their approximation is of interest as well. These amplitude ratios are given in Table 5.2.

Mode No., j	1	2	3	4	5
φ <sub>j</sub>    /    φ* <sub>j</sub>	1.0918	0.8617	1.0541	0.9688	0.8773
φ′ <sub>j</sub>    /    φ* <sub>j</sub>	1.0649	0.8346	1.0835	0.9683	0.8838
$\left\  \phi_{j}^{\prime\prime} \right\  / \left\  \phi_{j}^{*} \right\ $	1.0657	0.8323	1.0835	0.9605	0.8915

### Table 5.2: Amplitude ratios between the desired mode shapes and their approximations

The results in Tables 5.1 and 5.2 present a good agreement between the desired mode shapes and the modes obtained.

#### 5.5 Conclusions

The problem of constructing a mass-spring system with prescribed eigenvalues and mode shapes has been addressed. This is a non-linear approximation problem since the number of constraints, the eigendata, is larger than the number of free parameters, the number of masses and springs in the system.

It is shown that the problems of determining the mass and stiffness matrices can be solved separately. First, an optimal set of mode shapes associated with a physically realisable mass matrix is obtained. This is done by a convergent iterative algorithm. Then a physically realisable stiffness matrix is determined using the optimal mode shapes obtained in the previous stage.

Two methods of obtaining a physically realisable stiffness matrix have been suggested. One method determines a global optimal solution in a least square sense. This method involves non-linear optimisation of large matrices of order N for a problem with n degrees of freedom. The other method breaks the problem into n sub-problems of small dimensions and determines a local optimal solution for each sub-problem. The result is a computationally economical method of solution. It is shown through a numerical example that both methods lead to similar solutions.

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#### Section 5: Problem 2 - Design for Natural Frequencies and Mode Shapes

We note that in the given numerical example, the modal shapes of  $\Phi$ ,  $\Phi'$  and  $\Phi''$  do not have the same number of changes of sign among the elements of their columns as in the corresponding columns of  $\Phi^*$ . Based on this criteria, we may say that the mode shapes of  $\Phi$ ,  $\Phi'$  and  $\Phi''$  are *qualitatively* different from the corresponding mode shapes of  $\Phi^*$ . In the analysis we used to solve Problem 2, we have optimised the modal vectors to be as close as possible to the prescribed modal vectors in the Frobenius norm sense, without imposing the additional constraints of sign changes. Thus, extending the above method to allow for such sign change control would constitute a significant improvement to the method.

The results presented in this paper may be applied in designing physically realisable systems with prescribed spectral constrains, and in identifying realisable systems from modal test data.

# Section 6 PROBLEM 2(a): RECONSTRUCTION FROM TRUNCATED MODAL AND SPECTRAL DATA

In section 5 we have considered a problem associated with reconstruction of a physically realisable mass-spring system from the prescribed set of modal and spectral data. The main assumption on which we based our analysis is that the prescribed modal and spectral data are such that the resultant matrices  $\Phi^*$  and  $\Lambda^*$  are full matrices of size *nxn*. Thus in section 5 we have developed a solution to the problem of reconstruction from a *complete* set of data. In many (perhaps most) practical applications the desired modal and spectral properties are usually specified for only the first few lower modes, thus leading to a *truncated* set of prescribed modal and spectral data. In this section we show how to reconstruct various models of vibratory systems from such truncated sets of data. Since in all other respects, apart from the truncation of  $\Phi^*$  and  $\Lambda^*$ , this problem is identical to Problem 2, we designate it as Problem 2(a).

Partitioning  $\Phi^*$  and  $\Lambda^*$  in the form :

$$\Phi^* = [\Phi_1^* | \Phi_2^*], \quad \Phi_1^* \text{ is an } n \times l \text{ real matrix, } l < n \tag{6.1}$$

and

$$\Lambda^* = \left[\frac{\Lambda_1^*}{0} | \frac{0}{\Lambda_2^*}\right] , \Lambda_1 = \operatorname{diag}(\lambda_1^*, \dots, \lambda_l^*), \qquad (6.2)$$

we assume that  $\Phi_1^*$  and  $\Lambda_1^*$  are specified, while the submatrices  $\Phi_2^*$  and  $\Lambda_2^*$  remain unknown. Then the formulation of a problem we want to solve is as follows.

Problem 2(a): Reconstruction from truncated modal and spectral data.

Given  $\Phi_1^*$  and  $\Lambda_1^*$ , determine physically realisable mass and stiffness matrices **M** and **K**, such that the vibratory system contains modes which are as close as possible to the prescribed data.

We note that the orthogonality relations

$$\Phi_1^{*\mathrm{T}} \mathbf{M} \Phi_1^{*} = \mathbf{I}_l \tag{6.3}$$

and

$$\Phi_1^{*T} \mathbf{K} \Phi_1^{*} = \Lambda_1^{*} \tag{6.4}$$

are still valid. However, due to the effects of truncation, obtaining M and K which satisfy (6.3) and (6.4) respectively is not necessarily a solution to Problem 2(a). In fact, there may exist many different combinations of M and K that satisfy (6.3) and (6.4), but which are not solutions to Problem 2(a). Thus, in order for us to have a solution, M and K should satisfy

$$K\Phi_{1}^{*} = M\Phi_{1}^{*} \Lambda_{1}^{*} , \qquad (6.5)$$

and, to maintain the mass-normalisation properties, M should also satisfy (6.3).

As in the method presented in section 5, we realise that the solutions for M and K may be obtained separately. First, we find M satisfying (6.3), and then determine K such that (6.5) holds.

#### 6.1 Reconstruction of a Mass Matrix

The mass matrix M, may be written in the following form

$$M = \sum_{j=1}^{n} m_j B_j^{(M)}$$
(6.6)

where  $m_j$  is the mass of the j<sup>th</sup> element, and  $B_j^{(M)}$  is the mapping matrix. For a mass-spring model  $B_j^{(M)}$  is as follows

$$B_{j}^{(M)} = \begin{bmatrix} b_{pq}^{(M)} \end{bmatrix} = \begin{cases} b_{jj}^{(M)} = 1 \\ b_{pq}^{(M)} = 0 & elsewhere, p \neq q \end{cases}$$
(6.7)

Substituting equation (6.7) into equation (6.3), we obtain

$$I_{I} = \sum_{j=1}^{n} m_{j} \Phi_{1}^{*T} B_{j}^{(M)} \Phi_{1}^{*}$$
(6.8)

Each element of  $I_l$  is given by

$$\mathbf{I}_{l} = \begin{bmatrix} \delta_{pq} \end{bmatrix} = \begin{cases} \delta_{pq} = 1 , p = q \\ \delta_{pq} = 0 , p \neq q \end{cases}$$
(6.9)

Partitioning  $\Phi_1^*$  into column vectors as shown

$$\Phi_1^* = \left[ \phi_1^* \mid \phi_2^* \mid \dots \mid \phi_l^* \right]$$
(6.10)

then from (6.8) each element  $\delta_{pq}$  must be equal to

$$\delta_{pq} = \sum_{j=1}^{n} m_{j} \phi_{p}^{*T} B_{j}^{(M)} \phi_{q}^{*}$$
(6.11)

Let N=  $\frac{1}{2}(l^2 + l)$  and construct the vectors

$$\mathbf{y}_{\mathbf{M}} = (y_1^{(M)}, y_2^{(M)}, ..., y_N^{(M)})^{\mathrm{T}} = (\delta_{11}, \delta_{12}, \delta_{13}, ..., \delta_{1/}, \delta_{22}, \delta_{23}, ..., \delta_{2/}, \delta_{33}, ..., \delta_{1/})^{\mathrm{T}}$$
(6.12)  
and

$$\mathbf{m} = (m_1, m_2, ..., m_n)^T$$
 (6.13)

Denote

$$F_{M} = \left[f_{ij}^{(M)}\right] = \frac{\partial y_{i}^{(M)}}{\partial m_{i}}, \quad (i = 1, 2, ..., N; j = 1, 2, ..., n)$$
(6.14)

then all the elements of  $\mathbf{F}_{\mathbf{M}}$  can be evaluated using equation (6.11). Equation (6.8) can be written in a vector form

$$\mathbf{F}_{\mathbf{M}} \ \mathbf{m} = \mathbf{y}_{\mathbf{M}}.\tag{6.15}$$

Since  $\mathbf{F}_{\mathbf{M}}$  and  $\mathbf{y}_{\mathbf{M}}$  are known, (6.15) can be solved for **m**, and the mass matrix **M** can then be determined from the elements of vector **m** by equation (6.6).

We note that in order to obtain a solution for the system of size  $n \times l$  (l < n), we need to solve an augmented system (6.15) of size  $N \times n$  ( $N = \frac{1}{2}(l^2 + l)$ ). However, in this case augmentation is based on the smaller dimension l, whereas number of independent parameters available for optimisation is fixed at n. Therefore depending on the value of l there are three possibilities for the solution to (6.15). Set r = n-N, then if r > 0 there will be a family of solutions for **m**. This family of solutions is characterised by the following equation

$$\mathbf{m} = \mathbf{F}_{\mathbf{M}}^{\dagger} \mathbf{y}_{\mathbf{M}} + \mathbf{V}_{\mathbf{r}} \mathbf{b} \tag{6.16}$$

where  $\mathbf{F}_{M}^{\dagger}$  is the Moore-Penrose pseudoinverse of  $\mathbf{F}_{M}$ , **b** is an arbitrary vector of dimension  $rx_{1}$ , and  $\mathbf{V}_{r}$  is a matrix of dimension nxr which is obtained by a following procedure

Calculate singular value decomposition 
$$\mathbf{F}_{M} = \mathbf{U}\mathbf{S}\mathbf{V}^{T}$$
, and partition

the nxn matrix 
$$\mathbf{V} = [\mathbf{V}_{\mathbf{N}} \mid \mathbf{V}_{\mathbf{r}}]$$
, where  $\mathbf{V}_{\mathbf{N}}$  is nxN, and  $\mathbf{V}_{\mathbf{r}}$  is nxr. (6.17)

If r = 0, then  $\mathbf{F}_{\mathbf{M}}$  is a full square matrix, and there will be one unique solution for **m**. This unique solution is

$$\mathbf{m} = \mathbf{F}_{\mathbf{M}}^{-1} \mathbf{y}_{\mathbf{M}} . \tag{6.18}$$

And finally, if r < 0, then there are no solutions for **m**, and only an approximate solution (which is optimal in a least squares sense) can be obtained by

$$\mathbf{m} = \mathbf{F}_{\mathbf{M}}^{\dagger} \mathbf{y}_{\mathbf{M}} \ . \tag{6.19}$$

However, we also note that in order to satisfy the physical realisability criteria we require all the elements of  $\mathbf{m}$  to be positive. Therefore, if solutions of (6.16), (6.18) and (6.19) do not yield positive  $\mathbf{m}$ , it may have to be determined by solving the following *non-negative least squares problem* 

$$\min_{\mathbf{m}} \| \mathbf{F}_{\mathbf{M}} \mathbf{m} - \mathbf{y}_{\mathbf{M}} \|_{2}, \text{ subject to } \mathbf{m} \ge 0.$$
(6.20)

This will produce an optimal non-negative solution to the vector  $\mathbf{m}$  in a least square sense.

The above procedure is summarised by the following algorithm.

Algorithm 6.1: Determination of a Mass Matrix

*Input*: Desired modal data  $\Phi_1^*(nxl)$ .

#### Algorithm:

- 1) Column partition  $\Phi_1^*$  as in (6.10).
- 2) Set N=  $\frac{1}{2}(l^2 + 1)$ .
- 3) Construct vectors  $\mathbf{y}_{M}$  as in (6.12), via (6.9).

4) Form vector **m** of dimension nx1 as in (6.13).

5) Construct matrix  $\mathbf{F}_{\mathbf{M}}$  using (6.14) and (6.11).

6) (a) If n > N, then determine **m** by equation (6.16),

(b) if n = N, then determine **m** by (6.18),

(c) if n < N, then determine **m** by (6.19).

- 7) If **m** obtained in step 6 is not non-negative, then determine **m** by solving (6.20).
- 8) Construct **M** from the elements of **m** using (6.6).

Output: Physically realisable mass matrix M.

We note that Algorithm 6.1 can be used to determine **M** corresponding to any chosen analytical model, not just for a mass-spring system. All that is required is to use an appropriate mapping matrix  $\mathbf{B}_{i}^{(M)}$  in equation (6.6).

#### 6.2 Reconstruction of a Stiffness Matrix

Setting 
$$A^{T} = M\Phi_{1}^{*} \Lambda_{1}^{*}$$
, (6.21)  
equation (6.5) becomes  $K\Phi_{1}^{*} = A^{T}$ . (6.22)

Since  $\Phi_1^*$  and A are known, the stiffness matrix K can be determined. As in section 5, we may determine both the global and local optimal solutions to equation (6.22).

#### 6.2.1 Local optimal solution

Taking the transpose of (6.22) and using the fact that K is symmetric, we obtain

$$\mathbf{\Phi}_1^{*\mathrm{T}}\mathbf{K} = \mathbf{A}.\tag{6.23}$$

Partition A and K as follows

$$\mathbf{A} = [ \mathbf{a}_1 | \mathbf{a}_2 | \mathbf{a}_3 | \dots | \mathbf{a}_n ],$$
(6.24)  
$$\mathbf{K} = [\mathbf{k}_1 | \mathbf{k}_2 | \mathbf{k}_3 | \dots | \mathbf{k}_n ].$$
(6.25)

Then from equation (6.23), each column of A is given by

$$\Phi_1^{*T} \mathbf{k}_i = a_i \quad (j = 1, ..., n) .$$
(6.26)

Equation (6.26) is identical to equation (5.26), except that matrix  $\Phi_1^*$  has the dimensions of *nxl*. The same procedure as in section 5.3 can then be used to find a solution for **K**,

sequentially column by column. The resulting solution would be a local approximation for **K**. However, a general multi-connected mass-spring system of order *n* has  $\frac{1}{2}(n^2+n)$  independent spring elements, while the total number of constraints that need to be satisfied in equation (6.23) is *n* times *l* (and where l < n). Thus, if *l* is less than  $\frac{1}{2}(n+1)$ , then there will exist a family of exact solutions for **K**. By selecting a local optimisation method for calculating elements of **K**, we can only obtain approximate solutions to equation (6.23). Therefore, in this particular problem, a global approach for the solutions of **K** appears to be more suitable.

#### 6.2.2 Global solution for stiffness matrix

The stiffness matrix  $\mathbf{K}$  for a system with J independent spring elements, may be written in the following form

$$K = \sum_{q=1}^{J} s_{q} B_{q}^{(K)}$$
(6.27)

where  $s_q$  is the stiffness of the  $q^{\text{th}}$  spring, and  $\mathbf{B}_q^{(K)}$  is the mapping matrix corresponding to the chosen analytical model for the system. For example, equations (5.13) and (5.14) describe the **K** and  $\mathbf{B}_q^{(K)}$  for a multi-connected mass-spring system of order *n*, and for such system  $J = \frac{1}{2}(n^2 + n)$ .

Substituting (6.27) into (6.23), we obtain

$$\sum_{q=1}^{J} s_{q} \Phi_{1}^{*T} B_{q}^{(K)} = A \qquad (6.28)$$

Setting

$$\boldsymbol{B}_{+}^{(q)} = \begin{bmatrix} b_{ij}^{(q)} \end{bmatrix} = \boldsymbol{\Phi}_{1}^{*T} \boldsymbol{B}_{q}^{(K)} , \qquad (6.29)$$

then each  $ij^{th}$  element of A,  $a_{ij}$ , is given by

$$a_{ij} = \sum_{q=1}^{J} s_q \ b_{ij}^{(q)} \qquad (6.30)$$

Since there are  $n \times l$  independent elements in A, in order to obtain a global solution for K we need to solve simultaneously a system of  $n \times l$  linear equations of the form (6.30). We proceed in a similar fashion to the analysis described by equations (6.12) - (6.20).

Let 
$$\bar{N} = n \times l$$
 and construct the vectors  
 $\mathbf{y}_{K} = (\mathbf{y}_{1}^{(K)}, \mathbf{y}_{2}^{(K)}, ..., \mathbf{y}_{\bar{N}}^{(K)})^{T} = (a_{11}, a_{12}, ..., a_{1n}, a_{21}, ..., a_{2n}, a_{31}, ..., a_{ln})^{T}$ 
(6.31)  
and

$$\mathbf{s} = (\mathbf{s}_1, \, \mathbf{s}_2, \, ..., \, \mathbf{s}_J)^{\mathrm{T}}.$$
 (6.32)

Denote

$$F_{K} = \left[f_{ij}^{(K)}\right] = \frac{\partial y_{i}^{(K)}}{\partial s_{j}}, \quad (i = 1, 2, ..., \overline{N}; j = 1, 2, ..., J)$$
(6.33)

then all the elements of  $\mathbf{F}_{\mathbf{K}}$  can be evaluated using equation (6.30). Equation (6.22) can be written in a vector form

$$\mathbf{F}_{\mathbf{K}} \mathbf{s} = \mathbf{y}_{\mathbf{K}}.\tag{6.34}$$

Since  $\mathbf{F}_{\mathbf{K}}$  and  $\mathbf{y}_{\mathbf{K}}$  are known, (6.34) can be solved for s, and the stiffness matrix **K** can then be determined from the elements of vector s via equation (6.27).

As in a case of solution to (6.15), there are three possibilities for the solution to (6.34). Setting  $r = J - \bar{N}$ , then if r > 0 there will be a family of solutions for s. This family of solutions is characterised by the following equation

$$\mathbf{s} = \mathbf{F}_{\mathbf{K}}^{\dagger} \mathbf{y}_{\mathbf{K}} + \mathbf{V}_{\mathbf{r}} \mathbf{b} \tag{6.35}$$

where  $\mathbf{F}_{\mathbf{k}}^{\dagger}$  is the Moore-Penrose pseudoinverse of  $\mathbf{F}_{\mathbf{k}}$ , **b** is an arbitrary vector of dimension  $rx_1$ , and  $\mathbf{V}_r$  is a matrix of dimension Jxr which is obtained by the following procedure

Calculate singular value decomposition  $\mathbf{F}_{\mathbf{K}} = \mathbf{U}\mathbf{S}\mathbf{V}^{\mathsf{T}}$ , and partition the JxJ matrix  $\mathbf{V} = [\mathbf{V}_{\tilde{\mathbf{N}}} \mid \mathbf{V}_{\mathsf{r}}]$ , where  $\mathbf{V}_{\tilde{\mathbf{N}}}$  is Jx $\tilde{\mathbf{N}}$ , and  $\mathbf{V}_{\mathsf{r}}$  is Jxr. (6.36)

If r = 0, then  $\mathbf{F}_{\mathbf{K}}$  is a full square matrix, and there will be one unique solution for s. This unique solution is

$$\mathbf{s} = \mathbf{F}_{\mathbf{K}}^{-1} \mathbf{y}_{\mathbf{K}} \ . \tag{6.37}$$

And finally, if r<0, then there are no solutions for s, and only an approximate solution (which is optimal in a least squares sense) can be obtained by

$$\mathbf{s} = \mathbf{F}_{\mathbf{K}}^{\dagger} \mathbf{y}_{\mathbf{K}} \ . \tag{6.38}$$

However, we also note that in order to satisfy the physical realisability criteria we require all the elements of s to be positive. Therefore, if solutions of (6.35), (6.37) and (6.38) do not yield positive s, it may have to be determined by solving the following *non-negative least squares problem* 

$$\min_{\mathbf{s}} \| \mathbf{F}_{\mathbf{K}} \mathbf{s} - \mathbf{y}_{\mathbf{K}} \|_{2}, \text{ subject to } \mathbf{s} \ge 0.$$
(6.39)

This will produce an optimal non-negative solution to the vector s in a least square sense.

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The above process is summarised by the following algorithm.

Algorithm 6.2: Determination of a Stiffness Matrix

*Input*: Desired modal and spectral data  $\Phi_1^*$  and  $\Lambda_1^*$ , and mass matrix **M** obtained by Algorithm 6.1.

Algorithm:

- 1) Calculate the matrix **A** by equation (6.21).
- 2) Set  $\overline{N} = n \times I$ .
- 3) Construct vectors  $\mathbf{y}_{\mathbf{K}}$  as in (6.31).
- 4) Form a vector **s** of dimension Jx1 as in (6.32).
- 5) Construct a matrix  $\mathbf{F}_{\kappa}$  using (6.33) and (6.30).
- 6) (a) If  $J > \overline{N}$ , then determine **s** by equation (6.35),
  - (b) if  $J = \overline{N}$ , then determine **s** by (6.37),
  - (c) if  $J < \overline{N}$ , then determine **s** by (6.38).
- If s obtained in step 6 is not non-negative, then determine s by solving (6.39).
- 8) Construct K from the elements of s using (6.27).

Output: Physically realisable mass matrix K.

Algorithm 6.2 can be used to determine K corresponding to any chosen analytical model by substituting an appropriate mapping matrix  $\mathbf{B}_{q}^{(K)}$  in equation (6.27).

#### 6.3 Numerical Example

Suppose that the desired dynamic properties,  $\Lambda_1^*$  and  $\Phi_1^*$ , for a five degrees-of-freedom mass-spring system are:

$$\Lambda_1^* = \text{diag} (50, 100, 200)$$

and

	0.1	-0.1	0.2	
	0.1	0.1	0.2	
$\Phi_1^* =$	0.1	-0.1	0.3	
	0.1	-0.3	-0.1	
	0.3	0.2	-0.1	

and we wish to determine physically realisable M and K which have dynamic characteristics as close as possible to the above data.

In the above data l=3 and n=5. First calculating the mass matrix **M**, we note that parameter  $N=\frac{1}{2}(l^2+l)=6$ , and since n<N, there is no exact solution for **M**. The approximate solution for **M** is obtained by Algorithm 6.1 (using step 6(c)), and this solution is

 $\mathbf{M} = \text{diag} (7.3458, 5.2048, 3.6097, 7.2876, 7.0552)$ .

Calculating **K**, we note that  $J = \frac{1}{2}(n^2+n) = 15$  and  $\overline{N} = n \times l = 15$ , and since  $J = \overline{N}$ , there is one exact solution for **K**. Applying Algorithm 6.2 (with step 6(b)), this solution is

$$\boldsymbol{K} = \begin{bmatrix} 1062.6 & 45.5 & 85.3 & -222.9 & -184.4 \\ 45.5 & 540.9 & 219.4 & -128.9 & -105.0 \\ 85.3 & 219.4 & 436.5 & -45.7 & -142.3 \\ -222.9 & -128.9 & -45.7 & 753.2 & -35.6 \\ -184.4 & -105.0 & -142.3 & -35.6 & 513.1 \end{bmatrix}$$

However, since some of the off-diagonal elements of the above K are positive, this stiffness matrix is not physically realisable. To obtain the physically realisable K we perform step 7 of the Algorithm 6.2 and obtain

$$K = \begin{bmatrix} 1217.7 & 0 & 0 & -254.4 & -180.9 \\ 0 & 925.6 & 0 & 0 & -203.1 \\ 0 & 0 & 634.9 & -110.7 & -82.9 \\ -254.4 & 0 & -110.7 & 805.5 & -69.7 \\ -180.9 & -203.1 & -82.9 & -69.7 & 537.9 \end{bmatrix}$$

This K is physically realisable, and the mass-spring system corresponding to the obtained mass and stiffness matrices, M and K, is shown in Figure 6.1.

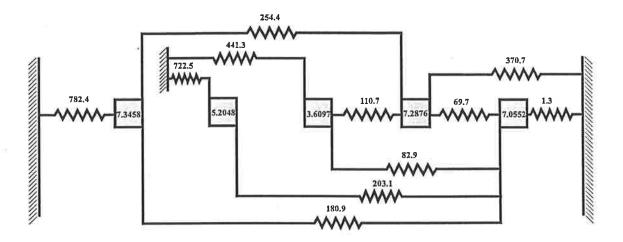


Figure 6.1: Mass-spring system corresponding to M and K.

The eigenvalues and the corresponding mass-normalised eigenvectors of this system are

 $\Lambda = \text{diag}$  (50.8213, 100.6356, 172.2435, 184.2177, 198.3586)

and

<b>—</b>				-
0.1087	-0.0997	-0.2450	0.1048	0.2083
0.0966	0.0835	0.0309	-0.3626	0.2084
0.0914	-0.0739	0.3870	0.1972	0.2730
0.1371	-0.3051	0.0426	-0.0980	-0.1181
0.3144	0.1653	0.0044	0.0593	-0.1096
	0.0966 0.0914 0.1371	0.0966 0.0835 0.0914 -0.0739 0.1371 -0.3051	0.09660.08350.03090.0914-0.07390.38700.1371-0.30510.0426	0.1087-0.0997-0.24500.10480.09660.08350.0309-0.36260.0914-0.07390.38700.19720.1371-0.30510.0426-0.09800.31440.16530.00440.0593

We note that the first, second and fifth modes in the  $\Lambda$  and  $\Phi$  above compare very well with the desired properties  $\Lambda_1^*$  and  $\Phi_1^*$ . For a good correlation it is required that the eigenvalue ratio, the amplitude ratio of the eigenvectors and the values of cosines between the two eigenvectors are all as close as possible to 1. The values of these ratios and cosines are presented in Table 6.1.

Desired Mode, i	Corresponding Obtained Mode, j	Eigenvalue Ratio, $\lambda_j/\lambda_i^*$	AmplitudeRatioofEigenvectors, $\  \phi_i \  / \  \phi_i^* \ $	Cosine of an angle between the two eigenvectors, $\cos(\angle \phi_i \phi_i^*)$
1	1	1.0164	0.9399	0.9956
2	2	1.0064	1.0587	0.9944
3	5	0.9918	1.0072	0.9966

Table 6.1 : Comparison between the desired and the obtained modes.

The results summarised in Table 6.1 demonstrate that a very good correlation is achieved between the desired and the obtained modes.

#### 6.4 Conclusions

In this section we have developed a method for reconstructing an analytical model of the vibratory system from a truncated set of desired modal properties. This method is general and is not restricted to any particular form of the mass and the stiffness matrices. Mass and stiffness matrices corresponding to any chosen analytical model can be reconstructed by this method.

It was shown that depending on the dimensions of the known desired data, we may obtain a family of solutions, a unique solution or an approximate solution which is optimal in a specified sense.

The presented numerical example demonstrated the application of the algorithm to a five degrees-of-freedom mass-spring system, and the obtained results showed a very good correlation between the desired and the obtained modal data.

## Section 7

### PROBLEM 2(b):

# INDEPENDENT PARAMETER DECOMPOSITION

In this section we define a special form for the mass and stiffness matrices, **M** and **K**, which are more general than these corresponding to a mass-spring system. It is demonstrated by examples that matrices of this form may correspond to various analytical models of vibratory systems, including a Finite Element model. We then show how to reconstruct these matrices from the prescribed modal and spectral data,  $\Phi^*$  and  $\Lambda^*$ . The problem we solve is identical to Problem 2, with the exception that the shapes of **M** and **K** are not necessarily correspond to a discrete mass-spring model. Thus, we designate this problem as Problem 2(b). The definition of the new matrix type for **M** and **K** is given below.

#### Definition 7.1: Independent Parameter Decomposition

Suppose that a symmetric mass matrix  $\mathbf{X}=[\mathbf{x}_{ij}]$  of size *n*x*n* is such that all of its elements  $\mathbf{x}_{ij}$  can be expressed as prescribed linear functions of *n* unknown

independent parameters { $x_1, x_2, ..., x_n$ }. Then we define this matrix to be an *independent parameter decomposable* if it can be described by a following product

$$\mathbf{X} = \mathbf{E}_{\mathbf{X}} \mathbf{D}_{\mathbf{X}} \mathbf{E}_{\mathbf{X}}^{\mathsf{T}}, \tag{7.1}$$

where the numerical values of all elements of  $E_x$  (*nxn*) are known, and  $D_x = \text{diag}\{d_1, d_2, ..., d_n\}, d_i \neq 0$ , with each diagonal element  $d_i$  equal to some known linear function of  $\{x_1, x_2, ..., x_n\}$ , i.e

$$d_{i} = \sum_{j=1}^{n} \beta_{ij} \chi_{j}, \quad (i = 1, 2, ..., n)$$
(7.2)

and  $\boldsymbol{\beta}_{ij}$  are known.

The above definition is demonstrated by the following examples.

#### Example 7.1: A simply-connected mass-spring system

Consider a simply-connected mass-spring system shown in Figure 7.1

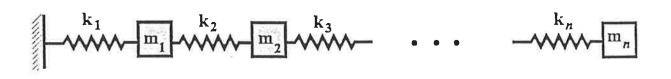


Figure 7.1: A simply-connected mass-spring system.

The mass and stiffness matrices corresponding to system have the following form  $M = \text{diag} (m_1, m_2, ..., m_n),$  and

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$$\mathbf{K} = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 & 0 & \cdots & 0 & 0 \\ -k_2 & k_2 + k_3 & -k_3 & 0 & \cdots & 0 & 0 \\ 0 & -k_3 & k_3 + k_4 & -k_4 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & -k_{n-2} & k_{n-2} + k_{n-1} & -k_{n-1} & 0 \\ 0 & 0 & \cdots & 0 & -k_{n-1} & k_{n-1} + k_n & -k_n \\ 0 & 0 & 0 & \cdots & 0 & -k_n & k_n \end{bmatrix}$$

Both matrices are IP decomposable because they can be expressed as products

$$\mathbf{M} = \mathbf{E}_{\mathbf{M}} \mathbf{D}_{\mathbf{M}} \mathbf{E}_{\mathbf{M}}^{\mathrm{T}}$$
(7.3)

$$\mathbf{K} = \mathbf{E}_{\mathbf{K}} \mathbf{D}_{\mathbf{K}} \mathbf{E}_{\mathbf{K}}^{\mathrm{T}} \tag{7.4}$$

where 
$$D_{M} = M = \text{diag}(m_{1}, m_{2}, ..., m_{n})$$
 (7.5)

$$\mathbf{E}_{\mathbf{M}} = \mathbf{I}_{\mathbf{n}} = \text{diag (1,1,...,1)},$$
 (7.6)

$$\mathbf{D}_{\mathbf{k}} = \text{diag}(\mathbf{k}_{1}, \mathbf{k}_{2}, ..., \mathbf{k}_{n}),$$
 (7.7)

7

and

r

$$\boldsymbol{E}_{\boldsymbol{K}} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 1 \end{bmatrix}$$

$$(7.8)$$

#### Example 7.2: A 2 d.o.f. finite element model of an axially vibrating rod

The mass and stiffness matrices of a 2 d.o.f. finite element model for a longitudinally vibrating rod are

$$M = \begin{bmatrix} \frac{m_1 + m_2}{3} & \frac{m_2}{6} \\ \frac{m_2}{6} & \frac{m_2}{3} \end{bmatrix}$$

and

$$\boldsymbol{K} = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} ,$$

where  $m_1$ ,  $m_2$ ,  $k_1$  and  $k_2$  are the masses and the stiffnesses of the two elements.

Both of these matrices are IP decomposable since they can be described by products of the form (7.3) and (7.4), respectively, with

$$\boldsymbol{D}_{M} = \begin{bmatrix} \frac{m_{1}}{3} + \frac{m_{2}}{4} & 0\\ 0 & \frac{m_{2}}{6} \end{bmatrix}, \qquad \boldsymbol{E}_{M} = \begin{bmatrix} 1 & \frac{1}{\sqrt{2}}\\ 0 & \sqrt{2} \end{bmatrix},$$

and

$$\boldsymbol{D}_{K} = \begin{bmatrix} k_{1} & 0 \\ 0 & k_{2} \end{bmatrix}, \qquad \boldsymbol{E}_{K} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}.$$

#### 7.1 Reconstruction of IP Decomposable Mass Matrices

In Problem 2 we want to determine the mass and stiffness matrices **M** and **K** corresponding to a physically realisable mass-spring system, such that its modal and spectral properties, described by modal matrix  $\Phi$  and spectral matrix  $\Lambda$ , are as close as possible to the prescribed modal and spectral matrices  $\Phi^*$  and  $\Lambda^*$ . In this section we generalise the solution of Problem 2 for the IP decomposable matrices **M** and **K**.

As in section 5, the two problems of determining  $\Phi$  and  $\Lambda$  corresponding to a realisable system can be solved separately. Consider now the problem of determining the optimal mode shape matrix  $\Phi$ .

If **M** is an IP decomposable mass matrix, then by definition 7.1 it may be expressed by a product of the form of equation (7.3), where  $\mathbf{E}_{\mathbf{M}}$  is a known matrix of size *nxn*, and

$$\boldsymbol{D}_{M} = diag \left( d_{1}^{(M)}, d_{2}^{(M)}, ..., d_{n}^{(M)} \right), d_{i}^{(M)} \neq 0,$$
(7.9)

is a diagonal matrix such that each element  $d_i^{(M)}$  (i=1,...,n) is equal to a known linear function of mass elements {m<sub>1</sub>, m<sub>2</sub>, ..., m<sub>n</sub> }, i.e

$$d_i^{(M)} = \sum_{j=1}^n \beta_{ij}^{(M)} m_j, \quad (i = 1, 2, ..., n)$$
(7.10)

where  $\beta_{ij}^{(M)}$  are known.

The mass matrix M must also satisfy the orthogonality equation

$$\mathbf{M} = \mathbf{\Phi}^{\mathbf{-T}} \mathbf{\Phi}^{\mathbf{-1}}.\tag{7.11}$$

Let Q be an orthonormal matrix, that is,  $QQ^{T} = I_n$ , and set matrix

$$D = D_M^{-1/2} = diag \left\{ \frac{1}{\sqrt{d_1^{(M)}}}, \frac{1}{\sqrt{d_2^{(M)}}}, ..., \frac{1}{\sqrt{d_n^{(M)}}} \right\} , \qquad (7.12)$$

then equations (7.11) and (7.3) will both be satisfied if the modal matrix  $\Phi$  is equal to

$$\Phi = E_M^{-T} D Q \qquad . \tag{7.13}$$

Thus a solution to our problem can be obtained by determining a diagonal matrix  $\mathbf{D}$  and an orthonormal matrix  $\mathbf{Q}$ , such that

$$\min_{\mathbf{D},\mathbf{Q}} \| \Phi^* - \mathbf{E}_{\mathbf{M}}^{-T} \mathbf{D} \mathbf{Q} \|$$
 (7.14)

In solving this problem we will make use of the algorithm 5.1 for solving an *orthogonal Procrustes problem*, which is described in section 5.

Thus setting

$$\mathbf{A} = \boldsymbol{\Phi}^* \tag{7.15}$$

$$\mathbf{B}_{t} = \mathbf{E}_{M}^{-T} \mathbf{D}_{t}$$
, (where t designates an iteration index), (7.16)

we may choose a diagonal matrix  $\mathbf{D}_0$  as an initial guess and obtain an orthonormal  $\mathbf{Q}_0$ which minimises  $\|\mathbf{A} - \mathbf{B}_0 \mathbf{Q}_0\|_F$ , by using Algorithm 5.1. We now show how to obtain a matrix  $\mathbf{D}_1$  such that

$$\| \Phi^{*} - \mathbf{E}_{M}^{-T} \mathbf{D}_{1} \mathbf{Q}_{0} \|_{F} \leq \| \Phi^{*} - \mathbf{E}_{M}^{-T} \mathbf{D}_{0} \mathbf{Q}_{0} \|_{F}.$$
(7.17)

The Frobenius norm is invariant under orthonormal multiplication. Hence

$$\| \Phi^* - \mathbf{E}_{M}^{-T} \mathbf{D}_1 \mathbf{Q}_0 \|_F = \| \Phi^* \mathbf{Q}_0^{-T} - \mathbf{E}_{M}^{-T} \mathbf{D}_1 \|_F,$$
(7.18)

Define

$$\mathbf{R} = \boldsymbol{\Phi}^* \mathbf{Q}_0^{\mathrm{T}},\tag{7.19}$$

and denote 
$$\epsilon = \| \mathbf{R} - \mathbf{E}_M^{-T} \mathbf{D}_1 \|_F^2$$
 (7.20)

Using the equality

$$\|R - E_M^{-T}D_1\|_F^2 = trace(R^TR) + trace(D_1^T E_M^{-1} E_M^{-T} D_1) - 2 trace(D_1^T E_M^{-1} R)$$
(7.21)

and setting

$$E = E_M^{-1} E_M^{-T}$$
(7.22)

$$F = E_M^{-1} R$$
 , (7.23)

equation (7.21) may be written as

$$\|\mathbf{R} - \mathbf{E}_{M}^{-T}\mathbf{D}_{1}\|_{F}^{2} = trace(\mathbf{R}^{T}\mathbf{R}) + trace(\mathbf{D}_{1}^{T}\mathbf{E}\mathbf{D}_{1}) - 2 trace(\mathbf{D}_{1}^{T}\mathbf{F}) \qquad (7.24)$$

Therefore, applying the knowledge that  $D_1$  is a diagonal matrix and its diagonal elements are defined in equation (7.12), we find that

$$\epsilon = trace \left( \mathbf{R}^{T} \mathbf{R} \right) + \sum_{i=1}^{n} \left[ \frac{e_{ii}}{d_{i}^{(M)}} - \frac{2f_{ii}}{\sqrt{d_{i}^{(M)}}} \right]$$
(7.25)

where  $e_{ii}$  and  $f_{ii}$  are the diagonal elements of matrices **E** and **F** respectively. Differentiating  $\varepsilon$  with respect to  $d_i^{(M)}$  and equating to zero to obtain the minimisation criteria, we obtain

$$\frac{\partial \varepsilon}{\partial d_i} = -\frac{e_{ii}}{d_i^{(M)^2}} + \frac{f_{ii}}{d_i^{(M)^{13}}} = 0$$
(7.26)

Then from (7.26) it is clear that  $\epsilon$  is minimised when

$$\frac{1}{\sqrt{d_i^{(M)}}} = \frac{f_{ii}}{e_{ii}} \,. \tag{7.27}$$

Thus, the residual error  $\epsilon$  is minimised when the diagonal elements of  $\mathbf{D}_1$  are equal to the ratios of the diagonal elements of  $\mathbf{F}$  and the corresponding diagonal elements of  $\mathbf{E}$ . Having determined a diagonal matrix  $\mathbf{D}_1$  satisfying (7.17), we can reapply the Algorithm 5.1 with  $\Phi^*$  and  $\mathbf{D}_1$  as an input and find an orthonormal matrix  $\mathbf{Q}_1$  such that

$$\| \Phi^{*} - \mathbf{E}_{M}^{-T} \mathbf{D}_{1} \mathbf{Q}_{1} \|_{F} \leq \| \Phi^{*} - \mathbf{E}_{M}^{-T} \mathbf{D}_{1} \mathbf{Q}_{0} \|_{F} .$$
(7.28)

Continuing in this manner iteratively, we obtain an approximate solution for the optimal modal matrix  $\Phi$ . The mass elements  $\{m_1, m_2, ..., m_n\}$  can then be determined by solving the system of linear equations (7.10) after substitution of the obtained values for  $\{d_1^{(M)}, d_2^{(M)}, ..., d_n^{(M)}\}$ . The following algorithm summarises this result.

#### Algorithm 7.1: Approximate solution for the modal matrix $\Phi$ and the mass elements

*Input*: An nxn modal matrix  $\Phi^*$ , an nxn matrix  $E_M$  and coefficients  $\beta_{ij}^{(M)}$  (i,j=1,...,n).

Algorithm: 1) Set initial guess  $D_0$  and a tolerance for convergence  $\epsilon$ .

- 2) For t=0, 1, 2, ...
  - a) Evaluate  $\mathbf{C} = \mathbf{D}_t^T \mathbf{E}_M^{-1} \Phi^*$ .
  - b) Compute the singular value decomposition  $C=U\Sigma V^{T}$ .
  - c) Evaluate  $\mathbf{Q}_{t} = \mathbf{U}\mathbf{V}^{\mathrm{T}}$ .

- d) Obtain  $\mathbf{F} = \mathbf{E}_{\mathbf{M}}^{-1} \Phi^* \mathbf{Q}_{\mathbf{t}}^{\mathbf{T}}$ .
- e) Set  $\mathbf{E} = \mathbf{E}_{\mathbf{M}}^{-1}\mathbf{E}_{\mathbf{M}}^{-T}$

f) Form 
$$D_{i+1} = diag \left\{ \frac{f_{11}}{e_{11}}, \frac{f_{22}}{e_{22}}, \dots, \frac{f_{nn}}{e_{nn}} \right\}$$

- g) Test convergence
  - (i) Set  $N_1 = \| \Phi^* E_M^{-T} D_t Q_t \|_F$ ,  $N_2 = \| \Phi^* E_M^{-T} D_{t+1} Q_t \|_F$ . (ii) If  $(N_1 - N_2) \le \epsilon$ , go to 3.
- 3)  $\Phi = \mathbf{E}_{M}^{-T}\mathbf{D}_{t+1}\mathbf{Q}_{t}$
- 4) Calculate

$$d_i^{(M)} = \left(\frac{e_{ii}}{f_{ii}}\right)^2$$
, *i*=1, ..., *n*.

5) Solve the following system of linear equations for  $m_j$  (j=1, ...,n).

$$\sum_{j=1}^{n} \beta_{ij}^{(M)} m_j = d_i^{(M)} ; \quad i=1, ..., n.$$

**Output:** An optimal modal matrix  $\Phi$  and a set of mass elements {m<sub>1</sub>, m<sub>2</sub>, ..., m<sub>n</sub>}.

It follows from (7.17) and (7.28) that the norm  $\| \Phi^* - \mathbf{E}_{\mathbf{M}}^{-T} \mathbf{D}_t \mathbf{Q}_t \|_F$  is a monotonic non-increasing function of an iteration index t. The Algorithm 7.1 thus necessarily converge.

#### 7.2 Reconstruction of IP Decomposable Stiffness Matrices

In this section we will use the results obtained from Algorithm 7.1 to reconstruct an IP decomposable stiffness matrix K, such that the norm  $\|\Lambda^* - \Lambda\|$  (which represents the difference between the desired and the obtained eigenvalues) is minimised.

If **K** is an IP decomposable stiffness matrix, then by definition 7.1 it may be expressed by a product of the form of equation (7.4), where  $\mathbf{E}_{\mathbf{K}}$  is a known matrix of size *nxn*, and

$$\boldsymbol{D}_{K} = diag \left( d_{1}^{(K)}, d_{2}^{(K)}, ..., d_{n}^{(K)} \right), d_{i}^{(K)} \neq 0,$$
(7.29)

is a diagonal matrix such that each element  $d_i^{(K)}$  (i=1,...,n) is equal to a known linear function of stiffness elements {k<sub>1</sub>, k<sub>2</sub>, ..., k<sub>n</sub> }, i.e

$$d_i^{(K)} = \sum_{j=1}^n \beta_{ij}^{(K)} k_j , \quad (i = 1, 2, ..., n)$$
(7.30)

where  $\beta_{ij}^{(K)}$  are known.

The orthogonality equation for the stiffness matrix is

$$\Phi^{\mathrm{T}}\mathbf{K}\Phi = \mathbf{\Lambda}.\tag{7.31}$$

Thus substituting (7.4) into (7.31), we obtain

$$\boldsymbol{\Phi}^{\mathrm{T}} \mathbf{E}_{\mathbf{K}} \mathbf{D}_{\mathbf{K}} \mathbf{E}_{\mathbf{K}}^{\mathrm{T}} \boldsymbol{\Phi} = \boldsymbol{\Lambda}. \tag{7.32}$$

Let

$$\mathbf{G} = \mathbf{E}_{\mathbf{K}}^{\mathbf{T}} \mathbf{\Phi} , \qquad (7.33)$$

then equation (7.32) becomes

$$\mathbf{G}^{\mathrm{T}}\mathbf{D}_{\mathrm{K}}\mathbf{G} = \mathbf{\Lambda} \,, \tag{7.34}$$

A diagonal matrix  $\mathbf{D}_{\mathbf{K}}$  may be expressed in terms of its elements as

$$D_{K} = \sum_{i=1}^{n} d_{i}^{(K)} H_{i}$$
(7.35)

where  $H_i$  is a *nxn* mapping matrix and it is equal to

$$\boldsymbol{H}_{i} = \begin{bmatrix} \boldsymbol{h}_{pq} \end{bmatrix} = \begin{cases} \boldsymbol{h}_{ii} = 1 \\ \boldsymbol{h}_{pq} = 0 & elsewhere \end{cases}$$
(7.36)

Substituting (7.35) into (7.34) we obtain

$$\Lambda = \sum_{i=1}^{n} d_i^{(K)} \left( \boldsymbol{G}^T \boldsymbol{H}_i \boldsymbol{G} \right) \quad .$$
 (7.37)

Partitioning G into column vectors

$$\boldsymbol{G} = \left[ \boldsymbol{g}_1 \mid \boldsymbol{g}_2 \mid \dots \mid \boldsymbol{g}_n \right], \tag{7.38}$$

then each  $pq^{th}$  element of  $\Lambda$  is given by

$$\lambda_{pq} = \sum_{i=1}^{n} d_i^{(K)} \left( \boldsymbol{g}_p^T \boldsymbol{H}_{\boldsymbol{g}} \boldsymbol{g}_{\boldsymbol{q}} \right) \quad . \tag{7.39}$$

Let N=  $\frac{1}{2}(n^2 + n)$  and construct the vectors

$$\mathbf{y} = (y_1, y_2, y_3, ..., y_N)^{\mathrm{T}} = (\lambda_{11}, \lambda_{12}, ..., \lambda_{1n}, \lambda_{22}, \lambda_{23}, ..., \lambda_{2n}, \lambda_{33}, ..., \lambda_{nn})^{\mathrm{T}}$$
(7.40)

and

$$\mathbf{d} = (\mathbf{d}_{1}^{(K)}, \, \mathbf{d}_{2}^{(K)}, \, ..., \, \mathbf{d}_{n}^{(K)})^{\mathrm{T}} \,.$$
(7.41)

Denote

$$\boldsymbol{P} = [p_{ij}] = \frac{\partial y_i}{\partial d_i^{(K)}}, \quad (i = 1, 2, ..., N; j = 1, ..., n)$$
(7.42)

then all the elements of P can be evaluated using equation (7.39).

Equation (7.37) can be written in a vector form

$$\mathbf{P} \, \mathbf{d} = \mathbf{y}.\tag{7.43}$$

Setting  $\Lambda = \Lambda^*$ , we may determine the vector **y** and solve equation (7.43) for **d**. However, since N>n for any n>1, there is, in general, no solution to (7.43). The approximate solution for **d**, which is optimal in a least squares sense, can be obtained by

 $\mathbf{d} = \mathbf{P}^{\dagger} \mathbf{y} \tag{7.44}$ 

where  $P^{\dagger}$  is the Moore-Penrose pseudo-inverse of P.

The stiffness elements  $\{k_1, k_2, ..., k_n\}$  can then be obtained by solving the system of linear equations defined by (7.30). This process is summarised by the following algorithm.

#### Algorithm 7.2: Approximate solution for the stiffness elements

*Input*: Modal matrix  $\Phi$  (nxn), desired eigenvalues matrix  $\Lambda^*$ (nxn), matrix  $E_{\kappa}$  (nxn) and coefficients  $\beta_{ij}^{(K)}$  (i,j=1,...,n).

**Algorithm:** 1) Calculate  $\mathbf{G} = \mathbf{E}_{\mathbf{K}}^{\mathsf{T}} \Phi$  and partition it as in equation (7.38).

2) Set  $\Lambda = \Lambda^*$  and form vector **y** as in (7.40)

3) Construct matrix P using equations (7.42),(7.49) and (7.36).

- 4) Calculate vector **d**, as defined by (7.41), using (7.44).
- 5) Solve the following system of linear equations for  $k_j$  (j=1, ...,n).

$$\sum_{j=1}^{n} \beta_{ij}^{(K)} m_j = d_i^{(K)}; \quad i=1, ..., n.$$

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**Output:** A set of stiffness elements  $\{k_1, k_2, ..., k_n\}$ , corresponding to an optimal **K**, which minimises the norm  $\|\mathbf{\Lambda}^* - \mathbf{\Lambda}\|$ .

We note that Algorithm 7.2 requires augmentation of the system from size n to size N. This is a computational barrier. In section 5 we presented a computationally less expensive method for obtaining a local optimal solution for K, but there we had to satisfy a system of N equations with N variable parameters. In this section we are required to satisfy a system of N equations with only n variables, therefore we are forced to seek a global optimal solution in order to achieve the best possible quality.

#### 7.3 Numerical Example

Consider a 2 d.o.f. finite element model for a longitudinally vibrating rod of Example 7.2. The mass and stiffness matrices for this model are of the following form

$$M = \begin{bmatrix} \frac{m_1 + m_2}{3} & \frac{m_2}{6} \\ \frac{m_2}{6} & \frac{m_2}{3} \end{bmatrix}$$

and

$$\boldsymbol{K} = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}$$

where  $m_1$ ,  $m_2$ ,  $k_1$  and  $k_2$  are the masses and the stiffnesses of the two elements in the model. The components of the independent parameter decomposition for these matrices were shown to be

$$D_{M} = \begin{bmatrix} \frac{m_{1}}{3} + \frac{m_{2}}{4} & 0\\ 0 & \frac{m_{2}}{6} \end{bmatrix}, \qquad E_{M} = \begin{bmatrix} 1 & \frac{1}{\sqrt{2}}\\ 0 & \sqrt{2} \end{bmatrix},$$

and

$$\boldsymbol{D}_{\boldsymbol{K}} = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix}, \qquad \boldsymbol{E}_{\boldsymbol{K}} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

Suppose that we wish to determine M and K, corresponding to the above form, such that the modal and spectral properties of the system be as close as possible to the desired properties described by

$$\Phi^* = \begin{bmatrix} 0.1 & 0.2 \\ 0.2 & -0.3 \end{bmatrix}, \quad \Lambda^* = \begin{bmatrix} 10 & 0 \\ 0 & 100 \end{bmatrix}$$

Applying Algorithm 7.1, we obtain an optimal modal matrix

$$\mathbf{\Phi} = \begin{bmatrix} 0.1300 & 0.1920 \\ 0.2114 & -0.2831 \end{bmatrix}$$

and the mass elements  $m_1$ = 35.6119 and  $m_2$ =26.9242, which correspond to a mass matrix

$$\boldsymbol{M} = \begin{bmatrix} 20.8454 & 4.4874 \\ 4.4874 & 8.9747 \end{bmatrix}$$

which satisfies the requirements of the prescribed form and the orthogonality properties of equation (7.11).

