THE ACQUISITION AND ANALYSIS OF CRANIOFACIAL DATA IN THREE DIMENSIONS

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Our gifts of knowledge and of inspired messages are only partial; but when what is perfect comes, then what is partial will disappear.

For what we see now is like a dim image in a mirror; then we shall see face to face. What I know now is only partial; then it will be complete, as complete as God's knowledge of me.

1 Corinthians 13: 9, 10, 12
New English Bible.

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DECLARATION

I declare that this thesis contains no material which has been accepted for the award of any other degree or diploma in any University and that, to the best of my knowledge and belief, the thesis contains no material previously published or written by another person, except where due reference is made in the text.

Amanda Helen Abbott.

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SUMMARY

Craniofacial osseous landmark coordinate data in three dimensions have been acquired in order to quantify regions of deformity in patients and also to establish methods for the development of syndrome specific and "normal" population standards.

Osseous landmark coordinate data have been determined using the techniques of biplanar radiogrammetry and computed tomography (CT).

The difficulties associated with osseous landmark identification for biplanar radiography have been overcome through the development of the "projection line" technique. This technique facilitates the identification of the same landmark on both films.

For the acquisition of craniofacial landmark coordinate data from CT, an off-line technique has been developed based on multiple sets of stereoscopic images of three dimensional CT reconstructions.

For each method the accuracy of the data has been well established.

Of the seventy-six osseous landmarks identified using the CT system and the thirty-four osseous landmarks using the biplanar system, twenty-five were common to both the measurement systems and these have been used for alignment of the CT and biplanar data thereby enabling the independently collected data to be combined (integrated) into a larger and more complete data set.

Osseous landmarks can be used to describe the essential features of a subject, as well as providing a basis for homology between subjects for shape and size comparison.

Distance and angle measurements which have long been used to study physical human growth and development have been determined from the coordinate data to provide the link with the well established techniques of craniometric and cephalometric measurement.

For direct comparison of homologous constellations of landmarks, some prior alignment of the constellations is required. In this thesis two alignment techniques were used. The first, based on the least squares criterion, orients, scales, and translates one subject relative to the other so as to minimize the sum of squares of differences between homologous landmarks. The second technique is an extension into three dimensions of Siegel's two dimensional repeated median approach where alignment is achieved by calculating the median translation vector, repeated median scale factor and repeated median orientation between the two shapes.

Strain analysis has also been used to describe shape change between homologous triangular and tetrahedral elements in terms of dilations and contractions along principal directions. The triangular and tetrahedral elements were constructed to describe bone surfaces and bone cavities respectively.

As no three dimensional standards exist, experimental reference standards have been developed for each analytic technique to facilitate the quantification of the extent of the deviation of an individual from a "normal" population.

A patient with a congenital syndrome was selected in order to assess the analysis techniques for the quantification of craniofacial deformity.

Significantly, all of the essential skeletal features characteristic of Treacher Collins Syndrome, and normally only qualitatively described, have been quantified for this patient by the analysis techniques. Further, the analyses have been applied to the same data, enabling comparison of the different shape analysis techniques.

CHAPTER 1

INTRODUCTION

1.1 Background To The Present Investigation

"Much of the current pediatric literature has been devoted to so-called "funny-looking kid" syndromes. This term disturbs us for personal as well as for scientific reasons....

Various fanciful terms have been employed to describe the faces of infants, viz., elfin, leprechaun-like, etc. Precision is obviously not the forte of this technique, since, we suspect, few can be consonant regarding the appearance of imaginary creatures. Often erroneous is the statement that the patient has ocular hypertelorism when, in fact, no measurement of interocular distance was ever attempted, the clinician having depended on gestalt alone. It is easy to be misled by clinical impression, since the distance between the eyes may appear to be abnormal depending upon the width of the face, the form of the glabellar area, the presence of epicanthal folds, the shape and width of the nose, etc."

Gorlin et al., 1976.

It has long been recognised by medical specialists that a need exists for a sound scientific basis in both pre-surgical planning and post operative evaluation for patients requiring correction of craniofacial deformities, not only to help improve present methods of management, but also to expand the knowledge of biology of craniofacial growth and its disorders.

An awareness of this need has led to the research programme reported in this thesis.

1.2 The Objectives Of The Project

Craniofacial deformities represent complex (three dimensional) problems varying with time, in type and severity, and displaying a vast range of characteristics. Using two dimensional data, derived from conventional cephalometric procedures, these traits are not readily described, and are often not appropriately analysed.

The approach adopted in the present work has, therefore, been concerned primarily with the development and application of methods for the acquisition and analysis of craniofacial data obtained in a three dimensional format, to enable the better representation and study of the craniofacial skeleton.

Specifically the objectives of the project have been to:

(i) acquire three dimensional craniofacial data,

(ii) explore the application of mathematical methods to the study of craniofacial shape, shape deformity and shape comparison, and

(iii) demonstrate how (i) and (ii) could be used to identify and quantify regions of deformity as a basis for understanding the ontogeny of craniofacial dysmorphology.

1.3 The Significance Of The Project

The potential significance of the project is such that from an understanding of the basic scientific principles related to the above objectives, the possibility arises of being able to provide a better description of craniofacial dysmorphology in quantitative as well as qualitative terms.

It is also anticipated the techniques developed will have broader application in many other fields, particularly anatomy, anthropology, genetics, orthodontics, oral surgery, forensic dentistry and road accident research.

CHAPTER 2

THE ACQUISITION OF THREE DIMENSIONAL CRANIOFACIAL DATA FROM BIPLANAR RADIOGRAPHS

2.1 Introduction

Standardized cephalometric radiographs are used in orthodontics, craniofacial surgery and growth studies to obtain data for morphological analysis. Typically, two orthogonal radiographs provide lateral and frontal images of the skull, which are usually analysed separately. With few notable exceptions (Savara, 1965; Selvik, 1974; Baumrind, 1975; Rune, 1980;), methods for acquiring data from cephalometric film pairs to construct three dimensional models of the skull have not been pursued; although the photogrammetric principles involved have been applied routinely in other disciplines. (McNeil, 1966; Hallett, 1970; Singh, 1970; Veress et al., 1977; Adams, 1981; Wolf, 1983; and Ghosh and Boulianne, 1984).

Attempts to obtain three dimensional data from radiographs are not new; in fact, it extends as far back as 1897 when Dennis successfully located a bullet in a brain by using "fluorographs" at 90° to each other, and exposed from known distances. At that time Dennis also commented on the "very deceptive" nature of two dimensional images.

Whilst the field of cephalometry continued to develop (Broadbent, 1931; Hofrath, 1931), very little progress was made in obtaining three dimensional osseous landmark coordinates from radiographs. It was not until the mid 1960s that further work was undertaken in this area, when Savara (1965), employing the work of Schwartz (1943), described a method of obtaining three dimensional coordinates from simultaneous biplanar radiographs. Savara obtained facial measurements free from magnification and distortion by using the three dimensional coordinates to calculate distances between metal markers and landmarks. The accuracy of the method was initially assessed by comparing direct distance measurements of metal markers, glued to the skull at anatomical positions (for example, condylion) with the corresponding distances derived from the three dimensional cephalometric data - in fact, the measurement error was calculated to be ± 0.3mm. Savara et al., (1966) later analysed errors of landmark location and found that the variability of location was much greater than the measurement variability. To reduce the landmark location error, Singh and Savara (1966), Tracy and Savara (1966), Savara et al. (1967, 1968, 1979), Sekiguchi and Savara (1972) and Takeguchi, Savara and Shadel (1980), defined new anatomical landmarks which could be identified on both films. However, most investigators find that many of the landmarks defined by Savara and associates are not readily identifiable, and as a consequence the landmarks and the method have not gained wide acceptance.

The accuracy of stereoscopic cephalometry was investigated by Hollender et al., (1968), using a test device containing steel balls. Their results, calculated from the stereo-radiographs, differed markedly from measurements taken directly from the test object. They suggested this problem could be rectified by increasing the stereo-base, but this introduced the further difficulty of obtaining acceptable stereo-imaging.

Likewise, Baumrind and Moffitt (1972); Baumrind et al., (1982) and Baumrind, Moffitt and Curry (1983a, 1983b) had anticipated that direct stereoscopic views of stereo-radiographs should improve the identification of skeletal landmarks. They too, however, found that this was not the case, and had to resort to separate analysis of each radiograph, followed by reconstitution of the data to three dimensional coordinates by classical photogrammetric principles.

The main source of error and frustration in the three dimensional approach to cephalometry is the inability of the experimenter to accurately locate the same landmark on the two films.

The ease of location of metallic implants in biplanar and stereo-radiographs, has resulted in the implant being widely used (Buck and Hodge, 1975; Rune, Sarnäs and Selvik, 1979; Rune, 1980; Adams, 1981; Garrison et al., 1982). While the implant method produces well defined landmarks, and continues to be popular under certain circumstances, (for example, the study of craniofacial deformity), its use in recent years for routine cephalometry has come under review, because its value to the patient is questionable.

2.2 Coordinate Determination From A Pair Of Projected Images

Although confronted with the same geometric problems, many of the above researchers proposed alternative mathematical solutions for obtaining three dimensional coordinates from a pair of projections. As an example of how to obtain three dimensional coordinates, the method described in this section is a generalization of the equations presented by Savara (1965).

Figure 2.1 shows the geometry of the radiographic set up. There is a source of X-rays, S, (assumed to be a point source), a subject, and a film plane. The origin is taken to be a point at a distance, d, from the source towards the film plane (nominally the centre of the head holder), and f is the source - film distance. A point **x** at (x, y, z) relative to the coordinate axes shown is centrally projected onto the point $\mathbf{x}_p = (x_p, y_p)$ in the film plane.

$$x_p = \frac{f}{d-z} \cdot x \qquad 2.1\,(a)$$

$$y_p = \frac{f}{d-z} \cdot y \qquad 2.1\ (b)$$

A rigid body movement of the subject can be specified by six parameters: three translation and three rotation.

Let R be the rotation matrix

$$R = \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix}$$

and $\mathbf{a}$ = (a, b, c) be the translation vector then the new position $\mathbf{x}'$ = (x', y', z') of the point $\mathbf{x}$ = (x, y, z) after a rigid body movement is given by

$$x' = R(x + a) \ . \qquad 2.2$$

If another exposure is taken, the new projected coordinates are $\mathbf{x}'_p = (x'_p, y'_p)$ given by

$$x'_p = \frac{f}{d - z'} \cdot x'$$

$$y'_p = \frac{f}{d - z'} \cdot y'$$

This situation is equivalent to leaving the subject fixed and moving the unit (or having a second X-ray unit) in the opposite sense (that is, a rotation of $R^{-1} = R^T$ followed by a translation of $-\mathbf{a}$).

If a second radiographic unit is used, the parameters f and d could be different; and the following equations would then apply:

$$x'_p = \frac{f'}{d' - z'} \cdot x' \qquad 2.3\ (a)$$

$$y'_p = \frac{f'}{d' - z'} \cdot y' \qquad 2.3\ (b)$$

where f' and d' are equivalent parameters to f and d for the second X-ray unit.

If the rotation, translation and X-ray unit parameters are predetermined, Equations 2.1, 2.2, and 2.3 can be solved for the three dimensional coordinates (x, y, z) from the measured coordinates (x_p, y_p) and (x'_p, y'_p).

A solution for x, y and z follows:

substitute Equation 2.2 into Equation 2.3 to give

$$x'_p = \frac{f'}{d' - z'} \left(R_{11}(x+a) + R_{12}(y+b) + R_{13}(z+c)\right) \qquad 2.4\ (a)$$

$$y'_p = \frac{f'}{d' - z'} \left(R_{21}(x+a) + R_{22}(y+b) + R_{23}(z+c)\right) \qquad 2.4\ (b)$$

substitute Equation 2.1 into Equation 2.4

$$x'_p = \frac{f'}{d' - z'} \left(R_{11}(\frac{d - z}{f} x_p + a) + R_{12}(\frac{d - z}{f} y_p + b) + R_{13}(z+c)\right) \qquad 2.5\ (a)$$

$$y'_p = \frac{f'}{d' - z'} \left(R_{21}(\frac{d - z}{f} x_p + a) + R_{22}(\frac{d - z}{f} y_p + b) + R_{23}(z+c)\right) \qquad 2.5\ (b)$$

from Equation 2.2

$$z' = R_{31}(x+a) + R_{32}(y+b) + R_{33}(z+c) \qquad 2.6$$

and substituting Equation 2.1 into Equation 2.6 gives

$$z' = R_{31}\left(\frac{d - z}{f} x_p + a\right) + R_{32}\left(\frac{d - z}{f} y_p + b\right) + R_{33}(z+c). \qquad 2.7$$

Next, Equation 2.7 is substituted into Equation 2.5 to solve for z in terms of x_p, y_p, x'_p.

$$x'_p\left(d' - R_{31}\left(\frac{d-z}{f}x_p + a\right) - R_{32}\left(\frac{d-z}{f}y_p + b\right) - R_{33}(z+c)\right)$$

$$= f'\left(R_{11}\left(\frac{d-z}{f}x_p + a\right) + R_{12}\left(\frac{d-z}{f}y_p + b\right) + R_{13}(z+c)\right)$$

$$z = \frac{f'\left(R_{11}\left(\frac{dx_p}{f} + a\right) + R_{12}\left(\frac{dy_p}{f} + b\right) + R_{13}c\right)}{x'_p\left(\frac{R_{31}x_p}{f} + \frac{R_{32}y_p}{f} - R_{33}\right) + f'\left(\frac{R_{11}x_p}{f} + \frac{R_{12}y_p}{f} - R_{13}\right)}$$

2.8

$$+ \frac{-x'_p\left(d - R_{31}\left(\frac{dx_p}{f} + a\right) - R_{32}\left(\frac{dy_p}{f} + b\right) - R_{33}c\right)}{x'_p\left(\frac{R_{31}x_p}{f} + \frac{R_{32}y_p}{f} - R_{33}\right) + f'\left(\frac{R_{11}x_p}{f} + \frac{R_{12}y_p}{f} - R_{13}\right)}$$

This expression for z can then be substituted into Equation 2.1 to solve the other two coordinates x and y.

For a rotation of 90° clockwise about the Y-axis with no translation, Equation 2.2 becomes

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \qquad 2.9$$

and Equation 2.8 simplifies to

$$z = \frac{-x'_p\,(fd' - dx_p)}{(ff' + x_p x'_p)} \qquad 2.10$$

In the biplanar case (that is, radiographs taken at 90° separation), the two radiographic orientations normally used are the lateral and coronal projections. The coronal view is either a PA (postero-anterior) or AP (antero-posterior) projection - views obtainable from the Adelaide Dental Hospital and the Adelaide Children's Hospital respectively. The unprimed and primed coordinates refer to a coordinate system relative to these views or orientations, and thus for convenience the projected points can be designated (x_L, y_L) and (x_{PA}, y_{PA}) or (x_{AP}, y_{AP}) as appropriate.

It should be noted that biplanar radiographs are obtained at the Adelaide Dental Hospital by rotating the patient between exposures while at the Adelaide Children's Hospital they are generated by simultaneous exposure of the subject by two orthogonal X-ray units.

For the Adelaide Dental Hospital the X, Y and Z-axes are defined as in Figure 2.2 (a) with the assignment of unprimed and primed coordinates to measurements specific for the lateral and postero-anterior films as follows

$$\begin{array}{ccc} \underline{\text{Lateral}} & \underline{\text{PA}} & \\ (x_L, y_L) = (x_p, y_p) & (x_{PA}, y_{PA}) = (x'_p, y'_p) & \\ d_L = d & d_{PA} = d' & \quad 2.11\ (a) \\ f_L = f & f_{PA} = f' & \end{array}$$

Likewise for the Adelaide Children's Hospital, the X, Y and Z-axes are defined as in Figure 2.2 (b) with

$$\begin{array}{ccc} \underline{\text{Lateral}} & \underline{\text{AP}} & \\ (x_L, y_L) = (x_p, y_p) & (x_{AP}, y_{AP}) = (x'_p, y'_p) & \\ d_L = d & d_{AP} = d' & \quad 2.11\ (b) \\ f_L = f & f_{AP} = f' & \end{array}$$

Comparison of Figures 2.2 (a) and 2.2 (b) and Equations 2.11 (a) and 2.11 (b) reveals that the geometry for the Adelaide Dental Hospital and Adelaide Children's Hospital is the same (although the orientation of the head between the two systems differs by 180°). Therefore, the same equations can be used to obtain the three dimensional coordinates of landmarks. However, to maintain consistent orientation of the coordinate data between the two radiographic systems, the Adelaide Children's Hospital coordinates are brought into alignment with the Adelaide Dental Hospital coordinate data by a rotation of 180° about the Y-axis. This rotation is simply implemented by multiplying the x and z coordinates by -1. For this reason, the equations that follow will only be given in terms of the Adelaide Dental Hospital measurement system.

Thus Equation 2.10 can be re-expressed as

$$z = \frac{-x_{PA}\left(f_L d_{PA} - d_L x_L\right)}{\left(f_L f_{PA} + x_L x_{PA}\right)} \qquad 2.12$$

and then substituting for z in Equation 2.1 gives

$$x = \frac{x_L\left(d_L f_{PA} + d_{PA} x_{PA}\right)}{\left(f_L f_{PA} + x_L x_{PA}\right)} \qquad 2.13\ (a)$$

$$y = \frac{y_L\left(d_L f_{PA} + d_{PA} x_{PA}\right)}{\left(f_L f_{PA} + x_L x_{PA}\right)} \; . \qquad 2.13\ (b)$$

These equations can be readily extended to other angles, such as for stereo pairs, by simply using the appropriate rotation matrix in the general formulae given above.

2.3 Coordinate Determination Of Difficult To Locate Landmarks

Although the methods described in Sections 2.1 and 2.2 for obtaining three dimensional coordinates are reported to be highly accurate (Savara, 1965; Selvik, 1974; Rune, 1980), it is interesting to note that three dimensional data acquisition has not been *routinely* used in cephalometric studies. One of the main reasons for this is the inability of the experimenter to satisfactorily locate the same biological reference point on both films - a necessary pre-requisite if three dimensional coordinates are to be calculated. For example, while the definition of the nasion is well defined and accepted for the lateral radiograph, there is no current definition for it on the coronal cephalogram.

In order to overcome this difficulty, the approach adopted in this work extends the aforementioned three dimensional coordinate reconstitution method by utilizing the geometry to assist in the location of reference points (Brown and Abbott, in press).

2.3.1 Equations of projected lines

It can be seen from Figure 2.3 that if the projection of a landmark can be located, for example, on the lateral film, this landmark must lie along a line which is projected on to the postero-anterior (PA) film. The location of this landmark on the PA film is facilitated because it is only then necessary to determine its position on this line.

The equation for this projected line on the PA film can be derived from the equations given in Section 2.2 as outlined below.

Substitution of Equation 2.8 into Equation 2.7 yields an expression for z' in terms of x_p, y_p and x_p'. Substitution of this expression for z' and the expression for z from Equation 2.8 into Equation 2.5 (b) gives an expression for y_p' in terms of x_p, y_p and x'_p. In terms of the PA and lateral notation, this is equivalent to y_{PA} being expressed as a function of x_L, y_L and x_{PA}. This function must be linear in x_{PA} due to the geometry as depicted in Figure 2.3.

Thus, solving the above derived general equations for the biplanar case gives

$$z = \frac{-x_{PA}\left(f_L d_{PA} - d_L x_L\right)}{\left(f_L f_{PA} + x_L x_{PA}\right)} \qquad 2.14$$

and

$$z' = \frac{x_L\left(d_L f_{PA} + d_{PA} x_{PA}\right)}{\left(f_L f_{PA} + x_L x_{PA}\right)} \qquad 2.15$$

Substitution of Equations 2.14 and 2.15 into Equation 2.5 (b) yields (note $y'_p = y_{PA}$)

$$y_{PA} = \frac{y_L\left(d_L f_{PA} + d_{PA} x_{PA}\right)}{\left(d_{PA} f_L - d_L x_L\right)} \qquad 2.16$$

Similarly, to obtain the equation for the alternative solution, that is, the projection of a landmark located on the PA film, and the line along which this landmark must lie projected onto the lateral film, the general equations may again be used except that the assignments of the unprimed and primed coordinates to the PA and lateral films become

PA	Lateral	
$(x_p, y_p) = (x_{PA}, y_{PA})$	$(x_L, y_L) = (x'_p, y'_p)$	
$d_{PA} = d$	$d_L = d'$	2.17
$f_{PA} = f$	$f_L = f'$	

and the transformation between coordinate systems is

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \qquad 2.18$$

which is a 90° rotation in the opposite sense to the previous situation.

Then
$$z = \frac{-x_L\left(d_L + \dfrac{d_{PA}x_{PA}}{f_{PA}}\right)}{\left(\dfrac{-x_L x_{PA}}{f_{PA}} - f_L\right)}$$

$$= \frac{x_L\left(d_L f_{PA} + d_{PA}x_{PA}\right)}{\left(f_L f_{PA} + x_L x_{PA}\right)} \qquad 2.19$$

and
$$z' = -\left(d_{PA} - \left(\frac{x_L\left(d_L f_{PA} + d_{PA}x_{PA}\right)}{f_L f_{PA} + x_L x_{PA}}\right)\right) \cdot \frac{x_{PA}}{f_{PA}}$$

$$= \frac{-x_{PA}\left(d_{PA}f_L - d_L x_L\right)}{f_L f_{PA} + x_L x_{PA}} \qquad 2.20$$

Substitution of Equations 2.19 and 2.20 into Equation 2.5 (b) gives (note $y'_p \equiv y_L$)

$$y_L = \frac{y_{PA}\left(d_{PA}f_L - d_L x_L\right)}{\left(d_L f_{PA} + d_{PA}x_{PA}\right)} \qquad 2.21$$

Compare with Equation 2.16.

2.3.2 Intersection of a projected line with a contour to determine the three dimensional location of a difficult to locate landmark

If, for example, a point on the lateral film is well defined, the projected line on the PA film can be calculated (Section 2.3.1). If, on the PA film, a contour can be defined along which the point is known to lie, its intersection with the projected line defines its three dimensional position in space.

After location of this well defined landmark on the lateral film, points along a contour on the PA film are tested until the projected line is crossed. The following method was applied to test whether the projected line had been crossed and to locate its intersection with the contour.

Let points x_1 and x_2 be two points on the contour and x_3 and x_4 two points on the projected line (Figure 2.4). The parametric equations for the two lines defined by these points can be written

$$x = \mu (x_2 - x_1) + x_1 \tag{2.22}$$

and

$$x' = \lambda(x_4 - x_3) + x_3 \tag{2.23}$$

where μ and λ are parameters which determine the positions of the points $\mathbf{x} = (x, y)$ and $\mathbf{x}' = (x', y')$ on the two lines respectively.

The lines intersect when $\mathbf{x} = \mathbf{x}'$ therefore from Equations 2.22 and 2.23 gives

$$\mu(x_2 - x_1) + x_1 = \lambda(x_4 - x_3) + x_3 \ . \tag{2.24}$$

Rearranging this equation gives

$$(x_2 - x_1)\mu + (x_3 - x_4)\lambda = (x_3 - x_1) \tag{2.25}$$

or, in matrix form,

$$\begin{pmatrix} (x_2 - x_1) & (x_3 - x_4) \\ (y_2 - y_1) & (y_3 - y_4) \end{pmatrix} \begin{pmatrix} \mu \\ \lambda \end{pmatrix} = \begin{pmatrix} x_3 - x_1 \\ y_3 - y_1 \end{pmatrix} .$$

The determinant of the matrix equation is

$$\Delta = (x_2 - x_1)(y_3 - y_4) - (x_3 - x_4)(y_2 - y_1) . \qquad 2.26$$

If $\Delta = 0$, the lines are parallel and there is no solution; however, in general, $\Delta \neq 0$ and therefore

$$\begin{pmatrix} \mu \\ \lambda \end{pmatrix} = \frac{1}{\Delta} \begin{pmatrix} (y_3 - y_4) & (x_4 - x_3) \\ (y_1 - y_2) & (x_2 - x_1) \end{pmatrix} \begin{pmatrix} x_3 - x_1 \\ y_3 - y_1 \end{pmatrix} . \qquad 2.27$$

The intersection is between (and can include) the two end points x_1 and x_2 if

$$\mu = \frac{\left((x_3 - x_1)(y_3 - y_4) + (x_4 - x_3)(y_3 - y_1)\right)}{\Delta} \qquad 2.28$$

is in the range $0 \leq \mu \leq 1$.

Then the point of intersection, from Equation 2.22,

$$x = \mu (x_2 - x_1) + x_1$$

$$y = \mu (y_2 - y_1) + y_1$$

is the required point on the PA film.

If μ is outside the range $0 \leq \mu \leq 1$, the points x_1 and x_2 do not straddle the projected line and the next pair of points on the contour are tested. This procedure can be made as accurate as desired by collecting points on the contour as close together as necessary. Suitable points for x_3 and x_4 on the projected lines are:

(i) for projection lines on the PA film, defined by a point on the lateral film

$$x_3 = \left(\frac{d_{PA}f_L - d_L f_{PA} - d_L x_L}{d_{PA}} , y_L \right)$$

and

$$x_4 = \left(\frac{- d_L f_{PA}}{d_{PA}} , 0 \right)$$

(ii) for projection lines on the lateral film, defined by a point on the PA film

$$x_3 = \left(\frac{d_{PA}f_L - d_L f_{PA} - d_{PA} x_{PA}}{d_L} , y_{PA} \right)$$

and

$$x_4 = \left(\frac{d_{PA}f_L}{d_L} , 0 \right) .$$

These points can be readily seen to satisfy Equations 2.16 and 2.21 respectively.

The above equations were implemented in the cephalometric coordinate data collection program series CEPHS 3D (see Section 2.4.2.4) and their evaluation using a test object, dried skulls and a patient (pre- and post-operative biplanar radiographs) is presented in Sections 2.5, 2.6, 2.7 and 2.8.

2.4 Experimental Method For The Acquisition Of Data

2.4.1 Description of equipment used

The radiology facilities of the Adelaide Dental Hospital (ADH) and the Adelaide Children's Hospital (ACH) were used in this study (Figures 2.5 (a) and (b)).

For both hospitals Table 2.1 summarizes the details of the radiographic equipment and projection parameters.

Tables 2.2 and 2.3 outline the equipment modifications for the Adelaide Dental Hospital and the Adelaide Children's Hospital necessary to ensure consistency with the considerations presented in Sections 2.2 and 2.3.

2.4.2 Data acquisition from biplanar radiographs

In order to obtain the three dimensional coordinate data from the biplanar radiographs, the following procedures were undertaken and are discussed below:

(i) establishment of the relative orientation of the films,

(ii) the identification of key features and landmarks on the radiographs and their transfer to the overlaid tracing film, and

(iii) digitization and storage on diskette of the coordinates of the landmarks.

2.4.2.1 Alignment of the Adelaide Dental Hospital biplanar radiographs

The fiducial markers (Table 2.2) were used to transfer the origin, which was readily determined on the lateral film, onto the postero-anterior film. They were also used to orient the film relative to the rotation axis, thereby avoiding the otherwise necessary assumption that the axis of rotation is parallel to the longer edge of the film. The alignment procedure was performed as follows:

(i) The PA and lateral radiographs were superimposed upon the images of the fiducial markers. Registration was maintained through the use of pins and the centre of the image of the right ear rod on the PA film was marked on to the lateral film (Figures 2.11 (a) and (b)).

In this way, the movement of the centre of the ear rod is directly marked on the film and the rotation axis is determined perpendicular to this movement. (This is not necessarily parallel to the edge of the film.)

(ii) The two films were then placed securely side by side, and acetate tracing film overlaid. The fiducial markers, origin, X-axis (determined in (i) above) and the reference markers or osseous landmarks were identified and marked onto the tracing paper (see Section 2.4.2.3). In addition, the major skeletal features were also traced for reference.

The point corresponding to the centre of the ear rod on the lateral tracing was digitized first. The coordinates of this point define the Cartesian origin for all subsequently digitized points. The next point digitized was that of the position, marked on the lateral radiograph during procedure (i) above, of the ear rod on the PA film. This defined the X-axis of the digitizer. These two points directly define the origin and the axes for the lateral film. However, not only does the PA film's origin need to be defined, but also its orientation relative to the lateral film, because alignment is only approximate prior to tracing. The origin and orientation of the PA film were determined by the location of the fiducial markers on each of the films. The coordinates of these points (x_1, y_1), (x_2, y_2) and (x_3, y_3), (x_4, y_4) on the lateral and PA films respectively were digitized and the lines joining left and right markers on both films, denoted l_1 and l_2 (Figure 2.11 (b)), were brought into registration by a rotation and translation as outlined below:

The angles of l_1 and l_2 to the X-axis, denoted θ_1 and θ_2 respectively, are given by

$$\tan\theta_1 = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{and} \quad \tan\theta_2 = \frac{y_4 - y_3}{x_4 - x_3} \ . \tag{2.29}$$

The angle $\theta = \theta_1 - \theta_2$ between l_1 and l_2 required to align the PA film with the lateral film, can be determined from the arctan of Equation 2.29 but, since it is $\cos\theta$ and $\sin\theta$ that are required for the registration, these can be most simply obtained through use of the following three formulae:

$$\begin{aligned} \tan\theta &= \tan(\theta_1 - \theta_2) \\ &= \frac{\tan\theta_1 - \tan\theta_2}{1 + \tan\theta_1 \tan\theta_2} \ , \end{aligned} \tag{2.30}$$

$$\cos\theta = \frac{1}{\sqrt{1 + \tan^2\theta}} \tag{2.31}$$

and

$$\sin\theta = \frac{\tan\theta}{\sqrt{1 + \tan^2\theta}} \ . \tag{2.32}$$

To bring l_2 parallel to l_1 , l_2 is rotated through angle θ about the digitizer origin: thus,

$$x_3' = x_3 \cos\theta - y_3 \sin\theta \qquad x_4' = x_4 \cos\theta - y_4 \sin\theta \tag{2.33 (a)}$$

$$y_3' = x_3 \sin\theta + y_3 \cos\theta \qquad y_4' = x_4 \sin\theta + y_4 \cos\theta \ . \tag{2.33 (b)}$$

To make (x_3', y_3') coincident with (x_1, y_1), line l_2 is translated by $(x_1 - x_3', y_1 - y_3')$.

The coordinates (x_{PA}, y_{PA}) of a landmark on the PA film are given in terms of the digitized coordinates (x,y) as

$$x_{PA} = x\cos\theta - y\sin\theta + (x_1 - x_3') \qquad 2.34\text{ (a)}$$

$$y_{PA} = x\cos\theta + y\sin\theta + (y_1 - y_3') \; . \qquad 2.34\text{ (b)}$$

2.4.2.2 Alignment of the Adelaide Children's Hospital biplanar radiographs

The method of radiographic film alignment developed for the Adelaide Dental Hospital system cannot be used to determine relative orientation of the lateral and AP (antero-posterior) films taken at the Adelaide Children's Hospital. This is because, at the Adelaide Children's Hospital biplanar radiography is performed using simultaneous orthogonal exposures so that a fiducial plate directly in front of one film cannot be seen on the other film. For this reason, a plumb line is employed at the Adelaide Children's Hospital which is positioned so that it is visible on both films (Figures 2.12 (a) and (b)).

The origin on the lateral film is taken to be the centre of the image of the ear rods, and a line is drawn through this point perpendicular to the vertical. The origin on the antero-posterior (AP) film is determined by bisecting the line joining the centres of the images of the two ear rods. This line must be kept perpendicular to the vertical as determined by comparison with the image of the plumb line on the AP film. The axis of rotation, the Y-axis, is parallel to the plumb line.

The two films are accurately aligned such that the two images of the plumb line are parallel and the lines through the origins coincide. The films are then securely retained in this orientation by adhesive tape.

2.4.2.3 Tracing of radiographs

The alignment procedures described in Sections 2.4.2.1 and 2.4.2.2 ensure that the coordinates derived from the lateral and coronal films are maintained in the correct geometric relationship to each other and to the radiographic system, regardless of the orientation of the radiographs on the viewing table.

The radiographs were overlaid with Acetate tracing film[1] and placed on a light box. A 0.5mm H pencil was used. Tracing was carried out in a darkened room. Opaque material was employed to mask areas of the radiograph not currently being traced in order to enhance the definition in areas of low X-ray absorption. Glasses with magnifying loops were worn as a further aid. The author found it particularly beneficial to have the company of the music of Mozart, Bach and Beethoven whilst tracing, thereby helping to reduce the tedium and thus increase the accuracy of tracing.

2.4.2.4 Biplanar radiographic coordinate data collection programs

The computing equipment used for data collection consisted of an Apple II plus[2], a Hewlett Packard[3] digitizing tablet (HP9874A) and a plotter (HP9872A) and these are shown in Figures 2.13 (a) to (c). A "verification plot" is shown in Figure 2.13 (c) of the landmarks digitized from the tracing shown in Figure 2.13 (b) - the tracing is subsequently superimposed on the "verification plot" to check that the landmarks have been faithfully recorded.

Table 2.4 lists and describes the functions of the biplanar radiographic coordinate data collection programs used in this study.

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1 General Aniline Film Corp., drafting-film 0.05mm
2 Apple Computer, Inc., California, U.S.A.
3 Hewlett Packard Company, Colorado, U.S.A.

2.4.2.5 Double determination

The author uses the term "double determination" to refer to the estimation of systematic and random errors by the replication of measurement. Systematic error is assessed by testing whether the mean difference between pairs of measurements differs significantly from zero. Usually this is determined using a t-test for one dimensional data (for example, length) but for the measurement of two or three dimensional coordinate data, tests based on $\chi^2(2)$ or $\chi^2(3)$ distributions respectively are more appropriate (see Section 6.2 for a discussion). If the mean is not zero at some high confidence level (say 95%), a systematic error is likely and its cause and affect must be evaluated. If there is no systematic error, the single measurement variance is determined from one half of the variance of the difference between two measurements and gives a measure of reproducibility.

2.4.2.6 Digitizing error

The contribution of the digitizing error to the error in location of a single point was estimated by digitizing a group of ten tracings of radiographs on two occasions one month apart. These ten radiographs were generated from the two radiographic determinations for each of the five test skulls (as described in Section 2.6). Only the lateral projections were used, as the lateral tracings had landmarks identified on them whereas the coronal tracings only had contours that contained landmarks (as per Section 2.3).

When it is considered that the ten lateral tracings were re-digitized one month later, it is reasonable to assume that the orientation of the two digitizing determinations for each of the ten radiographs would be different. Therefore, it was necessary to align the coordinate data of the second determination with the first determination. As the landmarks were identified by small marks (dots or crosses) on the tracing paper, it was expected that all twenty-five marks on each tracing would have the same probability distribution for the digitizing error. For this reason, least squares alignment was considered appropriate (Section 5.3 discusses the least squares method) rather than, say, the repeated median alignment fitting (Section 5.4 presents the repeated median approach), which is used when some landmarks are likely to have significantly different location errors. As there was no reason to expect scale differences, the alignment was performed without scaling.

