



Accurate Computation
of
Steady Nonlinear
Free-Surface Flows

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*Thesis submitted for the degree of
Doctor of Philosophy
in
Applied Mathematics
at
The University of Adelaide
(Faculty of Mathematical and Computer Sciences)*

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February, 1998

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ABSTRACT

A method of solution of steady-state nonlinear free-surface potential-flow problems is presented. The effects of various parameters on the speed and reliability of the method and the accuracy of the solutions are investigated. The method is subsequently applied to several problems involving flows around submerged bodies, and the results are presented.

Specifically, a distribution of discrete sources located external to the fluid domain is used to represent the velocity potential. An iterative procedure is employed which successively refines approximations to the free surface and potential until the desired solution is obtained. The iterative procedure demonstrates a quadratic rate of convergence.

Parameters of choice, such as resolution of the free surface discretisation, location of singularities, formulation and application of the radiation condition, length of computational domain, formulation of the applied boundary condition, selection of surface-update procedure, the scheme for shifting singularities between iterations, and the test of convergence, are investigated for their effects on accuracy, reliability and speed. Recommendations are made for the selection of each of these parameters.

To verify the accuracy and utility of the method, it is applied to flows about submerged bodies. The wave-making properties of circular cylinders, both with and without circulation, are investigated. Emphasis is placed on the choices of circulation that significantly reduce the amplitude of the waves produced. Two circular cylinders are considered in tandem, and the special choices of depth of submergence and separation for which each cylinder is free of horizontal force are determined.

In three-dimensional flow, the wave-making of a prolate spheroid is investigated. Observations of the change in wake pattern with speed are made, and a quantitative investigation of the forces is undertaken. Some interesting cases, where the waves produced are steep and the submergence of the body is shallow, are shown. Flows about spheres

and spheroids in a fluid of finite depth are then considered. Some interesting phenomena, including the widening of the wake, are observed. Comparison is made to the corresponding semi-infinite fluid cases, as well as to the appropriate linear theories. In each instance, the nonlinear effects produced by increasing body size are discussed.