Applying Algorithm 7.2 with the obtained modal matrix  $\Phi$ , we determine the stiffness elements  $k_1$ = 526.7766 and  $k_2$ = 356.3036, which correspond to a stiffness matrix

$$\boldsymbol{K} = \begin{bmatrix} 883.0803 & -356.3036 \\ -356.3036 & 356.3036 \end{bmatrix}$$

which is of the required form.

The modal and spectral properties of the system corresponding to the above M and K are

$$\boldsymbol{\Phi} = \begin{bmatrix} 0.1314 & 0.1911 \\ 0.2094 & -0.2847 \end{bmatrix}, \quad \boldsymbol{\Lambda} = \begin{bmatrix} 11.26 & 0 \\ 0 & 99.86 \end{bmatrix}$$

which correlate very well with the desired properties  $\Phi^*$  and  $\Lambda^*$ .

#### 7.4 Conclusions

In this section we have defined a special form for mass and stiffness matrices, which are more general than those corresponding to a mass-spring system. Methods for constructing such matrices to suit the prescribed modal and spectral properties were then developed.

The Algorithm 7.1 for reconstruction of the mass matrices is an extension of the theory developed in section 5, and is in fact a generalisation of that theory. The Algorithm 7.2 for reconstruction of the stiffness matrices is based on a matrix sensitivity analysis, and requires augmentation of the system in order to obtain a global optimal solution. This augmentation carries a significant computational penalty. However, due to an inherent deficiency in the number of variable parameters available for optimisation, the quality of a solution is problematic, and thus requires a global approach rather than a more computationally

efficient local optimisation approach.

The numerical example based on a 2 degrees-of-freedom finite element model of a longitudinally vibrating rod was presented. This example has demonstrated the application of the developed methods, and the results obtained correlated well with the prescribed modal and spectral properties.

## **Section 8**

### **PROBLEM 3:**

# MODIFICATIONS FOR DESIRED NATURAL FREQUENCIES<sup>3</sup>

In this section we present the analysis for the solution of Problem 3 formulated in section 2.3. In this problem the exact mass and stiffness matrices of the system, **M** and **K**, are assumed to be unknown. The only information which is assumed to be known about the system are the measured modal analysis data contained in the matrices  $\Lambda_1$  and  $\Phi_1$ . We then want to determine physically realisable modifications to the mass and stiffness (i.e  $\Delta M$  and  $\Delta K$ ), based only on  $\Lambda_1$  and  $\Phi_1$ , so that the modified system would have spectral properties as close as possible to the desired spectrum described by  $\Lambda^*$ .

It was shown in section 2.3 that an approximate solution to this problem may be obtained by solving the following norm minimisation problem

 $<sup>^3</sup>$  Material presented in this section has been published in [77].

$$\| (\mathbf{M} + \Delta \mathbf{M})^{-1/2} \mathbf{R} \|_{\mathrm{F}}$$
, subject to  $\overline{\Phi} \in \mathrm{span}(\Phi_1)$  (8.1)

where the residual matrix  $\mathbf{R}$  is given by

$$\boldsymbol{R} = [(\boldsymbol{K} + \Delta \boldsymbol{K}) \boldsymbol{\Phi} - (\boldsymbol{M} + \Delta \boldsymbol{M}) \boldsymbol{\Phi} \boldsymbol{\Lambda}]$$
(8.2)

and  $\overline{\Lambda}$  and  $\overline{\Phi}$  are some approximations to the desired  $\Lambda^*$  and the corresponding  $\Phi^*$  respectively.

It is shown in Parlett [42,pp.321-323] that if we determine the eigensolution,  $\Psi$  and  $\Omega$ , of

$$\mathbf{F}\boldsymbol{\Psi} - \mathbf{G}\boldsymbol{\Psi}\boldsymbol{\Omega} = \mathbf{0} \tag{8.3}$$

where

$$\mathbf{F} = \boldsymbol{\Phi}_{1}^{\mathrm{T}} (\mathbf{K} + \Delta \mathbf{K}) \boldsymbol{\Phi}_{1}$$
(8.4)

$$\mathbf{G} = \mathbf{\Phi}_{1}^{\mathbf{I}} (\mathbf{M} + \Delta \mathbf{M}) \mathbf{\Phi}_{1}, \tag{8.5}$$

then  $\overline{\Lambda} = \Omega$  and  $\overline{\Phi} = \Phi_1 \Psi$  minimise (8.1) under the required constraint that  $\overline{\Phi} \in \operatorname{span}(\Phi_1)$ .

The matrices K and M are not given, and cannot be determined. However, using the orthogonality properties

$$\boldsymbol{\Phi}_1^{\mathrm{T}} \mathbf{K} \boldsymbol{\Phi}_1 = \boldsymbol{\Lambda}_1 \tag{8.6}$$

and

$$\Phi_1^{\mathrm{T}} \mathbf{M} \Phi_1 = \mathbf{I}_{\mathrm{m}} , \qquad (8.7)$$

we have

$$\mathbf{F} = \mathbf{\Lambda}_1 + \mathbf{\Phi}_1^{\mathbf{T}} \mathbf{\Delta} \mathbf{K} \mathbf{\Phi}_1 \tag{8.8}$$

and

$$\mathbf{G} = \mathbf{I}_{m} + \boldsymbol{\Phi}_{1}^{T} \boldsymbol{\Delta} \mathbf{M} \boldsymbol{\Phi}_{1} . \tag{8.9}$$

Hence, if  $\Delta K$  and  $\Delta M$  are known, then F and G can be obtained and eigenproblem (8.3) can be solved for  $\Psi$  and  $\Omega$ . Our goal is to solve the *inverse eigenvalue problem* of determining  $\Delta K$  and  $\Delta M$  where  $\Omega = \Lambda^*$  is prescribed.

As shown in [42], the obtained solution is the Rayleigh-Ritz approximation of (2.14) from the subspace which is spanned by  $\Phi_1$ . Thus, it follows that the desired eigenvalues are upper bounds on the eigenvalues of the actual modified system (2.14).

First we consider a simpler case of modifications to a discrete mass-spring system, and then extend these results for more complex models. An important case where mass and stiffness matrices are interrelated is also considered.

#### 8.1 Modifying a Mass-Spring System.

A mass-spring system is the simplest model to analyse. In this model the mass and stiffness matrices are independent. Therefore, it is possible to change one without introducing changes to the other.

The global mass modifications matrix,  $\Delta M$ , can be written in terms of its elements as follows

$$\Delta M = \sum_{i=1}^{n} \delta m_i B_i^{(M)}$$
(8.10)

where  $\delta m_i$  represents the change in the *i*<sup>th</sup> mass element, and **B**<sub>i</sub><sup>(M)</sup> is the mapping matrix

$$\boldsymbol{B}_{i}^{(\mathcal{M})} = \begin{bmatrix} b_{pq}^{(\mathcal{M})} \end{bmatrix} = \begin{cases} b_{ii}^{(\mathcal{M})} = 1 \\ b_{pq}^{(\mathcal{M})} = 0 & elsewhere \end{cases}$$
(8.11)

The incremental stiffness matrix may be written in the following form

$$\Delta K = \sum_{i=0}^{n-1} \sum_{j=i+1}^{n} \delta s_{ij} B_{ij}^{(K)}$$
(8.12)

where  $\delta s_{ij}$  is the change in the stiffness of the spring connecting mass *i* to mass *j*, and  $\delta s_{oi}$  represents the change in the stiffness of the spring which connects mass *i* to the ground, and  $\mathbf{B}_{ij}^{(\mathbf{K})}$  is the matrix describing the spring connection between mass *i* and mass *j* 

$$\boldsymbol{B}_{ij}^{(K)} = \begin{bmatrix} b_{pq}^{(K)} \end{bmatrix} = \begin{cases} b_{ii}^{(K)} = b_{jj}^{(K)} = 1 \\ b_{ij}^{(K)} = b_{ji}^{(K)} = -1 \\ b_{pq}^{(K)} = 0 \text{ elsewhere} \end{cases}$$
(8.13)

It follows, therefore, that

$$G = I_m + \sum_{i=1}^{n} \delta m_i M_i$$
 (8.14)

and

$$F = \Lambda_{1} + \sum_{i=0}^{n-1} \sum_{j=i+1}^{n} \delta s_{ij} K_{ij}$$
(8.15)

where

$$\mathbf{K}_{ij} = \boldsymbol{\Phi}_1^{\mathrm{T}} \mathbf{B}_{ij}^{(K)} \boldsymbol{\Phi}_1 \tag{8.16}$$

and

$$\mathbf{M}_{i} = \boldsymbol{\Phi}_{1}^{T} \mathbf{B}_{i}^{(M)} \boldsymbol{\Phi}_{1} \quad . \tag{8.17}$$

Hence

$$\frac{\partial G}{\partial (\delta m_i)} = M_i \quad , \quad \frac{\partial G}{\partial (\delta s_{ij})} = 0 \tag{8.18}$$

$$\frac{\partial F}{\partial(\delta s_{ij})} = K_{ij} \quad , \quad \frac{\partial F}{\partial(\delta m_i)} = 0 \tag{8.19}$$

and, as in [16], we have

$$\frac{\partial \lambda_i^*}{\partial a_j} = \psi_i^T(a') \left[ \frac{\partial F}{\partial a_j} - \lambda_i(a') \frac{\partial G}{\partial a_j} \right] \psi_i(a')$$
(8.20)

where

$$a^{t} = (a_{1}, a_{2}, ..., a_{N}) = (\delta s_{01}, \delta s_{02}, ..., \delta s_{0n}, \delta s_{12}, ..., \delta s_{1n}, ..., \delta s_{n-1,n}, \delta m_{1}, ..., \delta m_{n}) , \qquad (8.21)$$

N =  $\frac{1}{2}(n^2+3n)$ , t is an iteration index,  $\lambda_i(a^t)$  is the  $i^{th}$  eigenvalue and  $\psi_i(a^t)$  is the corresponding eigenvector, both obtained in the t<sup>th</sup> iteration by using an iterate vector  $a^t$ .

The modification matrices  $\Delta M$  and  $\Delta K$  may then be found by an algorithm similar to Joseph [16]. This algorithm uses the Newton-Raphson method to find a new approximation for the structural modification parameters from some arbitrarily selected initial guess. The new approximation is calculated by determining the matrix derivatives and eigenvalue sensitivities as shown in (8.18), (8.19) and (8.20) above. These values determine a Jacobian matrix **J**, from which the eigenvalue sensitivity elements, calculated in (8.20), are found. Then a set of linear equations is solved to find the changes for improving the initially chosen structural modification parameters. The algorithm is summarised below, and its full derivation can be found in [16]. Algorithm 8.1: Approximate solution to Problem 3

Input: Measured modal analysis data  $\Lambda_i$  and  $\Phi_i$ 

#### Algorithm:

- 1) Set iteration index t = 0 and choose an initial guess for structural modification parameters  $a^{t}$ .
- 2) Form matrices  $\Delta \mathbf{K}$  and  $\Delta \mathbf{M}$ , by using structural modification parameters contained in the elements of  $a^{t}$  using (8.21).
- 3) Calculate F and G, by using (8.8) and (8.9).
- 4) By solving equation FΨ(a') = GΨ(a')Λ(a') compute the smallest m eigenvalues λ<sub>i</sub>(a') (i=1,...,m), λ<sub>1</sub>< λ<sub>2</sub><...< λ<sub>m</sub>, and the corresponding eigenvectors ψ<sub>i</sub>(a') normalised with respect to G.
- 5) Perform a convergence test, if  $\| \lambda_i(a') \lambda_i^* \|$  is sufficiently small, stop.
- 6) Compute the Jacobian matrix,  $\mathbf{J}=[J_{ij}]$ , i = 1,..., m; j=1,...,N {N =  $\frac{1}{2}(n^2+3n)$ }; where the (i,j)<sup>th</sup> element of **J** is given by

$$J_{ij} = \frac{\partial \lambda_i^*}{\partial a_j} = \psi_i^T(a') \left[ \frac{\partial F}{\partial a_j} - \lambda_i(a') \frac{\partial G}{\partial a_j} \right] \psi_i(a')$$

7) Calculate the singular value decomposition  $J=USV^{T}$ , where  $UU^{T}=I_{m}$ ,  $VV^{T}=I_{N}$ and **S** is the matrix containing r (r  $\leq$  m) singular values of **J**.

- 8) Form vector  $\mathbf{\Delta}$  from the diagonal elements of the matrix  $(\Lambda(a') \Lambda^*)$ , i.e  $\mathbf{\Delta}$ = diag  $(\Lambda(a') - \Lambda^*)$ .
- 9) Solve a set of linear equations  $Jd = \Delta$  and obtain a family of possible solutions for d

$$\mathbf{d} = \mathbf{J}^{\dagger} \mathbf{\Delta} + \mathbf{V}^{*} \mathbf{b} ,$$

where  $J^{\dagger}$  is a Moore-Penrose pseudoinverse of J,  $V^{\star}$  is a matrix consisting of the last (N-r) columns of V, and **b** is an arbitrary (N-r)x1 vector.

10) Calculate the next approximation for the structural modification parameters

$$a^{t+1} = a^{t} - d$$

11) Set t = t+1 and repeat from step 2.

Output: Vector a<sup>t+1</sup> containing the required structural modifications parameters.

In order to ensure the physical realisability of the solution, the elements of  $a^{t+1}$  must be such that the mass and stiffness elements of a modified structure are real and positive. Since the original matrices **M** and **K** are assumed to be unknown, precise limits for permissible reduction of the mass and stiffness elements are also unknown. However, if all elements of  $a^{t+1}$  are made non-negative, then the obtained modifications will not require reduction in any structural parameter, thus avoiding the problem described above. To make the elements of  $a^{t+1}$  non-negative it is required to select a vector **b**, such that the elements of **d** obtained in step 9 are less than or equal to the corresponding elements of  $a^t$ . This may be achieved by inserting the following procedure between steps 9 and 10 of Algorithm 8.1: Procedure 8.1:

- i) Calculate  $d = J^{\dagger} \Delta$
- ii) Calculate  $\beta = a^t d$
- iii) Set
  - $\gamma(i) = \begin{cases} \beta(i) &, \beta(i) \ge 0 \\ 0 &, \beta(i) < 0 \end{cases}; i = 1, 2, ..., N$
- vi) Set  $\alpha = \beta \gamma$
- vii) Calculate  $\mathbf{b} = V^{*+} \alpha$ , where  $V^{*+}$  is a Moore-Penrose pseudoinverse of  $V^{*}$
- viii) If  $\| b \|$  is sufficiently small, stop.
- ix) Set  $d = d + V^*b$ , and repeat from step (ii).

Algorithm 8.1 allows to determine the vector  $a^{t+1}$  which contains structural modification parameters. The mass and stiffness modification matrices  $\Delta M$  and  $\Delta K$  can then be determined from the elements of  $a^{t+1}$  by (8.21). The obtained solution is optimal in a Rayleigh-Ritz sense, and the residual (8.2) is minimised for all possible systems with truncated modal matrix  $\Phi$  taken from the subspace spanned by  $\Phi_1$  and where  $\Lambda \approx \Lambda^*$ .

#### 8.2 Special Case I: Modifications to mass only

Consider now the case where only the mass matrix is subject to modification (i.e  $\Delta K=0$ ). This problem consists of *m* equations with *n* unknowns,  $\delta m_1, \delta m_2, ..., \delta m_n$ . Here we have

$$\mathbf{F} = \mathbf{\Lambda}_1 \tag{8.22}$$

and

$$G = I_m + \sum_{i=1}^n \delta m_i M_i$$
 (8.23)

and the problem can be solved by Algorithm 8.1. Example 8.1 demonstrates this procedure.

#### 8.3 Special Case II: Interrelated System

Frequently, the mass and the stiffness matrices are interrelated. For example, the longitudinally vibrating rod may be modelled by finite difference model, with

$$\mathbf{m}_{i} = \rho \mathbf{A}_{i} l_{i} \tag{8.24}$$

$$s_i = \frac{EA_i}{l_i} \tag{8.25}$$

where:  $m_i$  and  $s_i$  are the mass and stiffness of the  $i^{th}$  element, respectively,

E and  $\rho$  are the Young's Modulus and density of the rod, and

 $A_i$  and  $l_i$  are, respectively, the cross-sectional area and length of the  $i^{th}$  element.

Here, the mass elements are interrelated to the stiffnesses via  $A_i$  and  $l_i$ . Hence, a change in **M** (or  $\delta$ m) causes a respective change in **K** (or  $\delta$ s).

If we wish to modify the natural frequencies of a rod by changing only the cross-sectional area (i.e  $A_i$ ), then the corresponding changes in the mass and stiffness elements are

$$\delta m_i = \rho l_i \delta A_i \tag{8.26}$$

$$\delta s_i = \frac{E \, \delta A_i}{l_i} \quad , \tag{8.27}$$

and we may write

$$\delta m_i = C_i^{(M)} \, \delta A_i \tag{8.28}$$

$$\delta s_i = C_i^{(K)} \, \delta A_i \tag{8.29}$$

where  $C_i^{(M)}$  and  $C_i^{(K)}$  are constants.

Thus, the global modification matrices are

$$\Delta \boldsymbol{K} = \sum_{i=1}^{n} \delta \boldsymbol{s}_{i} \boldsymbol{B}_{i}^{(\boldsymbol{K})}$$
(8.30)

$$\Delta M = \sum_{i=1}^{n} \delta m_i B_i^{(M)}$$
(8.31)

where  $\mathbf{B}_{i}^{(M)}$  is the same as in (8.11), and

$$\boldsymbol{B}_{i}^{(K)} = \begin{bmatrix} b_{pq}^{(K)} \end{bmatrix} = \begin{cases} b_{i-1}^{(K)} = b_{ii}^{(K)} = 1 \\ b_{i-1}^{(K)} = b_{ii-1}^{(K)} = -1 \\ b_{pq}^{(K)} = 0 \quad elsewhere \end{cases}$$
(8.32)

Substituting (8.28),(8.31) and (8.29),(8.30) into (8.10) and (8.9) respectively, we obtain

$$\boldsymbol{F} = \boldsymbol{\Lambda}_{1} + \boldsymbol{\Phi}_{1}^{T} \left[ \sum_{i=1}^{n} \delta \boldsymbol{A}_{i} C_{i}^{(K)} \boldsymbol{B}_{i}^{(K)} \right] \boldsymbol{\Phi}_{1}$$
(8.33)

and

$$\boldsymbol{G} = \boldsymbol{I}_{m} + \boldsymbol{\Phi}_{1}^{T} \left[ \sum_{i=1}^{n} \delta \boldsymbol{A}_{i} C_{i}^{(M)} \boldsymbol{B}_{i}^{(M)} \right] \boldsymbol{\Phi}_{1}$$
(8.34)

or equivalently

$$F = \Lambda_1 + \sum_{i=1}^n \delta A_i K_i$$
(8.35)

and

$$\boldsymbol{G} = \boldsymbol{I}_m + \sum_{i=1}^n \delta \boldsymbol{A}_i \boldsymbol{M}_i$$
(8.36)

where

$$\mathbf{K}_{i} = \boldsymbol{\Phi}_{1}^{T} \mathbf{C}_{i}^{(K)} \mathbf{B}_{i}^{(K)} \boldsymbol{\Phi}_{1}$$
(8.37)

$$\mathbf{M}_{i} = \boldsymbol{\Phi}_{1}^{T} \mathbf{C}_{i}^{(M)} \mathbf{B}_{i}^{(M)} \boldsymbol{\Phi}_{1} \qquad (8.38)$$

Thus, the problem may be solved by Algorithm 8.1.

A similar analysis may be applied when using finite element modelling. For the finite element model of the rod,  $\mathbf{B}_{i}^{(M)}$  is given by:

$$\boldsymbol{B}_{i}^{(M)} = \begin{bmatrix} b_{pq}^{(M)} \end{bmatrix} = \begin{cases} b_{i-1}^{(M)} = b_{ii}^{(M)} = \frac{1}{3} \\ b_{i-1}^{(M)} = b_{ii-1}^{(M)} = \frac{1}{6} \\ b_{pq}^{(M)} = 0 \quad elsewhere \end{cases}$$
(8.39)

and  $\mathbf{B}_{i}^{(K)}$  is the same as in (8.32).

We note that the method presented is flexible, and can be applied to other models of vibratory systems. The only change which is needed, is to use the appropriate mapping matrices associated with the chosen system.

#### **8.4 Numerical Examples**

#### Example 8.1: Mass only modifications

Consider a 10 degrees of freedom mass-spring system with

1	<b></b>									
	200	- 10	- 20	- 5	- 5	- 10	0	0	- 50	- 50
	- 10	100	0	0	0	0	- 20	- 10	- 20	- 10
	- 20	0	300	- 40	- 30	- 60	- 10	0	- 20	- 10
	- 5	0	- 40	400	- 30	- 40	- 50	- 20	- 10	- 70
	- 5	0	- 30	- 30	150	- 10	- 5	- 5	- 20	0
K =	- 10	0	- 60	- 40	- 10	250	0	0	0	- 80
	0	- 20	- 10	- 50	- 5	0	120	- 5	0	- 10
	0	- 10	0	- 20	- 5	0	- 5	250	0	-100
	- 50	- 20	- 20	- 10	- 20	0	0	0	350	- 40
	- 50	- 10	- 10	- 70	0	- 80	- 10	-100	- 40	400
	0 0 - 50	- 20 - 10 - 20	- 10 0 - 20	- 50 - 20 - 10	- 5 - 5 - 20	0 0 0	120 - 5 0	- 5 250 0	0 0 350	- 10 -100 - 40

and

and

 $\mathbf{M} = \text{diag} (1, 1, 1, 1, 1, 1, 1, 1, 1) .$ 

The smallest three eigenvalues and the corresponding mass-normalised eigenvectors of the system are

$$\mathbf{\Phi}_{1} = \text{diag} \quad (61.8300, \ 108.3525, \ 129.1425) \\ \mathbf{\Phi}_{1} = \begin{bmatrix} 0.2605 & -0.1625 & 0.3016 \\ 0.5453 & 0.7597 & 0.2873 \\ 0.2050 & -0.2277 & 0.0955 \\ 0.2301 & -0.1751 & -0.0392 \\ 0.2694 & -0.4256 & 0.1967 \\ 0.2558 & -0.2886 & 0.1673 \\ 0.5081 & -0.0279 & -0.8355 \\ 0.2159 & -0.1136 & 0.1054 \\ 0.1610 & -0.0617 & 0.1441 \\ 0.2668 & -0.1792 & 0.1385 \end{bmatrix}$$

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Assume that only  $\Phi_1$  and  $\Lambda_1$  are given, and that no other information is known. We wish to find modifications to the mass matrix only, such that the first three eigenvalues of the modified system are

$$\Lambda^* = diag(1, 2, 3)$$

Applying Algorithm 1 with  $a^0 = 0$  we obtain

 $\Delta M = diag(41.139, 82.580, 16.728, 7.127, 47.813, 32.122, 54.471, 14.350, 11.193, 24.213)$ .

Setting new mass matrix  $M_{mod} = M + \Delta M$ , we find that the eigenvalues of the modified system are

 $\Lambda_{mod} = diag(0.99, 1.96, 2.72, 3.92, 6.10, 11.58, 17.33, 21.14, 29.48, 50.50)$ .

The small discrepancy between the desired eigenvalues of  $\Lambda^*$  and the smallest three eigenvalues of  $\Lambda_{med}$  is due to the truncation error, which is unavoidable.

#### Example 8.2 : Longitudinally Vibrating Rod

Consider a uniform axially vibrating rod, fixed at x=0 and free to oscillate at x=L, with uniform properties  $\rho=E=L=A=1$ . The first three eigenpairs of the rod, obtained for a 10 degrees of freedom finite element model, are shown in Table 8.1.

Position from	Mode Shapes				
the Fixed End of the Rod	1 <sup>st</sup> mode	2 <sup>nd</sup> mode	3 <sup>rd</sup> mode		
0.1	0.2217	-0.6540	1.0527		
0.2	0.4379	-1.1655	1.4888		
0.3	0.6434	-1.4229	1.0527		
0.4	0.8330	-1.3701	0.0000		
0.5	1.0021	-1.0187	-1.0527		
0.6	1.1465	-0.4452	-1.4888		
0.7	1.2627 0.2254		-1.0527		
0.8	1.3478	0.8468	0.0000		
0.9	1.3997	1.2836	1.0527		
1.0	1.4171	1.4406	1.4888		
Eigenvalues	2.4725	22.6205	64.9165		

Table 8.1 : First Three Modes of a Uniform Cantilever Rod of Example 2

We wish to change the cross-sectional area of the rod, so that the eigenvalues of the modified rod will be

$$\Lambda^* = diag(1, 15, 100)$$

Applying Algorithm 8.1, we obtain the following area modifications for the finite elements model

 $\delta A = diag(-0.6698, -0.8567, -0.3661, 0.4642, 0.3919, -0.3530, -0.3213, 0.5501, 0.9244, 0.5870)$ 

When these changes are implemented, the lowest three eigenvalues of the finite element model of the modified rod are:

$$\Lambda_{mad} = diag (0.7, 12.7, 96.6)$$

Note that as predicted before, the desired eigenvalues  $\lambda_i^*$  are higher than their corresponding true eigenvalues  $\lambda_{mod i}$  of the modified system.

#### Example 8.3: Sensitivity Test

The sensitivity of Algorithm 8.1 to perturbations is now demonstrated.

Let

 $\Lambda_1 = \text{diag} ( 60, 105, 130 ),$ 

and

$$\boldsymbol{\Phi}_{1} = \begin{bmatrix} 0.3 & -0.2 & 0.3 \\ 0.5 & 0.8 & 0.3 \\ 0.2 & -0.2 & 0.1 \\ 0.2 & -0.2 & 0.0 \\ 0.3 & -0.4 & 0.2 \\ 0.3 & -0.3 & 0.2 \\ 0.5 & 0.0 & -0.8 \\ 0.2 & -0.1 & 0.1 \\ 0.2 & -0.1 & 0.1 \\ 0.3 & -0.2 & 0.1 \end{bmatrix}$$

which may be obtained by "rounding off" the elements of the eigenpairs in Example 8.1.

Repeating Example 8.1 with these values, we obtain

 $\Delta M = diag(36.478, 50.249, 15.308, 14.211, 15.418, 32.902, 92.94, 24.081, 24.081, 50.464)$ 

which is quite different from the modifications obtained in Example 8.1. However, we find that the eigenvalues of the modified system are

 $\Lambda_{mod} = diag(0.97, 1.96, 3.09, 5.79, 7.64, 8.83, 12.06, 14.57, 19.03, 27.53)$ 

and the three smallest eigenvalues of  $\Lambda_{mod}$  represent a good estimate to the desired eigenvalues. Thus the introduction of perturbations to the given data, caused convergence to a different possible solution.

#### 8.5 Conclusions

In this section we have defined an optimisation problem, which allows us to overcome the effect of truncation. It was shown that this optimisation problem may be solved by applying the algorithm of Joseph [16] (with some minor alterations) to obtain a physically realisable solution. The obtained solutions are optimal in a Rayleigh-Ritz sense. The desired eigenvalues are thus higher than the eigenvalues of the actual modified system.

Using this approach we may also modify vibratory systems with interrelated mass and stiffness matrices. Some examples were given and the sensitivity of the problem to perturbation has been numerically demonstrated.

## Section 9

### **PROBLEM 4:**

# MODIFICATIONS FOR DESIRED NATURAL FREQUENCIES AND MODE SHAPES

In this section we present the analysis of Problem 4, which was formulated in section 2.3. Our Problem 4 is identical to the problem investigated by Ram and Braun in [46]. They have shown that a family of optimal solutions (in a Rayleigh-Ritz sense) to this problem is characterised by the following equations

$$\Delta M = \Phi_{1}^{T\dagger} \left( \Psi^{-T} \Psi^{-1} - I_{m} \right) \Phi_{1}^{\dagger} + Y - \Phi_{1}^{T\dagger} \Phi_{1}^{T} Y \Phi_{1} \Phi_{1}^{\dagger}$$
(9.1)

$$\Delta K = \Phi_1^{T\dagger} \left( \Psi^{-T} \Lambda^* \Psi^{-1} - \Lambda_1 \right) \Phi_1^{\dagger} + X - \Phi_1^{T\dagger} \Phi_1^{T} X \Phi_1 \Phi_1^{\dagger}$$
(9.2)

where  $\Phi_1^{\dagger}$  denotes the Moore-Penrose pseudoinverse of  $\Phi_1$ ,  $\Psi = \Phi_1^{\dagger} \Phi^{\star}$ , and X and Y are arbitrary  $m \times m$  real symmetric matrices.

We note that all elements in the equations (9.1) and (9.2) are known, with the exception of matrices **X** and **Y**. Since **X** and **Y** can be arbitrarily assigned, a solution for  $\Delta M$  and  $\Delta K$  can be evaluated. However, in general, such arbitrarily selected **X** and **Y** do not result in a physically realisable solution. In other words, the obtained mathematical solutions do not give us a hint on how to change the geometry or the material properties of the structure in order to get the required modifications. Therefore, here our main aim is to develop a method for extracting a *physically realisable* solutions for  $\Delta M$  and  $\Delta K$  from the general family of solutions defined by equations (9.1) and (9.2).

Setting

$$\boldsymbol{P} = \boldsymbol{\Phi}_{1}^{T\dagger} \left( \boldsymbol{\Psi}^{-T} \boldsymbol{\Psi}^{-1} - \boldsymbol{I}_{m} \right) \boldsymbol{\Phi}_{1}^{\dagger}$$
(9.3)

$$T = \Phi_{1}^{T\dagger} \left( \Psi^{-T} \Lambda^{*} \Psi^{-1} - \Lambda_{1} \right) \Phi_{1}^{\dagger}$$
(9.4)

$$H = \Phi_1 \Phi_1^{\dagger} \tag{9.5}$$

equations (9.1) and (9.2) become

$$\Delta \mathbf{M} = \mathbf{P} + \mathbf{Y} - \mathbf{H}^{\mathrm{T}} \mathbf{Y} \mathbf{H}$$
(9.6)

$$\Delta \mathbf{K} = \mathbf{T} + \mathbf{X} - \mathbf{H}^{\mathrm{T}} \mathbf{X} \mathbf{H} .$$
 (9.7)

We note that equations (9.6) and (9.7) have identical form, and also that this form is very similar to the well known *Discrete Lyapunov Equation (DLP)*. The solution for DLP is available in the Control Toolbox of MATLAB under the function name **dlyap**. The **dlyap** algorithm of MATLAB allows to determine Y, such that equation (9.6) holds for any given matrices  $\Delta M$ , P and H (or determine X for any given  $\Delta K$ , T and H in a case of equation

(9.7)). Our problem is different, since neither  $\Delta M$  nor  $\Delta K$  are known. However, it appears that we should be able to arbitrarily choose any  $\Delta M$  and  $\Delta K$ , and then calculate Y and X which satisfy (9.6) and (9.7). The implication of this is that we may choose  $\Delta M$  and  $\Delta K$  to be zero matrices, and then obtain Y and X which still, supposedly, give a Rayleigh-Ritz approximation to the desired natural frequencies and mode shapes. Clearly, this is not physically possible. A detailed investigation of this apparent contradiction has shown that both (9.6) and (9.7) are ill-conditioned to be solved by **dlyap** algorithm, and therefore the contradiction does not really exist. And interestingly, the reasons behind this ill-conditioning have also provided a key element in deriving a solution to our problem. The following analysis describes this solution.

The matrix  $\mathbf{H}$ , defined by (9.5), is a product of a matrix by its pseudoinverse. Calculating the singular values decomposition of  $\mathbf{H}$ , we obtain

$$\mathbf{H} = \mathbf{U} \mathbf{Z} \mathbf{U}^{\mathrm{T}}$$
(9.8)

where  $UU^{T} = U^{T}U = I_{n}$ , and the partitioned singular values matrix Z is as follows

$$Z = \left[\frac{I_m}{O} \mid \frac{O}{O}\right]$$
(9.9)

and where O represents submatrices with all elements equal to zero.

Substituting (9.8) into (9.6) and (9.7), we obtain

$$\Delta \mathbf{M} = \mathbf{P} + \mathbf{Y} - \mathbf{U}\mathbf{Z}\mathbf{U}^{\mathrm{T}}\mathbf{Y} \ \mathbf{U}\mathbf{Z}\mathbf{U}^{\mathrm{T}}$$
(9.10)

$$\Delta \mathbf{K} = \mathbf{T} + \mathbf{X} - \mathbf{U}\mathbf{Z}\mathbf{U}^{\mathrm{T}}\mathbf{X} \ \mathbf{U}\mathbf{Z}\mathbf{U}^{\mathrm{T}} \ . \tag{9.11}$$

Multiplying both sides of (9.10) and (9.11) by  $U^{T}$  and U, we get

$$\mathbf{U}^{\mathrm{T}} \Delta \mathbf{M} \ \mathbf{U} = \mathbf{U}^{\mathrm{T}} \mathbf{P} \ \mathbf{U} + \mathbf{U}^{\mathrm{T}} \mathbf{Y} \ \mathbf{U} - \mathbf{Z} \mathbf{U}^{\mathrm{T}} \mathbf{Y} \ \mathbf{U} \mathbf{Z}$$
(9.12)

$$\mathbf{U}^{\mathrm{T}} \Delta \mathbf{K} \ \mathbf{U} = \mathbf{U}^{\mathrm{T}} \ \mathbf{T} \ \mathbf{U} + \mathbf{U}^{\mathrm{T}} \mathbf{X} \ \mathbf{U} - \mathbf{Z} \mathbf{U}^{\mathrm{T}} \mathbf{X} \ \mathbf{U} \mathbf{Z} \ , \tag{9.13}$$

Setting

$$\mathbf{P}^* = \mathbf{U}^{\mathrm{T}} \mathbf{P} \mathbf{U} \tag{9.14}$$

$$\Gamma^* = \mathbf{U}^{\mathrm{T}} \mathbf{T} \mathbf{U} \tag{9.15}$$

$$\mathbf{Y}^* = \mathbf{U}^{\mathrm{T}} \mathbf{Y} \ \mathbf{U} \tag{9.16}$$

$$\mathbf{X}^* = \mathbf{U}^{\mathrm{T}} \mathbf{X} \ \mathbf{U},\tag{9.17}$$

then equations (9.12) and (9.13) become

$$\mathbf{U}^{\mathrm{T}} \Delta \mathbf{M} \ \mathbf{U} = \ \mathbf{P}^{*} + \ \mathbf{Y}^{*} - \ \mathbf{Z} \ \mathbf{Y}^{*} \mathbf{Z}$$
(9.18)

$$\mathbf{U}^{\mathrm{T}} \Delta \mathbf{K} \ \mathbf{U} = \mathbf{T}^{*} + \mathbf{X}^{*} - \mathbf{Z} \ \mathbf{X}^{*} \mathbf{Z} \ , \tag{9.19}$$

Partitioning U,  $P^*$ ,  $T^*$ ,  $Y^*$  and  $X^*$  as follows

$$\mathbf{U} = [\mathbf{U}_1 | \mathbf{U}_2], \mathbf{U}_1 \text{ is } n \times m \text{ real matrix } m < n$$
(9.20)

$$P^{*} = \left[\frac{P_{1}^{*}}{P_{2}^{*T}} \frac{|P_{2}^{*}|}{|P_{3}^{*}|}\right], P_{1}^{*} \text{ is } mxm \qquad (9.21)$$

$$T^{*} = \left[\frac{T_{1}^{*}}{T_{2}^{*T}} \frac{|T_{2}^{*}|}{|T_{3}^{*}|}\right], T_{1}^{*} \text{ is } mxm \qquad (9.22)$$

$$Y^{*} = \left[\frac{Y_{1}^{*}}{Y_{2}^{*T}} \left| \frac{Y_{2}^{*}}{Y_{3}^{*}} \right], Y_{1}^{*} \text{ is } mxm \qquad (9.23)$$

$$X^{*} = \left[\frac{X_{1}^{*}}{X_{2}^{*T}} \frac{|X_{2}^{*}|}{|X_{3}^{*}|}\right], X_{1}^{*} \text{ is } mxm \qquad (9.24)$$

then, also using (9.9), equations (9.18) and (9.19) may be written as follows:

$$\begin{bmatrix} U_1^T \Delta M U_1 & | U_1^T \Delta M U_2 \\ U_2^T \Delta M U_1 & | U_2^T \Delta M U_2 \end{bmatrix} = \begin{bmatrix} P_1^* & | P_2^* \\ P_2^{*T} & | P_3^* \end{bmatrix} + \begin{bmatrix} Y_1^* & | Y_2^* \\ Y_2^{*T} & | Y_3^* \end{bmatrix} - \begin{bmatrix} I_m & | O \\ O & | O \end{bmatrix} \begin{bmatrix} I_m & | O \\ Y_2^{*T} & | Y_3^* \end{bmatrix} \begin{bmatrix} I_m & | O \\ O & | O \end{bmatrix}$$
(9.25)

and

$$\begin{bmatrix} U_{1}^{T} \Delta K U_{1} & | & U_{1}^{T} \Delta K U_{2} \\ \hline U_{2}^{T} \Delta K U_{1} & | & U_{2}^{T} \Delta K U_{2} \end{bmatrix} = \begin{bmatrix} T_{1}^{*} & | & T_{2}^{*} \\ \hline T_{2}^{*T} & | & T_{3}^{*} \end{bmatrix} + \begin{bmatrix} X_{1}^{*} & | & X_{2}^{*} \\ \hline X_{2}^{*T} & | & X_{3}^{*} \end{bmatrix} - \begin{bmatrix} I_{m} & | & O \\ \hline O & | & O \end{bmatrix} \begin{bmatrix} X_{1}^{*} & | & X_{2}^{*} \\ \hline X_{2}^{*T} & | & X_{3}^{*} \end{bmatrix} \begin{bmatrix} I_{m} & | & O \\ \hline O & | & O \end{bmatrix}.$$
(9.26)

Therefore we obtain

$$\left[\frac{U_1^T \Delta M U_1}{U_2^T \Delta M U_1} \left| \frac{U_1^T \Delta M U_2}{U_2^T \Delta M U_2} \right] = \left[\frac{P_1^*}{P_2^{*T}} \left| \frac{P_2^*}{P_3^*} \right] + \left[\frac{Y_1^*}{Y_2^{*T}} \left| \frac{Y_2^*}{Y_3^*} \right] - \left[\frac{Y_1^*}{O} \left| \frac{O}{O} \right] \right]$$
(9.27)

and

$$\left[\frac{U_1^T \Delta K U_1}{U_2^T \Delta K U_1} + \frac{U_1^T \Delta K U_2}{U_2^T \Delta K U_2}\right] = \left[\frac{T_1^*}{T_2^{*T}} + \frac{T_2^*}{T_3^*}\right] + \left[\frac{X_1^*}{X_2^{*T}} + \frac{X_2^*}{X_3^*}\right] - \left[\frac{X_1^*}{O} + \frac{O}{O}\right] \quad . \quad (9.28)$$

Simplifying (9.27) and (9.28) further we obtain

$$\left[\frac{U_{1}^{T}\Delta MU_{1}}{U_{2}^{T}\Delta MU_{1}} \left| \frac{U_{1}^{T}\Delta MU_{2}}{U_{2}^{T}\Delta MU_{2}} \right] = \left[\frac{P_{1}^{*}}{P_{2}^{*T} + Y_{2}^{*T}} \left| \frac{P_{2}^{*} + Y_{2}^{*}}{P_{3}^{*} + Y_{3}^{*}} \right]$$
(9.29)

and

$$\left[\frac{U_1^T \Delta K U_1}{U_2^T \Delta K U_1} \left| \frac{U_1^T \Delta K U_2}{U_2^T \Delta K U_2} \right] = \left[\frac{T_1^*}{T_2^{*T} + X_2^{*T}} \left| \frac{T_2^* + X_2^*}{T_3^* + X_3^*} \right] \right]$$
(9.30)

Thus, the reasons behind the ill-conditioning of the equations (9.6) and (9.7) for the function **dlyap** of MATLAB now become clear. Separating the first elements from the partitioned matrices in (9.29) and (9.30), we note that the following relations must be satisfied

$$\mathbf{U}_{1}^{\mathrm{T}} \Delta \mathbf{M} \ \mathbf{U}_{1} = \mathbf{P}_{1}^{\star} \tag{9.31}$$

and

 $\mathbf{U}_{1}^{\mathrm{T}} \Delta \mathbf{K} \ \mathbf{U}_{1} = \mathbf{T}_{1}^{*}. \tag{9.32}$ 

Equations (9.31) and (9.32) are independent of Y and X. Since P, T and H are known,  $P_1^*$ ,  $T_1^*$  and  $U_1$  are also predetermined. Thus, selecting arbitrary  $\Delta M$  and  $\Delta K$ , would not, in general, satisfy (9.31) and (9.32), and therefore, the fundamental condition for a successful application of the **dlyap** algorithm is violated.

We note that in equations (9.31) and (9.32) the only unknowns are  $\Delta M$  and  $\Delta K$ , and therefore these matrices can be calculated. It also follows from the **dlyap** algorithm, that for any so obtained  $\Delta M$  and  $\Delta K$ , all other elements of the equations (9.29) and (9.30)

(except for these described in (9.31) and (9.32)) may, in general, be satisfied by some particular matrices **Y** and **X**. Thus, it is a sufficient condition for determining a solution to our problem, if we obtain physically realisable  $\Delta M$  and  $\Delta K$  which satisfy (9.31) and (9.32). The following analysis shows the necessary procedures for achieving this aim.

#### 9.1 Mass Modifications

In general, for a *n* degrees-of-freedom system, the mass matrix **M** would contain *n* independent parameters corresponding to the masses of each of the elements which are part of the system. However, when evaluating the necessary modifications to the system's mass (i.e  $\Delta$ **M**), a designer may wish to restrict any such modification to only l (l < n) elements.

The global mass modification matrix  $\Delta M$  can then be expressed as

$$\Delta \boldsymbol{M} = \sum_{q=1}^{I} \delta m_q \, \boldsymbol{B}_q^{(M)} \tag{9.33}$$

where  $\delta m_q$  is a modification to the mass of the q<sup>th</sup> element, and  $\mathbf{B}_q^{(M)}$  is the *nxn* mapping matrix corresponding to a chosen analytical model.

Substituting equation (9.33) into equation (9.31), we obtain

$$\boldsymbol{P}_{1}^{*} = \sum_{q=1}^{l} \delta m_{q} \boldsymbol{U}_{1}^{T} \boldsymbol{B}_{q}^{(M)} \boldsymbol{U}_{1} \qquad (9.34)$$

Also from (9.14), (9.20) and (9.21),  $\mathbf{P_1}^*$  is equal to

$$P_{1}^{*} = \left[p_{ij}^{*}\right] = U_{1}^{T} P U_{1}$$
(9.35)

Partitioning  $U_1$  into column vectors as shown

$$\boldsymbol{U}_1 = \begin{bmatrix} \boldsymbol{u}_1 & \boldsymbol{u}_2 & \dots & \boldsymbol{u}_m \end{bmatrix}, \qquad (9.36)$$

then each element  $\boldsymbol{p}_{ij}^{*}$  must be equal to

$$p_{ij}^{*} = \sum_{q=1}^{l} \delta m_{q} u_{i}^{T} B_{q}^{(M)} u_{j} \qquad (9.37)$$

Let N=  $\frac{1}{2}(m^2 + m)$  and construct the vectors

$$\mathbf{y}_{\mathbf{M}} = (\mathbf{y}_{1}^{(\mathbf{M})}, \mathbf{y}_{2}^{(\mathbf{M})}, ..., \mathbf{y}_{\mathbf{N}}^{(\mathbf{M})})^{\mathrm{T}} = (\mathbf{p}_{11}^{*}, \mathbf{p}_{12}^{*}, \mathbf{p}_{13}^{*}, ..., \mathbf{p}_{1m}^{*}, \mathbf{p}_{22}^{*}, ..., \mathbf{p}_{mm}^{*})^{\mathrm{T}}$$
(9.38)

and

$$\delta \mathbf{m} = (\delta m_1, \, \delta m_2, \, ..., \, \delta m_I)^{\mathrm{T}}.$$
 (9.39)

Denote

$$\boldsymbol{F}_{M} = \left[ f_{ij}^{(M)} \right] = \frac{\partial y_{i}^{(M)}}{\partial (\delta m_{j})} , \quad (i = 1, 2, ..., N; j = 1, 2, ..., l)$$
(9.40)

then all the elements of  $\mathbf{F}_{\mathbf{M}}$  can be evaluated using equation (9.37). Equation (9.31) can be written in a vector form

$$\mathbf{F}_{\mathbf{M}} \,\, \boldsymbol{\delta \mathbf{m}} = \mathbf{y}_{\mathbf{M}}.\tag{9.41}$$

Since  $\mathbf{F}_{M}$  and  $\mathbf{y}_{M}$  are known, (9.41) can be solved for  $\delta \mathbf{m}$ , and the mass modification matrix  $\Delta \mathbf{M}$  can then be determined from the elements of vector  $\delta \mathbf{m}$  by equation (9.33).

We note that in order to obtain a solution for the system of size  $n \times m$  (m < n), we need to solve an augmented system (9.41) of size Nxl (N=  $\frac{1}{2}(m^2 + m)$ ). However, in this case augmentation is based on the smaller dimension m, whereas number of independent parameters available for optimisation is fixed at l. Therefore depending on the values of l and m there are three possibilities for the solution to (9.41).

Set r = l-N, then if r > 0 there will be a family of solutions for  $\delta m$ . This family of solutions is characterised by the following equation

$$\delta \mathbf{m} = \mathbf{F}_{\mathbf{M}}^{\dagger} \mathbf{y}_{\mathbf{M}} + \mathbf{V}_{\mathbf{r}} \mathbf{b}$$
(9.42)

where  $\mathbf{F}_{\mathbf{M}}^{\dagger}$  is the Moore-Penrose pseudoinverse of  $\mathbf{F}_{\mathbf{M}}$ , **b** is an arbitrary vector of dimension  $r \times 1$ , and  $\mathbf{V}_{\mathbf{r}}$  is a matrix of dimension  $l \times r$  which is obtained by a following procedure

Calculate singular value decomposition  $\mathbf{F}_{\mathbf{M}} = \mathbf{W}\mathbf{S}\mathbf{V}^{\mathsf{T}}$ , and partition the lxl matrix  $\mathbf{V} = [\mathbf{V}_{\mathsf{N}} \mid \mathbf{V}_{\mathsf{r}}]$ , where  $\mathbf{V}_{\mathsf{N}}$  is lxN, and  $\mathbf{V}_{\mathsf{r}}$  is lxr. (9.43)

If r = 0, then  $\mathbf{F}_{\mathbf{M}}$  is a full square matrix, and there will be one unique solution for  $\delta \mathbf{m}$ . This unique solution is

$$\delta \mathbf{m} = \mathbf{F}_{\mathbf{M}}^{-1} \mathbf{y}_{\mathbf{M}} . \tag{9.44}$$

And finally, if r<0, then there are no solutions for  $\delta \mathbf{m}$ , and only an approximate solution (which is optimal in a least squares sense) can be obtained by

$$\delta \mathbf{m} = \mathbf{F}_{\mathbf{M}}^{\dagger} \mathbf{y}_{\mathbf{M}} \ . \tag{9.45}$$

If it is desired that all the elements of  $\delta \mathbf{m}$  to be positive, and if solutions of (9.42), (9.44) and (9.44) do not yield positive  $\delta \mathbf{m}$ , than it may be obtained by solving the following *non*- negative least squares problem

$$\min_{\delta \mathbf{m}} \| \mathbf{F}_{\mathbf{M}} \delta \mathbf{m} - \mathbf{y}_{\mathbf{M}} \|_{2}, \text{ subject to } \delta \mathbf{m} \ge \mathbf{0}.$$
(9.46)

This will produce an optimal non-negative solution to the vector  $\delta \mathbf{m}$  in a least square sense. It should be noted, however, that  $\Delta \mathbf{M}$  which is a solution of (9.1) is itself only a Rayleigh-Ritz approximation to the solution of the modification problem. Therefore, an approximate solution to (9.1), obtained by (9.45) and (9.46), is in reality "an approximation to an approximation", which may not be acceptable in applications based on possible poor quality of the solutions. Thus, from practical considerations, it appears that it may be best to restrict the application of this method to systems where  $\delta \mathbf{m}$  can be determined by either (9.42) or (9.44), which requires that  $l \geq N$ .

The above procedure ensures that the form of the obtained mass modification matrix  $\Delta M$  corresponds to a physically realisable system via equation (9.33). The procedure is also independent of an arbitrary choice for the matrix **Y**, and it is summarised by the following algorithm.

#### Algorithm 9.1: Determination of a Mass Modification Matrix

*Input*: Modal test data  $\Phi_1(nxm)$  and  $\Lambda_1(mxm)$ , and desired modal data

 $\Phi^*(nxm)$  and  $\Lambda^*(mxm)$ .

Algorithm:

1) Calculate **P** and **H** using (9.3) and (9.5).

2) Obtain the singular value decomposition  $H = UZU^{T}$ .

- 3) Column partition  $U = [u_1, u_2, ..., u_m, ..., u_n].$
- 4) Set  $U_1 = [u_1, u_2, ..., u_m]$ .
- 5) Calculate  $\mathbf{P}_1^* = \mathbf{U}_1^T \mathbf{P} \cdot \mathbf{U}_1$ .
- 6) Set N=  $\frac{1}{2}(m^2 + m)$ .
- 7) Construct vectors  $\mathbf{y}_{M}$  as in (9.38).
- 8) Form vector  $\delta m$  of dimension lx1 as in (9.39).
- 9) Construct matrix  $F_{M}$  using (9.40) and (9.37).
- 10) (a) If l > N, then determine  $\delta m$  by equation (9.42),
  - (b) if l = N, then determine  $\delta m$  by (9.44),
  - (c) if l < N, then determine  $\delta m$  by (9.45).
- If desire non-negative *δm* and the one obtained in step 10 is not,
   then determine *δm* by solving (9.46).
- 12) Construct  $\Delta M$  from the elements of  $\delta m$  using (9.33).