The mean and standard deviation of the residuals from the ten least squares alignments for the double determinations are given in Tables 2.5 (a) and (b) respectively. A $\chi^2(2)$ distribution was used to assess whether any of the means differed significantly from zero (that is, whether there was any significant systematic difference between the two digitizing determinations). (The rationale for using a χ^2 distribution is discussed further in Section 6.2 with respect to the $\chi^2(3)$ distribution as applied to the analysis of the three dimensional coordinate data). Table 2.5 (a) also presents the $d\sqrt{n}/\sigma$ scores (whose squares have a $\chi^2(2)$ distribution) and these indicate that only two landmarks differed significantly in their digitized position between determinations at the 95% confidence level ($d\sqrt{n}/\sigma$ = 1.731), while none were significantly different at the 99% confidence interval ($d\sqrt{n}/\sigma$ = 2.145). The magnitude of the average residual for each point is less than 40 microns which is less than the thickness of an average human hair. The standard deviations are less than 0.1mm, substantially less than the thickness of the pencil lines on the tracings.

The single point digitizing measurement variance is given by

$$S^2 = \frac{1}{2n}\sum_{i=1}^{n} |\mathbf{x}_i|^2$$

where $\mathbf{x}_i$ is the residual vector after alignment of the first digitizing determination with the second digitizing determination for a landmark for the i^{th} radiograph (tracing) and n is the number of radiographs (tracings). The factor two in the denominator arises because the digitizing error can be equally ascribed to either determination (see Dahlberg,1940 with respect to estimating the distance measurement error associated with a single determination).

Table 2.5 (c) gives the standard deviation of the single point digitizing error for each of the twenty-five points. The pooled digitizing measurement standard deviation was found to be 0.046mm and at the 95% confidence level digitizing error was 0.08mm. (The manufacturer Hewlett Packard gives the resolution of the digitizer as 0.025mm).

The osseous landmark location error is of the order 0.7mm to 0.9mm for biplanar radiographic data (Section 2.6) and of the order 1.7mm for computed tomography data (Section 3.6). The digitizing errors are extremely small in relation to landmark location errors, and negligible with respect to the morphological variation. Therefore, they have minimal influence on the distribution of the final coordinate data collected.

2.5 Determination Of The Accuracy Of The Mathematical Model

The consistency of the mathematical model with the physical geometries of the radiographic set-ups of the Adelaide Dental Hospital and the Adelaide Children's Hospital was assessed using a specially constructed test object that fulfilled the following criteria:

(i) constructed of an appropriate radiolucent material with the incorporation of metal markers of known diameter, enabling the direct and indirect calibration of the test object,

(ii) precise control of orientation with reference to the central beams of the radiographic equipment,

(iii) similar dimensions to that of the human head, and

(iv) adaptable to the existing radiographic equipment.

2.5.1 Manufacture and calibration of a suitable test object

As a three dimensional box-shaped test object would be difficult to accurately prepare and its subsequent calibration would require specialized photogrammetric equipment, it was decided to construct a planar test object, which was so designed that it could be angled to sample the three dimensional space that would normally be occupied by a subject's head.

To produce the planar test object, an acrylic sheet of dimensions 250mm x 200mm x 10mm was machined so that it could be attached independently of the head holder (Figure 2.14 (a)). Within the acrylic sheet, metal spheres (diameter 0.7mm) were glued into prepared holes of constant depth. The test object markers covered an area larger than the projected human head and were distributed evenly across the field (Figure 2.14 (b)). In order to distinguish left from right and top from bottom an asymmetric point was embedded in the pattern (Figure 2.14 (c)).

The test object was aligned so that its mid-point was approximately at the centre of the X-ray beam (Figures 2.15 (a) and (b)).

The direct calibration of the coordinate positions of the test object markers was determined using a travelling microscope[1]. As the holes that contained the metal markers were drilled to equal depths, the z coordinates were set to zero. The replicability of marker location is shown in Table 2.6.

2.5.2 Biplanar radiographic determination of test object marker locations

Biplanar radiographs of the test object were taken at five angles at the Adelaide Dental Hospital and eight angles at the Adelaide Children's Hospital. An example of a radiograph of the acrylic test object is shown in Figure 2.15 (c). The separation between the test angles was approximately thirty degrees. Thus, an adequate sampling of the three dimensional space that would be occupied by a human head was ensured.

The radiographs were aligned, traced, digitized (see Section 2.4.2) and the location of each of the metal markers in the test object was determined using the following three modes:

(i) Mode 1 - landmark/marker located on both the lateral and coronal films,

(ii) Mode 2 - landmark/marker located on the lateral film and on a contour on the coronal film, and

(iii) Mode 2 - landmark/marker located on the coronal film and on a contour on the lateral film.

The landmark/marker location for Modes 2 and 3 is determined from the intersection of a projected line with the contour[2] (as described in Sections 2.3.1 and 2.3.2).

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1 Travelling Microscope Cam Metric Ltd., Cambridge, England.

2 As no contours exist for location of metal markers on the test object, suitable "contours" that were approximately vertical and passed through the markers were chosen for this purpose.

2.5.3 Comparison of the biplanar radiographic determination of the test object's marker location with the calibrated coordinates for the test object using t-tests

The accuracy of coordinate data obtained using the biplanar radiographic technique was assessed by comparison with the coordinate data determined using the travelling microscope. The two sets of coordinate data were aligned using the least squares method discussed in Section 5.3. The least squares procedure minimizes the sum of squared distances between corresponding points.

In this case t-tests (see for example, Sokal and Rohlf, 1981) were performed on each component of the residual, rather than using a χ^2 test on the magnitude of the residual (as used in Section 2.4.2.6 for evaluation of the digitizing error) so that systematic errors along each coordinate axis could be assessed. While the t-tests on the x and y residuals showed no significant difference from zero, the t-tests on the z coordinates of the residuals revealed that there was a systematic error in this component for some of the markers (Tables 2.7 (a) to (c)). Only in a few instances, however, did this error lie outside $\pm$ 0.1mm. Direct observation of the test object disclosed that the glue used to adhere the markers had in some cases displaced the balls in the narrow drill holes, thereby altering the z coordinates from the assumed value of zero. By setting the height of each of the affected markers to the average z coordinate, determined from the thirty nine measurements of each affected point over both systems, a good estimate of their height was obtained. The final acrylic test object coordinates with the corrected z components are shown in Table 2.8.

2.5.4 Scale factor correction of Adelaide Children's Hospital data

It had been anticipated that a scale factor of unity would have been calculated by the process of least squares fitting (Section 5.3) the Adelaide Children's Hospital three dimensional radiographic coordinate data of the acrylic test object to the calibrated coordinates, had all the geometric parameters of the equipment been precisely determined; instead, a scale factor of approximately 1.005 (1.00498) was derived. This is a significant factor, which could account for a 0.5mm coordinate difference at a distance 100mm away from the centre of the test object. Such a difference is much greater than the standard deviation of the coordinate measurements and is a reflection of model parameters. More precisely, the scaling of the Adelaide Children's Hospital data is dependent upon the distance between the X-ray source, the film and mid-sagittal plane or transporonic plane. The scale factor 1.005 has been applied to all subsequent Adelaide Children's Hospital data.

On the other hand, the scale factor between the Adelaide Dental Hospital radiographically determined three dimensional coordinate data for the acrylic test object and the calibrated coordinates for the same was found to be 1.0005, giving a displacement error of 0.05mm at a distance of 100mm from the centre of the acrylic test object. This is not a significant scale factor, as it is much less than the standard deviation of coordinate measurements.

2.5.5 Accuracy of the biplanar radiographic determination of the marker positions

After correcting of the scale factor for the Adelaide Children's Hospital biplanar data and some of the z coordinate positions of the acrylic test object markers, the mean and standard deviation of the residuals following least squares alignment of the radiographic data with the calibrated marker positions were again determined for both hospitals and for each of the three modes of data collection, and are given in Tables 2.9 (a) to (f). A $\chi^2(3)$ distribution was used to assess whether there was any significant difference between the biplanar radiographic three dimensional coordinate data and the calibrated coordinate positions. For Mode 1, only one biplanar radiographic marker position was found to differ significantly from the calibrated position, for both hospitals, although their mean differences (less than $\approx$ 0.08mm) were less than other mean differences that were found non-significant (up to 0.10mm). This is significantly less than the osseous landmark location errors determined in Section 2.6 (0.72 – 0.92mm). For Mode 2, no systematic errors were detected, whereas for Mode 3 a number of systematic errors were found of the order 0.2mm. The root mean square (rms) value of the differences between the biplanar radiographically determined positions and the calibrated true position of each metal marker gives the biplanar radiographic location error, which includes the effect of these small systematic errors (Tables 2.10 (a) to (f)).

Identification of reduced fidelity in the biplanar radiographic coordinate data at the extremities of the test object

Relative to the best and worst location errors for each mode and for both the Adelaide Children's Hospital and the Adelaide Dental Hospital (Tables 2.10 (a) to (f)), the F test revealed that in all cases there was a significant difference at the 95% confidence interval (Table 2.11).

The best points were observed at the centre, that is, close to the central beam, while the worst points were towards the edge of the pattern. When it is considered that the location error of the identical metal markers as imaged on the radiographic film should be the same regardless of position, this slightly worsening of results towards the edge can be explained by approximations in the model, such as the assumption of perfect orthogonality of the biplanar system and errors associated with the location of the origin on the lateral film and the coronal films.

Comparison of the accuracy of the three modes of data collection

The overall standard deviation of the biplanar radiographic marker location errors for each mode and for both systems were calculated by pooling the individual marker location errors (square root of the mean square marker location error) (Table 2.12 (a)). A comparison was performed using the F test (see for example, Sokal and Rohlf, 1981) to determine whether the marker location errors were significantly different between the three modes of coordinate determination (Table 2.12 (b)). The appropriate number of degrees of freedom for the F test for each mode is the number of markers multiplied by the number of test angles multiplied by three. This last factor of three arises from the consideration that each component independently contributes to the marker location error and has a Gaussian distribution with a variance of one third of the squared marker location error.

The results of Tables 2.9 to 2.12 can be summarized as follows :

(i) There is no significant difference between modes 1 and 2 (at the 95% confidence interval) using the F test.

(ii) For modes 1 and 2 the worst marker location errors are under 0.25mm, with the pooled error approximately 0.16mm. This is a positive result, especially as the metal markers have a diameter of 0.7mm.

(iii) Mode 3 results are significantly worse than modes 1 and 2 but are still under 0.3 mm for the Adelaide Dental Hospital and 0.4mm for the Adelaide Children's Hospital. This result is a reflection of the reduced accuracy in locating the origin of the coronal film, due to indirect location methods. Fortunately, there was no need in this work to use Mode 3 as all the landmarks used in this study were more appropriately determined using Mode 2.

(iv) The worst results are at edge positions just outside the average head area for adult male Caucasians.

2.5.6 Summary of Section 2.5

Of the three modes employed in this study, Mode 1 required exact landmark identification on both lateral and coronal films. Use of Mode 1 therefore implies the ability to accurately locate the same anatomical or reference marker on both films. Modes 2 and 3, on the other hand, do not have the same stringent requirement of exact identification of anatomical landmarks or reference markers, as they utilize the landmark location from one film to calculate a projection line on the other film. If the contour or structure on which the anatomical landmark or reference marker lies can be identified on the second film, the intersection between the contour and the projection line is used to calculate the three dimensional coordinates of the point.

For the reasons outlined above, it was decided that, provided a choice was available, the most appropriate mode to obtain three dimensional coordinate data for osseous landmarks would be Mode 2 for both the Adelaide Dental Hospital and the Adelaide Children's Hospital systems.

For this mode, the maximum marker location error was 0.25mm. The overall landmark location error was 0.16mm and at the 95% confidence level for a $\chi^2(3)$ distribution, the landmark location error was 1.614 x 0.16mm = 0.26mm. The metal markers have a radius of 0.35mm so the markers were always located to within the size of the metal marker. Thus, this method is excellent for the purposes of identifying metallic markers or implants. This accuracy is well within the landmark location errors of the order 1mm determined in the next section. Therefore, while small systematic errors were discernible in the data for metal markers, these errors were not significant relative to the errors for which the system was used.

2.6 Reproducibility Of Osseous Landmark Identification For Dried Skulls Using Biplanar Radiography.

In this section, the reproducibility of obtaining three dimensional coordinates of osseous landmarks is described. The procedures adopted to determine the three dimensional coordinates of the acrylic test object were applied to the location of osseous landmarks on dried skulls. Of course, in using skulls, there is the additional problem of landmark identification.

It was not the intention of this investigation to produce population statistics, but rather to produce test material for the validation of different methods for collecting three dimensional data and its subsequent analysis. It was decided therefore, that five skulls would be an adequate number for this purpose.

Five skulls of Australian Aboriginal origin with intact cranium and mandible were selected from the South Australian Museum's skeletal collection (Figure 2.16). The skulls were radiographed (biplanar) twice, a week apart, at the Adelaide Dental Hospital (Figures 2.17 (a) to (c)). Six months later the skulls were radiographed (biplanar) at the Adelaide Children's Hospital, and again seven days later (Figures 2.18 (a) and (b)). The alignment, tracing and digitizing of these radiographs were carried out according to the procedures outlined in Section 2.4.2. The anatomical landmarks and their definitions used in this study are given in Tables 2.13 (a) and (b) and Figure 2.19. Initially, forty-four reference landmarks were considered for the study of reproducibility, from which a final thirty-four were retained. Table 2.13 (b) and subsequent tables referring to these landmarks retain the original numeric identifications assigned to the reference landmarks.

Whilst great care was taken to align the skulls along the Frankfort Horizontal, it was probable that the skull orientation within the head holder differed slightly at the next exposure seven days later. For each of the five skulls, it was therefore necessary to use an alignment procedure. The procedure selected was the more robust repeated median alignment procedure described in Section 5.4 rather than the least squares alignment procedure discussed in Section 5.3, because it was expected that the location error of all the landmarks would not be identical.

The residuals of the fits were used to test the null hypothesis of zero mean difference between the first and second determination of three dimensional coordinates for an osseous landmark. The residual of each landmark for each skull of the double determination are reproduced in Tables 2.14 (a) to (e) (Adelaide Dental Hospital) and 2.15 (a) to (e) (Adelaide Children's Hospital).

2.6.1 Landmark relocation accuracy for both the Adelaide Dental Hospital and the Adelaide Children's Hospital

Tables 2.16 (a) and (b) list for both hospitals the average residuals for each component, the magnitude of the average residuals, the number of observations and the score $d\sqrt{n}/\sigma$ (whose square has a $\chi^2(3)$ distribution as discussed in Section 6.2). This score was used to test for significant deviations of a mean coordinate position from an expected value of zero.

The χ^2 results tabulated in Tables 2.16 (a) and (b) show that of the thirty-four variables measured, five were significantly different from zero at the 95% confidence interval for both radiographic systems. These variables are:

(i) Adelaide Dental Hospital

Landmk No.	Landmark Name	Mag of Ave Residual
10	external auditory meatus right	1.927mm
11	external auditory meatus left	1.514mm
14	articulare right	0.692mm
28	lower molar point right	1.227mm
40	optic foramen right	0.786mm

(ii) Adelaide Children's Hospital

Landmk No.	Landmark Name	Mag of Ave Residual
4	vertex	3.457mm
7	mastoid tip left	0.419mm
9	basion	1.105mm
33	coronoid tip left	2.253mm
42	nasale	0.345mm

Of these results five remained significant at the 99% confidence interval and can be explained as follows:

(i) The vertex (ACH), being the highest point on a surface of large radius of curvature, is very dependent on the orientation of the skull within the cephalostat. It is likely, therefore, that separate determinations located different points.

(ii) The inability to accurately locate the appropriate contour line on the PA film for the external auditory meatus right and left and lower molar point right (ADH) led to inconsistent determination of their three dimensional coordinates.

(iii) The average difference between determinations for the nasale is only 0.345mm, the thickness of a pencil line. While a change in landmark identification is indicated, the difference is small.

The reproducibility of location of a single landmark was calculated from the difference between the two determinations over the five skulls. This single landmark relocation error (or reproducibility), S, (Dahlberg,1940 with respect to estimating the distance measurement error associated with a single determination) is given by

$$S^2 = \frac{1}{2n}\sum_{i=1}^{n} |x_i|^2$$

where x_i is the residual vector for a landmark for the i^{th} skull and n is the number of skulls. The factor two in the denominator arises because measurement error can be equally ascribed to either determination.

The calculation of the landmark relocation error incorporates all possible experimental errors (for example, repositioning the subject, tracing from two sets of radiographs, digitizing, landmark identification and system error). The relocation errors determined for the Adelaide Dental Hospital and Adelaide Children's Hospital are shown in Tables 2.17 (a) and (b) respectively while Table 2.18 summarizes these results.

A relatively large number of variables (34) were used to test the three dimensional method and some are included having significantly larger location errors than others. For this reason, the median error is given as an indication of the general osseous landmark relocation error. Additionally, the two determinations for each skull were averaged to provide a better estimate of the landmark coordinates.

2.6.2 Comparison of landmark location between the Adelaide Dental Hospital and the Adelaide Children's Hospital

In order to detect any differences in landmark location between the two radiographic systems, repeated median fits were applied between the Adelaide Dental Hospital's and Adelaide Children's Hospital's averaged three dimensional biplanar coordinate data for each skull. Once again, the residuals of the fits were used to test the null hypothesis of zero mean difference between the Adelaide Dental Hospital's and Adelaide Children's Hospital's determinations of the three dimensional coordinates of a landmark.

It was expected that the positions of the landmarks determined on both systems should be the same. But this was not the case, with approximately 40% of landmarks being significantly different at the 95% confidence interval (Table 2.19). Moreover, the landmark relocation errors for each individual system are in general better than those obtained from the comparison between systems (compare Tables 2.17 (a) and (b) with Table 2.20).

These results are perhaps a reflection of a slight change in the author's definitions of various landmarks during the six month period between data collection for each system. The one month median osseous landmark relocation error, averaged between the two systems, is 0.82mm while the median osseous landmark location error between the Adelaide Dental Hospital and Adelaide Children's Hospital systems is 1.5mm. It would appear that the author became more familiar with the anatomy and procedures for locating landmarks in three dimensions over this period of time. For example, the external auditory meati (left and right) were poorly defined on the PA and it was found difficult to be consistent. At this stage its definition on the PA was evolving (ADH radiographs being traced first). In the ACH data, obtained six months later, the relocation errors for the external auditory meati (left and right) were found to be non-significant. The mastoid tip was used as an aid to enable the more consistent location of the external auditory meati on the AP film. (This measurement will be minimally larger than its craniometric equivalent). It would be expected that the osseous landmark location error between systems would decrease towards the one month landmark location error if the procedures were repeated now that the definitions and location methods have been developed.

It might be thought that the greater accuracy obtained with the double determination between radiographic tracings one month apart is due to the author's ability to remember cues not related to the definition of the landmark in question. This is not the case. It should be reiterated that the second determination is based on its own cephalograms, so there are no extraneous cues.

It should also be noted that the initial assumption that landmark identification should be the same is questionable, as the two systems are not identical, there being a 180° rotation of the subject; that is, left lateral and postero-anterior films are taken at the Adelaide Dental Hospital, compared with right lateral and antero-posterior films at the Adelaide Children's Hospital. The combination of this problem together with dissimilar source to target film distances not only results in differences in the superimpositioning of structures but also differently magnified radiographic images. For these reasons, it is perhaps not unreasonable to find that 40% of landmarks have been relocated slightly differently between the two systems.

2.7 Comparison Of Craniometric And Biplanar Roentgenographic Osseous Landmark Location For Dried Skulls

This study had as its aim the comparison of three dimensional coordinate data obtained from the above biplanar radiographs with craniometric measurement of the same five skulls.

2.7.1 Craniometric distance determination

The craniometric landmarks used in this study are also listed in Table 2.13. For each of the five skulls, a total of thirty-nine anthropometric distances covering all regions of the skull were measured using calipers (Figure 2.20). One month later the distances were re-measured for the purpose of error assessment using the method of double determination. The average and standard deviations of the differences between the two determinations of the distances for the five skulls are given in Table 2.21.

The measurement error for each craniometric distance was assessed by applying Student's t-test (see for example, Sokal and Rohlf, 1981) to determine whether the mean difference between determinations differed significantly from zero, that is, if there was a significant change in definition or measurement technique between the two sets of determinations. The t-scores of the mean differences are also given in Table 2.21.

It was observed that five distances differed significantly from zero at the 95% confidence interval and of these distances, the standard deviation of the difference was less than approximately 0.5mm. This finding indicated a consistent difference in the measurement of these particular distances between the two determinations. However, their distance measurement errors were less than 0.6mm. The D-statistic given in Table 2.21 is the distance measurement error associated with a single determination (Dahlberg, 1940), and is calculated using

$$D^2 = \frac{1}{2n} \sum_{i=1}^{n} (d_{1i} - d_{2i})^2$$

where d_{1i} and d_{2i} are the first and second determinations of the distances between a landmark pair for the i^{th} skull and n is the number of skulls. The factor of two arises in the denominator because the distance measurement error can be equally ascribed to each of the determinations.

The measurement errors associated with a single distance determination were found to lie in the range 0.08mm (nasal breadth) to 1.16mm (bi-zygomaticomaxillary breadth).

In addition, the results of the two determinations were averaged to provide a better estimate of the distances (Tables 2.22 (a) to (e)).

2.7.2 Comparison

From the average three dimensional coordinates for each osseous landmark for each skull and for both radiographic systems, it was possible only to calculate eighteen of the forty distances determined directly with calipers. The reason for this involves the inability of the viewer to identify the additional craniometric landmarks (necessary to calculate the other distances) from the radiographs.

To assess any differences in landmark definitions and/or measurement techniques between direct and indirect measurement systems, t-tests were applied to determine whether the mean difference between craniometric and biplanar radiographic measurements for the same distances over the five skulls differed significantly from zero. The results of these comparisons are presented in Tables 2.23 (a) for craniometric versus Adelaide Dental Hospital and (b) for craniometric versus Adelaide Children's Hospital.

Four average differences were found to be significant at the 95% confidence interval. These are:

(i) The small standard deviations for the distance glabella to opisthocranion for both the Adelaide Dental Hospital's and the Adelaide Children's Hospital's biplanar radiographic data versus the craniometric distance data (Tables 2.23 (a) and (b)) are indicative of consistent measurement approaches. The biplanar methods have over-estimated the craniometric measurement of the distance by 2.8mm (Table 2.23 (a)) and 2.9mm (Table 2.23 (b)) respectively. When it is considered that this distance represents the maximum head length, its error in determination using either approach is only approximately 2%.

The craniometric measurement of glabella is defined as the most prominent point in the midline between the two eyebrow ridges, a little above the fronto-nasal suture, whereas glabella for the lateral radiographic projection is defined as the most anterior point on the frontal bone (Martin, 1928). It is highly conceivable that these two points may not necessarily coincide, due to superimposition of the supra-orbital ridges when viewed in the lateral projection, thereby causing the viewer to locate glabella more superiorly on the lateral radiograph. As the location of opisthocranion is dependent upon the location of glabella (that is, the most distant point from glabella in the mid-sagittal plane), it is not surprising that a significant difference has been observed.

(ii) The average difference between the craniometric and the Adelaide Dental Hospital's biplanar radiographic measurements for the bi-mastoid tip breadth is only 0.442mm (Table 2.23 (a)), the thickness of a pencil line. While this result was found to be significant, indicating a difference in definition or measurement technique, the difference is substantially smaller than the biplanar distance measurement error of the order of one millimetre.

(iii) Whilst a significant difference was observed between the craniometric and Adelaide Children's Hospital's biplanar radiographic determinations of the distance basion to bregma (Table 2.23 (b)), it should be noted that this measurement could be determined for only two of the five skulls for both methods. The standard deviation in this case, therefore, has little meaning. However, the observed average residual is 1.54mm and this was considered an acceptable difference between the two approaches for these particularly difficult to locate radiographic landmarks.

Of the remaining non-significant differences, the magnitude of the mean difference ranged from 0.002mm to 2.874mm for the Adelaide Dental Hospital's biplanar radiographic data versus the craniometric distance data (Table 2.23 (a)) and from 0.032mm to 2.878mm for the Adelaide Children's Hospital's biplanar radiographic data versus the craniometric distance data (Table 2.23 (b)).

It is concluded from the above experiments that three dimensional cephalometric osseous landmark definitions are consistent with direct craniometric measurements, with the exception of one or both of glabella and opisthocranion. Nevertheless, glabella and opisthocranion are useful landmarks to determine, using biplanar radiography, as the definition difference was only of the order 2.8mm and the distance measurement represents the maximum head length.

2.8 Reproducibility Of Osseous Landmark Identification For Patients Using Biplanar Radiography

To evaluate the influence of soft tissue and osseous abnormality on landmark identification, biplanar radiographic data was required of patients presenting for treatment to the Australian Cranio-Facial Unit, Adelaide Children's Hospital. While the radiographic records of suitable patients were available, it was found most patients were in the mixed phase of treatment, that is, initial corrective surgery had occurred prior to the installation of the biplanar equipment described in Section 2.4.1 (Table 2.1). In the course of the present study, there was a scarcity of subjects presenting for the first time with untreated Treacher Collins Syndrome (the selected syndrome for study in this thesis) and it was only possible to procure one new adult patient presenting with this rare clefting syndrome (Figures 2.21 (a) and (b)).

Ideally, it would have been desirable to first assess the relationship between soft tissue and landmark identification using biplanar radiography on subjects with no pathology (that is, a normal population) rather than assess, as presented here, the combined influence of soft tissue and pathology on osseous landmark location.

Pre- and post-operative biplanar radiographs were taken of the patient presenting with Treacher Collins Syndrome (see Figure 2.22 for an example of patient positioning, and Figures 2.23 (a) and (b) for pre- and post-operative radiographs of the patient with Treacher Collins Syndrome) and these radiographs were aligned, traced (Figures 2.24 (a) and (b)) and digitized as described in Section 2.4.2. Repeat tracings of these radiographs were performed one month later for the purpose of determining the reproducibility of measurement. The two determinations for both the pre- and post-operative data were aligned using the repeated median fitting approach (Section 5.4).

The results of this investigation showed that the average residual for each landmark ranged from 0.134mm to 1.985mm (Table 2.24). The osseous landmark relocation errors given in Table 2.25 are indicative values only, because only two residuals contributed to the calculation of each error. Overall, the pooled osseous landmark location error was 1.28mm, while the median landmark location error was 0.66mm.

These findings are comparable with the results obtained for the Adelaide Children's Hospital's biplanar data for the five dried test skulls and comply with the statistics given in Section 2.6.

There is little doubt that pathological conditions, if present, can make the identification of radiographic osseous landmarks difficult. For example, radiographic landmarks are sometimes absent or grossly malpositioned in instances of craniofacial malformations.

The estimated osseous landmark location errors calculated in this section correspond favourably with those obtained for the dried skulls, albeit that all the facial bones are known to be affected in Treacher Collins Syndrome. It is reasonable to conclude, for this patient, that the soft tissue and the osseous abnormality have not adversely influenced landmark reproducibility.

2.9 Summary

The preliminary work presented in this chapter delineated a method for the determination of the three dimensional coordinate data using biplanar radiography. Later experiments confirmed the accuracy and reproducibility of the technique, by applying it to an acrylic test object, five dried test skulls, and an adult patient with Treacher Collins Syndrome. The theory behind the technique, together with its practical application, is summarised below.

From two orthogonal radiographs, three dimensional coordinates of landmarks were obtained using the "projection line" technique. The geometry of the radiographic configuration was exploited to facilitate identification of difficult to locate landmarks that could not be readily identified on both films. This "projection line" technique required the landmark to be exactly established on one projection and a contour identified on the other projection along which the landmark was known to lie.

The radiographs were aligned, traced, landmarks and/or contours identified and digitized. The digitizing accuracy was found to be 0.08mm.

The method was initially evaluated using a specially constructed acrylic test object, with 0.7mm diameter metal markers imbedded into its surface. For the Mode 1 method, which involved the precise identification of landmarks on both lateral and coronal radiographs, the three dimensional coordinates of the metal markers were determined with an accuracy of 0.16mm. Identical accuracy was obtained for the Mode 2 method, which involved exact landmark location on the lateral film and location of the same landmark on a contour on the coronal radiograph. The accuracy of 0.16mm was well within the diameter of the markers, thus validating the "projection line" technique.

Having established the efficacy of the "projection line" method for biplanar radiography using the test object, the approach was further investigated by determining the relocation accuracy of thirty-four osseous landmarks for five dried test skulls. The landmarks selected included many of the standard landmarks used in two dimensional cephalometry. The three dimensional coordinates of these osseous landmarks were determined with a median reproducibility of 0.91mm for the Adelaide Dental Hospital and 0.72mm for the Adelaide Children's Hospital biplanar radiographic equipment.

In order to corroborate that the landmarks were correctly located using the biplanar radiographic method, craniometric measurements using calipers were taken for comparison.

To this end, thirty-nine craniometric distance measurements were made for each of the five skulls. These measurements were repeated for a double determination and it was calculated that they had a median reproducibility of 0.389mm. Eighteen of these distances could be compared with distances determined from the three dimensional coordinates collected using the biplanar method. Seventeen of the radiographic distances were found to be concordant with those determined from the craniometric data, only one distance differing significantly (by 2.8mm) owing to definition differences between the two measurement systems.

The technique was lastly applied to an adult patient with Treacher Collins Syndrome to evaluate the influence of soft tissue and pathology on osseous landmark location.

In this experiment, the landmark location errors gathered from the double determination of the pre- and post-operative biplanar radiographs were indicative only, due to the limited data set. Nevertheless, the results were consistent with those derived for the dried skulls.

In conclusion, the present study has introduced the "projection line" technique as a method for accurate determination of three dimensional coordinate data of known reproducibility from biplanar radiographs.

CHAPTER 3

THE ACQUISITION OF THREE DIMENSIONAL CRANIOFACIAL DATA FROM COMPUTERISED TOMOGRAPHY RECONSTRUCTIONS

3.1 Introduction

The theoretical principles on which computerised tomography (CT) are based were first expounded in 1917 by an Austrian mathematician named Radon. The original calculations necessary to theoretically reconstruct an object were formidable and remained impractical until the development of sophisticated computerised technology.

It wasn't until the late 1950's that Cormack, a physicist, proposed that the internal spacial detail of an object could be obtained, provided an adequate number of X-ray views were taken. Further work by Cormack on the theory of this concept led to successful results being published in 1963 (Cormack, 1963), with a practical model developed independently in 1968 by Godfrey Hounsfield (Hounsfield, 1973).

The work of both Cormack and Hounsfield was acknowledged in 1979, when together they were awarded the Nobel Prize in Medicine for their pioneering research on the theory and application of computerised tomography (Cormack, 1979; Hounsfield, 1979).

The technology behind computerised tomography is now well established, with four generations of scanners having been built for clinical use in the past decade.

In order to appreciate the nature of computerised tomography, it is perhaps best to first briefly outline some of the characteristics of conventional radiography and complex motion tomography.

Essentially, conventional radiography compresses all the information inherent in a three dimensional (3D) object onto a two dimensional (2D) film, with the resultant image representing the summation of densities lying between the X-ray source and the film.

In an effort to overcome the problem of superimposed structures, conventional tomography or complex motion tomography has been used (see for example, Resnick, 1981). In this technique, the X-ray source and the film move in a synchronous manner, resulting in a generalised blurring of superficial and deep structures, leaving only the area of interest in focus. However, in reality this is very difficult to attain, as the the X-ray beam is relatively broad in nature, resulting in a loss of contrast resolution, as not only is the area of interest irradiated, but also the surrounding tissues.

The advantage, then, of CT over conventional radiography and conventional tomography, is that computerised tomography provides the capability of visualizing tissues of interest in sequential layers without the problem of superimposed structures which are not of direct interest to the observer.

While multiple computerised tomography images are available with minimal superimpositioning of structures (dependent on the slice thickness), they are only two dimensional representations, and the three dimensional nature of the object must still be inferred. However, Herman and Liu (1977) integrated a series of CT axial slice images to produce the so called "3D CT reconstructions". The "life-like" appearance of these three dimensional CT reconstructions represented a major advance in diagnostic medicine, as such images conveyed very quickly the spatial relationships of the structures under examination.

The output of the CT scanner provides the input for "3D image" production programs such as Display82 developed by the Medical Imaging Processing Group, Department of Radiology, Pennsylvania (Udupa, 1983, Chen et al., 1984) for General Electric.

Three dimensional imaging has become an active area of medical research and recently at a forum entitled "3D imaging in Medicine" in Philadelphia, December 1987, several papers (Zinreich et al., 1986; Burk et al., 1986; Sontag, 1987; Cutting et al., 1987; Udupa, 1987; Hemmy and Lindquist, 1987; Herman, 1988a) were presented detailing such aspects of computerised tomography as clinical applications of three dimensional imaging, software/hardware developments, different system comparisons and future trends. Whilst several of the presenters discussed the ability to quantify data (Sartoris et al., 1986; Cutting et al., 1987), the focus of papers tended towards improvements in image quality by the use of advanced software, in conjunction with independent or workstation computers. Another focal point was the interactive nature of computer graphics to enable surgeons to simulate surgical procedures.

While there has been much interest as to the value of quantification from three dimensional CT images, to the best of the author's knowledge no data have been published as to the precision of the location of osseous landmarks in the craniofacial region. It is against this background that the present work was initiated, to determine the feasibility of obtaining three dimensional coordinates of osseous landmarks from three dimensional CT reconstructions.

3.2 Description Of Equipment And Scanning Method

A General Electric[1] (GE) CT/T 8800 Scanner housed in the Department of Radiology, Adelaide Children's Hospital, in conjunction with a Data General[2] Nova 5140 was used to generate axial scans according to the Hospital's routine operating protocol (Figure 3.1). The protocol which was employed for the acrylic test object, dried skulls, and for the patients selected in this study, involved a current of 80mA, 120KV, pulse width code of 3msec, slice thickness of 5mm and a table shift interval of 3mm. For the patients, a slice thickness and a table shift of 1.5mm through the orbits was used.

For each CT examination, care was taken to ensure that the subjects scanned did not move. It was necessary, however, to sedate or give general anaesthesia to children under seven years and other patients whose cooperation could not be assured. The dried skulls and the patients' heads were oriented so that the orbito-meatal line (Frankfort Horizontal Line) was perpendicular to the floor. This position was maintained by use of the head strap fixed rigidly to an acrylic head holder. Figures 3.2 (a) and (b) shows how this was achieved while Figure 3.2 (c) depicts the manner in which the acrylic test object was fastened to the scanning table.

Only one three dimensional CT reconstruction per subject was available due to imposed time restrictions for non-clinical work. Therefore, the author chose to sub-region the CT data file to exclude information superior to the supra-orbital ridge and posterior to the foramen magnum. The rationale for this was to permit viewing of the otherwise non-visible cranial base landmarks. The few landmarks excluded as a result of the sub-regioning were readily obtained

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[1] General Electric Company, Medical Systems Group,Wisconsin, U.S.A.
[2] Data General, U.S.A.

using the biplanar radiogrammetry technique. For two and three skulls respectively, the mastoid tips and opisthion were excluded as a result of non-exact adherence by the radiographers to the above protocol.

The threshold level determines the minimum density of material to be included in the three dimensional CT reconstruction. For dried skulls, a level of -550 to -450 is suitable and for patients a level of 150 to 200 is appropriate. Unfortunately for one of the dried skulls (A38778), the radiographers did not adhere to the appropriate level resulting in holes in the three dimensional CT reconstructions. As this was not recognised until well into the data analysis phase of this thesis, the subsequently repeated reconstructions have not as yet been analysed. This work has emphasised the need for strict adherence to protocols in order to maintain the quality of the reconstructions and facilitate interpretation of images.

