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**GROUNDWATER MODELING AND  
MANAGEMENT**

**USING**

**THE FINITE ELEMENT METHOD  
AND  
EVOLUTIONARY OPTIMISATION  
TECHNIQUES**

**By**

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## Synopsis

For realistic modeling of a groundwater system, two problems namely: (1) the forward problem (simulation) and (2) the inverse problem (calibration) must be solved. The forward problem predicts the unknown system states given the known system parameters and boundary conditions. The inverse problem determines the unknown physical parameters of the system by fitting observed system states. The inverse problem must first be solved to determine the appropriate model structure and model parameters in order to obtain reliable results from the forward solution. Solution of an inverse problem or a management decision problem requires the use of optimisation techniques. For the past two decades, the techniques that have been used for such problems have been blocked by several inherent difficulties. Firstly the solution of the inverse problem may be nonunique and unstable as a result of observation errors. Secondly, the quality and quantity of observation data are insufficient. Thirdly, the model structure error which is difficult to estimate, dominates other errors. Within the inverse model itself, the computation of derivatives as required by the gradient-based techniques also introduce the problem of ill-conditioning and instability for highly nonlinear problems and very sensitive parameters.

The present research study was initiated with the objective of using evolutionary techniques instead of the gradient-based methods to solve the optimisation problems embodied in both management and inverse models. The research study was completed in two phases. The work accomplished during the first phase of the study included the development of an inverse model for the identification of groundwater flow and solute transport parameters through the use of the shuffled complex evolutionary technique with finite element models developed to solve two- and three-dimensional problems. The resulting inverse model was tested extensively with field pumping test, synthesised data and corrupted synthesised data. The results of the test indicated that the shuffled complex

evolutionary technique of optimisation was robust, efficient, and effective, and could serve as a useful alternative to the more traditional gradient-based methods. The attractiveness of the technique lies in the fact that it performs its search without the need for computation of derivatives; a feature that makes the shuffled complex evolution superior to the gradient-based methods in the event of highly nonlinear problems and parameter sensitivity and interaction. A critical test carried out to investigate the weakness of the shuffled complex evolutionary technique indicated that the population of points in the search space must be well selected in order to achieve a more reliable and cost effective solution.

The second phase of the research study was the development of management-decision model for the optimisation of pollutant extraction from contaminated aquifers using the genetic algorithm evolutionary technique. Once again, this optimisation technique was linked with a finite element numerical model. An additional technique developed and incorporated into the standard genetic algorithm model was a time-variation approach that allowed the location and numbers of pumping rates to vary as the contaminant plume moves along with time. This precaution was taken to avoid redundancy of extraction wells (an hence excessive remedial costs) as the plumes move away from their original location with the progress of time during the forward simulation process. A theoretical analysis using both the standard genetic algorithm and the time-variation technique indicated that the latter was more efficient resulting in about 27% and 16% cost savings respectively, for both one- and two-dimensional potential arrangements of extraction wells.

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