**Output:** Physically realisable mass modification matrix  $\Delta M$ .

#### 9.2 Stiffness Modifications

The number of independent spring elements in a *n* degrees-of-freedom mass-spring model may vary from (n-1) in a case of a *free-free simply-connected* system to  $\frac{1}{2}(n^2+n)$  for a *multiconnected* system. Thus assuming that J (where  $J \leq \frac{1}{2}(n^2+n)$ ) of the spring elements are available for modifications, the global stiffness modification matrix  $\Delta K$  can then be expressed as

$$\Delta \mathbf{K} = \sum_{q=1}^{J} \delta s_q \, \mathbf{B}_q^{(\mathbf{K})} \tag{9.47}$$

where  $\delta s_q$  is a modification to the stiffness of the q<sup>th</sup> spring element, and  $\mathbf{B}_q^{(K)}$  is the *nxn* mapping matrix corresponding to a chosen analytical model.

We note that the form of equations (9.31) and (9.32), and also of (9.33) and (9.47), are identical. Therefore, by substituting matrix **T**, defined by (9.4), for matrix **P**, and also substituting vector

$$\boldsymbol{\delta s} = (\delta s_1, \, \delta s_2, \, \dots, \, \delta s_J \,) \tag{9.48}$$

for vector  $\delta \mathbf{m}$ , we may use the same procedure for evaluating  $\Delta \mathbf{K}$  as was used for calculating  $\Delta \mathbf{M}$ . The required procedure is described by the following algorithm.

Algorithm 9.2: Determination of a Stiffness Modification Matrix Input: Modal test data  $\Phi_1(nxm)$  and  $\Lambda_1(mxm)$ , and desired modal data

 $\Phi^*(n \times m)$  and  $\Lambda^*(m \times m)$ .

#### Algorithm:

- 1) Calculate T and H using (9.4) and (9.5).
- 2) Obtain the singular value decomposition  $\mathbf{H} = \mathbf{U}\mathbf{Z}\mathbf{U}^{\mathsf{T}}$ .
- 3) Column partition  $U = [u_1, u_2, ..., u_m, ..., u_n]$ .
- 4) Set  $U_1 = [u_1, u_2, ..., u_m]$ .
- 5) Calculate

$$\boldsymbol{T_1}^* = \begin{bmatrix} \boldsymbol{t}_{ij}^* \end{bmatrix} = \boldsymbol{U_1}^T \boldsymbol{T} \boldsymbol{U_1}$$

- 6) Set N=  $\frac{1}{2}(m^2 + m)$ .
- 7) Construct vector

$$\mathbf{y}_{\mathsf{K}} = (\mathbf{y}_{1}^{(\mathsf{K})}, \mathbf{y}_{2}^{(\mathsf{K})}, \dots, \mathbf{y}_{\mathsf{N}}^{(\mathsf{K})})^{\mathsf{T}} = (\mathbf{t}_{11}, \mathbf{t}_{12}, \mathbf{t}_{13}, \dots, \mathbf{t}_{1m}, \mathbf{t}_{22}, \dots, \mathbf{t}_{mm})^{\mathsf{T}}$$

- 8) Form vector  $\delta s$  of dimension Jx1 as in (9.48).
- 9) Construct matrix

$$F_{K} = [f_{ij}^{(K)}] = \frac{\partial y_{i}^{(K)}}{\partial (\delta s_{j})}, \quad (i = 1, 2, ..., N; j = 1, 2, ..., J)$$

using equation

$$t_{ij}^{*} = \sum_{q=1}^{J} \delta s_{q} u_{i}^{T} B_{q}^{(K)} u_{j}$$
.

10) (a) If J > N, then  $\delta s = F_{\kappa}^{\dagger} y_{\kappa} + V_{r} b$ ,

( $V_r$  is obtained by a procedure similar to (9.43), **b** is an arbitrary vector).

- (b) if J = N, then  $\delta s = F_{\kappa}^{-1}y_{\kappa}$
- (c) if J < N, then  $\delta s = F_{\kappa}^{\dagger} y_{\kappa}$

11) If desire non-negative **os** and the one obtained in step 10 is not, then

determine *d***s** by solving

$$\min_{\delta s} \| \mathbf{F}_{K} \delta s - \mathbf{y}_{K} \|_{2}, \text{ subject to } \delta s \geq 0$$

12) Construct  $\Delta K$  from the elements of  $\delta s$  using (9.47).

**Output:** Physically realisable stiffness modification matrix  $\Delta K$ .

#### 9.3 Numerical Examples

#### Example 9.1

Consider a mass spring system shown in Figure 9.1.

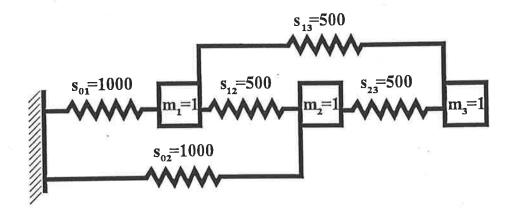


Figure 9.1: A three-degree-of-freedom mass-spring system

The mass and stiffness matrices of this system are as follows

$$M = diag(1,1,1)$$

and

$$\boldsymbol{K} = \begin{bmatrix} 2000 & -500 & -500 \\ -500 & 2000 & -500 \\ -500 & -500 & 1000 \end{bmatrix}$$

The spectral and modal properties associated with this system are as follows:

 $\Lambda = \text{diag}$  (500, 2000, 2500)

and

$$\Phi = \begin{bmatrix} 0.4082 & -0.5774 & -0.7071 \\ 0.4082 & -0.5774 & 0.7071 \\ 0.8165 & 0.5774 & 0.0000 \end{bmatrix}$$

Now, assume that the physical properties of this system, namely its mass and stiffness matrices, are not known, and also assume that the only available information about the system are the first two of its modes, i.e

$$\Lambda_1 = \text{diag} (500, 2000),$$

and		Г	- T	
anu		0.4082	-0.5774	
	$\Phi_1 =$	0.4082	-0.5774	
	1	0.8165	0.5774	

Suppose that we want to modify the system so that all elements of  $\Phi_1$  are not larger than 0.5, but we also want to achieve this without increasing the magnitude of the existing elements. Under these constraints the desired modal matrix,  $\Phi^*$ , is

	0.4	-0.5	
Φ* =	0.4	-0.5	
	0.5	0.5	

We also want to modify the spectral properties of the system so that the desired eigenvalues of the system are

$$\Lambda^* = \text{diag} (500, 1500)$$
.

Based on the dimensions of  $\Phi_1$  and  $\Lambda_1$ , we realise that in the above system there are a maximum of three independent mass elements and a maximum of six independent spring elements available for modification. The number of constraints to be satisfied by the solutions to (9.31) and (9.32) is equal to three for both mass and stiffness modifications. Therefore, we expect that there exists one unique solution for  $\delta m$ , and a family of solutions for  $\delta s$ .

Applying Algorithm 9.1, using step 10(b), we obtain the following unique solution for the mass modification matrix,  $\Delta M$ , corresponding to a mass-spring analytical model

$$\Delta M = \text{diag} (0.2346, 0.2346, 1.0247).$$

Since there exists a family of solutions for  $\delta s$ , we choose a minimal norm solution (determined by using step 10(a) in the Algorithm 9.2 with *b* being zero vector), and obtain the following stiffness modification matrix  $\Delta K$ 

$$\Delta K = \begin{bmatrix} -197.5 & 0.0 & -67.9 \\ 0.0 & -197.5 & -67.9 \\ -67.9 & -67.9 & 938.3 \end{bmatrix}$$

The modified mass and stiffness matrices for the system are then as follows

$$\mathbf{M}_{mod} = \mathbf{M} + \Delta \mathbf{M} = \text{diag} (1.2346, 1.2346, 2.0247)$$

and

$$\boldsymbol{K}_{\text{mod}} = \boldsymbol{K} + \boldsymbol{\Delta}\boldsymbol{K} = \begin{bmatrix} 1802.5 & -500.0 & -567.9 \\ -500.0 & 1802.5 & -567.9 \\ -567.9 & -567.9 & 1938.3 \end{bmatrix}$$

The mass-spring system corresponding to  $M_{mod}$  and  $K_{mod}$  is shown in Figure 9.2.

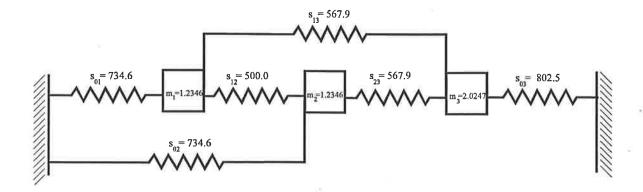


Figure 9.2: A modified mass-spring system

The eigenvalues and mass-normalised eigenvectors corresponding to this modified system are

 $\Lambda_{mod} = diag (495.8, 1516.5, 1865.0)$ 

and

$$\Phi_{mod} = \begin{bmatrix} 0.4279 & -0.4710 & -0.6364 \\ 0.4279 & -0.4710 & 0.6364 \\ 0.5202 & 0.4726 & 0.0000 \end{bmatrix}$$

The visual comparison between the two desired modes and the first two modes of the modified system show good correlation. However, a good correlation requires that the eigenvalue ratio, the amplitude ratio of the eigenvectors and the values of cosines between the two eigenvectors are all as close as possible to 1. The values of these ratios and cosines are presented in Table 9.1.

Desired Mode, i	Corresponding Obtained Mode, j	Eigenvalue Ratio, λ <sub>j</sub> /λ <sub>i</sub> *	AmplitudeRatioofEigenvectors, $\  \phi_j \  / \  \phi_i^* \ $	Cosine of an angle between the two eigenvectors, $\cos(\angle \phi_j \phi_i^*)$
1	1	0.9916	1.0570	0.9999
2	2	1.0110	0.9431	1.0000

Table 9.1 : Comparison between the desired and the obtained modes.

Results in Table 9.1 demonstrate that a very good correlation is achieved between the desired and the obtained modes.

#### Example 9.2: Sensitivity test

In this example we examine the sensitivity of the developed method to small perturbations in the measured data. Suppose that the matrices  $\Lambda_1$  and  $\Phi_1$  of Example 9.1 were measured with some perturbations, and are as follows

$$\Lambda_{1} = \text{diag} (450, 2050)$$
and
$$\Phi_{1} = \begin{bmatrix} 0.4 & -0.6 \\ 0.4 & -0.6 \\ 0.8 & 0.6 \end{bmatrix}$$

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Repeating Example 9.1 with the new  $\Lambda_1$  and  $\Phi_1$ , we obtain the following mass and stiffness modification matrices

$$\Delta M = \text{diag} (0.2701, 0.2701, 1.0216)$$

and

$$\Delta K = \begin{bmatrix} -142.7 & 0.0 & -44.4 \\ 0.0 & -142.7 & -44.4 \\ -44.4 & -44.4 & 946.0 \end{bmatrix}$$

The modified mass and stiffness matrices for the system then are

 $\mathbf{M}_{mod} = \mathbf{M} + \Delta \mathbf{M} = \text{diag}$  (1.2701, 1.2701, 2.0216)

and

$$K_{\text{mod}} = K + \Delta K = \begin{vmatrix} 1857.3 & -500.0 & -544.4 \\ -500.0 & 1857.3 & -544.4 \\ -544.4 & -544.4 & 1946.0 \end{vmatrix}$$

The mass-spring system corresponding to the above  $\mathbf{M}_{mod}$  and  $\mathbf{K}_{mod}$  is shown in Figure 9.3.

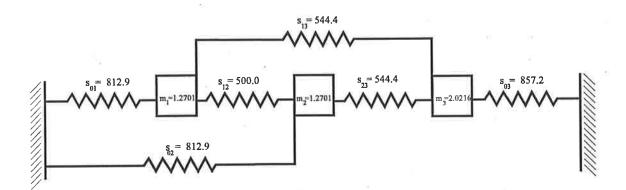


Figure 9.3: A modified mass-spring system from sensitivity test

The eigenvalues and mass-normalised eigenvectors corresponding to this modified system are

$$\Lambda_{mod} = diag (532.3, 1499.0, 1856.0),$$

and

$$\boldsymbol{\Phi}_{mod} = \begin{bmatrix} 0.4186 & -0.4674 & -0.6274 \\ 0.4186 & -0.4674 & 0.6274 \\ 0.5239 & 0.4692 & 0.0000 \end{bmatrix}$$

We note that the resulting solution differs marginally from the solution of Example 9.1. However, the differences are small, and the correlation between the desired and the obtained modes is very good. Table 9.2 shows the eigenvalue ratio, the amplitude ratio of the eigenvectors and the values of cosines between the two eigenvectors.

Desired Mode, i	Corresponding Obtained Mode, j	Eigenvalue Ratio, λ <sub>j</sub> /λ <sub>i</sub> *	Amplitude Ratio of Eigenvectors, $\ \phi_j\ /\ \phi_i^*\ $	Cosine of an angle between the two eigenvectors, $\cos(\angle \phi_j \phi_i^*)$
1	1	1.0646	1.0471	1.0000
2	2	0.9993	0.9360	1.0000

Table 9.2 : Comparison between the desired and the obtained modes.

Results in Table 9.2 show that despite the introduction of perturbations into the measured data, the quality of the obtained solution is not greatly affected. Therefore we conclude that the developed method is sufficiently robust to perform adequately when perturbations are relatively small.

#### 9.4 Conclusions

A method for determining physically realisable mass and stiffness modifications has been developed. The method is broadly based on the results of Ram and Braun in [30], and it allows determination of the mass and stiffness modification matrices corresponding to any chosen analytical model (i.e the method is general, and is not restricted to any specific analytical model).

Depending on the dimensions of the measured modal data contained in  $\Lambda_1$  and  $\Phi_1$ , the method allows to obtain a family of solutions, an unique solution, or an optimal approximate solution for the mass modification matrix  $\Delta M$ , and the stiffness modification matrix  $\Delta K$ . However, since  $\Delta M$  and  $\Delta K$  themselves constitute only an approximation to the desired solution, it is recommended that the method is applied only in situations where exact solutions for  $\Delta M$  and  $\Delta K$  are available.

The method was tested on a numerical example, and a solution obtained showed a good correlation between the desired and the obtained modal properties. The sensitivity of the method to small perturbations was also performed, and the method was found to be sufficiently robust to cope adequately with introduced perturbations without noticeable deterioration in its performance.

# Section 10

### **EXPERIMENT**

The main aim of the experimental work described in this section was to check if the developed theoretical results can be used in practical design applications. It is well known that the dynamic behaviour of a discrete system is fundamentally different from the behaviour of a continuous system. In practice, all measured modal analysis data is obtained from a real structure, which behaves like a continuous system. Thus, there is an obvious possibility that the measured modal data may be incompatible with the chosen analytical model of a test structure. In general, a finite element model gives a good correlation with the behaviour of a continuous system for approximately a third of its modes. A discrete mass-spring system would probably give a reasonable correlation for even less number of modes. These "well-correlated" modes correspond to the lower natural frequency end of the spectrum, and the lower the natural frequency of a mode, the better is the correlation.

Section 10: Experiment

Provided the measured modal data and the desired modes are within this range of good correlation, the performance of the algorithms should, in principle, be acceptable. However, this condition may prove to be too restrictive for many practical cases.

Most of the theoretical results described in the previous sections were based on the assumption that a vibratory system may be modelled as a conservative discrete mass-spring system. It was the prime objective of this experiment to test whether such assumption may be successfully applied to a practical engineering structure. A simple "desk-top" test rig, which could be used for testing and demonstrating the developed theory, was deemed sufficient to achieve our objectives.

Because our aim was to test a practical engineering structure, we specifically did not want to use an experimental model which consisted of lumped masses connected by light springs. To use such model is equivalent to testing a physical mass-spring system, which is not representative of any obvious engineering application. At the same time, we wanted to use an experimental model which would give a good correlation with the behaviour of a massspring system. To do otherwise would have created a large uncertainty in testing the performance of the algorithms.

The two systems considered appropriate for our test model were the torsional shaft-pulley system (see figure 2.1) and a "building" model which is shown in figure 10.1. Both of these test models may represent a large number of real engineering structures. The torsional system is clearly representative of any rotational machinery power transmission trains or

gear box assemblies. The "building" model may represent any cantilever structure, such as buildings, aeroplane wing, and many other.

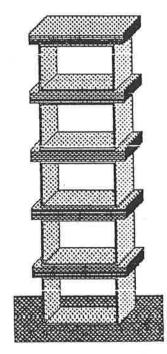


Figure 10.1: Example of a "building" model

The "building" model was chosen as best suitable because of cost, simplicity and safety considerations. To measure the torsional modes would have required the use of a more sophisticated equipment and a more complicated test set-up. Also, to demonstrate the resonance of this torsional system, it had to be driven at high rotational speeds (while resonating) which was considered too unsafe.

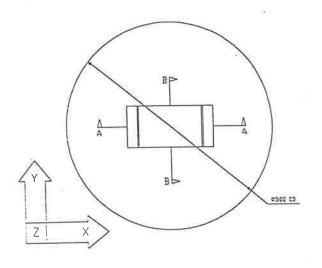
The Algorithm 4.2 for solving Problem 1 (see section 5) was then chosen as most suitable for the experimental assessment. This algorithm was selected because, unlike algorithms for Problems 2 and 4 (see sections 6 and 9), it only involved changing the masses of the structure, thus greatly simplifying the design of the test model. Also, unlike algorithm for Problem 3 (see section 8), it is not sensitive to perturbations in the measured modal analysis data, thus giving a more stable platform for the experimental assessment of its performance.

#### **10.1 Test Model Description**

The test model consisted of nine extruded aluminium box sections, which represented the "walls" of a building, and a large number of steel plates of various thickness (and hence mass), which were sandwiched between the adjacent box sections to obtain the necessary "floor" mass at each location. The aluminium box sections had a uniform thickness of 3mm throughout, and its dimensions were 160mm(long) x 100mm(wide) x 100mm(high). The steel plates had dimensions of 215mm(long) x 100mm(wide) and were made in various thicknesses to allow for different mass configurations. The adjacent box sections with plates in between were joined together by mild steel, M6x1.0, hexagonal head bolts of appropriate lengths.

The overall, general layout of the assembled model is shown in Figure 10.2. Figure 10.3 shows details of a joint connection between the two adjacent box sections with steel plates in-between.

Section 10: Experiment



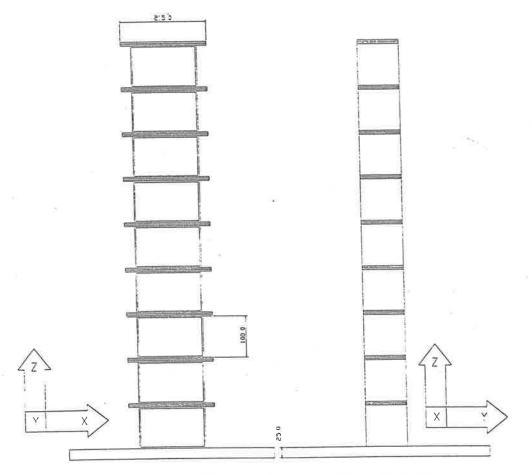
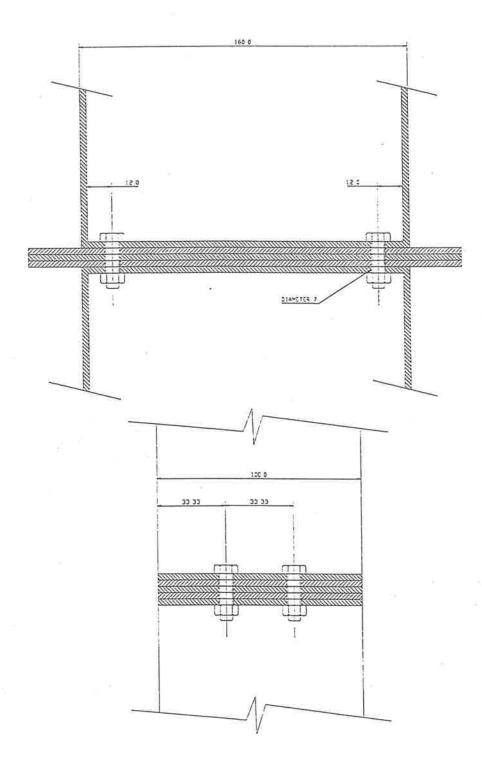


Figure 10.2 : The overall general layout of the test structure.





#### 10.2 Mass Elements of the Test Model

Figure 10.4 shows a schematic layout between two adjacent "floors" of the test structure.

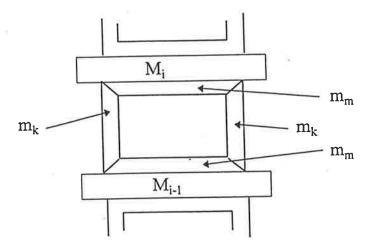


Figure 10.4 : A schematic layout between two adjacent "floors".

In the theoretical mass-spring system, the springs, which connect each mass element to others, are themself have no mass. In a real physical structure this is clearly not the case. Each aluminium box section had a finite mass, although, in general, this mass was very small relative to the mass of the steel plates at each "floor". A choice had to be made whether to ignore the mass of the aluminium box sections, or to include it in the calculations of the "floor" mass. To maintain the accuracy of the test model, it was decided to include the mass of the box sections in our calculations. The effective mass of each "floor" was calculated based on the well known Rayleigh's method (see e.g. Thomson [78], pp. 24-25). The resulting mass matrix for our test structure was then assumed to have the following form

	$M_1 + 2(m_m + \frac{m_k}{3})$	0	0	0	0	0	0	0	0	
	0	$0$ $M_2 +$ $2(m_m + \frac{m_k}{3})$	0	0	0	0	0	0	0	
	0	0	$\frac{M_3^+}{2(m_m^+ \frac{m_k}{2})}$	0	0	0	0	0	0	
	0	0	0	$\frac{M_4^+}{2(m_m^+ \frac{m_k}{3})}$	0	0	0	0	0	
<b>M</b> =	0	0	0	0	$\frac{M_5^+}{2(m_m^+ \frac{m_k}{2})}$	0	0	0	0	(10.1)
	0	0	0	0	0	$M_6^+$ $2(m_m^+ \frac{m_k}{3})$	0	0	0	
	0	0	0		0	0	$\frac{M_7}{2(m_+ + \frac{m_k}{-})}$	0	0	
	0	0	0	0	0	0	0	$\frac{M_8^+}{2(m_m^+ \frac{m_k}{3})}$	0	
	0	0	0	0	0	0	0	0	$\frac{M_9^+}{(m_m^+ \frac{m_k}{3})}$	

where  $M_i$  (i=1,2,...,9) was the mass of the steel plates and connecting bolts at the i<sup>th</sup> "floor",  $m_m$  is the mass of a horizontal segment of a box, and  $m_k$  is the mass of a vertical segment of a box.

From measurement it was found that the mass of each box section was approximately equal to 540 grams, and also, from the dimensions of the box section, we know that

$$m_m = 1.6m_k$$
 . (10.2)

The mass of each box section is equal to

$$2(m_{\rm m} + m_{\rm k}) = 540. \tag{10.3}$$

Thus from (10.2) and (10.3) we find that

$$m_m \approx 166 \text{ grams}$$
 (10.4)

and 
$$m_k \approx 104$$
 grams. (10.5)

And, consequently, the sum

$$m_m + \frac{m_k}{3} = 201 \ gramms$$
 (10.6)

This value was then used to determine the additional mass from steel plates which would give us the desired total mass at each "floor". In general, by carefully manipulating with the plates of different masses, we were able to achieve the mass at each "floor" which was nominally within  $\pm 2$  grams of the desired value.

#### **10.3 Determination of the Stiffness**

The stiffness matrix corresponding to our test model, was assumed to have the following form

$$\boldsymbol{K} = \begin{bmatrix} 2k & -k & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -k & 2k & -k & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -k & 2k & -k & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -k & 2k & -k & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -k & 2k & -k & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -k & 2k & -k & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -k & 2k & -k & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -k & 2k & -k & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -k & 2k & -k \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -k & 2k & -k \\ \end{bmatrix}$$
(10.7)

where k was the stiffness constant of each box section.

The value of k was estimated theoretically using the listed properties of aluminium and the information about the dimensions and the shape of the box section. Using Thomson [78, p.178] the stiffness constant k is given by

$$k = 24 \frac{EI}{l^3} \tag{10.8}$$

where

E = Young's Modulus of Elasticity of aluminium

I = moment of inertia of a box section

l = height of the box section.

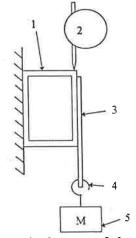
It was not clear which value had to be used for height l of the box section, i.e 94mm or 100mm. Therefore, both values were used to determine the upper and lower bounds for k. Substituting the listed values for E = 70 - 75 GPa, and using other dimensions of a box section to calculate I, the theoretical value of k was found to be

$$k = 378 \text{ kN/m} - 488 \text{ kN/m}.$$
 (10.9)

Such a large uncertainty (over 20%) was considered too great for our experiment, and therefore a simple procedure was carried out to measure the value of k experimentally.

#### 10.3.1 Stiffness determination experiment

Figure 10.5 shows the schematic layout of the experimental set-up for stiffness measurement.



#### APPARATUS

1. Aluminium Box Section

2. Dial Indicator

3. Mounting Plate

4. Hook

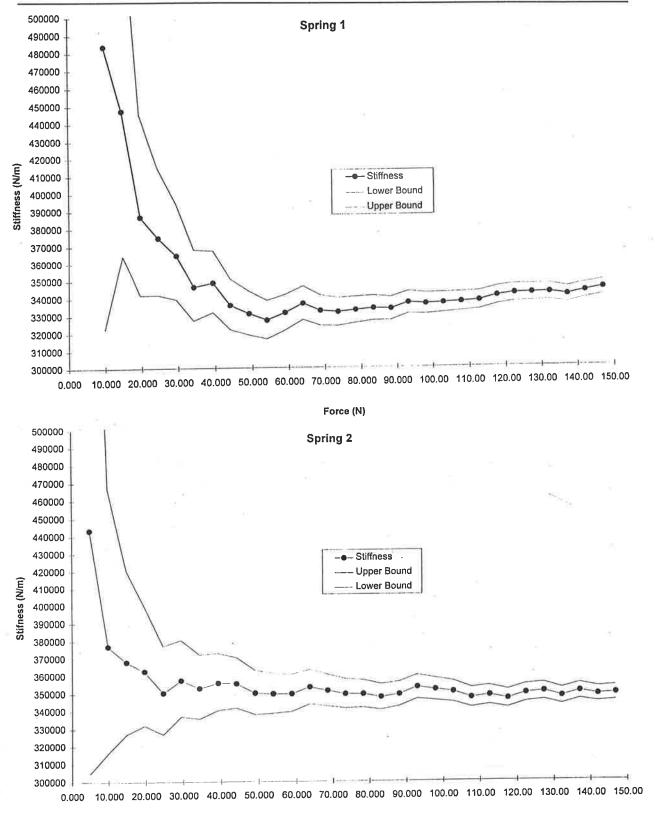
5. Load Masses

Figure 10.5: Schematic layout of the stiffness measurement experiment

## The method was:

- 1. Mount the box section onto a solid, straight vertical surface as shown in Figure 10.5.
- 2. Attach the mounting plate to the other side of the box section as shown, and place the hook through the hole in the mounting plate.
- 3. Position the dial indicator so that its tip is touching the front edge of the box section, and set the dial reading to zero.
- 4. Add 0.5kg masses, one at a time, and measure the deflections from the dial indicator.

This process was repeated for three randomly selected box sections, and the results from this experiment are shown in Figure 10.6. The raw data from this experiment is given in the Appendix A. We note the value of the stiffness is significantly higher under small loads (and hence small deflections). However, experimental uncertainty in measured values of small deflections was very large, mainly due to dial resolution limitations. At higher loads the value of the stiffness seemed to "settle" around the approximately 350 kN/m mark.



Force (N)

Figure 10.6: Measured stiffness values

## 10.3.2 Optimal value for the stiffness, k

It was anticipated that during the modal analysis testing of the "building" model the deflections of the "walls" would be relatively small. Therefore, a large uncertainty in what value should be used for stiffness k still existed, despite the availability of the measured data. In the end, the optimal value for the stiffness k was determined by performing a Chi-squared test, which is defined as

$$\chi_0^2 = \sum_{i=1}^9 \frac{(a_i - b_i)^2}{a_i}$$
(10.10)

where

 $\kappa_0^2$  = Chi-squared value  $a_i = i^{th}$  measured experimental natural frequency  $b_i = i^{th}$  analytically determined natural frequency.

Clearly, in the equation (10.10) if  $a_i = b_i$  (for i=1,...,9), then  $\varkappa_0^2 = 0$ . Thus, the objective was to find the stiffness value, k, which produced analytical natural frequencies  $b_i$  (i=1,...,9), such that the magnitude of the  $\varkappa_0^2$  were minimised. The procedure was then as follows. The modal analysis tests were performed on several configurations of a test structure (i.e different mass configurations at each floor), and the measured natural frequencies of each configuration recorded. The analytical values for the natural frequencies of the model with the same mass configurations were calculated using a number of different values for the stiffness k. Then, setting  $a_i$  to be the measured natural frequencies and  $b_i$  to be the corresponding analytically determined natural frequencies, the values of the  $\varkappa_0^2$  were

calculated using equation (10.10). The value of the stiffness k, which consistently gave the lowest  $\kappa_0^2$  value was

$$k = 378 \text{ kN/m}$$
 (10.11)

and this value was then used in all subsequent experiments. The raw data from these Chisquared tests is given in the Appendix B.

### 10.4 Experimental Testing of the Algorithm 4.2

The schematic layout of the experimental set-up is shown in Figure 10.7. The equipment used for these experiments are listed below.

### EQUIPMENT USED

1. The "Building" Model.

2. Brüel and Kjær Accelerometer - model 9040.

3. Brüel and Kjær Signal Analyser - model 2032.

4. Brüel and Kjær Charge Amplifier - model 5666.

5. Brüel and Kjær Impulse Hammer - model 1234.

6. IBM Compatible Personal Computer.

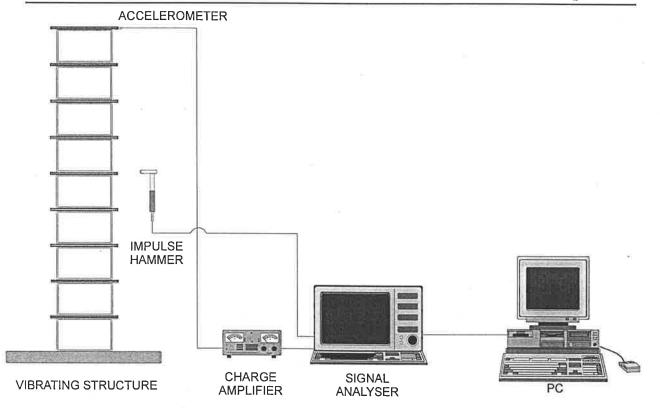


Figure 10.7 : Schematic layout of the Algorithm Testing Experiment.

The experimental procedure was then as follows:

- 1. Nine different natural frequencies were arbitrarily chosen.
- 2. Using the stiffness k = 378 kN/m, the stiffness matrix of the "building" model was constructed via equation (10.7).
- 3. Algorithm 4.2 for solving Problem 1 (see section 4) was then applied to determine the necessary mass matrix. (Note: Sometimes several repetitions of the algorithm (with different starting values for the initial guess of the mass matrix) were required, in order to obtain the natural frequencies which were adequately close to the desired frequencies.)

- 4. Using the obtained mass matrix from step (3) and equations (10.1) and (10.6), the masses of the steel plates to be added at each "floor" were determined.
- 5. The physical test structure was then assembled with the determined amount of plates at each "floor".
- 6. The structure was lightly struck by the impulse hammer, consecutively at each "floor", each time recording the natural frequencies of the structure.
- 7. The measured natural frequencies at each "floor" were averaged, and compared with the desired natural frequencies and the natural frequencies of the analytical system determined by the Algorithm 4.2.

The raw data from these experiments is given in the Appendix C. In Figure 10.8 we present the graphical comparison between the measured, the desired and the analytically determined (by Algorithm 4.2) natural frequencies.

In all results shown in Figure 10.8, the "Frequency" axis is set between the same limits (from 0-220Hz), thus allowing easy visual comparison of the data from all tests. We also want to emphasise the following point. Although in theory Algorithm 4.2 should permit unrestricted assignment of arbitrarily chosen natural frequencies, in our experiment the achievable natural frequency range was approximately 5Hz to 220Hz. This limitation was a direct consequence of the physical constraints on the smallest and the largest mass that we could have at the "floors". Clearly, the smallest mass was simply the mass of the aluminium box sections with no steel plates added. The maximum obtainable mass was governed by the available supply of the steel plates (which was approximately 70kg).

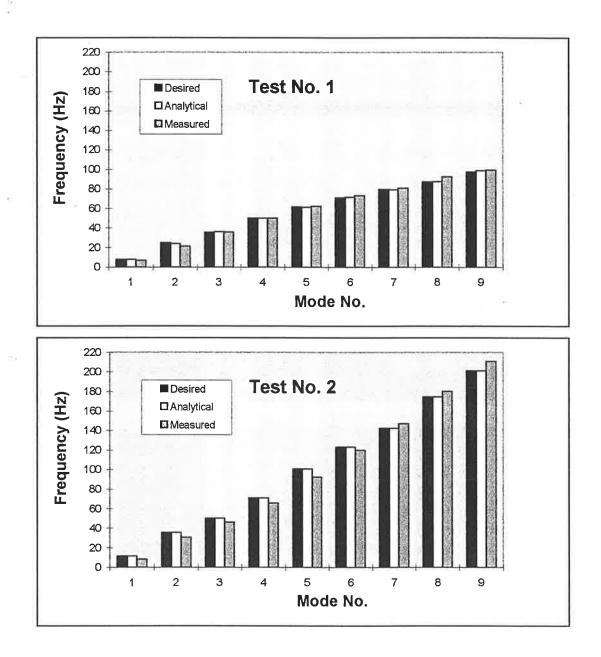


Figure 10.8 : Comparison between the desired, measured and analytical (using Algorithm 4.2) natural frequencies

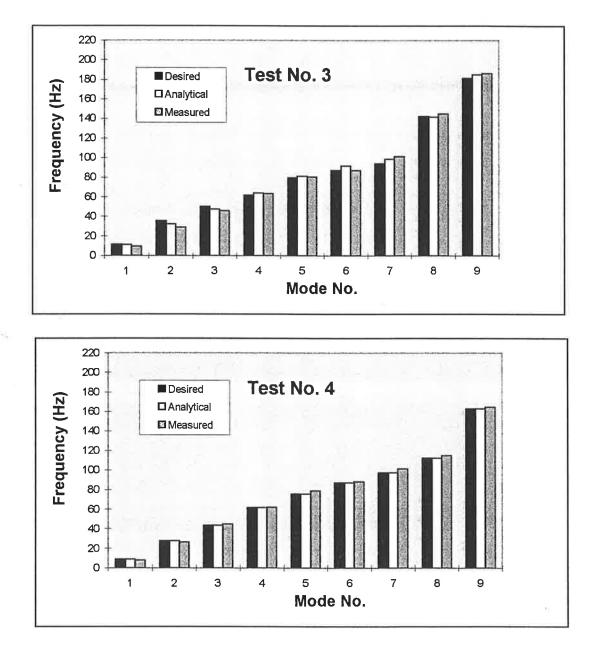


Figure 10.8 (cont'd): Comparison between the desired, measured and analytical (using Algorithm 4.2) natural frequencies.

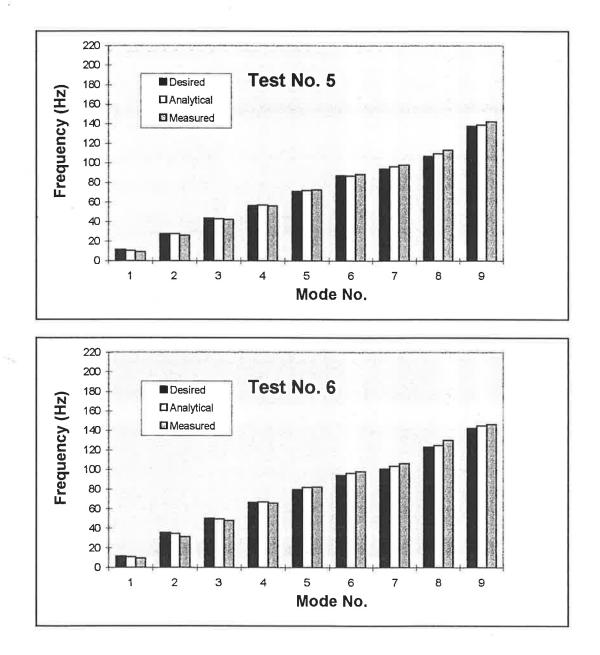


Figure 10.8 (cont'd): Comparison between the desired, measured and analytical (using Algorithm 4.2) natural frequencies.

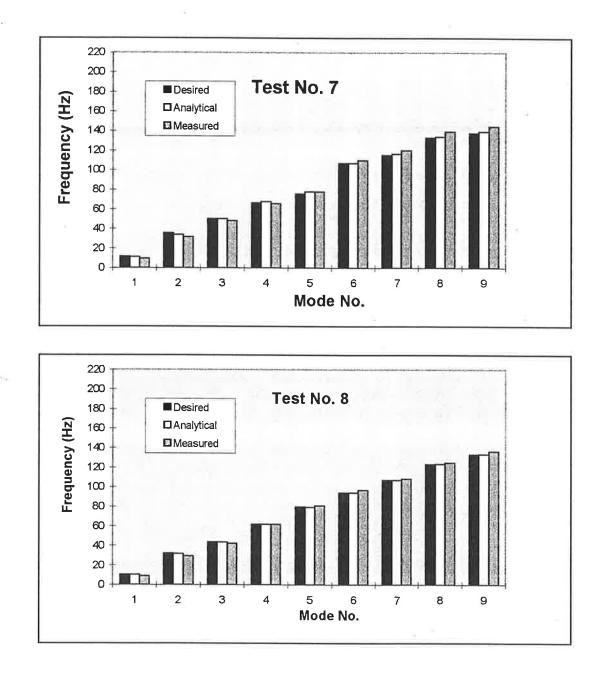


Figure 10.8 (cont'd): Comparison between the desired, measured and analytical (using Algorithm 4.2) natural frequencies.

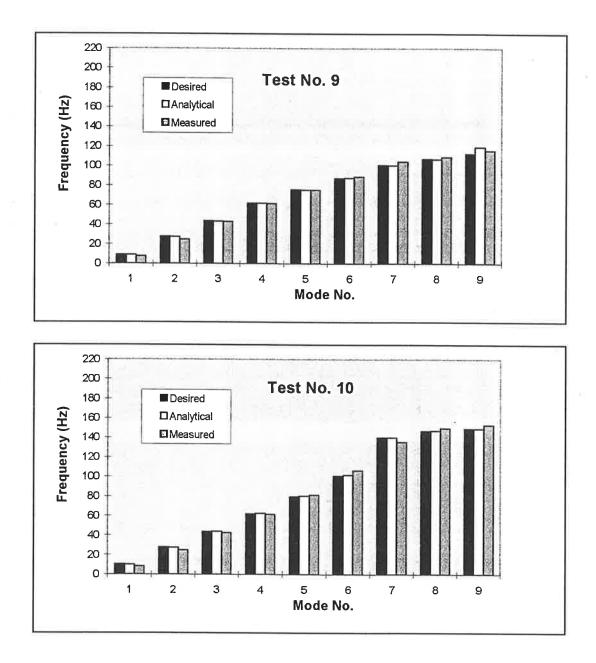


Figure 10.8 (cont'd): Comparison between the desired, measured and analytical (using Algorithm 4.2) natural frequencies.

#### **10.5 Conclusions**

The results presented in Figure 10.8 show a good correlation between the desired natural frequencies and the natural frequencies both of the test structure and of the analytical model determined by Algorithm 4.2. Therefore, the main aim of this experimental program, which was to test whether the assumption of conservative mass-spring system is acceptable in the practical engineering applications, was achieved and the answer is positive. However, we are fully aware that the chosen test structure was highly "tailored" and optimised for conformance with such analytical model, and that most "real-life" structures would not be so successful. The experimental test model was, however, well suited to the stated scope of our experiment, and it is representative of some useful engineering structures. The Algorithm 4.2 was found to work well in applications to a real physical structure, and it has a potential of being a very useful tool for the design of vibratory systems to suit natural frequency requirements.

# Section 11

## CONCLUSIONS

The conclusions pertaining specifically to the particular problems investigated, were given at the end of the appropriate sections dealing with those problems. Here we present the general conclusion, which are applicable to all results given in this thesis.

The material presented in this thesis contains a logically complete set of solutions to practical problems dealing with the design and structural modifications of structures, which may be adequately modelled by a mass-spring analytical system. In section 4 we developed a method which allows determination of a mass matrix when the stiffness matrix and a complete set of the desired natural frequencies of a system are known. A derivative of a method by Joseph [16] allows a similar procedure to be followed when the desired natural frequency set is truncated, i.e when not every natural frequency is precisely specified. In section 5 we presented a method for optimal reconstruction of a mass-spring system from

### Section 11: Conclusions

a complete set of prescribed spectral and modal data. The analysis given in section 6 then allows us to do the same when the prescribed spectral and modal data are incomplete. In section 7 we have extended the solution method of section 5 to a more general class of the mass and stiffness matrices (i.e. which not necessarily correspond only to a mass-spring system). In section 8 we used the method of Joseph [16] as a basis for developing a new algorithm for obtaining the necessary mass and stiffness modifications to an existing structure, so that the natural frequencies of a modified system are as close as possible to the prescribed values. And finally, in section 9 we developed a method for extracting the physically realisable set of solutions for a problem of structural modifications where both spectral and modal constraints are present. A family of solutions to this problem was originally characterised by Ram and Braun [46], but no method of obtaining a physically realisable solution was developed. Our result thus complements and completes the solution given in [46].

The physical realisability of a solution was the main criteria that had to be satisfied in all of the methods developed in this thesis. All of the presented methods aim at being useful in practical engineering applications, rather than just being of purely mathematical interest. The author hopes that the main contribution of this work would be to make available a useful practical set of design tools which may be applied to "real-life" problems. To some extent this contribution was recognised by publication and the feedback from the three refereed papers [75,76,77], which deal respectively with the material of section 4, section 5 and section 8. Two pending papers [79,80], containing the material developed in sections 6 and 9, will also soon be submitted for a journal publication. The practical application of

the method developed in section 4 has also been demonstrated by the experimental results, which are given in section 10.

The author also believes that the work developed in this thesis has filled a small void in the knowledge of inverse vibration problems, particularly in applications with conservative mass-spring systems. However, several of the developed methods (e.g. algorithms of sections 6, 7, 8 and 9) do have an "in-build" ability to cope with the mass and stiffness matrices corresponding to vibratory systems other than the mass-spring model. For example, they may be applied to the mass and stiffness matrices corresponding to a finite element model. Some open problems concerned with improving the developed methods were identified (for example: How to control the sign changes in the obtained modal vectors in Problem 2?), but we leave those problems for later investigations.

Last, but not least, it should be emphasised that engineering solutions must not only be *physically realisable* but *practical* as well. This means that additional constrains may need to be taken into account (e.g. the maximal allowed mass, geometrical and spatial restrictions, etc.). Hence, as expected, the design process involves a combination of experience, intuition and science. In this thesis we have focused on the latter only. A great improvement to the developed methods would be to enable direct prescriptions of *practical* solutions. However, we also leave this important task for later study.

Section 12

## REFERENCES

- [1] Gladwell G.M.L. (1986), *Inverse Problems in Vibration*, Martinus Nijhoff Publishers, Dordrecht.
- [2] Gladwell G.M.L (1986), *Inverse Problems in Vibration*, Applied Mechanical Review, vol. 39, pp. 1013-1018.
- [3] Gladwell G.M.L. (1996), Inverse Problems in Vibration II, To appear.
- [4] Chu M.T. (1996), Inverse Eigenvalue Problems, To appear.
- [5] Downing A.C. and Householder A.S. (1956), Some Inverse Characteristic Value Problems, Journal of the Association of Computational Machines, vol. 3, no. 3, pp. 203-207.
- [6] Hadeler K.P. (1969), *Multiplikative Inverse Eigenwertprobleme*, Linear Algebra and Its Applications, vol. 2, pp. 65-86.

- [7] Kublanovskaja W.N. (1970), On an Approach to the Solution of the Inverse
   Eigenvalue Problem, Zapiski Nauchnih Seminarov Leningradskogo Otdelenia
   Matematicheskogo Instituta imeni V.A. Steklova Akademii Nauk SSSR, pp. 138-149.
- [8] De Oliveira G.N. (1972), On the Multiplicative Eigenvalue Problem, Canadian Mathematical Bulletin, vol. 15, no. 2, pp. 189-190.
- [9] Friedland S. (1975), On Inverse Multiplicative Eigenvalue Problem for Matrices,
   Linear Algebra and Its Applications, vol. 12, pp. 127-137.
- [10] Friedland S. (1977), Inverse Eigenvalue Problems, Linear Algebra and Its Applications, vol. 17, pp. 15-51.
- [11] Dias da Silva J.A. (1986), On the Multiplicative Inverse Eigenvalue Problem, Linear Algebra and Its Applications, vol. 78, pp. 133-145.
- [12] He Xuchu and Dai Hua (1989), Sufficient Conditions for the Solubility of the Multiplicative Inverse Eigenvalue Problems, Nanjing University Mathematical Journal, vol. 6, no. 2, pp. 1-7.
- [13] Biegler-König F.W. (1981), Sufficient Conditions for the Solubility of Inverse Eigenvalue Problem, Linear Algebra and Its Applications, vol. 40, pp. 89-100.
- [14] Nocedal J. and Overton M.L. (1983), Numerical Methods for Solving Inverse
   Eigenvalue Problems, Lecture Notes in Mathematics, No. 1005, Springer-Verlag,
   pp. 212-226.
- [15] Friedland S., Nocedal J. and Overton M.L. (1987), The Formulation and Analysis of Numerical Methods for Inverse Eigenvalue Problems, SIAM Journal of Numerical Analysis, vol. 24, no. 3, pp. 634-667.

- [16] Joseph K.T. (1992), Inverse Eigenvalue Problem in Structural Design, AIAA Journal,vol. 30, no. 12, pp. 2890-2896.
- [17] Baldwin J.F. and Hutton S.G. (1985), Natural Modes of Modified Structures, AIAA Journal, vol. 23, no. 11, pp. 1737-1743.
- [18] McCalley R.B. Jr (1960), Error Analysis for Eigenvalue Problems, ASCE Proceedings of the 2<sup>nd</sup> Conference on Electronic Computation, pp. 523-550.
- [19] Wittrick W.H. (1962), Rates of Change of Eigenvalues with Reference to Buckling and Vibration Problems, Journal of the Royal Aeronautical Society, vol. 66, pp. 590-591.
- [20] Lancaster P. (1964), On Eigenvalues of Matrices Dependent on a Parameter, Numerische Mathematik, vol. 6, no. 5, pp. 377-387.
- [21] Fox R.L. and Kapoor M.P. (1968), Rates of Change of Eigenvalues and Eigenvectors, AIAA Journal, vol. 6, no.12, pp. 2426-2429.
- [22] Nelson R.B. (1976), Simplified Calculation of Eigenvector Derivatives, AIAA Journal, vol. 14, pp. 1210-1205.
- [23] Van Belle H. (1976), Theory of Adjoint Structures, AIAA Journal, vol. 14, pp. 977-979.
- [24] Van Honacker P. (1980), Differential and Difference Sensitivities of Natural Frequencies and Mode Shapes of Mechanical Structures, AIAA Journal, vol. 18, no.12, pp.1511-1514.
- [25] Rudisill C.S. (1974), Derivatives of Eigenvalues and Eigenvectors for a General Matrix, AIAA Journal, vol. 12, no. 2, pp. 721-722.
- [26] Rudisill C.S. (1975), Numerical Method for Evaluating the Derivatives of Eigenvalues and Eigenvectors, AIAA Journal, vol. 13, pp. 834-836.

- [27] Muira H. and Schmit L.A. (1978), Second Order Approximation of Natural Frequency
   Constraints in Structural Design, International Journal for Numerical Methods in Engineering, vol. 13, pp. 337-351.
- [28] Van Belle H. (1982), Higher Order Sensitivities in Structural Systems, AIAA Journal, vol. 20, pp. 286-288.
- [29] Rizai N.M. and Bernard J.E. (1982), An Efficient Method for Predicting the Dynamic Effects of a Redesign, Proceedings of the 1<sup>st</sup> International Modal Analysis Conference, pp. 436-442.
- [30] Wang J., Heylen W. and Sas P. (1987), Accuracy of Structural Modification Techniques, Proceedings of the 5<sup>th</sup> International Modal Analysis Conference, pp. 65-75.
- [31] To W.M. and Ewins D.J. (1991), Non-linear Sensitivity Analysis of Mechanical Structures Using Modal Data, Proceedings of Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science, vol. 205, pp. 67 - 75.
- [32] Zimoch Z. (1987), Sensitivity Analysis of a Vibrating System, Journal of Sound and Vibration, vol. 115, no.3, pp.447-458.
- [33] Zimoch Z. (1991), Algorithm for Modification of Parameters in Vibrating Systems, AIAA Journal, vol. 29, no. 9, pp. 1525-1527.
- [34] Lord Rayleigh (1945), Theory of Sound (Volume 1), 2<sup>nd</sup> Edition, Dover Publications.
- [35] Jones R.P.N.(1960), The Effect of Small Changes in Mass and Stiffness on the Natural Frequencies and Modes of Vibrating Systems, International Journal of Mechanical Science, vol. 1, no. 4, pp. 350-355.