The three dimensional reconstruction program used in this study was Display82, developed by the Medical Image Processing Group of the University of Pennsylvania (Udupa, 1983). Display82 was accessed by the program 3D83 (Chen et al., 1984) for GE Medical Systems. Essentially, this program simplifies the creation of three dimensional displays by reducing the number of interactions on the part of the user. Additionally, the program 3DMS was employed for direct distance measurement from the screen of the Independent Physicians Display Console (GE Medical Systems). The data so obtained was initially stored on a Data General Disc (190 Megabytes) and then later archived for permanent record onto a 9 track, 6400 foot magnetic tape.

The Independent Physician Display Console, resolution 320x320 matrix, was used to view axial, two dimensional reformats and three dimensional CT reconstructions. Photographic images of the axial slices and three dimensional

CT reconstructions were recorded with the GE multiformat camera using Agfa[1] Scopix film and developed in a Kodak[2] M5 (90 second) processor.

3.3 Accuracy Of Computerised Tomography Axial Slice Data

At the outset, it was desirable to confirm the manufacturer's claimed submillimetre accuracy for axial slices (GE Brochure, 1983). This verification involved scanning the acrylic test object on two separate occasions, using a similar scanning protocol to that employed routinely for patients, except for a reduced pulse width code (as a reduced X-ray density is sufficient to satisfactorily image the object). The data from these scans were measured on five different occasions at approximately six monthly intervals. The coordinates of the metal markers were determined using software facilities available on the Independent Physicians Display Console, while viewing the appropriate axial slice (Figure 3.3). It should be noted that only the axial slice where all metal markers were visible was selected for analysis, therefore the z component of all the CT determined marker positions and the calibrated acrylic test object marker positions were set to zero.

Coordinates obtained for the determinations of the acrylic test object's fourteen metal markers were recorded and then least squares fitted (see Section 5.3) to the coordinates derived directly using the travelling microscope. $\chi^2(2)$-tests (see Section 6.2) were performed on these data to assess whether the mean residuals of the fit differed significantly from zero, that is, whether the CT coordinates differed significantly from the travelling microscope measurements.

Table 3.1 gives the average residuals and standard deviations respectively. Of the fourteen markers, only marker number nine was found to differ

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1 Agfa-Gevaert Ltd., West Germany.
2 Kodak, U.S.A.

significantly from its calibrated location. As there was no reason to expect an accuracy difference in the determination of the position of the identical metal markers, statistical fluctuation due to the small sample size was most likely responsible for the result observed for marker number nine.

The root mean square value for each marker gives its relocation error (Table 3.2). This is effectively the standard deviation of the residuals assuming zero mean difference and is given by

$$S_{rms} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} d_i^2} \quad . \qquad 3.1$$

The marker location errors (S_{rms}) are in the range 0.295mm to 0.859mm with a pooled marker location error of 0.526mm. At the 95% confidence level for a $\chi^2(2)$ distribution, this corresponds to an accuracy of 0.91mm. This is comparable to the pixel width of 0.8mm.

The marker location error calculated from Equation 3.1 differs from the Dahlberg statistic used in Section 2.4.2.6 by a factor of $\sqrt{2}$. This is because the determination of the acrylic test object's coordinates using the travelling microscope is approximately thirty times (0.9/0.03) more accurate than the equivalent CT determination. Thus, it is unreasonable to equally ascribe the error between the two methods.

The biplanar radiographic technique described in Chapter 2 located the metal markers with an accuracy of 0.26mm at the 95% confidence level, while the accuracy of the CT scanner using the Independent Physician Display Console was found to be 0.91mm. The CT accuracy using the Independent Physician Display Console appeared to be limited by the coarseness of control over the cursor position.

These results reflect the faithfulness of the CT scanner to produce reliable axial slice information and the reliability of the program on the Independent Physician's Display Console to report x, y position for a given slice (z).

The version of the CT reconstruction program used in this investigation did not allow a useful three dimensional reconstruction of the acrylic test object. The discreet nature of the acrylic test object (0.7mm metal markers embedded in acrylic) is unlike the continuous structure of skeletal material and consequently the surface algorithm performed poorly. As the parameters were set to reconstruct the acrylic sheet it was anticipated that holes would be left where the metal markers should have been. It should be noted, however, that the acrylic test object had been designed originally to assess the biplanar radiographic technique and not the three dimensional CT reconstruction algorithm. It was felt that the construction of another test object of sufficient accuracy for the three dimensional reconstruction case would have been difficult and expensive.

Under the conditions of the experiment reported above, it was reasonable to conclude that the CT scanner has the submillimetre accuracy for axial slices claimed by the manufacturer.

The above preliminary study on the accuracy of coordinate data obtained from the CT scanner was background to a more fundamental investigation on the accuracy and reproducibility of osseous landmark identification for dried skulls.

3.4 Accuracy And Reproducibility Of Osseous Landmark Identification From Distance Measurement Using Display82 And 3DMS Programs For Dried Skulls

The CT scanner software Display82 and 3DMS provide the facility to obtain three dimensional coordinate data of landmarks specified by the position of a cursor, and in addition gives distances between pairs of landmarks. When the distances between landmarks on several test skulls were compared with direct caliper measurements, some potential difficulties in using the CT scanner for landmark determination became apparent. These were:

(i) the time involved.

The appropriate image of the three dimensional CT reconstruction is viewed by an operator who places the cursor over the osseous landmark in question. Its position is reported via the use of a function key in terms of element (in pixels), line (in pixels), and image number (giving relative orientation). This information is recorded and the procedure repeated for all landmarks of interest. By feeding the recorded information into the 3DMS program, the distance between and/or the location of selected landmarks could be obtained. This operation was found to be extremely time consuming and was not practical on the Adelaide Children's Hospital's CT scanner as it was in heavy clinical demand.

(ii) the location of the landmarks from monoscopic images.

Even though the images displayed on the console were of superior quality to the standard hard copy images, the depth perception produced by stereoscopic viewing is of greater benefit to landmark identification.

(iii) many landmarks were defined for identification based traditional cephalometry.

As a consequence, a lateral or near lateral CT image was chosen to identify these landmarks and it was found that frequently the designated position did not intersect the surface or was located on the surface behind the edge of the surface of interest. This meant that an orientation of the reconstruction should be chosen in which the landmark was not at the very edge of the reconstruction. This requirement necessitated increased interactive use of the CT scanner, which heavy clinical demand constrained.

For these reasons, it was decided that for the initial evaluation of CT coordinate data, landmark determination would be off-line, using the technique described in the next section. With the greater access privileges that have become available with the completion of this work, coordinate determination using Display82 and 3DMS will be compared in the near future with the coordinate data obtained using the off-line technique.

3.5 Coordinate Determination From Multiple Stereo Pairs Of Three Dimensional Computerised Tomography Reconstructions

In this section, the mathematical theory necessary to determine three dimensional coordinates from two orthographic[1] views is presented followed by the application of this approach to multiple stereo pairs of three dimensional CT reconstructions.

3.5.1 Determination of the depth coordinate from two orthographic views

Figures 3.4 (a) and (b) show the geometric situation as a marker is rotated through an angle θ about the Z-axis, with the Y-axis as the direction of

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[1] projection by lines perpendicular to the plane of projection

projection. The quantities to be determined are the depths y_1 and y_2 from measurements x_1 and x_2 with known angle θ.

From the geometry

$$x_2 = x_1\cos\theta - y_1\sin\theta \qquad 3.1$$

$$y_2 = x_1\sin\theta + y_1\cos\theta \qquad 3.2$$

Therefore

$$y_1 = \frac{(x_1\cos\theta - x_2)}{\sin\theta} \qquad 3.3$$

and substituting Equation 3.3 into Equation 3.2

$$y_2 = x_1\sin\theta + \frac{(x_1\cos\theta - x_2)}{\sin\theta}\cos\theta \qquad 3.4$$

From two orthographic stereo CT images, the coordinates (x_1, z_1) and (x_2, z_2) can be measured for a particular landmark. The angular separation of the stereo pair is known, therefore the depths y_1 and y_2 can be calculated using Equations 3.3 and 3.4, with a better estimate for the z component being the average of z_1 and z_2.

Influence of errors

The influence of location errors in x_1 and x_2 on y_1 can be determined by differentiation of Equation 3.4.

$$dy_1 = \frac{\partial y_1}{\partial x_1}dx_1 + \frac{\partial y_1}{\partial x_2}dx_2$$

$$= \frac{(\cos\theta dx_1 - dx_2)}{\sin\theta}$$

For errors Δx_1 and Δx_2 in x_1 and x_2 respectively, the depth error is,

$$\Delta y_1 \cong \frac{\cos\theta}{\sin\theta}\Delta x_1 - \frac{1}{\sin\theta}\Delta x_2$$

Therefore, the larger the angulation separation, θ, the smaller the error Δy_1 in the depth y_1.

For example,

for $\theta = 9°$, $\qquad \Delta y_1 = 6.31\Delta x_1 - 6.39\Delta x_2$

and for $\theta = 45°$, $\qquad \Delta y_1 = \Delta x_1 - \Delta x_2\sqrt{2}$

So clearly the larger angle gives greater depth accuracy.

3.5.2 Three dimensional coordinate data from three dimensional computerised tomography reconstructions

In order to obtain three dimensional osseous landmark coordinate data from the three dimensional CT reconstructions, forty images of the subjects separated by 9° about each of the X-axis and Z-axis were generated.

The CT images are orthographic and therefore the equations for determining three dimensional coordinates given in the previous Section are applicable, provided each landmark can be identified in more than one orientation of the reconstruction about the same rotation axis.

Between adjacent orientations (that is, 9° separation) stereo imaging was used, as this gave excellent depth perception, thereby greatly facilitating the identification and location of landmarks.

The location of the centre of the image was determined by positioning the cursor at (0,0), and the rotation axes were taken to be parallel to the edges of the image. The orientation of the image about the rotation axis was displayed on the console. Initially, images of approximately 50% real size were used, but it was found that images of approximately 70% of real size gave more accurate results and all data in this work are derived from images of this size.

A stereo comparator of the kind employed by a cartographer, where a floating mark is used that does not impede depth perception and allows direct digitization of the data, was not available for the present investigation, so the following approach was developed.

After evaluation of all possible stereo pairs for (a) ease of identification of osseous landmarks and (b) number of osseous landmarks visible in each pair of stereo images (to reduce the number of tracings required to obtain the data), stereo pairs (two images 9° apart) at intervals of 45° were selected as follows:

X-axis pairs: [27°,36°], [72°,81°], [117°,126°], [225°,234°], [270°,279°], [315°,324°];

Z-axis pairs: [18°,27°], [63°,72°], [108°,117°], [234°,243°], [279°,288°], [324°,333°].

For 0°, the head is in the anatomical position and facing the viewer. Angles are anti-clockwise looking down the X-axis of rotation and clockwise looking down the Z-axis of rotation. The axes are oriented as follows: with the origin at the centre of the head, the Z-axis is through the top of the head, the X-axis is through the subject's left ear and the Y-axis is through the back of the head. All the landmarks could be located in more than one of the above stereo pairs.

The three dimensional CT reconstructions were traced in a fashion similar to the biplanar radiographs. That is, the images were overlaid with Acetate tracing film[1], placed on a light box and traced in a darkened room using a 0.5mm H pencil. Opaque material was employed to mask areas of excess light. These stereo pairs were viewed using a Wild[2] stereoscope during tracing (Figure 3.5). Unfortunately, tracing and marking the image in one view tended to impede stereo perception when attempting to trace and mark the associated pair. In the light of this, only one image of a pair could be traced at one time.

The errors associated in the subsequent digitizing of marked osseous landmarks are the same as for the radiographic case (see Section 2.4.2.6 for a discussion on digitizing error). To reduce the potential problem of depth error associated with small errors in landmark relocation error, three dimensional coordinates of osseous landmarks were generated, using the method outlined above, from pairs of images at intervals of at least 45°. For these reasons, only the left image of each stereo pair was traced and digitized (Figures 3.6 and 3.7).

For each rotation axis, many landmarks had been digitized on more than two images and this meant that their three dimensional coordinates could be calculated from any pair of images on which that landmark had been identified. The three dimensional coordinates calculated using any orientations about either axis were expressed relative to the frontal view (0°: subject facing viewer). The final three dimensional coordinates for each osseous landmark were taken to be the median coordinates of all determinations (from the multiple pairs of images) of the landmark for a particular subject. The median was used in preference to the mean, as attempts were made to identify a given landmark from as many views as possible;

[1] General Aniline Film Corp., drafting film 0.05mm thickness.
[2] Wild ST4-made by Wild in Singapore for Wild, Switzerland.

perhaps views that were inappropriate for the landmark, possibly or potentially giving rise to larger location errors for some determinations. This is of importance as the mean is influenced by all estimates of a given landmark and a large location error in one determination could significantly affect the final estimate of the landmark's coordinates. Hence, the median, in this situation, was a more reliable estimate.

3.6 Reproducibility Of Osseous Landmark Identification From Multiple Stereo Pairs Of Three Dimensional Computerised Tomography Reconstructions For Dried Skulls

The anatomical osseous landmarks considered in this investigation are given in Tables 3.3 (a) and (b) and their location shown in Figures 3.6 and 3.7. Initially one hundred and eight osseous landmarks were considered for the study of reproducibility from which a final seventy-six were used. As a result of incorrect thresholding or subregioning, not all of the originally selected landmarks were visible in the stereo images because protocols were not precisely followed by radiographers. The results of this work show the need for exact adherence to a protocol and future work will include all the required data. Table 3.3 (b) and subsequent tables referring to these landmarks retain the original numeric identification assigned to the osseous landmarks.

The five test skulls were positioned and scanned as described in Section 3.2. Figures 3.8 to 3.12 show for each of the five skulls the orientations of the CT images used for determination of osseous landmark three dimensional coordinates. An assessment of the landmark relocation error from CT data was made using the method of double determination (in a similar manner to that used for the biplanar results described in Section 2.6). However, due to limited access to the CT scanner, it was not possible to re-scan the five test skulls and then retrace their three dimensional CT reconstruction images. Thus, the double determination was based on the same CT stereo image pairs retraced after a time interval of one month. This interval ensured that one did not remember non-image cues that would aid in identifying the same position on the film.

The coordinate data of the two determinations for the same skull were aligned using repeated median fitting, and the resulting residuals (Tables 3.4 (a) to (e)) were used for the determination of the landmark location errors. The average residuals and standard deviations, across the five skulls for each landmark, are given in Tables 3.5 (a) and (b). Significant average residuals, using the $\chi^2(3)$ test described in Section 6.2, are identified if their $d\sqrt{n}/\sigma$ score exceeds 1.614 (95% confidence interval). For some landmarks, the $\chi^2(3)$ test indicated that the average residual is significantly different from zero. This meant that the definition of the landmark (in the author's mind) had changed in the one month period between determinations. This implied evolution of the landmark definitions and suggested that if the measurements were repeated again, with the more stable definitions, an even greater consistency and accuracy would be attained. Of the seventy-six osseous landmarks measured, twelve had changed significantly but for the most part, were displaced by less than 2mm (which is within the width of 2 to 3 voxels, where one voxel = 0.8 x 0.8 x 0.8 mm).

Table 3.6 lists the osseous landmark relocation errors using the CT data. These errors are in the range of 0.4llmm for the right coronoid notch to 5.165mm for left posterior clinoid. The median landmark relocation error is 1.7mm, that is, approximately the width of 2 voxels in CT terms. When it is considered that the images show only moderate contrast (due to incorrect camera adjustment; later serviced), were scanned 5/3 as opposed to 1.5/1.5, and the images were less than life size, the result reported for the median landmark relocation accuracy is excellent. The deficiencies in the images were operator related and because these have been identified (with the help of Professor Gabor Herman, 1988b) and can be easily overcome, one can expect to improve on the already excellent accuracy of three dimensional coordinate data for landmarks.

3.7 Comparison Of Craniometric And Computerised Tomography Osseous Landmark Location For Dried Skulls

The distances between landmarks located using CT data were compared with the corresponding craniometric measurements to ascertain whether there were any significant landmark definition differences between the two methods of measurement. The results of this comparison for each of the five skulls are shown in Tables 3.7 (a) to (e). The average and standard deviation of the differences for each distance were calculated to determine the significance of the average differences using t-tests (Table 3.8).

Of the thirty-one distances compared, ten were found to be significantly different at the 95% confidence interval (lower portion of Table 3.8). The results can be summarized as follows:

(i) The measurement demonstrating the greatest discrepancy between direct and indirect determinations was observed for the bicondylar breadth, where the CT measurement had over estimated the true breadth by 9.34mm. However, the craniometric measurement of the bicondylar breadth was taken using landmarks consistent with the definition used for biplanar radiography. See Table 3.3 (a) regarding the landmark definition for condylion. Comparison with Tables 2.23 (a) and (b) shows the difference between craniometric measurement and biplanar measurement for bicondylar breadth to be not significantly different. The two determinations (CT and craniometric) of bicondylar breadth are not strictly comparable and a measurement difference was expected. The analysis of this distance is included in the Table to indicate the consequence of differences in definition.

(ii) Small standard deviations of the differences between the measurement techniques for the distances nasion to anterior nasal spine and nasal breadth, are indicative of the well defined nature of these distances, both craniometrically and using CT. Nevertheless, there is a difference in location between the two systems, that is, the craniometric measurement, nasion to anterior nasal spine is larger than its CT equivalent by 2.13mm, while the CT determination of the nasal breadth is larger than its equivalent craniometric measurement by 1.43mm. These differences are most likely a result of the influence of thresholding, which determines the minimum density of material to be included in the three dimensional CT reconstruction. Both the bone of the anterior nasal spine and nasal aperture come to a sharp point or edge respectively. It is probable that, while the general threshold is suitable for most bone densities, it is unlikely to fully image thin bone projections.

(iii) The craniometric measurements for the height and breadth of the right and left orbits are larger than the comparable CT determination. The direct caliper measurement of the orbital height reflects the distance from the most superior to the most inferior point of the orbital rim, whereas the CT determination is between the landmarks superior orbitale and orbitale. The CT measurement was more vertical, while the caliper measurement was more diagonal, hence the observed smaller value for the CT determination. In retrospect, the craniometric measurement using calipers could have been between the landmarks used for the CT determination. The use of different definitions reflects the evolution of the landmarks for the CT. The craniometric caliper measurements were taken before the CT measurements and during CT measurement the landmark definitions were altered, to be more suitable for the CT.

Similar arguments apply to the measurement of CT breadth and the width of the frontal process of the zygoma.

(iv) A significant difference was observed in the distance measurement of the bigonial breadth and this was again indicative of definition differences between the two measurement systems. The positions of the gonions were determined on the CT image using the bisector of the lines that define tangent gonion to project back onto the surface of the mandible (in a similar fashion to that employed with the biplanar technique).

The craniometric measurement reflects the maximum width in the local environment of the gonions and there can be small bony projection (due to muscle insertions) that are not encompassed by the CT definition used.

(v) Again, one appears to be identifying different positions on the coronoid tips with the two methods. The craniometric measurement was taken between the external surfaces of the coronoid tips to give the maximum separation of the calipers. For CT, the views are limited by the presence of the zygomatic arch which means that while the top of the coronoid tip is identified, it does not necessarily coincide with the points of maximum width. If the zygomatic arch was removed by suitable subregioning, it might be expected that the point of maximum width would be more easily identified.

(vi) The landmarks left and right zygomatic frontal were included to measure the external width of the anterior cranial fossa, however the CT determination of this breadth was found to be significantly larger than the craniometric measurement. The accuracy of 0.15mm for the direct caliper measurement (Table 2.21) probably reflects that a local minimum along the superior temporal line was used instead of the specified landmark definitions (see Table 3.3 (a)).

In this region, interactive identification of landmarks is probably preferable, as either or both the local minimum along the superior temporal line in the region of the frontal bone and the landmarks left and right zygomatic frontal could be identified.

Apart from the thresholding differences in (ii) above, the distances that were significantly different from the craniometric measurement resulted from definition differences between the measurement systems. That is, the same landmarks were not being measured although referred to by the same name, see Table 3.3 (a). Recognition of these differences will lead to consistency of definitions used in future work.

The average differences that were found to be non-significant between craniometric and CT data (upper portion of Table 3.8) were in the range 0.180mm to 2.636mm, with one rogue value of 5.59mm. This rogue value was for the bi-articular eminence distance for which the landmarks, articular eminence right and left, were ill defined for three of the five skulls due to flattened and/or worn eminences. Essentially, for two of the skulls there was no difference between the CT and craniometric measurements and it is only the impaired articular eminences on the remaining skulls that has led to a large variance being calculated. Similarly, the distance, nasion to basion, was found to have a relatively large standard deviation due to the influence of one observation (Table 3.7 (e) compared with Tables 3.7 (a) to (d)).

Comparison of Tables 2.23 and 3.8 show that CT accuracy is comparable to biplanar accuracy - reproducibilities being 1.7mm for CT and for the biplanar technique 0.7mm at the Adelaide Children's Hospital and 0.9mm at the Adelaide Dental Hospital.

The results of this section confirm that the three dimensional CT landmark coordinates derived using the multiple stereo imaging technique are consistent with craniometric measurement, provided that the same landmark definitions can be followed for the two measurement techniques.

3.8 Reproducibility Of Osseous Landmark Identification From Multiple Stereo Pairs Of Three Dimensional Computerised Tomography Reconstructions For Patients

To assess the influence of soft tissue and osseous abnormality on landmark identification, multiple stereo pairs of three dimensional CT reconstructions were generated and landmark coordinate data determined in a similar manner to that already described in Section 3.6. The same number and orientation of stereo pairs were used as for landmark determination on the dried skulls.

Three patients were selected - an adult and an eleven year old, both with Treacher Collins Syndrome (Figures 3.13 (a) and (b)), and a twelve month old child with Apert's syndrome (Figure 3.13 (c)). Images from the three dimensional CT reconstructions for each patient (Figures 3.14, 3.15 and 3.16) were traced, digitized and the three dimensional coordinates of osseous landmarks determined. One month later the images were re-traced, digitized and three dimensional coordinates again determined to ascertain the reproducibility of landmark location using the technique of double determination (as discussed in Section 2.4.2.5).

The coordinate data of the two determinations for the same patient were aligned using the repeated median fitting approach discussed in Section 5.4 (Tables 3.9 (a) to (c)). This alignment revealed the average residual for each landmark ranged from 0.114mm to 5.673mm (Table 3.10) with 88% (sixty-seven out of seventy-six) below 3mm.

Single landmark location errors (determined as in Section 2.6.1) are given in Table 3.11. These should be considered indicative only, as there were only three (and sometimes less) patients contributing to their determination. In explanation, in some cases the three dimensional CT reconstruction images had not been generated according to the subregioning specifications and in these instances it was not possible to obtain all of the landmarks (for example, the patient with Apert's syndrome). Other missing landmarks were related to the presence of pathological conditions, resulting in the absence of bone and therefore the associated landmarks could not be identified. The overall or pooled landmark location error taken over seventy-six landmarks was 2.0mm (approximately the width of 2.5 voxels) while the median landmark location error was 1.2mm (approximately the width of 1.5 voxels). These results are comparable to those obtained for the landmark relocation errors calculated in Section 3.6 for the five dried skulls and indicate, for these patients, that soft tissue and osseous abnormality have had no apparent affect on landmark identification.

It should be noted that the adult patient under current discussion is the same patient for whom landmark data had also been determined using the biplanar techniques (see Section 2.8).

3.9 Summary

The conclusion derived from studies of the kind discussed in this Chapter is that landmarks from three dimensional CT reconstructions can be located with a degree of precision that is comparable to that of the biplanar method. Additionally, the location of osseous landmarks and the subsequent production of three dimensional coordinate data from three dimensional CT reconstructions is not adversely affected by the presence of soft tissue or pathological conditions.

In response to the severe time restrictions during the early stages of this work, a method was developed to determine three dimensional coordinate data of osseous landmarks from multiple stereo images of three dimensional CT reconstructions. This had the advantage that stereoscopic viewing of these radiographs could be used, providing enhanced definition to the image being analysed and facilitating landmark location.

The reproducibility of the coordinate data using this method was assessed by applying the method of double determination to the five test skulls. The resulting median landmark relocation error was calculated to be 1.7mm (approximately the width of 2 voxels). The accuracy of the coordinate data using this method was assessed by comparison with craniometric distance measurements, and was found to be in the range 0.18mm to 2.64mm for twenty-one distances. These results were comparable to the accuracies found for the biplanar radiographic technique, although the relocation errors are larger for the CT determination. However, many more landmarks were determined using the CT technique than with the biplanar technique.

Similar results for the landmark relocation accuracy were found for the patients, with average and median relocation accuracies of 2.0mm and 1.2mm respectively. For patients, lack of data for a landmark needs to be interpreted with care, as absence of particular landmarks may be a reflection of the pathological condition; for example, lack of zygomatic arches in patients with Treacher Collins Syndrome, or as a result of threshold selection. Absence due to the influence of the pathologic condition also conveys information.

In conclusion, accurate three dimensional coordinate data from three dimensional CT reconstructions using the multiple stereo imaging technique have been obtained with known reproducibility for seventy-six osseous landmarks.

CHAPTER 4

THE COMPARISON AND INTEGRATION OF THREE DIMENSIONAL COORDINATE DATA FROM BIPLANAR RADIOGRAPHY AND COMPUTERISED TOMOGRAPHY

4.1 Introduction

Three dimensional coordinate data of osseous landmarks has been collected for the same dried skulls and a patient using both biplanar radiogrammetry and CT reconstructions. The landmarks in common can be used for alignment of the landmarks derived from the two data collection modes and for comparison and verification of the consistency of the landmark definitions between systems. Further, the landmarks from the two modes can be integrated to provide an expanded data base of three dimensional coordinate data.

The Australian Cranio-Facial Unit is physically located within the Adelaide Children's Hospital, and patients who are deemed to require a complete radiographic assessment, following a thorough case history and examination by the Unit's staff, are always radiographed at the Adelaide Children's Hospital rather than at the Adelaide Dental Hospital. It is for this reason that, from this point on, only the biplanar data collected using the Adelaide Children's Hospital's system is utilized. A comparison of the Adelaide Children's Hospital's and Adelaide Dental Hospital's biplanar data were presented in Section 2.6.

4.2 Comparison Of Biplanar And Computerised Tomography Osseous Landmark Location

Of the seventy-six osseous landmarks identified using the CT system and the thirty-four osseous landmarks identified using the biplanar system, twenty-five[1] were common to both measurement systems and it is these that were used for alignment and comparison of the two sets of coordinate data.

For each of the five test skulls, the two sets of three dimensional coordinates, determined by the biplanar and the CT methods, were aligned using the repeated median fitting procedure (Section 5.4) and the resulting twenty-five residuals used to test for any significant differences between the two systems in the location of the osseous landmarks.

The averages and standard deviations of the residuals are given in Tables 4.1 (a) and (b) respectively. Listed also in Table 4.1 (a) is the $d\sqrt{n}/\sigma$ value for each landmark and this was used to assess the significance of the residuals, using the $\chi^2(3)$ test described in Section 6.2. Ten of the residuals were significantly different from zero at the 99% confidence interval ($d\sqrt{n}/\sigma > 1.945$).

Most of those landmark locations observed to be significantly different between the CT and the biplanar systems were also noted to be significant in earlier comparisons of craniometric with biplanar radiography and CT landmark locations (Sections 2.7 and 3.7 respectively).

Explanations for the observed significant results are:

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[1] Although opisthion and infradentale have been identified using both modalities this was for one skull only, and therefore an assessment of the significance of differences between their landmark definitions cannot be made. This reduces the potential number of assessable landmarks from twenty-seven to twenty-five. For some skulls this number is reduced further where landmarks could not be identified using one or other of the modes of imaging.

(i) As reported in Section 3.7, the biplanar definition of condylion (right or left) differs from that of the CT definition, with the CT location lateral to that of the biplanar position. More precisely, for the biplanar system condylion is defined for the lateral projection as the most superior point on the condylar head and for the AP projection as the mid-point of the most superior surface on the condylar head. For the CT system, condylion is defined as the most superior point on the visible lateral surface of the articulated condylar head. While the two definitions are in the same neighbourhood, a significant difference between the two systems for this landmark (right and left) was to be expected.

(ii) The radiographic position of the gonion (right or left) is determined using the bisector of the lines that define tangent gonion to project back on to the surface of the mandible. Tangent gonion is defined as the point of intersection between the mandibular plane line (tangent to the lower border of the mandible through gnathion) and the ramus line (tangent to the posterior border of the mandible through articulare). However, the key reference positions of gnathion and articulare could not be readily defined on the CT images, as the three dimensional CT reconstructions did not quite include the most inferior point on the mandibular symphysis (gnathion). Further, articulare is a derived cephalometric landmark based on the intersection of medial and lateral radiographic shadows and it can therefore only be estimated on the CT reconstruction. For CT, the ramus line was chosen to be the tangent to the posterior aspect of the condylar head. The significant difference observed between the two radiographic systems for these landmarks showed that the gonions were located in a consistently different manner for the two systems.

This reflects the difficulty in locating articulare in the traditional way using CT, and hence indicates the inappropriateness of the landmark definition for CT.

(iii) The significant difference noted for the location of the optic foramina between the CT and the biplanar systems was not surprising. The location of the optic foramina using biplanar radiography was extremely difficult, requiring a high degree of proficiency and experience on the part of the viewer. Even with great care and skill, the final location of the optic foramina can only be considered to represent an educated guess. (It was possible, however, to be consistent between the five skulls for the biplanar system because the same criteria, that is, reference structures, could be used.) On the other hand, the location of the optic foramina as determined from the three dimensional CT reconstruction was found to be much easier and therefore more consistent because of the "life-like" appearance of the image.

(iv) The significant difference recorded in the location of the right molar points (upper and lower) between the CT and the biplanar methods was also not unexpected. In part, this significance can be attributed to the geometry of the Adelaide Children's Hospital's biplanar system, where the left side of the subject's face and skull is always closest to the lateral film. Thus the structures of the left hand side are less affected by magnification than those of the right. The identification of upper and lower molar points is further compounded by the superimpositioning of the teeth onto themselves, resulting in the right molar points being much more difficult to locate than the left.

The location of the molar points from three dimensional CT images is also difficult, as there is a reduction in CT resolution in the region of the teeth due to the unexpectedly high X-ray density of the "sticky wax" used to fix the position of the mandible relative to the maxilla. This made it harder to identify the exact location of the molar points. Despite this problem, it is reasonable to assume that the landmarks will be identified in the general region of their true position.

(v) Nasion and nasale have been located differently between the two radiographic systems. This result was unexpected on the basis of landmark definitions, and would appear to be related to difficulty in determining the exact contour to trace on the antero-posterior (AP) film. A contour was consistently located, but this was probably a reflection of the use of the mid-sagittal line in the absence of a clearly visible contour containing these landmarks on the AP film.

The landmarks found significantly different between the two systems fall into two classes. The first category includes those landmarks where distinct definition differences could be identified between the two systems, such as for condylion and gonion. Significant differences between the CT and craniometric measurements involving these landmarks were also found in Section 3.7, due to the craniometric measurement being taken to reflect the biplanar definitions. By selecting the most appropriate definition, the differences could be rectified.

The second group, such as the right molar points, optic foramina, nasion and nasale, reflect the lack of distinguishable features on the AP film, where even the use of the projection line technique is not sufficient to overcome landmark identification difficulties. The features in this class are more easily identified from the stereo CT images, due to the benefits of depth perception and the non-superimpositioning of structures.

The remaining fifteen osseous landmarks show no significant difference in location between the biplanar and the CT approaches.

4.3 Integration Of Biplanar and Computerised Tomography Osseous Landmarks

One of the obvious advantages of being able to obtain reliable three dimensional coordinate data from different imaging systems is that it opens the way for data integration and the formation of expanded coordinate data bases. Integration of two or more three dimensional coordinate data sets involves alignment of the data sets using the landmarks in common (and perhaps the averaging of these landmarks after alignment), followed by augmentation of one data set with the additional landmarks from the other data set.

The value of integration is clear when one is confronted with numerous diagnostic images, with no single image containing the entire complement of information. The ability to integrate the complementary information into one data set allows for a more complete analysis and interpretation of the data. This has obvious advantages in the clinical evaluation of patients.

However, a pre-requisite to data integration is that sufficient landmarks must be in common between the imaging modes, to allow for alignment of the data sets and, furthermore, each should have known accuracy.

As mentioned in Section 4.2, it was established that of the seventy-six CT and thirty-four biplanar osseous landmarks derived for the five test skulls, twenty-five were common to both radiographic systems. Of these twenty-five, fifteen osseous landmarks were found not to have significantly different locations.

With this in mind, the three dimensional biplanar coordinate data were aligned with the three dimensional CT coordinate data using repeated median fitting. Integration of the data were performed prior to determination of differences between the two radiographic systems, alignment being performed by using all landmarks in common. As more than 50% of the landmarks were found not to differ significantly in position (between the two systems), alignment using repeated median fitting would not have been unduly influenced by those landmarks found to be significantly different.

After alignment, the landmarks in common to both systems could be averaged to improve the estimate of their location. However, it was elected to retain the CT coordinates of the landmark positions. This is primarily due to consideration of landmark identification in the presence of a pathological condition. Landmark determination using the biplanar technique is very dependent on the observer's knowledge of anatomy. For normal skulls, the most likely contours are used on the AP film, but in the presence of a pathological condition, anatomic relationships can be very different, making identification of suitable contours difficult. So, while good reproducibilities can be obtained through selection of an apparently appropriate contour, the landmark may not be located in the desired position. However, for three dimensional CT reconstructions, their is greater confidence in the observer's ability to determine landmark locations in the presence of pathology due to the life-like appearance of the image. A statistical comparison between landmark positions determined using the two techniques on patients has not yet been performed, as common data have been collected for only one patient. For these reasons, it was decided not to average the positions of the landmarks in common for the patient, but rather to retain the CT determined positions and augment the data with the biplanar technique determined positions. Further, for consistency, it was elected to do the same for the dried skull data, even though the same considerations did not apply.

For the five test skulls and the patient with Treacher Collins Syndrome, the biplanar and CT three dimensional coordinate data were integrated. The CT data were augmented by the biplanar data and the resultant integrated coordinates for the five test skulls are listed in Tables 4.2 (a) to (e), with the biplanar derived landmarks indicated by the suffix '(bi)'. For the patient, the integrated coordinate data are given in Table 4.3. In addition, more biplanar landmarks were utilized in the integrated patient data than in the integrated dry skull data, as posterior nasal spine and all four molar landmarks were obscured in the CT images due to "streaking" produced by amalgam restorations. The landmark location errors corresponding to the integrated data are given in Table 4.4.

For each skull and for the patient, the integrated osseous landmarks have been used to produce three dimensional wire frame models, plotted as stereo pairs in Figures 4.1 (a) to (f). The landmarks have been grouped into regions representing the mandible (red), maxilla and nasal bones (green), orbits (purple), zygomas and zygomatic process of the temporal bone (orange), the cranium (blue) and cranial base (brown). The landmarks can be identified by reference to Figure 6.1 (f) which shows a set of labelled wire frame diagrams for the skull.

4.4 Summary and Discussion

The comparison between biplanar and CT osseous landmark locations for the five test skulls determined that approximately 60% of the landmarks did not have significantly different locations. The accuracy and reproducibility of these landmarks have been well determined (see Sections 2.6, 2.7, 2.8, 3.6, 3.7, 3.8) and their use has made it possible to integrate the data obtained from the biplanar technique with that obtained using the CT method. In this respect, the work presented here represents a major advance on the only other research published on three dimensional coordinate data integration (Cutting et al., 1986a, 1986b, 1987), where no details were given on the methods of data collection, integration or accuracy of the data. A more detailed comparison of the two approaches is presented in Chapter 8.

