Generalized Quadrangles and Associated Structures

Matthew Ross Brown, B.Sc (Hons) (Adelaide)

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Contents

Abstract

Our aim in this thesis has been to consider questions concerning the relationship between a Generalized Quadrangle (GQ) and various substructures, with a view to proving characterisation and classification results. We also lay the groundwork for new GQ construction methods, although no new GQs are constructed here.

In Chapter 1 we introduce preliminary concepts and results required for the rest of the thesis, involving graphs, quadrics, geometries, GQs, algebraic topology on a simplicial complex and covers of geometries.

Chapter 2 contains a detailed investigation of the ovoid $K1(\sigma)$ of $Q(4,q)$ constructed by Kantor in [30], including construction of non-elation rosettes of $Q(4,q)$ containing only $K1(\sigma)$ ovoids and rosettes containing both $K1(\sigma)$ ovoids and elliptic quadric ovoids.

In Chapter 3 we show that if $\mathcal{S}$ is a GQ of order (s,s^2) and $\mathcal{S}'$ is a subquadrangle of order s doubly subtended in $\mathcal{S}$, then the subtended ovoid/rosette structure is a Semi-Partial Geometry (SPG). A new SPG is constructed from a GQ of Kantor ([31]) and a $Q(4,q)$ subquadrangle. For a q-clan GQ $\mathcal{S}$, q even, Payne constructed a family of subquadrangles $\mathcal{S}_\alpha$ of order q ([45]). We derive the algebraic conditions under which $\mathcal{S}_\alpha$ is doubly subtended in $\mathcal{S}$, and hence gives an SPG.

In Chapter 4 it is shown that if q is even a non-classical GQ of order (q,q^2) containing a subquadrangle isomorphic to $Q(4,q)$ implies the existence of a new ovoid of $\mathrm{PG}(3,q)$. Also, by a homology calculation, it is shown that if $\mathcal{S}$ is a GQ of order (q,q^2), q odd, such that $\mathcal{S}$ contains a $Q(4,q)$ subquadrangle, with each ovoid of $Q(4,q)$ subtended by $\mathcal{S}$ an elliptic quadric ovoid, then $\mathcal{S}$ is isomorphic to $Q(5,q)$.

In Chapter 5 we show a GQ $\mathcal{S}$ of order s with a regular point (∞) gives rise to a cover of the affine plane constructed from $\mathcal{S}$ and (∞), as in [49, 1.3.1]. Given an affine plane π of order s and an s-fold cover of π satisfying special conditions we construct a GQ of order s with a regular point. If the cover of π is algebraic the condition on the cover is interpreted in cohomological terms; we investigate these for the remainder of the Chapter 5.