- [36] Romstad K.M., Hutchinson J.R. and Runge K.H. (1973), Design Parameter Variation and Structural Response, International Journal for Numerical Methods in Engineering, vol. 5, pp. 337-349.
- [37] Stetson K.A. and Harrison I.R. (1981), Redesign of Structural Vibration Modes by Finite Element Inverse Perturbation, ASME Journal of Engineering for Power, vol. 103, pp. 319-325.
- [38] Sandstrom R.E. and Anderson W.J. (1982), Modal Perturbation Methods for Marine Structures, Transactions of the Society of Naval Architects and Marine Engineers, vol. 90, pp. 41-54.
- [39] Kim K., Anderson W.J. and Sandstrom R.E. (1983), Nonlinear Inverse Perturbation Method in Dynamic Analysis, AIAA Journal, vol. 21, no. 9, pp. 1310-1316.
- [40] Hoff C.J., Bernitsas M.M., Sandstrom R.E. and Anderson W.J. (1984), Inverse Perturbation Method for Structural Redesign with Frequency and Mode Shape Constraints, AIAA Journal, vol. 22, no. 9, pp. 1304-1309.
- [41] Zhang Q., Wang W., Allemang R.J. and Brown D.L. (1988), Prediction of Mass Modification for Desired Natural Frequencies, Proceedings of the 6<sup>th</sup> International Modal Analysis Conference, pp. 1026-1032.
- [42] Parlett B.N. (1980), The Symmetric Eigenvalue Problem, Prentice-Hall.
- [43] Ram Y.M., Braun S.G. and Blech J. (1988), Structural Modifications in Truncated Systems by the Rayleigh-Ritz Method, Journal of Sound and Vibration, vol. 125, no. 2, pp. 203-209.

- [44] Ram Y.M. and Braun S.G. (1990), Upper and Lower Bounds for the Natural Frequencies of Modified Structures Based on Truncated Modal Testing Results, Journal of Sound and Vibration, vol. 137, no. 1, pp. 69-81.
- [45] Ram Y.M. and Braun S.G. (1993), Eigenvector Error Bounds and Their Applications to Structural Modification, AIAA Journal, vol. 31, no. 4, pp. 759-764.
- [46] Ram Y.M. and Braun S.G. (1991), An Inverse Problem Associated with Modification of Incomplete Dynamic System, ASME Journal of Applied Mechanics, vol. 58, pp. 233-237.
- [47] Berman A. (1984), System Identification of Structural Dynamic models Theoretical and Practical Bounds, AIAA paper 84-0929, pp. 123-129.
- [48] Zhang Q. and Lallement G. (1987), Comparison of Normal Eigenmodes Calculation Methods Based on Identified Complex Modes, Journal of Spacecraft and Rockets, vol. 24, no. 1, pp. 69-73.
- [49] Ahmadian H., Gladwell G.M.L. and Ismail F. (1995), Extracting Real Modes from Complex Measured Modes, Proceedings of the 13<sup>th</sup> International Modal Analysis Conference.
- [50] Tsuei Y.G. and Yee E.K.L. (1989), A Method for Modifying Dynamic Properties of Undamped Mechanical Systems, ASME Journal of Dynamic Systems, Measurement and Control, vol. 111, pp. 403-408.
- [51] Yee E.K.L and Tsuei Y.G. (1991), Method for Shifting Natural Frequencies of Damped Mechanical Systems, AIAA Journal, vol. 29, no. 11, pp. 1973-1977.
- [52] Ram Y.M. (1994), Enlarging a Spectral Gap by Structural Modification, Journal of Sound and Vibration, vol. 176, no. 2, pp. 225-234.

- [53] Coppolino R.N. (1981), Structural Mode Sensitivity to Local Modification, SAETechnical Paper 81-1044.
- [54] Zhang Q., Allemang R.J. and Brown D.L. (1990), Modal Filter: Concept and Application, Proceedings of the 8<sup>th</sup> International Modal Analysis Conference, pp. 487-496.
- [55] Bucher I. and Braun S. (1993), The Structural Modification Inverse Problem: An Exact Solution, Mechanical Systems and Signal Processing, vol. 7, no. 3, pp. 217-238.
- [56] Bucher I. and Braun S. (1994), Efficient Optimization Procedure for Minimising Vibratory Response via Redesign or Modification, Part I: Theory, Journal of Sound and Vibration, vol. 175, no. 4, pp. 433-453.
- [57] Bucher I. and Braun S. (1994), Efficient Optimization Procedure for Minimising Vibratory Response via Redesign or Modification, Part II: Examples, Journal of Sound and Vibration, vol. 175, no. 4, pp. 455-473.
- [58] Friswell M.I. and Mottershead J.E. (1995), Finite Element Model Updating in Structural Dynamics, Kluwer Academic Publishers, Dordrecht.
- [59] Mottershead J.E. and Friswell M.I. (1993), Model Updating in Structural Dynamics, Journal of Sound and Vibration, vol. 167, pp. 347-375.
- [60] Boley D. and Golub G.H. (1987), A Survey of Matrix Inverse Eigenvalue Problems, Inverse Problems, vol. 3, pp. 595-622.
- [61] Ram Y.M. and Coldwell J. (1992), Physical Parameters Reconstruction of a Free-Free Mass-Spring System from Its Spectra, SIAM Journal of Applied Mathematics, vol. 52, no. 1, pp. 140-152.

- [62] Gladwell G.M.L. and Movahhedy M. (1995), Reconstruction of a Mass-Spring System
   from Spectral Data I: Theory, Inverse Problems in Engineering, vol. 1, pp. 179-189.
- [63] Movahhedy M., Ismail F. and Gladwell G.M.L. (1995), Reconstruction of a Mass-Spring System from Spectral Data II: Experiment, Inverse Problems in Engineering, vol. 1, pp. 315-327.
- [64] Ram Y.M. and Gladwell G.M.L. (1994), *Constructing a Finite-Element Model of a Vibrating Rod from Eigendata*, Journal of Sound and Vibration, vol. 169, pp. 229-237.
- [65] Ram Y.M. (1994), An Inverse Mode Problem for a Continuous Model of an Axially Vibrating Rod, ASME Journal of Applied Mechanics, vol. 61, pp. 624-628.
- [66] Ram Y.M. (1994), Inverse Mode Problems for the Discrete Model of a Vibrating Beam, Journal of Sound and Vibration, vol. 169, pp. 239-252.
- [67] Starek L. and Inman D.J. (1991), On the Inverse Vibration Problem with Rigid-Body Modes, ASME Journal of Applied Mechanics, vol. 58, pp. 1101-1104.
- [68] Starek L. and Inman D.J. (1991), Inverse Problem in Vibration for Generating Symmetric Coefficient Matrices, Vibration Analysis-Analytical and Computational, ASME DE-Vol. 37, pp. 13- 17.
- [69] Starek L., Inman D.J. and Kress A. (1992), *A Symmetric Inverse Vibration Problem*, ASME Journal of Vibration and Acoustics, vol. 114, pp. 564 - 568.
- [70] Starek L. and Inman D.J. (1995), A Symmetric Positive Definite Inverse Vibration Problem with Underdamped Modes, ASME Design Engineering Technical Conference, vol. 3, Part C, DE-Vol. 84-3, pp. 1089-1094.
- [71] Starek L. and Inman D.J. (1995), *A Symmetric Inverse Vibration Problem with Overdamped Modes*, Journal of Sound and Vibration, vol. 181, pp. 893-903.

- [72] Golub G.H. and Van Loan C.F. (1989), Matrix Computations, 2<sup>nd</sup> Edition, The John
   Hopkins University Press.
- [73] Lawson C.L. and Hanson R.J. (1974), Solving Least Squares Problems, Prentice-Hall.
- [74] Chu M.T. (1992), Numerical Methods for Inverse Singular Value Problems, SIAM Journal of Numerical Analysis, vol. 29, no. 3, pp. 885-903.
- [75] Sivan D.D. and Ram Y.M. (1995), On the Inverse Multiplicative Eigenvalue Problem with an Engineering Application, ASME Design Engineering Technical Conference, vol. 3, Part C, DE-Vol. 84-3, pp. 1135-1142.
- [76] Sivan D.D. and Ram Y.M. (1996), Optimal Construction of a Mass-Spring System with Prescribed Modal and Spectral Data, Journal of Sound and Vibration, In press.
- [77] Sivan D.D. and Ram Y.M. (1996), Mass and Stiffness Modifications to Achieve Desired Natural Frequencies, Communications in Numerical Methods in Engineering, vol. 12, no. 9, pp. 531-542.
- [78] Thomson W.T. (1993), Theory of Vibration with Application, 4th Edition, Prentice Hall.
- [79] Sivan D.D. and Ram Y.M (1996), Optimal Construction of Conservative Vibratory Systems from the Truncated Set of Prescribed Modal Data, To be submitted.
- [80] Sivan D.D. and Ram Y.M.(1996), Extraction of Physically Realisable Solutions for an Inverse Modification Problem, To be submitted.