The approach adopted in the present investigation allowed for the augmentation of the CT data with biplanar landmarks not determined using the CT system. Further, the CT osseous landmark data were retained in preference to the biplanar data, when the landmark had been located using both systems. It would have been possible to have averaged the CT and biplanar osseous landmarks with common definitions, or use the more accurate determination, but for patients where pathological conditions make landmark location more difficult using biplanar radiography, the CT data were preferred.

While the median relocation error for CT landmarks was greater than for biplanar landmarks (1.7 mm compared to 0.7 mm, although direct comparisons with craniometric distance measurements show similar accuracies, (see Sections 2.7 and 3.7), there were, however, many more landmarks identified from the three dimensional CT reconstruction than could even be attempted from the biplanar radiographs. Further, the viewer requires less experience to determine landmarks from three dimensional CT reconstructions as opposed to the projected images of the biplanar technique. This is because the "life-like" nature of the CT images gives the viewer more confidence in locating landmarks, compared with the biplanar technique, where the landmarks have to be identified from a superimposed differentially enlarged radiographic image.

For patients with severe syndromal features, landmarks can be extremely difficult to locate confidently, using biplanar radiography. On the other hand, the "life-like" appearance of the three dimensional CT reconstructions facilitates osseous landmark identification, even in the presence of gross pathological conditions. This is a particularly vital pre-requisite, when the object is to quantify the extent of the pathology.

With greater access to the CT scanner (or off-line access to the data), all the biplanar landmarks could have been determined using the CT (where definition differences exist, new definitions appropriate for CT that convey the desired information could be used). The biplanar technique, however, provides a suitable alternative method when CT scans cannot be used. It has been demonstrated that where both techniques are available, it is possible to combine the data.

Between sixty and eighty-five osseous landmarks with known location errors were identified for each of the five test skulls and the patient. The known accuracy and reliability of this integrated three dimensional coordinate data allows the craniofacial complex to be well described, providing for the first time the key to proceed with the analysis and quantification of shape in three dimensions.

CHAPTER 5

SHAPE ANALYSIS TECHNIQUES

5.1 Introduction

Man has long been fascinated with the representation of form and symmetry. Early examples of this interest may be seen in prehistoric cave and bark paintings which demonstrate man's ability to visualize a shape in his mind and then physically reproduce his interpretation of the image.

Of course, shape interpretation is highly subjective, and as a consequence, is extremely difficult to quantify scientifically.

It was not until the Renaissance that a more exact approach to the study of shape was adopted. Prominent in this field was Leonardo da Vinci (1452-1519), whose great interest in anatomy, together with his artistic ability, allowed him to represent form realistically. Leonardo appreciated that the quantification of shape depended upon an awareness of perspective.

One method which he developed relied upon the use of grids for size comparisons of different regions of the same object (Figure 5.1). Argentieri (1956) quotes a description of this technique from Leonardo's "Treatise on Painting".

> "Set a frame with a network of thread in it between your eye and the nude model you are drawing, and draw these same squares on the paper . . . Then place a pellet of wax on a spot of the net which will serve as a fixed point . . . Afterward, remember when drawing

> figures to use the rule of the corresponding proportions of the limbs as you have learned it from the frame and the net . . ."

The principles of classical perspective were further advanced by Albrecht Dürer (1471-1528). Dürer's technique involved the use of a glass screen and a vertical rod to fix the position of the eye. The subject was viewed through the screen and drawn point by point on to the glass. The vertical rod allowed perspective to be maintained (Figure 5.2).

Dürer also popularized the use of mesh grids to define facial proportions. Figure 5.3 depicts drawings by Dürer showing differences in facial types, as determined by his dynamic rectangular grid network. As the grid system supporting a face is distorted by expansion or contraction, the corresponding points on the face are proportionately moved.

In the early twentieth century the attempts to scientifically represent shape gained greater momentum, with the work of D'Arcy Thompson being particularly notable. Thompson (1917) analysed biological processes from their mathematical and physical aspects without undertaking actual experimentation. His approach can best be explained by considering Figure 5.4. In this figure, a Diodon has been transformed by manipulation of the cartesian coordinates to an Orthagoriscus. Having achieved this transformation, it was Thompson's objective to mathematically express the relationship between one shape and the other. Moreover, Thompson suggested that it would be "comparatively easy" to identify the "force" (in magnitude and direction) necessary to produce the transformation depicted. Thompson considered that a coordinate grid enabled the viewer to see the overall simplicity of an organism, as well as the simple transform relationship. According to Thompson (1917):

"The coordinate throws into relief the integral solidarity of the organism, and enables us to see how simple a certain kind of correlation is which had been apt to seem a subtle and a complex thing.

But if, on the other hand, diverse and dissimilar fishes can be referred as a whole to identical functions of very different coordinate systems, this fact will of itself constitute a proof that variation has proceeded on definite and orderly lines, that a comprehensive "law of growth" has pervaded the whole structure in its integrity, and that some more or less simple and recognisable system of forces has been in control. It will not only show how real and deep-seated is the phenomenon of "correlation", in regard to form, but it will also demonstrate the fact that a correlation which had seemed too complex for analysis or comprehension is, in many cases, capable of very simple graphical expression."

Whilst Thompson's book is beautifully illustrated, several of his diagrams show inconsistencies. For example, in Figure 5.4, the tail of the Orthagoriscus is not in proportion to that which would be expected from the diagram of the Diodon. Since the grid is symmetrical, there is no accounting for the tail fin difference. Unfortunately, this problem cannot be resolved, as Thompson left no detailed notes on the construction of the drawings. It is not known, therefore, how he derived the axes on which the figures appear.

Despite these shortcomings, Thompson's elegant concepts have provided the foundation and stimulus for more detailed studies into the analysis and quantification of shape and shape change (Bonner, 1961).

Sneath (1967), in an attempt to resolve the apparent perplexities of Thompson's grid method, used trend surface analysis. Incorporated into this approach was a proposal for least squares matching of sets of homologous points. If a number of homologous points can be defined on two related forms, the forms may be compared on the basis of the "match" between homologous points. After compensating for differences in size between the forms, if required, the two sets of homologous points are superimposed so that the summed squares of differences between the points in set 1 and set 2 are minimized. The residual sum of squares provides a measure of the "similarity" between the forms. Sneath then used these data in another fitting procedure, where the x and y differences between homologous points were fitted separately by linear, quadratic and cubic polynomials. These polynomials were then used to calculate the displacement of the grid points in order to produce the Thompson-like transformation grids. These polynomials were also used to produce separate contour maps of x and y differences, to enable the "trends in x and y" to be studied.

This technique provides an analytical method of producing Thompson-like grids and quantifying the shape differences; however, the coefficients of these polynomials are not independent and cannot be ascribed any biological significance.

Rather than least squares matching, Siegel (1982a, 1982b) advocated the use of robust and resistant statistical techniques. Siegel's robust method has the same objective as Sneath's, that is, the superimposition of two related sets of homologous points. However, Siegel's repeated median fitting procedure avoids the inherent limitation of the least squares fitting method of allowing regions with large deformations to have a significant impact on the fit, thereby possibly obscuring some of the true shape differences. Furthermore, Siegel's approach tends to limit the influence of large differences between homologous points, and the resulting fit is closer in similar regions and not as close in dissimilar areas. According to Siegel, the resistant fit is superior to the least squares fit, in that the differences are more readily identified and agree more closely with the perceived structural differences.

Healy and Tanner (1981) developed a new method for the study of biological shapes. Their method is based on homologous points (or homologous distances) that can be measured on each member of a family of related biological forms. Healy and Tanner proposed that, because comparative size is multiplicative rather than additive, all linear variables be transformed to log values. The transformed log-measurements, when subject to principal component analysis, can be used to disclose sources of variation, independent of size which is held constant by the previous transformation. These sources of variation, then, can be regarded as indicative of variations in shape within the population.

Another method of shape analysis is Booksteins's biorthogonal grid approach (1978), in which grids are overlaid on both the initial and final shapes such that they reflect local shape change in terms of direction and magnitude (dilation or contraction). At each point, the two shapes can be related to each other by scale factors along the two orthogonal grid directions. Bookstein suggested that the use of biorthogonal grids, as opposed to Thompson-like grids, simplifies the representation of shape change, as there are no components of shear or twist.

However, the method of biorthogonal grids has not gained wide acceptance, despite extensive interest (Humphries et al., 1981). This may, in part, be due to the complex manner in which Bookstein described his approach!

Essentially, Bookstein's technique is one of strain analysis of finite elements. This relatively new method of morphological analysis has been applied by Cheverud et al., (1983) to the quantification of three dimensional cranial morphology of the rhesus macaque. The cranium is defined by twelve elements, each of which is constructed with eight nodes which were selected from forty-two reference points located on frontal, lateral, or basal views of the cranium (Figure 5.5). A mean cranial form is derived by averaging the x, y, and z coordinates of the forty-two reference points measured on fifty specimens. The cranial shapes of all fifty specimens are then obtained by the transformation of the mean cranial form into each of the fifty individual crania. From these results, statistics on the stretch ratios are obtained for each element.

More recently, several other reports dealing with finite element analysis of craniofacial morphology and growth have appeared - for example, Richtsmeier and Cheverud (1986); Moss et al., (1987); Richtsmeier (1987); and Bookstein (1987). Even so, the technique of finite element analysis has not gained wide acceptance, although the above proponents use forceful language advocating their method and denigrating all traditional methods.

5.2 Distance and Angle Calculation

Measurements of distances and angles directly from skeletal material or living subjects has long formed the major basis of shape analysis in anthropology. With the advent of radiology in the late nineteenth century (Röntgen, 1895), it became possible to measure internal features of the skull.

However, three dimensional internal features are projected onto the two dimensional film plane. As a result, the distances and angles measured depend not only on the features of the subject, but also on the parameters of the radiographic equipment and the subject's relative orientation and position. To enable the use of cephalometric data from different sources, standardised cephalometric techniques were developed (Broadbent, 1931; Hofrath, 1931).

With the use of the biplanar and CT techniques discussed in Chapters 2 and 3 respectively, three dimensional coordinate data can be obtained and then used for the generation of distance and angle data. Such calculations are based on the following considerations:

The distance, d_{ij}, between two landmarks whose positions are specified by the vectors $\mathbf{x}_i = (x_i, y_i, z_i)$ and $\mathbf{x}_j = (x_j, y_j, z_j)$ is simply the magnitude of the vector difference between the points

$$d_{ij} = | \mathbf{x}_i - \mathbf{x}_j | \qquad 5.1$$

that is,
$$d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2}$$

where x_i, y_i, and z_i are the Cartesian coordinate components of the vector $\mathbf{x}_i$ and similarly for $\mathbf{x}_j$.

The angle, θ, between two vectors defined by three landmarks $\mathbf{x}_i$, $\mathbf{x}_j$, and $\mathbf{x}_k$ (that is, the vectors $(\mathbf{x}_i - \mathbf{x}_j)$ and $(\mathbf{x}_k - \mathbf{x}_j)$), is calculated from the dot product of difference vectors

$$\cos\theta = \frac{(\mathbf{x}_i - \mathbf{x}_j)\,(\mathbf{x}_k - \mathbf{x}_j)}{|\mathbf{x}_i - \mathbf{x}_j|\,|\mathbf{x}_k - \mathbf{x}_j|} \, . \qquad 5.2$$

Similarly the angle, θ , between two vectors, specified by the landmarks $\mathbf{x}_i$, $\mathbf{x}_j$, $\mathbf{x}_k$, and $\mathbf{x}_l$ (that is, the vectors $(\mathbf{x}_j - \mathbf{x}_i)$ and $(\mathbf{x}_l - \mathbf{x}_k)$), is given by

$$\cos\theta = \frac{(\mathbf{x}_j - \mathbf{x}_i)\,(\mathbf{x}_l - \mathbf{x}_k)}{|\mathbf{x}_j - \mathbf{x}_i|\,|\mathbf{x}_l - \mathbf{x}_k|} \, . \qquad 5.3$$

Distance and angle measurements form the simplest bases for shape comparison. By comparison of the distance between homologous pairs of landmarks, it is immediately apparent which structure is the larger and by how much.

5.3 Point Configuration Alignment By Least Squares Fitting

For comparison of two or more point configurations (sets of coordinate data in two or three dimensions), some alignment of the configurations is necessary - for example, it would be difficult to compare skulls, one facing postero-anterior and the other facing antero-posterior.

A general approach is to scale, orient and translate one point configuration such that it aligns with the other in a least squares sense, that is, to minimize the sum of the squared differences between homologous points.

Assuming that points from one configuration $\{\mathbf{x}_i, i = 1,2,...,n\}$ have been matched to the corresponding points of the other $\{\mathbf{y}_i, i = 1,2,...,n\}$, the relationship between homologous points can be written as

$$\mathbf{y}_i = sR\mathbf{x}_i + \mathbf{t} + \mathbf{d}_i \qquad 5.4$$

where R, s, and $\mathbf{t}$, are respectively the rotation matrix, scale factor, and translation vector to be determined, and $\mathbf{d}_i$ is the residual difference between the configurations at the i^{th} point.

For least squares alignment, R, s, and $\mathbf{t}$, are found such that

$$d^2 = \sum_{i=1}^{n} d_i^2 = \sum_{i=1}^{n} |\mathbf{y}_i - (sR\mathbf{x}_i + \mathbf{t})|^2 \qquad 5.5$$

is minimized.

By taking partial derivatives of Equation 5.5 with respect to each component of t in turn, d^2 is found to be minimized when

$$t = \frac{1}{N}\sum_{i=1}^{n}(y_i - sRx_i) = \bar{y} - sR\bar{x} \quad . \qquad 5.6$$

By translating each point configuration such that its centroid is at the origin, before least squares fitting, t becomes zero and the number of parameters to be found is accordingly reduced. Equation 5.5 can therefore be rewritten as

$$d^2 = \sum_{i=1}^{n} |y_i' - sRx_i'|^2 \qquad 5.7$$

where $y_i' = y_i - \bar{y}$ and $x_i' = x_i - \bar{x}$.

The scale and rotation parameters used to minimize d^2 were obtained through the use of procedure ZXSSQ, a routine from the IMSL Library, which is a package of mathematical and statistical routines available on the University of Adelaide's Vax 11/785[1].

ZXSSQ required a subroutine to be written, which calculated each

$$d_i = y_i' - sRx_i' \qquad 5.8$$

from the data x_i' and y_i' with scale and rotation parameters provided by ZXSSQ. Initial guesses for the scale and rotation parameters are used to start ZXSSQ (starting values are always 1.0 and 0° respectively). The iterative minimization of Equation 5.7 continues until the scale and rotation parameters are found to be within a specified accuracy (5 significant figures). Three angles are required, in three dimensions, to uniquely specify the rotation matrix, but the angles themselves are not unique and can be specified as rotations about any

[1] Digital Equipment Corporation, Maynard, Massachusetts, U.S.A.

convenient axes. In this thesis, the angles are specified as rotations about the Z, Y, and X-axes. The required rotation matrix can then be generated by concatenating the separate matrices for each rotation.

In two dimensions, only one angle parameter is required to specify the rotation matrix.

Since developing the above method for point configuration alignment, another approach to minimizing Equation 5.5 has come to notice using singular value decomposition (Huber, 1980 (unpublished); Arun, Huang and Blodstein, 1987). The method is elegant and non-iterative. A possible advantage of this approach over the author's could be the speed of calculation. However, as the time of execution of the entire program using the author's method was of a few seconds duration only, this latter method was not implemented.

To illustrate the least squares fitting approach, a rectangular figure was rotated, scaled and translated (Figures 5.6 (a) and (b)). The green rectangle was fitted to the red rectangle using the least squares method (Table 5.1). The residuals at each vertex were less than 0.0005mm with $d^2 = 3 \times 10^{-12} mm^2$.

When one vertex of the object is deformed, the fit is not exact. Figures 5.7 (a) and (b) and Table 5.2 illustrate the results for two dimensions. The sum of squares of the residuals is $d^2 = 3.80 mm^2$. It is not clear. however, from the residuals or the illustration which vertex was deformed.

Similarly, an orthorhombic figure was rotated, scaled by 0.75 and translated to test the algorithm in three dimensions (Figures 5.8 (a) and (b) and Table 5.3). The residuals in this case were less than 0.001mm with d^2 less than $2 \times 10^{-6} mm^2$. Note that the input coordinate data was to three decimals places so that an exact fit, on this data, is not necessarily possible. The scale factor was found to be 1.333 as expected.

The results for the scaled three dimensional orthorhombic figure with two deformed vertices are illustrated in Figures 5.9 (a) and (b) and Table 5.4. The scale factor was found to be 1.2588 compared with the scale factor of 1.333 found for the non-deformed figure. The difference is due to the increased size of the deformed figure.

5.4 Point Configuration Alignment By Repeated Median Fitting

> "Two shapes can rarely be superimposed perfectly; different fitting criteria will generally yield different results. By allowing regions with large deformations to have a large impact on the fit, least squares methods can minimize true shape differences and thereby obscure them. A resistant technique, however, limits the influence of large deformations and the resulting fit is close in similar regions and not close in relatively deformed regions. In this way, resistant techniques can help to identify similarities and differences in form more effectively than least squares methods."
>
> Siegel and Benson, 1982.

A particularly resistant approach to the alignment of point configurations is that based on repeated medians (Siegel, 1982a, 1982b; Siegel and Benson, 1982).

Shape differences associated with just one landmark will affect the parameters estimated by a least squares fit. In the repeated median fitting approach, nearly 50% of the landmarks can be significantly altered with no change in the estimated fitting parameters. The alignment on this 50% of the landmarks makes the technique particularly useful in identifying similarities and differences between point configurations.

5.4.1 Repeated median fitting in two dimensions

The method developed by Siegel for two dimensional coordinate data is outlined in this section.

The repeated median algorithm produces different estimates from the least squares fitting approach for s, R, and **t** in Equation 5.4, reproduced here for the reader's convenience,

$$\mathbf{y}_i = s R \mathbf{x}_i + \mathbf{t} + \mathbf{d}_i \qquad 5.9$$

for i = 1, 2,....n pairs of homologous points. The points $\mathbf{x}_i$ are from one homologue and the points $\mathbf{y}_i$ are from the other. In two dimensions, the rotation matrix, R, is determined by one parameter, the rotation angle, θ, about the normal to the plane.

Scale Factor

For each pair of homologous points, say i and j, there corresponds an estimate of the scale factor, s_{ij}, between the homologues

$$s_{ij} = |\mathbf{y}_i - \mathbf{y}_j| \,/\, |\mathbf{x}_i - \mathbf{x}_j| \qquad 5.10$$

The repeated median estimate of the scale factor is

$$s = \operatorname*{med}_{i} \left(\operatorname*{med}_{j \neq i} s_{ij} \right) \qquad 5.11$$

which represents the median of the n medians of the n-1 scale factors.

Rotation Matrix

The rotation matrix is determined by a repeated median estimate, θ, of the rotation angle about the z axis (plane normal) defined as

$$\theta = \operatorname*{med}_{i} \left(\operatorname*{med}_{j \neq i} \theta_{ij} \right) \qquad 5.12$$

where θ_{ij} is the angle required to align vector $(\mathbf{x}_i - \mathbf{x}_j)$ with $(\mathbf{y}_i - \mathbf{y}_j)$.

Translation

The median estimate for the translation between the configurations is

$$\mathbf{t} = \underset{i}{\text{med}}\,(\mathbf{t}_i) \tag{5.13}$$

where each point, $\mathbf{t}_i$, is given by

$$\mathbf{t}_i = \mathbf{y}_i - sR(\theta)\mathbf{x}_i \tag{5.14}$$

where the median operation is taken over each Cartesian coordinate component in turn.

To illustrate the very different fits that can be produced by the least squares and the repeated median methods, similar geometric figures to those of Siegel and Benson (1982) have been used. The results, shown in Figures 5.10 (a) to (f) and Tables 5.5 (a) and (b), are comparable to those obtained by Siegel and Benson. With the repeated median approach, it is clear from Tables 5.5 (a) and (b) and Figures 5.10 (c) and (f) that the shape differences are associated with only a few points, the other points being superimposed. The least squares approach (Figures 5.10 (b) and (e)) produces a fit where the residuals are clustered around an average distance and thus the shape differences are not as distinct.

5.4.2 Extension of repeated median fitting to three dimensions

The repeated median fitting approach can be extended to three dimensions. The Equations 5.9 to 5.14, presented above, are in vector notation and clearly remain valid in three dimensions, except that the rotation matrix, R, now depends on three parameters, and not just one. These three parameters can be taken to be the rotations α, β, and γ, about the coordinate axes. In general, a single repeated median estimate of each of these angles is not sufficient for alignment, as the coordinate axes may not be the most appropriate for the particular orientations of the homologues. The rotation matrix can, however, be determined by an iterative procedure where repeated median estimates of α, β, and γ, are recalculated as alignment proceeds.

Let $$R^{(k)} = R_x^{(k)} R_y^{(k)} R_z^{(k)} \tag{5.15}$$

where $R_x^{(k)}$, $R_y^{(k)}$, and $R_z^{(k)}$ are rotations matrices for rotations of $\alpha^{(k)}$, $\beta^{(k)}$, and $\gamma^{(k)}$ about the X, Y, and Z-axes respectively. The superscript (k) refers to the k^{th} iteration.

Let $\gamma_{ij}^{(1)}$ be the angle about the Z-axis required to align the projection of vector $(\mathbf{x}_i - \mathbf{x}_j)$ with the projection of vector $(\mathbf{y}_i - \mathbf{y}_j)$ on the X-Y plane, then the repeated median estimate of $\gamma^{(1)}$ is given by

$$\gamma^{(1)} = \underset{i}{\text{med}} \, (\underset{j \neq i}{\text{med}} \, \gamma_{ij}^{(1)}) \tag{5.16}$$

The $\mathbf{x}_i$ coordinate data is then rotated using $R_z^{(1)}$ to give

$$\mathbf{x}_i^{(1)} = R_z^{(1)} \mathbf{x}_i \qquad \text{where } i = 1,2,\ldots,n. \tag{5.17}$$

Let $\beta_{ij}^{(1)}$ be the angle about the Y-axis required to align the projection of vector $(\mathbf{x}_i^{(1)} - \mathbf{x}_j^{(1)})$ with the projection of vector $(\mathbf{y}_i - \mathbf{y}_j)$ on the X-Z plane, then the repeated median estimate of $\beta^{(1)}$ is given by

$$\beta^{(1)} = \underset{i}{\text{med}} \, (\underset{j \neq i}{\text{med}} \, \beta_{ij}^{(1)}). \tag{5.18}$$

The $\mathbf{x}_i^{(1)}$ coordinate data is then rotated using $R_y^{(1)}$ to give

$$\mathbf{x}_i^{(2)} = R_y^{(1)} \mathbf{x}_i^{(1)} = R_y^{(1)} R_z^{(1)} \mathbf{x}_i \quad . \tag{5.19}$$

Let $\alpha_{ij}^{(1)}$ be the angle about the X-axis required to align the projection of vector $(\mathbf{x}_i^{(2)} - \mathbf{x}_j^{(2)})$ with the projection of vector $(\mathbf{y}_i - \mathbf{y}_j)$ on the Y-Z plane, then the repeated median estimate of $\alpha^{(1)}$ is given by

$$\alpha^{(1)} = \underset{i}{\text{med}}\ (\underset{j \neq i}{\text{med}}\ \alpha_{ij}^{(1)}) \ . \tag{5.20}$$

The $\mathbf{x}_i^{(2)}$ coordinate data is then rotated using $R_x^{(1)}$ to give

$$\mathbf{x}_i^{(3)} = R_x^{(1)}\mathbf{x}_i^{(2)} = R_x^{(1)}R_y^{(1)}R_z^{(1)}\mathbf{x}_i \ . \tag{5.21}$$

Thus after the first iteration the net rotation, $R^{(1)}$, can be represented by

$$R^{(1)} = R_x^{(1)}R_y^{(1)}R_z^{(1)} \ . \tag{5.22}$$

Generally, alignment is not complete at this stage and it is necessary to iteratively recalculate the rotation parameters until $R^{(k)}$ is very nearly the identity matrix; for example, when $\alpha^{(k)}$, $\beta^{(k)}$, and $\gamma^{(k)}$ are less than 0.2°. For K iterations, the rotation matrix, R, becomes

$$R = \prod_{k=1}^{K} R^{(k)} = \prod_{k=1}^{K} R_x^{(k)}R_y^{(k)}R_z^{(k)} \tag{5.23}$$

where $\alpha^{(k)}$, $\beta^{(k)}$, and $\gamma^{(k)}$ are given by

$$\begin{aligned} \alpha^{(k)} &= \underset{i}{\text{med}}\ (\underset{j \neq i}{\text{med}}\ \alpha_{ij}^{(k)}) \\ \beta^{(k)} &= \underset{i}{\text{med}}\ (\underset{j \neq i}{\text{med}}\ \beta_{ij}^{(k)}) \\ \gamma^{(k)} &= \underset{i}{\text{med}}\ (\underset{j \neq i}{\text{med}}\ \gamma_{ij}^{(k)}) \end{aligned} \tag{5.24}$$

where $\gamma_{ij}^{(k)}$, $\beta_{ij}^{(k)}$ and $\alpha_{ij}^{(k)}$ are the angles about the Z, Y and X axes respectively required to align the projection of the vectors $(x_i^{(3k-3)} - x_j^{(3k-3)})$, $(x_i^{(3k-2)} - x_j^{(3k-2)})$ and $(x_i^{(3k-1)} - x_j^{(3k-1)})$ respectively with the projection of the vector $(y_i - y_j)$ on the X-Y, X-Z and Y-Z planes respectively. For the first iteration, $k = 1$, and $x_i^{(0)}$ corresponds to the initial data.

When vectors $(x_i^{(n)} - x_j^{(n)})$ or $(y_i - y_j)$ were within 5.7° of the active rotation axis, the associated angle was excluded from the calculation of the repeated median estimate of the rotation angle.

Examples of three dimensional repeated median fitting

The three dimensional repeated median fitting approach was used also to align the orthorhombic shapes in Figure 5.9 (a), and reproduced in Figure 5.11 (a), with the results shown in Figure 5.11 (b). The residuals for the repeated median alignment are shown on the right side of Table 5.6 where it can be seen that the residuals are all less than 0.001mm, except for the two deformed points; the residuals for these two latter points differ from the expected value of 8.660mm by less than 0.02mm. The scale factor was found to be 1.3333 as expected. After three iterations, the angle adjustment was less than $\delta\theta = 0.25°$, which would correspond to adjustments of less than approximately

$$r\delta\theta = 30 * 0.25 * \frac{\pi}{180}$$
$$= 0.13\text{mm}$$

at the positions of the vertices, where $r = 30$mm is approximately one half of the maximum length of the figure. Seven iterations were performed for the data in Table 5.6, after which time the angle adjustments were less than 0.0001°. On the left side of Table 5.6, the least squares alignment results are repeated from Table 5.4 for direct comparison of the two alignment methods. (Compare Figures 5.9 (b) and 5.11 (b)). The shape difference is defined more clearly after the repeated median alignment because the fit is nearly exact for all but two of the vertices.

To illustrate how repeated median fitting in three dimensions behaves when the object and its homologue are best aligned along axes far removed from the coordinate axes, two long thin rectangles were generated as in Figure 5.12. Clearly, rotational alignment would be best carried out about the object's longitudinal axis. The repeated median rotational alignment required some ten iterations before the angular rotations settled down to less than 0.25°. The magnitude of the residuals for each vertex after fifteen iterations (angular adjustments less than 0.03°) was less than 0.01mm. For least squares fitting, the residuals were less than 10^{-5}mm ($d^2 < 10^{-10}$mm^2). Both procedures align the identically shaped but rotated, translated, and scaled figures with no perceptible difference, that is, within the thickness of the lines.

The results for least squares and repeated median fitting when one vertex is shifted to distort the rectangle, are given in Tables 5.7 and 5.8 and illustrated in Figures 5.13 (a) to (c). For least squares, the residuals for each vertex are quite large. From the residuals alone, it is not clear which point was deformed. In fact, the point with the largest residual was not the deformed point. For the repeated median fit, three of the residuals are less than approximately 0.3mm, and the residual of the deformed point differs from the expected value of 20mm by less than 0.2mm. There were fourteen iterations before the angle adjustments were less than 0.25°. However, if least squares fitting is applied first, only two iterations are required by the repeated median method before the angle adjustments are less than 0.25°. In this case, the residuals are impressively low, differing from their expected values by less than 0.002mm (Table 5.9 and Figure 5.13 (d)).

Therefore, it is preferable to precede the repeated median fit by a least squares fit to approximately align the homologous figures. The least squares method is significantly faster than the repeated median approach, so that least squares fitting can be used to reduce the required number of repeated median iterations.

5.4.3 Summary

On evaluating Siegel's repeated median alignment technique in two dimensions and its extension to three dimensions, the author agrees with Siegel and Benson's concluding remarks in their paper entitled "A robust comparison of biological shapes":

> "Least squares methods have proven themselves in many situations and we are not suggesting that they be entirely replaced by robust techniques. In fact, least squares methods produce an overall fit whose residual sum of squares is a useful single-number measure of how different two specimens are. However, if in analyzing the comparative morphology of animal skeletons, we are interested in the detailed identification of similarities and allometric shape differences, then a resistant method, such as the repeated median algorithm presented here, would be preferable, This approach tends to produce a close fit which allows ready identification of parts with similar shapes. Since the influence of deformed parts upon the fit itself is limited, the regions of morphologic differences can be easily identified by their poor fit."
>
> Siegel and Benson, 1982.

5.5 Shape Comparison Using Strain Analysis

The techniques of strain analysis can be utilized to give another means of representing shape change, other than distance or angle measurement. In this technique, the skull is partitioned into a number of "finite elements". Triangular or tetrahedral elements are used in this thesis, although elements of other shapes can be employed. The vertices of the elements are defined by the three dimensional coordinates of the osseous landmarks identified and described in previous chapters. Strain analysis applied to homologous elements describes the shape change that transforms one element into its homologue. The principal strains resulting from this analysis are not dependent on the orientation of either homologue (just as distance and angle measurements do not require alignment of the homologous skulls). The principal strain directions for an element are essentially "fixed" to the element and if the element is rotated. the principal strain directions move with it. The principal strains and directions are the eigenvalues and eigenvectors of the strain tensor (matrix) that describe the "deformation" or shape change from the first homologue (initial) to the second (final or deformed).

In the fields of engineering, physics and mathematics, strain analysis is applied to continua to describe deformation due to applied forces, or stresses. In the context of shape comparison, strain analysis is used simply to describe the shape change from one element to its homologue, without any suggestion of a mechanism related to the actual stresses on bones.

Both triangular and tetrahedral elements can be generated from three dimensional data. Many of the structures in the skull that are of interest are approximately planar surfaces and it is appropriate to analyse these using plane strain (two dimensional analysis), although the coordinates describing the vertices are, of course, three dimensional. Some structures, particularly the bony cavities, (for example, the orbital and nasal cavities), are more appropriately analysed as three dimensional tetrahedral elements.

The following subsections will outline the method of strain analysis as applied to individual elements, but described in its original context of deformation continua.

5.5.1 Lagrangian description of deformation

Deformation refers to a change in the shape between some initial (undeformed) configuration and a subsequent (deformed) configuration. The emphasis in deformation studies is on the initial and final configurations. No consideration is given to the process of how the deformation occurred or to intermediate configurations.

When a continuum undergoes deformation, the particles move along various paths in space. This motion may be expressed by equations of the form

$$\mathbf{x}' = \mathbf{x}'(\mathbf{x}, t) \qquad 5.25$$

which give the current location, $\mathbf{x}'$, of the particle that was at $\mathbf{x}$ at time $t=0$.

This equation indicates that the initial configuration may be mapped into the current or final configuration. It is assumed that such mapping is one to one and continuous, with continuous partial derivatives to whatever order is required. This description of motion or deformation is known as the Lagrangian formulation (as described in Mase, 1970).

5.5.2 The material deformation gradient

By partial differentiation of Equation 5.25 with respect to each component of **x**, in three-dimensional space, it can be seen that a small line element, **dx'**, in the deformed object, is related to the undeformed line element, **dx**, by

$$\begin{pmatrix} dx_1' \\ dx_2' \\ dx_3' \end{pmatrix} = \begin{pmatrix} \frac{\partial x_1'}{\partial x_1} & \frac{\partial x_1'}{\partial x_2} & \frac{\partial x_1'}{\partial x_3} \\ \frac{\partial x_2'}{\partial x_1} & \frac{\partial x_2'}{\partial x_2} & \frac{\partial x_2'}{\partial x_3} \\ \frac{\partial x_3'}{\partial x_1} & \frac{\partial x_3'}{\partial x_2} & \frac{\partial x_3'}{\partial x_3} \end{pmatrix} \begin{pmatrix} dx_1 \\ dx_2 \\ dx_3 \end{pmatrix}$$

or, in matrix form,

$$\mathbf{dx}' = \mathrm{F}\,\mathbf{dx} \qquad 5.26$$

where $F_{ij} = \frac{\partial x_i'}{\partial x_j}$. The matrix F is known as the material deformation gradient.

An intuitive interpretation of F is given later (Section 5.5.9) in terms of its principal values, the stretch ratios along the principal directions, and rotation matrices associated with the orientation of the initial and final elements.

5.5.3 The Lagrangian finite strain tensor

Consider the squared length of a small line element, **dx'**, in the deformed configuration.

$$\begin{aligned}(\mathbf{dx}')^2 &= \mathbf{dx}'^{T}\,\mathbf{dx}' \\ &= (\mathrm{F}\,\mathbf{dx})^{T}\,(\mathrm{F}\,\mathbf{dx})\end{aligned}$$

$$= \mathbf{dx}^T F^T F\, \mathbf{dx} \qquad 5.27$$

The superscript T denotes matrix transpose.

The difference between the square of two homologous small line elements in the final and initial configurations can be used as a measure of deformation or shape changes. That is,

$$\begin{aligned}(\mathbf{dx}')^2 - (\mathbf{dx})^2 &= \mathbf{dx}^T F^T F\, \mathbf{dx} - \mathbf{dx}^T \mathbf{dx} \\ &= \mathbf{dx}^T (F^T F - I)\, \mathbf{dx}\end{aligned} \qquad 5.28$$

where I is the identity matrix.

Define

$$L = \frac{1}{2}(F^T F - I) \qquad 5.29$$

then

$$(\mathbf{dx}')^2 - (\mathbf{dx})^2 = \mathbf{dx}^T\, 2L\, \mathbf{dx} \qquad 5.30$$

L is known as the Lagrangian finite strain tensor. The difference in squared length is dependent on the initial line element, **dx**, and L, therefore L must describe the deformation.

L is independent of the orientation and position of the final (deformed) configuration. Consider a new final configuration which has simply been rotated and translated so that

$$\mathbf{x}'' = R\mathbf{x}' + \mathbf{b}$$

then

$$\begin{aligned}\mathbf{dx}'' &= R\, \mathbf{dx}' \\ &= RF\, \mathbf{dx}\end{aligned}$$

$$= F'' \, dx$$

where the new material deformation gradient, F'', is given by

$$F'' = RF$$

Therefore the new Lagrangian finite strain tensor is given by

$$\begin{aligned} L'' &= \frac{1}{2}(F''^T F'' - I) \\ &= \frac{1}{2}((RF)^T(RF) - I) \\ &= \frac{1}{2}(F^T R^T RF - I) \\ &= \frac{1}{2}(F^T F - I) \\ &= L \end{aligned}$$

so that rotation and translation of the deformed shape relative to the original shape do not alter the Lagrangian finite strain tensor.

5.5.4 The material displacement gradient

A more familiar form of the Lagrangian finite strain tensor is that in which this tensor appears as a function of the displacement gradients. The relative displacement of homologous points, at $\mathbf{x}$ in the initial configuration and at $\mathbf{x}'$ in the final configuration, is denoted by the vector $\mathbf{u}$, given by

$$\mathbf{u} = \mathbf{x}' - \mathbf{x} \qquad 5.31$$

In the neighbourhood of $\mathbf{x}$ in the initial configuration and $\mathbf{x}'$ in the deformed configuration

$$\begin{aligned} du &= dx' - dx \\ &= F\,dx - dx \end{aligned}$$

$$= (F - I)\, dx$$

therefore

$$du = J\, dx \qquad 5.32$$

where

$$J = F - I\,. \qquad 5.33$$

J is known as the material displacement gradient and $J_{ij} = \dfrac{\partial u_i}{\partial x_j}$.

The Lagrangian finite strain tensor can be expressed in terms of the material displacement gradient by substitution of Equation 5.33 into Equation 5.29 to give

$$\begin{aligned} L &= \frac{1}{2}(F^T F - I) \\ &= \frac{1}{2}((J + I)^T (J + I) - I) \\ &= \frac{1}{2}((J^T + I)(J + I) - I) \\ &= \frac{1}{2}(J^T J + J + J^T + I - I) \end{aligned}$$

and this gives the more familiar form of L

$$L = \frac{1}{2}(J + J^T + J^T J) \qquad 5.34$$

or

$$L_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \frac{\partial u_j}{\partial x_i}\frac{\partial u_i}{\partial x_j}\right)$$

5.5.5 Principal strains and principal strain directions

In order to simplify the interpretation of the Lagrangian finite strain tensor, directions can be found such that L can be described in terms of dilations and/or contractions alone. These directions are given by the eigenvectors of L, while the dilations and/or contractions are related to the eigenvalues of L.

Define the stretch ratio, Λ, by the ratio of the lengths of a deformed line element to the length of the corresponding undeformed line element so that

$$\Lambda^2 = \frac{(dx')^2}{(dx)^2} \tag{5.35}$$

Using Equation 5.30 this becomes

$$\begin{aligned}\Lambda^2 &= \frac{dx^T 2L\, dx + (dx)^2}{(dx)^2} \\ &= \frac{dx^T 2L\, dx}{(dx)^2} + 1\end{aligned} \tag{5.36}$$

If the direction of a line element dx is such that dx is an eigenvector of L, with eigenvalue (principal strain) λ, then

$$L\, dx = \lambda dx \tag{5.37}$$

and substitution into Equation 5.36 gives the squared principal stretch ratios in terms of the principal strains as

$$\Lambda^2 = 2\lambda + 1 \tag{5.38}$$

The principal stretch ratios are an important measure of the extensional strain and provide a basis for interpretation of the finite strain tensor.

For small deformations Equation 5.38 becomes

$$\begin{aligned}\Lambda &= \sqrt{2\lambda + 1} \\ &\cong \lambda + 1\end{aligned} \tag{5.39}$$

and the principal strains can be expressed in the more intuitive form

$$\lambda \cong \Lambda - 1 \qquad 5.40$$

In the infinitesimal approximation, the principal strains are a measure of the change of length per unit original length.

5.5.6 The cubical dilation

The volume change per original volume that results from the deformation can be calculated and is known as the cubical dilation. Consider a small volume element with sides along the (orthogonal) principal strain directions, then the ratio of the final (deformed) to initial volume is the product of the principal stretch ratios

$$\frac{\text{final volume}}{\text{initial volume}} = \Lambda_1 \Lambda_2 \Lambda_3 \qquad 5.41$$

and the cubical dilation or fractional volume change is

$$\frac{\Delta V}{V} = \Lambda_1 \Lambda_2 \Lambda_3 - 1 \quad . \qquad 5.42$$

5.5.7 Strain analysis of triangular (2d) and tetrahedral (3d) elements

For triangular elements in two dimensions and tetrahedral elements in three dimensions, a linear transformation of the form

$$\mathbf{x}' = \mathrm{A}\mathbf{x} + \mathbf{b} \qquad 5.43$$

exists between the initial and final (deformed) elements.

The matrix, A, and the translation vector, **b**, can be determined uniquely from the (n + 1) vertices of the initial and final elements, where n is the dimension (2 or 3), provided that the vertices are not colinear (in two dimensions) or coplanar (in three dimensions). Equation 5.43 is a vector equation so that there are n equations for each vertex, one for each coordinate component, thus providing the necessary number of independent equations needed to solve for the $n^2 + n$ unknown parameters combined in the matrix, A, and the translation vector, **b**.

For convenience, one vertex of the final element and its homologous vertex on the initial element can be translated to the origin so that $\mathbf{b} = \mathbf{0}$ and

$$\mathbf{x}' = A\mathbf{x} \tag{5.44}$$

From partial differentiation of this equation and from Equation 5.26, it can be seen that in this case A is equal to the material deformation gradient

$$A = F \tag{5.45}$$

The vectors describing the positions of the vertices of the initial and deformed elements (excluding the vertices at the origin) can be compiled in columnar fashion to form the matrices X and X' respectively and hence

$$X' = F X \ . \tag{5.46}$$

The vectors of X are linearly independent because of the non-colinear (two dimensions) or non-coplanar (three dimensions) nature of the vertices; therefore the inverse of X exists, and

$$F = X' X^{-1} \tag{5.47}$$

The displacement, **u**, between homologous points, Equation 5.31, is defined as

$$\mathbf{u} = \mathbf{x}' - \mathbf{x} \tag{5.48}$$

so that for the elements defined here

$$\begin{aligned}\mathbf{u} &= (A - I)\,\mathbf{x} \\ &= (F - I)\mathbf{x}\end{aligned}$$

and from Equation 5.33,

$$u = J\,x \tag{5.49}$$

where J is the material displacement gradient. Vectors describing the displacements of the non-superimposed vertices can be combined columnwise to form the matrix U and as the inverse of X exists, as before, then

$$J = U\,X^{-1} \tag{5.50}$$

Since J and F are related via Equation 5.33, either may be calculated from the coordinates of the vertices, and then from them, the Lagrangian strain tensor.

5.5.8 Triangulation of matrices X, X' and U

The matrices X, X' and U can be put into triangular form to simplify the calculation of F or J.

Let the vertices of a tetrahedral element at x_1, x_2, x_3 and x_4, be denoted as A, B, C, and D. Triangulation is achieved by alignment of each element (individually) in the following way.

(1) The element is translated so that A is at the origin (Figure 5.14 (a))

$$\mathbf{x}_i^{(1)} = \mathbf{x}_i - \mathbf{x}_1 \;.$$

(2) The element is rotated about the Z axis so that the line segment $\bar{A}B$ is in the X-Z plane (Figure 5.14 (b))

$$\mathbf{x}_i^{(2)} = R_1\,\mathbf{x}_i^{(1)} \;.$$

(3) The element is rotated about the Y-axis to bring $\bar{A}B$ into coincidence with the X-axis (Figure 5.14 (c))

$$\mathbf{x}_i^{(3)} = R_2\,\mathbf{x}_i^{(2)} \;.$$

(4) Finally the element is rotated about the X-axis to bring C and thus the base of the tetrahedral element into the X-Y plane (Figure 5.14 (d))

$$\mathbf{x}_i^{(4)} = R_3\, \mathbf{x}_i^{(3)} \ .$$

The homologous element is similarly translated and oriented (although the element will, in general, have a different translation vector and different orientation angles).

X, X' and U are therefore upper triangular matrices. That is

$$X = \begin{pmatrix} x_2^{(4)} & x_3^{(4)} & x_4^{(4)} \\ 0 & y_3^{(4)} & y_4^{(4)} \\ 0 & 0 & z_4^{(4)} \end{pmatrix}$$

and similarly for X' and U.

In two dimensions, all that is required is a rotation about the Z-axis to bring the line segment $\bar{AB}$ into coincidence with the X-axis (that is, only steps (1) and (2) above).

5.5.9 Singular value decomposition of the material deformation gradient

By singular value decomposition, the material deformation gradient, F, can be expressed in the form

$$F = R_2\, \Lambda\, R_1^T \qquad 5.51$$

where R_1 and R_2 are orthogonal matrices, that is, $R_1^T R_1 = R_2^T R_2 = I$ and Λ is a diagonal matrix (see for example, Golub and Reinsch, 1970; Lawson and Hanson, 1974; Klemma and Laub, 1980).

The Lagrangian finite strain tensor is given, then, by substitution of Equation 5.51 into 5.29

$$\begin{aligned} 2L &= R_1 \Lambda R_2^T R_2 \Lambda R_1^T - I \\ &= R_1 \Lambda^2 R_1^T - I \\ &= R_1 (\Lambda^2 - I) R_1^T \ . \end{aligned} \qquad 5.52$$

Post multiplying both sides by R_1 and dividing by two

$$LR_1 = \frac{1}{2} R_1 (\Lambda^2 - I) \ . \qquad 5.53$$

Therefore, the columns of R_1 are the eigenvectors of L and the eigenvalues, λ_i of L are related to the singular values, Λ_i (the diagonal elements of the matrix Λ), of F by

$$\lambda_i = \frac{1}{2} (\Lambda_i^2 - 1) \qquad 5.54$$

which is equivalent to Equation 5.38, therefore the Λ_i are the principal stretch ratios.

To calculate the principal strain directions for the (inverse) transformation from the final shape back to the initial shape consider

$$dx = F' dx' \qquad 5.55$$

where from Equation 5.26

$$F' = F^{-1} \qquad 5.56$$

and then from Equation 5.51

$$F' = R_1 \Lambda^{-1} R_2^T \qquad 5.57$$

The finite strain tensor, L', that describes the transformation from final to initial shape can be found by expressing the right hand side of Equation 5.28 in terms of dx' rather than dx .

$$\begin{aligned}(dx')^2 - (dx)^2 &= dx'^T dx' - dx'^T F'^T F' dx' \\ &= dx'^T (I - F'^T F') dx' \end{aligned} \qquad 5.58$$

so that

$$L' = \frac{1}{2}(I - F'^T F') \qquad 5.59$$

Substitution of Equation 5.57 into 5.59 gives

$$\begin{aligned} 2L' &= I - (R_1 \Lambda^{-1} R_2^T)^T (R_1 \Lambda^{-1} R_2^T) \\ &= I - R_2 \Lambda^{-1} R_1^T R_1 \Lambda^{-1} R_2^T \\ &= R_2 (I - \Lambda^{-2}) R_2^T \end{aligned} \qquad 5.60$$

and therefore the columns of R_2 are the eigenvectors of L' and the eigenvalues λ_i', of L' are related to the singular values, Λ_i, of F by

$$\lambda_i' = \frac{1}{2}(1 - \Lambda_i^{-2}) \qquad 5.61$$

If the principal strain directions for transformation of the initial element to the final element have been determined by eigenvector analysis of the strain matrix L to give R_1, the principal strain directions for the transformation from the final element back to the initial element (the columns of R_2) can be determined directly from Equation 5.51

$$R_2 = F R_1 \Lambda^{-1} \qquad 5.62$$

rather than by eigenvector analysis of L'.

R_1 and R_2 can be determined directly by singular value decomposition of F, (Equation 5.51) using, for example, the appropriate IMSL routine. Programs developed for this thesis to evaluate R_1 and R_2 were based on the eigenvalue solution of L rather than direct singular value decomposition of F as the initial development was in terms of the Lagrangian finite strain tensor.

The material deformation gradient, F, can be interpreted, using Equation 5.51, as

(1) a rotation of the initial element to align the principal strain directions with the coordinate axes,

(2) dilations and/or contractions along these directions to produce the final (deformed) shape, and

(3) a rotation to orient the element with its final position.

Thus, as well as describing the deformation required to transform the initial into the final shape, F also contains information on the orientation of the initial and final elements. Additionally, Equation 5.52 shows L to depend on the orientation of the initial element and Equation 5.60 shows L' to depend on the orientation of the final element.

Examples

Two examples are given in order to illustrate the strain analysis technique.

For the first example, Figure 5.15 (a) and Table 5.10 (a) show the results of a strain analysis on a triangle that simply has been deformed by extension along both the X and Y-axes. Vertex 1 is at the origin, vertex 3 is along the X-axis and the line from vertex 3 to vertex 2 is parallel to the Y-axis.

The major and minor principal strain directions were found to be along the X-axis (colour coded green in the figure) and the Y-axis (colour coded red in the figure) respectively. The major and minor principal stretch ratios were found to be 1.25 and 1.0667, which is in agreement with the expected results of 50/40 and 32/30 respectively. The percentage area change associated with the deformation is an increase of 33.3%. The principal strain directions are drawn at the centroids of the initial and final triangles. The lengths drawn reflect the stretch ratios. On the initial triangle, the principal strain directions and stretch ratios reflect the change required to transform the initial triangle to have the shape of the final triangle (but not necessarily its position or orientation).

In Figure 5.15 (b), the triangles are superimposed on their centroids and oriented to superimpose the principal strain directions of the initial and final triangles. (Only the principal strain directions and principal stretch ratios for the initial triangle are plotted.) This makes it a little easier to see that the deformation of the initial triangle to the final triangle is in accordance with the magnitude and direction of the principal strains, particularly when the triangles are not as simply oriented relative to the coordinate axes as in this case.

The analysis shows clearly that the deformation is simply dilations parallel to the two orthogonal sides of the triangle which, for convenience, were oriented to be parallel to the X and Y-axes.

In Figure 5.15 (c), the initial triangle is the same as for Figure 5.15 (a) and the final triangle is almost the same as in Figure 5.15 (a), the difference being that there is a small displacement of vertex 2 parallel to the X-axis. The corresponding strain analysis is given in Table 5.10 (b). In this case, the principal strain directions are no longer parallel to any side of the triangle or the coordinate axes. Figure 5.15 (d) shows the two shapes aligned on the principal strain directions, where it can be seen that the given dilations along the principal strain directions will transform the initial into the final shape (with a little imagination).

One could describe the shape change in terms of distance changes between vertices or angle differences at the vertices, but the transformation is most simply represented by the dilations along the principal strain directions.

A three dimensional example is given in Figure 5.16 and Table 5.11. The initial figure is a tetrahedron with the normals to three of the four sides oriented parallel to the coordinate axes. The deformed tetrahedron was created by extending the base of the initial tetrahedron by 10mm parallel to the X-axis. Stereo images are given to facilitate the visualization .

Again the principal strain directions are plotted at the centroids of the figures colour coded red, green and purple to correspond to minor, semi-major, and major principal strain directions which were found to be parallel to the Z, Y and X-axes respectively. The figure was rotated for plotting to give a better perspective view. The minor and semi-major principal stretch ratios are identical and equal to unity, as expected, while the major principal stretch ratio was found to be 1.25, also as expected according to the ratio 50:40 corresponding to the extension of the base from 40mm to 50mm along the X-axis.

The principal strain directions and principal stretch ratios are a concise method of representing the shape difference between the two tetrahedra.

5.6 Summary

The desire to quantify shape and shape differences arises from the need to be able to describe an object accurately for comparative purposes and to facilitate the accurate and consistent communication of the characteristic features of the object.

Distance and angle measurements between homologous landmarks represent one of the simplest and most easily applied techniques for shape comparison. Distance comparisons indicate size differences while angular comparisons indicate changes in relative displacement of landmarks. Further, distance and angle measurements do not require alignment of the structures being compared, rather they simply require an homology to exist between the landmarks being compared.

Another method for shape comparison is superimposition. In this way differences between two objects are highlighted. The most commonly used method of superimposition is alignment on features that are similar between the two objects. The best features for alignment, however, are not always clear. Two alignment approaches have been described - least squares and repeated median.

Least squares alignment minimizes the sum of squares of differences between homologous landmarks by scaling, translating and rotating one homologue to align with the other homologue. Least squares alignment is most appropriately used when the landmarks of one shape are expected to have the same statistical variation about the homologous landmarks of the other shape (for example, double determination of digitizing error). Where some landmarks have a larger measurement variance, it is possible to weight the residual with the inverse measurement of the standard deviation for that landmark, to maintain the same statistical distribution for each landmark. In this way, landmarks with large errors do not adversely influence the alignment. This requires pre-determined landmark location errors. Weighting according to landmark location errors before fitting was utilized early in the development of this thesis, but was found not to significantly alter the alignment and so was discontinued.

Repeated median alignment positions one shape relative to the other by calculating the median translation vector, repeated median scale factor, and repeated median orientation. The scale factor and orientation were based on the relative scale factor and orientation of all homologous line segments. The repeated median is calculated by firstly determining a median scale factor or orientation for each landmark from all line segments associated with that landmark. The repeated median scale factor or orientation is then given by the median of the median values determined for each landmark. By using repeated medians, the technique has the potential for exact alignment on those landmarks that do not differ in shape between the homologues, provided they number more than 50% of the landmarks.

Repeated median alignment is most appropriately used when it is expected that only some of the landmarks may differ significantly from the homologous landmarks of the other shape (for example, comparison of data for a patient pre and post operatively).

Neither least squares or repeated median alignment methods are "the correct" approach to align two shapes for comparison. They simply represent two different alignment procedures. Other alignment techniques have been to constrain alignment to "stable" landmarks such as alignment on implants in longitudinal studies, or to reference lines, such as the sella to nasion line.

The mathematics and interpretation of the technique of strain analysis have been described. Strain analysis is used to describe the shape change between homologous triangular or tetrahedral "finite elements". Shape change is quantified in terms of dilations and contractions along principal directions.

The techniques for shape comparison developed and introduced in this chapter are applied in the following chapters to the craniofacial osseous landmark data collected in the preceding chapters, in order to assess their relative merits as descriptors of biological shape difference.

CHAPTER 6

CREATION OF EXPERIMENTAL REFERENCE STANDARDS

6.1 Introduction

While a "patient", or indeed, any individual in question, can be compared with another individual, it is much more meaningful to compare the patient with a population standard of known mean and variability, so that some assessment can be made of the significance of the differences.

One of the most widely cited sets of cephalometric standards is that derived from the Michigan Longitudinal Growth Study (Riolo et al., 1974). This standard gives statistics relative to distance and angular measurements taken between landmarks identified on lateral cephalograms, for male and female Caucasians aged between six and sixteen. Unfortunately, there is no comparable three dimensional cephalometric standard. A three dimensional coordinate standard needs to be created to fully exploit three dimensional craniofacial data. This represents an enormous task, requiring the acquisition of three dimensional radiographic data, specific for age and sex, and preferably representing a reasonable number of ethnic groups. Of course, this would take many years of compilation of data from many institutions, assuming that ethical and technical problems could be resolved. The establishment of such a comprehensive standard is beyond the scope of the present investigation, which deals with the evaluation of three dimensional coordinate data acquisition and shape analysis techniques.

Therefore, in this thesis, experimental reference standards are produced using the four female dried test skulls, which had been used previously to assess the accuracy of three dimensional coordinate data collection. Whilst it may be considered a sample size of four is relatively small, it is, however, sufficient for evaluation of methods of shape analysis. For convenience, the experimental reference standards are termed population standards, as they are used in exactly the same way as true standards drawn from large samples.

6.2 Quantification Of The Deviation Of An Individual From A Population Mean In Terms Of The Population Standard Deviation

Population statistics can be used to test whether an individual is likely to belong to that population. If the population mean and standard deviation for a particular random variable are known, the deviation of that random variable from the mean can be expressed in terms of the standard deviation.

6.2.1 The Z-score for the Gaussian distribution

For a Gaussian distribution, the Z-score is defined as

$$Z = \frac{x - \mu}{\sigma} \qquad 6.1$$

where x is a random variable, and μ and σ are respectively the population mean and standard deviation for that random variable.

Significant deviations from the standard are identified when the measurement differs from the mean by more than Z = 1.96 standard deviations at the 95% confidence interval for a Gaussian distribution. (For further discussion, see any standard statistics text book, for example, Sokal and Rohlf, 1981).

6.2.2 A $d\sqrt{n}/\sigma$-score for the χ^2 distribution

In the case of a single three dimensional coordinate measurement (x,y,z), a $\chi^2(3)$ distribution is required (Papoulis, 1984) to determine the significance of the distance of a coordinate position from a mean coordinate position $\mu = (\mu_x, \mu_y, \mu_z)$ with population variance σ^2. This is due to the following considerations:

Consider a vector $\mathbf{x} = (x,y,z)$ a distance

$$d = |\mathbf{x} - \mu| \tag{6.2}$$

from the mean position μ, where each component of the vector difference $(\mathbf{x} - \mu)$ has a Gaussian distribution with zero mean, and variances σ_x^2, σ_y^2 and σ_z^2 respectively. The Gaussian distribution is normalised by dividing each component by its standard deviation, to give the appropriate Z-scores. A new variable X^2 can be defined by taking the sum of squares of these three independent standard Normal variables (that is, Gaussian with zero mean and unity variance)

$$X^2 = \left(\frac{x - \mu_x}{\sigma_x}\right)^2 + \left(\frac{y - \mu_y}{\sigma_y}\right)^2 + \left(\frac{z - \mu_z}{\sigma_z}\right)^2 \tag{6.3}$$

X^2 has a $\chi^2(3)$ distribution.

The population variance for a landmark is defined to be

$$\sigma^2 = \sum_{i=1}^{n} \frac{d_i^2}{n} = \sigma_x^2 + \sigma_y^2 + \sigma_z^2 \tag{6.4}$$

where $d_i = |\mathbf{x}_i - \mu|$ is the distance of the i^{th} measurement $\mathbf{x}_i$, for that landmark, from the mean. Each component of $\mathbf{x}$ is expected to be independent and identically distributed so that $\frac{\sigma^2}{3}$ is a pooled estimator of $\sigma_x^2 = \sigma_y^2 = \sigma_z^2$, and then from Equation 6.3

$$X^2 = \frac{d^2}{\sigma_x^2} . \tag{6.5}$$

In the tables that follow d/σ is tabulated, from which the χ^2 distributed variable X^2 can be calculated:

$$X^2 = \frac{3d^2}{\sigma^2} . \qquad 6.6$$

Similarly, when only two dimensional coordinate data are available, a $\chi^2(2)$ distribution is required and the test statistic becomes

$$X^2 = \frac{2d^2}{\sigma^2} . \qquad 6.7$$

For determination of the significance of the deviation of a population mean from an expected true mean, (for example, Section 2.4.2.4) the standard deviation, σ, of a single random variable is replaced by the standard deviation for the mean, $\sigma/\sqrt{n}$ (also known as the standard error) and the residual, d, becomes the distance between the population and expected true means. In this case, $d\sqrt{n}/\sigma$ values are quoted from which the χ^2 distributed variable X^2 equal to $\frac{2d^2n}{\sigma^2}$ or $\frac{3d^2n}{\sigma^2}$ can be determined for two or three dimensions respectively.

Significant deviations from the standard are identified when $X^2 > 7.815$ or $d\sqrt{n}/\sigma > 1.614$ at the 95% confidence level for a $\chi^2(3)$ distribution. For two dimensions, a $\chi^2(2)$ distribution is appropriate and significant deviations are noted when $X^2 > 5.991$ or $d\sqrt{n}/\sigma > 1.731$ at the 95% confidence level.

The values d/σ or $d\sqrt{n}/\sigma$ are tabulated in preference to the X^2-score, as their significance values are closer to the more familiar significance values for the standard normal and t distributions.

6.3 Generation Of Distance And Angle Experimental Reference Standards For A Dried Skull Population

Tables 6.1 (a) to (e) list the number of observations, mean, minimum, maximum, range, standard deviation and expected standard deviation (see Section 6.3.2) of distances and angles derived from the three dimensional coordinate data for the mandible, maxilla, orbits, zygomas, and cranium (although there is some overlap). These variables have been selected so as to encompass the essential features of the standard two dimensional distance and angle analyses as well as to utilise the many additional landmarks identified using the biplanar and CT approaches to enable the above craniofacial regions to be described more completely.

6.3.1 Influence of small sample size on the population mean and standard deviation

A distance measurement performed on each member of the sample can result in the measurement being either lower or higher than the true population mean. With a population size of four there is a probability of one eighth that all four measurements are either lower or higher than the true population mean and this would result in the underestimation of the population standard deviation. As there are many more than eight distances and angles calculated, there is a good chance that a significant number of them will have a lower standard deviation than that of the population.

For the purpose of determining the significance of differences from the population mean, the expected minimum standard deviation, based on landmark location errors, has been for the determination of Z-scores, or $d\sqrt{n}/\sigma$–scores, if it was larger than the measured standard deviation of the sample.

6.3.2 Estimation of expected minimum standard deviations for distances and angles based on landmark location errors

The observed population variance should exceed the variance expected, due to landmark location variance being the sum of the landmark location variance and the true population variance. Accordingly, estimates of the expected variance of distance and angle measurements have been calculated from the landmark location variance in the following manner:

Let x_1 and x_2 be the positions of two landmarks with standard deviations $\sigma_1/\sqrt{3}$ and $\sigma_2/\sqrt{3}$ for each component of x_1 and x_2 respectively (assuming the standard deviations of each component to be equal). The distance between the landmarks, $|x_1 - x_2|$, will have an expected standard deviation between $\sqrt{\frac{\sigma_1^2 + \sigma_2^2}{3}}$ and $\sqrt{\sigma_1^2 + \sigma_2^2}$.

The first value arises by considering that the change in distance measurement arises from variation in coordinate positions along the line of the vector only, whereas the second value is obtained by considering that the variation of each component contributes equally . The second value is selected as an appropriate estimate, since it represents the upper limit of the range of the expected distance standard deviation, based on landmark location error.

Thus, for the distance between two landmarks, the expected minimum standard deviation, S_d, is defined as

$$S_d = \sqrt{\sigma_1^2 + \sigma_2^2} \qquad 6.8$$

where σ_1^2 and σ_2^2 are the variances of the two landmarks.

For the angle between the vectors $(\mu_2 - \mu_1)$ and $(\mu_4 - \mu_3)$, the expected minimum standard deviation, S_θ, is defined as

$$S_\theta = \left(\frac{(\sigma_1^2 + \sigma_2^2)}{3|\mu_2 - \mu_1|^2} + \frac{(\sigma_3^2 + \sigma_4^2)}{3|\mu_4 - \mu_3|^2} \right)^{0.5} \qquad 6.9$$

where μ_1, μ_2, μ_3 and μ_4 and σ_1, σ_2, σ_3 and σ_4 are the mean positions and standard deviations of the four landmarks (landmarks 2 and 3 can be the same to give the angle defined by three landmarks). This equation is derived from the following considerations :

> The angular variation of the vector $(x_2 - x_1)$ due to a variance σ_2^2 in the location of x_2 is approximately
>
> $$\delta\theta \sim \frac{\sigma_2}{3|x_2 - x_1|}$$
>
> where the factor of three arises from the consideration that variances perpendicular to the vector $(x_2 - x_1)$ will dominate the angular variance, as variances parallel to $(x_2 - x_1)$ will not change the angle. There is a similar contribution to the standard deviation due to landmark location errors associated with the other end landmarks. The variances of the four contributions are summed to give Equation 6.9.

It should be re-emphasised that these estimates for expected distance and angle standard deviations calculated from the landmark location errors are for indicative purposes only.

6.3.3 A comparison of symmetric features of the experimental reference standards for the detection of potentially anomalous data

Initially, as an aid for checking for potentially anomalous data, the four female skulls used to generate the standards were compared with the newly created distance and angle standards. No significant Z-scores were observed. However, due to the small sample size, a potential anomaly may not necessarily be identified by its Z-score alone. This is because the potential anomaly could modify the mean and standard deviation sufficiently to mask its presence. However, when the sample size is sufficiently large, this is an appropriate approach and the individuals used to create the standard can be compared with the standard, in order to identify potentially anomalous data via Z-score excursions of greater than 1.96 at the 95% confidence interval.

While individual skulls show a degree of bilateral asymmetry, it might be expected that a population standard will show considerably less asymmetry (one could test this with a sufficient data base). As the experimental reference standards reflect a small specific population (four skulls), one needs to ensure that the source of any specific characteristic that introduces asymmetry into the standard is known. Therefore, the bilateral features of the experimental reference standards have been compared in order to determine whether there are any large discrepancies and whether these are due to anomalous data or accurately reflect the population being described. The experimental reference standards were thus ensured to more than likely reflect a reasonable representation of a true population standard, from which inferences drawn from comparisons with individuals would reflect biological differences.

Statistics for the left and right sides of the experimental reference standards have been analysed by comparing the differences between the means, minima and maxima. If any of these statistics for a particular variable differed between left and right sides by more than 5mm or 7.5°, the values for that variable, for both the left and right sides for *each* skull, were compared in order to determine whether the origin of the observed asymmetry is measurement error or a reflection of a true characteristic of a skull. The bilateral comparisons for each of the distance and angle experimental reference standards are given in Tables 6.2 (a) to (e) and those exceeding the above criterion are asterisked. Of the ninety bilateral comparisons, fourteen exceeded the above criteria (Tables 6.3 (a) to (e)).

This technique detected a number of potential anomalies which received special examination, however, these experimental reference standards were not modified as a consequence, although their presence was noted.

Two of these detected features became apparent in later testing of the standard through analysis of the male skull. These were the optic foramen and the ramus height. The asymmetry in position of the optic foramen arises from measurement error and is discussed in detail in Section 6.7.3.

Examination of skull A90 revealed that while the right condylar head had normal appearance, the left displayed perforations onto the cancellous bone, as well as surface remodelling characteristic of degenerative arthritis of the temporo-mandibular joint. The effect of the pathological condition can be seen in the population statistics for the ramus heights given in Table 6.3 (a). The left ramus height for A90 is noticeably smaller than the right, probably due to the disease process. If the measurement for the left is replaced by that of the right, the statistics for the left mean and standard deviation would be 58.05 and 1.97 respectively and the left and right means would be more similar, with the difference in minima for ramus heights being 0.08mm. For this reason, it is believed that the larger value recorded for the right is within the normal population variance and that the lower value for the left was pathologically small for this skull.

The asymmetry detected for the optic foramen and the ramus height arose from two distinct sources - measurement error and pathology. The optic foramen measurement could be corrected, while the effects of the latter excluded from the standard.

The remaining detected potential anomalies did not become apparent in subsequent analyses, although their effect of increasing the population variance may have masked detection of potentially significant features in subjects compared with the standards.

In creating a true population standard based on a larger number of subjects, tests such as those described in this section are essential for screening the data.

6.4 Generation Of Osseous Landmark Experimental Reference Standards For A Dried Skull Population

Assuming homology, comparison of a pair of three dimensional shapes on a point by point basis can be achieved by aligning the two point configurations and using the residuals to quantify differences. In two dimensions, several people have compared the differences between individuals using either least squares or repeated median alignment (Sneath, 1967; Siegel and Benson, 1982). However, in order to test whether an individual belongs to a specific population, it is preferable to compare that individual with a known population mean and variance, rather than with individual members of the population, as this allows some assessment to be made of the significance of the differences of the individual from the standard.

For this reason, population standards have been created from the three dimensional coordinate data, presented in Chapters 2, 3 and 4, by alignment of each member of the population and calculating the statistics of the coordinate positions for each landmark. Wire frame models constructed from these three dimensional coordinate landmark data are utilised in the following sections to facilitate visualization. Figures 6.1 (a) to (f) illustrate their relationship to the individual bone regions. As these three dimensional alignment techniques have never been used before with cranial analyses, two sets of standards were created to provide useful comparative data; one using the least squares alignment procedure (see Section 5.3), the other using the repeated median alignment procedure (see Section 5.4). The methods used to create these standards are outlined in Sections 6.4.1 and 6.4.2.

6.4.1 Least squares experimental reference bone standards

For least squares alignment the standard was created in the following manner:

(1) Select any skull for initial alignment (A90).

(2) Least squares fit all skulls to the alignment skull, using scaling.
If the initial alignment skull is smaller, for example, than the rest, the other skulls will be reduced in size by the fitting procedure and a small average skull would result. Such an effect is removed by calculating an average scale factor and scaling by its inverse. By the use of scaling during alignment, general size differences between the skulls do not increase the final standard deviation about landmark positions.

(3) Calculate the mean and standard deviation of aligned skull coordinates.

(4) Calculate the average scale factor and apply its inverse to the means and standard deviations.

(5) Replace the initial alignment skull with the mean skull data.

(6) Repeat steps (2) to (5) until no effective changes occur in the average skull (three times was found to be sufficient).

(7) The scale factor of each of the skulls relative to the standard was utilised to determine the standard deviation of the differences of the scale factors from unity.

The skull standard is shown in Figure 6.2 while Tables 6.4 (a) and (b) give the coordinates and the standard deviations. The landmark location errors are reproduced from Table 4.4 for comparison. As expected, the standard deviation about the position of each landmark is generally greater than the landmark location error, due to the variance of the population.

On comparing an individual to the standard skull, it is possible that large differences could be obtained for a bone, and, although it may not differ in shape, it could have a different orientation or size relative to the rest of the skull. It is therefore also desirable to be able to align single bones of the individual with their corresponding bone standards. The individual osseous landmarks used to represent the five major craniofacial regions of the mandible, maxilla, orbits, zygomas and cranium are given in Table 6.5 (a) to (e). Individual bone standards were created using exactly the same method as for the creation of the skull standard. Tables 6.6 (a) to (e) give the coordinates and the standard deviations for the mandible, the maxilla, the orbits, the zygomas, and the cranium. Obviously, the defined bones are a subset of the skull and, as expected, the population variance of the landmarks associated with the individual bone standards are smaller than the corresponding variances for the skull standard, with the exception of condylion right (just).

Comparisons between the individual bone standards and the skull standard are given in Tables 6.7 (a) to (e) and illustrated in Figures 6.3 (a) to (e). As expected, there are slight differences, but these are generally less than the landmark location error.

Each of the skulls used for creation of the standards can be compared with the standards to test for large differences (based on the d/σ-score) in individual landmark locations that may indicate anomalous data. However, the small sample size limits the value of this test for reasons similar to those given in Section 6.3.3. Techniques for comparison of bilateral landmark symmetry in the standard can also be used, although these have not been applied to this data. However, it is expected the asymmetries revealed by the bilateral comparison of distances and angles of the standard in Section 6.3.3 would again be highlighted.

6.4.2 Repeated median experimental reference bone standards

For reasons discussed in detail in Section 5.4, least squares fitting procedures are unduly influenced by rogue landmarks (landmarks with large location errors). Standards have also been created to reduce the potential impact of these landmarks, using the resistant repeated median alignment procedure. This involved a similar approach to that given for the creation of the least squares standard, and is as follows:

(1) Select one skull for alignment (A90).

(2) Repeated median fit all skulls to the alignment skull, with scaling allowed.

(3) Calculate the median, average and standard deviation of the aligned, scaled skull coordinates.

(4) Calculate the median scale factor and apply its inverse to the medians, averages, and standard deviations.

(5) Replace the initial alignment skull with the median skull.

(6) Repeat steps (2) to (5) until no significant changes occur in the repeated median skull (three times was found to be sufficient).

(7) Retain the average and standard deviation as the "repeated median" standard. (The average was selected, rather than the median, because it is more appropriate for use with the standard deviation in calculating d/σ-scores). The scale factor of each of the skulls relative to the standard was utilised to determine the standard deviation of the differences of the scale factors from unity.

The repeated median skull standard is shown in Figure 6.4, while Tables 6.8 (a) and (b) give the coordinates and the standard deviations.