# **APPENDIX A:**

## Raw Data from Stiffness Measurement Tests

Task: (Cap. Fees. 91)         Dat. (m)         Data (m)	Spring 1					Error			Spring 2					Error	
0.667         4.834         0.01         1.002/65         48303         997266         322422           1.039         9.800         0.022         2.026         447247         0.677         6.46676         316135           1.68         1.6455         0.088         3.80-65         386721         44525         34173         1.5         14.71         0.04         4.0052-65         326247         331877           2.49         2.4247         0.076         6.70E-65         346357         331877         331889         33188		Force (N)	Dist (mm)	Dist (m)	Suffness		Upper Bound			Force	Dist (mm)	Dist (m)	Suffness	Lower Bound	Upper Bound
TODS         TODS         TODS         Constraint         STREPS         STREPS <td></td> <td></td> <td></td> <td></td> <td></td> <td>and the state of the second se</td> <td></td> <td></td> <td></td> <td>4.876</td> <td>0.011</td> <td>1,10E-05</td> <td></td> <td>812595</td> <td>304723</td>						and the state of the second se				4.876	0.011	1,10E-05		812595	304723
1:65         1:65         0:64         0:05:05							A 2652 A 1056 A 1				0.026	2.60E-05	376930	466676	316135
1:968         19.423         0.022         5.20E-05         374677         414525         331011         1         1976         19.26         5.40E-05         5.40E-15         33982         339262         327270         0.023         8.2726         5.3150         3.49         3.412         3.416         3.416         3.416         5.416.55         3.453         3.417         30532         33926         33927         30270         0.022         8.2726         31551         3.49         3.417         30532         33524         31551         3.49         3.417         30532         33524         31551         3.49         4.67         1.14         1.426-64         355375         377070         34601           5.003         3.68.90         0.151         1.656-40         33119         341644         321330         5.447         5.665         33528         352687         331623         34569         332627         0.141         1.466-64         349629         31767         322314         341199         323471         5.644         6.622         6.518         0.163         1.862-44         35449         352687         3326287         3326287         332637         5.647         5.116         3262887         3326284         3327									1.5	14.715	0.04				327000
12.99         24.427         0.007         6.70E-05         306217         377157         328683           3         23.440         0.005         6.50E-05         362257         371677         328683         371677         328687         336810         34.89         34.277         0.007         6.70E-05         336217         377157         336810         336810         34.89         34.227         0.007         6.70E-05         336217         377157         336810         336810         336120         34.89         34.227         0.007         6.70E-05         336217         377157         336810         336810         3311764         336810         3311764         336810         337157         336810         3311764         326217         377157         337601         346401         30117         377167         336810         332623         346811         341644         34541         34564         35583         335583         335583         335583         335583         335583         335523         336523         336623         346221         34641         345693         336263         345721         345733         34573         345733         34573         34573         34573         34573         34573         34573         3								11	1.996	19.581	0.054	5.40E-05	362607	399607	331877
3         29.400         0.005         6.506-05         387247         330552         333550           4.004         39.279         0.117         1.776-05         33267         0.007         9.766-05         3372747         330552         333550           4.004         39.279         0.117         1.776-05         33264         3.061         3.99         33.142         0.111         1.1264         335500         335600           5.001         40.060         0.15         1.302-04         332114         541764         4.594         44.997         0.141         1.424-24         353570         333642         341651         4.594         4.697         0.141         1.546-0         310237         335500         335362         336427         54124         54199         54123         341553         33647         34154         54199         54124         34199         335500         335362         336362         34247         1.556-04         3556         35764         335504         342125         336437         34212         34194         36198         54224         2.245-44         346718         35504         34225         35517         1.4224         32524         355173         353620         353622         3					364581	393982	339263		2.499	24,515	0.07				
3 442 44.68 0.008 9.80763 34555 867295 31165 3469 44.27 0.077 9.7062 325247 37204 33550 340304 44.54 44.07 0.124 1.224 35575 37007 341601 44.454 44.07 0.124 1.224 35575 37007 341601 44.454 44.07 0.124 1.224 35575 37007 341601 44.454 44.07 0.124 1.244 35575 37007 341601 1.01604 35507 0.014 1.01624 35575 37007 341601 1.01624 35575 37007 341601 1.01624 35575 37007 341601 1.01624 35575 37007 341601 1.01624 35575 37007 341601 1.01624 35575 37007 341601 1.01624 35575 37007 341601 1.01624 35575 37007 341601 1.01624 35575 37007 341601 1.01624 35007 0.016 1.01624 34561 0.017 1.2264 0.0151 0.0164 0.016 1.01624 0.016 1.01624 0.016 0.015 0.0072 0.0016 0.015 0.000 0.016 0.015 0.0000 0.000 0.000 0.000 0.0000 0.000 0.0					346235	367875	327000		2.987	29.302	0,082				
4.004       39.279       0.117       1.178.204       335720       550708       321661       3.399       33142       0.111       1.1024       1.2424       35533       3702780       340601         5.001       40.660       0.15       1.502-64       327055       338244       316615       4.494       4.497       0.141       1.4024       348373       3326973       333740         5.033       5.840       0.151       1.502-64       31514       341614       321379       6.497       5.262       0.166       1.652-64       345504       350361       333862       333762       333862       333862       333862       333862       333862       333862       333862       333862       333862       333862       33464       6.422       6.5386       0.161       1.652-64       345516       350641       443454       355041       443454       345784       346934       354414       341388       522       522       2.522-42       4444       347718       342625       342631       34776       342873       337443       34414       341388       562       74707       35244       355130       342677       353374       344272       326273       353374       344271       345133       <				9.80E-05	348555	367295	331635		3.489	34,227					
10.07         40.020         0.16         1.002-04         22025         333844         316515           5.03         55.944         0.164         1.002-04         346527         321336           5.03         55.944         0.154         1.002-04         339570         3323679           5.04         65.04         0.122         1.022-04         3323571         346408         327764           5.696         66.801         0.217         2.072-04         333550         333776         332379           6.966         66.801         0.217         2.072-04         333550         333766         323379           7.939         75.05         0.221         2.216-04         33562         340067         322548           7.979         7.8274         0.224         2.248-04         349516         35014         34139           8.038         8.012         0.227         2.026-4         33562         340067         340237         330247         34235         340061         341037         34272         35037         34272         35037         34277         35239         34277         35239         34272         35037         34242         34276         350563         34272			0,117	1.17E-04											
5.503         5.804         0.193         1.515-0         335153         341774         321336           5.003         5.889         0.175         1.756-0         335151         34606         3237164           5.696         65.810         0.077         0.276-0         33526         333730         6.482         6.586         0.173         35149         36559         32322         343721           5.697         65.810         0.277         0.2776-0         33526         333730         6.482         6.513         0.195         1.586-0         335859         335329         335385         335349         305595         326222         334471         53149         305595         342235         300801         9.779         78277         72777         72777         72777	4,485	43,998	0.133												
2.603         26.889         0.175         1.75E-04         332811         344008         327141         5.986         56.723         0.168         1.08E-04         332437           5.696         65.604         0.102         1.02E-04         333575         33379         5.3367         5.3264         5.33269         35269         342715         364237         364237         364237         364237         364237         364237         364262	5,001	49,060	0.15												
6.6.3       2.6.2       2.3214       3.41199       322879       6.422       6.3.689       0.18       1.80E-4       353229       353362       344265         7.937       75.606       0.221       2.21E-64       332506       330750       325249       7.479       78.274       2.24       2.24E-44       345451       338041       341898         8.938       83.316       0.25       2.56E-44       333265       340067       325731       8.471       8.101       0.229       2.38E-44       347701       35513       340275         9.488       93.077       0.277       2.77E-44       335019       342156       330061       9.472       2.8202       0.262       2.58E-44       356373       344272         9.488       93.077       0.277       2.77E-44       335019       342156       330262       9.473       7.645       3313       3162-44       353704       340276         10.478       10.2789       0.305       3.056-44       347714       342421       10.673       10.764       313       3162-44       353704       340067       322420       10.673       10.764       331       3162-44       345265       347717       11.476       11.476       11.476	5,503	53,984													
6.6.6.7         6.6.8.1         0.103         1.0.27         2.075.6         333766         323730         6.8.4         6.8.13         0.105         1.9.65.04         35149         3800585         342055           7.901         78.392         0.235         2.215.64         333562         340084         326842         7.482         7.3398         0.21         2.10-64         335140         1.4444474         335141         341388           8.493         8.316         0.25         2.55.64         333563         340057         326741         8.471         8.101         0.229         2.352.64         44771         355130         340576           9.488         80.07         0.277         2.776.44         336019         342166         330061         9.472         9.228         2.852.64         350156         344733           10.479         10.776         0.319         3.162.44         341553         344056         332420         11.976         11.765         1.331678         340056         332472         3.235.44         345624         345624         345623         344733           1.977         1.74.44         3.44553         3.46651         335765         335765         340576         332373         345															
7.03       0.3.06       0.221       2.215.04       332001       340307       325.49       7.49       7.897       7.897       7.897       7.897       7.897       7.897       7.897       7.877       0.221       2.24.04       344714       34109         9.93       8.316       0.25       2.505.04       333265       300607       326731       8.471       8.101       0.229       2.385.04       344701       35513       34472         9.922       2.972.0       2.916.04       338019       342166       330061       9.472       2.2200       0.228       2.852.04       353039       380156       344775         10.778       10.278       0.276.0       3.916.04       337014       342631       31576       10.973       10.764       0.313       3.162.04       337014       344666       335775       11.976       11.976       11.476       10.280       3.32.24.4       346243       344243         11.976       11.724       0.217       0.217       0.228       3.276.04       341751       344666       335765       11.976       11.476       11.350       3.325.04       34656       34473         11.976       11.776       0.313       3.166.04       346661															
7:930       7:8302       2:325       2:325 4       2:335:24       3:408:34       3:268:22       7:379       7:279       7:274       0:224       2:24:24-04       3:44:93       3:55:130       3:40:57         9:838       83:110       0:229       2:82:16-04       3:35:530       3:40:57       3:55:130       3:40:57       3:55:130       3:40:57       3:55:130       3:40:57       3:55:130       3:40:57       3:55:130       3:40:57       3:55:130       3:40:57       3:55:130       3:40:57       3:55:130       3:40:57       3:55:130       3:40:57       3:55:130       3:40:57       3:45:130       3:47:33       3:47:33       3:47:33       3:47:33       3:47:33       3:47:33       3:47:33       3:47:33       3:47:24       3:35:64       3:55:65       3:17:75       1:47:57															
9.833       6.335       0.25       2.80E.04       333265       340067       328731       8.471       8.3101       0.279       2.38E.04       347701       355130       340276         9.862       89.3077       0.277       2.77E.04       330519       342196       330061       8972       82280       353305       342390       300212       9.868       9.772       2.77E.04       355373       342472         9.828       9.027.9       0.205       3.05E.04       337014       342831       3131678       30016       347701       355377       345707       345707       345707       345707       345707       355170       344731       31017       10.778       0.238       338E.04       35763       340238       33763       310673       32282.04       335064       34573       11.976       11.269       0.238       338E.04       345024       342024       342215       11.976       11.876       11.269       0.238       338E.04       345024       342024       342024       342024       342024       342024       342024       342024       342024       342024       342024       342024       342024       342024       342024       342024       342024       342024       332056.4								1							
b 0002         08.212         0.222         2.222.04         335685         3.43235         3.0300         8.472         8.615         0.222         2.222.00         3.36567         3.44733           9.685         9.7023         0.201         2.915.04         335507         3.22390         3.00122         9.472         9.2520         0.263         2.561.04         3.5517         3.44733           10.678         102.799         0.035         3.556.4         3.57101         3.40005         3.32420         10.673         10.678         0.2780         2.285.04         3.5350.54         3.44733           11.977         117.46         0.243         3.446.04         3.415.53         3.446.56         3.517.50         3.44733           11.977         117.2580         0.323         3.586.04         3.505.04         3.565.04         3.46562         3.517.50         3.44231           12.472         12.22.350         0.337         3.385.04         3.455.44         3.46522         3.44733           13.472         13.272.11         0.328         3.856.04         3.455.44         3.4652.24         3.465.24         3.465.24         3.465.24         3.465.24         3.465.24         3.465.24         3.465.24         3.465.24								L .							
9.868         9.3077         2.772         2.772.4         335019         332196         3300261         9.472         9.2030         2.632.04         333309         360156         345707           9.872         9.702         0.205         3.056.44         337014         342831         331378         9.735         0.278         2.726.24         350230         345707         34707           10.979         107.704         0.319         3.156.44         340231         331578         340006         332406         335176         10776.45         0.313         3156.44         345616         334701           11.977         117.484         0.344         3.445.04         34553         345651         335621         1077.4         0.323         326.64         436562         347731           12.968         127.216         0.312         3.316.44         345655         345015         3362.26         3357.65         3366.44         3566.45         3356.44         3566.45         3356.44         34565         34501           13.47         12.214         0.328         3.806.44         4.406.74         4.226.34         34273         14.325         1.426.71         14.56         0.467.75         355176         35171								L .							
Sec.         Construction         Construction         Sec.         Sec.<								L							
10.47         10.475         10.472 </td <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td>1</td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td>								1							
10.070       10.073       10.073       10.073       10.073       10.074       0.014       327242       322950       341731         11.145       112.619       0.311       3.316_04       344204       34283       342645       335070       32260       336624       341321         11.977       117444       0.343       3.46_04       341751       346633       337443       11976       1177       112.680       0.333       3.266-04       346526       341751         12.663       11976       1177       112.480       0.323       3.266-04       346526       347700         13.77       132.141       0.383       3.866-04       341525       347011       336236       13.472       12.265       0.33       3.866-04       345565       347744         13.77       132.141       0.383       3.866-04       345573       350264       345523       345624       345623       345624       345623       345624       345624       345624       345624       345624       345624       345624       345624       345624       345624       345624       345624       345624       345624       345624       345624       345624       345627       35564       34564       345627 <td></td>															
111,48       102,19       0.33       102,19       0.33       114,48       114,47       112,80       0.323       3.23E-04       245426       341553         111,977       1117,44       0.344       3.44E-04       341553       3466501       335650       11,976       117,485       0.333       3.33E-04       346523       341525         12,926       127,2716       0.372       3.72E-04       341571       346633       33743       11,476       112,870       0.353       3.50E-04       346523       34555       34471         13,877       137,075       0.55       0.4       4.00E-04       342658       347001       33833       11,471       112,972       127,715       0.331       331E-04       355175       355175       355177       344524         14,467       14,1521       0.412       4.12E-04       342E34       3420701       330339       14,411       14,1961       0.402       4.34720       34524       345027       353175       345427         14,964       14,607       0.44       4.002E       2.602-05       374567       453373       314238       14,965       14,807       0.42       4.20E-04       349540       353751       344527         14															
11:000       11:1744       0.344       3.44:0.4       2.44:0.3       3.46:04       3.38:0.4       3.38:0.4       3.46:04       3.38:0.4       3.46:04       3.38:0.4       3.46:04       3.38:0.4       3.46:04       3.38:0.4       3.46:04       3.38:0.4       3.46:04       3.38:0.4       3.46:04       3.38:0.4       3.46:04       3.38:0.4       3.46:04       3.38:0.4       3.46:04       3.38:0.4       3.46:04       3.46:05       3.38:0.4       3.46:04       3.								L							
12.472       122.323       0.358       1.58E-0.4       344628       334625       334645         12.428       127.216       0.372       3.72E-0.4       349528       335462       355462       355462       355462       355462       355462       355462       355462       355462       355462       345013       34603       380E-04       347760       352428       342010       340331       13.472       122.72       122.72       122.72       122.72       122.75       0.381       3.80E-04       347760       352428       342014         13.472       132.77       137.055       0.44       4.00E-04       342563       347011       340331       13.472       127.77       0.391       3116-04       35547       35517       36143         14.964       14.6797       0.424       4.24E-04       342218       323076       14.471       14.167       0.474       4.4551       34540       353452       353136       344542         14.963       1.4607       0.04       4.00E-05       355770       321123       380073       321820       32076       321822       32182       34543       345427         1.984       19.4639       0.151       1516.0       355542       33716								L							
12.938       127.216       0.372       1272.10       0.372       1272.10       0.372       1272.10       0.372       1272.10       0.3072       1272.10       0.3072       1272.10       0.3072       1272.10       1272.10       1272.11       1272.11       0.30556       335462       345015       335265       335266       335462       34224         13.97       137.065       0.4       4.00E-04       34263       34701       338433       13.472       13.073       3.91E-04       350576       355117       34454         14.467       141.921       0.412       4.12E-04       342274       322184       14.477       141.961       0.407       4.07E-04       348797       353136       344542         14.467       146.07       0.44       4.2604       969228       323076       353751       34454       363273       34454       345427         1984       14.607       0.04       4.00E-05       365777       4713.32       32802       37313       37025       328982       32402       345437       344273         3.865       34188       0.094       9.00E-05       365780       337143       34433       34532       34532       342427       345427       345427 <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td>L</td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td>								L							
13.47       132.14       0.388       3.88E-04       340569       345015       336236         13.872       137.05       0.4       4.00E-04       342633       347001       338433       31373       132.05       0.38       3.80E-04       342767       353136         14.867       141.921       0.412       4.122-04       344213       340701       340339       314701       340339         14.467       141.921       0.412       4.122-04       344213       350330       32118       34571       35136       34454         13.47       141.921       0.424       4.24E-04       346213       314224       347501       34454         14.951       Force       Dist.(mm)       Dist.(m)       Stringe       322076       353136       34454         0.933       9.741       0.026       48614       969228       323076       32812       34501       34532       345242         1.849       19.433       0.054       5.40E-05       365701       390176       344713       346332       347254       34532       373156       344671       346532       367260       34716       4.4395       0.664       355527       353751       345147       346332 <t< td=""><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>L</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></t<>								L							
13572       137.05       0.4       4.00E-04       342863       347001       338433         14.467       141.921       0.412       4.122-04       342619       350350       342014         14.467       146,787       0.424       4.24E-04       345219       350350       342184         Spring 3       Error         Mass (Kg)       Force       Dist. (mm)       Dist. (mi)       Stimess       Lower Bound       0.42       4.20E-04       349540       353751       3465427         Mass (Kg)       Force       Dist. (mi)       Dist. (mi)       Stimess       Lower Bound       10.987       346544       353751       345427         1984       14.607       0.04       4.002-65       355771       34767       463873       314236         2.881       24.81       24.339       0.696       6.90E-65       357733       328026       373194       344273         3.885       34.188       0.004       9.40E-65       355710       344273       342427         3.885       39.122       0.151       1.51E-04       355238       373196       344671         5.88       58.866       0.152       1.92E-04       355232       373196								L				3.80E-04	347790	352428	343274
14.467       141.921       0.412       4.12E-04       342429       348701       340339       14.471       141.961       0.407       4.07E-04       348797       353136       344564         14.964       146.07       0.424       4.24E-04       346219       350350       32114       14.965       146.807       0.42       4.20E-04       349540       353751       344564         Mass (Kg)       Force       Dist. (mm)       Dist. (m)       Suffness       Lower Bound Upper Bound       14.965       146.807       0.42       4.20E-04       349540       353751       344564         0.933       9.741       0.022       2.060-55       374657       453873       314236       4489       40.07       0.024       4.00E-05       355777       417345       324602         1.844       19.453       0.056       6.00E-05       35570       390176       344273       3380241       328900       2.983       2.9263       0.094       9.052       0.137       1.37E-04       356023       37136       344671       353761       34784       344671       35376       347144       4.89       4.037       0.123       1.28E-04       355263       367183       344532       347184       344671       3								L	13.973	137.075	0.391				
14.964       146.797       0.424       4.24E-04       346219       350350       342184       14.965       146.807       0.42       4.20E-04       349540       353751       345427         Mass (Kg)       Force       Dist. (mm)       Dist. (m)       Stiftness       Lower Bound Upper Bound       0.923       3.741       0.025       2.60E-05       374657       417345       324602       323076         1.884       1.4607       0.044       4.00E-05       355177       417345       324602       329876         2.983       2.92.63       0.065       5.05E-05       355177       417345       324602       349340       34532         3.485       34.188       0.094       9.40E-05       365177       3417345       324602       349378       344732         3.485       34.188       0.094       9.40E-05       355273       300176       343178       344732         3.485       34.188       0.094       9.40E-05       35520       376176       343178       344671         4.89       4.037       0.151       1.51E-04       355253       367280       347110       6477       65.39       0.121       2.71E-04       355255       360798       346524       361327<						348701	340339	н	14.471	141.961	0.407				
Spring 3         Error           Mass (KG)         Force         Dist. (mm)         Dist. (m)         Suffness         Lower Bound Upper Bound           0.484         4.846         0.01         1.00E-05         #44614         969228         323076           0.993         9.741         0.026         2.00E-05         374557         314236           1.489         14.607         0.024         4.00E-05         326727         337205         329802           2.481         24.333         0.059         6.90E-05         355770         339176         324273         32800           2.983         22.923         0.069         6.90E-05         355707         391716         344273           3.465         34.188         0.054         4.02e-05         355707         394133         345332           3.485         34.188         0.054         4.02e-05         355707         394133         345332           3.485         38.128         0.137         1.37E-04         355625         3731195         344601           4.99         48.952         0.151         1.5E-04         355633         366249         345091           6.477         68.395         0.122         1.2E-04				4 24F-04	346219	350350	2/219/		14 965	146 B07	7 0.42	4 20F-04	349540	353751	345427
Mass (Kg)         Force         Dist. (m)         Dist. (m)         Stiffness         Lower Bound           0.484         4.846         0.01         1.00E-05         484614         969228         323076           0.993         9.741         0.026         2.60E-05         37457         453873         314236           1.489         14.607         0.04         4.00E-05         355177         417345         324602           2.481         24.330         0.056         6.90E-05         355720         390176         344273           3.485         34.188         0.094         9.40E-05         355700         390176         344273           3.988         31.122         0.109         1.09E-04         358520         376176         343178           4.489         44.037         0.123         1.23E-04         358225         373196         344040           4.99         48.952         0.137         1.37E-04         35538         36649         345081           5.481         53.769         0.151         1.51E-04         35593         367280         34710           6.477         63.839         0.178         1.78E-04         35593         367280         34718 <tr< td=""><td></td><td></td><td>0.424</td><td></td><td></td><td></td><td>042104</td><td></td><td>14 505</td><td>1401001</td><td>0.14</td><td>4.2.00.0</td><td></td><td>1 000101</td><td></td></tr<>			0.424				042104		14 505	1401001	0.14	4.2.00.0		1 000101	
0.444         4.846         0.01         1.00E-05         484614         969228         323076           0.993         9.741         0.025         2.60E-05         374657         463873         314236           1.489         19.463         0.054         5.40E-05         36077         417345         324602           2.481         24.339         0.056         6.90E-05         352733         380291         328900           2.983         29.263         0.08         8.00E-05         353701         384133         345332           3.988         3.122         0.109         1.09E-04         355023         376176         343178           4.499         44.037         0.123         1.23E-04         355023         377195         344040           4.99         48.952         0.137         1.37E-04         355033         366249         345081           5.481         53.759         0.151         1.5E-04         355533         366249         345081           6.972         68.395         0.122         1.22E-04         35225         356750         34710           6.972         68.395         0.249         2.49E-04         353523         366432         34725 <td>11.001</td> <td></td> <td>0,424</td> <td>4.242.04</td> <td>1 0.02.0</td> <td></td> <td>342104</td> <td>1</td> <td>14.505</td> <td>1401001</td> <td>0.12</td> <td>1.2.00.0</td> <td>1</td> <td>1 000.01</td> <td></td>	11.001		0,424	4.242.04	1 0.02.0		342104	1	14.505	1401001	0.12	1.2.00.0	1	1 000.01	
0.993       9.741       0.026       2.60E-05       374657       463873       314236         1.489       14.607       0.04       4.00E-05       385177       417345       324602         2.983       2.9481       24.339       0.056       6.90E-05       352733       380291       328902         2.983       2.9263       0.098       8.00E-05       352733       380291       328902         3.485       34.188       0.094       9.40E-05       353701       384133       345332         3.988       39.122       0.109       1.09E-04       358203       376176       343178         4.489       44.037       0.123       1.23E-04       355205       377313       370848       344732         5.481       53.769       0.151       1.51E-04       355253       366749       345081         6.477       63.539       0.178       1.78E-04       355283       367280       347210         6.972       66.395       0.192       1.92E-04       355264       345323       347255         7.971       78.196       0.221       2.21E-04       35524       366750       347184         8.974       80.035       0.249       2.48E-0			0.424		1	Error			14 505	140.001	0.12	1.202.0	1	5	S.,
1.489       14.607       0.04       4.00E-05       365177       417345       324602         1.984       19.463       0.054       5.00E-05       355273       320291       329802         2.983       29.263       0.08       8.00E-05       355790       390176       344273         3.985       34.188       0.094       9.40E-05       355701       384133       345332         3.988       39.12       0.199       1.09E-04       35520       37176       344732         3.988       39.12       0.193       1.09E-04       35520       37176       344732         3.988       39.12       0.131       1.51E-04       356034       365278       344671         5.98       58.664       0.165       1.65E-04       356533       367280       347210         6.977       68.393       0.178       1.78E-04       356254       366452       347210         6.977       68.393       0.192       1.02E-04       35526       367180       347184         7.971       78.196       0.221       2.21E-04       35526       367280       347210         6.972       68.393       0.182       1.28E-04       35524       366199	Spring 3	1		n) Dist. (m)	Stiffness	Error Lower Bour	nd Upper Bound		14.505	140,007	0.10	1.202 0		5	S.,
1.926       19.453       0.054       5.40E-05       352733       397205       329802         2.481       24.339       0.059       6.90E-05       352733       380291       328900         2.983       29.263       0.08       8.00E-05       355790       390176       344273         3.485       34.188       0.094       9.40E-05       355700       397176       344333         3.988       39.122       0.109       1.09E-04       358202       376176       343178         4.489       44.037       0.123       1.23E-04       356225       373196       344640         4.99       48.952       0.131       1.37E-04       356025       37280       344732         5.481       53.769       0.151       1.51E-04       356256       365750       347140         6.972       68.395       0.172       1.22E-04       356226       365750       347210         6.972       68.395       0.122       1.22E-04       355226       346514       34594         8.473       83.120       0.235       2.3EE-04       353703       361392       346334         9.477       78.496       0.261       2.5EE-04       35554       360799<	Spring 3 Mass (K	g) Force	Dist. (mn	n) Dist (m) 1.00E-05	Stiffness 484614	Error Lower Bour 969228	nd Upper Bound 323076			140,007		1.202.0		<u></u>	
2.481       24.339       0.069       6.90E-05       352733       380291       328900         2.983       29.263       0.08       8.00E-05       355790       390176       344233         3.485       34.188       0.994       9.40E-05       355720       390176       34433         3.988       39.122       0.109       1.09E-04       358220       376176       343178         4.489       44.037       0.123       1.23E-04       358225       373186       344432         5.481       53.769       0.151       1.51E-04       356034       368278       3444732         5.481       53.769       0.151       1.51E-04       356034       368278       344671         5.98       58.664       0.165       1.65E-04       355633       367280       347210         6.972       68.395       0.192       1.92E-04       356226       365750       347184         7.469       73.271       0.206       2.06E-04       355824       364532       347255         7.971       78.166       0.221       2.21E-04       35524       365142       347505         9.477       82.969       0.261       2.46E-04       355234       34591	Spring 3 Mass (Kr 0.494	g) Force 4.846	Dist. (mn 0.01 0.026	n) Dist (m) 1.00E-05 2.60E-05	Stiffness 484614 374557	Error Lower Bour 969228 463873	nd Upper Bound 323076 314236			110,001		1.202.0	1	<u></u>	S.,
2.98329.2630.088.00E-053657903901763442733.48534.1880.094 $9.40E-05$ 3637013841333453323.98839.1220.1091.09E-043585203761763431784.48944.0370.1231.23E-043580253731963440404.9948.9520.1371.37E-043573133708483447325.48153.7690.1511.51E-043560343682783446715.9858.6640.1651.65E-043555333672803472106.97268.3950.1921.92E-043552363657503471847.46973.2710.2062.06E-043552543620163459988.47383.1200.2352.35E-043531823463348.97488.0350.2492.48E-04355243631629.97697.8650.2782.78E-04352313544459.47792.9690.2612.61E-04355243631629.97697.8650.2782.78E-043523135447810.478102.7890.2952.95E-0434827834397711.476112.5800.3223.22E-043552435514134428011.975117.4750.3833.38E-0434574535277734249212.47212.3500.3613.558435561434428011.975117.4750.3833.38E-04345743350512	Spring 3 Mass (Kr 0.494 0.993	g) Force 4.845 9.741	Dist. (mn 0.01 0.026 0.04	n) Dist (m) 1.00E-05 2.60E-05 4.00E-05	Stiffness 484614 374557 365177	Error Lower Bour 969228 463873 417345	nd Upper Bound 323076 314236 324602		14.505	110.001		1.206-0	1	<u> </u>	S
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Spring 3 Mass (Kr 0.494 0.993 1.489	g) Force 4.845 9.741 14.607 19.463	Dist. (mn 0.01 0.026 0.04 0.054	1) Dist. (m) 1.00E-05 2.60E-05 4.00E-05 5.40E-05	Stiffness 484614 374557 365177 360427	Error Lower Bour 969228 463873 417345 397205	nd Upper Bound 323076 314236 324602 329882		14.505	140,007		1.206-0	1		5.5 <sub>5</sub>
3.988 $33.122$ $0.109$ $1.09E-04$ $35920$ $376176$ $343178$ $4.499$ $44.037$ $0.123$ $1.23E-04$ $358925$ $373195$ $344040$ $4.99$ $48.952$ $0.137$ $1.37E-04$ $357313$ $370848$ $344732$ $5.481$ $53.769$ $0.151$ $1.51E-04$ $356934$ $368278$ $344671$ $5.98$ $58.664$ $0.165$ $1.65E-04$ $355533$ $367280$ $347210$ $6.477$ $63.539$ $0.178$ $1.78E-04$ $355253$ $367280$ $347210$ $6.972$ $68.395$ $0.192$ $1.92E-04$ $355253$ $367280$ $347184$ $7.469$ $73.271$ $0.206$ $2.06E-04$ $355834$ $364532$ $347255$ $7.971$ $78.196$ $0.221$ $2.21E-04$ $353254$ $360799$ $346594$ $8.473$ $83.120$ $0.235$ $2.35E-04$ $353554$ $360799$ $346594$ $9.477$ $92.969$ $0.261$ $2.61E-04$ $352204$ $363162$ $349509$ $9.976$ $97.865$ $0.278$ $2.78E-04$ $352329$ $343977$ $11.476$ $112.580$ $0.322$ $3.22E-04$ $349526$ $355141$ $342801$ $10.975$ $107.665$ $0.308$ $3.88E-04$ $34758$ $352777$ $32492$ $12.472$ $12.350$ $0.351$ $3.51E-04$ $34554$ $342631$ $12.97$ $127.236$ $0.368$ $3.68E-04$ $347558$ $352777$ $32492$ $12.472$ <td< td=""><td>Spring 3 Mass (Kr 0.494 0.993 1.489 1.984 2.481</td><td>g) Force 4.845 9.741 14.607 19.463 24.339</td><td>Dist_ (mn 0.01 0.026 0.04 0.054 0.069</td><td>n) Dist (m) 1.00E-05 2.60E-05 4.00E-05 5.40E-05 6.90E-05</td><td>Stiffness 484614 374557 365177 360427 352733</td><td>Error Lower Bour 969228 463873 417345 397205 380291</td><td>nd Upper Bound 323076 314236 324602 329882 328900</td><td></td><td>14.505</td><td>140.007</td><td></td><td></td><td></td><td></td><td>5.5<sub>5</sub></td></td<>	Spring 3 Mass (Kr 0.494 0.993 1.489 1.984 2.481	g) Force 4.845 9.741 14.607 19.463 24.339	Dist_ (mn 0.01 0.026 0.04 0.054 0.069	n) Dist (m) 1.00E-05 2.60E-05 4.00E-05 5.40E-05 6.90E-05	Stiffness 484614 374557 365177 360427 352733	Error Lower Bour 969228 463873 417345 397205 380291	nd Upper Bound 323076 314236 324602 329882 328900		14.505	140.007					5.5 <sub>5</sub>
4.499 $44.037$ $0.123$ $1.23E-04$ $358025$ $373196$ $344040$ $4.99$ $48.952$ $0.137$ $1.37E-04$ $55713$ $370848$ $344732$ $5.481$ $53.769$ $0.151$ $1.51E-04$ $356034$ $368278$ $344671$ $5.98$ $58.664$ $0.165$ $1.65E-04$ $355533$ $366649$ $345081$ $6.477$ $63.539$ $0.178$ $1.78E-04$ $356253$ $367280$ $347210$ $6.972$ $68.395$ $0.192$ $1.92E-04$ $356226$ $365750$ $347184$ $7.469$ $73.271$ $0.206$ $2.06E-04$ $3525634$ $364532$ $347255$ $7.971$ $78.166$ $0.221$ $2.21E-04$ $353226$ $362512$ $347255$ $7.971$ $78.166$ $0.221$ $2.21E-04$ $353254$ $362523$ $362512$ $3477$ $92.969$ $0.218$ $2.49E-04$ $353254$ $360799$ $346594$ $9.477$ $92.969$ $0.261$ $2.61E-04$ $352231$ $358478$ $345511$ $10.975$ $107.665$ $0.308$ $3.08E-04$ $349551$ $355229$ $343977$ $11.476$ $112.580$ $0.322$ $3.22E-04$ $349525$ $355141$ $344280$ $11.975$ $107.665$ $0.308$ $3.08E-04$ $34758$ $352777$ $342492$ $12.472$ $122.350$ $0.351$ $3.51E-04$ $345749$ $350512$ $341114$ $13.472$ $132.160$ $0.388$ $3.68E-04$ $345749$ $350512$	Spring 3 Mass (K 0.494 0.993 1.489 1.984 2.481 2.983	g) Force 4.845 9.741 14.607 19.463 24.339 29.263	Dist_ (mn 0.01 0.026 0.04 0.054 0.069 0.08	n) Dist. (m) 1.00E-05 2.60E-05 4.00E-05 5.40E-05 6.90E-05 8.00E-05	Stiffness 484614 374557 365177 360427 352733 355790	Error Lower Bour 969228 463873 417345 397205 380291 390176	nd Upper Bound 323076 314236 324602 329882 328900 344273		14.000	1000		1.202.0			S
4.99       49.952       0.137       1.37E-04       357313       37D848       344732         5.481       53.769       0.151       1.5FE-04       356034       368278       344671         5.98       58.664       0.165       1.65E-04       355538       366649       345081         6.477       63.599       0.178       1.7E-04       356226       347210         6.972       68.395       0.192       1.92E-04       356226       365750       347184         7.469       73.271       0.206       2.0EE-04       355834       364332       347255         7.971       78.196       0.221       2.21E-04       355254       360799       346594         8.473       83.120       0.235       2.35E-04       353703       361392       346334         8.974       88.035       0.249       2.49E-04       35254       360799       346594         9.477       92.969       0.261       2.61E-04       352231       354478       345511         10.975       107.665       0.308       3.08E-04       349523       355141       345911         10.975       107.665       0.308       3.68E-04       347558       355277       34	Spring 3 Mass (K: 0.494 0.993 1.489 1.984 2.481 2.983 3.485	g) Force 4.845 9.741 14.607 19.463 24.339 29.263 34.188	Dist_ (mn 0.01 0.026 0.04 0.054 0.069 0.08 0.094	n) Dist (m) 1.00E-05 2.60E-05 4.00E-05 5.40E-05 6.90E-05 8.00E-05 9.40E-05	Stiffness 484614 374557 365177 360427 352733 355790 363701	Error Lower Bour 969228 463873 417345 397205 380291 390176 384133	nd Upper Bound 323076 314236 324602 329882 328900 344273 345332		14.000	110,001					S
5.481       53.769       0.151       1.51E-04       356034       368278       344671         5.98       58.664       0.165       1.65E-04       355338       366649       345081         6.477       63.539       0.178       1.78E-04       356953       367280       347210         6.972       68.395       0.192       1.92E-04       356253       367280       347184         7.469       73.271       0.206       2.06E-04       355684       364532       347255         7.971       78.196       0.221       2.21E-04       353276       362016       345998         8.473       83.120       0.235       2.35E-04       353703       361392       346344         8.974       88.035       0.249       2.49E-04       355254       360799       346594         9.477       92.969       0.261       2.61E-04       355203       358478       345811         10.478       102.789       0.295       2.95E-04       348438       354445       342631         10.975       107.665       0.308       3.08E-04       345581       352277       342921         11.975       117.475       0.338       3.38E-04       345749       <	Spring 3 Mass (K) 0.494 0.993 1.489 1.984 2.481 2.481 2.983 3.485 3.988	g) Force 4.845 9.741 14.607 19.463 24.339 29.263 34.188 39.122	Dist. (mn 0.01 0.026 0.04 0.054 0.069 0.08 0.094 0.109	<ul> <li>Dist. (m)</li> <li>1.0DE-05</li> <li>2.60E-05</li> <li>4.00E-05</li> <li>5.40E-05</li> <li>6.90E-05</li> <li>8.00E-05</li> <li>9.40E-05</li> <li>1.09E-04</li> </ul>	Stiffness 484614 374557 365177 360427 352733 365790 363701 358520	Error Lower Bour 959228 453873 417345 387205 380291 390176 384133 376176	nd Upper Bound 323076 314236 324602 329882 328900 344273 345332 345332 343178		14.000	10000					5-4 -
5.98       59.664       0.165       1.65E-04       35E538       366649       345081         6.477       63.539       0.178       1.7E-04       356953       367280       347210         6.972       68.395       0.192       1.92E-04       355226       366750       347184         7.469       73.271       0.206       2.06E-04       35584       364532       347255         7.971       78.196       0.221       2.21E-04       35326       360790       34634         8.473       83.120       0.235       2.35E-04       353703       361392       34634         8.974       88.035       0.249       2.49E-04       35554       366799       346594         9.477       92.969       0.261       2.61E-04       355203       363162       349509         9.976       97.865       0.278       2.78E-04       352031       35445       342631         10.975       107.665       0.308       3.08E-04       349561       355329       345977         11.975       117.475       0.338       3.38E-04       345749       35512       341114         12.97       122.350       0.351       3.51E-04       345563       349630	Spring 3 Mass (K: 0.494 0.993 1.984 2.481 2.983 3.485 3.988 4.489	g) Force 4.845 9.741 14.607 19.463 24.339 29.263 34.188 39.122 44.037	Dist. (mn 0.01 0.026 0.04 0.054 0.069 0.08 0.094 0.109 0.123	n) Dist (m) 1.00E-05 2.60E-05 4.00E-05 5.40E-05 6.90E-05 8.00E-05 9.40E-05 1.09E-04 1.23E-04	Stiffness 484614 374557 365177 360427 352733 355790 363701 358520 358025	Error Lower Bour 969228 463873 417345 397205 380291 390176 384133 376176 373196	1d Upper Bount 323076 314236 324602 329882 328900 344273 345332 343178 344040		14.000						S-4.
6.477       63.539       0.178       1.78E-04       356933       367280       347210         6.972       68.395       0.192       1.92E-04       356226       365750       347184         7.469       73.271       0.206       2.06E-04       355824       345232       347255         7.971       78.196       0.221       2.21E-04       353626       362016       345998         8.473       83.120       0.235       2.35E-04       353554       360799       346594         9.477       92.969       0.261       2.61E-04       355254       360799       346594         9.477       92.969       0.261       2.61E-04       355213       358478       345509         9.976       97.865       0.278       2.78E-04       325221       358478       345811         10.478       102.789       0.295       2.95E-04       348438       354445       342631         10.975       107.665       0.302       3.22E-04       349576       355141       343977         11.975       117.475       0.338       3.38E-04       34758       352777       342492         12.472       122.350       0.351       3.51E-04       345561	Spring 3 Mass (K 0.494 0.993 1.489 1.984 2.481 2.983 3.485 3.988 4.489 4.99	g) Force 4.846 9.741 14.607 19.463 24.339 29.263 34.188 39.122 44.037 48.952	Dist_ (mn 0.01 0.026 0.04 0.054 0.069 0.08 0.094 0.109 0.123 0.137	n) Dist. (m) 1.00E-05 2.60E-05 4.00E-05 5.40E-05 6.90E-05 8.00E-05 9.40E-05 1.09E-04 1.23E-04 1.37E-04	Stiffness 484614 374557 365177 3652733 355790 365790 363701 358920 358025 357313	Error Lower Bour 969228 463873 417345 397205 380291 390176 364133 376176 373196 370848	1d Upper Bound 323076 314236 324602 329882 328900 344273 345332 345332 345332 345332 345332 345332 345332 345332 345332		14.505						S-4,
6.972       68.395       0.192       1.92E-04       356226       365750       347184         7.469       73.271       0.206       2.0EE-04       355824       364532       347255         7.971       78.196       0.221       2.21E-04       353226       362016       345998         8.473       83.120       0.235       2.35E-04       353703       361392       346334         8.974       88.035       0.249       2.49E-04       353554       360799       346594         9.477       92.969       0.261       2.61E-04       352031       358478       345511         10.478       102.789       0.295       2.95E-04       348438       354445       342631         10.975       107.665       0.308       3.08E-04       349551       355329       343977         11.476       112.580       0.322       3.22E-04       349526       355141       344280         11.975       117.475       0.338       3.38E-04       345543       345141         12.472       122.350       0.351       3.51E-04       345563       345758         12.472       122.350       0.351       3.51E-04       345564       345361      12.4	Spring 3 Mass (K, 0.494 0.993 1.489 1.984 2.481 3.485 3.988 4.489 4.99 5.481	g) Force 4.846 9.741 14.607 19.463 24.339 29.263 34.188 39.122 44.037 48.952 53.769	Dist. (mm 0.01 0.026 0.04 0.059 0.08 0.094 0.109 0.123 0.137 0.151	<ul> <li>Dist (m)</li> <li>1.00E-05</li> <li>2.60E-05</li> <li>4.00E-05</li> <li>5.40E-05</li> <li>6.90E-05</li> <li>8.00E-05</li> <li>9.40E-05</li> <li>1.09E-04</li> <li>1.23E-04</li> <li>1.37E-04</li> <li>1.51E-04</li> </ul>	Stiffness 484614 374557 365177 360427 352733 355790 363701 358920 358025 357313 356034	Error Lower Bour 969228 453873 417345 397205 380291 390176 384133 376176 377648 370848 368278	1d Upper Bound 323076 314236 324602 329882 328900 344273 345332 343178 344040 344732 344671		14.505						S
7.469       73.271       0.206       2.06E-04       355684       364532       347255         7.971       78.196       0.221       2.21E-04       353826       362016       345998         8.473       83.120       0.235       2.35E-04       353703       361392       346334         8.974       88.035       0.249       2.49E-04       353554       360799       346594         9.477       92.969       0.261       2.61E-04       352031       358478       345811         10.478       102.789       0.295       2.95E-04       348438       354445       342631         10.975       107.665       0.308       3.08E-04       345514       344280         11.975       117.475       0.338       3.38E-04       34575       353614       343681         12.97       122.350       0.351       3.51E-04       34556       355012       341114         13.472       132.160       0.383       3.88E-04       345749       350512       341114         13.472       132.160       0.383       3.88E-04       34556       349630       340619         13.958       137.026       0.4       4.00E-04       342555       346901	Spring 3 Mass (K: 0.494 0.993 1.984 2.481 2.983 3.485 3.988 4.489 4.99 5.481 5.98	g) Force 4.845 9.741 14.607 19.463 24.339 29.263 34.188 39.122 44.037 48.952 53.769 58.664	Dist. (mm 0.01 0.026 0.04 0.059 0.08 0.094 0.109 0.123 0.137 0.151	1) Dist (m) 1.00E-05 2.60E-05 4.00E-05 5.40E-05 8.00E-05 9.40E-05 1.09E-04 1.23E-04 1.37E-04 1.51E-04 1.65E-04	Süffness 484614 374567 365177 365477 352733 355790 358520 358520 358525 357313 4356054 355326	Error Lower Bour 969228 463873 417345 397205 380291 390176 384133 376176 373196 375196 37648 366649	1d Upper Bound 323076 314236 324602 328802 328900 344273 345332 343178 344040 344732 344671 345081	9	14.505						
7.971       78.196       0.221       2.21E-04       353826       362016       345998         8.473       83.120       0.235       2.35E-04       353703       361392       346334         8.974       88.035       0.249       2.49E-04       35354       360799       346594         9.477       92.969       0.261       2.61E-04       352031       358478       345811         10.478       102.789       0.295       2.95E-04       348438       354445       342631         10.975       107.665       0.308       3.08E-04       349561       35529       343977         11.975       117.475       0.338       3.38E-04       34758       352777       342492         12.472       122.350       0.351       3.51E-04       34556       35512       341114         13.472       132.160       0.388       3.68E-04       345749       350512       341114         13.472       132.160       0.383       3.83E-04       345749       350512       341114         13.472       132.160       0.383       3.83E-04       345749       350512       341114         13.452       0.44       44506       345565       349630	Spring 3 Mass (K) 0,494 0,993 1,984 1,984 2,481 2,983 3,485 3,988 4,489 4,89 5,481 5,98 6,477	g) Force 4,845 9,741 14,607 19,463 24,339 29,263 34,188 39,122 44,037 48,952 53,769 58,664 63,539	Dist. (mn 0.01 0.026 0.04 0.054 0.09 0.09 0.123 0.137 0.151 0.165 0.178	n) Dist (m) 1.00E-05 2.60E-05 5.40E-05 5.40E-05 8.00E-05 9.40E-05 1.09E-04 1.23E-04 1.37E-04 1.51E-04 1.51E-04 1.55E-04 1.78E-04	Stiffness 484614 374557 365177 360427 352733 355790 358520 358520 358520 358520 358524 358534 358534 358534	Error Lower Bour 965228 463873 417345 380291 390176 384133 376176 375196 375196 375196 370848 36629 366549 367280	nd Upper Bound 323076 314236 324602 329882 328900 344273 345332 345332 345332 344573 344040 344732 344671 345081 347210		14.505				>		
8473       83.120       0.235       2.35E-04       353703       361392       346334         8.974       88.035       0.249       2.49E-04       353554       360799       346594         9.477       92.969       0.261       2.61E-04       35254       363162       349509         9.976       97.865       0.278       2.78E-04       35221       358478       345811         10.478       102.789       0.295       2.95E-04       348438       354445       342631         10.975       107.665       0.308       3.08E-04       349551       355141       343977         11.975       117.75       0.338       3.38E-04       349558       355141       343977         11.975       17.747       0.338       3.38E-04       349526       355141       343977         12.472       122.350       0.351       3.51E-04       349526       355141       343081         12.97       127.236       0.368       3.68E-04       345749       350512       341114         13.472       132.160       0.383       3.83E-04       345561       339630         13.958       137.026       0.4       400E-04       342555       3466301	Spring 3 Mass (K, 0.494 0.993 1.984 2.481 2.983 3.485 3.988 4.489 4.489 4.499 5.481 5.98 6.477 6.972	g) Force 4.846 9.741 14.607 19.463 24.339 29.263 34.188 39.122 44.037 48.952 53.769 58.664 463.539 68.395	Dist. (mn 0.01 0.026 0.04 0.054 0.08 0.094 0.103 0.123 0.137 0.151 0.165 0.178 0.192	<ol> <li>Dist (m)</li> <li>1.00E-05</li> <li>2.60E-05</li> <li>4.00E-05</li> <li>5.40E-05</li> <li>6.90E-05</li> <li>9.40E-05</li> <li>9.40E-05</li> <li>1.09E-04</li> <li>1.23E-04</li> <li>1.51E-04</li> <li>1.51E-04</li> <li>1.78E-04</li> <li>1.92E-04</li> </ol>	Stiffness 484614 374557 365177 360427 352733 365790 365790 365790 355790 355790 355790 355790 355790 355793 3555953 4 355953	Error Lower Bour 959228 433873 417345 387205 380291 390176 37848 373196 370848 368278 366249 366249 366249 366278 366549 3667280	1d Upper Bound 323076 314236 324602 329882 328900 344273 345332 345332 345332 344533 344040 344732 344671 344719 347184		14.000				2		S-4,
8.974       88.035       0.249       2.49E-04       353554       360799       346594         9.477       92.969       0.261       2.61E-04       352204       353162       349509         9.976       97.865       0.278       2.78E-04       352031       358478       345811         10.478       102.789       0.295       2.95E-04       348438       354445       342631         10.975       107.665       0.308       3.08E-04       349551       355329       343977         11.476       112.580       0.322       3.22E-04       349526       355141       342800         11.975       117.475       0.338       3.38E-04       34554       342292         12.472       122.350       0.351       3.51E-04       345576       355141       343681         12.97       127.236       0.368       3.68E-04       345749       350512       341114         13.472       132.160       0.383       3.68E-04       345565       3496301       338336         13.968       137.026       0.4       4.00E-04       342555       346901       338336         14.464       141.892       0.411       4.11E-04       345236       349447 <td>Spring 3 Mass (tk, 0.494 0.993 1.489 1.984 2.481 2.983 3.485 3.988 4.489 4.99 5.481 5.98 6.477 6.972 7.469</td> <td>g) Force 4.846 9.741 14.607 19.463 24.339 29.263 34.188 39.122 44.037 48.952 53.769 58.664 63.539 68.395 73.271</td> <td>Dist. (mm 0.01 0.026 0.04 0.054 0.089 0.084 0.094 0.109 0.123 0.137 0.151 0.165 0.178 0.192 0.206</td> <td>1) Dist (m) 1.00E-05 2.60E-05 4.00E-05 5.40E-05 6.90E-05 9.40E-05 1.09E-04 1.23E-04 1.37E-04 1.51E-04 1.52E-04 1.5</td> <td>Stiffness 484614 374567 365177 360427 352733 545790 535790 535790 535791 4358220 535731 4356534 356534 355535 4355693 355535</td> <td>Error Lower Bour 969228 453873 417345 397205 380291 390176 384133 376176 376176 377848 368278 366549 366549 3665750 364532</td> <td>1d Upper Bound 323076 314236 329882 329882 328900 344273 345332 343178 344040 344732 344671 344671 345081 347210 347184 347255</td> <td></td> <td></td> <td></td> <td></td> <td></td> <td>2</td> <td></td> <td>S</td>	Spring 3 Mass (tk, 0.494 0.993 1.489 1.984 2.481 2.983 3.485 3.988 4.489 4.99 5.481 5.98 6.477 6.972 7.469	g) Force 4.846 9.741 14.607 19.463 24.339 29.263 34.188 39.122 44.037 48.952 53.769 58.664 63.539 68.395 73.271	Dist. (mm 0.01 0.026 0.04 0.054 0.089 0.084 0.094 0.109 0.123 0.137 0.151 0.165 0.178 0.192 0.206	1) Dist (m) 1.00E-05 2.60E-05 4.00E-05 5.40E-05 6.90E-05 9.40E-05 1.09E-04 1.23E-04 1.37E-04 1.51E-04 1.52E-04 1.5	Stiffness 484614 374567 365177 360427 352733 545790 535790 535790 535791 4358220 535731 4356534 356534 355535 4355693 355535	Error Lower Bour 969228 453873 417345 397205 380291 390176 384133 376176 376176 377848 368278 366549 366549 3665750 364532	1d Upper Bound 323076 314236 329882 329882 328900 344273 345332 343178 344040 344732 344671 344671 345081 347210 347184 347255						2		S
9.477       92.969       0.261       2.61E-04       356204       363162       349509         9.976       97.865       0.278       2.78E-04       352031       358478       345811         10.478       102.789       0.295       2.99E-04       348438       354445       345631         10.975       107.665       0.308       3.08E-04       349561       355329       343977         11.476       112.580       0.322       3.22E-04       349561       355329       343977         11.975       117.475       0.338       3.38E-04       347558       352777       342492         12.472       122.350       0.351       3.51E-04       348576       355614       343651         12.97       127.236       0.368       3.68E-04       345749       350512       341114         13.472       132.160       0.383       3.83E-04       345765       346951       340619         13.965       137.026       0.4       4.00E-04       342565       346901       338336         14.464       141.892       0.411       4.11E-04       345236       349487       341086	Spring 3 Mass (K: 0.494 0.993 1.984 2.481 2.983 3.485 3.988 4.489 4.99 5.481 5.98 6.477 6.972 7.469 7.971	g) Force 4.846 9.741 14.607 19.463 24.339 29.263 34.188 39.122 44.037 48.952 53.769 58.664 63.539 68.395 73.271 78.196	Dist. (mn 0.01 0.026 0.04 0.059 0.08 0.094 0.109 0.123 0.137 0.151 0.165 0.178 0.192 0.206 0.221	1) Dist (m) 1.00E-05 2.60E-05 4.00E-05 5.40E-05 8.00E-05 8.00E-05 8.00E-05 1.09E-04 1.23E-04 1.37E-04 1.51E-04 1.51E-04 1.52E-04 2.0EE-0 2.21E-04	Stiffness 484614 374557 365177 360427 353701 3585290 363701 358025 357312 4356034 355538 4356538 4356538 4355538 4355528 4355528	Error Lower Bour 969228 463873 417345 397205 380291 390176 384133 376176 373196 375196 375188 366549 366549 36675280 36675280 36675280 36675280 36675280 36675280 36675280	nd Upper Bound 323076 314236 324602 329882 328900 344273 345332 345332 345332 3447432 344671 344732 344671 345081 347184 347255 345998		14.505				2		
9.976       97.865       0.278       2.78E-04       352C31       358478       345811         10.478       102.789       0.295       2.95E-04       348438       354445       342631         10.975       107.665       0.308       3.08E-04       349561       355329       345377         11.975       117.475       0.338       3.38E-04       349561       355329       343977         11.975       117.475       0.338       3.38E-04       347558       352777       342492         12.472       122.350       0.351       3.51E-04       348576       353614       343681         12.97       127.236       0.388       3.68E-04       345749       350512       341114         13.472       132.160       0.383       3.83E-04       345065       349630       340619         13.968       137.026       0.4       4.00E-04       342555       346901       338336         14.464       141.892       0.411       4.11E-04       345236       349A87       341086	Spring 3 Mass (K) 0.494 0.993 1.984 2.481 2.983 3.485 3.988 4.489 4.99 5.481 5.98 6.477 6.972 7.469 7.971 8.473	g) Force 4.845 9.741 14.607 19.463 24.339 29.263 34.188 39.122 44.037 48.952 53.769 58.664 63.539 68.395 73.271 78.196 83.120	Dist. (mm 0.01 0.026 0.04 0.054 0.069 0.09 0.123 0.137 0.151 0.165 0.178 0.192 0.206 0.221	1) Dist (m) 1.00E-05 2.60E-05 4.00E-05 6.90E-05 8.00E-05 8.00E-05 9.40E-05 1.09E-04 1.23E-04 1.37E-04 1.37E-04 1.37E-04 1.37E-04 1.37E-04 1.37E-04 1.32E-04 2.26E-05 2.25E-05 2.2	Sitfness 484614 374567 365177 365177 352733 355790 353731 355520 357313 4355538 4355538 4355538 4355538 4355538 4355538 4355554 4355558	Error Lower Bour 969228 463873 417345 390176 384133 376176 375196 375196 375196 376184 366249 367280 365750 365750 365750 364532 3664532 3664532 3652016 3652016 36361392	nd Upper Bound 323076 314236 324602 329882 328900 344273 345332 345332 345332 344274 344732 344671 3447240 3447732 344671 347240 347240 347245 345988 346334		14.505				2		
10.478       102.789       0.295       2.95E-04       348438       354445       342631         10.975       107.665       0.308       3.08E-04       349551       355329       343977         11.476       112.580       0.322       3.22E-04       349551       355141       344280         11.975       117.475       0.338       3.38E-04       349526       355141       344280         11.975       117.475       0.338       3.38E-04       347558       352777       342492         12.472       122.350       0.351       3.51E-04       348576       353614       343681         12.97       127.236       0.368       3.68E-04       345749       350512       341114         13.472       132.160       0.383       3.83E-04       345565       349630       340619         13.958       137.026       0.4       4.00E-04       342555       3496301       338336         14.464       141.892       0.411       4.11E-04       345236       349487       341086	Spring 3 Mass (K% 0.494 0.993 1.984 2.481 2.983 3.485 3.988 4.489 4.489 4.489 5.481 5.98 6.477 6.972 7.469 7.971 8.473 8.473	g) Force 4.846 9.741 14.607 19.463 24.339 29.263 34.188 39.122 44.037 48.952 53.769 58.664 463.539 68.395 73.271 78.196 88.035	Dist. (mm 0.01 0.026 0.04 0.054 0.069 0.08 0.094 0.109 0.123 0.137 0.151 0.165 0.178 0.192 0.206 0.221 0.235 0.249	<ol> <li>Dist (m)</li> <li>1.00E-05</li> <li>2.60E-05</li> <li>4.00E-05</li> <li>5.40E-05</li> <li>6.90E-05</li> <li>9.40E-05</li> <li>9.40E-05</li> <li>1.09E-04</li> <li>1.23E-04</li> <li>1.51E-04</li> <li>1.5EE-04</li> <li>1.92E-04</li> <li>2.06E-04</li> <li>2.21E-04</li> <li>2.35E-04</li> <li>2.49E-04</li> </ol>	Stiffness 484614 374557 365177 365273 352733 352733 355320 535320 535325 4355532 4355532 4355532 4355532 4355532 4355532 4355532 4355532 4355532 4355532 43555532 43555532 43555553 43555553 43555555 43555555 4355555555	Error Lower Bour 959228 433873 417345 337205 380291 390176 374176 373196 370848 368278 368278 366549 366549 366549 366549 366550 4366522 366550 3661392 4360799	1d Upper Bound 323076 314236 324602 329882 328900 344273 345332 345332 345332 344040 344732 344671 347210 3447184 347255 345998		14.00				2		
10.975       107.665       0.308       3.08E-04       349561       355329       343977         11.476       112.580       0.322       3.22E-04       349626       355141       344280         11.975       117.475       0.338       3.38E-04       347558       352777       342492         12.472       122.350       0.351       3.51E-04       345576       355614       34681         12.97       127.236       0.368       3.68E-04       345749       350512       341114         13.472       132.160       0.383       3.88E-04       34556       349630       340619         13.968       137.026       0.4       4.00E-04       342565       346901       338336         14.464       141.892       0.411       4.11E-04       345236       349487       341086	Spring 3 Mass (K, 0.494 0.993 1.489 1.984 2.481 2.983 3.485 3.988 4.489 4.99 5.481 5.98 6.477 6.972 7.469 7.971 8.473 8.974	g) Force 4.845 9.741 14.607 19.463 24.339 29.263 34.188 39.122 44.037 48.952 53.769 58.664 63.539 68.395 73.271 78.196 38.3120 88.035 92.969	Dist. (mm 0.01 0.026 0.04 0.054 0.089 0.084 0.094 0.109 0.123 0.137 0.151 0.165 0.178 0.192 0.206 0.221 0.235 0.249 0.251	1) Dist (m) 1.00E-05 2.60E-05 4.00E-05 5.40E-05 6.90E-05 8.00E-05 9.40E-05 1.09E-04 1.23E-04 1.37E-04 1.51E-04 1.51E-04 1.52E-04 1.52E-04 2.66E-0 2.21E-0 2.48E-0 2.48E-0 2.61E-0	Stiffness 484614 374567 365177 365427 352733 545790 363701 358520 358520 358520 358526 355334 356533 4 356533 4 356533 4 356533 4 356533 4 356533 4 356533 4 356533 4 356534 4 353254 4 353254 4 355204 4 355204 4 355204 4 355204 4 355204 4 355204 4 355204 4 355204 356204 4 355205 4 356204 4 356204 4 356204 4 356204 4 356204 4 356204 4 356204 356204 4 356204 4 356204 4 356204 356204 356204 356204 356204 356204 356204 356204 356204 356204 356204 356204 35773 355720 355773 355773 355773 355773 355773 355773 355773 355753 3557573 3557573 3557573 3557573 3557573 3557573 3557573 3557573 355777 355773 355773 355775 357773 355775 355775 357773 355775 357777 355777 355775 357777 357777 355775 357777 355777 357777 357777 355777 357777 3577777 3577777777	Error Lower Bour 969228 463873 417345 337205 380291 390176 37848 376176 376176 376176 376176 376176 37648 368278 366649 366549 366549 366550 3665550 366550 365550 366550 3655500 3655500 3655500 365550000000000	1d Upper Bound 323076 314236 329882 329802 328900 344273 345332 343178 344040 344732 344671 345081 347210 347184 347255 345984 346594 346594 349509						2		
11.476       112.580       0.322       3.22E-04       349626       355141       344280         11.975       117.475       0.338       3.38E-04       347558       352777       342492         12.472       122.350       0.351       3.51E-04       348576       353614       340881         12.97       127.236       0.368       3.68E-04       345749       350512       341114         13.472       132.160       0.383       3.88E-04       345565       349630       340619         13.958       137.026       0.4       4.00E-04       342565       346901       338336         14.464       141.892       0.411       4.11E-04       345236       349487       341086	Spring 3 Mass (K; 0.494 0.993 1.489 1.984 2.481 2.983 3.485 3.988 4.489 4.99 5.481 5.98 6.477 7.459 7.971 8.473 8.974 9.477 9.477	g) Force 4.846 9.741 14.607 19.463 24.339 29.263 34.188 39.122 44.037 48.952 53.769 58.664 63.539 68.395 73.271 78.196 83.120 88.035 92.959 59.265 97.865	Dist. (mm 0.01 0.026 0.04 0.054 0.089 0.094 0.109 0.123 0.131 0.165 0.178 0.120 0.206 0.221 0.235 0.249 0.261	1) Dist (m) 1.00E-05 2.60E-05 4.00E-05 5.40E-05 8.00E-05 8.00E-05 9.40E-05 1.09E-04 1.37E-04 1.51E-04 1.51E-04 1.51E-04 1.52E-04 2.21E-04 2.35E-04 2.43E-04 2.45E-04 2.4	Siiffness 484614 374557 365427 365427 355730 355790 355730 355720 355822 355634 356224 355634 356224 35558584 355585858585858585858585858585858585858	Error Lower Bour 969228 463873 417345 397205 380291 390176 384133 376176 373196 376176 376176 376176 376176 376188 368452 366649 3665750 3665750 3665750 3665750 3665750 3661392 43661392 43661392	nd Upper Bound 323076 314236 324602 329882 328900 344273 345332 345332 344573 344040 344732 344571 345081 347210 347184 347250 345599 345509 345509						2		
11.975       117.475       0.338       3.38E-04       347558       352777       342492         12.472       122.350       0.351       3.51E-04       348576       353614       343681         12.97       127.236       0.368       3.68E-04       345749       350512       341114         13.472       132.160       0.383       3.83E-04       345565       349630       340619         13.958       137.026       0.4       4.00E-04       342565       346901       338336         14.464       141.892       0.411       4.11E-04       345236       349487       341086	Spring 3 Mass (K, 0,494 0,993 1,984 2,481 2,983 3,485 3,988 4,489 4,99 5,481 5,98 6,477 6,972 7,469 7,971 6,972 7,469 7,971 8,974 9,477 9,977	g) Force 4.845 9.741 14.607 19.463 24.339 29.263 34.188 39.122 44.037 48.952 58.664 63.539 68.395 73.271 78.196 88.035 92.865 88.035 92.865 88.035 92.865 88.035 92.865 88.035 92.865 88.035 92.865 88.035 92.865 88.035 92.865 88.035 92.865 88.035 92.865 88.035 92.865 88.035 92.865 88.035 92.865 88.035 92.865 88.035 92.865 88.035 92.865 88.035 92.865 88.035 92.865 88.035 92.865 92.865 92.865 92.865 92.865 92.865 92.865 88.035 92.865 92.865 92.865 92.865 88.035 92.855 92.855	Dist. (mm 0.01 0.026 0.04 0.054 0.069 0.09 0.123 0.137 0.151 0.165 0.178 0.192 0.206 0.221 0.235 0.249 0.261 0.278	Dist (m)           1.00E-05           2.60E-05           4.00E-05           5.40E-05           6.00E-05           9.40E-05           9.40E-05           1.09E-04           1.09E-04           1.03E-04           1.03E-04           1.37E-04           1.51E-04           1.65E-04           1.78E-04           2.06E-05           2.49E-0           2.49E-0           2.61E-0           2.78E-0           3.08E-0	Stiffness 484614 374557 365177 365273 352733 55790 358520 358520 358520 358520 358520 4355532 4355532 4355532 4355532 4355532 4355523 4355520 35820 4355520 35820 4355520 35820 435520 35820 4355520 35820 435520 35820 4355520 35820 435520 35820 4355520 35820 435520 35820 435520 35820 4355520 35820 435555 35820 435555 35820 435555 35820 435555 35820 435555 35820 435555 35820 35820 35820 35820 35820 35820 35820 35820 35820 35820 35820 35820 35820 35555 35820 35555 35820 35555 35820 35555 35820 35555 35820 35555 35555 35555 35555 35555 35555 355555 355555 355555 355555 355555 355555 355555 355555 355555 355555 355555 355555 355555 355555 3555555	Error Lower Bour 969228 463873 417345 390176 384133 376176 375196 375196 37618 366279 3662780 367580 36758	1d Upper Bound 323076 314236 324602 329882 328900 344273 345332 345332 345332 344671 345381 344640 344732 344671 347281 347285 345998 346594 346594 345811 345999 345811 345931		14.00				2		
12.472       122.350       0.351       3.51E-04       348576       343581         12.97       127.236       0.368       3.68E-04       345749       350512       341114         13.472       132.160       0.383       3.83E-04       345565       349630       340619         13.968       137.026       0.4       4.00E-04       342555       349630       340619         14.464       141.892       0.411       4.11E-04       345235       349487       341086	Spring 3 Mass (4%) 0.494 0.993 1.984 2.481 2.983 3.485 3.988 4.489 4.489 4.489 4.489 5.481 5.98 6.477 6.972 7.469 7.971 8.473 8.473 8.473 8.473 8.473 8.474 9.976 10.47 10.97	g) Force 4.845 9.741 14.607 19.463 24.339 29.263 34.188 39.122 44.037 48.952 53.769 58.664 463.539 68.395 73.271 78.196 88.035 92.959 58.854 88.035 92.959 58.102.785 102.785 102.785	Dist. (mm 0.01 0.026 0.04 0.054 0.099 0.123 0.137 0.151 0.165 0.178 0.192 0.206 0.221 0.226 0.278 0.278 0.278 0.278 0.278	Dist (m)           1.00E-05           2.60E-05           4.00E-05           5.40E-05           6.00E-05           9.40E-05           9.40E-05           1.09E-04           1.09E-04           1.03E-04           1.03E-04           1.37E-04           1.51E-04           1.65E-04           1.78E-04           2.06E-05           2.49E-0           2.49E-0           2.61E-0           2.78E-0           3.08E-0	Stiffness 484614 374557 365177 360427 350427 358705 363701 3585205 363701 3585205 363701 358520 358025 353701 43562538 43562538 4356254 358224 35825 35824 35825 35824 35825 35824 35825 35824 35825 35824 35825 35824 35825 35824 35825 35824 35825 35824 35825 35824 35825 35824 35825 35824 35825 35824 35825 35824 35825 35825 35824 35825 35824 35825 358	Error Lower Bour 969228 463873 417345 337205 380291 390176 37848 376176 375176 375176 375186 376176 375188 368278 366549 366549 366549 366549 366549 366549 366549 366550 3665750 3655750 3655750 3655750 3655750 3655750 3655750 3655750 3655750 3655750 3655750 3655750 3655750 3655750 3655750 365750 3757500 3757500 3757500 375750000000000	1d Upper Bound 323076 314236 324602 329802 328900 344273 345332 345332 344040 344732 344671 3446471 344671 344732 344671 344732 344671 347255 345981 347215 346594 346594 346594 346594 345599 345811 342651 34273 346534 346534 346534 346534 346551 342651 342651 342651 346554 346556 346554 346555 346555 3465556 3465556 3465						×		
12.97       127.236       0.368       3.69E-04       345749       350512       341114         13.472       132.160       0.383       3.83E-04       345566       349630       340619         13.968       137.026       0.4       4.00E-04       342555       3466901       33836         14.464       141.892       0.411       4.11E-04       345236       349487       341086	Spring 3 Mass (K, 0.494 0.993 1.489 1.984 2.481 2.983 3.485 3.988 4.489 4.399 5.481 5.98 6.477 7.6.972 7.469 7.971 8.473 8.974 9.477 9.976 10.47	g) Force 4.845 9.741 14.607 19.463 24.339 29.263 34.188 39.122 44.037 48.952 53.769 58.664 63.539 68.395 73.271 78.196 88.120 88.035 73.271 78.196 83.120 88.102 83.126	Dist. (mm 0.01 0.026 0.04 0.054 0.089 0.084 0.094 0.109 0.123 0.137 0.151 0.165 0.178 0.221 0.206 0.221 0.225 0.249 0.251 0.248 0.248 0.248 0.248 0.248 0.255 0.249 0.255 0.249 0.255 0.249 0.255 0.249 0.255 0.249 0.255 0.249 0.255 0.249 0.255 0.240 0.255 0.240 0.255 0.240 0.255 0.240 0.255 0.25	Dist (m)           1.00E-05           2.60E-05           4.00E-05           6.90E-05           8.00E-05           9.40E-05           6.90E-05           1.09E-04           1.23E-04           1.37E-04           1.57E-04           1.58E-04           2.26E-05           2.49E-04           2.35E-04           2.78E-04           2.78E-04           3.28E-04           3.38E-04           3.38E-04	Siiffness 484614 374557 365177 360427 355730 355790 355730 355202 355202 355320 35558200 35558	Error Lower Bour 969228 463873 417345 397205 380291 390176 384133 376176 376176 375196 376176 375196 376176 375196 366549 366549 366549 366550 366520 5365750 3661392 43661392 43661392 4365750 5355141 3553478 3555141	nd Upper Bound 323076 314236 324602 329882 328900 344273 345332 345332 345332 344040 344732 344671 345081 347210 347184 347255 345998 346334 346594 346599 345591 345591 345591 345591 345597 3424202 342420 342420		14.000				2		
13.968 137.026 0.4 4.00E-04 342565 346901 338336 14.454 141.892 0.411 4.11E-04 345236 349487 341086	Spring 3 Mass (K, 0,494 0,993 1,984 2,481 2,983 3,485 3,988 4,489 4,89 5,481 5,98 6,477 6,972 7,469 7,971 8,473 8,974 9,477 9,977 10,47 10,97 11,47	g) Force 4.846 9.741 14.607 19.463 24.339 29.263 34.188 39.122 44.037 48.952 53.769 58.664 63.539 68.395 73.271 78.196 88.035 97.865 8 102.786 5 107.666 6 112.588 5 117.47 2 122.35	Dist. (mm 0.01 0.026 0.04 0.054 0.099 0.123 0.137 0.151 0.165 0.178 0.192 0.206 0.225 0.249 0.225 0.229 0.225 0.308 0.322 5 0.308	Dist (m)           1.00E-05           2.60E-05           4.00E-05           5.40E-05           6.90E-05           8.00E-05           9.40E-05           1.03E-04           1.03E-04           1.03E-04           1.37E-04           1.51E-04           1.52E-04           1.62E-04           2.06E-0           2.35E-0           2.48E-0           2.78E-0           3.08E-0           3.28E-0           3.38E-0           3.38E-0           3.38E-0           3.38E-0           3.51E-0	Stiffness 484614 374557 365177 365177 352733 355790 53527313 3555320 5357313 355532 43555532 435555532 435555532 43555532 435555532 435555532 435555532 435555532 4355555532 435555555555	Error Lower Bour 969228 463873 417345 380291 390176 384133 376176 376176 376176 376176 366278 366278 366278 366278 366549 3665492 365750 355144 355329 355144 355329 355144 355329 355144 355329 355144 355329	14 Upper Bound 323076 314236 324602 329882 328900 344273 345332 345332 345332 345332 344732 344671 344732 344671 3447210 347184 347253 345988 345334 345598 345594 345598 345594 345595 345595 345595 345597 345597 345595 345595 345597 345597 345595 345595 345597 345597 345595 345597 345597 345597 345595 345597 345595 345597 345595 345597 345595 345597 345595 345597 345595 345595 345597 345595 345595 345597 345597 345595 345597 34559						2		
14.464 141.892 0.411 4.11E-04 345236 349487 341086	Spring 3 Mass (K, 0.494 0.993 1.984 2.481 2.983 3.485 3.988 4.489 4.99 5.481 5.98 6.477 6.972 7.469 7.971 8.473 8.974 9.477 9.976 10.47 10.97 11.47 11.97	g) Force 9, Force 9, Force 9, Force 9, 741 14,607 19,463 24,339 29,263 34,188 39,122 44,037 48,952 53,769 58,664 463,539 68,395 73,271 78,196 88,035 92,969 58,102,781 5107,663 6112,585 5107,663 6112,585 5107,663 6112,585 5117,47,21 22,357 127,23 7127,23	Dist. (mm 0.01 0.026 0.04 0.054 0.099 0.123 0.137 0.151 0.165 0.178 0.192 0.206 0.221 0.226 0.278 0.225 0.249 0.261 0.278 0.278 0.278 0.278 0.278 0.278 0.278 0.278 0.278 0.275 0.385 0.275 0.385 0.225 0.338 0.035 0.35	Dist (m)           1.00E-05           2.60E-05           4.00E-05           5.40E-05           6.00E-05           9.40E-05           9.40E-05           1.03E-04           1.03E-04           1.03E-04           1.23E-04           1.51E-04           1.51E-04           1.23E-04           2.06E-0-           2.49E-0-           2.49E-0-           3.08E-0-           3.08E-0-           3.51E-0-           3.51E-0-           3.68E-0-	Stiffness 484614 374557 365177 365177 352733 525730 535320 535320 535320 535320 4355532 4355532 4355532 4355532 4355532 4355532 4355532 4355532 4355532 4355520 35820 43535520 35820 43535520 35820 43535520 35820 435452 35820 435452 35820 435452 35820 435452 35820 435452 35820 435553 435520 35820 435553 435520 35820 435553 435553 435520 35820 435553 435520 35820 435553 35820 35553 35820 35820 35553 35820 35553 35820 35553 35820 35553 35820 35553 35820 355532 3555352 355532 355553 3555553 355553 355553 355553 355553 355553 355553 355553 355553 355553 3555553 3555553 3555553 3555553 3555553 3555553 3555555	Error Lower Bour 969228 433873 417345 337205 380291 390176 373196 376176 373196 370848 368278 366549 366549 366549 366549 366549 366549 366549 366549 3661392 4366750 366549 3661392 43661392 43661392 43661392 43661392 4365750 365750 4365750 365132 5355141 8355225 5355141 8355274 5355141	10 Upper Bound 323076 314236 324602 329882 3288900 344273 345332 345332 345332 344040 344732 344671 344280 347184 347255 345998 344584 346594 34581 34581 34280 3428377 344280 342877 344280 343977 344280 343977 344280 34534 34534 34534 34534 345595 345595 345555 345555 345555 3455555 3455555 3455555 345555555555		14.000				2		
	Spring 3 Mass (K, 0.494 0.993 1.489 1.984 2.481 2.983 3.485 3.988 4.489 4.99 5.481 5.98 6.477 7.6592 7.469 7.971 8.473 8.473 8.473 8.974 9.477 9.976 10.47 11.97 11.47 11.97 12.49	g) Force 4.845 9.741 14.607 19.463 24.339 29.263 34.188 39.122 44.037 48.952 53.769 58.654 63.539 68.395 73.271 78.196 88.120 88.125 92.969 5.854 4.835 73.271 78.196 8.8120 8.8120 8.8125 107.66 5.107.66 5.107.66 5.117.67 2.122.35 7.127.23 1.22.35 7.127.23 1.22.35 1.27.23 1.22.35	Dist. (mm 0.01 0.026 0.04 0.054 0.069 0.08 0.094 0.109 0.123 0.137 0.151 0.165 0.178 0.192 0.206 0.221 0.225 0.249 0.251 0.225 0.338 0.0351 5.0356 0.0351 5.0355	Dist (m)           1.00E-05           2.60E-05           4.00E-05           5.40E-05           6.00E-05           8.00E-05           9.40E-05           1.03E-04           1.03E-04           1.03E-04           1.03E-04           1.03E-04           1.37E-04           1.51E-04           1.52E-04           1.82E-04           2.06E-0           2.21E-0           2.35E-0           2.48E-0           2.61E-0           3.08E-0           3.38E-0           3.68E-0	Stiffness 484614 374557 365177 365427 352733 535700 363701 358520 358520 358520 358520 358520 358524 355532 43555532 435555532 435555532 435555532 435555532 435555532 435555532 4355555532 435555555555	Error Lower Bour 969228 463873 417345 387205 380291 390176 375176 375176 375176 375184 368278 366549 3665490 3665750 3665750 3665750 3665750 3665750 3665750 3665750 3665750 3665750 3665750 3665750 3665750 3665750 3665750 3665750 355750 35570 355750 35570 3	1d Upper Bound 323076 314236 324602 328802 328900 344273 345332 345332 3443732 3446471 3446471 344671 344732 344671 344732 344671 344732 344674 347255 345981 347215 345949 345949 345949 345941 345949 345947 344280 7344280 7344280 7344280 7344280 7344280 734292 434584 134586 1345866 134586 134586 134586 134586 134586 134586 134586 13458						×		
14.965 145.807 0.422 4.22E-04 347883 352054 343809	Spring 3 Mass (K, 0,494 0,993 1,489 1,984 2,481 2,983 3,485 3,988 4,489 4,89 5,481 5,98 6,477 6,972 7,469 7,971 8,473 8,974 9,477 9,977 10,47 10,97 11,47 11,97 12,47 13,47 13,47	g) Force 4,845 9,741 14,607 19,463 24,339 29,263 34,188 39,122 44,037 48,952 53,769 58,664 63,539 68,395 73,271 78,196 8,8120 8,8120 8,8120 8,8120 8,127,85 5,117,47 2,122,35 7,127,23 2,132,16 8,137,02 1,32,16 8,137,02 1,32,16 8,137,02 1,32,16 1,37,02 1,32,16 1,37,02 1,32,16 1,37,02 1,32,16 1,37,02 1,32,16 1,37,02 1,32,16 1,37,02 1,32,16 1,37,02 1,32,16	Dist. (mm 0.01 0.026 0.04 0.054 0.069 0.094 0.123 0.137 0.151 0.165 0.178 0.192 0.206 0.221 0.235 0.249 0.255 0.249 0.255 0.308 0.0351 5 0.368 0 0.363 5 0.363	Dist (m)           1.00E-05           2.60E-05           4.00E-05           6.90E-05           8.00E-05           9.40E-05           6.90E-05           1.03E-04           1.03E-04           1.03E-04           1.37E-04           1.51E-04           1.73E-04           2.23E-04           2.75E-04           2.75E-04           3.05E-04           3.05E-04      3.05E-04           3.05E-04 <td>Siiffness           484614           374557           365177           365177           365177           352733           355790           353701           3557313           355534           35554           355584           355584           355584           355544           355534           355544           355534           355544           355534           355544           355544           34855           348431           348554           348574           348574           348574           348574           348574           348574           34754           34754           34256</td> <td>Error Lower Bour 965928 463873 417345 390176 384133 376176 375196 375196 375196 375196 375196 375196 375196 366549 366549 366549 366550 3665750 3675750 35777760 3575700 3575700 3575700 3575700 3575700 3575700 357570000</td> <td>nd Upper Bound 323076 314236 324602 329882 328900 344273 345332 345332 345332 345332 34534040 344732 344574 345581 347210 347184 347255 345598 346534 346594 346594 346594 346594 342631 342633 342631 342633 3426</td> <td></td> <td></td> <td></td> <td></td> <td></td> <td>2</td> <td></td> <td></td>	Siiffness           484614           374557           365177           365177           365177           352733           355790           353701           3557313           355534           35554           355584           355584           355584           355544           355534           355544           355534           355544           355534           355544           355544           34855           348431           348554           348574           348574           348574           348574           348574           348574           34754           34754           34256	Error Lower Bour 965928 463873 417345 390176 384133 376176 375196 375196 375196 375196 375196 375196 375196 366549 366549 366549 366550 3665750 3675750 35777760 3575700 3575700 3575700 3575700 3575700 3575700 357570000	nd Upper Bound 323076 314236 324602 329882 328900 344273 345332 345332 345332 345332 34534040 344732 344574 345581 347210 347184 347255 345598 346534 346594 346594 346594 346594 342631 342633 342631 342633 3426						2		
	Spring 3 Mass (k%) 0.494 0.993 1.984 2.481 2.983 3.485 3.988 4.489 5.481 5.98 6.477 6.972 7.469 7.971 8.473 8.974 9.976 10.47 10.97 11.47 11.97 12.47 13.96 13.47 13.96 13.47	g) Force 4.846 9.741 14.607 19.463 24.339 29.263 34.188 39.122 44.037 48.952 53.769 58.664 63.359 68.395 73.271 78.196 88.035 79.265 8 102.781 5 107.661 6 112.588 5 107.662 5 107.662 5 107.662 5 107.662 6 112.588 5 117.47 2 122.35 7 127.23 127.25 127.25 127.25 127.25 127.25 127.25 127.25 127.25 127.25 127.25 127.25 127.25 127.25 127.25 127.25 127.25 127.25	Dist. (mm 0.01 0.026 0.04 0.054 0.099 0.123 0.137 0.151 0.165 0.788 0.192 0.206 0.221 0.255 0.249 0.225 0.278 0.025 0.308 0.032 0.351 5.0.338 0.032 0.0351 5.0.385 0.0382 0.0351 5.0.385 0.0382 0.0442 0.0382 0.0382 0.0382 0.0442 0.0382 0.0442 0.0382 0.0442 0.0382 0.0442 0.0382 0.0442 0.0442 0.0442 0.0442 0.0444 0.0442 0.04444 0.04444 0.04444 0.04444 0.04444 0.04444 0.04444 0.04444 0.04444 0.04444 0.04444 0.04444 0.04444 0.04444 0.04444 0.04444 0.04444 0.0444444 0.044444444 0.04444444444	Dist (m)           1.00E-05           2.60E-05           4.00E-05           5.40E-05           6.90E-05           8.00E-05           9.40E-05           1.07E-04           1.07E-04           1.07E-04           1.07E-04           1.37E-04           1.51E-04           1.52E-04           1.78E-04           2.06E-0-           2.49E-0           2.49E-0           2.5EE-0           3.08E-0           3.51E-0           3.51E-0           3.51E-0           3.68E-0           3.51E-0           3.68E-0           3.68E-0           3.51E-0           3.68E-0           3.58E-0           3.58E-0           3.58E-0           3.58E-0           3.68E-0	Stiffness           484614           374557           365177           365177           365177           365177           352733           355532           355532           355532           355532           355532           355532           355532           355224           355224           355224           355224           355224           355224           355224           355224           355224           355224           355224           355224           355224           355224           355224           355223           345433           34956           34956           34574           34574           34523           34523           34523           34557           34557           34556           34557           34557           34557           34557           34557           34557	Error Lower Bour 969228 433873 417345 380291 390176 384133 37176 373196 376176 373196 376176 376176 36649 366549 366549 366549 366549 366549 366549 366549 366549 366549 366549 366549 366549 366549 365750 355126 355126 355126 355126 355126 355126 355126 355126 355126 355126 355126 355126 355126 355126 355126 355126 355126 34690555 34690555 34690555555555555555555555555555555555555	1d Upper Bound 323076 314236 324602 329882 328900 344273 345332 345332 345332 344273 345332 344712 344732 344671 3447210 344732 344671 3447210 344784 3447255 345998 345594 34659566 36666666666666666666666666666666666		14.000				2		