Individual repeated median bone standards have been created using exactly the same method as for the creation of the skull standard. (See Tables 6.5 (a) to (e) for a list of the osseous landmarks used to represent the individual bones.) Tables 6.9 (a) to (e) give the average coordinates and the standard deviations for the mandible, the maxilla, the orbits, the zygomas, and the cranium.

Comparisons between the repeated median individual bone standards and the repeated median skull standard are given in Tables 6.10 (a) to (e) and Figures 6.5 (a) to (e).

Again, each of the skulls used for the creation of the standards can be compared with the standard in order to detect and examine large differences. Comparison of bilateral features would also be of value in this regard, but has not been performed on this data. Potentially anomalous data has been noted previously in Section 6.3.3.

6.4.3 Comparison of least squares and repeated median standards

In a normal population the expected mean and median are the same and this is reflected in Figures 6.6 (a) to (f) and Tables 6.11 (a) to (f).

While there are small differences between the standards these are generally within the landmark location error.

6.5 Selection Criteria And Definition Of Finite Elements For Strain Analysis Of The Craniofacial Complex

In order to apply the techniques of strain analysis (as discussed in Section 5.5) to the craniofacial complex, an homology must be able to be described between the structures being compared. The homologous landmarks can then be used as a basis for subdividing the skull into a number of elements of finite size. The strain analysis technique describes the shape difference between homologous elements in terms of a uniform strain within the finite elements. Therefore, for these elements to be sensitive to subtle shape differences of biological significance the elements should:

(i) be small enough to describe, or be contained within, a single biological unit but large enough such that the influence of the landmark location errors on the vertex positions does not adversely affect the analysis,

(ii) not overlap, so that each describes a unique environment, otherwise the description of areas of overlap becomes more complicated,

(iii) have angles at vertices not too small (say approximately $\geq 15°$). In this way the matrix inversion of Equation 5.50 is well conditioned.

Many of the bones of the craniofacial complex are thin in one of their dimensions. Thus it is more appropriate for these structures to be described by their external surfaces by defining triangular, rather than tetrahedral, elements. The vertices of the triangular elements are still determined in three dimensional space but two dimensional (or planar) strain analysis is used to describe their deformation.

Some structures, particularly cavities, are biological regions that have a significant depth component and can therefore be appropriately analysed using tetrahedral elements and three dimensional strain analysis.

The elements used to investigate the use of strain analysis for the quantitative description of the shape differences of the craniofacial complex between an individual and a standard were defined using the osseous landmarks determined in Chapters 2, 3, and 4 and are shown in Figures 6.7 (a) to (f) and listed in Tables 6.12 (a) to (f).

It can be seen that these elements give a good representation of the bones, especially when one considers that there are only a finite number of (consistently) distinctly recognisable landmarks on a smoothly varying structure.

6.6 Generation Of Strain Analysis Experimental Reference Standards For A Dried Skull Population

In an endeavour to clarify and enhance the interpretation of parameters derived from the strain analysis, it was decided to create experimental reference strain standards from which tests could be made to determine whether strains calculated for a new individual, belonged to the same population.

For each female dried skull, a strain analysis was performed with each of the repeated median bone standards. The percentage stretches and the percentage area changes are given in Tables 6.13 (a) to (e). The percentage stretch is defined by

$$\% \text{ stretch} = (\text{principal stretch ratio} - 1) \times 100 \qquad 6.10$$

and the percentage area/volume change is defined by

$$\% \text{ area/volume change} = (\text{product of principal stretch ratios} - 1) \times 100 \qquad 6.11$$

Strain standards were produced by calculating the mean and standard deviation of the data presented in Tables 6.13 (a) to (e). These statistics, along with the minimum, maximum, range and number of observations, are given in Tables 6.14 (a) to (e).

The probability distributions of the minor and major strains for the experimental reference standards are non-Gaussian. This behaviour is related to the fact that the eigenvalues of the strain analysis are ordered such that the smaller and larger eigenvalues are always associated with the minor and major principal strains respectively. This results in skewed probability distributions, with the mean for the minor and major strains being negative and positive respectively. A mathematical description of the distributions is given in the following section.

Probability distribution of the minor and major strains for the experimental reference strain standards

If the principal strains calculated for the standard were not ordered by the eigenvalue routine EIGRS, one could postulate that each eigenvalue would have a Gaussian distribution with zero mean. The probability distributions of the major and minor strains (ordered eigenvalues) can then be determined by considering two variables x and y (the unordered eigenvalues) with independent and identical probability distributions. The probability that X is in the range x to $(x + dx)$ and Y is in the range y to $(y + dy)$ is $f(x)\,f(y)\,dx\,dy$, where $f(x)$ is the probability density function.

Then the probability that

$$\max(x, y) < m \tag{6.12}$$

is given by

$$H(m) = \int_{-\infty}^{m} \int_{-\infty}^{m} f(x)\, f(y)\, dx\, dy \tag{6.13}$$

where H(m) is the probability distribution function for the variable m. This equation essentially sums all the probabilities that $x < m$ and $y < m$. H(m) can be rewritten as follows :

$$\begin{aligned} H(m) &= \int_{-\infty}^{m} f(y)dy \int_{-\infty}^{m} f(x)dx \\ &= \left(\int_{-\infty}^{m} f(x)dx \right)^2 . \end{aligned} \tag{6.14}$$

The probability density function h(m) is the derivative of the probability distribution function, therefore

$$h(m) = 2f(m) \int_{-\infty}^{m} f(x)dx . \tag{6.15}$$

If the probability density function f(x) is taken to be Gaussian, the probability density function h(m) for the major principal strain is twice the product of a Gaussian function and its related distribution function. The expected density function for the major principal strains is shown in Figure 6.8 where it can be seen that its peak and mean are shifted to the right. The density function for the minor principal strain is the mirror image about the Y-axis so that it peaks at a negative strain value.

The probability density of the area/volume change is also non-Gaussian, as each variable is the product of two/three Gaussian variables (essentially the two/three eigenvalues).

6.6.1 Criteria for detection of significant percentage stretches and area/volume changes

While the probability densities of the principal strains and the area/volumes changes are non-Gaussian, "Z-scores" have been calculated from sample means and variances which may be indicative of significant differences, although the exact confidence levels for significance have not been calculated. Thus, because of the non-Gaussian nature of these distributions, it is important to exercise judgement in the interpretation of results. The major criterion used for assessment of significance by the author have been values of percentage stretch and percentage area/volume change exceeding 20%, with the magnitude of the "Z-score" available in the generated tables[1] for reference (and possible future use).

6.7 Comparison Of A Male Skull With The Experimental Reference Standards

In order to demonstrate how the standards presented earlier in this Chapter could be used, another skull of identical ethnic background, similar age, but of different sex was compared with the various bone standards. It had been expected that the results of these comparisons would highlight differences in sex related characteristics only.

The comparison of the mandible of this latter skull with the standards is presented in considerable detail, simply to indicate the type and extent of

1 planar strain analyses only

information that is obtainable by these methods. The other skull components, however, are discussed more generally.

It should be noted that for the individual osseous landmark analysis four alignment methods were used. These involved both the least squares and repeated median alignment techniques discussed in Sections 5.3 and 5.4, with the option of scaling during alignment activated or inhibited. As these three dimensional alignment techniques have not been used before with craniofacial analyses, all were utilised for the evaluation of shape differences.

In this section, the biological inferences drawn from the analyses are compared with the well accepted traits of males relative to females. If these inferences are in accordance with the sex differences, more credence can be given to the techniques. Additional biological inferences, that might be expected to be drawn because of the originality of the application of the analysis techniques to the comparatively large number of three dimensional landmarks, should not be taken out of the context of this thesis (that is, seen to be generally applicable) given the restricted sample size.

6.7.1 The Mandible

Individual osseous landmark analysis

The results for the least squares and repeated median alignment of the male mandible with the mandible standard, with and without scaling, are shown in Figures 6.9 (a) to (d). For alignment with scaling, the individual d/σ-scores for the mandible (Tables 6.15 (a) and (b)) indicate there is very little difference after scaling between the male mandible and the experimental mandible standard. Of these d/σ-scores, only the lower molar left is significant. While the visual impression is that the scaled fits are much closer than the corresponding non-scaled fits, the analysis shows that the scale factors are non-significant in terms of population variance. However, scale factors, 1.053 and 1.058 for the least squares and repeated median fits respectively, show that the male mandible is larger than the female mean.

Alignment without scaling shows true size relationships (Figures 6.9 (c) and (d)) and the corresponding d/σ-scores calculated from the residuals show the expected increase due to the difference in size between the male and the experimental mandible standard (Tables 6.15 (c) and (d)). In addition to the lower molar left being significant, with scaling, the gonion right (least squares and repeated median) and infradentale (repeated median only) were also found significant without scaling. Comparison of the scaled and non-scaled results indicates that the observed significance of these latter landmarks is size related.

For least squares alignment the root-mean-square (rms) value of the residuals can be calculated, representing a single measure which is indicative of the degree of similarity between the male mandible and the experimental mandible standard. For the scaled least squares alignment, the root-mean-square (rms) residual (Equation 3.1) is 2.66mm (Figure 6.9 (a)), and for non-scaled least squares alignment, the rms residual is slightly larger at 3.93mm, reflecting both the size and shape components (Figure 6.9 (c)).

Distance and angle analysis

Of the thirty-six distances and twenty-two angles used to define the mandible, seven were found to be significant (Table 6.16). On scanning down the column of Z-scores for distances, it can be seen that there are many more positive scores than negative - indicative of the increased size of the male mandible relative to the female mandible standard.

Three of the measurements found significant involved the dental arch, with the distances, lower molar left to infradentale and lower molar right to infradentale, being larger than the standard (mll-id(11): $Z = 2.41$, difference = 5.7mm; mlr-id(12): $Z = 3.33$, difference = 4.3mm) while the dental arch angle was significantly reduced (mlr-id-mll(49): $Z = -2.13$, difference = -2.6°).

The remaining significant results were the distance gnathion to pogonion, which was significantly smaller than the standard (gn-pg(3): $Z = -2.35$, difference = -3.96mm) and the angles, gonion left to condylion left to coronoid notch left, condylion left to coronoid notch left to coronoid tip left, and coronoid notch right to coronoid tip right to external oblique line right, all significantly larger than the standard (gol-cdl-cnl(41): $Z = 2.09$, difference = 7.7°; cdl-cnl-ctl(43): $Z = 3.76$, difference = 18.8°; cnr-ctr-eolr(44): $Z = 3.54$, difference = 11.3°).

Strain analysis

Table 6.17 lists the results of the strain analysis for each triangular element matched between the male and the standard, while Figures 6.10 (a) and (b) show these "matched" mandibular elements with their principal strains and strain directions.

The right anterior ramus is defined by the triangular element, coronoid notch right, external oblique line right to coronoid tip right (Δ 68, 70, 32), while the right middle ramus is represented by the triangle, gonion right, coronoid notch right, external oblique line right (Δ 16, 68, 70) with the triangle of the right posterior ramus consisting of condylion right, gonion right, and coronoid notch right (Δ 12, 16, 68).

Study of the **right posterior ramus** triangle, showed that a minimal increase of 2.9% had occurred in the minor principal strain direction, which essentially paralleled the line joining the landmarks gonion right to condylion right (posterior ramus height). This was associated with a 15.9% dilation in the width of the ramus, resulting in an overall area increase of 19.3%.

The magnitude and direction of the principal strains for the **right middle ramus** triangle followed a similar pattern to that described for the right posterior ramus triangle, with a negligible height increase, together with a large width expansion along the major principal strain direction of 23.9%, leading to a net area difference of 25.1%.

An 8.4% reduction was observed along the minor principal strain direction for the **right anterior ramus** with a 16.1% dilation along the major principal strain axis. The net area difference is 6.3%.

These results for the **right male ramus** indicate that while the height of the posterior ramus is minimally larger and the anterior ramus height is smaller, the major difference is associated with the larger ramus width, resulting in a total area increase of the right ramus of 19.4% relative to the female mandible standard.

Analysis of the results obtained for the triangle, gonion right to external oblique line right to lower molar right, which effectively represents the **junction of the right ramus to the right body of the mandible**, reveal that there is a 7.5% reduction relative to the standard along the minor principal strain direction. This is associated with a 22.5% expansion along the line joining gonion right to external oblique line right (major principal strain direction) and results in an overall area increase of 13.3%.

Two triangular elements represent the right body of the mandible; an upper element (lower molar right, gnathion, infradentale (Δ 28, 20, 22)) which effectively defines the dental alveolus and anterior body height, and a lower element representing the length and height of the body (gonion right, lower molar right, gnathion (Δ 16, 28, 20)).

Whilst the **upper body element right** showed negligible difference in the anterior body height, a 13.2% expansion was observed along the major principal strain direction which was approximately parallel to the line joining lower molar right to infradentale. This result signifies an increase in length of the male dental arch on the right, compared to the female standard and is consistent with equivalent results noted in the distance and angle analysis. Overall, a 13.1% increase in area was noted. The **lower body element right** showed a 6.8% increase in height and a 12.5% increase in length, resulting in a general increase in area of 20.2% relative to the female mandible standard.

Elements have been defined for the left side of the mandible, in a similar manner to those of the right, that is anterior (Δ 69, 71, 33), middle (Δ 17, 69, 71), and posterior (Δ 13, 17, 69) ramus triangles, junction of left ramus to the left body triangle (Δ 17, 71, 29), and upper (Δ 29, 20, 22) and lower (Δ 17, 29, 20) triangular elements of the body.

Consideration of the **left posterior ramus** reveal a similar pattern to that of the right posterior ramus, that is, negligible difference in posterior ramus height and an increase in width of 24.1% with a net area change of 25.0%. The **left middle ramus** triangle shows an expansion along both principal strain directions, specifying a general increase in area of 24.0%. Likewise, the **left anterior ramus** triangle displays a similar pattern to that of the right anterior ramus triangle, with 6.6% reduction along the minor principal strain axis and an 11.7% expansion along the major principal direction, leading to a slight area increase of 4.3%.

Results obtained for the **left ramus** mirror those of the right, in that the height of the posterior ramus increased minimally while the anterior ramus height was observed to decrease. The most notable change was seen in the width of the ramus, which showed a total area increase of 21%. Ostensibly, the 20% larger rami width observed for the right and left sides of the male mandible, compared with the female standard, are consistent with increased muscular development in these regions - an observation which alludes to a sex related characteristic.

Once again, in comparison to the right, the triangle representing the **junction of the left ramus to the body of the mandible** shows a similar direction of shape change. A reduction of 20.6% occurred along the line joining the lower molar left to external oblique line left, with a 16.1% expansion along the major principal strain direction. This produced a net area reduction of 7.8%.

Study of the results for the **left body of the mandible** reveal that the **upper body element** shows negligible change in anterior body height, while a 17.4% dilation was observed along the principal strain axes which essentially parallels the line joining infradentale to lower molar left. This result signifies an increase in length of the left dental arch and once again is consistent with the distance and angle results and the individual osseous landmark analysis noted above. Similarly, the **lower body** triangle shows a minimal reduction in the height of the body and an expansion of 14.7% along the major principal strain direction. Both triangles show area increases of 17.2% upper and 12.7% lower, which represents a net area difference of the left body of the male mandible of 14.2%. Again, agreement is noted for the results obtained between the left and right sides of the body of the male mandible in comparison to the female standard, with the key features being an increase in dental arch length, stable anterior body height and a general increase in the area of the body.

As the chin is defined by the small triangle, gnathion to pogonion to infradentale, care must be exercised in the interpretation of the strain analysis for this particular triangle. Care is necessary because the size of the landmark location errors, associated with the vertices of the triangle, are increased relative to the length of its sides, and in this way, it is possible to obtain relatively large stretch ratios simply because of landmark location errors.

6.7.2 The Maxilla

Individual osseous landmark analysis

Assessment of the d/σ scores for the male maxilla, using the least squares and repeated median approaches, both without scaling, show that only the upper molar right and upper molar left are significant. In addition, medial orbitale left has been found to be significant for the least squares fit (Tables 6.18 (c) and (d) and Figures 6.11 (c) and (d)).

However, when scaling is employed, there is no significant difference between the male maxilla and the female standards for either alignment technique (Tables 6.18 (a) and (b) and Figures 6.11 (a) and (b)). The scale factors, although non-significant in terms of population variance, indicate for both least squares and repeated median fitting approaches (1.047 and 1.033 respectively) that the male maxilla is larger than the female standard.

Comparison of non-scaled with scaled results suggested that the significant d/σ-scores observed were a reflection of size and that in general there was very little shape difference between the male maxilla and the female standards. The root-mean-square residual with scaling (2.59mm) was considerably smaller than the root-mean-square residual without scaling (3.1mm).

Distance and angle analysis

Thirty-nine distances and twenty-four angles were used to describe the maxilla, and of these, only four distances and one angle differed significantly from the standard at the 95% confidence level (Table 6.19). Three of these measurements were concerned with dental arch size, with the other measurements related to the breadth of the maxilla and the nasofrontal angle.

In comparison to the standard, the male skull demonstrated significantly increased dental arch length bilaterally (mur-pr(5): Z = 2.84, difference = 6.0mm; mul-pr(6): Z = 3.07, difference = 6.1mm) and the dental arch breadth was also significantly increased (mur-mul(31): Z = 5.46, difference = 7.4mm). The bi-zygomaxillary breadth for the male skull was also found to be significantly larger than the female standard (zmr-zml(30): Z = 3.13, difference = 4.9mm). The nasofrontal angle was found to be significantly smaller for the male skull (na-n-g(59): Z = -2.59, difference = -12.0°).

Strain analysis

Table 6.20 and Figure 6.12 shows that of the twenty-one triangles used to describe the maxilla only five triangles demonstrate percentage stretches and/or area changes greater than 20%, thereby indicating the similarity of this particular male maxilla to the experimental reference standard. There are, however, fifteen triangles showing area increases and this alludes to the generally larger size of the male maxilla relative to the female mean.

In keeping with this tendency towards a larger male maxilla, the dental arch lengths show dilations of 14.2% and 14.8% for the right and left respectively, and are consistent with the significantly larger distance measurements from prosthion to the upper molar landmarks. The location of the upper molar landmarks were also found to differ significantly from the experimental maxilla standard using the non-scaled fitting approaches (Tables 6.18 (c) and (d)).

The triangles used to define the right and left nasal bones are particularly small and care must be exercised in interpretation of the results. As discussed previously, care is necessary because the size of the landmark location errors associated with the vertices of the triangle, relative to the length of its sides, are increased.

6.7.3 The Orbits

Individual osseous landmark analysis

The results for the least squares and repeated median alignment of the male orbit with the orbit standard, with and without scaling, are shown in Figures 6.13 (a) to (d). From the plots produced by the repeated median approach, it can be seen that where there are more landmarks, such as in the region of the orbital rim, preferential alignment of structures has occurred.

Of interest is the observation that the left optic foramen for the male skull appears to be displaced inferiorly. For this reason, an examination of the location of the optic foramina was performed for each of the subjects studied (that is, the four female skulls used to create the standard (Tables 4.2 (b) to (e)), the male skull (Tables 4.2 (a)) and the patient (Table 4.3). The coordinates of the left and right optic foramina for the male skull and the female patient show no appreciable asymmetry. For the standard, the right and left optic foramina had been located for only two of the four female skulls, due to inappropriate thresholding (see Chapter 3). Indeed, for one of the female skulls (A57590 Table 4.2 (e)) there was a marked height discrepancy between the left and right optic foramina. The height discrepancy between the optic foramina for A57590 was approximately 9mm and the final height difference between right and left optic foramina for the average orbits was about 5mm, which is consistent with averaging over two skulls. The standard deviations for the right and left optic foramina are 2mm and 5mm respectively. This result implies that A57590's left optic foramen has been located too superiorly, rather than A57590's right optic foramen being placed too inferiorly. The optic foramina are very important landmarks, as they enable the posterior limit of the orbital cones to be defined, providing the basis for quantitative measurements, (for example, distance and angle, least squares, repeated median, and two and three dimensional strain analysis), to be made for the orbital cavity. However, the production of appropriate descriptive population statistics for the optic foramina was not feasible because suitable images for study were only available for two of the four female skulls, and it was therefore not possible to calculate a reliable population variance. Nevertheless, this variance is still used to demonstrate the methodology, but with full knowledge and understanding of the implications of the limited sample size. The same methodology would apply when a large sample size is utilised. Thus the final standard deviations for the optic foramina are based on only two measurements and therefore should be considered indicative only.

Except for the optic foramen right (least squares and repeated median) and infraorbital foramen left (repeated median only), the individual d/σ-scores, for alignment with scaling, indicate that there is very little difference after scaling between the male orbits and the experimental orbital standard (Tables 6.21 (a) and (b)).

The scale factors, 1.053 and 1.051 for least squares and repeated median fits respectively, although non-significant in terms of population variance, indicate that the male orbits are larger than the female mean.

Alignment without scaling shows true size relationships and the corresponding d/σ-scores calculated from the residuals show the expected increase in value due to the difference in size between the male and the standard orbit (Tables 6.21 (c) and (d)).

Although non-significant after scaled alignments, the zygomatic corner right, nasion, superior orbitale right, and orbitale right, are significant for one or both of the non-scaled fitting procedures, and therefore the difference could most likely be attributed to a size phenomenon. The optic foramen right and to a lesser extent, the infraorbital foramen left, noted to be significant after scaled alignment, are also significant for non-scaled fitting and this suggests both a size and shape difference.

The root-mean-square values for this analysis were 3.0mm (scaled) and 3.7mm (non-scaled). These root-mean-square residuals allow one to make a "rough" estimate of the relative importance of size versus shape differences.

Distance and angle analysis

Table 6.22 lists the thirty-eight distances and seventeen angles used to describe the orbits. Study of the significant results reveals that four of the distances used to describe the right orbital cone (from optic foramen right to medial orbitale right, superior orbitale right, lateral orbitale right, and orbitale right) are all significantly larger than the standard (morr-ofr(8): Z = 3.90, difference = 6.2mm; sorr-ofr(9): Z = 4.47, difference = 8.9mm; lorr-ofr(10): Z = 4.51, difference = 5.0mm; orr-ofr(12): Z = 2.84, difference = 6.6mm). These results, when considered in conjunction with an examination of the plots of the non-scaled fits, Figures 6.13 (c) and (d), suggest that the male's right optic foramen is displaced posteriorly and that the orbital rim is more anterior than the experimental standard. This latter finding, while it can only be considered indicative, is nevertheless in accordance with the expected male trait of more developed supra-orbital ridge and zygomatic process about the orbit. These male attributes are possibly masked on the left by the higher standard deviation associated with the location of the left optic foramen (as discussed earlier).

The only other significant distance involved the measurement between the right and left optic foramina and this is significantly smaller for the male when compared with the experimental standard (ofr-ofl(36): $Z = -2.71$, difference = -4.1mm). It should be noted that the 5mm discrepancy in height between the female standard's right and left optic foramina discussed earlier will not have affected this result, as the influence of this difference is only 0.6mm (The difference = $(\sqrt{21.72^2 + 5^2} - 21.72) = (22.3 - 21.72) \sim 0.6$mm) using data from Table 4.2 (e).

The angle, medial orbitale right to optic foramen right to lateral orbital right is significantly reduced relative to the female standard, due to the medio-posterior displacement of the right optic foramen (morr-ofr-lorr(46): $Z = -3.58$, difference = -4.7°).

The reduced separation of the optic foramen also accounts for the significant decrease in the angle, optic foramen right to nasion to optic foramen left (ofr-n-ofl(54): $Z = -5.0$, difference = -6.8°), while the observed increase in the angle, optic foramen right to sella to optic foramen left, can be accounted for by sella being located more anteriorly in the male skull than the experimental standard (ofr-s-ofl(55): $Z = 3.42$, difference = 22.7°).

Strain analysis

The results of the strain analysis (Table 6.23 and Figures 6.14 (a) to (c)) highlight the larger size of the male orbits relative to the female reference standard, as shown by the percentage area increases in all but one triangle contributing to the definition of the orbits. The exception to this trend is triangle (Δ 49 55 47) which has a 3.4% area reduction.

More specifically, four triangles from the sixteen considered for the orbits have been found to have stretches and/or area differences greater than 20%. For example, the triangle enclosed by the osseous landmarks medial orbitale right to superior orbitale right to optic foramen right, which effectively describes the right superior medial orbital wall and roof, shows a 20.8% dilation in the direction parallel to the line joining superior orbitale right to the optic foramen right. This is consistent with the increased distances medial orbitale right to optic foramen right and superior orbitale right to optic foramen right, as well as displacements of the landmarks superior orbitale right and optic foramen right as described above.

The triangle enclosed by the osseous landmarks, optic foramen right, superior orbitale right and lateral orbitale right, which describes the right superior lateral orbital roof and wall, reveals a 22.0% dilation in the postero-anterior direction. This result conforms with increased distances observed for lateral orbitale right to optic foramen right and superior orbitale right to optic foramen right and the displacement of the osseous landmarks superior orbitale right and optic foramen right. There is also a 6.9% dilation in the minor principal strain direction, leading to a net area increase of 30.4% (see Figures 6.13 (c) and 6.14 (c)).

A 23.9% area enlargement occurred in the triangle (orbitale right, optic foramen right, and medial orbitale right), describing the right medio-inferior orbital wall and floor. The location of the osseous landmarks optic foramen right and orbitale right were found significant, together with the distances medial orbitale right to optic foramen right and orbitale right to optic foramen right, as noted above.

Finally, the triangle, optic foramen left, opposite orbitale left and orbitale left, which essentially outlines the left lateral orbital floor, shows a 26.5% dilation around the orbital floor along its major principal strain direction.

6.7.4 The Zygoma

Individual osseous landmark analysis

The male zygoma and the female standard after the various alignment procedures, are shown in Figures 6.15 (a) to (d). Small d/σ-scores were observed for the comparison between the male zygoma and the experimental zygoma standard, using scaled least squares and repeated median fitting (Tables 6.24 (a) and (b)). Once again, the scale factors, 1.046 and 1.039 for the least squares and repeated median fits respectively, although non-significant in terms of population variance, have been increased relative to the standard and are indicative of a larger zygoma for the male skull.

In the case of the non-scaled least squares and repeated median approaches, the landmarks orbitale right, zygomatic corner right and zygomatic corner left (repeated median only) are significant (Tables 6.24 (c) and (d)). When it is considered that the results for the same landmarks using the scaled least squares and scaled repeated median fitting approaches are non-significant, it would appear that the differences for these landmarks are size related. This finding is also consistent with the results discussed for the orbits, although the bone standards used were different.

For the scaled and non-scaled least squares alignments, the root-mean-square (rms) residuals were 2.52mm and 3.72mm respectively, indicating a substantial size contribution to the differences observed between the male and the standard.

Distance and angle analysis

Some twenty-nine distances and fourteen angles were used to describe the zygoma and only three measurements were found to be significant (Table 6.25). Examination of these results show that the bi-zygomaxillary breadth and the breadth between the zygomatic corners were significantly larger than the standard (zmr-zml(23): Z = 3.13, difference = 5.0mm; zcr-zcl(26): Z = 3.27, difference = 6.3mm). The distance lateral orbitale right to zygomatic corner right, which reflects the height and width of the right frontal process of the zygoma (lateral orbital wall), was also significantly larger than the experimental standard (lorr-zcr(1): Z = 2.29, difference = 2.7mm). These findings are consistent with the generally accepted larger size of male zygomatic breadths relative to females.

Strain analysis

The results of the strain analysis for the male zygoma against the reference standard are presented in Figure 6.16 and Table 6.26. The similarity of the male zygoma to the experimental zygoma standard is reflected in the result that none of the percentage stretches or area differences were greater than 20%. All the triangles used to describe the zygomas demonstrate area increases, indicating that the male zygomas are larger than the female mean. This finding is consistent with the known male trait of more prominent and robust zygomas. This tendency towards increased size was also observed in the distance and angle analysis and individual osseous landmark analysis.

6.7.5 The Cranium

Individual osseous landmark analysis

In the case of the cranium, for both the least squares and repeated median fitting approaches with and without scaling, the individual d/σ-scores reveal minimal differences between the male cranium and the female standards (Tables 6.27 (a) to (d) and Figures 6.17 (a) to (d)). Only the osseous landmarks glabella, bregma, and zygomatic frontal right (non-scaled only) had significant d/σ-scores. However, no significance can be attached to the bregma d/σ-score, as the female standard for this landmark comprised only one observation, whereas zygomatic frontal right appears to be size related, while glabella shows both a size and shape component to its variance.

Increased scale factors were observed for both the least squares (1.048) and repeated median (1.040) fitting approaches and, although non-significant in terms of population variance, indicate that the male cranium is larger than the female mean.

The root-mean-square residuals for least squares alignment, with and without scaling, were 4.47mm and 5.66mm respectively. These two numbers are indicative of the size and shape difference between the male and the experimental reference standard.

Distance and angle analysis

The cranium was described by thirty-seven distances and fifteen angles and of these measurements nine appear significant from the Z-scores (Table 6.28). However, of these nine measurements eight involved the landmark bregma.

As previously noted, the standard contains only one measurement for bregma and thus there are no population statistics relating to distances and angles which involve this landmark. The Z-scores have been calculated from the expected standard deviation based on landmark location accuracy alone, so no comment related to population variance can be made.

However, the distance glabella to bregma shows a 21mm difference from the standard. This is indicative of the more prominent glabella of the male and the greater height of the bregma (indicated by the 13mm height difference from basion to bregma). The distances zygomatic frontal right to bregma and zygomatic frontal left to bregma, are 10mm larger than the standard although the width zygomatic frontal right to zygomatic frontal left differs from the standard by less than 1mm. This is indicative of the taller head for this male. The 8mm difference in glabella to zygomatic frontal right coupled with the small differences between glabella to zygomatic frontal left and zygomatic frontal right to zygomatic frontal left, indicates that region of the glabella is more prominent for the male (Figures 6.17 (c) and (d)).

The individual landmark coordinate analysis above indicated that the glabella was significantly more prominent than the female reference standard. However, the distance and angle analysis did not show significance in distances involving the glabella (except in conjunction with the bregma, which appears to differ from the standard).

The other significant result is for the angle opisthocranion to opisthion to basion which is larger than the standard (op-o-ba(44): $Z = 3.17$, difference = 8.3°).

Strain analysis

The cranium was divided into triangles which described the calvaria and the cranial base and the results of the strain analysis are given in Table 6.29 and Figures 6.18 (a) to (c). Area increases were noted for all of the triangles but none were found to have dilations greater than 20%. Once again, these results lend support to a larger male cranium.

6.7.6 The Skull

Individual osseous landmark analysis

The results of the scaled least squares and repeated median fits indicate that there is very little shape difference between the male skull and the female standards (Figures 6.19 (a) and (b) and Tables 6.30 (a) and (b)). The d/σ-scores for each landmark indicate that only a few landmarks differ significantly from the experimental reference skull standard.

While the scale factor results for both the least squares (1.031) and repeated median (1.033) fits indicate size increase, they are non-significant.

Similar results were observed for the non-scaled least squares and repeated median fits (Tables 6.30 (c) and(d) and Figures 6.19 (c) and (d)).

The root-mean-square residuals for the least squares analyses were 4.01mm (scaled) and 4.84mm (non-scaled). These are slightly larger than those obtained for the individual bone analyses. This increase is possibly due to a contribution from slight differences in the relationships or articulation of the bones comprising the skull.

Distance and angle analysis

The distance and angle analysis of the male skull against the experimental reference standard is identical to that for the individual bone distance and angle analysis reported in Sections 6.7.1 to 6.7.5. The standards are the same regardless of how the distances and angles are grouped.

Strain analysis

A strain analysis of the male skull against the average coordinate data of the experimental reference standard show similar results to those presented in Sections 6.7.1 to 6.7.5 and will not be further discussed.

6.8 Summary and Discussion

Morphological standards pertinent to the mathematical methods described in Chapter 5 were created from the four female dried skulls to act as experimental reference standards against which individuals could be compared, with the significance of any differences being assessed using either the Z-score or $d\sqrt{n}/\sigma$-score test statistics. This had the additional advantage of allowing the assessment of the sensitivity of each of the analysis techniques to differences between the individual and the standards.

The most fundamental craniometric descriptors of morphology are distances and angles between osseous landmarks. Accordingly, population statistics for the four female skulls were determined for a large number of distances and angles describing the five major regions of the craniofacial complex, as indicated in Table 6.31.

It is natural with the availability of three dimensional coordinate data to want to compare the relative positions of coordinates directly; however, some alignment of the data is necessary before such comparisons have meaning.

Two three dimensional alignment techniques were used for this purpose: the well established least squares alignment technique and a three dimensional repeated median alignment technique the author specifically developed by extending the two dimensional method of Siegel (1982a, 1982b). After alignment of all four female skulls, population statistics of the coordinate positions were determined to create both "least squares" and "repeated median" standard bones for the same craniofacial regions listed above. Comparison of the root-mean-square residuals allows one to make a "rough" estimate of the relative importance of size and shape differences. Also, if the root-mean-square residual is approximately the landmark location error, one may say there was very little difference between the two shapes.

In order to facilitate the interpretation of results derived from the newly developed strain analysis technique, "strain standards" were created. These were produced by determining the strains of the defined triangular elements for each of the four female skulls relative to the appropriate three dimensional coordinate repeated median bone standard. For each triangular element, population statistics were calculated for percentage stretches and area changes for the mandible, maxilla, orbits, zygoma, and cranium.

As an aid in demonstrating the method of use of the standards, another skull of identical ethnic background, similar age but of different sex was compared to the aforementioned bone standards. The five test skulls, used in the present study were drawn from a population of Narrinyeri skulls, whose sex had been previously determined by several researchers, including Richards (1983) using discriminant function analysis, (Giles and Elliot, 1963) in conjunction with criteria described by Larnach and Freedman (1964) and Larnach and Macintosh (1971). The validity of the discriminant function approach in determining the sex of Australian Aboriginal subjects of known sex from radiographs was demonstrated by Townsend, Richards and Carroll (1982), when they correctly classified ninety-two subjects from a population of one hundred individuals. Table 6.32 shows the sex distribution for the Narrinyeri skulls, including the five test skulls (Richards, 1983). It should be noted that of the five skulls used in this thesis, one of the female skulls was marginally on the male side of the female mean while the male skull was on the female side of the male mean. Thus, observation of subtle sex differences between the male skull and the female experimental standard were not anticipated; instead, the comparison was expected to highlight the more obvious sex differences.

The scaled fit of the coordinate data of the male mandible to the experimental reference standard indicated that the male mandible was larger, although this was not significant at the 95% confidence level. Similarly, the distance and angle analysis showed that most distances were larger for the male, although only the distances associated with the mandibular dental arch length were significantly larger. This general increase in size of the male mandible relative to the standard was also observed in the results of the strain analysis, where most triangles showed positive area changes. The strain analysis, too, shows the increase in mandibular dental arch length bilaterally through the magnitude and direction of the major principal strains. It also quantifies significant area increases in the widths of the rami and bodies of the mandible. These features can be seen clearly in the plots of the non-scaled male mandible, compared with the experimental reference standard.

The larger size and increased width of the rami and bodies of the mandible are indicative of male traits.

Comparison of the distance and angle measurements of the male maxilla relative to the maxilla's experimental distance and angle standard suggested that the male had a larger maxillary complex with the bi-zygomaxillary breadth, the dental arch breadth and lengths being significantly larger. In general, the strain analysis results supported the trend of a larger male maxilla relative to the female standard, with most triangles (15 out of 21) showing positive area increases. The scale factors obtained from the coordinate data fits also lent further support to a larger male maxilla relative to the experimental standard, although these were non-significant in terms of the population variance.

The distance and angle analysis showed that the distances from the right orbital rim to the right optic foramen for the male were significantly larger than for the female standard. This can also be seen in the strain analysis, where three triangular elements involving the right orbital cavity have percentage stretches or area increases greater than 20%. This finding is consistent with the male trait of more developed supra-orbital ridge and zygoma about the orbit. Similar features relating to the left orbit were not evident, although the left lateral orbital floor showed a large dilation in the direction around the orbital floor. The location of the left optic foramen of the standard is probably 5mm too high, as discussed in Section 6.7.3. With hindsight, it would have been better to discard the measurement causing this large discrepancy, but the rogue measurement was not noticed until the completion of the analysis.

With respect to the male zygomas, a comparison of the scaled coordinate fits with the experimental reference standard revealed the the male zygomas were approximately 4% larger, although this result was non-significant in terms of population variance. Area increases were observed in all triangular elements used for the strain analysis, but none were greater than 20%. Positive Z-scores were encountered for most of the distances, indicating the generally larger size of the male zygomas. Further, the distances, bi-zygomaxillary breadth, bi-zygomatic corner and the height and width of the right lateral orbital wall, were all found to be significantly larger than the standard. Larger, broader, more robust zygomas are consistent with accepted male characteristics.

In the case of the cranium, the majority of the distance Z-scores from the distance and angle analysis, and percentage area changes from the strain analysis, were positive, indicating that the male cranium is larger than the female reference standard. This finding is also supported by the scaled fits of the three dimensional coordinate data which give scale factors of 1.05 and 1.04 for the least squares and repeated median alignments respectively. Although these results are not significant in terms of being outside the population variance of the created female reference standard at the 95% confidence level, it does indicate the larger size of the male cranium.

The individual coordinate analysis applied to the entire male skull showed a size increase relative to the female standard (again not at the 95% confidence level). Many more significant differences were found between the male and standards for individual bone analysis compared with the entire skull analysis, indicating the increased sensitivity when one focuses on an individual bone. The increased root-mean-square residuals (residuals for the least squares alignments of the male skull relative to the standard), in comparison with the root-mean-square residuals of the individual bone analyses, is possibly a reflection of slight differences in articulation of the bones.

For some landmarks, only one or two measurements were available to compile the standards (rather than four if the landmark was identifiable on each skull). This effectively reduces the subsequent analyses to comparisons of individuals (for the landmarks in question), so that while statistics are not available, valuable comparisons can still be made. The perspective, however, is more limited. One does not know how the skull, used for the standard, relates to the population from which it is drawn.

Nevertheless, the author has persisted using these measurements in the reference standard. When only one measurement is available, the standard deviation is set to zero so that the limitations in this case are clear. For two measurements, a standard deviation can be calculated - it relates to the separation of the landmarks - but there is no guarantee that these two measurements will reflect the true population statistics. Regardless of these problems, it is better to have one measurement for comparison than none, as long as this is acknowledged and its limitations are recognized.

This chapter has described several techniques that quantify craniofacial relationships in three dimensions. These methods include:

(i) metric analyses based on distances and angles,

(ii) comparison of individual osseous landmarks in three dimensions, after suitable alignment of homologous structures, and

(iii) strain analysis of homologous triangular and tetrahedral elements.

As the analyses were applied to the same data, each analysis technique should reveal the same structural differences and lead to the same interpretation and conclusions. The approaches, however, throw the data into different perspective and while each reflect the same differences, the ease of interpretation vary depending on the feature.

The results obtained from each analysis were consistent with the known differences between male and females, implying that each analysis technique is potentially useful for the quantification of shape and size differences and that the experimental reference standard is acceptable. Although not based on a large population, the experimental reference standards can be used, therefore, with reasonable confidence for evaluation of the analysis techniques for the quantification of craniofacial deformity.

CHAPTER 7

A COMPARISON OF A PATIENT PRESENTING WITH TREACHER COLLINS SYNDROME (MANDIBULOFACIAL DYSOSTOSIS) WITH THE EXPERIMENTAL REFERENCE STANDARDS

7.1 Introduction

The clinical, radiographic and anatomical manifestations of patients presenting with Treacher Collins Syndrome has been documented by many authors (Berry, 1889; Treacher Collins, 1900; Lockhart, 1929; Franceschetti and Klein, 1949; Rogers, 1964; Dahl, Kreiborg and Björk, 1975; Kawamoto, 1976; Marsh et al., 1986a; and David, 1986). The following soft tissue and skeletal features have been cited by Gorlin et al., (1976) in their widely acclaimed text book on syndromes of the head and neck for patients with Treacher Collins Syndrome (mandibulofacial dysostosis). These can be summarised as follows:

- **Facial Characteristics**
  - laterally downward sloping palpebral fissures
  - coloboma in outer third of lower lid
  - depressed cheekbones
  - nose appears larger because of the lack of zygoma development
  - nasal nares often narrow, alar cartilages hypoplastic, choanal atresia
  - pinna often deformed, crumpled forward, misplaced, 30% have absent external auditory meati, or ossicle defects associated with conductive deafness
  - receding chin
  - large fish-like mouth (macrostomia, approximately 15%)
  - tongue-shaped process of hair extending towards the cheek

Skeletal Characteristics

Mandible

- hypoplastic mandible
- angle of the mandible more obtuse
- ramus may be deficient
- coronoid and condylar processes may be flat or aplastic
- undersurface of the body of the mandible has a concave appearance

Maxilla

- poor development
- high arched palate associated with cleft palate in approximately 30% of cases
- dental malocclusion associated with open bite; additionally, teeth may be widely separated, hypoplastic or displaced
- nasofrontal angle is usually obliterated and the bridge of the nose raised

Orbits

- lower margin of the orbit noted to be defective
- roof inclining downward and outward
- orbital cavity oval shaped

Zygomas

- may be totally absent
- most often grossly and symmetrically underdeveloped
- non-fusion of the zygomatic arches

Calvaria

- essentially normal
- supra-orbital ridges may be poorly developed
- may be increased digital markings on radiographs in the presence of normal sutural relationships

Other

- mastoids frequently sclerotic and not pneumatized
- paranasal sinuses are often small or completely absent

The analysis techniques developed in Chapter 5 were applied to quantify the shape differences between a patient with Treacher Collins Syndrome and the experimental reference standards discussed in Chapter 6.

As well as identifying the better known qualitative features of the syndrome, it is expected that the quantity and accuracy of the landmark data will perhaps identify other more subtle features of the syndrome for this particular patient. In addition, the characteristics of the syndrome are quantified relative to a "normal" population. This population, although of a different ethnic group and developed from a limited data sample, nevertheless provides an experimental reference standard that not only allows the methodology developed in Chapter 6 to be further assessed, but also enables its applicability to the clinical situation to be determined.

The information gained from the various analyses of the patient is summarised in Table 7.18, where a comparison is also drawn with the classic description of the syndrome by Gorlin et al., 1976, referred to above. A detailed morphological description, however, of the findings of each analytic technique is given in the following sections.

7.2 The Mandible

The mandible was defined by the following landmarks - condylion right, condylion left, gonion right, gonion left, gnathion, pogonion, infradentale, lower molar right, lower molar left, coronoid tip right, coronoid tip left, coronoid notch right, coronoid notch left, external oblique line right and external oblique line left.

7.2.1 Individual osseous landmark analysis of the patient's mandible relative to the experimental reference mandible standard

The results of least squares and repeated median fits between the patient and the standard are shown in Figures 7.1 (a) to (d) and Tables 7.1 (a) to (d). The scale factor Z-scores under Tables 7.1 (a) and (b) indicate that the patient is smaller than the standard (5.8% and 8.4% smaller respectively), although not outside the normal population variance. Figures 7.1 (a) and (b) show the mandible fitted with scaling allowed to enhance shape differences, while Figures 7.1 (c) and (d) show the fits without scaling to show true size and shape differences.

A general impression from Figures 7.1 (a) to (d) is that the patient has bilaterally smaller posterior ramus height (go-cd), less developed condylar process on the left (not assessed on the right due to the landmark coronoid notch being obscured (as discussed in Section 7.2.2)), smaller mandibular body length bilaterally (gn-go), more obtuse ramus to body angle (gn-go-cd), narrower dental arch, more laterally displaced coronoid tips (ctr-ctl), and larger bigonial breadth (gor-gol). The mandibular symphysis (gn, pg, id) however, appears similarly shaped to the standard. Fewer landmarks have been identified on the right, making it difficult to determine if one side is more affected than the other.

The Tables 7.1 (a) to (d) quantify the extent of the deviation from the standard, while the Figures 7.1 (a) to (d) provide a visual impression and highlight the direction of the deviation.

At the 95% confidence interval, all the patient's mandibular landmarks were found to differ significantly from the homologous landmarks of the experimental reference standard, although for a few landmarks this depended on whether the comparisons were made with or without scaling and on whether least squares or repeated median fits were used.

It is interesting to note that the position of the patient's right coronoid tip is significant from that of the standard when scaling is invoked, but non-significant when scaling is inhibited. This is attributable to the flaring of the right coronoid tip laterally and the smaller size of the patient relative to the standard. When the patient's data are scaled up in size to match the standard, the coronoid tip is moved further (laterally) away from the standard. This reasoning also applies to the left external oblique line.

A general impression of the similarity between the patient and the standard is given by the root-mean-square (rms) of the residuals. The root-mean-square values for the scaled and non-scaled least squares fits were 8.46mm and 8.93mm respectively, indicating the general difference in size and shape between the patient and the experimental mandible standard. The corresponding root-mean-square values for the male mandible were 2.66mm and 3.93mm. This is indicative of the greater difference between the patient and the experimental reference standard, rather than between a "normal" male and the experimental reference standard.

7.2.2 Distance and angle analysis of the patient's mandible relative to the experimental reference mandible standard

The distance and angle measurements of the patient and their Z-scores are listed in Table 7.2. (The population statistics of the standard are also listed for reference). Of the thirty-six distances and twenty-two angles measured, twenty-nine were found to be significant. Most of the Z-scores for the distances were negative, indicating that the patient was smaller than the standard.

The total mandibular length as measured from condylion to gnathion is significantly reduced bilaterally (cdr-gn(21): Z = -2.35, difference = -16.0mm; cdl-gn(27): Z = -2.85, difference = -16.1mm). This finding is also supported by the distance measurements condylion right to pogonion and condylion left to pogonion (cdr-pg(22): Z = -2.52, difference = -14.5mm; cdl-pg(28): Z = -2.43, difference = -17.0mm).

The right ramus was defined by the following landmarks - gonion right, condylion right, coronoid notch right, coronoid tip right, external oblique line right and by the distance measurements 1, and 15 to 19 and the angles 40, 42, and 44 (Table 7.2).

Consideration of these Z-scores revealed that six of these measurements were absent because the patient's right coronoid tip had deviated laterally, obscuring the location of the coronoid notch, and the patient's dysplastic right zygoma obscured the region of the external oblique line right. With hindsight, it would have been preferable to have estimated the positions of these landmarks. Of course, there would have been a larger location error for each, but nevertheless the estimates would have been within a few millimetres.

The right posterior ramus height (gonion right to condylion right), and the distance gonion right to coronoid tip right were both significantly smaller than the standard (cdr-gor(1): Z = -4.84 difference = -12.6mm; gor-ctr(18): Z = -2.04, difference = -9.0mm), while the distance between condylion right and the coronoid tip right was non-significant.

Similarly, the left ramus can be defined by the following landmarks - gonion left, condylion left, coronoid notch left, coronoid tip left, external oblique line left and by the distance measurements 6 to 9, and 23 to 25 and the angles 41, 43, and 45 (Table 7.2).

The Z-scores revealed that the patient's left posterior ramus height (gonion left to condylion left) was significantly smaller than the standard (cdl-gol(6): Z = -3.13 difference = -13.0mm). It was observed that the Z-score of the right posterior ramus height was substantially larger than the left. This difference in Z-scores is a result of pathology in the left condylar head of one of the skulls used in creating the standard (as discussed in Section 6.3). The pathology has influenced the population statistics by reducing the mean and increasing the variance. This increased variance has resulted in a smaller Z-score than would have been expected for the left posterior ramus.

Unlike the distance from gonion left to condylion left, the distances from gonion left to coronoid notch left and gonion left to coronoid tip left were both non-significant.

An analysis of the distances and angles defining the left condylar and coronoid region showed that the distance from condylion left to the coronoid notch left and the distance from condylion left to the coronoid tip left were significantly smaller (cdl-cnl(7): Z = -4.40, difference = -11.8mm; cdl-ctl(25): Z = -2.54, difference = -10.2mm). The angle, gonion left to condylion left to coronoid notch left, was significantly larger (gol-cdl-cnl(41): Z = 2.96, difference = 10.9°), whereas the distance from the coronoid notch left to the coronoid tip left and the angles, condylion left to coronoid notch left to coronoid tip left, and, coronoid notch left to coronoid tip left to external oblique line left, were all non-significant.

The relationship of the ramus to the body of the mandible was defined by the angles 37, 38, 46, 47, 50 and 51 (Table 7.2). The angle between the posterior ramus and the body of the mandible as defined by the angles, condylion right to gonion right to gnathion, and condylion left to gonion left to gnathion, were significantly more obtuse (cdr-gor-gn(37): Z = 2.58, difference = 12.9°; cdl-gol-gn(38): Z = 3.07, difference = 10.4°). This was supported by the essentially equivalent measurements, condylion right to gonion right to pogonion, and condylion left to gonion left to pogonion (cdr-gor-pg(50): Z = 3.04, difference = 14.0°; cdl-gol-pg(51): Z = 3.92, difference = 11.7°). The angle of the left anterior border of the ramus to the dental alveolar crest in the molar region also reflects this pattern (that is, significantly more obtuse (ctl-eoll-mll(47): Z = 4.76, difference = 26.4°). An equivalent measurement could not be determined for the right because the landmark external oblique line right was obscured by the patient's right zygoma.

Measurements taken of the lower border of the mandible indicated that the patient's mandibular plane length, as measured from gonion right to pogonion and gonion left to pogonion, was significantly smaller than the standard (gor-pg(29): Z = -2.33, difference = -14.6mm; gol-pg(30): Z = -2.09, difference = -16.2mm). These results were supported by the essentially equivalent measurements, gonion right to gnathion, and gonion left to gnathion, with the difference from the standard being approximately 13mm. The lower border angle as measured from gonion right to gnathion to gonion left, and also gonion right to pogonion to gonion left, showed a significantly increased angle relative to the standard (gor-gn-gol(39): Z = 3.99, difference = 18.7°; gor-pg-gol(52): Z = 4.64, difference = 18.1°; Figure 7.1 (d), repeated median fit).

Lower face height measurements, gnathion to pogonion, pogonion to infradentale, chin line to mandibular plane line right, and chin line to mandibular plane line left, were all non-significant, as were the breadth measurements, with the exception of the coronoid tip width which was significantly larger than the standard (ctr-ctl(34): Z = 2.08, difference = 9.7mm) This exception was a result of the right coronoid tip being flared laterally.

With respect to the dental arch, the left dental arch length was smaller than the right side and significantly smaller than the standard (mll-id(11): Z = -3.32, difference = -7.8mm). Arch breadth and arch angle were also significantly smaller (mlr-mll(36): Z = -2.69, difference = -8.5mm; mlr-id-mll(49): Z = -4.48, difference = -5.5°).

To complete the mandibular distance and angle analysis, the angular relationship of the mandibular planes (right and left) to the cranial base and the angular relationship of the mandibular planes (right and left) to the hard palate (or nasal line) were also considered. It was found, in each case, that the angle was significantly more obtuse than the standard (ML(r)/NSL(54): Z = 8.05, difference = 17.1°; ML(l)/NSL(53): Z = 6.89, difference = 20.5°; NL/ML(r)(56): Z = 5.83, difference = 15.5°; NL/ML(l)(55): Z = 3.69, difference = 17.3°). This was consistent with the finding reported above on the relationship of the ramus to the body of the mandible.

7.2.3 Strain analysis of the patient's mandible relative to the experimental reference mandible standard

The results of the triangular strain analysis are given in Table 7.3, and Figures 7.2 (a) and (b) show the matched mandibular elements with their principal strains and strain directions.

The left anterior ramus was defined by the triangular element, coronoid notch left to external oblique line left to coronoid tip left (Δ 69, 71, 33), while the left middle ramus was represented by the triangle, gonion left, coronoid notch left, and external oblique line left (Δ 17, 69, 71), with the triangle of the left posterior ramus consisting of condylion left, gonion left, and coronoid notch left (Δ 13, 17, 69) (see Table 7.3).

Consideration of the **left posterior ramus** triangle, revealed that a 45.0% reduction occurred in the minor principal strain direction, which was approximately parallel to the line joining the condylar head left to the coronoid notch left. This was combined with an 7.3% reduction along the major principal strain axis, resulting in an overall area reduction of 49.0% relative to the standard. Additionally the **left middle ramus** triangle demonstrated a net area reduction of 18.8%, while the **left anterior ramus** triangle showed a slight area decrease of 3.1%.

These results describe an increasing underdevelopment, from the anterior to posterior border of the left ramus and are indicative of a hypoplastic condition in the left posterior ramus.

The **junction of the left ramus to the left body of the mandible** was effectively represented by the triangle, gonion left to external oblique line left to lower molar left (Δ 17, 71, 29). Of interest is that the overall area change was small (+5.8%), compared with the large reduction along the minor principal strain direction (20.3%), and the large expansion along the major principal strain direction (32.8%). Inspection of Figure 7.2 (a) reveals that the direction of deformation of the major principal strain direction was essentially a continuation, although more obtuse, of the major principal strain direction of the middle ramus triangle. These results imply that while the area of the bone remains unaltered, the patient relative to the standard demonstrates a large deformation in the region of the angle of the left mandible. The direction of this deformation indicates that the patient has an increased ramus to body angle relative to the standard.

The **left body of the mandible** was divided into two triangular elements, an **upper body element** (lower molar left, gnathion, infradentale (Δ 29, 20, 22)) which effectively represents the dental alveolus and anterior body height, and a **lower body element** representing the length and height of the body (gonion left, lower molar left, gnathion (Δ 17, 29, 20). Both triangular elements showed large area reductions of 15.7% and 30.2% respectively. The minor principal strain direction of the upper triangle was along the line joining infradentale to the lower molar left and showed a reduction along this direction of 18.3%. This result signifies a reduced dental arch length. The lower triangle shows similar contractions along both principal strain directions, specifying a general decrease in area.

As noted in the Section 7.2.2, the coronoid notch right and the external oblique line right could not be located in the patient due to the coronoid notch being obscured by the coronoid tip and the dysplastic zygoma obscuring the region of the external oblique line right. As a consequence, only two triangles, the **upper and lower** triangular elements of the **body of the right mandible**, were matched to the standard's right mandible. The two triangles were unlike the left side, with the **upper body** triangle **right**, which contains information pertaining to the dental alveolus and anterior body height, exhibiting little change from the standard and the **lower body** triangle **right** displaying marked deformation, with an overall area reduction of 17.9%. The marked deformation is a result of a reduction in the right mandibular body length (25.5%) with an increased body height (10.2%). However, this latter increase may be attributed to over-eruption of a tooth in the area of the lower molar right, thereby raising the measured height of the dental alveolus.

As mentioned in Section 6.7.1 the chin was defined by the small triangle, gnathion to infradentale to pogonion (Δ 20, 22, 21), and the results obtained from this particular triangle's strain analysis must be interpreted with care. This is because the size of the landmark location errors associated with the vertices of the triangle relative to the length of its sides, are increased. Thus, it is possible to obtain relatively large stretch ratios simply because of landmark location errors.

The external surface area of the mandible can be estimated by the sum of the areas of the triangles used to describe the mandible. The total area of the patient's mandible was 38.3cm^2, which was 8.8cm^2 smaller than the standard (these areas are based only on those triangles in common between the patient and the standard, that is, left ramus, left and right bodies of the mandible).

7.3 The Maxilla

The maxilla was defined by the following landmarks - nasion, prosthion, anterior nasal spine, posterior nasal spine, molar upper right, molar upper left, zygomaxillare right, zygomaxillare left, palatine tubercle right, palatine tubercle left, nasale, medial orbitale right, medial orbitale left, orbitale right, orbitale left, infraorbital foramen right, infraorbital foramen left, maximum nasal breadth right, maximum nasal breadth left, incision superius right and incision superius left.

7.3.1 Individual osseous landmark analysis of the patient's maxilla relative to the experimental reference maxilla standard

The results of the least squares and repeated median fits for the maxilla between the patient and the standard are shown in Figures 7.3 (a) to (d) and Tables 7.4 (a) to (d). The scale factor Z-scores beneath Tables 7.4 (a) and (b) indicate that the patient is smaller than that of the standard (8.4% and 4.7% smaller respectively), although not outside the population variance. Figures 7.3 (a) and (b) show the maxilla fitted with scaling allowed, highlighting any shape differences, while Figures 7.3 (c) and (d) show the fit without scaling, highlighting size differences.

It is obvious from the plots that some of the landmarks used to define the maxilla have not been identified from the patient's radiographs. The non-identified landmarks are zygomaxillare right and left, palatine tubercle right and left, medial orbitale left and infraorbital foramen right and left. The landmarks zygomaxillare right and left were not measured, as appropriate views were not available from the standard six X-axis rotations selected. In the case of the other landmarks, thin or poor quality bone surrounding the expected position of these landmarks resulted in these landmarks being impossible to identify. A threshold level of 190 was used, and as this level has been found suitable for most clinical three dimensional CT reconstructions, it was inferred that the holes in the reconstructions for the patient were a consequence of the poorer quality and/or thinner and/or absence of bone in these regions reflecting the hypoplastic nature of the syndrome.

Inspection of Figures 7.3 (a) to (d) led to the following impression of the patient's maxilla:

(i) Size
- smaller than the standard.

(ii) Nasal complex
- the nasal length as defined by the distance between nasion to nasale, is larger than the standard, with nasale more antero-superiorly located than in the standard.
- the nasal aperture (defined by nasale, nasal breadth left and right, and anterior nasal spine) appears more prominent at nasale but medio-posteriorly displaced at nasal breadth right.

(iii) Dental arch dimensions
- the dental arch breadth and angle appear reduced relative to the standard.
- the angulation to the patient's nasal line parallels the standard's dento-alveolar line, while the patient's dento-alveolar line parallels the standard's nasal line.

(iv) Face height
- the patient's anterior face height is almost identical to that of the standard.
- the patient's lateral face height (bilaterally), as measured between orbitale and upper molar, is reduced relative to the standard.

(v) Anterior lower orbital rim
- the angle between nasion, medial orbitale right and orbitale right is more acute for the patient than for the standard, indicating a change in angulation of the right lower orbital rim.

All the measured landmarks, with the exception of nasion, prosthion, and nasal breadth left, differed significantly at the 95% confidence interval, using a $\chi^2(3)$ test in at least one of the Tables 7.4 (a) to (d).

The root-mean-square of the residuals for least squares fits of the patient's maxilla to the experimental maxilla standard were 6.61mm and 7.02mm for the scaled and non-scaled fits respectively. For the male, the corresponding root-mean-square values were 2.59mm and 3.10mm. This is indicative of the considerable difference between the patient's maxilla and the experimental standard.

7.3.2 Distance and angle analysis of the patient's maxilla relative to the experimental reference maxilla standard

The distance and angle measurements for the patient's maxilla and their Z-scores are reported in Table 7.5.

In comparison to the standard, the patient demonstrates significantly reduced dental arch length bilaterally (mur-pr(5): Z = -2.92, difference = -6.2mm; mul-pr(6): Z = -2.61, difference = -5.2mm), and the dental arch breadth is significantly decreased (mur-mul(31): Z = -11.68, difference = -15.9mm). Additionally, the dental arch angle is significantly smaller than the standard (mur-pr-mul(44): Z = -4.83, difference = -16.1°). This result is a reflection of the fact that the dental arch breadth is approximately three times smaller than the arch length.

The patient also demonstrates reduced lateral maxillary height - a reduction which is more pronounced on the right than the left (mur-orr(20): Z = -3.30, difference = -16.2mm; mul-orl(21): Z = -2.34, difference = -11.0mm). However, the height of the anterior maxilla (that is, anterior face height distance and angle measurements) is within normal variance as are the nasal cavity lengths and the naso-pharynx measurements.

The nasofrontal angle is significantly more obtuse for the patient than for the standard indicating a flatter angle at nasion (na-n-g(59): Z = 3.70, difference = 17.2°).

It is interesting to note that the angle between the hard palate (nasal line) and prosthion is significantly smaller than the skeletal (Australian Aboriginal) standard (pns-ans-pr(58): Z = -3.35, difference = -28.3°). This result is probably a racial difference as it has been recorded that Australian Aboriginals commonly display dental alveolar prognathism, whereas Asian populations do not generally demonstrate this feature to the same extent (Brown, in press).

The root mean square of the Z-scores for all measured distances and angles was 2.6. This is indicative of a significant variation of the patient's maxillary complex from the standard.

7.3.3 Strain analysis of the patient's maxilla relative to the experimental reference maxilla standard

The results of the triangular strain analyses are summarized in Table 7.6 and displayed in Figure 7.4. The strain analysis revealed that most of the triangles decreased in area, which is consistent with the result of reduced size of the maxilla, determined using the other analysis techniques (Sections 7.3.1 and 7.3.2). Inspection of the Figures revealed that nearly all of the minor principal strain directions were approximately vertical, indicating a general reduction in height of the maxilla, especially laterally. (The exceptions are the upper dental alveolar triangles that show the dental alveolar height is increased relative to the standard. This, however, is a reflection of the loss of central incisors in the standard and subsequent resorption of the dental alveolar in a supero-posterior direction. The anterior right nasal aperture shows a general size decrease which is also consistent with a reduction in height of the maxilla).

The nasal complex was described by eight triangular elements - the right nasal bone (nasion, nasale, and medial orbitale right (Δ 2, 42, 46)), the right lateral aspect of the nasal aperture (nasale, medial orbitale right, and nasal breadth right (Δ 42, 46, 66)), the anterior right nasal aperture (nasal breadth right, nasale and anterior nasal spine (Δ 66, 42, 24)), the right hard palate (palatine tubercle right, posterior nasal spine, anterior nasal spine (Δ 36, 25, 24)), the left nasal bone (nasion, nasale, and medial orbitale left (Δ 2, 42, 47)), the left lateral aspect of the nasal aperture (nasale, medial orbitale left, and nasal breadth left (Δ 42, 47, 67)), the anterior left nasal aperture (nasal breadth left, nasale and anterior nasal spine (Δ 67, 42, 24)), the left hard palate (palatine tubercle left, posterior nasal spine, anterior nasal spine (Δ 37, 25, 24)), and the nasal septum (anterior nasal spine, nasion, and posterior nasal spine (Δ 24, 2, 25)).

Of these, five were measured for the patient and can be seen in Figure 7.4. The **right nasal bone** triangle was found to be substantially larger (54.1% area increase) than the standard with a 12.8% and 36.6% increase along the principal strain directions. The principal strain directions are oriented such that they suggest width and height dilations of the right nasal bone triangle.

The **right lateral aspect of the nasal aperture** triangle demonstrated marked shape deformation with a 12.3% contraction in its height (minor principal strain direction) and a 30.3% dilation in its major principal strain direction (essentially corresponding to its width). Overall, there was a net area increase of 14.3%.

Unfortunately, the left nasal bone and left lateral aspect of the nasal aperture were not measured, because the osseous landmark medial orbitale left could not be definitively identified from the patient's radiography.

The **anterior right nasal aperture** triangle showed a general size decrease resulting from contractions of 15.8% and 8.9% along the minor and major principal strain directions respectively, with a net area decrease of 23.3%. In contrast, the **anterior left nasal aperture** showed minimal area change (5.6% increase), but demonstrated a shape deformation with a contraction of 13.0% in the minor principal strain direction (essentially the height of the aperture) and 21.3% dilation in the major principal strain direction across the aperture.

The **nasal septum** triangle showed a 4.6% decrease in height and a 14.3% decrease in depth resulting in a net area decrease of 18.2%.

The increased size of the right nasal bone, coupled with the general height decrease of the remaining triangles describing the nasal complex, gives an overall impression of a smaller nose but with a prominent nasal bone.

Three triangles were used to describe the **anterior surface of the maxillary sinus right**. These were - the supero-medial anterior maxillary sinus triangle right (medial orbitale right, nasal breadth right, orbitale right (Δ 46, 66, 54)), the infero-medial anterior maxillary sinus triangle right (upper molar right, orbitale right, nasal breadth right (Δ 26, 54, 66)) and the lateral anterior maxillary sinus triangle right (upper molar right, orbitale right, zygomaxillare right (Δ 26, 54, 30)). Unfortunately, as discussed earlier, the landmarks zygomaxillare right and left could not be measured from the standard views, so the lateral anterior maxillary sinus triangle right is absent.

While there is minimal area change (0.8% increase) of the **supero-medial anterior maxillary sinus** triangle **right**, this triangle displays a distortion because of the 7.9% contraction in the minor principal strain direction (height) and the 9.4% dilation in the major principal strain direction (width).

The **infero-medial anterior maxillary sinus** triangle **right** particularly emphasises the lack of height between the lower margin of the orbital rim and upper molar right, because there is a 39.3% contraction in this direction and a 11.2% dilation across the width. This has resulted in a net area reduction of 32.6%.

Similarly, the **left anterior surface of the maxillary sinus** was defined by supero-medial anterior maxillary sinus triangle left (medial orbitale left, nasal breadth left, orbitale left (Δ 47, 67, 55)), the infero-medial anterior maxillary sinus triangle left (upper molar left, orbitale left, nasal breadth left (Δ 27, 55, 67)) and the lateral anterior maxillary sinus triangle left (upper molar left, orbitale left, zygomaxillare left (Δ 27, 55, 31)). Of these three triangles, only the **infero-medial anterior maxillary sinus** triangle **left** could be determined for the patient, as the osseous landmark medial orbitale left was missing. This triangle displayed a similar pattern of shape change to the right but with reduced magnitude, that is, showing a 26.9% reduction in height between the lower margin of the orbital rim and upper molar left and minimal contraction of 0.7% in the major principal strain direction, with a net area reduction of 27.5%.

The **dental alveolar complex** was defined by four triangles - the superior maxillary dental alveolar triangle right (nasal breadth right, anterior nasal spine, upper molar right (Δ 66, 24, 26)), inferior maxillary dental alveolar triangle right (anterior nasal spine, upper molar right, prosthion (Δ 24, 26, 23)), the superior maxillary dental alveolar triangle left (nasal breadth left, anterior nasal spine, upper molar left (Δ 67, 24, 27)), and inferior maxillary dental alveolar triangle left (anterior nasal spine, upper molar left, prosthion (Δ 24, 27, 23)).

The **left superior maxillary dental alveolar** triangle also showed marked shape deformation, with a 24.2% contraction along the minor principal strain direction and a 9.9% expansion in the major principal strain direction, with a net area reduction of 17.0%. The **left inferior maxillary dental alveolar** triangle showed a 9.5% area reduction associated with a 14.0% contraction along the minor principal strain direction and a 4.8% dilation along the major principal strain direction.

The **superior maxillary dental alveolar** triangle **right** displayed marked shape deformation with a 40.1% contraction in posterior height (minor principal strain direction) and a 10.7% contraction in length (major principal strain direction), with a net area reduction of 46.5%. The **inferior maxillary dental alveolar** triangle **right** displayed a 15.3% reduction in dental arch length and a minimal contraction of 0.5% in the anterior dental alveolar height, with a net area reduction of 15.7%.

Both the right dental alveolar triangles showed 10% to 15% reduction of the dental arch length, with the anterior dental alveolar height remaining essentially unchanged and the posterior dental alveolar height showing marked reduction.

The net anterior maxillary bone surface of the patient was 4.5cm^2 smaller than the maxilla standard for surface area measured.

7.4 The Orbits

The orbits were defined by the following landmarks - optic foramen right, medial orbitale right, superior orbitale right, lateral orbitale right, opposite orbitale right, orbitale right, zygomatic corner right, optic foramen left, medial orbitale left, superior orbitale left, lateral orbitale left, opposite orbitale left, orbitale left and zygomatic corner left.

7.4.1 Individual osseous landmark analysis of the patient's orbits relative to the experimental reference orbit standard

Plots of the patient's orbits superimposed on plots of the reference orbits after least squares and repeated median alignment, with and without scaling, are given in Figures 7.5 (a) to (d). Examination of these plots reveal the following features:

(i) not all landmarks are identified. The missing landmarks are medial orbitale left and the left and right infraorbital foramen.

(ii) relative prominence of the nasal bone - the perspective is the comparison of the orbits with the experimental reference orbit standard. When the whole skull is used (see Section 7.7.1), it becomes apparent that it is not so much the prominence of the nasal bone, but the considerable inferior, posterior, lateral displacement of the orbital rim.

(iii) the lower orbital rims are displaced posteriorly relative to the experimental reference orbit standard (right and left).

(iv) the lateral aspect of the superior orbital roof is longer (superior orbitale to lateral orbitale longer (right and left)).

(v) more elongate nature of the orbits.

The differences between the patient and the experimental reference orbit standard are quantified in Tables 7.7 (a) to (d). All the measured landmarks, except optic foramen left, are significantly different at the 95% confidence interval using a $\chi^2(3)$ test in at least one of these Tables. Both the least squares and the repeated median alignment approaches display significant scale factors indicating that the patient's orbits are significantly smaller (5.6% and 7.0% smaller respectively) than the experimental orbits standard. These results are indicative of the considerable size and shape differences between the patient's orbits and the experimental standard.

The root-mean-square of the residuals for the least squares fits, with and without scaling, of the patient's orbits with the orbital reference standard were 6.60mm and 7.44mm respectively. The corresponding root-mean-square values for the male were 3.01mm and 3.70mm. Again, these results are indicative of the substantial differences between the patient's orbits and the experimental reference orbit standard.

7.4.2 Distance and angle analysis of the patient's orbits relative to the experimental reference orbit standard

The distance measurements 3 to 12, 26, and 27 and the angles 40 to 46 (Table 7.8) were used to describe the right orbital cone.

Consideration of these Z-scores revealed that the distance between medial orbitale right and superior orbitale right was significantly reduced (morr-sorr(3): Z = -3.11, difference = -8.0mm), with the distance between superior orbitale right and lateral orbitale right being significantly increased (sorr-lorr(4): Z = 3.37, difference = 8.0mm); while the other distances which completed the anterior border of the orbit showed no significant change in length.

Whilst the distances from the optic foramen right to the lateral orbitale right, optic foramen right to the opposite orbitale right, and the optic foramen right to orbitale right were significantly reduced (ofr-lorr(10): Z = -2.72, difference = -3.0mm; ofr-oorr(11): z = -2.99, difference = -6.1mm; orr-ofr(12): Z = -2.34, difference = -5.45), the distances from the optic foramen right to the medial orbitale right, and the optic foramen right to superior orbitale right, were non-significant.

These results imply that the landmarks, medial orbitale right and superior orbitale right are in the same plane as the standard, whereas lateral orbitale right, opposite orbitale right and orbitale right are more posteriorly positioned.

The height of the right orbit as measured from superior orbitale right to orbitale right and the breadth of the right orbit as measured from medial orbitale right to lateral orbitale right are not significantly different from the standard.