# **APPENDIX B:**

## Raw Data from Chi-squared Tests

	on 1										
Node	Mass	Expt Freq	k= 378000			K=350000		1	K=340000		
	(kg)	(Hz)	Freq (Hz)	ΔF	Chi squ'd	Freq (Hz)	ΔF		Freq (Hz)	ΔF	Chi squ'd
1	6.195	6.800	8.090	1.290	0.245	7.788	0.988	0.143	7.788	0.988	0.14
2	4.867	21.800	24.450	2.650	0.322	23.531	1.731	0.137	23.192	1.392	0.08
3	3.772	35.300	36.570	1.270	0.046	35.185	0.115	0.000	34.679	0.621	0.01
4	4.641	50.600	51.080	0.480	0.005	49.152	1.449	0.041	43.444	7.156	1.01
5	6.503	62.000	62.170	0.170	0.000	59.825	2.175	0.076	58.964	3.036	0.14
6	5.361	73.700	72.790	0.910	0.011	70.044	3.656	0.181	69.036	4.664	0.29
7	3.628	80.500	80.390	0.110	0.000	77.359	3.141	0.123	76.246	4.254	0.22
8	3.291	91.200	89.910	1.290	0.018	86.517	4.683	0.240	85.272	5.928	0.38
9	2.078	103.500	100.630	2.870	0.080	96.834	6.666	0.429	95.441	8.060	0.62
			5	Sum ≈	0.727		Sum =	1.373		Sum =	2.93
(P.O. P. D. P. Starter			Carlin Consta			包容制的建设	<b>法公</b> 的资源	EXPECTAL F		ENCOUTING	
Configuratio											
Node	Mass		k= 378000			K=350000			K=340000		
	(kg)	(Hz)	Freq (Hz)	ΔF		Freq (Hz)	ΔF		Freq (Hz)	ΔF	Chi squ'a
1	5.928	7.000	7.958	0.958		7.672	0.672	0.065	7.562	0.562	0.04
2	4.600	21.458	24.085	2.626	0.321	23.166	1.708	0.136	22.833	1.375	0.08
3	3.505	35.757	36.083	0.326		34.726	1.031	0.030	34.227	1.530	0.06
4	4.374	50.350	50.380	0.030			1.872	0.070	47.781	2.569	0.1:
5	6.236	62.167	61.352	0.815		59.035	3.132	0.158		3.982	0.2
6	5.094	73.450		1.636			4.343	0.257		5.337	0.3
7	3.361	81.000		1.789			4.786	0.283	75.117	5.883	0.4
8	3.024	92.563	87.882	4.681			7.378	0.588	83.958	8.604	0.8
9	1.944	99.500	98.792	0.708			4.435			5.802	0.3
				Sum ≃	0.784	And the Party of t	Sum =	1.783		Sum =	2.5
	ALCONOMINE IN MICH.			法律法律		C. Walter	的主义的问题	我的家的影响			
Configuration	on 3										
	Mana	Cust Case	1 070000								
Node	Mass		k= 378000		011 U	K=350000			K=340000		
	(kg)	(Hz)	Freq (Hz)	ΔF	The Party New York, Name	Freq (Hz)	۵F		Freq (Hz)	ΔF	Chi squ'
1	(kg) 4.735	(Hz) 8.194	Freq (Hz) 11.254	3.060	1.142	Freq (Hz) 10.828	2.633	0.846	Freq (Hz) 10.672	ΔF 2.477	0.7
1 2	(kg) 4.735 1.196	(Hz) 8.194 30.708	Freq (Hz) 11.254 35.553	3.060 4.844	1.142 0.764	Freq (Hz) 10.828 34.225	2.633 3.517	0.846	Freq (Hz) 10.672 33.733	ΔF 2.477 3.024	0.7
1 2 3	(kg) 4.735 1.196 0.220	(Hz) 8.194 30.708 46.286	Freq (Hz) 11.254 35.553 50.354	3.060 4.844 4.069	1.142 0.764 0.358	Freq (Hz) 10.828 34.225 48.462	2.633 3.517 2.177	0.846 0.403 0.102	Freq (Hz) 10.672 33.733 47.765	ΔF 2.477 3.024 1.479	0.7- 0.2 0.0-
1 2 3 4	(kg) 4.735 1.196 0.220 0.805	(Hz) 8.194 30.708 46.286 65.750	Freq (Hz) 11.254 35.553 50.354 71.158	3.060 4.844 4.069 5.408	1.142 0.764 0.358 0.445	Freq (Hz) 10.828 34.225 48.462 68.473	2.633 3.517 2.177 2.723	0.846 0.403 0.102 0.113	Freq (Hz) 10.672 33.733 47.765 67.487	ΔF 2.477 3.024 1.479 1.737	0.7 0.2 0.0 0.0
1 2 3 4 5	(kg) 4.735 1.196 0.220 0.805 0.464	(Hz) 8.194 30.708 46.286 65.750 92.583	Freq (Hz) 11.254 35.553 50.354 71.158 100.671	3.060 4.844 4.069 5.408 8.088	1.142 0.764 0.358 0.445 0.707	Freq (Hz) 10.828 34.225 48.462 68.473 96.866	2.633 3.517 2.177 2.723 4.282	0.846 0.403 0.102 0.113 0.198	Freq (Hz) 10.672 33.733 47.765 67.487 95.472	ΔF 2.477 3.024 1.479 1.737 2.888	0.7 0.2 0.0 0.0 0.0
1 2 3 4 5 6	(kg) 4.735 1.196 0.220 0.805 0.464 0.874	(Hz) 8.194 30.708 46.286 65.750 92.583 .119.922	Freq (Hz) 11.254 35.553 50.354 71.158 100.671 123.281	3.060 4.844 4.069 5.408 8.088 3.359	1.142 0.764 0.358 0.445 0.707 0.094	Freq (Hz) 10.828 34.225 48.462 68.473 96.866 118.269	2.633 3.517 2.177 2.723 4.282 1.653	0.846 0.403 0.102 0.113 0.198 0.023	Freq (Hz) 10.672 33.733 47.765 67.487 95.472 116.922	ΔF 2.477 3.024 1.479 1.737 2.888 3.000	0.7 0.2 0.0 0.0 0.0 0.0 0.0
1 2 3 4 5 6 7	(kg) 4.735 1.196 0.220 0.805 0.464 0.874 2.265	(Hz) 8.194 30.708 46.286 65.750 92.583 .119.922 146.861	Freq (Hz) 11.254 35.553 50.354 71.158 100.671 123.281 142.353	3.060 4.844 4.069 5.408 8.088 3.359 4.509	1.142 0.764 0.358 0.445 0.707 0.094 0.138	Freq (Hz) 10.828 34.225 48.462 68.473 96.866 118.269 136.960	2.633 3.517 2.177 2.723 4.282 1.653 9.901	0.846 0.403 0.102 0.113 0.198 0.023 0.668	Freq (Hz) 10.672 33.733 47.765 67.487 95.472 116.922 134.989	ΔF 2.477 3.024 1.479 1.737 2.888 3.000 11.872	0.7 0.2 0.0 0.0 0.0 0.0 0.0 0.0
1 2 3 4 5 6 7 8	(kg) 4.735 1.196 0.220 0.805 0.464 0.874 2.265 0.759	(Hz) 8.194 30.708 46.286 65.750 92.583 .119.922 146.861 180.438	Freq (Hz) 11.254 35.553 50.354 71.158 100.671 123.281 142.353 174.331	3.060 4.844 4.069 5.408 8.088 3.359 4.509 6.107	1.142 0.764 0.358 0.445 0.707 0.094 0.138 0.207	Freq (Hz) 10.828 34.225 48.462 68.473 96.866 118.269 136.960 167.750	2.633 3.517 2.177 2.723 4.282 1.653 9.901 12.687	0.846 0.403 0.102 0.113 0.198 0.023 0.668 0.892	Freq (Hz) 10.672 33.733 47.765 67.487 95.472 116.922 134.989 165.336	ΔF 2.477 3.024 1.479 1.737 2.888 3.000 11.872 15.101	0.7 0.2 0.0 0.0 0.0 0.0 0.0 0.0 1.2
1 2 3 4 5 6 7	(kg) 4.735 1.196 0.220 0.805 0.464 0.874 2.265	(Hz) 8.194 30.708 46.286 65.750 92.583 .119.922 146.861 180.438	Freq (Hz) 11.254 35.553 50.354 71.158 100.671 123.281 142.353 174.331 201.348	3.060 4.844 4.069 5.408 8.088 3.359 4.509 6.107 9.443	1.142 0.764 0.358 0.445 0.707 0.094 0.138 0.207 0.423	Freq (Hz) 10.828 34.225 48.462 68.473 96.866 118.269 136.960 167.750 193.728	2.633 3.517 2.177 2.723 4.282 1.653 9.901 12.687 17.063	0.846 0.403 0.102 0.113 0.198 0.023 0.668 0.892 1.381	Freq (Hz) 10.672 33.733 47.765 67.487 95.472 116.922 134.989 165.336 190.941	ΔF 2.477 3.024 1.479 1.737 2.888 3.000 11.872 15.101 19.851	0.7 0.2 0.0 0.0 0.0 0.0 0.0 1.2 1.8
1 2 3 4 5 6 7 8 9	(kg) 4.735 1.196 0.220 0.805 0.464 0.874 2.265 0.759 3.852	(Hz) 8.194 30.708 46.286 65.750 92.583 .119.922 146.861 180.438 210.792	Freq (Hz) 11.254 35.553 50.354 71.158 100.671 123.281 142.353 174.331 201.348	3.060 4.844 4.069 5.408 8.088 3.359 4.509 6.107 9.443 Sum =	1.142 0.764 0.358 0.445 0.707 0.094 0.138 0.207 0.423 4.278	Freq (Hz) 10.828 34.225 48.462 68.473 96.866 118.269 136.960 167.750 193.728	2.633 3.517 2.177 2.723 4.282 1.653 9.901 12.687 17.063 Sum =	0.846 0.403 0.102 0.113 0.198 0.023 0.668 0.892 1.381 4.626	Freq (Hz) 10.672 33.733 47.765 67.487 95.472 116.922 134.989 165.336 190.941	ΔF 2.477 3.024 1.479 1.737 2.888 3.000 11.872 15.101 19.851 Sum =	0.7 0.2 0.0 0.0 0.0 0.0 0.0 0.0 1.2 1.8 5.3
1 2 3 4 5 6 7 8 9	(kg) 4.735 1.196 0.220 0.805 0.464 0.874 2.265 0.759 3.852	(Hz) 8.194 30.708 46.286 65.750 92.583 .119.922 146.861 180.438 210.792	Freq (Hz) 11.254 35.553 50.354 71.158 100.671 123.281 142.353 174.331 201.348	3.060 4.844 4.069 5.408 8.088 3.359 4.509 6.107 9.443 Sum =	1.142 0.764 0.358 0.445 0.707 0.094 0.138 0.207 0.423 4.278	Freq (Hz) 10.828 34.225 48.462 68.473 96.866 118.269 136.960 167.750 193.728	2.633 3.517 2.177 2.723 4.282 1.653 9.901 12.687 17.063 Sum =	0.846 0.403 0.102 0.113 0.198 0.023 0.668 0.892 1.381 4.626	Freq (Hz) 10.672 33.733 47.765 67.487 95.472 116.922 134.989 165.336 190.941	ΔF 2.477 3.024 1.479 1.737 2.888 3.000 11.872 15.101 19.851 Sum =	0.7 0.2 0.0 0.0 0.0 0.0 0.0 0.0 1.2 1.8 5.3
1 2 3 4 5 6 7 8 9	(kg) 4.735 1.196 0.220 0.805 0.464 0.874 2.265 0.759 3.852	(Hz) 8.194 30.708 46.286 65.750 92.583 .119.922 146.861 180.438 210.792	Freq (Hz) 11.254 35.553 50.354 71.158 100.671 123.281 142.353 174.331 201.348	3.060 4.844 4.069 5.408 8.088 3.359 4.509 6.107 9.443 Sum =	1.142 0.764 0.358 0.445 0.707 0.094 0.138 0.207 0.423 4.278	Freq (Hz) 10.828 34.225 48.462 68.473 96.866 118.269 136.960 167.750 193.728	2.633 3.517 2.177 2.723 4.282 1.653 9.901 12.687 17.063 Sum =	0.846 0.403 0.102 0.113 0.198 0.023 0.668 0.892 1.381 4.626	Freq (Hz) 10.672 33.733 47.765 67.487 95.472 116.922 134.989 165.336 190.941	ΔF 2.477 3.024 1.479 1.737 2.888 3.000 11.872 15.101 19.851 Sum =	0.7 0.2 0.0 0.0 0.0 0.0 0.0 0.0 1.2 1.8 5.3
1 2 3 4 5 6 7 8 9	(kg) 4.735 1.196 0.220 0.805 0.464 0.874 2.265 0.759 3.852 0.759 3.852	(Hz) 8.194 30.708 46.286 65.750 92.583 .119.922 146.861 180.438 210.792 Expt Freq	Freq (Hz) 11.254 35.553 50.354 71.158 100.671 123.281 142.353 174.331 201.348 k= 378000	3.060 4.844 4.069 5.408 8.088 3.359 4.509 6.107 9.443 Sum =	1.142 0.764 0.358 0.445 0.707 0.094 0.138 0.207 0.423 4.278	Freq (Hz) 10.828 34.225 48.462 68.473 96.866 118.269 136.960 167.750 193.728 K=350000	2.633 3.517 2.177 2.723 4.282 1.653 9.901 12.687 17.063 Sum =	0.846 0.403 0.102 0.113 0.198 0.023 0.668 0.892 1.381 4.626	Freq (Hz) 10.672 33.733 47.765 67.487 95.472 116.922 134.989 165.336 190.941 K=340000	ΔF 2.477 3.024 1.479 1.737 2.888 3.000 11.872 15.101 19.851 Sum =	0.7 0.2 0.0 0.0 0.0 0.0 0.0 1.2 1.8 5.3
1 2 3 4 5 6 7 8 9 9 Configurati Node	(kg) 4.735 1.196 0.220 0.805 0.464 0.874 2.265 0.759 3.852 ion 4 Mass (kg)	(Hz) 8.194 30.708 46.286 65.750 92.583 .119.922 146.861 180.438 210.792 Expt Freq (Hz)	Freq (Hz) 11.254 35.553 50.354 71.158 100.671 123.281 142.353 174.331 201.348 k= 378000 Freq (Hz)	3.060 4.844 4.069 5.408 8.088 3.359 4.509 6.107 9.443 Sum =	1.142 0.764 0.358 0.445 0.707 0.094 0.138 0.207 0.423 4.278 4.278 Chi squ'd	Freq (Hz) 10.828 34.225 48.462 68.473 96.866 118.269 136.960 167.750 193.728 K=350000 Freq (Hz)	2.633 3.517 2.177 2.723 4.282 1.653 9.901 12.687 17.063 Sum =	0.846 0.403 0.102 0.113 0.198 0.023 0.668 0.892 1.381 4.626	Freq (Hz) 10.672 33.733 47.765 67.487 95.472 116.922 134.989 165.336 190.941 K=340000 Freq (Hz)	ΔF 2.477 3.024 1.479 1.737 2.888 3.000 11.872 15.101 19.851 Sum =	0.7 0.2 0.0 0.0 0.0 0.0 0.9 1.2 1.8 5.3 5.3 Chi squ
1 2 3 4 5 6 7 8 9 9 Configurati Node	(kg) 4.735 1.196 0.220 0.805 0.464 0.874 2.265 0.759 3.852 0.759 3.852 0.759 3.852 0.759 3.852 0.759 3.852 0.759 3.852	(Hz) 8.194 30.708 46.286 65.750 92.583 .119.922 146.861 180.438 210.792 Expt Freq (Hz) 9.569	Freq (Hz) 11.254 35.553 50.354 71.158 100.671 123.281 142.353 174.331 201.348 k= 378000 Freq (Hz) 11.141	3.060 4.844 4.069 5.408 8.088 3.359 4.509 6.107 9.443 Sum = ΔF 1.571	1.142 0.764 0.358 0.445 0.707 0.094 0.138 0.207 0.423 4.278 4.278 Chi squ'd	Freq (Hz) 10.828 34.225 48.462 68.473 96.866 118.269 136.960 167.750 193.728 K=350000 Freq (Hz) 10.738	2.633 3.517 2.177 2.723 4.282 1.653 9.901 12.687 17.063 Sum = 000000000000000000000000000000000000	0.846 0.403 0.102 0.113 0.198 0.023 0.668 0.892 1.381 4.626 Chi squ'd	Freq (Hz) 10.672 33.733 47.765 67.487 95.472 116.922 134.989 165.336 190.941 K=340000 Freq (Hz) 10.583	ΔF 2.477 3.024 1.479 1.737 2.888 3.000 11.872 15.101 19.851 Sum = ΔF 1.014	0.7 0.2 0.0 0.0 0.0 0.0 0.9 1.2 1.8 5.3 5.3 5.3 5.3 5.3 5.3 5.3 5.3 5.3 5.3
1 2 3 4 5 6 7 8 9 9 Configurati Node	(kg) 4.735 1.196 0.220 0.805 0.464 0.874 2.265 0.759 3.852 0.759 3.852 0.759 3.852 0.759 3.852 0.759 3.852 0.759 3.852	(Hz) 8.194 30.708 46.286 65.750 92.583 .119.922 146.861 180.438 210.792 Expt Freq (Hz) 9.569 28.944	Freq (Hz) 11.254 35.553 50.354 71.158 100.671 123.281 142.353 174.331 201.348 k= 378000 Freq (Hz) 11.141 32.187	3.060 4.844 4.069 5.408 8.088 3.359 4.509 6.107 9.443 Sum = ΔF 1.571 3.243	1.142 0.764 0.358 0.445 0.707 0.094 0.138 0.207 0.423 4.278 Chi squ'd 0.258 0.363	Freq (Hz) 10.828 34.225 48.462 68.473 96.866 118.269 136.960 167.750 193.728 K=350000 Freq (Hz) 10.738 30.964	2.633 3.517 2.177 2.723 4.282 1.653 9.901 12.687 17.063 Sum = ΔF 1.168 2.020	0.846 0.403 0.102 0.113 0.198 0.023 0.668 0.892 1.381 4.626 Chi squ'd 0.143 0.141	Freq (Hz) 10.672 33.733 47.765 67.487 95.472 116.922 134.989 165.336 190.941 K=340000 Freq (Hz) 10.583 30.524	ΔF 2.477 3.024 1.479 1.737 2.888 3.000 11.872 15.101 19.851 Sum = ΔF 1.014 1.579	0.7 0.2 0.0 0.0 0.0 0.0 0.9 1.2 1.8 5.3 5.3 5.3 5.3 5.3 5.3 5.3 5.3 5.3 5.3
1 2 3 4 5 6 7 8 9 9 Configurati Node	(kg) 4.735 1.196 0.220 0.805 0.464 0.874 2.265 0.759 3.852 0.759 3.852 0.759 3.852 0.759 3.852 0.759 3.852 0.759 3.852 0.759 3.852 0.759 3.852 0.759 3.852 0.759 3.852 0.759 3.852 0.759 3.852 0.759 3.852 0.759 3.852 0.759 3.852 0.759 3.855 0.759 3.855 0.759 3.855 0.759 3.855 0.759 3.855 0.759 3.855 0.759 3.855 0.759 3.855 0.759 3.855 0.759 3.855 0.759 3.855 0.759 3.855 0.759 3.855 0.759 3.855 0.759 3.855 0.759 3.855 0.759 3.855 0.759 3.855 0.759 0.759 3.855 0.759 3.776 0.759 3.776	(Hz) 8.194 30.708 46.286 65.750 92.583 .119.922 146.861 180.438 210.792 Expt Freq (Hz) 9.569 28.944 45.972	Freq (Hz) 11.254 35.553 50.354 71.158 100.671 123.281 142.353 174.331 201.348 k= 378000 Freq (Hz) 11.141 32.187 47.560	3.060 4.844 4.069 5.408 8.088 3.359 4.509 6.107 9.443 Sum = ΔF 1.571 3.243 1.588	1.142           0.764           0.358           0.445           0.707           0.094           0.138           0.207           0.423           4.276           2.276           0.138           0.207           0.423           4.276           0.2058           0.363           0.363           0.055	Freq (Hz) 10.828 34.225 48.462 68.473 96.866 118.269 136.960 167.750 193.728 K=350000 Freq (Hz) 10.738 30.964 45.745	2.633 3.517 2.177 2.723 4.282 1.653 9.901 12.687 17.063 Sum = 0 ΔF 1.168 2.020 0.227	0.846 0.403 0.102 0.113 0.198 0.023 0.668 0.892 1.381 4.626 Chi squ'd 0.143 0.141 0.001	Freq (Hz) 10.672 33.733 47.765 67.487 95.472 116.922 134.989 165.336 190.941 K=340000 Freq (Hz) 10.583 30.524 45.087	ΔF 2.477 3.024 1.479 1.737 2.888 3.000 11.872 15.101 19.851 Sum = ΔF 1.014 1.579 0.886	0.7 0.2 0.0 0.0 0.0 0.0 0.0 1.2 1.8 5.3 5.3 5.3 5.3 5.3 5.3 5.3 5.3 5.3 5.3
1 2 3 4 5 6 7 8 9 9 Configurati Node	(kg) 4.735 1.196 0.220 0.805 0.464 0.874 2.265 0.759 3.852 0.759 3.852 0.759 3.852 0.759 3.855 0.759 3.855	(Hz) 8.194 30.708 46.286 65.750 92.583 .119.922 146.861 180.438 210.792 Expt Freq (Hz) 9.569 28.944 45.972 63.056	Freq (Hz) 11.254 35.553 50.354 71.158 100.671 123.281 142.353 174.331 201.348 k= 378000 Freq (Hz) 11.141 32.187 47.560 63.920	3.060 4.844 4.069 5.408 8.088 3.359 4.509 6.107 9.443 Sum = ΔF 1.571 3.243 1.588 0.865	1.142           0.764           0.358           0.445           0.707           0.094           0.138           0.207           0.423           4.276           2.276           0.138           0.207           0.423           4.276           0.2055           0.363           0.363           0.363           0.055           0.012	Freq (Hz) 10.828 34.225 48.462 68.473 96.866 118.269 136.960 167.750 193.728 K=350000 Freq (Hz) 10.738 30.964 45.745 61.475	2.633 3.517 2.177 2.723 4.282 1.653 9.901 12.687 17.063 Sum = 0 ΔF 1.168 2.020 0.227 1.581	0.846 0.403 0.102 0.113 0.198 0.023 0.668 0.892 1.381 4.626 Chi squ'd 0.143 0.141 0.001 0.040	Freq (Hz) 10.672 33.733 47.765 67.487 95.472 116.922 134.989 165.336 190.941 K=340000 Freq (Hz) 10.583 30.524 45.087 60.591	ΔF 2.477 3.024 1.479 1.737 2.888 3.000 11.872 15.101 19.851 Sum = ΔF 1.014 1.579 0.886 2.465	0.7 0.2 0.0 0.0 0.0 0.0 0.0 1.2 1.8 5.3 5.3 5.3 5.3 5.3 5.3 5.3 5.3 5.3 5.3
1 2 3 4 5 6 7 8 9 9 Configurati Node	(kg) 4.735 1.196 0.220 0.805 0.464 0.874 2.265 0.759 3.852 0.759 3.852 0.759 3.852 0.759 3.855 0.759 3.855 0.759 3.856 4.332	(Hz) 8.194 30.708 46.286 65.750 92.583 .119.922 146.861 180.438 210.792 Expt Freq (Hz) 9.569 28.944 45.972 63.056 80.597	Freq (Hz) 11.254 35.553 50.354 71.158 100.671 123.281 142.353 174.331 201.348 k= 378000 Freq (Hz) 11.141 32.187 47.560 63.920 81.185	3.060 4.844 4.069 5.408 8.088 3.359 4.509 6.107 9.443 Sum = ΔF 1.571 3.243 1.588 0.865 0.587	1.142           0.764           0.358           0.445           0.707           0.094           0.138           0.207           0.423           4.278           2.278           2.278           0.207           0.423           0.207           0.423           0.207           0.423           0.207           0.423           0.207           0.423           0.207           0.423           0.207           0.303           0.207           0.423           0.207           0.303           0.207           0.303           0.205           0.0055           0.004	Freq (Hz) 10.828 34.225 48.462 68.473 96.866 118.269 136.960 167.750 193.728 K=350000 Freq (Hz) 10.738 30.964 45.745 61.475 77.972	2.633 3.517 2.177 2.723 4.282 1.653 9.901 12.687 17.063 Sum = ΔF 1.168 2.020 0.227 1.581 2.626	0.846 0.403 0.102 0.113 0.198 0.023 0.668 0.892 1.381 4.626 Chi squ'd 0.143 0.141 0.044 0.046 0.086	Freq (Hz) 10.672 33.733 47.765 67.487 95.472 116.922 134.989 165.336 190.941 K=340000 Freq (Hz) 10.583 30.524 45.087 60.591 76.850	ΔF 2.477 3.024 1.479 1.737 2.888 3.000 11.872 15.101 19.851 Sum = ΔF 1.014 1.579 0.886 2.465 3.748	0.7 0.2 0.0 0.0 0.0 0.0 0.0 1.2 1.8 5.3 5.3 5.3 5.3 5.3 5.3 5.3 5.3 5.3 5.3
1 2 3 4 5 6 7 8 9 9 Configurati Node 1 2 3 4 5 6	(kg) 4.735 1.196 0.220 0.805 0.464 0.874 2.265 0.759 3.852 0.759 3.852 0.759 3.852 0.759 3.855 0.759 3.855 0.759 3.856 4.332 0.802	(Hz) 8.194 30.708 46.286 65.750 92.583 .119.922 146.861 180.438 210.792 Expt Freq (Hz) 9.569 28.944 45.972 63.056 80.597 87.056	Freq (Hz) 11.254 35.553 50.354 71.158 100.671 123.281 142.353 174.331 201.348 <b>k= 378000</b> Freq (Hz) 11.141 32.187 47.560 63.920 81.185 92.049	3.060 4.844 4.069 5.408 8.088 3.359 4.509 6.107 9.443 Sum = ΔF 1.571 3.243 1.588 0.865 0.587 4.993	1.142           0.764           0.358           0.445           0.707           0.094           0.138           0.207           0.423           4.278           Chi squ'd           0.258           0.363           0.363           0.363           0.363           0.055           0.012           0.004           0.286	Freq (Hz) 10.828 34.225 48.462 68.473 96.866 118.269 136.960 167.750 193.728 K=350000 Freq (Hz) 10.738 30.964 45.745 61.475 77.972 88.561	2.633 3.517 2.177 2.723 4.282 1.653 9.901 12.687 17.063 Sum = ΔF 1.168 2.020 0.227 1.581 2.626 1.505	0.846 0.403 0.102 0.113 0.198 0.023 0.668 0.892 1.381 4.626 7.1381 4.626 7.1381 0.143 0.143 0.144 0.044 0.044 0.045 0.005 0.026	Freq (Hz) 10.672 33.733 47.765 67.487 95.472 116.922 134.989 165.336 190.941 K=340000 Freq (Hz) 10.583 30.524 45.087 60.591 76.850 87.286	ΔF 2.477 3.024 1.479 1.737 2.888 3.000 11.872 15.101 19.851 Sum = ΔF 1.014 1.579 0.886 2.465 3.748 0.231	0.7 0.2 0.0 0.0 0.0 0.0 0.0 1.2 1.8 5.3 5.3 5.3 5.3 5.3 5.3 5.3 5.3 5.3 5.3
1 2 3 4 5 6 7 8 9 9 Configurati Node 1 2 3 4 5 6 7	(kg) 4.735 1.196 0.220 0.805 0.464 0.874 2.265 0.759 3.852 0.759 3.852 0.759 3.852 0.759 3.855 0.759 3.856 4.332 0.802 1.430	(Hz) 8.194 30.708 46.286 65.750 92.583 .119.922 146.861 180.438 210.792 200.792 20	Freq (Hz) 11.254 35.553 50.354 71.158 100.671 123.281 142.353 174.331 201.348 <b>k= 378000</b> Freq (Hz) 11.141 32.187 47.560 63.920 81.185 92.049 98.868	3.060 4.844 4.069 5.408 8.088 3.359 4.509 6.107 9.443 Sum = ΔF 1.571 3.243 1.588 0.865 0.587 4.993 2.538	1.142           0.764           0.358           0.445           0.707           0.094           0.138           0.207           0.423           4.278           2.278           2.278           0.255           0.363           0.255           0.363           0.363           0.363           0.363           0.055           0.012           0.004           0.286           0.064	Freq (Hz) 10.828 34.225 48.462 68.473 96.866 118.269 136.960 167.750 193.728 K=350000 Freq (Hz) 10.738 30.964 45.745 61.475 77.972 88.561 95.101	2.633 3.517 2.177 2.723 4.282 1.653 9.901 12.687 17.063 Sum = 0 ΔF 1.168 2.020 0.227 1.581 2.626 1.505 6.305	0.846 0.403 0.102 0.113 0.198 0.023 0.668 0.892 1.381 4.626 Chi squ'd 0.143 0.143 0.144 0.044 0.044 0.046 0.006 0.006 0.026 0.026	Freq (Hz) 10.672 33.733 47.765 67.487 95.472 116.922 134.989 165.336 190.941 K=340000 Freq (Hz) 10.583 30.524 45.087 60.591 76.850 87.286 93.733	ΔF 2.477 3.024 1.479 1.737 2.888 3.000 11.872 15.101 19.851 Sum = ΔF 1.014 1.579 0.886 2.465 3.748 0.231 7.673	0.7 0.2 0.0 0.0 0.0 0.0 0.0 1.2 1.8 5.3 5.3 5.3 5.3 5.3 5.3 5.3 5.3 5.3 5.3
1 2 3 4 5 6 7 8 9 9 Configurati Node 1 2 3 4 5 6 7 8	(kg) 4.735 1.196 0.220 0.805 0.464 0.874 2.265 0.759 3.852 0.759 3.852 0.759 3.852 0.759 3.856 4.332 0.802 1.430 0.351	(Hz) 8.194 30.708 46.286 65.750 92.583 .119.922 146.861 180.438 210.792 200.792 20	Freq (Hz) 11.254 35.553 50.354 71.158 100.671 123.281 142.353 174.331 201.348 <b>k= 378000</b> Freq (Hz) 11.141 32.187 47.560 63.920 81.185 92.049 98.868 141.809	3.060 4.844 4.069 5.408 8.088 3.359 4.509 6.107 9.443 Sum = ΔF 1.571 3.243 1.588 0.865 0.587 4.993 2.538 3.265	1.142           0.764           0.358           0.445           0.707           0.094           0.138           0.207           0.423           4.278           2.224           Chi squ'd           0.255           0.363           0.255           0.363           0.255           0.004           0.256           0.055           0.012           0.004           0.286           0.064           0.074	Freq (Hz) 10.828 34.225 48.462 68.473 96.866 118.269 136.960 167.750 193.728 K=350000 Freq (Hz) 10.738 30.964 45.745 61.475 77.972 88.561 95.101 136.445	2.633 3.517 2.177 2.723 4.282 1.653 9.901 12.687 17.063 Sum = 000000000000000000000000000000000000	0.846 0.403 0.102 0.113 0.198 0.023 0.668 0.892 1.381 4.626 Chi squ'd 0.143 0.143 0.144 0.044 0.044 0.046 0.006 0.006 0.006 0.026 0.0392 0.514	Freq (Hz) 10.672 33.733 47.765 67.487 95.472 116.922 134.989 165.336 190.941 K=340000 Freq (Hz) 10.583 30.524 45.087 60.591 76.850 87.286 93.733 134.481	ΔF 2.477 3.024 1.479 1.737 2.888 3.000 11.872 15.101 19.851 Sum = ΔF 1.014 1.579 0.886 2.465 3.748 0.231 7.673 10.597	0.7 0.2 0.0 0.0 0.0 0.0 0.0 1.2 1.8 5.3 5.3 5.3 5.3 5.3 5.3 5.3 5.3 5.3 5.3
1 2 3 4 5 6 7 8 9 9 7 Configurati Node 1 2 3 4 5 6 7	(kg) 4.735 1.196 0.220 0.805 0.464 0.874 2.265 0.759 3.852 0.759 3.852 0.759 3.852 0.759 3.856 4.332 0.802 1.430 0.351	(Hz) 8.194 30.708 46.286 65.750 92.583 .119.922 146.861 180.438 210.792 200.792 20	Freq (Hz) 11.254 35.553 50.354 71.158 100.671 123.281 142.353 174.331 201.348 <b>k= 378000</b> Freq (Hz) 11.141 32.187 47.560 63.920 81.185 92.049 98.868 141.809 184.983	3.060 4.844 4.069 5.408 8.088 3.359 4.509 6.107 9.443 Sum = ΔF 1.571 3.243 1.588 0.865 0.587 4.993 2.538	1.142           0.764           0.358           0.445           0.707           0.094           0.138           0.207           0.423           4.278           2.224           Chi squ'd           0.255           0.363           0.255           0.363           0.255           0.004           0.256           0.055           0.012           0.004           0.286           0.064           0.074	Freq (Hz) 10.828 34.225 48.462 68.473 96.866 118.269 136.960 167.750 193.728 K=350000 Freq (Hz) 10.738 30.964 45.745 61.475 77.972 88.561 95.101 136.445 178.026	2.633 3.517 2.177 2.723 4.282 1.653 9.901 12.687 17.063 Sum = 000000000000000000000000000000000000	0.846 0.403 0.102 0.113 0.198 0.023 0.668 0.892 1.381 4.626 Chi squ'd 0.143 0.143 0.144 0.044 0.044 0.046 0.006 0.006 0.026 0.0392 0.514	Freq (Hz) 10.672 33.733 47.765 67.487 95.472 116.922 134.989 165.336 190.941 K=340000 Freq (Hz) 10.583 30.524 45.087 60.591 76.850 87.286 93.733 134.481 175.465	ΔF 2.477 3.024 1.479 1.737 2.888 3.000 11.872 15.101 19.851 Sum = ΔF 1.014 1.579 0.886 2.465 3.748 0.231 7.673 10.597	0.7 0.2 0.0 0.0 0.0 0.0 0.0 1.2 1.8 5.3 5.3 5.3 5.3 5.3 5.3 5.3 5.3 5.3 5.3

Configuratio	on 5										
Node	Mass	Expt Freq	k= 378000			K=350000			K=340000		
	(kg)	(Hz)	Freq (Hz)	ΔF		Freq (Hz)	ΔF		Freq (Hz)	15	011
	6.797	9.000	10.473	1.473	0.241	10.079	1.079	0,129	9.934		Chi squ'o
2	3.557	26.000	27.731	1.731	0.241	26.683	0.683	0.129	26.299	0.934	0.09
3	3.684	42.083	43.163	1.080	0.028	41.534	0.549	0.018	40.937	0.299	0.00
4	3.235	56.167	57.025	0.859	0.028	54.872	1.295	0.030	54.082	1.147	0.03
5	2.347	72.771	72.018	0.753	0.008	69.299	3.472	0.030	68.301	2.085	0.07
6	2.584	88.667	86.793	1.874	0.040	83.517	5.150	0.299	82.315	4.469	0.27
7	1.757	98.188	96.339	1.848	0.035	92.701	5.487	0.207	91.367	6.352	0.4
8	1.086	113.479	109.465	4.015	0.142	105.331	8.149	0.585	103.815	6.821	0.4
9	1.204	142.313	139.292	3.021	0.064	134.033	8.280	0.482	132.104	9.664	0.83
				Sum =	0.685		Sum =	2.022		10.208 Sum =	0.7
-DATE STATE		A CONTRACTOR	LISSINGER STREET	R-mailtonesin	0.000	TTANK COLUMN	Strite Diverse	MIRDINERSES.	TAX SALATE ACHORE		2.96
Configuratio	on 6		Construction of the Party	and the Cristian Color	un a la singli de la	and a second state	et a state water	NINKE SALVE		A CARLEND AND AND AND AND AND AND AND AND AND A	
Node	Mass	Expt Freq	k= 378000			K=350000			K-240000		
	(kg)	(Hz)	Freq (Hz)	ΔF	Chi squ'd		۵F	Chi squ'd	K=340000		
1	3.498	9.500	10.747	1.247		Freq (Hz)			Freq (Hz)		Chi squ'
2	2.017	31.281	34.541	3.259	0.164	10.336	0.836	0.074	10.188	0.688	0.0
3	1.558	48.031	49.625	1.593	0.340 0.053	32.382 47.753	1.101	0.039	31.916	0.635	0.0
4	2.019	65.766		1.393	0.053	47.753 64.553	0.278 1.213	0.002	47.066	0.966	0.0
5	3.064	82.031	81.788	0.244	0.020	78.700	3.332	0.022		2.142	0.0
6	3.070	98.000		1.600	0.001	92.761	5.239	0.135		4.464	0.2
7	2.285	106.422		2.705	0.028	99.804				6.574	0.4
8	0.960	129.906		5.021	0.089	120.171	6.618 9.735	0.412 0.730		8.054	0.6
9	0.877	146.156		1.014	0.194	139.672	9.735 6.485	0.730		11.465	1.0
				Sum =	0.879	135.072	Sum =	1.981		8.494	0.4
UNICASING REAL			STATES NE			CARACTERS SKO		1.501	and the second s	Sum =	2.9
Configurati	00.7	ST THE FPART COURT OF ST	A DOVE A CASH INTO A REAL OF	- Martin - Froz Martin	and a state of the second second second	Constant of a constant	See Double Annual March	all and the state	2.1.2.2.2.2.2.1		
					10120-00				and the second se	and the second law is	Collection of the
Node	Mass	Expt Freq	k= 378000			2					inis and a
	Mass		k= 378000 Freg (Hz)	٨F	Chi sou'd	K=350000		Chi sou'd	K=340000		
	Mass (kg)	(Hz)	Freq (Hz)	ΔF 1 481		K=350000 Freq (Hz)	۵F		K=340000 Freq (Hz)	ΔF	Chi squ
Node 1	Mass (kg) 4.011	(Hz) 9.500	Freq (Hz) 10.981	1.481	0.231	K=350000 Freq (Hz) 10.565	∆F   1.065	0.119	K=340000 Freq (Hz) 10.413	∆F 0.913	Chi squ 0.0
Node 1 2	Mass (kg) 4.011 1.332	(Hz) 9.500 31.472	Freq (Hz) 10.981 33.807	1.481 2.335	0.231 0.173	K=350000 Freq (Hz) 10.565 32.528	∆F 1.065 1.056	0.119 0.035	K=340000 Freq (Hz) 10.413 32.050	∆F 0.913 0.588	Chi squ 0.0 0.0
Node 1	Mass (kg) 4.011 1.332 1.829	(Hz) 9.500 31.472 48.444	Freq (Hz) 10.981 33.807 50.168	1.481 2.335 1.723	0.231 0.173 0.061	K=350000 Freq (Hz) 10.565 32.528 48.272	∆F 1.065 1.056 0.172	0.119 0.035 0.001	K=340000 Freq (Hz) 10.413 32.050 47.578	∆F 0.913 0.588 0.867	Chi squ 0.0 0.0 0.0
Node 1 2 3	Mass (kg) 4.011 1.332 1.829 1.205	(Hz) 9.500 31.472 48.444 65.969	Freq (Hz) 10.981 33.807 50.168 67.788	1.481 2.335 1.723 1.819	0.231 0.173 0.061 0.050	K=350000 Freq (Hz) 10.565 32.528 48.272 65.228	∆F 1.065 1.056 0.172 0.741	0.119 0.035 0.001 0.008	K=340000 Freq (Hz) 10.413 32.050 47.578 64.289	∆F 0.913 0.588 0.867 1.679	Chi squ 0.0 0.0 0.0 0.0
Node 1 2 3 4 5	Mass (kg) 4.011 1.332 1.829 1.205 3.019	(Hz) 9.500 31.472 48.444 65.969 78.016	Freq (Hz) 10.981 33.807 50.168 67.788 77.945	1.481 2.335 1.723 1.819 0.070	0.231 0.173 0.061 0.050 0.000	K=350000 Freq (Hz) 10.565 32.528 48.272 65.228 75.001	∆F 1.065 1.056 0.172 0.741 3.015	0.119 0.035 0.001 0.008 0.117	K=340000 Freq (Hz) 10.413 32.050 47.578 64.289 73.921	∆F 0.913 0.588 0.867 1.679 4.094	Chi squ 0.0 0.0 0.0 0.0 0.0
Node 1 2 3 4	Mass (kg) 4.011 1.332 1.829 1.205 3.019 1.769	(Hz) 9.500 31.472 48.444 65.969 78.016 109.819	Freq (Hz) 10.981 33.807 50.168 67.788 77.945 106.789	1.481 2.335 1.723 1.819 0.070 3.030	0.231 0.173 0.061 0.050 0.000 0.084	K=350000 Freq (Hz) 10.565 32.528 48.272 65.228 75.001 102.752	∆F 1.065 1.056 0.172 0.741 3.015 7.068	0.119 0.035 0.001 0.008 0.117 0.455	K=340000 Freq (Hz) 10.413 32.050 47.578 64.289 73.921 101.273	∆F 0.913 0.588 0.867 1.679 4.094 8.546	Chi squ 0.0 0.0 0.0 0.0 0.2 0.6
Node 1 2 3 4 5 6 7	Mass (kg) 4.011 1.332 1.829 1.205 3.019 1.769 2.929	(Hz) 9.500 31.472 48.444 65.969 78.016 109.819 120.431	Freq (Hz) 10.981 33.807 50.168 67.788 77.945 106.789 116.734	1.481 2.335 1.723 1.819 0.070 3.030 3.696	0.231 0.173 0.061 0.050 0.000 0.084 0.113	K=350000 Freq (Hz) 10.565 32.528 48.272 65.228 75.001 102.752 112.319	∆F 1.065 1.056 0.172 0.741 3.015 7.068 8.112	0.119 0.035 0.001 0.008 0.117 0.455 0.546	K=340000 Freq (Hz) 10.413 32.050 47.578 64.289 73.921 101.273 110.702	∆F 0.913 0.588 0.867 1.679 4.094 8.546 9.729	Chi squ 0.0 0.0 0.0 0.0 0.2 0.6 0.7
Node 1 2 3 4 5 6	Mass (kg) 4.011 1.332 1.829 1.205 3.019 1.769 2.929 0.974	(Hz) 9.500 31.472 48.444 65.969 78.016 109.819 120.431 139.450	Freq (Hz) 10.981 33.807 50.168 67.788 77.945 106.789 116.734 134.260	1.481 2.335 1.723 1.819 0.070 3.030 3.696 5.190	0.231 0.173 0.061 0.050 0.000 0.084 0.113 0.193	K=350000 Freq (Hz) 10.565 32.528 48.272 65.228 75.001 102.752 112.319 129.185	∆F 1.065 1.056 0.172 0.741 3.015 7.068 8.112 10.265	0.119 0.035 0.001 0.008 0.117 0.455 0.546 0.756	K=340000 Freq (Hz) 10.413 32.050 47.578 64.289 73.921 101.273 110.702 127.327	ΔF 0.913 0.588 0.867 1.679 4.094 8.546 9.729 12.124	Chi squ 0.0 0.0 0.0 0.2 0.6 0.7 1.0
Node 1 2 3 4 5 6 7 8	Mass (kg) 4.011 1.332 1.829 1.205 3.019 1.769 2.929 0.974	(Hz) 9.500 31.472 48.444 65.969 78.016 109.819 120.431 139.450	Freq (Hz) 10.981 ·33.807 50.168 67.788 77.945 106.789 116.734 134.260 139.376	1.481 2.335 1.723 1.819 0.070 3.030 3.696 5.190 5.236	0.231 0.173 0.061 0.050 0.000 0.084 0.113 0.193 0.190	K=350000 Freq (Hz) 10.565 32.528 48.272 65.228 75.001 102.752 112.319 129.185 134.111	△F 1.065 1.056 0.172 0.741 3.015 7.068 8.112 10.265 10.500	0.119 0.035 0.001 0.008 0.117 0.455 0.546 0.756 0.762	K=340000 Freq (Hz) 10.413 32.050 47.578 64.289 73.921 101.273 110.702 127.327 132.181	ΔF 0.913 0.588 0.867 1.679 4.094 8.546 9.729 12.124 12.430	Chi squ 0.0 0.0 0.0 0.0 0.2 0.6 0.7 1.0 1.0
Node 1 2 3 4 5 6 7 8 9	Mass (kg) 4.011 1.332 1.829 1.205 3.019 1.769 2.929 0.974 1.175	(Hz) 9.500 31.472 48.444 65.969 78.016 109.819 120.431 139.450 144.611	Freq (Hz) 10.981 33.807 50.168 67.788 77.945 106.789 116.734 134.260 139.376	1.481 2.335 1.723 1.819 0.070 3.030 3.696 5.190 5.236 Sum =	0.231 0.173 0.061 0.050 0.000 0.084 0.113 0.193 0.190 1.095	K=350000 Freq (Hz) 10.565 32.528 48.272 65.228 75.001 102.752 112.319 129.185 134.111	∆F 1.065 1.056 0.172 0.741 3.015 7.068 8.112 10.265 10.500 Sum =	0.119 0.035 0.001 0.008 0.117 0.455 0.546 0.756 0.762 2.799	K=340000 Freq (Hz) 10.413 32.050 47.578 64.289 73.921 101.273 110.702 127.327 132.181	ΔF 0.913 0.588 0.867 1.679 4.094 8.546 9.729 12.124 12.430 Sum =	Chi squ 0.0 0.0 0.0 0.2 0.6 0.7 1.0 1.0 3.9
Node 1 2 3 4 5 6 7 8 9	Mass (kg) 4.011 1.332 1.829 1.205 3.019 1.769 2.929 0.974 1.175	(Hz) 9.500 31.472 48.444 65.969 78.016 109.819 120.431 139.450 144.611	Freq (Hz) 10.981 ·33.807 50.168 67.788 77.945 106.789 116.734 134.260 139.376	1.481 2.335 1.723 1.819 0.070 3.030 3.696 5.190 5.236 Sum =	0.231 0.173 0.061 0.050 0.000 0.084 0.113 0.193 0.190 1.095	K=350000 Freq (Hz) 10.565 32.528 48.272 65.228 75.001 102.752 112.319 129.185 134.111	∆F 1.065 1.056 0.172 0.741 3.015 7.068 8.112 10.265 10.500 Sum =	0.119 0.035 0.001 0.008 0.117 0.455 0.546 0.756 0.762 2.799	K=340000 Freq (Hz) 10.413 32.050 47.578 64.289 73.921 101.273 110.702 127.327 132.181	ΔF 0.913 0.588 0.867 1.679 4.094 8.546 9.729 12.124 12.430 Sum =	Chi squ 0.0 0.0 0.0 0.2 0.6 0.7 1.0 1.0 3.9
Node 1 2 3 4 5 6 7 8 9 9 0 00000000000000000000000000000	Mass (kg) 4.011 1.332 1.829 1.205 3.019 1.769 2.929 0.974 1.175	(Hz) 9.500 31.472 48.444 65.969 78.016 109.819 120.431 139.450 144.611	Freq (Hz) 10.981 33.807 50.168 67.788 77.945 106.789 116.734 134.260 139.376	1.481 2.335 1.723 1.819 0.070 3.030 3.696 5.190 5.236 Sum =	0.231 0.173 0.061 0.050 0.000 0.084 0.113 0.193 0.190 1.095	K=350000 Freq (Hz) 10.565 32.528 48.272 65.228 75.001 102.752 112.319 129.185 134.111	∆F 1.065 1.056 0.172 0.741 3.015 7.068 8.112 10.265 10.500 Sum =	0.119 0.035 0.001 0.008 0.117 0.455 0.546 0.756 0.762 2.799	K=340000 Freq (Hz) 10.413 32.050 47.578 64.289 73.921 101.273 110.702 127.327 132.181	ΔF 0.913 0.588 0.867 1.679 4.094 8.546 9.729 12.124 12.430 Sum =	Chi squ 0.0 0.0 0.0 0.2 0.6 0.7 1.0 1.0 3.9
Node 1 2 3 4 5 6 7 8 9	Mass (kg) 4.011 1.332 1.829 1.205 3.019 1.769 2.929 0.974 1.175 ion 8 Mass	(Hz) 9.500 31.472 48.444 65.969 78.016 109.819 120.431 139.450 144.611	Freq (Hz) 10.981 33.807 50.168 67.788 77.945 106.789 116.734 134.260 139.376 K= 378000	1.481 2.335 1.723 1.819 0.070 3.030 3.696 5.190 5.236 Sum =	0.231 0.173 0.061 0.050 0.000 0.084 0.113 0.193 0.190 1.095	K=350000 Freq (Hz) 10.565 32.528 48.272 65.228 75.001 102.752 112.319 129.185 134.111	∆F 1.065 1.056 0.172 0.741 3.015 7.068 8.112 10.265 10.500 Sum =	0.119 0.035 0.001 0.008 0.117 0.455 0.546 0.756 0.762 2.799	K=340000 Freq (Hz) 10.413 32.050 47.578 64.289 73.921 101.273 110.702 127.327 132.181 K=340000	ΔF 0.913 0.588 0.867 1.679 4.094 8.546 9.729 12.124 12.430 Sum =	Chi squ 0.0 0.0 0.0 0.2 0.6 0.7 1.0 1.0 3.9
Node 1 2 3 4 5 6 7 8 9 9 Configurat	Mass (kg) 4.011 1.332 1.829 1.205 3.019 1.769 2.929 0.974 1.175 ion 8 Mass (kg)	(Hz) 9.500 31.472 48.444 65.969 78.016 109.819 120.431 139.450 144.611	Freq (Hz) 10.981 33.807 50.168 67.788 77.945 106.789 116.734 134.260 139.376 k= 378000 Freq (Hz)	1.481 2.335 1.723 1.819 0.070 3.030 3.696 5.190 5.236 Sum = 232.47 (5.3) ΔF	0.231 0.173 0.061 0.050 0.000 0.084 0.113 0.193 0.190 1.095	K=350000 Freq (Hz) 10.565 32.528 48.272 65.228 75.001 102.752 112.319 129.185 134.111 K=350000 Freq (Hz)	ΔF 1.065 1.056 0.172 0.741 3.015 7.068 8.112 10.265 10.500 Sum = 2.255 ΔF	0.119 0.035 0.001 0.008 0.117 0.455 0.546 0.756 0.762 2.799	K=340000 Freq (Hz) 10.413 32.050 47.578 64.289 73.921 101.273 110.702 127.327 132.181 K=340000 Freq (Hz)	ΔF 0.913 0.588 0.867 1.679 4.094 8.546 9.729 12.124 12.430 Sum = Control Control Cont	Chi squ 0.0 0.0 0.0 0.2 0.6 0.7 1.0 1.0 3.9 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5
Node 1 2 3 4 5 6 7 8 9 Configurat Node 1	Mass (kg) 4.011 1.332 1.829 1.205 3.019 1.769 2.929 0.974 1.175 ion 8 Mass (kg) 6.525	(Hz) 9.500 31.472 48.444 65.969 78.016 109.819 120.431 139.450 144.611 Expt Freq (Hz) 8.781	Freq (Hz) 10.981 33.807 50.168 67.788 77.945 106.789 116.734 134.260 139.376 k= 378000 Freq (Hz) 10.078	1.481 2.335 1.723 1.819 0.070 3.030 3.696 5.190 5.236 Sum = 232 MF 500 ΔF 1.297	0.231 0.173 0.061 0.050 0.000 0.084 0.113 0.193 0.190 1.095 Chi squ'd 0.192	K=350000 Freq (Hz) 10.565 32.528 48.272 65.228 75.001 102.752 112.319 129.185 134.111 K=350000 Freq (Hz) 9.692	ΔF 1.065 1.056 0.172 0.741 3.015 7.068 8.112 10.265 10.500 Sum = ΔF 0.911	0.119 0.035 0.001 0.008 0.117 0.455 0.546 0.756 0.762 2.799	K=340000 Freq (Hz) 10.413 32.050 47.578 64.289 73.921 101.273 110.702 127.327 132.181 K=340000 Freq (Hz) 9.553	ΔF 0.913 0.588 0.867 1.679 4.094 8.546 9.729 12.124 12.430 Sum = ΔF 0.772	Chi squ 0.0 0.0 0.2 0.2 0.6 0.7 1.0 1.0 3.5 5555555 0.7 7 1.0 1.0 0.7 0.7 1.0 1.0 0.7 1.0 1.0 0.7 1.0 1.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
Node 1 2 3 4 5 6 7 8 9 Configurat Node 1 2	Mass (kg) 4.011 1.332 1.829 1.205 3.019 1.769 2.929 0.974 1.175 0.974 1.175 ion 8 Mass (kg) 6.529 1.496	(Hz) 9.500 31.472 48.444 65.969 78.016 109.819 120.431 139.450 144.611 Expt Freq (Hz) 8.781 29.250	Freq (Hz) 10.981 33.807 50.168 67.788 77.945 106.789 116.734 134.260 139.376 k= 378000 Freq (Hz) 10.078 31.691	1.481 2.335 1.723 1.819 0.070 3.030 3.696 5.190 5.236 Sum = 232 47 504 ΔF 1.297 2.441	0.231 0.173 0.061 0.050 0.000 0.084 0.113 0.193 0.190 1.095 Chi squ'd 0.192 0.204	K=350000 Freq (Hz) 10.565 32.528 48.272 65.228 75.001 102.752 112.319 129.185 134.111 K=350000 Freq (Hz) 9.692 30.492	ΔF 1.065 1.056 0.172 0.741 3.015 7.068 8.112 10.265 10.500 Sum = ΔF 0.911 1.242	0.119 0.035 0.001 0.008 0.117 0.455 0.546 0.756 0.762 2.799 524 Chi squ'd 0.094 0.053	K=340000 Freq (Hz) 10.413 32.050 47.578 64.289 73.921 101.273 110.702 127.327 132.181 K=340000 Freq (Hz) 9.553 30.053	ΔF 0.913 0.588 0.867 1.679 4.094 8.546 9.729 12.124 12.430 Sum = ΔF 0.772 0.803	Chi squ 0.0 0.0 0.2 0.2 0.2 0.7 1.0 1.0 3.5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5
Node 1 2 3 4 5 6 7 8 9 Configurat Node 1 2 3	Mass (kg) 4.011 1.332 1.829 1.205 3.019 1.769 2.929 0.974 1.175 0.974 1.175 ion 8 Mass (kg) 6.529 1.496 1.371	(Hz) 9.500 31.472 48.444 65.969 78.016 109.819 120.431 139.450 144.611 Expt Free (Hz) 8.781 29.250 42.000	Freq (Hz) 10.981 33.807 50.168 67.788 77.945 106.789 116.734 134.260 139.376 k= 378000 Freq (Hz) 10.078 31.691 43.818	1.481 2.335 1.723 1.819 0.070 3.030 3.696 5.190 5.236 Sum = 232 47 534 ΔF 1.297 2.441 1.818	0.231 0.173 0.061 0.050 0.000 0.084 0.113 0.193 0.190 1.095 Chi squ'd 0.192 0.204 0.079	K=350000 Freq (Hz) 10.565 32.528 48.272 65.228 75.001 102.752 112.319 129.185 134.111 K=350000 Freq (Hz) 9.692 30.492 42.163	ΔF 1.065 1.056 0.172 0.741 3.015 7.068 8.112 10.265 10.500 Sum = ΔF 0.911 1.242 0.163	0.119 0.035 0.001 0.008 0.117 0.455 0.546 0.756 0.762 2.799 Chi squ'd 0.094 0.053 0.001	K=340000 Freq (Hz) 10.413 32.050 47.578 64.289 73.921 101.273 110.702 127.327 132.181 K=340000 Freq (Hz) 9.553 30.053 41.555	ΔF 0.913 0.588 0.867 1.679 4.094 8.546 9.729 12.124 12.430 Sum = ΔF 0.772 0.803 0.445	Chi squ 0.0 0.0 0.2 0.6 0.7 1.0 1.0 3.5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5
Node 1 2 3 4 5 6 7 8 9 Configurat Node 1 2 3 4 4 5 6 7 8 9 1 1 2 3 4 5 6 7 8 9 1 2 3 4 5 6 7 8 9 1 2 3 4 5 6 7 8 9 1 2 3 4 5 6 7 8 9 1 2 3 4 5 6 7 8 9 1 2 3 4 5 6 7 8 9 1 2 3 4 5 6 7 8 9 2 2 5 6 7 8 9 2 5 6 7 8 9 2 5 6 7 8 9 2 5 6 7 8 9 7 7 8 9 7 7 8 9 7 7 8 9 7 7 8 9 7 7 8 9 7 7 8 9 7 7 7 8 9 7 7 8 9 7 7 8 9 7 7 8 9 7 7 8 9 7 7 8 9 7 7 8 9 7 7 8 9 7 8 9 7 7 8 9 7 7 8 9 7 7 8 9 7 7 7 8 9 7 7 8 9 7 7 8 7 7 8 7 7 8 7 7 8 7 8 7 7 8 7 7 8 7 7 8 7 7 8 7 7 8 7 7 8 7 8 7 7 8 7 7 8 7 7 8 7 7 7 8 7 7 7 8 7 7 7 8 7 7 7 8 7 7 7 8 7 7 7 7 7 7 7 7 7 7 7 7 7	Mass (kg) 4.011 1.332 1.829 1.205 3.019 1.769 2.929 0.974 1.175 0.974 1.175 0.974 1.175 0.974 1.175 0.974 1.175	(Hz) 9.500 31.472 48.444 65.969 78.016 109.819 120.431 139.450 144.611 Expt Free (Hz) 8.781 29.250 42.000 61.714	Freq (Hz) 10.981 33.807 50.168 67.788 77.945 106.789 116.734 134.260 139.376 k= 378000 Freq (Hz) 10.078 31.691 43.818 61.441	1.481 2.335 1.723 1.819 0.070 3.030 3.696 5.190 5.236 Sum = 232 47 534 ΔF 1.297 2.441 1.818 0.273	0.231 0.173 0.061 0.050 0.000 0.084 0.113 0.193 0.190 1.095 Chi squ'd 0.192 0.204 0.079 0.001	K=350000 Freq (Hz) 10.565 32.528 48.272 65.228 75.001 102.752 112.319 129.185 134.111 K=350000 Freq (Hz) 9.692 30.492 42.163 59.118	ΔF 1.065 1.056 0.172 0.741 3.015 7.068 8.112 10.265 10.500 Sum = ΔF 0.911 1.242 0.163 2.597	0.119 0.035 0.001 0.008 0.117 0.455 0.546 0.756 0.762 2.799 Chi squ'd 0.094 0.053 0.001 0.105	K=340000 Freq (Hz) 10.413 32.050 47.578 64.289 73.921 101.273 110.702 127.327 132.181 K=340000 Freq (Hz) 9.553 30.053 41.555 58.267	ΔF 0.913 0.588 0.867 1.679 4.094 8.546 9.729 12.124 12.430 Sum = ΔF 0.772 0.803 0.445 3.447	Chi squ 0.0 0.0 0.2 0.2 0.2 0.2 0.2 0.2 0.2 0.2
Node 1 2 3 4 5 6 7 8 9 Configurat Node 1 2 3 4 5 5 5 5 5 5 5 5 5 5 5 5 5	Mass (kg) 4.011 1.332 1.829 1.205 3.019 1.769 2.929 0.974 1.1757 1.17577 1.17577 1.17577 1.175777 1.1757777777777	(Hz) 9.500 31.472 48.444 65.969 78.016 109.819 120.431 139.450 144.611 Expt Free (Hz) 8.781 29.250 42.000 61.714 80.688	Freq (Hz) 10.981 33.807 50.168 67.788 77.945 106.789 116.734 134.260 139.376 <b>k= 378000</b> Freq (Hz) 10.078 31.691 43.818 61.441 379.361	1.481 2.335 1.723 1.819 0.070 3.030 3.696 5.190 5.236 Sum = 22.245 ΔF 1.297 2.441 1.818 0.273 1.327	0.231 0.173 0.061 0.050 0.000 0.084 0.113 0.193 0.190 1.095 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5	K=350000 Freq (Hz) 10.565 32.528 48.272 65.228 75.001 102.752 112.319 129.185 134.111 K=350000 Freq (Hz) 9.692 30.492 42.163 59.118 76.360	ΔF 1.065 1.056 0.172 0.741 3.015 7.068 8.112 10.265 10.500 Sum = ΔF 0.911 1.242 0.163 2.597 4.327	0.119 0.035 0.001 0.008 0.117 0.455 0.546 0.756 0.762 2.799 Chi squ'd 0.094 0.053 0.001 0.105	K=340000 Freq (Hz) 10.413 32.050 47.578 64.289 73.921 101.273 110.702 127.327 132.181 K=340000 Freq (Hz) 9.553 30.053 41.555 58.267 75.262	ΔF 0.913 0.588 0.867 1.679 4.094 8.546 9.729 12.124 12.430 Sum = ΔF 0.772 0.803 0.445 3.447 5.426	Chi squ 0.0 0.0 0.2 0.2 0.2 0.2 0.2 0.2 0.2 0.2
Node 1 2 3 4 5 6 7 8 9 Configurat Node 1 2 3 4 5 6 7 8 9 1 2 3 4 5 6 7 8 9 1 2 3 4 5 6 7 8 9 1 2 3 4 5 6 7 8 9 1 2 3 4 5 6 7 8 9 1 2 3 4 5 6 7 8 9 1 2 5 6 7 8 9 1 2 5 6 7 8 9 1 5 6 7 8 9 1 5 6 7 8 9 1 5 6 7 8 9 1 5 6 7 8 9 1 5 6 7 8 9 1 5 6 7 8 9 1 5 6 7 8 9 1 5 6 7 8 9 1 5 6 7 8 9 5 5 7 7 8 9 1 5 7 7 8 9 1 9 1 5 7 7 8 9 1 5 7 7 8 9 7 7 8 9 7 7 8 9 7 7 8 9 7 7 8 8 9 7 7 8 8 9 7 7 8 7 7 8 9 7 7 8 8 9 7 7 8 8 7 7 8 7 7 8 7 8 7 8 7 8 7 8 8 7 7 8 7 8 7 8 7 8 7 8 7 8 7 8 7 8 7 8 7 8 7 8 7 7 8 8 7 8 8 7 8 8 8 7 8 7 8 7 8 8 8 7 8 8 8 8 8 8 8 8 8 8 8 8 8	Mass (kg) 4.011 1.332 1.829 1.205 3.019 1.769 2.929 0.974 1.175 0.974 1.371 1.175	(Hz) 9.500 31.472 48.444 65.969 78.016 109.819 120.431 139.450 144.611 Expt Free (Hz) 8.781 29.250 42.000 61.714 80.688 96.906	Freq (Hz) 10.981 33.807 50.168 67.788 77.945 106.789 116.734 134.260 139.376 <b>k= 378000</b> Freq (Hz) 10.078 31.691 43.818 61.441 379.361 94.290	1.481 2.335 1.723 1.819 0.070 3.030 3.696 5.190 5.236 Sum = 22.245 2.245 1.297 2.441 1.818 0.273 1.327 2.616	0.231 0.173 0.061 0.050 0.000 0.084 0.113 0.193 0.190 1.095 5.01203 Chi squ'd 0.192 0.204 0.079 0.001 0.022 0.071	K=350000 Freq (Hz) 10.565 32.528 48.272 65.228 75.001 102.752 112.319 129.185 134.111 K=350000 Freq (Hz) 9.692 30.492 42.163 59.118 76.360 90.726	ΔF 1.065 1.056 0.172 0.741 3.015 7.068 8.112 10.265 10.500 Sum = ΔF 0.911 1.242 0.163 2.597 4.327 6.180	0.119 0.035 0.001 0.008 0.117 0.455 0.546 0.756 0.762 2.799 Chi squ'd 0.094 0.053 0.001 0.109 0.232 0.394	K=340000 Freq (Hz) 10.413 32.050 47.578 64.289 73.921 101.273 110.702 127.327 132.181 K=340000 Freq (Hz) 9.553 30.053 41.555 58.267 75.262 89.420	ΔF 0.913 0.588 0.867 1.679 4.094 8.546 9.729 12.124 12.430 Sum = ΔF 0.772 0.803 0.445 3.447 5.426 7.486	Chi squ 0.0 0.0 0.2 0.2 0.2 0.2 0.2 0.2 0.2 0.2
Node 1 2 3 4 5 6 7 8 9 Configurat Node 1 2 3 4 5 6 7 8 9 1 2 3 4 5 6 7 8 9 1 2 3 4 5 6 7 8 9 1 2 3 4 5 6 7 8 9 1 2 3 4 5 6 7 8 9 1 1 2 1 1 1 1 1 1 1 1 1 1 1 1 1	Mass (kg) 4.011 1.332 1.829 1.205 3.019 1.769 2.929 0.974 1.175 0.974 1.175 ion 8 Mass (kg) 6.529 1.496 1.371 1.673 3.094 3.827 2.426	(Hz) 9.500 31.472 48.444 65.969 78.016 109.819 120.431 139.450 144.611 Expt Freq (Hz) 8.781 29.250 42.000 61.714 80.688 96.906 108.188	Freq (Hz) 10.981 33.807 50.168 67.788 77.945 106.789 116.734 134.260 139.376 <b>k= 378000</b> Freq (Hz) 10.078 31.691 43.818 61.441 379.361 94.290 106.756	1.481 2.335 1.723 1.819 0.070 3.030 3.696 5.190 5.236 Sum = 22.441 1.297 2.441 1.818 0.273 1.327 2.616 1.431	0.231 0.173 0.061 0.050 0.000 0.084 0.113 0.193 0.190 1.095 5.012032 Chi squ'd 0.192 0.204 0.079 0.001 0.022 0.071 0.019	K=350000 Freq (Hz) 10.565 32.528 48.272 65.228 75.001 102.752 112.319 129.185 134.111 K=350000 Freq (Hz) 9.692 30.492 42.163 59.118 76.360 90.726 102.724	ΔF 1.065 1.056 0.172 0.741 3.015 7.068 8.112 10.265 10.500 Sum = ΔF 0.911 1.242 0.163 2.597 4.327 6.180 5.463	0.119 0.035 0.001 0.008 0.117 0.455 0.546 0.756 0.762 2.799 Chi squ'd 0.094 0.053 0.001 0.109 0.232 0.394 0.276	K=340000 Freq (Hz) 10.413 32.050 47.578 64.289 73.921 101.273 110.702 127.327 132.181 K=340000 Freq (Hz) 9.553 30.053 41.555 58.267 75.262 89.420 101.243	ΔF 0.913 0.588 0.867 1.679 4.094 8.546 9.729 12.124 12.430 Sum = ΔF 0.772 0.803 0.445 3.447 5.426 7.486 6.944	Chi squ 0.0 0.2 0.2 0.2 0.7 1.0 1.0 3.5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5
Node 1 2 3 4 5 6 7 8 9 2 Configurat Node 1 2 3 4 5 6 7 8 9 9 1 2 3 4 5 6 7 8 9 9 1 1 2 3 4 5 6 7 8 9 1 1 2 1 1 1 1 1 1 1 1 1 1 1 1 1	Mass (kg) 4.011 1.332 1.829 1.205 3.019 1.769 2.929 0.974 1.175 ion 8 Mass (kg) 6.529 1.496 1.371 1.673 3.094 3.827 2.426 2.005	(Hz) 9.500 31.472 48.444 65.969 78.016 109.819 120.431 139.450 144.611 Expt Free (Hz) 8.781 29.250 42.000 61.714 80.688 96.906 108.188 124.781	Freq (Hz) 10.981 33.807 50.168 67.788 77.945 106.789 116.734 134.260 139.376 <b>k= 378000</b> Freq (Hz) 10.078 31.691 43.818 61.441 379.361 94.290 106.756 123.373	1.481 2.335 1.723 1.819 0.070 3.030 3.696 5.190 5.236 Sum = 22.441 1.818 0.273 1.327 2.616 1.431 1.408	0.231 0.173 0.061 0.050 0.000 0.084 0.113 0.193 0.190 1.095 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5	K=350000 Freq (Hz) 10.565 32.528 48.272 65.228 75.001 102.752 112.319 129.185 134.111 K=350000 Freq (Hz) 9.692 30.492 42.163 59.118 76.360 90.726 102.724 118.627	ΔF 1.065 1.056 0.172 0.741 3.015 7.068 8.112 10.265 10.500 Sum = ΔF 0.911 1.242 0.163 2.597 4.327 6.180 5.463 6.155	0.119 0.035 0.001 0.008 0.117 0.455 0.546 0.756 0.762 2.799 Chi squ'd 0.094 0.053 0.001 0.109 0.232 0.394 0.236 0.304	K=340000 Freq (Hz) 10.413 32.050 47.578 64.289 73.921 101.273 110.702 127.327 132.181 K=340000 Freq (Hz) 9.553 30.053 41.555 58.267 75.262 89.420 101.243 116.920	ΔF 0.913 0.588 0.867 1.679 4.094 8.546 9.729 12.124 12.430 Sum = ΔF 0.772 0.803 0.445 3.447 5.426 7.486 6.944 7.862	Chi squ 0.0 0.0 0.0 0.2 0.6 0.7 1.0 1.0 3.9 3.5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5
Node 1 2 3 4 5 6 7 8 9 2 2 2 2 2 2 2 2 3 4 5 5 6 7 8 9 2 2 3 4 5 6 7 8 9 2 2 3 4 5 6 7 8 9 2 2 2 3 4 5 6 7 8 9 2 2 2 3 4 5 6 7 8 9 2 2 2 2 3 4 5 6 7 8 9 2 2 2 2 2 2 2 2 2 2 2 2 2	Mass (kg) 4.011 1.332 1.829 1.205 3.019 1.769 2.929 0.974 1.175 ion 8 Mass (kg) 6.529 1.496 1.371 1.673 3.094 3.827 2.426 2.005	(Hz) 9.500 31.472 48.444 65.969 78.016 109.819 120.431 139.450 144.611 Expt Freq (Hz) 8.781 29.250 42.000 61.714 80.688 96.906 108.188 124.781	Freq (Hz) 10.981 33.807 50.168 67.788 77.945 106.789 116.734 134.260 139.376 <b>k= 378000</b> Freq (Hz) 10.078 31.691 43.818 61.441 379.361 94.290 106.756 123.373 133.562	1.481 2.335 1.723 1.819 0.070 3.030 3.696 5.190 5.236 Sum = 22.441 1.297 2.441 1.818 0.273 1.327 2.616 1.431	0.231 0.173 0.061 0.050 0.000 0.084 0.113 0.193 0.190 1.095 5 5 5 5 6 6 7 7 7 7 7 7 7 7 7 7 7 7 7	K=350000 Freq (Hz) 10.565 32.528 48.272 65.228 75.001 102.752 112.319 129.185 134.111 K=350000 Freq (Hz) 9.692 30.492 42.163 59.118 76.360 90.726 102.724 118.627 128.508	ΔF 1.065 1.056 0.172 0.741 3.015 7.068 8.112 10.265 10.500 Sum = ΔF 0.911 1.242 0.163 2.597 4.327 6.180 5.463 6.155	0.119 0.035 0.001 0.008 0.117 0.455 0.546 0.756 0.762 2.799 Chi squ'd 0.094 0.053 0.001 0.109 0.232 0.394 0.232 0.394	K=340000           Freq (Hz)           10.413           32.050           47.578           64.289           73.921           101.273           110.702           127.327           132.181           K=3400000           Freq (Hz)           9.553           30.053           41.555           58.267           75.262           89.420           101.243           116.920           126.660	ΔF 0.913 0.588 0.867 1.679 4.094 8.546 9.729 12.124 12.430 Sum = ΔF 0.772 0.803 0.445 3.447 5.426 7.486 6.944 7.862	Chi squ 0.0 0.0 0.0 0.2 0.6 0.7 1.0 1.0 3.9 0.7 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0

## **APPENDIX C:**

Raw Data from Experimental Tests of Algorithm 4.2

Appendix (
Ω
Raw
Data 1
from
Data from Experimental
nental
Tests o
-
112
Algorithm
4.2

#### **RESULTS FROM TESTING OF ALGORITHM**

NOTES:

Dested

0.025

0 250

0 500

1.000

1.500

2 000

2 500

1 750

0.500

1.000

2.000

4.000

6,000

8.000

12,000

18 000

0.050

0 500

1 000

1.500

2 500

3 000

3 500

8 000

13 000

A (radis)

0.03

0 300

0.750

1 500

2 250

3 000

3 750

5 000

10 500

0.050

0 300

0,750

1,250

2 000

3 000

3 500

4.500

A (red/s) (Hz)

Desred

the state the selection

Experiment 5

Desired

AND REAL PROPERTY OF

Deste

Experiment 2

Experiment 3

約4月時日 中心をする。

Experiment 4

3

Experiment 1

These results have used the spring constant K = 376000 N/m

The lumped parameter model has been used is one thed of the spring mass has been lumped at the nodes

7.96

24.08

38.08

50 38

81.35

71.81

79 21 87.86

98 79

35 55

50.35

71.16

100.67

123,28

142.35

174.33

201.35

11,14

32 19 47.56

63.92

81.18

92 05

98.87

141.81

184 98

672 3.79

27.57

43.59

61.64

75.49

87.17

97.46

112.54

163.09

一上一方法 经月月 日月 日子 公司 二

(ed)m (titz) (etbas)

10.47 7.19

27.73

43.15

57.03

72 02

65 79

96.34

109.46

139.29

CONTRACT NOTIFIE

tracted to about the starting of the start

All lambda values below have been multiplied by the constant E(-5)

Given by sigorthm

0.025

0 229

0.514

1.002

1.486

2 038

2.477

3 049

3 853

0.499

1.001

1.999

4.001

8 000

8,000

11,998

16.005

0.049

0 409

0.893

1.613

2 802

3 345

3.859

7.939

13 509

0.030

0.300

0 750

1 500

2 250

3.000

3 750

5.000

10 500

0.043

0.304

0.736

1.284

2.048

2.974

3.664

4.731

7.660

Green by elgorithm

Given by algorithm

(Hz) A (red/s) ((Hz) m (kg)

Given by algorithm A (rad/s) (Hz) A (rad/s) (Hz)

liven by sigorithm A (rad/s) [[Hz] A (rad/s) [[[Hz] m(kg] 0.050 11.25 5.137

A (rad/s) (P4z) A (rad/s) ( (P4z)

7.96

25.16

35.59

50 33

61.64

71.18

79.58

87.17

07 46

35 59

50 33

71.18

100 66

123.28

142.35

201.32

11.25

35 59

50 33

61.64

79.50

87.17

04.16

142.35

101.48

# 72

27.57

43.59

61.64

75 40

87.17

97,48

112.54

11,25

27.57

43.59

58.27

71.18

87,17

94 16

105.76

137.03

11.24 家子の学校があるというに、「「「ならん」「おんちょう」

m (%g) 6.330

5.007

3 907

4.776

6 638

5 4 96

3,763

3.426

2 145

1.598 1.196

0.022

1 207

0.656

1 276

2.667

1.162

4.053

m (kg)

4 17

3.081

3.892

4.258

4,734

1.832

0.753

0 904

3,728

2.537

1,866

0 911

2 620

3.436

3 857

3.523

3.959

4.050

3.637

2.749

2,985

1.488

.

Experimental Results (Hz) mass at Striking Location

21.5

35.75

62 25

735

61

92.5

09.5

iking Locatic

8.25

30 75

48.5

65.75

92.5

119.75

148,75

8 75

29

46

63

60 5

67

101,5

145 25

Leven Annews and the statistic

Striking Location

7.75

26 5

61.75

78.75

88 25

101,5

114,75

184.5

ung Location

done

pot

45

180 5.188 75

Experimental Results (Hz)

Experimental Results (Hz)

如此 的战 目标记录系

自由的中央的特性和中国学校中的特殊。和于古地行政

50 25-50 5

node (to)

5.928

4 600

3 505

4.374

6 236

5.094

3,361

3.024

1.944

mass at

node (kg

4.735

0.220

0 805

0 464

0.074

2 265

0.759

3.852

mess at

377

2 659

3.490

3,855

4 332

0 607

1.430

0 351

0 703

mass at

node (kg)

3,295

3.326

2 135

1.464

0 500

2.218

3.034

3.455

3 372

10.215 5 - 3 4 40

mass at

node (kg

6 79

3 557

3.684

3.235

2.347

2 584

1.757

1.085

node (kg

Experimental Results (Hz)

Striking Localio

Experimental Results (Hz)

All entries marked with a \* exher had poor coherence (below #5%) or were poorly defined in the Frequency Responce Function

Entries labled 'not done' were not done because the results were either not necessary or because the results had poor coherence for several entries

21.5

35.75

62

81

92.25

99.5

8 25

30.75

65 75

120

147

0 5.9 75

29

63

80,5

66 75

7.75

20.5

61,75

78,75

101.5

114.75

104 75

26

42 25

56.25

72.75

88.75

98 25

113.5

VIBES N

45

101 25

45 75

50.5-50.25

done

8.25

30.75

48.25

65.75

147

25

29

46 25

63

80.5

7.75

26,5

61.75

88.25

101.5

26

42

58 25

72 75

68.75

98 25

113.5

114.75

出来。而来到现代重新在市**市**田355

45

79

87-87 25

101-101.25

210.75 210 5-210.75

92-92 25

119,75-120

~ 97

> 29 46 46 63 25 62 5 63.5 63 80.125 60,75 80.5 61 87.25 87.25 87,125 86,75 145.25 145.5 144.75 144,75 105 875 185 75 188 25 7.75 7.75 175 7.75 28.5 26.5 26,5 28.5 45 45 45 61.75 62 61,875 61.75 78.75 78,75 79 78.75 88.125 88.25 87.5 88,5 101.5 101 5 101.5 115 101.5 115.25 115 114.75 164.75

> > 26

58 56 25

42.25

72 75

88,75

98 25

113.5

142 25

(H:) 26.5 26.5 61.8 78.9 88.2 101.75 101.5 114.9 164 6 184 25 -----Ay, Freq Spread (14) 9.0 26 0 26 42

72.625

98.125

142 25

73

88.5 95

113.25 113 825

142 25

Av, Freq.

(Hz)

21.5

35.75

50 25

62

73.25

99 5

30.75

48 25

93.5

120.5

25

63.25

80 5

87.25

145.25

NATION NO.

not

done

8.125

30.75

65.5

118.5

146.5

182.25

0.625

29

not

done

48 25

65.75

119.75

148.75

92

ふうかん ひかんない ちかく だんなからう ひょうかん うちょうかん

29

not

fone

45.75

21.5

35 785

62.25

73 5

61

99.5

8.25 8 125

30.75 30 625

48.25

65,75

92.75

120 25

147

28 5

「中心」を行うたいないための「日本の日本の日本のなかたであ

35 75

50.5

62 25

73 5

81

92.5

09.5

8.25

30.75

48 25

65.75

119.75

- 178.625

147

711

29

46

63

81 87

7.75

26,5

45

79

68.25

101.25

114.75

61.75

101.75

144 625

21.5

50.25

62 25

73.5

81

995

8 25

30.5

48.25

65,75

93 92 25

120

148.75

7.0

21.5

35.8

50.4

52.2

73.5

81.0 81

92.6

Ay, Freq.

(H2)

8.2

30.7 45.3

65.8

92.6

119.9

145.0

180.4 210 ft

100 1000

AV. FIRS

(Hz)

9.6

28.9 46.0 63.1 80.6

87.1

101.4

145.1

Av. Fred

Spread

(He)

0 25

0 035

0 25

0.25

0.25

0.75

SHE

Spread

(14)

0.25

0.25

0.5

1.5

05

3.625

0 375

Spread

(Ht) 0.25

0.5

0.875

0.625

0.875

Spread

(Hz)

0.25

0.25

0.5

0.5

05

12.01

(Hz)

0.25

0.25

0.25

0.25

0 375

0.375

56 .

72.8

88.7

98.2

113.5

0.5

-				- 2
	9	not	9	P
	26	done	26	28
	42		42.25	42
	58 25		56 25	58.25
	72 75		72 75	2.75
	BB 75		88.75	8 75
	98 25		98 25	8 25
	113.5		113.5	13.5
	142.5			-

Value Used

21 5

35 75

73.5

92.5

99.5

8.25

30.75

65.75

Value Used

29 46

63

80.5

87 87.125

101.5

7.75

26 5

88.25

101.5

114,75

164.5

Value Used

nol

45

145.25

92.5 **B2 125** 

119.75 119.875

46.5

Value Used

81

50 375

done

8.75

30.75

46 25

65.75

147

0.5

29

63

80.5

7.75

26 5

61.75

88.25

101.5

114,75

79 78.75

45

101.125

46 25

210.75 210.625

NO. OF THE OWNER WATCHING

21.5 21.25

35.75

62

81

ØZ 25

99.5

8.25

30.75

65.75

92.75 92 375

14

147

29

63

80.5

86.75

101 25

145.25

26.5

61.75

101.5

114,75

45

45.75

50.375

not

done

A.A 25

30,75

65.5

92 25

1195

148.5

182.25

0.5-9.75

29

46

63 5

81

86.75

144.75

7.75

26.5

45

61.75

78.75

88.25

101.5

164.5

28

42

88.5 88 5

98 98-98 25

113.25 .25-113.5

142 25 142 25

115 25

21 5

50 25

62 25

73.5

99.5

8 25

30.5

46 25

65.75

93

120

2.5

29

46

62 5

80 5

67-87.25

145.5

7.75

26.5

81.75-62

88-88 25

101 5

42 25

72.75

88.75

98 25

113.5

142 25

56

115

45

79

148 75

White an experiment

81

21.5

35.75

50.25

73.25

62 62 25

81

99.5

30,75

40.25

R3.5

120.5

147 146.75

200'

29

63.25

80.5

87.25

145 25

106.5 166.625

7.75

26.5

45

62 61,75

79 78,75

88.5

101,75

164 25

20 42 done

56 56 25 56 73 2 5-72 75

not

done

8-8.25

48.25

65.75

119.75

148.75

92

05

29

63

45.75

80.75

87.25

145-145.5

「市場」は、読まれた日本

7.15

26 5

45

62

78.75

88.5

101.5

not

done

資料の設計はたい

115

185 75-100

30.5-30.75

21.5

35,785

50 25°

62 25

73.5

81

93

99.5

8.25

30,75

46 25

85,75

82.75

120.25

147

0.5

48

28.5

63 25

87.25

144.75

7.75

26.5

61.75

78,75

87.5

101.5

114.75

104.5

28

42

58.25

72,75

88.75

98.25

113.5

142.5

45

80-80 25

21.25

35 75

50 5

62 25

735

81

92.5

99.5

8.25

30.75

48.25

65.75

119.75

147

211

CREATE SHITE

0.5

29

46

63

81

87

101.75

188.75

7.75

26.5

61.75

88.25

101.25

114.75

164 75

OSTA P

not

done

78

45

145 25 144 5-144 75

174.25\* 173.75-174\* 178.5-179.25 179.25-179.75\* 179-179.75\* 180.25-180.75\*

ENDERING THE PATHER AND

92 75 92 25-92 5

### **RESULTS FROM TESTING OF ALGORITHM**

NOTES: These results have used the spring constant X = 378000 N/m The surped parameter model has been used: i.e. one third of the spring mass has been tumped at the nodes All entury marked weth a " either had poor coherence (balver 65%) or were poorly defined in the Frequency Responce Function Entures labled "not dons" were not done because the results were enture action and the results had poor coherence for several entures All lambde values below have been multipled by the constant E(-5)

	Al lambda v		
  |  |   |   
  |  |  |   |  
  |  |   |   
  |  |  |  |   
  |  |  |  |  |  
   |   |   |   
  |
|---|---|--|---
--	--	---
---	---	--
---	--	--
--	--	--
--	--	--
---	---	--
perment	6	-
  | 10   | Experiment  | at Results (Hz)   
  |  |  |   | |
  |  | -   |   
  |  | Value Used   |  |   
  |  |  |  |  |  
   |   | 100000000000000000000000000000000000000   | Spread (Hz)   
  |
|   | Önsred  | 1  | Given by alg  | orthm   
  | 1  | mass at   | Stuking Location  
  |  |  |   |  
  | 6  |   | 8   
  | 0  | 1  | 2  | 3   
  | 4  | 5  | 6  | 1  | 8  
   |   | 95  | (54)  
  |
|   | A ((ad/s)   | (Hz)   | A (rad/s)   | 1(012)  
  | m (kg)   | noda (kg)   | 1   
  | 2  | 3  | 95  | 95   
  | 25   | 95  | 95  
  | 95   | net  | 95   | 95  
  | 95   | 95   | 95   | 95   | 0 5<br>31 25   
   | 95  | 313   | 0 25  
  |
| 1   | 0.050   | 11 25  | 0 046   | 10.75   
  | 3 900  | 3 498   | not   
  | 95   | 95   | 31 25   | 31 25  
  | 31.5   | 31 25   | 31 25   
  | 31 25  | done   | 31 25  | 31 25   
  | 31 25  | 31 25 48   | 31.5   | 48 25  | 48   
   | 46  | 48 0  | 0.25  
  |
| 2   | 0 500   | 35 59  | 0 471   | 34 54   
  | 2.419  | 2 017   | done  
  | 31 25  | 31 25  | 48  | 48   
  | 48   | 48 25   | 48  
  | 48   |  | 48   | 48  
  | 48<br>65 75  | 48   | 65 75  | 65 75  | 85 875   
   | 65 75   | 65 8  | 0 125   
  |
| 2   | 1 000   | 50 33  | 0 972   | 49 52   
  | 1,960  | 1.558   |   
  | 65.75  | 65.75  | 65.75   | 65,75  
  | 65 75  | 85 75   | 65 75 66  
  | 65 75  |  | 65 75  | 65 75   
  | 81 625   | 82   | 62   | 82 875   | 82 25  
   | 81 75   | 82 0  | 1 25  
  |
| 4   | 1 750   | 66 58  | 1.777   | 67 09   
  | 2 421  | 2.019   |   
  | 82   | 81.75  | 81.5-81.75  | 82   
  | 82   | 81,75-82  | 82 25   
  | 81,75  |  | 52   | 98  
  | 98   | 98 25  | 98   | 98   | 97.75  
   | 96  | 98.0  | 05  
  |
| 5   | 2 500   | 79 55  | 2 641 3 669   | 81 79<br>96 40  
  | 3 472  | 3 070   |   
  | 98   | 98   | 98  | 98 25  
  | 98   | 98  | 87 75   
  | 105.5  |  | 106 25   | 106.5   
  | 106 25   | 106 375  | 105.5  | 106.75   | 106 25   
   | 106 5   | 106.4   | 0.5   
  |
| 6   | 3 500   | 100.66   | 4 247   | 103.72  
  | 2.687  | 2 285   |   
  | 108 25   | 106.5  | 106 25  | 106 25-106 5   
  | 106 5  | 106 75  | 108 25  
  | 100.5  |  | 129.75   | 129 75  
  | 129 875  | 130 25   | and the second   | ¥ .  |  
   |   | 129.9   | 0 5   
  |
| 1   | 8 000   | 123 28   | 6 157   | 124 89  
  | 1,362  | 0.960   |   
  | 129 75   | 129 75   | 129 75-130  | 130 25   
  | 130.25*<br>146 25  | 146-145 25  | 145 75-146  
  | 140  |  | 146 25   | 148 25  
  | 146 25   | 148 25   | 145 25   | 148 125  | 145 875  
   | 146   | 145 2   |   
  |
| 0   | 8 000   | 142.35   | 8.317   | 145.14  
  | 1.076  | 0.877   |   
  | 148 25   | 140.25   | 148 25  | 145 25   
  | 140 23   | B (*84 LOS 5 C 4  | CONTRACTOR OF   
  | NUMBER OF  | 0.65.2 96  | A CERTIFICATION OF   | 心运作实际情况   
  | 的时间  | t also to t  | 1.1  | 1日月1日日   | 5.1010111111111  
   | 化的现代  | TUNE  |   
  |
| Sector  | Ingeneral.  | 11.114-14  | 10 10 10 10   | Link ports  
  | · Carlesteril  | er den se   | 10000000000000000000000000000000000000  
  | 564 634/040  | 山市省省市市   | A PROVINCE  | ISTO-HINDRALH UN   
  | NAMES OF STREET  | COLUMN TO PO  | | |
  |  |  |  |   
  | _  |  |  | _  |  
   | LAN   | v. Freq   | Spread  
  |
| perment   | 1   |  |   |   
  |  | Experiment  | In Resource (rich   
  | _  |  |   | 10000  
  |  |   |   
  |  | Value Used   |  |   
  |  |  |  | 2  | 8  
   |   | (Hz)  | (22)  
  |
| lode  | Desred  |  | Given by alg  |   
  |  |   | Striking Location   
  |  | 3  | 4   | 5  
  | 6  | 1   | 8   
  | 9  | 1  | 2  | 3   
  | 4  | 5  | 95   | 95   | 95   
   | 95  | 0.5   | 0   
  |
|   | A (rad/s)   | (Htt)  | A (/ad/s)   | 1(Hz)   
  |  | node (kg)   |   
  | 2  | 9.5  | 95  | 9.5  
  | 95   | 95  | 9.5   
  | 9.5  | 95   | 9.5  | 9.5   
  | 95   | 95   | 31 25  | 31.5   | 31.5   
   | 31.5  | 31.5  | 0 25  
  |
| 1   | 0.050   | 11.25  | 0.048   | 10 58   
  | 4,413  | 4 011   | 95<br>31.5  
  | 315  | 31.5   | 31.5  | 31,5   
  | 31 25  | 31.5  | 315   
  | 31.5   | 31.5   | 315  | 315   
  | 48 5   | 48 5   | 48.5   | 48   | 48 5   
   | 48 5  | 40.4  | 05  
  |
| 2   | 0,500   | 35 59  | 0.451   | 33 61   
  | 1.734  | 1.332   | 48.5  
  | 48.5   | 48.5   | 48 5  | 48 5   
  | 48 5   | 48  | 48,5  
  | 48.5   | 48 5   | 48 5   | 68  
  | 68   | 66   | 68   | 66   | 68   
   | 65 75   | 66 0  | 0 25  
  |
| 3   | 1,000   | 50 33<br>66 58   | 0 994   | 50 17<br>67 79  
  | 1 607  | 1.205   | 66  
  |  | 66   | 66  | 68   
  | 66   | 66  | 68  
  | 65.75<br>78  | 78   | 78   | 78  
  | 78   | 78 25  | 77.75  | 78 125   | ×.   
   | 78  | 78.0  | 05  
  |
| 1   | 1,750   | 75.49  | 2 399   | 77.95   
  | 3.421  | 3 019   | 78  
  | 78   | 76   | 78  | 78.25  
  | 77.75  | 78-78.25  | 109.5   
  | 110  | 110  | 109.75   | 109 75  
  | 109 75   | 109 875  | 109 75   | 110  | 109 5  
   | 110   | 109 8<br>120 4  | 05  
  |
| 0   | 4 500   | 105 76   | 4.502   | 106 79  
  | 2.171  | 1,769   | 110   
  | 109.75   | 109.75   | 109.75  | 109,75-110   
  | 109 75   | 121   | 120   
  | 120 5  | 120 375  | 120 25   | 120 5   
  | 120 125  | 120 5  | 120 625  | 121  | 120  
   | 120.5   | 129.5   | 05  
  |
| 7   | 5 250   | 115 32   | 5 380   | 116 73  
  | 3,331  | 2 929   | 120 25-120 5  
  | 120 25   | 120 5  | 120-120 25  | 120,5  
  | 120 5-120 75   | 121   |   
  |  | 139.5  | 139 25   | 139 5   
  | 139 25   | 139 75   |  | 144 5  | 144 25   
   | 144 25  | 144.0   | 0.75  
  |
| á   | 7.000   | 133 10   | 7.116   | 134 28  
  | 1.376  | 0.974   | 139.5   
  | 139 25   | 139 5  | 139.25  | 144.75   
  | 144.5  | 144.5   | 144.25  
  | 144.25   | 145  | 144.75   | 145   
  | 144.5  | 144.75   | 144.5  | 144.0  | ANT AM IN 192  
   | 10.000  |   | Acces   
  |
| 9   | 7.500   | 137 83   | 7.669   | 139.38  
  | 1,378  | 1.175   | 145   
  | 144.75   | 145  |   |  
  |  | LA TONI - APA   | STAN TAK  
  | Vare -   | 的家姓东西  | = 1.000  | NOV SELECT  
  | SUPPLIE  | C. MILLORD   | 3.649.24   | CHILL PRIME  | 0,107,100  
   |   |   |   
  |
| 10.72 4.83  | 20 404 KM   | Station 1  | 同時度の設定  | <b>以外加州</b> 1月  
  | NUMBER OF  | COLUMN SH   | tal Results (Hz)  
  | 64241PH844   | ALC: NO. APRIL   | ALC: N.L. OLLOW   |  
  |  |   |   
  |  |  |  | _   
  |  |  |  |  |  
   | 1   | v Freq  | Spread  
  |
| Experimen   |   |  |   | a ball and  
  |  | mass at   | Striking Location   
  |  |  |   |  
  |  |   |   
  |  | Value Used   | 2  |   
  | 4  | 5  | 6  | 7  | 8  
   | 9   | (Hz)  | (142)   
  |
| lode  | Desired   |  | Given by als  | 1(14)   
  | m (kg)   | node (kg)   | distanting Recently in  
  | 2  | 3  | 4   | 5  
  | 6  | 1   | 0   
  | - 11   | not  | 8.75   | 8.75  
  | 9  | 8.75   | 8.75   | 8.75   | 8,75   
   | 8.75  | 8.8   | 0 25  
  |
| <u> </u>  | A (rad/s)   | (HJ)<br>10.07  | A (rad/s)   | 10 05   
  | 6.931  | 6,529   | ing!  
  | 8.75   | 8.75   | 9   | 8.75   
  | 0.75   | 8.75  | 8,75  
  | 8.75<br>29.25  | dona   | 29 25  | 29.25   
  | 29 25  | 29 25  |  | 29 25  | 29.25  
   | 29 25   | 29 3  | 2   
  |
| 1   | 0.400   | 31.83  | 0 397   | 31.69   
  | 1.898  | 1,498   | done  
  | 29 25  | 29 25  | 29 25   | 29 25  
  | 29*<br>42  | 29 25<br>42 25*   | 29 25<br>42   
  | 42   | 00110  | 42   | 201   
  | 42   | 42   | 42   | 1.5  | 42   
   | 42  | 42.0  | 0 25  
  |
| 1   | 0.750   | 43 59  | 0.758   | 43.82   
  | 1.774  | 1.371   |   
  | 42   | 41.75*   | 42  | 42   
  |  |   |   
  |  |  | 61.75  | 61.75   
  | 81.75  |  | 61.75  | 61.75  | 61 75  
   | 61.5  |   |   
  |
1			
  |  |   |   
  |  |  |   |  
  |  | 61.75   | 61.75   
  | 61.5   |  |  |   
  |  |  |  | 37.00  | 80.76  
   | 80.53   |   |   
  |
|   | 1.500   | 61.64  | 1.490   | 61,44   
  | 2.075  | 1 673   |   
  | 61.75  | 61.75  | 61,75   | 61,25°<br>80,75  
  | 61 75  | 61,75   | 81.75   
  | 60.5   |  | 80.75  | 80 5  
  | 80 75  | 80 75  | 80 75  | 80.75  | 80.75  
   | 80.5  | 80.7  | 0 25  
  |
| 5   | 1,500   | 61.64<br>79.55   | 1.490<br>2.486  | 61.44<br>79.36  
  | 2.075<br>3.496   | 1.673<br>3.094  |   
  | 80.75  | 80 5   | 61,75<br>80,75<br>87  | 61,25*<br>80,75<br>96,75   
  |  |   |   
  | 60.5<br>90.75  |  | 80.75<br>97  | 80 5<br>97  
  | 80.75<br>97  | 96.75  | 97   | 87   | 80 75<br>96 75<br>107 75   
   | 80.5<br>96.75<br>108.5  |   |   
  |
| 5   | 2.500<br>3.500  | 79 55<br>84.15   | 1 490<br>2 486<br>3 510   | 61,44<br>79,36<br>94,29   
  | 2 075<br>3 496<br>4 230  | 1 873<br>3 094<br>3 827   |   
  | 80,75<br>97  | 80 5<br>97   | 80.75   | 80.75  
  | 80 75  | 80,75<br>97<br>108 5  | 80.75<br>96.75<br>107.75  
  | 60.5<br>96.75<br>108.5   |  | 80,75<br>97<br>108 25  | 80 5<br>97<br>108 375   
  | 80 75<br>97<br>107 875   | 96.75<br>108   | 97<br>108 25   |  | 96.75  
   | 90.75   | 96.9<br>108.2<br>124.8  | 0 25<br>0 75<br>0 75  
  |
| 5 6 7   | 2 500<br>3 500<br>4 500   | 79 55<br>94.15<br>105 76   | 1.490<br>2.486<br>3.510<br>4.499  | 61.44<br>79.36<br>94.29<br>106.76   
  | 2 075<br>3 498<br>4 230<br>2 628   | 1.873<br>3.094<br>3.827<br>2.426  |   
  | 80,75<br>97<br>108 25 1  | 80 5   | 80.75<br>87   | 80.75<br>96.75<br>108<br>125   
  | 80 75<br>97  | 80,75<br>97<br>108 5<br>124,75  | 80 75<br>96 75<br>107 75<br>124 75  
  | 60.5<br>96.75<br>108.5<br>124.75   |  | 80 75<br>97<br>108 25<br>124 25  | 80 5<br>97<br>108 375<br>125  
  | 80 75<br>97<br>107 875<br>125  | 96.75<br>108<br>125  | 97<br>108 25<br>124 75   | 97<br>108 5<br>124 75  | 96.75<br>107.75<br>124.75<br>136.875   
   | 90.75<br>108.5<br>124.75  | 96.9<br>108.2<br>124.8<br>136.5   | 0 25<br>0 75<br>0 75<br>0 625   
  |
| 5 6 7 8   | 2 500<br>3 500<br>4 500<br>6 000  | 79 55<br>94.15<br>108 76<br>123 28   | 1 490<br>2 486<br>3 510<br>4 499<br>6 009   | 61 44<br>79 36<br>94 29<br>106 76<br>123 37   
  | 2 075<br>3 496<br>4 230<br>2 628<br>2 407  | 1 873<br>3 094<br>3 827<br>2 426<br>2 005   |   
  | 80 75<br>97<br>108 25 1<br>124 25  | 80 5<br>97<br>08 25-108 5<br>125<br>136 5  | 80.75<br>97<br>107.75-108<br>125<br>136.25  | 80.75<br>96.75<br>108<br>125<br>136.25   
  | 80 75<br>97<br>108 25<br>124 75  | 80,75<br>97<br>108 5<br>124,75<br>136 5*  | 80.75<br>96.75<br>107.75<br>124.75<br>136.75-137*   
  | 60.5<br>96.75<br>108.5<br>124.75<br>136.75*  |  | 80 75<br>97<br>108 25<br>124 25  | 80 5<br>97<br>108 375<br>125  
  | 80 75<br>97<br>107 875<br>125  | 96.75<br>108<br>125  | 97<br>108 25<br>124 75   | 97<br>108 5<br>124 75  | 96.75<br>107.75<br>124.75<br>136.875   
   | 90.75<br>108.5<br>124.75  | 96.9<br>108.2<br>124.8<br>136.5   | 0 25<br>0 75<br>0 75<br>0 625   
  |
| 5<br>6<br>7<br>8<br>9   | 2 500<br>3 500<br>4 500<br>6 000  | 79 55<br>94.15<br>108 76<br>123 28   | 1 490<br>2 486<br>3 510<br>4 499<br>6 009   | 61 44<br>79 36<br>94 29<br>106 76<br>123 37   
  | 2 075<br>3 496<br>4 230<br>2 628<br>2 407  | 1 873<br>3 094<br>3 827<br>2 426<br>2 005   | aller a le contrati a succession de la contrati de  
  | 80 75<br>97<br>108 25 1<br>124 25  | 80 5<br>97<br>08 25-108 5<br>125<br>136 5  | 80.75<br>97<br>107.75-108<br>125<br>136.25  | 80.75<br>96.75<br>108<br>125<br>136.25   
  | 80 75<br>97<br>108 25<br>124 75  | 80,75<br>97<br>108 5<br>124,75  | 80.75<br>96.75<br>107.75<br>124.75<br>136.75-137*   
  | 60.5<br>96.75<br>108.5<br>124.75<br>136.75*  | ध्ये-कोर्य्याल्य मे  | 80 75<br>97<br>108 25<br>124 25  | 80 5<br>97<br>108 375<br>125  
  | 80 75<br>97<br>107 875<br>125  | 96.75<br>108<br>125  | 97<br>108 25<br>124 75   | 97<br>108 5<br>124 75  | 96.75<br>107.75<br>124.75<br>136.875   
   | 90.75<br>108.5<br>124.75  | 96 9<br>108 2<br>124 8<br>136 5   | 0 25<br>0 75<br>0 75<br>0 625   
  |
|   | 2 500<br>3 500<br>4 500<br>6 000<br>7 000   | 79 55<br>94.15<br>108 76<br>123 28   | 1 490<br>2 486<br>3 510<br>4 499<br>6 009   | 61 44<br>79 36<br>94 29<br>106 76<br>123 37   
  | 2 075<br>3 496<br>4 230<br>2 828<br>2 407<br>1.087   | 1 673<br>3 094<br>3 827<br>2 426<br>2 005<br>0 866  | al Results () (2)   
  | 80 75<br>97<br>108 25 1<br>124 25  | 80 5<br>97<br>08 25-108 5<br>125<br>136 5  | 80.75<br>97<br>107.75-108<br>125<br>136.25  | 80.75<br>96.75<br>108<br>125<br>136.25   
  | 80 75<br>97<br>108 25<br>124 75  | 80,75<br>97<br>108 5<br>124,75<br>136 5*  | 80.75<br>96.75<br>107.75<br>124.75<br>136.75-137*   
  | 80.5<br>96.75<br>108.5<br>124.75<br>136.75*  |  | 80 75<br>97<br>108 25<br>124 25  | 80 5<br>97<br>108 375<br>125  
  | 80 75<br>97<br>107 875<br>125  | 96.75<br>108<br>125  | 97<br>108 25<br>124 75   | 97<br>108 5<br>124 75  | 96.75<br>107.75<br>124.75<br>136.875   
   | 90.75<br>108.5<br>124.75  | 96.9<br>108.2<br>124.8<br>136.5<br>136.5<br>137.453-35  | 0 25<br>0 75<br>0 75<br>0 625<br>7455 5 - 4<br>Spread   
  |
| Experimen   | 2 500<br>3 500<br>4 500<br>6 000<br>7 000<br>4 43 5 500   | 79 58<br>94 15<br>106 76<br>123 28<br>133 16   | 1 490<br>2 466<br>3 510<br>4 499<br>6 009<br>7 043  | 61,44<br>79,36<br>94,29<br>106,76<br>123,37<br>133,56   
  | 2 075<br>3 496<br>4 230<br>2 828<br>2 407<br>1.087   | 1 673<br>3 094<br>3 827<br>2 426<br>2 005<br>0 866  | elaca a storin sua<br>nal Results (Hz)<br>Straing Location  
  | 80,75<br>97<br>108 25 1<br>124 25<br>138 5   | 80 5<br>97<br>08 25-108 5<br>125<br>136 5  | 80.75<br>87<br>107.75-108<br>125<br>136.25  | 80,75<br>96,75<br>108<br>125<br>130,25   
  | 80 75<br>97<br>108 25<br>124 75  | 80,75<br>97<br>108 5<br>124,75<br>136 5*  | 80.75<br>96.75<br>107.75<br>124.75<br>136.75-137*   
  | 80.5<br>96.75<br>108.5<br>124.75<br>136.75*  | 와 아이에게 사<br>Value Used<br>1  | 80 75<br>97<br>108 25<br>124 25  | 80 5<br>97<br>108 375<br>125  
  | 80.75<br>97<br>107.875<br>125<br>130.25  | 96.75<br>108<br>125<br>136.25  | 97<br>108 25<br>124 75   | 67<br>108 5<br>124 75  | 96.75<br>107.75<br>124.75<br>136.875<br>2.5 - Hoke<br>8  
   | 90.75<br>108.5<br>124.75  | 96 9<br>108 2<br>124 8<br>136 5<br>136 5<br>136 5<br>136 7<br>124 8<br>136 5  | 0 25<br>0 75<br>0 75<br>0 625<br>785 2 6 - 4  
  |
|   | 2 500<br>3 500<br>4 500<br>6 000<br>7 000<br>6 00 5 5 5 5 5<br>7 00<br>0 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5  | 79 58<br>94 15<br>106 76<br>123 28<br>133 16   | 1 490<br>2 486<br>3 510<br>4 499<br>6 009   | 61,44<br>79,36<br>94,29<br>106,76<br>123,37<br>133,56   
  | 2 075<br>3 496<br>4 230<br>2 828<br>2 407<br>1 087<br>71.007<br>71.007   | 1 873<br>3 094<br>3 827<br>2 428<br>2 005<br>0 888<br>Experiment  | in ressous tras   
  | 80,75<br>97<br>108 25 1<br>124 25<br>138 5   | 80 5<br>97<br>08 25-108 5<br>125<br>136 5  | 80.75<br>87<br>107.75-108<br>125<br>136.25  | 80.75<br>96.75<br>108<br>125<br>130.25   
  | 80 75<br>97<br>108 25<br>124 75  | 80,75<br>97<br>108 5<br>124,75<br>136 5<br>(4)1-94 s-1291   | 80.75<br>96.75<br>107.75<br>124.75<br>136.75-137*   
  | 80.5<br>96.75<br>108.5<br>124.75<br>136.75*  |  | 80.75<br>97<br>108.25<br>124.25<br>136.5<br>   | 80 5<br>97<br>108 375<br>125<br>138 5<br>138 5<br>137 5<br>138 5<br>138 5<br>137 5<br>137 5<br>138 5<br>137 | 80.75<br>97<br>107.875<br>125<br>136.25<br>136.25<br>136.25<br>136.25   
  | 96.75<br>108<br>125<br>136.25<br>136.25<br>5<br>7.5  | 97<br>108 25<br>124 75<br>6<br>7.5   | 87<br>108 5<br>124 75  | 96.75<br>107.75<br>124.75<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875  | 90.75<br>108.5<br>124.75<br>   
  | 96.9<br>108.2<br>124.8<br>136.5<br>100.412-50<br>NV Freq<br>(Hz)<br>7.5   | 0 25<br>0 75<br>0 75<br>0 625<br>74556-4<br>Spread<br>(H2)<br>0  |
| Experimen   | 2 500<br>3 500<br>4 500<br>6 000<br>7 000<br>4 43 5 500   | 79 58<br>94.15<br>106 76<br>123 28<br>133 16<br>133 16<br>139(9)(C h<br>1)(Hy)<br>8.72   | 1 490<br>2 486<br>3 510<br>4 499<br>8 009<br>7 043<br>4 2 5 6 7 8 6<br>0 021  | 61.44<br>79.36<br>94.29<br>106.76<br>123.37<br>133.56<br>   
  | 2 075<br>3 496<br>4 230<br>2 628<br>2 407<br>1.067<br>71.067<br>71.067<br>71.067<br>71.067<br>71.067<br>71.067<br>71.067<br>71.067<br>71.067   | 1.873<br>3.094<br>3.827<br>2.426<br>2.005<br>0.868<br>Start pub<br>Experiment<br>mass et<br>node (kg)<br>1.800  | in ressous tras   
  | 80.75<br>97<br>108 25 1<br>124 25<br>138 5<br>138 5<br>126 1<br>138 5<br>138 5<br>110<br>1101<br>1015<br>100<br>100<br>100000000000000                | 80 5<br>97<br>08 25-108 5<br>125<br>136 5<br>136 5<br>136 5<br>136 5<br>136 5<br>136 5<br>136 5<br>136 5<br>136 5<br>137 5   | 80.75<br>97<br>107.75-108<br>125<br>136.25<br>136.25  | 80.75<br>96.75<br>108<br>125<br>130.25<br>130.25<br>130.25<br>130.25<br>130.25<br>130.25<br>130.25  | 80 75<br>97<br>108 25<br>124 75   
  | 80,75<br>97<br>108 5<br>124,75<br>136 5*  | 80.75<br>96.75<br>107.75<br>124.75<br>136.75-137*<br>14.55-137*  | 80.5<br>96.75<br>108.5<br>124.75<br>136.75<br>136.75<br>136.75<br>136.75<br>136.75<br>136.75<br>136.75<br>136.75<br>136.75<br>136.75   |  
   | 80.75<br>97<br>108.25<br>124.25<br>136.5<br>24.75<br>24.75   | 80 5<br>97<br>108 375<br>125<br>136 5<br>5<br>33 9 + 7<br>3<br>7 5<br>24 75  | 80.75<br>97<br>107.875<br>125<br>136.25<br>7.13.71<br>4<br>7.5<br>24.75   
  | 96.75<br>108<br>125<br>136.25<br>136.25<br>5<br>7.5<br>24.75   | 97<br>108 25<br>124 75<br>6<br>7.5<br>24 75  | 87<br>108 5<br>124 75<br>7<br>7<br>7 5<br>25   | 96.75<br>107.75<br>124.75<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875   | 90.75<br>108.5<br>124.75   
  | 96 9<br>108 2<br>124 8<br>136 5<br>136 5<br>136 5<br>136 7<br>124 8<br>136 5  | 0 25<br>0 75<br>0 625<br>74 5 5 4<br>5 5 7 6 4<br>(H2)<br>0<br>0 25<br>0 25  |
| Experimen   | 2 500<br>3 500<br>4 500<br>6 000<br>7.000<br>4 14 1 502<br>1 9<br>Desred<br>A (rad/s)   | 79 55<br>94.15<br>108 76<br>123 28<br>133 16<br>133 16<br>134 94 57<br>8 72<br>27 57   | 1 490<br>2 486<br>3 510<br>4 499<br>8 009<br>7 043<br>4 2 5 7 8 6<br>6 040<br>7 043<br>4 2 5 7 8 6<br>8 040<br>7 043<br>4 2 5 7 8 6<br>8 040<br>7 043<br>4 2 5 7 8 6<br>8 049<br>7 043<br>4 2 5 7 8 6<br>8 049<br>7 043<br>4 2 5 7 8 6<br>7 043<br>4 2 5 7 8 7 8 6<br>7 043<br>4 2 5 7 8 7 8 7 8 7 8 7 8 7 8 7 8 7 8 7 8 7  | 61.44<br>79.36<br>94.29<br>106.76<br>123.37<br>133.56<br>   
  | 2 075<br>3 496<br>4 230<br>2 628<br>2 407<br>1.687<br>71.47 18,05<br>m (kg)<br>2 202<br>2 760  | 1.673<br>3.094<br>3.827<br>2.426<br>2.005<br>0.886<br>5.944 (pab)<br>Experiment<br>mass et<br>node (kg)<br>3.800<br>2.378   | in ressous tras   
  | 80.75<br>97<br>108.25<br>124.25<br>138.5<br>7.6<br>7.5<br>24.75  | 80 5<br>97<br>108 25-108 5<br>136 5<br>136 5<br>136 5<br>136 5<br>136 5<br>136 5<br>136 5<br>7.5<br>24 75  | 80.75<br>87<br>107.75-108<br>125<br>136.25  | 80.75<br>96.75<br>108<br>125<br>130.25   
  | 80 75<br>97<br>108 25<br>124 75<br>124 75<br>0<br>7 5  | 80,75<br>97<br>108 5<br>124.75<br>136 5<br>44.44<br>44.524<br>7<br>7,5<br>7,5<br>25<br>42 5-42,75   | 80.75<br>96.75<br>107.75<br>124.75<br>136.75-137<br>150:5-4<br>7.5<br>25<br>42.75   
  | 80.5<br>96.75<br>108.5<br>124.75<br>136.75<br>136.75<br>136.75<br>136.75<br>136.75<br>136.75<br>136.75<br>136.75<br>136.75<br>136.75<br>136.75<br>136.75<br>136.75<br>136.75<br>136.75   |  | 80.75<br>97<br>108.25<br>124.25<br>136.5<br>34.5<br>24.75<br>42.625  | 80 5<br>97<br>108 375<br>125<br>138 5<br>138 | 80.75<br>97<br>107.875<br>125<br>136.25<br>136.25<br>4.75<br>24.75<br>42.5   
   | 96.75<br>108<br>125<br>136.25<br>5<br>7.5<br>24.75<br>42.75  | 97<br>108 25<br>124 75<br>6<br>7.5<br>24 75<br>42 75   | 87<br>108 5<br>124 75  | 96.75<br>107.75<br>124.75<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875  | 90.75<br>108.5<br>124.75<br>200,0334<br>9<br>7.5<br>24.75   
   | 96.9<br>108.2<br>124.8<br>136.5<br>w.Freq<br>(Hz)<br>7.5<br>24.8<br>42.7<br>61.3  | 0 25<br>0 75<br>0 625<br>0 625<br>0 625<br>0 625<br>0 625<br>0 25<br>0 25<br>0 25<br>0 25  |
| Experimen   | 2 500<br>3 500<br>4 500<br>6 000<br>7 000<br>7 000<br>Charleboll<br>19<br>Desred<br>A (rad/s)<br>0 030<br>0 300<br>0 750  | 79 56<br>94.15<br>106 76<br>123 26<br>133 16<br>3399570 %  | 1 490<br>2 486<br>3 510<br>4 499<br>6 009<br>7 043<br>1 2 5 6 7 4 9<br>A (red/s)<br>0 031<br>0 299<br>0 742   | 61,44<br>79 36<br>94 29<br>106,76<br>123 37<br>133 56<br>133 56<br>143 35   
  | 2 075<br>3 496<br>4 230<br>2 628<br>2 407<br>1.067<br>TL& IE(2)<br>TL& IE(2)<br>0<br>2 202<br>2 760<br>3 459   | 1.873<br>3.094<br>3.827<br>2.426<br>2.005<br>0.886<br>5.446<br>Esperamen<br>mass et<br>node (kg)<br>5.800<br>2.378<br>3.067   | in ressous tras   
  | 80.75<br>97<br>108 25 1<br>124 25<br>138 5<br>7<br>20 100 100 100<br>2<br>7.5<br>24.75<br>42 5-42 75   | 80 5<br>97<br>08 25-108 5<br>125<br>136 5<br>136 5<br>136 5<br>136 5<br>136 5<br>136 5<br>136 5<br>136 5<br>136 5<br>137 5   | 80.75<br>97<br>107.75-108<br>125<br>136.25<br>136.25<br>4<br>7.5<br>24.75   | 80.75<br>96.75<br>108<br>125<br>130.25<br>5<br>5<br>7.5<br>24.75   
  | 80 75<br>97<br>108 25<br>124 75<br>144 75<br>6<br>7 5<br>24 75<br>61.5   | 80,75<br>97<br>108 5<br>124,75<br>136 5*<br>74 136 5*<br>7,5<br>25<br>42 5-42 75<br>61,25   | 80.75<br>96.75<br>107.75<br>124.75<br>136.75-137<br>136.75-137<br>15<br>25<br>25<br>42.75<br>61.25  
  | 80.5<br>96.75<br>108.5<br>124.75<br>136.75<br>136.75<br>136.75<br>136.75<br>24.75<br>24.75<br>42.5<br>61.25  |  | 80.75<br>97<br>108 25<br>124 25<br>136 5<br>136 5<br>24 75<br>42 625<br>61 25  | 80 5<br>97<br>108 375<br>125<br>138 5<br>5<br>5<br>5<br>5<br>5<br>7<br>5<br>24 75<br>42 75<br>61 5  
  | 80.75<br>97<br>107.875<br>125<br>130.25<br>130.25<br>44.75<br>44.75<br>42.5<br>61.25   | 96.75<br>108<br>125<br>136.25<br>5<br>7.5<br>24.75<br>42.75<br>61.25   | 97<br>108 25<br>124 75<br>6<br>7.5<br>24 75  | 87<br>108 5<br>124 75<br>7<br>7<br>7 5<br>25<br>42 625   | 96.75<br>107.75<br>124.75<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875  
   | 90.75<br>108.5<br>124.75<br>24,75<br>42.5   | 96.9<br>108.2<br>124.8<br>136.5<br>104.402-55<br>104.402-55<br>(Hz)<br>7.5<br>24.8<br>42.7<br>61.3<br>75.3  | 0 25<br>0 75<br>0 625<br>0 625<br>0 625<br>0 625<br>0 625<br>0 25<br>0 25<br>0 125  
  |
| Experimen   | 2 500<br>3 500<br>4 500<br>6 000<br>7 000<br>4 500<br>7 000<br>7 000<br>1 000<br>0 0000<br>0 0000<br>0 000<br>0 000<br>0 000<br>0 000<br>0 000<br>0 000<br>0 000  | 79 55<br>94,15<br>106 76<br>123 26<br>133 16<br>3595577 %<br>11H2)<br>8,72<br>27,57<br>43 59<br>61 64  | 1 490<br>2 486<br>3 510<br>4 499<br>8 009<br>7,043<br>4 2 5 6 6 6<br>1 6 7 45<br>1 (red/s)<br>0 031<br>0 031<br>0 299<br>0 742<br>1 493   | 61.44<br>79.36<br>94.29<br>106.76<br>123.37<br>133.56<br>7.33.56<br>7.33.56<br>7.51<br>8.62<br>27.51<br>43.35<br>61.49   | 2 075<br>3 496<br>4 200<br>2 828<br>2 407<br>1.067<br>51.46 19,35<br>m (kg)<br>2 202<br>2 760<br>3 469<br>2 530  
   | 1.673<br>3.094<br>3.827<br>2.426<br>2.005<br>0.886<br>5.445 e.45<br>Esperamen<br>mass et<br>node (kg)<br>3.800<br>2.378<br>3.067<br>2.128   | in ressous tras  | 80.75<br>97<br>108.25<br>124.25<br>138.5<br>7.6<br>7.5<br>24.75  
   | 80 5<br>97<br>08 25-108 5<br>185<br>5<br>185 5<br>385<br>7<br>5<br>24 75<br>42 75  | 80,75<br>87<br>107,75-108<br>125<br>136,25<br>44<br>7,5<br>24,75<br>42,5  | 80.75<br>98.75<br>108<br>125<br>134.25<br>57.5<br>7.5<br>24.75<br>42.75<br>61.25<br>75.25  
  | 80 75<br>97<br>108 25<br>124 75<br>••••••••••••••••••••••••••••••••••••  | 80,75<br>97<br>108 5<br>124,75<br>135 5<br>7<br>125 7<br>7<br>25<br>7,5<br>25<br>42 5-42 75<br>81,25<br>75 25-75 5  | 80.75<br>96.75<br>107.75<br>124.75<br>136.75-137*<br>159.75-137*<br>7.5<br>25<br>42.75<br>61.25<br>75.25   | 60.5<br>96.75<br>108.5<br>124.75<br>136.75<br>136.75<br>136.75<br>136.75<br>124.75<br>24.75<br>61.25<br>5.25-75 5  |   
  | 80.75<br>97<br>108.25<br>124.25<br>130.5<br>24.25<br>24.75<br>24.75<br>24.75<br>42.625<br>61.25<br>75.25   | 80 5<br>97<br>108 375<br>125<br>138 5<br>138 | 80.75<br>97<br>107.875<br>125<br>136.25<br>136.25<br>4.75<br>24.75<br>42.5   | 96.75<br>108<br>125<br>136.25<br>5<br>7.5<br>24.75<br>42.75  | 97<br>108 25<br>124 75<br>6<br>7 5<br>24 75<br>42 75<br>81 5   
   | 97<br>108 5<br>124 75<br>124 75<br>7<br>7<br>7<br>5<br>25<br>42 625<br>61.25<br>75 375<br>89 125   | 96.75<br>107.75<br>124.75<br>136.875<br>136.875<br>136.875<br>136.875<br>126.475<br>126.475<br>25<br>42.75<br>61.25<br>75.25<br>88.75  | 90.75<br>108.5<br>124.75<br>24.75<br>7.5<br>24.75<br>42.5<br>51.25<br>75.375<br>89  |
96.9<br>108.2<br>124.8<br>136.5<br>ww.Freq<br>(Hz)<br>7.5<br>24.8<br>4.2<br>5.3<br>75.3<br>89.0   | 0 25<br>0 75<br>0 625<br>74555   |
| Experimen   | 2 500<br>3 500<br>4 500<br>6 000<br>7 000<br>45 53 2 5000<br>1 9<br>0 030<br>0 0300<br>0 0300<br>0 7500<br>1 500<br>2 250   | 79 55<br>94 15<br>108 76<br>133 26<br>133 16<br>isperset: h<br>1(Hu)<br>8.72<br>27.57<br>43 59<br>81 54<br>75 49   | 1 490<br>2 486<br>3 510<br>4 499<br>8 009<br>7 643<br>4 2 6 6 6 7 8 6<br>A (red/s)<br>0 031<br>0 299<br>0 742<br>1 493<br>2 235   |
61.44<br>79.36<br>94.29<br>106.76<br>123.37<br>133.56<br>133.56<br>133.56<br>133.56<br>145.00<br>15.00<br>145.00<br>145.00<br>145.00<br>145.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00<br>149.00       | 2 075<br>3 496<br>4 230<br>2 628<br>2 407<br>1.067<br>TL& IE(2)<br>TL& IE(2)<br>0<br>2 202<br>2 760<br>3 459   | 1.873<br>3.094<br>3.827<br>2.426<br>2.005<br>0.886<br>5.446<br>Esperamen<br>mass et<br>node (kg)<br>5.800<br>2.378<br>3.067   | in ressous tras  
   | 80.75<br>97<br>108 25 1<br>124 25<br>138 5<br>7.5<br>24.75<br>42 5-42 75<br>81.25<br>75 25<br>89   | 80 5<br>97<br>08 25-108 5<br>125<br>136 5<br>24 75<br>24 75<br>42 75<br>61.5<br>75 25<br>88.75-69  | 80,75<br>87<br>107,75-108<br>125<br>136,25<br>136,25<br>136,25<br>44<br>5<br>24,75<br>42,5<br>61,25<br>75,25<br>89  
   | 80.75<br>96.75<br>108<br>125<br>130.25<br>5<br>7.5<br>24.75<br>42.75<br>61.25<br>75.25<br>89  | 80 75<br>97<br>108 25<br>124 75<br>7.5<br>7.5<br>24 75<br>42 75<br>61.5<br>75 25<br>89   | 80,75<br>97<br>108 5<br>124,75<br>136 5*<br>7<br>136 5*<br>7<br>136 5*<br>7<br>2<br>2<br>4<br>2<br>5<br>4<br>2<br>5<br>4<br>2<br>5<br>4<br>2<br>5-75<br>5<br>5-75<br>5<br>5<br>75 25-75<br>5<br>8<br>9-89,25  | 80.75<br>96.75<br>107.75<br>124.75<br>136.75-137*<br>156.75-137*<br>156.75<br>25<br>42.75<br>61.25<br>75.25<br>66.25<br>75.25<br>86.75   
   | 60.5<br>90.75<br>108.5<br>124.75<br>136.75<br>9<br>7.5<br>24.75<br>61.25<br>5.25.75.5<br>89  |  | 80.75<br>97<br>108 25<br>124 25<br>136 5<br>136 5<br>24 75<br>42 625<br>61 25  | 80 5<br>97<br>108 375<br>125<br>136 5<br>5<br>136 5<br>5<br>5<br>5<br>5<br>7 5<br>7<br>5<br>24 75<br>6 1 5<br>75 25  
   | 80.75<br>97<br>107.875<br>125<br>136.25<br>136.25<br>24.75<br>24.75<br>42.5<br>61.25<br>75.25  | 96.75<br>108<br>125<br>136.25<br>5<br>7.5<br>24.75<br>42.75<br>61.25<br>75.25<br>75.25<br>89<br>104.5  | 6<br>7 108 25<br>124 75<br>6<br>7 5<br>24 75<br>81.5<br>75 25<br>89<br>104 25  | 97<br>108 5<br>124 75<br>75<br>25<br>42 625<br>61.25<br>75 375<br>69.125<br>104 25   
   | 96.75<br>107.75<br>124.75<br>136.875<br>136.875<br>136.875<br>136.875<br>136.875<br>25<br>42.75<br>81.25<br>75.25<br>81.25<br>75.25<br>81.25<br>75.25<br>88.75<br>104.25   | 90.75<br>108.5<br>124.75<br>124.75<br>24.75<br>42.5<br>61.25<br>75.375<br>89<br>104.5   | 96.9<br>108.2<br>124.8<br>136.5<br>136.5<br>136.5<br>136.5<br>142.4<br>155<br>24.8<br>42.7<br>81.3<br>75.3<br>89.0<br>104.3   | 0 25<br>0 75<br>0 75<br>0 625<br>1 825 2 5 - 1<br>(H2)<br>0<br>0 25<br>0 25<br>0 125<br>0 375<br>0 25  
   |
| Experimen   | 2 500<br>3 500<br>4 500<br>6 000<br>7 000<br>4 00<br>7 000<br>4 00<br>7 000<br>4 00<br>7 000<br>4 00<br>7 000<br>4 00<br>7 000<br>4 00<br>7 000<br>4 000<br>7 000<br>6 000<br>7 000<br>6 000<br>7 000<br>6 000<br>7 000<br>6 000<br>7 000<br>6 000<br>0 000<br>7 000<br>0 0000<br>0 000<br>0 000<br>0 000<br>0 000<br>0 000<br>0 000<br>0 000<br>0 000<br>0 000  | 79 58<br>94 18<br>106 76<br>133 28<br>133 16<br>135 16<br>15<br>15<br>15<br>15<br>15<br>15<br>15<br>15<br>15<br>15<br>15<br>15<br>15   | 1 490<br>2 486<br>3 510<br>4 499<br>8 009<br>7,043<br>4 2 5 6 6 6<br>1 6 7 45<br>1 (red/s)<br>0 031<br>0 031<br>0 299<br>0 742<br>1 493   | 61.44<br>79.36<br>94.29<br>106.76<br>123.37<br>133.56<br>7.33.56<br>7.33.56<br>7.51<br>8.62<br>27.51<br>43.35<br>61.49   | 2 075<br>3 496<br>4 230<br>2 407<br>2 407<br>1.087<br>1.087<br>1.087<br>1.087<br>2 202<br>2 780<br>3 459<br>2 530<br>3 137  
  | 1 673<br>3 094<br>3 627<br>2 426<br>2 005<br>0 886<br>5 426<br>2 005<br>0 886<br>5 426<br>5 4<br>5 40<br>5 426<br>5 40<br>5 40<br>5 40<br>5 40<br>5 40<br>5 40<br>5 40<br>5 40  | in ressous tras  | 80.75<br>97<br>108 25 1<br>124 25<br>138 5<br>7<br>5<br>24.75<br>42 5-42 75<br>81 25<br>75 25<br>81 25<br>75 25<br>89<br>104 25  | 80 5<br>97<br>108 25-108 5<br>125<br>136 5<br>136 5<br>136 5<br>136 5<br>136 5<br>135 5 | 80,75<br>87<br>107,75-108<br>125<br>136,25<br>136,25<br>136,25<br>44,25<br>42,5<br>61,25<br>75,25<br>89<br>9104 6  
  | 80,75<br>96,75<br>108<br>125<br>136,25<br>5<br>7,5<br>7,5<br>7,5<br>7,5<br>7,5<br>61,25<br>75,25<br>89<br>9104 5  | 80 75<br>97<br>108 25<br>124 75<br>••••••••••••••••••••••••••••••••••••  | 80,75<br>97<br>108 5<br>124,75<br>135 5<br>61,24<br>7<br>7<br>5<br>25<br>42 5-42 75<br>61,25<br>75 25-75 6<br>89-89,25<br>104,25  
   | 80.75<br>96.75<br>107.75<br>124.75<br>136.75-137*<br>159.75-137*<br>7.5<br>25<br>42.75<br>61.25<br>75.25   | 60.5<br>96.75<br>108.5<br>124.75<br>136.75<br>136.75<br>136.75<br>136.75<br>124.75<br>24.75<br>61.25<br>5.25-75 5  |  | 80 75<br>97<br>108 25<br>124 25<br>136 5<br>24 75<br>24 75<br>42 625<br>61 25<br>75 25<br>89   
               | 80 5<br>97<br>108 375<br>125<br>138 5<br>24 75<br>42 75<br>61 5<br>76 25<br>88 875<br>104 25   | 80.75<br>97<br>107.875<br>125<br>136.25<br>136.25<br>24.75<br>24.75<br>42.5<br>61.25<br>75.25<br>89  | 96.75<br>108<br>125<br>136.25<br>5<br>7.5<br>24.75<br>42.75<br>61.25<br>75.25<br>89  | 97<br>108 25<br>124 75<br>6<br>7 5<br>24 75<br>42 75<br>81 5<br>75 25<br>89  | 97<br>108 5<br>124 75<br>124 75<br>7<br>7<br>7<br>5<br>25<br>42 625<br>61.25<br>75 375<br>89 125  
  | 96.75<br>107.75<br>124.75<br>136.875<br>136.875<br>136.875<br>136.875<br>126.475<br>126.475<br>25<br>42.75<br>61.25<br>75.25<br>88.75  | 90.75<br>108.5<br>124.75<br>24.75<br>7.5<br>24.75<br>42.5<br>51.25<br>75.375<br>89  | 96.9<br>108.2<br>124.8<br>136.5<br>136.5<br>136.5<br>136.5<br>136.5<br>24.8<br>42.7<br>81.3<br>75.3<br>89.0<br>104.3<br>106.6   
   | 0 25<br>0 75<br>0 625<br>74 5 2 3 4<br>(H2)<br>0 0 25<br>0 25<br>0 25<br>0 125<br>0 125<br>0 375<br>0 25<br>0 375<br>0 25  |
| Experimen<br>Node<br>1<br>2<br>3<br>4<br>5<br>6<br>7  | 2 500<br>3 500<br>4 500<br>6 000<br>7 000<br>45 43 5 5 5 5<br>7 000<br>4 (rad/s)<br>0 030<br>0 030<br>0 030<br>0 030<br>0 750<br>1 500<br>2 250<br>3 000<br>4 000<br>4 000<br>0 0000<br>0 0000<br>0 0000<br>0 00000<br>0 0000  | 79 58<br>94 18<br>108 76<br>123 28<br>133 16<br>355952 %<br>134952 %<br>135952 %<br>135952 %<br>135952 %<br>135952 %<br>135952 %<br>155757<br>43 59<br>61 54<br>75 49<br>87 75 49<br>75 75 75 75 75 75 75 75 75 75 75 75 75 7  | 1 490<br>2 486<br>3 510<br>4 499<br>6 009<br>7 643<br>4 2 6 6 009<br>7 6 4 3<br>7 6 4 5<br>7 6 7 6 7 6<br>7 6 7 6 7 6<br>7 6 7 6 7 6 7 6  | 61.44<br>79.36<br>94.29<br>106.76<br>123.37<br>133.56<br>  | 2 075<br>3 496<br>4 230<br>2 626<br>2 407<br>1.067<br>5146 (#) 32<br>2 702<br>2 760<br>3 469<br>2 530<br>3 137<br>4 050<br>3 870   | 1 673<br>3 094<br>3 827<br>2 426<br>2 005<br>0 886<br>2 846 a 2<br>8 867<br>2 867<br>2 108<br>2 378<br>3 067<br>2 128<br>2 735<br>3 648<br>3 648   
  | Straing Location   | 80.75<br>97<br>108 25 1<br>124 25<br>138 5<br>7<br>5<br>24.75<br>42 5-42 75<br>81.25<br>7 5 25<br>89<br>104 25   | 80 5<br>97<br>08 25-108 5<br>125<br>136 5<br>136 5<br>136 5<br>136 5<br>24 75<br>42 75<br>42 75<br>61.5<br>75 25<br>88.75-89<br>104 25   
   | 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  | 80.75<br>96.75<br>107.75<br>124.75<br>136.75-137*<br>136.75-137*<br>135.75<br>25<br>42.75<br>61.25<br>75.25<br>88.75<br>104.25<br>104.25<br>109  | 60 5<br>96 75<br>108 5<br>124 75<br>136 75<br>136 75<br>136 75<br>24 75<br>61 25<br>5 25 75 5<br>61 25<br>5 25 75 5<br>109<br>104 5<br>109   | Vakue Used   | 80,75<br>97<br>108,25<br>124,25<br>136,5<br>124,25<br>136,5<br>136,5<br>136,5<br>136,5<br>136,5<br>115,75<br>115,75<br>115,75<br>115,75   
  | 80 5<br>97<br>108 375<br>136 5<br>35<br>36 5<br>36 5<br>36 5<br>24 75<br>42 75<br>42 75<br>61 5<br>76 25<br>88 875<br>104 25   | 80,75<br>97<br>107,875<br>125<br>130,25<br>130,25<br>130,25<br>130,25<br>24,75<br>42,75<br>42,75<br>42,75<br>42,75<br>42,75<br>56,1,25<br>89<br>104,5<br>108,5   | 967,75<br>108<br>125<br>136,25<br>7,5<br>24,75<br>61,25<br>75,25<br>89<br>9104,5<br>108,75   | 97<br>108 25<br>124,75<br>75<br>24,75<br>24,75<br>81,5<br>75 24<br>24,75<br>81,5<br>75 25<br>89<br>104,25<br>108,75   
  | 87<br>108 5<br>124 75<br>7<br>7<br>7<br>5<br>7<br>5<br>75<br>75<br>75<br>375<br>89,125<br>104 25<br>109  | 96,75<br>107,75<br>124,75<br>138,875<br>138,875<br>138,875<br>138,875<br>25<br>42,75<br>81,25<br>75,25<br>81,25<br>75,25<br>81,25<br>75,25<br>81,25<br>75,104,25<br>104,25<br>109  | 90.75<br>108.5<br>124.75<br>24.75<br>24.75<br>42.5<br>01.25<br>75.375<br>39<br>104.5<br>109   |
96.9<br>108.2<br>124.8<br>136.5<br>762,702,55<br>24.8<br>42.7<br>61.3<br>75.3<br>89.0<br>104.3<br>105.7   | 0 25<br>0 75<br>0 625<br>73557   |
| Experimen<br>Node<br>1<br>2<br>3<br>4<br>5<br>6<br>7<br>7<br>8<br>9<br>9  | 2 500<br>3 500<br>4 500<br>6 000<br>7 000<br>0 300<br>0 4 500<br>5 000<br>4 500<br>5 000<br>1 500<br>1 500  | 79 58<br>94,18<br>108 76<br>123 28<br>123 28<br>123 16<br>123 28<br>123 28<br>123 28<br>123 28<br>123 28<br>123 28<br>123 28<br>123 28<br>123 28<br>125 49<br>87,17<br>100 66<br>112 54<br>125 55<br>125 55<br>125<br>125 55<br>125 55<br>1          | 1 490<br>2 486<br>3 510<br>4 499<br>7 043<br>7 043<br>7 043<br>0 031<br>0 0000000000   | 61 44<br>79 36<br>94 29<br>106 76<br>123 37<br>133 56<br>133 56<br>133 56<br>133 56<br>133 56<br>133 56<br>143 35<br>61 49<br>16 50<br>149<br>17 100 39<br>106 50<br>119 21<br>106 50<br>119 21  | 2 075<br>3 490<br>4 2300<br>2 828<br>2 407<br>1.087<br>1.087<br>2 702<br>2 760<br>3 469<br>2 530<br>3 469<br>2 530<br>3 469<br>2 530<br>3 469<br>2 533<br>3 423<br>3 423   | 1 673<br>3 094<br>3 627<br>2 426<br>2 005<br>0 866<br>3 2005<br>0 866<br>3 800<br>2 376<br>3 067<br>2 128<br>3 067<br>2 128<br>3 067<br>2 128<br>3 068<br>3 468<br>2 431<br>3 222<br>2 428<br>3 468<br>2 431<br>3 222<br>2 428<br>3 648<br>3 468<br>2 431<br>3 222<br>2 428<br>3 648<br>3 468<br>2 431<br>3 222<br>2 428<br>3 65<br>2 45<br>3 65<br>2 45<br>4<br>5 7<br>4<br>5 7<br>5<br>7<br>5<br>7<br>5<br>7<br>5<br>7<br>5<br>7<br>5<br>7<br>5<br>7<br>5<br>7<br>5<br>7   | 1115.5  
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| 80 75<br>97<br>108 25<br>124 75<br>124 75<br>61<br>75<br>24 75<br>61,5<br>75 25<br>89<br>104 25<br>108 75  | 80,75<br>97<br>108 5<br>124,75<br>136 5<br>7<br>7,5<br>75<br>42 5-42,75<br>61,25<br>75 25-75 5<br>89-89,25<br>104,25<br>104,25  
   | 80,75<br>96,75<br>107,75<br>124,75<br>136,75-137<br>14,65,65,4<br>14,55,75<br>25<br>42,75<br>42,75<br>61,25<br>75,25<br>86,75<br>104,25<br>104,25<br>104,25<br>109<br>109<br>109   | 60.5<br>96.75<br>108.55<br>124.75<br>136.75<br>136.75<br>136.75<br>7.5<br>24.75<br>42.5<br>61.25<br>5.25.75.5<br>5.5<br>5.25.75.5<br>89<br>104.5<br>109<br>9<br>104.5<br>109<br>9<br>9<br>9<br>9<br>9<br>9<br>9<br>9<br>9<br>9<br>9<br>9<br>7<br>5<br>24.75<br>108.75  | Value Used<br>1<br>-<br>-<br>-<br>1155<br>EmOletiSky<br>Value Used<br>1  | 80.75<br>97<br>108.25<br>124.25<br>136.5<br>124.25<br>136.5<br>7.5<br>24.75<br>42.675<br>61.25<br>75.25<br>89<br>104.25<br>108.5<br>104.25<br>108.5<br>104.25<br>108.5<br>115-0005<br>115-0005   
   | 80 5<br>97<br>108 375<br>136 5<br>35<br>36 5<br>36 5<br>36 5<br>24 75<br>42 75<br>42 75<br>61 5<br>76 25<br>88 875<br>104 25   | 80,75<br>97<br>107,875<br>125<br>130,25<br>130,25<br>130,25<br>130,25<br>24,75<br>42,75<br>42,75<br>42,75<br>42,75<br>42,75<br>56,1,25<br>89<br>104,5<br>108,5   | 967,75<br>108<br>125<br>136,25<br>7,5<br>24,75<br>61,25<br>75,25<br>89<br>9104,5<br>108,75   | 97<br>108 25<br>124,75<br>75<br>24,75<br>24,75<br>81,5<br>75 24<br>24,75<br>81,5<br>75 25<br>81,5<br>75 25<br>89<br>104,25<br>108,75   
   | 87<br>108 5<br>124 75<br>7<br>7<br>7<br>5<br>7<br>5<br>75<br>75<br>75<br>375<br>89,125<br>104 25<br>109  | 96,75<br>107,75<br>124,75<br>138,875<br>138,875<br>138,875<br>138,875<br>25<br>42,75<br>81,25<br>75,25<br>81,25<br>75,25<br>81,25<br>75,25<br>81,25<br>75,104,25<br>104,25<br>109  | 90.75<br>108.5<br>124.75<br>24.75<br>24.75<br>42.5<br>01.25<br>75.375<br>39<br>104.5<br>109   | 90.9<br>108.2<br>124.8<br>136.5<br>136.5<br>136.5<br>136.5<br>136.7<br>24.8<br>42.7<br>61.3<br>75.3<br>89.0<br>104.3<br>106.8<br>115.7<br>76.3<br>89.0<br>104.3<br>106.8<br>115.7   
   | 0 25<br>0 75<br>0 625<br>10 25<br>0 25<br>0 25<br>0 25<br>0 25<br>0 25<br>0 25<br>0 25   |
| Node<br>1<br>2<br>3<br>4<br>5<br>6<br>7<br>7<br>8<br>9<br>2225<br>1<br>8<br>9<br>2225<br>1<br>8<br>9<br>2225<br>1<br>8<br>1<br>2225<br>1<br>8<br>1<br>2225<br>1<br>22<br>1<br>22<br>1 | 2 500<br>3 500<br>4 500<br>6 000<br>7,000<br>0 Beared<br>A (rad/s)<br>1 0,030<br>0,750<br>1 500<br>2 250<br>3 000<br>4 500<br>5 000<br>4 000<br>0 3 000<br>4 000<br>1 500<br>3 000<br>4 000<br>1 500<br>3 000<br>4 000<br>1 500<br>1 500  | 79 56<br>94,16<br>108 76<br>123 28<br>133 16<br>133 16<br>135 57<br>27,57<br>43 59<br>61 54<br>75,49<br>61 54<br>75,49<br>71 00 67<br>61 100 67<br>61 100 76<br>100 76<br>75<br>75<br>75<br>75<br>75<br>75<br>75<br>75<br>75<br>75<br>75<br>75<br>75   | 1 490<br>2 486<br>3 510<br>4 499<br>8 7,043<br>7,043<br>7 20 4724 or<br>7 20 472<br>3 007<br>3 978<br>4 478<br>5 610<br>0 476 5 56452<br>6 478 4 478<br>5 610<br>1 0 476 56452<br>6 478 4 478<br>5 610  | 61 44<br>79 36<br>94 29<br>106 76<br>123 37<br>133 56<br>************************************  | 2 075<br>3 496<br>4 230<br>2 828<br>2 827<br>1 067<br>1 067<br>2 828<br>2 780<br>3 469<br>2 530<br>3 670<br>2 633<br>3 672<br>2 633<br>3 675<br>2 635<br>2 647<br>2 635<br>2 647<br>2 655<br>2 647<br>2 655<br>2 6555<br>2 6555<br>2 6555<br>2 6555<br>2 65555<br>2 65555<br>2 655555<br>2 65555555555                                     | 1 673<br>3 094<br>3 627<br>2 426<br>2 005<br>0 686<br>2 426 42<br>2 106<br>2 376<br>3 067<br>2 128<br>2 735<br>3 648<br>3 468<br>2 431<br>3 222<br>7 425 4269<br>Experiment<br>mass al<br>noss (xg)   
   | Straing Location<br>1<br>115 5<br>115 5<br>11           | 80.75<br>97<br>108 25 1<br>124 25<br>138 5<br>7<br>5<br>24.75<br>42 5-42 75<br>81.25<br>7 5 25<br>89<br>104 25   | 80 5<br>97<br>98 25-108 5<br>138 5<br>24 4 92 5<br>3<br>7.5<br>24 75<br>6 1.5<br>75 25<br>88.759<br>104 25<br>115 75<br>25<br>34 15 75<br>26   | 80.75<br>97<br>107.75-108<br>125<br>139.25<br>247.5<br>4<br>247.5<br>42.5<br>61.25<br>75.25<br>75.25<br>75.25<br>75.25<br>710.46<br>108.5<br>104.6<br>108.5   
   | 80.75<br>96.75<br>96.75<br>108<br>125<br>138.25<br>7.5<br>24.75<br>42.75<br>61.25<br>75.25<br>89<br>9104 5<br>108.75<br>2   | 80 75<br>108 25<br>124 75<br>124 75<br>125 75<br>125 75<br>125 75<br>125 75<br>125 75<br>125 75<br>125 75<br>126 75  | 80.75<br>97<br>108.5<br>124.75<br>124.75<br>126.5<br>124.75<br>125.75<br>25<br>25<br>25<br>25<br>25<br>25<br>25<br>25<br>25<br>25<br>25<br>25<br>25   | 80.75<br>96.75<br>107.75<br>127.475<br>136.75-137<br>15.75.75<br>7.75<br>42.75<br>61.25<br>75.25<br>75.25<br>75.25<br>104.25<br>109<br>109<br>109<br>109<br>109<br>109<br>109<br>109<br>109<br>109   
   | 60.5<br>96.75<br>108.55<br>124.75<br>136.75<br>136.75<br>24.75<br>24.75<br>42.5<br>61.25<br>5.25-75.5<br>89<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>109<br>109<br>109<br>109<br>109<br>109<br>109<br>109<br>109   | Value Used<br>1<br>-<br>-<br>-<br>-<br>-<br>-<br>-<br>-<br>-<br>-<br>-<br>-<br>-<br>-<br>-<br>-<br>-<br>-  | 80,75<br>97<br>108 25<br>124 25<br>124 25<br>124 25<br>124 25<br>124 75<br>42 625<br>61 25<br>7 52<br>42 625<br>61 25<br>7 52<br>108 5<br>115 75<br>115 75<br>115 75<br>125 88 87<br>108 75<br>108 75<br>108 75<br>108 75<br>108 25<br>108 25<br>108<br>108 25<br>108 25<br>108 25<br>108 25<br>108 25<br>108<br>108 25<br>108<br>108 25<br>108<br>108<br>108<br>108<br>108<br>108<br>108<br>108<br>108<br>108   | 80 5<br>97<br>108 375<br>125<br>136 5<br>7 5<br>7 5<br>7 5<br>42 75<br>61 5<br>7 5 25<br>88 875<br>104 25<br>115 75<br>104 25<br>115 75<br>104 25<br>115 75  |
00.75<br>97<br>107.875<br>130.25<br>130.25<br>130.25<br>130.25<br>130.25<br>130.25<br>130.25<br>130.25<br>130.25<br>130.25<br>130.25<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>105.5<br>10          | 967.75<br>108<br>125<br>136.25<br>136.25<br>136.25<br>75.24<br>75<br>24.75<br>42.75<br>61.25<br>89<br>104.55<br>108.75<br>108.75<br>5<br>108.75<br>5<br>8<br>8<br>9  | 97<br>108 25<br>124,75<br>75<br>24,75<br>24,75<br>81,5<br>75 24<br>24,75<br>81,5<br>75 25<br>81,5<br>75 25<br>89<br>104,25<br>108,75   | 87<br>108 5<br>124 75<br>7<br>7<br>7<br>5<br>7<br>5<br>75<br>75<br>75<br>375<br>89,125<br>104 25<br>109  |
96,75<br>107,75<br>124,75<br>135,875<br>155,875<br>155,875<br>25<br>42,75<br>42,75<br>42,75<br>42,75<br>42,75<br>42,75<br>42,75<br>42,75<br>42,75<br>42,75<br>42,75<br>42,75<br>42,75<br>42,75<br>42,75<br>42,75<br>42,75<br>109<br>42,5<br>109<br>42,5<br>109<br>42,5<br>109<br>42,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>109<br>40,5<br>10,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>40,5<br>100<br>100<br>100<br>100<br>100<br>100<br>100<br>100<br>100<br>10 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75<br>42 75<br>8<br>15<br>24 75<br>8<br>15<br>24 75<br>15<br>24 75<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>25<br>10<br>10<br>10<br>10<br>10<br>10<br>10<br>10<br>10<br>10  | 87<br>108 5<br>124 75<br>7<br>25<br>42 625<br>69 125<br>104 25<br>75 375<br>89 125<br>109 2<br>109<br>2<br>2/4<br>7<br>7<br>8  | 96,75<br>107,75<br>124,75<br>136,875<br>136,875<br>25<br>42,75<br>25<br>42,75<br>26<br>42,75<br>26<br>42,75<br>26<br>42,75<br>26<br>42,75<br>26<br>42,75<br>26<br>42,75<br>26<br>42,55<br>26<br>7,675<br>24,75   | 99.75<br>106.5<br>122.75<br>227.75<br>24.75<br>42.5<br>61.25<br>61.25<br>75.375<br>109<br>-<br>-<br>-<br>-<br>-<br>-<br>-<br>-<br>-<br>-<br>-<br>-<br>-<br>-<br>-<br>-<br>-<br>-<br>-  
  | 96.8<br>108.2<br>124.8<br>136.5<br>136.5<br>136.5<br>136.5<br>136.5<br>24.8<br>42.7<br>81.0<br>7.5<br>24.8<br>42.7<br>81.0<br>7.5<br>24.8<br>42.7<br>81.0<br>7.5<br>24.8<br>42.7<br>80.0<br>104.3<br>115.7<br>7<br>84.5<br>24.8<br>42.5<br>8<br>80.0<br>24.8<br>42.5<br>8<br>8.0<br>24.8<br>42.5<br>8<br>8<br>9<br>9<br>10<br>8<br>10<br>8<br>10<br>8<br>10<br>8<br>10<br>8<br>10<br>8<br>1   | 0 25<br>0 75<br>0 625<br>0 625<br>0 625<br>0 625<br>0 25<br>0 25<br>0 25<br>0 125<br>0 125<br>0 125<br>0 375<br>0 25<br>0 375<br>0 25<br>0 25<br>0 25<br>0 25<br>0 25<br>0 25<br>0 25<br>0 2   |
| Node<br>1<br>2<br>3<br>4<br>5<br>6<br>7<br>7<br>8<br>9<br>2225<br>1<br>8<br>9<br>2225<br>1<br>8<br>9<br>2225<br>1<br>8<br>1<br>2225<br>1<br>8<br>1<br>2225<br>1<br>22<br>1<br>22<br>1 | 2 500<br>3 500<br>4 500<br>7.000<br>Desired<br>A (rad/s)<br>0.000<br>0.750<br>1.500<br>2 250<br>3 000<br>4 500<br>5.000<br>4 000<br>0.750<br>1.500<br>5.000<br>0.000<br>4 000<br>0.000<br>2 250<br>3 000<br>4 000<br>0.000<br>2 250<br>3 000<br>4 000<br>0.000<br>2 500<br>0 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58<br>94,16<br>108,76<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>123,28<br>124,28<br>123,28<br>124,28<br>123,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>14,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,28<br>124,2             | 1 490<br>2 480<br>3 510<br>4 499<br>6 009<br>7 043<br>7 043<br>7 043<br>7 043<br>7 043<br>7 043<br>0 031<br>0 090<br>0 742<br>1 483<br>3 007<br>3 978<br>4 478<br>5 3 007<br>3 978<br>4 478<br>5 3 007<br>2 978<br>4 478<br>5 3 007<br>2 978<br>4 478<br>5 3 007<br>5 978<br>5 009<br>5 000<br>5 00000000  | 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  | 80.5<br>90.75<br>108.5<br>124.75<br>136.75<br>136.75<br>42.5<br>42.5<br>42.5<br>42.5<br>5.25.75.6<br>89<br>104.5<br>109<br>7.75<br>24.75<br>24.75<br>24.75<br>24.75<br>24.75<br>24.75<br>24.75<br>24.75  | Value Used<br>1<br>-<br>-<br>-<br>-<br>-<br>-<br>-<br>-<br>-<br>-<br>-<br>-<br>-   | 80,75<br>97<br>108,25<br>124,25<br>134,65<br>124,25<br>134,65<br>24,75<br>42,625<br>61,25<br>75,25<br>89<br>104,25<br>104,25<br>105,75<br>115,75<br>24,75<br>2<br>8<br>8<br>9<br>104,25<br>115,75<br>2<br>8<br>8<br>9<br>104,25<br>125<br>125<br>126<br>125<br>124<br>126<br>124<br>126<br>124<br>126<br>124<br>126<br>124<br>126<br>124<br>124<br>124<br>124<br>124<br>124<br>124<br>124<br>124<br>124  | 80 5<br>97<br>108 375<br>135<br>136 5<br>75 5<br>42 75<br>42 75<br>42 75<br>61 5<br>76 25<br>76 25 | 60 75<br>97<br>107 875<br>138 25<br>61 24 35<br>42 5<br>61 25<br>75 25<br>89<br>104 5<br>108 5<br>415 24 75<br>42 475  
   | 967.75<br>108<br>125<br>136.25<br>136.25<br>5<br>7.5<br>247.75<br>61.25<br>75.25<br>75.25<br>75.25<br>75.25<br>75.25<br>75.25<br>75.25<br>75.25<br>75.25<br>75.25<br>75.25<br>75.25<br>75.25<br>75.25<br>75.25<br>75.24<br>75.5<br>108<br>75.24<br>75.5<br>75.24<br>75.5<br>75.24<br>75.5<br>75.24<br>75.5<br>75.24<br>75.5<br>75.24<br>75.5<br>75.24<br>75.5<br>75.24<br>75.5<br>75.24<br>75.5<br>75.24<br>75.5<br>75.24<br>75.5<br>75.24<br>75.5<br>75.24<br>75.5<br>75.24<br>75.5<br>75.24<br>75.5<br>75.25<br>75.24<br>75.5<br>75.24<br>75.5<br>75.24<br>75.5<br>75.24<br>75.5<br>75.24<br>75.5<br>75.24<br>75.5<br>75.24<br>75.5<br>75.25<br>75.25<br>75.24<br>75.5<br>75.25<br>75.24<br>75.5<br>75.25<br>75.25<br>75.5<br>75.5<br>75.5<br>75.25<br>75.5<br>75. | 97<br>108 25<br>124 25<br>124 25<br>124 25<br>815<br>75 24 275<br>815<br>75 25<br>75 25<br>85<br>104 25<br>108 25<br>104 25<br>108 25<br>85<br>104 25<br>85<br>104 25<br>85<br>104 25<br>85<br>104 25<br>85<br>104 25<br>85<br>104 25<br>85<br>104 25<br>85<br>104 25<br>85<br>85<br>104 25<br>85<br>85<br>104 25<br>85<br>85<br>104 25<br>85<br>85<br>85<br>85<br>85<br>85<br>85<br>85<br>85<br>8   | 87<br>108 5<br>104 75<br>104 75<br>104 75<br>104 75<br>105<br>104 25<br>104 25<br>104 25<br>104 25<br>104 25<br>104 25<br>104 25<br>104 25<br>105<br>105<br>105<br>105<br>105<br>105<br>105<br>10  | 96,75<br>107,75<br>124,75<br>136,875<br>136,875<br>136,875<br>136,875<br>136,875<br>136,875<br>142,75<br>61,25<br>75,25<br>88,75<br>104,25<br>109<br>2,475<br>109<br>2,475<br>109<br>2,475<br>109<br>2,475<br>109<br>109<br>109<br>109<br>109<br>109<br>109<br>109<br>109<br>109   | 99 75<br>106 5<br>124 75<br>124 75<br>124 75<br>124 75<br>42 5<br>82 75<br>104 5<br>109<br>104 5<br>109<br>104 5<br>109<br>104 5<br>109<br>104 5<br>109<br>109<br>109<br>109<br>109<br>109<br>109<br>109  
   | 96.8<br>108.2<br>124.8<br>136.5<br>136.5<br>104.042-55<br>104.042-55<br>24.8<br>42.7<br>81.3<br>75.3<br>89.0<br>104.3<br>-106.8<br>115.7<br>24.8<br>42.7<br>81.3<br>-106.8<br>105.8<br>24.8<br>42.7<br>81.0<br>8.0<br>24.8<br>42.5<br>61.2<br>81.1<br>8.0<br>24.5<br>61.2<br>81.2<br>81.2<br>81.2<br>81.2<br>81.2<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.8<br>80.  | 0 25<br>0 75<br>0 625<br>5 0 625<br>5 0 625<br>0 625<br>0 25<br>0 25<br>0 375<br>0 25<br>0 375<br>0 25<br>0 375<br>0 25<br>0 375<br>0 25<br>0 375<br>0 25<br>0 375<br>0 25<br>0 25<br>0 25<br>0 25<br>0 25<br>0 375<br>0 25<br>0 25<br>0 25<br>0 25<br>0 25<br>0 25<br>0 25<br>0 2   |
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  | 1 673<br>3 094<br>3 627<br>2 426<br>2 005<br>0 886<br>2 005<br>0 886<br>2 005<br>0 886<br>2 005<br>0 886<br>2 005<br>0 886<br>2 005<br>2 126<br>2 126<br>2 126<br>2 126<br>2 126<br>2 405<br>2 400<br>2 405<br>2 405     | 5traing 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  | 96,75<br>107,75<br>124,75<br>136,875<br>8<br>7,5<br>25<br>42,75<br>81,25<br>84,275<br>81,25<br>84,275<br>81,25<br>104,25<br>104,25<br>104,25<br>104,25<br>104,25<br>104,25<br>81<br>104,25<br>81<br>106  | 99.75<br>106.5<br>124.75<br>124.75<br>7.5<br>7.5<br>7.5<br>75.375<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>104.5<br>109<br>109<br>109<br>109<br>109<br>109<br>109<br>109   | 96.8<br>108.2<br>124.8<br>138.5<br>124.8<br>138.5<br>124.8<br>4.7<br>5.2<br>24.8<br>4.7<br>5.2<br>24.8<br>4.7<br>5.3<br>89.0<br>104.3<br>115.7<br>106.5<br>115.7<br>105.5<br>105.5<br>105.7<br>105.5<br>105.7<br>105.5<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.7<br>105.  | 0 25<br>0 75<br>0 625<br>73 25 2 4<br>(Hz)<br>0 0 25<br>0 25<br>0 25<br>0 375<br>0 25<br>0 375<br>0 25<br>0 375<br>0
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0075<br>97<br>107.875<br>13025<br>13025<br>13025<br>425<br>61.25<br>61.25<br>61.25<br>61.25<br>61.25<br>61.25<br>61.25<br>89<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>89<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>104.5<br>100.5<br>100.5<br>100.5<br>100.5<br>100.5<br>100.5<br>100.5<br>100.5<br>100.5<br>100.5<br>100.5<br>100.5<br>100.5<br>100.5<br>100.5<br>100.5<br>100.5<br>100.5<br>100.5<br>100.5<br>100.5<br>100.5<br>1000 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Appendix Ω Raw Data from Experimental Tests of Algorithm 4.2

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