Consideration of the angles between the landmarks defining the right orbit revealed that only the angle at the superior orbitale right (morr-sorr-lorr(40): Z = -2.37, difference = -11.4°) was significant. This angle has decreased, relative to the standard, from being an obtuse angle to being almost a right angle. This factor, in combination with the distance measurements, medial orbitale right to superior orbitale right, superior orbitale right to lateral orbitale right, optic foramen right to lateral orbitale right, and optic foramen right to opposite orbitale right, suggested an infero-lateral-posterior displacement of the patient's right lateral orbital wall relative to the standard.

Similarly, the distance measurements 15 to 24, 28, and 29 and the angles 47 to 53 (Table 7.8) were used to describe the left orbital cone.

The distance superior orbitale left to lateral orbitale left was longer than the standard by 9.3mm and this result was comparable to that observed on the right. However, because the expected standard deviation based on landmark location error was larger on the left, this result was non-significant.

The other measurements which define the left orbit showed no significant difference in either length or angulation.

It should be noted that the measurement medial orbitale left could not be taken due holes in the reconstruction in this region as consequence of the thin bone in the region of the orbit. (In retrospect, the medial orbitale left is a key landmark, as demonstrated from the right orbital analysis, in determining the relative position of the patient's orbit to the standard. Perhaps an educated guess of this landmark would have highlighted the trend. Of course, its accuracy would not have been as good as other orbital landmarks). The absence of the medial orbitale left landmark has limited the number of measurements that can be made for the left orbit. In this case, the reason for absence of this landmark is as important as its measurement - the bone is abnormally thin in this region.

The breadth measurements, lateral orbitale right to lateral orbitale left, opposite orbitale right to opposite orbitale left, orbitale right to orbitale left, and optic foramen right to optic foramen left, indicated that the general separation of the orbits was reduced by approximately 5mm relative to the standard. The greatly reduced separation of the left and right superior orbitale landmarks (sorr-sorl(31): $Z = -3.79$, difference = -14.3mm) is a reflection of the shorter distance between medial orbitale right and superior orbitale right (and a probably similar shortness can be inferred on the left from the increased distance superior orbitale left to lateral orbitale left).

The reduced width of the frontal processes of the zygomas (lateral orbital walls), more so on the right (oorr-zcr(37): Z = -6.20, difference = -12.1mm; oorl-zcl(38): Z = -3.68, difference = -7.7mm), is indicative of a hypoplastic condition within these bones.

The distance nasion to sella is within normal variation (see Section 7.6 Cranium). The angle at the nasion, between the optic foramina is smaller, consistent with the reduction in the distance between the optic foramina by 5mm (ofr-n-ofl(54): Z = -5.04, difference = -6.9°) but the angle at sella, between the optic foramina is substantially larger than the standard (ofr-s-ofl(55): Z = 4.72. difference = 31.33°). These results suggested that the patient's optic foramina were more medio-posteriorly positioned relative to the experimental standard.

7.4.3 Strain analysis of the patient's orbits relative to the experimental reference orbit standard

In the case of strain analysis of the orbits, either triangles (reflecting the orbital surface) or tetrahedra (reflecting the orbital cavity) can be used to describe the shape changes between the standard and the patient.

The results of the triangular strain analysis are given in Table 7.9 while Figures 7.6 (a) to (c) show the matched orbital elements with their principal strains and strain directions.

Because of the possible error in the location of the left optic foramen of the experimental reference standard discussed in Section 6.7.3, a comparison of the patient's left optic cone with the standard is excluded. The analysis of the right optic cone is indicative of the methodology and type of results that can be expected.

The **anterior border of the right orbital cavity** can be described by three triangles: the right medial anterior orbital triangle (superior orbitale right, orbitale right, medial orbitale right (Δ 48, 54, 46)), the right central anterior orbital triangle (opposite orbitale right, superior orbitale right, orbitale right (Δ 52, 48, 54)), and the right lateral anterior orbital triangle (superior orbitale right, lateral orbitale right, opposite orbitale right (Δ 48, 50, 52)).

The **right medial anterior orbital** triangle exhibited marked shape change with a 39.1% reduction occurred in the minor principal strain direction, with a 9.8% enlargement in the major principal strain axis, the net area change being a contraction of 33.1%. Similarly, the **right central anterior orbital** triangle also demonstrated a large shape deformation, 27.2% reduction in the minor principal strain direction, with a 22.1% dilation along the major principal strain axis. This triangle showed a smaller area reduction of 11.2%. Likewise, the **right lateral anterior orbital** triangle demonstrated appreciable shape deformation with a slight contraction of 0.6% along the minor principal strain direction and a 33.4% expansion in the major principal strain direction leading to an overall area enlargement of 32.6%.

Close study of these triangles revealed that a clockwise movement of the principal strain directions had occurred with the right medial orbital triangle's major principal strain axis paralleling the line through one-seven o'clock, and the right central orbital triangle's major principal strain direction being collateral to the two-eight o'clock line, while the right lateral orbital triangle's principal strain direction approaches the three-nine o'clock line (Figure 7.6 (b)).

Scrutiny of the above findings suggest that, relative to the standard, the patient displays a small right medial anterior orbital triangle, a slightly smaller right central anterior orbital triangle, and a noticeably elongate right lateral anterior orbital triangle. This information combined with the directions of the major principal strain axes conveys an impression of the patient's right orbit being skewed infero-laterally.

The **surface of the cone of the right orbit** can be defined by five triangular elements - the right supero-medial orbital wall and roof (medial orbitale right, superior orbitale right, and optic foramen right (Δ 46, 48, 40)), the right supero-lateral orbital roof and wall (optic foramen right, superior orbitale right, and lateral orbitale right (Δ 40, 48, 50)), the right infero-lateral orbital wall (optic foramen right, lateral orbitale right, and opposite orbitale right (Δ 40, 50, 52)), the right lateral orbital floor (optic foramen right, opposite orbitale right, and orbitale right (Δ 40, 52, 54)), and the right infero-medial wall and floor (orbitale right, optic foramen right, and medial orbitale right (Δ 54, 40, 46)).

The results of the **right supero-medial orbital wall and roof** triangle reflect a large shape change with a contraction along the minor principal strain direction of 36.0% and a dilation along the major principal strain direction of 9.8%. These deformations contributed to a net area reduction of 29.8%. Similarly, the **right supero-lateral orbital roof and wall** exhibited a noticeable shape change, with a 4.7% contraction and a 30.4% dilation along the minor and major principal strain directions respectively, producing a 24.3% area enlargement in the patient relative to the standard. Consideration of the strain directions of both triangles suggest that the superior orbitale right is displaced more medially in the patient relative to the standard.

The **right infero-lateral orbital wall** of the cone exhibited moderate shape deformation with a 13.6% contraction and a 13.2% dilation along the minor and major principal strain directions respectively, resulting in a minimal area reduction of 2.1% By comparison, the **right lateral orbital floor** triangle demonstrated a large area reduction of 20.8%, but displayed only minimal shape change.

Analysis of the **right infero-medial orbital wall and floor** showed marked shape deformation with a reduction of 16.0% along the minor principal strain axis and a dilation of 20.0% along the major principal strain axis, with a negligible area increase of 0.8%.

The impression from the above analysis of the cone of the right orbit is of marked shape deformation of the patient relative to the standard.

The **anterior border of the left orbital cavity** can be described by three triangular elements - the left medial anterior orbital triangle (superior orbitale left, orbitale left, medial orbitale left (Δ 49, 55, 47)), the left central anterior orbital triangle (opposite orbitale left, superior orbitale left, orbitale left (Δ 53, 49, 55)) and the left lateral anterior orbital triangle (superior orbitale left, lateral orbitale left, opposite orbitale left (Δ 49, 51, 53)). As noted previously, the medial orbitale left could not be located, due to holes in the reconstruction in this region, so that only two of the left polygons of the patient were available for analysis.

The strain analysis of the **anterior border of the left orbital cavity** showed a similar pattern to that of the right, that is, the direction of the major principal strain directions show a change from an infero-lateral orientation to a more lateral orientation moving laterally.

Both the **left central and lateral anterior orbital** triangles show marked shape deformation with area changes of a contraction of 16.4% and an expansion of 22.7% respectively. The directions and magnitude of the principal strains give the impression of an infero-laterally skewed left orbit.

The right and left orbital cavities can be partitioned into tetrahedra suitable for volumetric strain analysis. Four landmarks on the orbital rim and the optic foramen were chosen to form the tetrahedra. It was possible to form superior and inferior orbital tetrahedra or medial and lateral tetrahedra, but the latter option was chosen, as the landmark medial orbitale left for the patient was unavailable. The two tetrahedra are, for each orbit, the medial orbital tetrahedron (optic foramen, medial orbitale, superior orbitale, orbitale), and the lateral orbital tetrahedron (optic foramen, superior orbitale, lateral orbitale, orbitale).

However, because of the possible error in the location of the left optic foramen of the experimental reference standard (discussed in Section 6.7.3), a comparison of the patient's left orbital cavity with the standard was excluded. The analysis of the right orbital cavity is indicative of the methodology and type of results that can be expected (Table 7.10).

Consideration of the **right lateral** tetrahedron in Figures 7.7 (a) and (b), (comprised of the four faces: right supero-lateral orbital roof and wall (Δ 40, 48, 50), right infero-lateral orbital wall and floor (Δ 40, 50, 54), right lateral anterior orbital plane (Δ 48, 50, 54) and right orbital height plane (Δ 48, 40, 54)) revealed that 17.9% contraction occurred essentially in the direction from right orbitale to the right supero-lateral orbital roof and wall (the minor principal strain direction marked in red), a slight contraction of 4.6% from the optic foramen to the lateral anterior orbital plane (the semi-major principal strain direction marked in green), and a large dilation of 30.4% approximately in the direction from right superior orbitale to the right lateral orbital wall (the major principal strain direction marked in purple). Although the patient's right lateral tetrahedron showed marked shape deformation relative to the standard, there was only a minimal change in comparative volume (+2.2%).

Analysis of the **right medial** tetrahedron, Figures 7.7 (a) and (c), comprised of right supero-medial orbital wall and roof (Δ 46, 48, 40), the right orbital height plane (Δ 48, 40, 54), the right medial anterior orbital plane (Δ 48, 46, 54) and right infero-medial orbital wall and floor (Δ 54, 40, 46), showed that a large contraction of 45.6% occurred in the minor principal strain direction, marked in red, which has a large orbital width component, a 8.2% contraction occurred in the semi-major principal strain direction (green) and a 23.8% expansion in the major principal strain direction (purple). Associated with the marked deformation of the right medial orbital tetrahedron is a large volume reduction of 38.2%.

The net volume reduction for the right orbit is 1.5ml (16.5%) which verifies the impression that the patient's right orbital cavity is smaller than the standard. There was a height reduction between the supero-lateral orbital roof and wall and orbitale, a slightly posterior displacement of the lateral anterior orbital plane and an increased separation of the lateral orbital wall and the superior orbitale, leading to an impression of elongation of the patient's right orbit in an infero-lateral-posterior direction.

7.5 The Zygomas

The right and left zygomas were defined by the following landmarks - orbitale right and left, opposite orbitale right and left, lateral orbitale right and left, zygomatic corner right and left, external auditory meatus right and left and zygomaxillare right and left.

The patient's CT radiography revealed that the left and right zygomas were severely hypoplastic (Figures 7.8 (a) and (b)). Both zygomatic arches were absent and the left and right bodies of the zygoma were dysplastic (more so on the right).

7.5.1 Individual landmark analysis of the patient's zygoma relative to the experimental reference zygoma standard

Figures 7.9 (a) to (d) show the plots of the patient's zygoma relative to plots of the reference standards after the four alignment procedures (that is, least squares and repeated median, with and without scaling). It should be noted that for the patient, the line joining the zygomatic corner to the external auditory meatus is indicative only, as the zygomatic arches are absent. It is apparent from the plots that some of the landmarks used to define the zygoma have not been identified from the patient's radiography. The non-identified landmarks are the zygomaxillare right and left, infraorbital foramen right and left, and articular eminence right and left. The reason for the absence of the landmarks zygomaxillare right and left are given in Section 7.5.2. Infraorbital foramen right and left were unable to be precisely located due to the thin bone surrounding these foramina, giving rise to holes in the three dimensional CT reconstructions. In the case of the articular eminence right and left, the zygomatic process of the temporal bone was bilaterally absent (Figures 7.8 (a) and (b)).

Generally, the impression gained from inspection of Figures 7.9 (a) to (d) is that the patient relative to the standard showed: marked narrowing of the frontal process of the zygomatic bone (right and left, essential the lateral orbital walls); orbitale right and left are located more posteriorly; the right and left frontal processes of the zygomas (lateral orbital walls) are displaced infero-medially-posteriorly (non-scaled) and infero-posteriorly (scaled); zygomatic corners right and left are positioned more anteriorly and the external auditory meatus (right and left) and the mastoid tips are located more supero-anteriorly (non-scaled only). Overall, the patient's right zygoma appears to be more affected than the left, especially in the region of the frontal process of the zygoma.

Tables 7.11 (a) to (d) quantify the difference between the patient and the reference standard. Comparison of the non-scaled results with the scaled results revealed that the osseous landmarks lateral orbitale left and opposite orbitale left were significant after non-scaled alignment, implying that the observed significance of these landmarks is size related (that is, lack of development). In contrast, all the other measured landmarks with the exception of orbitale left (repeated median only) were significantly different at the 95% confidence interval using a $\chi^2(3)$ test for both the scaled and non-scaled least squares and repeated median alignments. Significant scale factor Z-scores (-3.7 least squares and -4.5 repeated median) were calculated, indicating that the patient's zygomas are considerably smaller (9.1% least squares and 11.2% repeated median) than the experimental reference standard. These results, when viewed in conjunction with the plots, suggest that the patient's right zygoma is more affected (less developed and differently shaped) than their left zygoma.

The root-mean-square of the residuals for the least squares fits, with and without scaling, of the patient's zygomas with the zygoma reference standard were 7.72mm and 9.17mm respectively. The corresponding root-mean-square of the residuals for the male were 2.52mm and 3.72mm. Once again these residuals are indicative of the substantial differences between the patient's zygomas and the experimental zygoma reference standard.

7.5.2 Distance and angle analysis of the patient's zygoma relative to the experimental reference zygoma standard

For this patient, it should be noted that both zygomatic arches (that is, the zygomatic process of the temporal bone) are absent, which implies by definition that the articular eminence (right and left) are also absent (Figures

7.8 (a) and (b)). However, the "zygomatic arch" length can still be considered to be the distance between the zygomatic corner and the external auditory meatus.

The distance measurements 1 to 7, and 15 to 18 and the angles 30, and 32 to 37 (Table 7.12) were used to describe the right zygoma.

The Z-scores, related to the analysis of the patient's right zygoma, revealed that the distance defined by lateral orbitale right to zygomatic corner right was significantly smaller than the standard (lorr-zcr(1): Z = -5.41, difference = -6.3mm). While all the other distances used to determine the outline of the right zygomatic bone were also smaller, they were non-significant. The right "zygomatic arch" length was also non-significant, even though, as noted in the last section on the orbit, there was infero-lateral-posterior displacement of the orbit. This can be attributed to the significantly reduced thickness of the right frontal process of the zygoma (lateral orbital wall) (effectively moving the zygomatic corner right more anteriorly) and also to some extent the displacement of the right external auditory meatus (see Figures 7.9 (c) and (d)).

To obtain information on the height of the zygoma, measurements were taken from the zygomaxillare right to opposite orbitale right, zygomaxillare right to zygomatic corner right and zygomaxillare right to external auditory meatus right. Unfortunately, the landmarks zygomaxillare right and left were not measured, as appropriate views were unavailable from the standard six X-axis rotations selected. As mentioned previously, stereo pairs were selected that displayed the maximum number of landmarks for a particular orientation. Again, with hindsight, it would have been better to have requested non-standard stereo pairs appropriate for the identification of the landmarks zygomaxillare right and left.

The distance representing the thickness of the right frontal process of the zygoma (lateral orbital wall) was significantly thinner than the standard (zcr-oorr(18): Z = -6.20, difference = -12.1mm). This result is indicative of a hypoplastic condition within this region.

The angular measurement, lateral orbitale right to zygomatic corner right to external auditory meatus right, was significantly reduced and approached a right angle compared with the more obtuse angle observed in the standard (lorr-zcr-eamr(32): Z = -4.96, difference = -29.1°). The angle, opposite orbitale right to lateral orbitale right to zygomatic corner right, was also significantly smaller than the standard (oorr-lorr-zcr(37): Z = -7.02, difference = -47.0°). These two angular results also relate to the right frontal process of the zygoma (lateral orbital wall) and are further indicative of hypoplastic condition within this region. The remaining angles either could not be measured because of missing landmarks or were found to be non-significant.

The distance measurements 8 to 14, and 19 to 22 and the angles 31, and 38 to 43 (Table 7.12) were used to describe the left zygoma.

Analysis of the patient's left zygoma's Z-scores showed that the distance defined by zygomatic corner left to the external auditory meatus left ("zygomatic arch" length) was significantly shorter than the standard (zcl-eaml(9): Z = -3.39, difference = -9.0mm), while the other distances used to determine the outline of the zygomatic bone were all non-significant.

As noted above in relation to the measurement of zygomaxillare right and left, appropriate views were unavailable to allow satisfactory location of these landmarks. Hence measurements which include these landmarks, for example, height measurements of the left zygoma, are absent.

The distance zygomatic corner left to opposite orbitale left, which is a measure of the thickness of the left frontal process of the zygoma (lateral orbital wall), was significantly thinner than the standard (zcl-oorl(22): Z = -3.68, difference = -7.7mm). Once again, this is indicative of a hypoplastic condition within the frontal process of the left zygoma.

The angular measurement lateral orbitale left to zygomatic corner left to external auditory meatus left was significantly smaller than the standard (lorl-zcl-eaml(38): Z = -2.25, difference = -21.3°). The remaining angles were non-significant or could not be determined because of missing landmarks.

The ear and condylar separations are both 101mm, but while the ear separation was significantly greater than the standard (eamr-eaml(25): Z = 2.19, difference = 6.4mm), the condylar separation was not significantly different. It was expected that the ear separation would be several millimetres smaller than the condylar separation, because the lateral poles were used to define the position of the condylar heads. This indicates that there has been some abnormal development around the ears - a finding which is consistent with the significance found for the individual Z-scores for the left and right external auditory meatus (Section 7.5.1).

The significantly reduced separation of the zygomatic corners (zcr-zcl(26): Z = -10.40, difference = -20.2mm) and the shape of the orbits (as discussed in Section 7.4) would indicate that the zygomatic corners are positioned more medio-posteriorly than the standard, resulting in the temporal fossae and the frontal processes of the zygomatic bones being almost flush for this patient (Figure 7.8 (b)).

The root-mean-square of the Z-scores for all measured distances and angles is 3.6. This indicates significant variance of the patient's zygomatic bones from the standard.

7.5.3 Strain analysis of the patient's zygoma relative to the experimental reference zygoma standard

The dysplastic nature of the patient's zygomas has resulted in fewer landmarks being present and/or indentifiable, so that only two triangles were matched to those of the zygoma strain standard. These were the triangles that effectively represented the left (Δ 53, 51, 63) and right (Δ 52, 50, 62) frontal processes of the zygomas (effectively the left and right lateral orbital walls). The results of the analysis are given in Table 7.13 and the triangles with their principal strains and strain directions are shown in Figures 7.10 (a) and (b).

On the left (Figure 7.10 (a)), there was size reduction associated with both principal strains 39.6% and 6.2% reduction in the minor and major principal strains respectively. The net area change was a 43.3% reduction.

The magnitude and direction (see Figure 7.10 (b)) of the right minor principal strain indicated a 69.0% reduction in the width of the frontal process of the zygoma (lateral orbital wall), while the major principal strain (4.8%) indicated no significant change in the height. The net effect was a 67.5% reduction in the area of the region enclosed by the landmarks defining the triangle.

The large area reductions are consistent with a marked hypoplastic condition within the frontal process of the zygoma (lateral orbital walls), more so on the right which is consistent with an incomplete Tessier type 8 cleft. This conclusion is supported by the three dimensional CT reconstructions shown in Figures 7.8 (a) and (b).

7.6 The Cranium

The cranium was defined by the following landmarks - sella, nasion, glabella, vertex, opisthocranion, opisthion, mastoid tip left, mastoid tip right, basion, external auditory meatus left, external auditory meatus right, bregma, zygomatic frontal left, zygomatic frontal right, foramen magnum breadth left and foramen magnum breadth right.

7.6.1 Individual landmark analysis of the patient's cranium relative to the experimental reference cranium standard

The plots of the patient's cranium relative to the plots of the experimental reference cranium standard after the four alignment procedures are shown in Figures 7.11 (a) and (d). All the landmarks used to create a standard cranium were also identified for the patient.

The visual impression gained from examination of these plots was that the patient, relative to the standard, had a slightly larger skull height, but the other major skull dimensions of breadth and length were similar. The foramen magnum was also similarly positioned. The major observed differences involve the patient's forehead, which gave the appearance of being steeper (nasion to glabella, sella to nasion to glabella) and underdeveloped laterally (as measured from glabella to zygomatic frontal left and right).

The differences between the patient and the standard are quantified in Tables 7.14 (a) to (d). It is of interest to note that opisthion and the mastoid tip left are significantly different when least squares scaling is invoked, but non-significant when least squares scaling is inhibited (and non-significant for both scaled and non-scaled repeated median alignments). The landmarks nasion, glabella, vertex, opisthocranion, zygomatic frontal left and right, and external auditory meatus left (least squares only) were all significantly different at the

95% confidence interval using a $\chi^2(3)$ test for both the scaled and non-scaled repeated median and least squares alignments. The observed significance for these landmarks can be attributed to both a size and a shape component.

The scale factors, although non-significant in terms of population variance, indicate for both the least squares and repeated median alignments that the patient's cranium was larger than the standard (3.8% and 3.3% respectively).

The root-mean-square of the residuals for the least squares fits, with and without scaling of the patient's cranium with the cranium reference standard, were 11.96mm and 12.26mm respectively and this indicative of the large variation in landmark positions of the patient's cranium relative to the standard's cranium. The corresponding root-mean-square residuals for the male were 4.47mm and 5.66mm.

7.6.2 Distance and angle analysis of the patient's cranium relative to the experimental reference cranium standard

While more landmarks were defined to describe the limits of the anterior and middle cranial fossae, the population size for the standard was too small to enable these new landmarks to be utilized for the purpose of statistical comparison. In one of the four skulls comprising the standard, the three dimensional CT reconstruction of the cranial base showed holes due to inappropriate thresholding and the other three had been inappropriately sub-regioned (as discussed in Section 3.2). The cranial base analysis was, therefore, confined to the cephalometric landmarks of - sella, nasion, basion, opisthion, glabella, with only the foramen magnum breadth right and foramen magnum breadth left being additionally included.

Traditionally, the cranial base has been defined by the distance measurements - sella to nasion, sella to basion, nasion to basion, and the cranial base angle nasion to sella to basion. Garn et al., (1984) also included the measurements - sella to glabella, sella to opisthion, glabella to opisthion, in their pattern profile analysis of skull dimensions. A comparison of the Z-scores for these measurements revealed that there was no significant difference between the patient and the standard.

The distance measurements 13-16, 27, 28, and the angles 44, 45, and 48-51 (Table 7.15) were used to describe the foramen magnum.

Distance measurements opisthion to foramen magnum breadth right, foramen magnum breadth right to basion, basion to foramen magnum breadth left, and foramen magnum breadth left to opisthion, which describe the perimeter of the foramen magnum, were all non-significant. The length (basion to opisthion) and breadth (foramen magnum breadth right to foramen magnum breadth left) measurements of the foramen magnum were also non-significant. The angular measurements opisthion to foramen magnum breadth right to basion, opisthion to foramen magnum breadth left to basion, foramen magnum breadth right to opisthion to foramen magnum breadth left, foramen magnum breadth right to basion to foramen magnum breadth left, and opisthion to foramen magnum breadth right to basion, were all non-significant. Only the angle opisthocranion to opisthion to basion was found to be significantly larger than the standard (op-o-ba(44): $Z = 3.53$, difference = 9.2°). This result suggests either that the external occipital protuberance is more prominent in this patient, or that the patient's foramen magnum is tilted inferiorly at basion relative to the standard.

The calvaria is classically defined as the dome-like superior portion of the cranium, composed of the superior portions of the frontal, parietal, and occipital bones. However, the lack of non-sagittal landmarks in the true region of the calvaria result in a poor representation of this structure. Consequently, in the present investigation, the definition of the calvaria was extended to include the following landmarks - nasion, glabella, bregma, vertex, lambda, opisthocranion, mastoid tip right, mastoid tip left, opisthion, external auditory meatus right, external auditory meatus left, zygomatic frontal right, zygomatic frontal left and by the distance measurements 2-12, 18-26, and the angles 39-43, 46, 47, and 52 (Table 7.15).

It should be noted that both lambda and bregma could only be defined in one of the four skulls used to create the standard. Thus, there exists no population statistics relative to distances and angles which involve these landmarks.

Analysis of the Z-scores defining the calvaria revealed that the distances, nasion to glabella (n-g(2): Z = 4.71, difference = 18.0mm), glabella to zygomatic frontal right (g-zfr(4): Z = 3.29, difference = 15.3mm), opisthocranion to opisthion (op-o(10): Z = 2.75, difference = 16.6mm), opisthocranion to mastoid tip left (op-mtl(12): Z = 2.34, difference = 18.7mm), external auditory meatus right to vertex (eamr-v(20): Z = 2.70, difference = 13.3mm), were all significantly larger than the standard, while the external auditory meatus right to zygomatic frontal right (eamr-zfr(19): Z = -5.45, difference = -24.6mm) and external auditory meatus left to zygomatic frontal left (eaml-zfl(22): Z = -3.88, difference = -22.3mm) were both significantly smaller. The remaining distance measurements were non-significant.

With respect to the distance measurement of nasion to glabella (n-g(2): Z = 4.71, difference = 18.0mm), a large positive Z-score was recorded for the patient. This result can be attributed to the patient's forehead describing a large radius of curvature, making the position of glabella more variable. Additionally, the angle sella to nasion to glabella was significantly smaller than the standard (s-n-g(39): Z = -2.00, difference = -16.1°) and reinforces the impression of a broad, flat forehead.

Significantly smaller distances involving the external auditory meatus right to zygomatic frontal right (eamr-zfr(19): Z = -5.45, difference = -24.6mm) and the external auditory meatus left to zygomatic frontal left (eaml-zfl(22): Z = -3.88, difference = -22.3mm), were observed and are evidence of an hypoplastic condition in the region of the temporal fossa.

It is worth mentioning, however, that the distance external auditory meatus left to the vertex (difference = 11.9mm), while non-significant, displays a similar difference to the equivalent significant distance on the right. These results, together with the distance measurements, basion to vertex (ba-v(37): Z = 2.00, difference = 14.0mm), and sella to vertex (s-v(35): Z = 5.43, difference = 19.3mm), suggest that the patient has an increased cranial height relative to the standard. The breadth measurements external auditory meatus right to external auditory meatus left and mastoid tip right to mastoid tip left were both significantly greater than the standard (eamr-eaml(25): Z = 2.19, difference = 6.4mm; mtr-mtl(24): Z = 2.03, difference = 8.4mm). As mentioned in Section 7.5.2, these results are indicative of abnormal development around the ears, rather than, perhaps, a true increase in breadth.

Cranial length as measured from glabella to opisthocranion was non-significant. All other angles for the calvaria were non-significant.

7.6.3 Strain analysis of the patient's cranium relative to the experimental reference cranium standard

Strain analysis of the cranial base was not attempted due to insufficient landmarks (as outlined in Section 7.6.2). Therefore, strain analysis of the cranium was confined to the description of the calvaria and these results are reported in Table 7.16 and displayed in Figures 7.12 (a) to (c). The triangles used to describe the calvaria are relatively large due to the scarcity of distinguishing features that can be reproducibly located (due to the difficulty in defining reproducible landmarks in this ovoid region). Consequently, the triangles only give a rough guide to the position of the surface between the vertices. They still, however, contain all the shape information regarding the displacement of the landmarks between the patient and the standard.

The most striking feature of the strain analysis is the very symmetric orientation of the principal strain directions about the mid-sagittal plane (Figure 7.12 (c)).

The calvaria was defined by eight triangles - the right frontal (glabella, vertex, zygomatic frontal right (Δ 3, 4, 64)), the right anterior temporal parietal (vertex, external auditory meatus right, zygomatic frontal right (Δ 4, 8, 64)), the right posterior temporal parietal (vertex, external auditory meatus right, opisthocranion (Δ 4, 8, 5)), the right occipital (opisthocranion, external auditory meatus right, opisthion (Δ 5, 8, 6)), the left frontal (glabella, vertex, zygomatic frontal left (Δ 3, 4, 65)), the left anterior temporal parietal (vertex, external auditory meatus left, zygomatic frontal left (Δ 4, 7, 65)), the left posterior temporal parietal (vertex, external auditory meatus left, opisthocranion (Δ 4, 7, 5)), and the left occipital (opisthocranion, external auditory meatus left, opisthion (Δ 5, 7, 6)).

The **right and left frontal** triangles showed a similar pattern (their principal strain directions were symmetric about the mid-sagittal plane), but their magnitudes revealed that the right was more affected than the left. The large area increases (57.4% and 29.5% for the right and left respectively) between the patient and the standard were essentially a result of the displacement of the landmarks zygomatic frontal right and left infero-laterally, which reflects an hypoplastic condition in the region of the anterior temporal fossae (that is, sphenoid bone, the frontal process of the zygoma and the zygomatic process of the frontal bone).

The **right and left anterior and posterior temporal parietal** triangles show contractions in their minor principal strain directions all parallel to the line joining the vertex to opisthocranion, with contractions ranging from 10.9% to 27.6%. In the major principal strain directions, dilations in the range of 16.4% to 31.4% were found. Larger values of contraction and dilation were found for the right. Although the triangles displayed large shape differences, the area differences between the patient and the standard were minimal, that is, between 6.4% decrease and 5.6% increase.

Both right and left occipital triangles reflect the superior displacement of the patient's opisthocranion relative to the standard. The resulting area increases were 27.1% on the right and 34.3% on the left.

The net area difference between the patient and the standard for all the triangles describing the calvaria was 3,470.8 mm^2.

7.7 The Skull

The skull was defined by including all landmarks of the mandible, maxilla, orbits, zygomas, and cranium and additionally the cranial base landmark numbers 91 to 106.

7.7.1 Individual osseous landmark analysis of the patient's skull relative to the experimental reference skull standard

In this section, it is important to note that the repeated median and least squares skull standards were not merely a collection of the individual bone standards used in the previous sections, but were generated by using all available landmarks defined for this study (as described in Section 6.4).

The plots of the patient's skull superimposed on those of the experimental skull standards after least squares and repeated median alignment with and without scaling are given in Figures 7.13 (a) to (d). The essential difference between these plots and those of the individual bone analyses is that the plots shown in Figure 7.13 allow the actual spatial relationships of the bones to each other to be visualised. In this way, additional information concerning the orientation and position of neighbouring bones can be gained, together with an overall impression of the shape and size differences between the patient and the standard.

A description of the non-scaled plots is presented first, as this represents a direct comparison of the patient relative to the standard.

From inspection of Figures 7.13 (c) and (d) the following key features were noted:

(i) Mandible

- the entire mandible is much smaller than the standard
- the gonial angle bilaterally is more obtuse
- the receding chin
- the patient's condylar heads are more antero-superiorly and medially positioned
- narrower dental arch
- larger bigonial breadth
- the condylar and coronoid processes are less developed on the left (the equivalent measurements on the patient's right can not be assessed because the osseous landmark coronoid notch right could not be identified (see Section 7.2.2))
- the whole mandible appears to be rotated, moving the chin upwards and backwards

(ii) Maxilla

- the entire maxilla is smaller than the standard
- anterior face height almost identical to that of the standard
- lateral face heights (right and left), as measured between orbitale and upper molar, are reduced
- the dental arch depth, breadth and width are reduced
- the angle between the nasal bone and the frontal bone is almost 160° (most obvious from the left 45° plot)
- the nasal tip appears more prominent, the landmarks nasal breadth left and right are more posteriorly and slightly superiorly displaced, leading to an overall appearance of a more retroclined nasal aperture
- the nasal line or hard palate (anterior nasal spine to posterior nasal spine) of the patient and the standard are essentially parallel and almost superimposed
- the patient's nasal bone (nasion to nasale) is almost superimposed upon the standards
- the occlusal plane line of the standard parallels the patient's dento-alveolar line

  (However, the observed difference in the dento-alveolar line orientations between the patient and the standard is most likely a reflection of the absence of central incisors in the skulls used to create the standard. The loss of dento-alveolar bone in this region has displaced the position of prosthion in the standard in a superior posterior direction.)

(iii) Orbits

- the lower orbital rim and the lateral orbital wall are displaced in an infero-lateral posterior direction bilaterally relative to the standard

- the distance between superior orbitale and lateral orbitale is increased bilaterally
- when combined, these two effects give the patient's orbits a more elongated appearance compared to the orbit standard
- the relationship of medial orbitale to nasion is almost identical to that of the standard, but the position of the remaining landmarks defining the patient's orbit become increasingly infero-laterally posteriorly displaced the more laterally the orbit is transversed
- the overall impression gained is that the patient's orbits are elongated in an infero-lateral posterior direction relative to the standard

(iv) Zygomas

- note the zygomatic arches are absent in the patient but the upper borders appear in Figures 7.13 (a) to (d) for indicative purposes only
- the patient's zygomas are much smaller, differently shaped, and displaced in an infero-lateral direction

(v) Calvaria

- the calvaria is approximately the same length and similar breadth, but the height of the patient's calvaria is slightly increased
- the major differences are in the region of the forehead, with the patient being flatter in the midline and the infero-posterior displacement of the landmarks zygomatic frontal left and right coupled with the infero-lateral inclination of the supra-orbital roof from superior orbitale are indicative of lack of supra-orbital ridge development bilaterally
- external auditory meatus (right and left) and mastoid tip (right and left) are slightly more anteriorly positioned

- foramen magnum similarly positioned (repeated median and least squares) and oriented (repeated median only)

Scaled comparison

Both the repeated median and the least squares scale factors indicated that the patient was smaller than the standard - but of interest is the degree of difference between the estimates - the repeated median estimate was almost significant (8.6% smaller, Z = -1.88) while the least squares estimate was well within normal variance (3.4% smaller, Z = -0.77). All bones except the cranium were found to be smaller than their corresponding bone standard and while the least squares scale factor for the skull is influenced by all landmarks, the repeated median scale factor estimate for the skull is relatively unaffected by the slightly larger cranium. Therefore the repeated median estimate was more consistent with the estimate for the individual components in the facial complex.

The patient's skull is smaller than the standard, so when scaling is applied to the alignment procedure, the patient's skull is enlarged. In this way, individual features of the patient which were larger than the standard (such as the calvaria) on the non-scaled plots become even larger in comparison with the standard when scaling is introduced. While the values of the d/σ-scores for many landmarks were reduced after scaled alignment, the vast majority remained significant, alluding to not only a size difference between the patient and the standard but also substantial shape differences, especially in the regions of the angle of the mandible, chin, orbits and zygomas.

The individual landmark coordinate analysis for fitting with and without scaling indicated that all of the measured osseous landmarks, except for the mid-sagittal cranial base landmarks (nasion non-scaled only), foramen magnum and hard palate, were significantly different at the 95% confidence interval using a $\chi^2(3)$ test in a least one of Tables 7.17 (a) to (d). The plots and the residuals give the direction and magnitude of the differences and are indicative of the scope of the abnormality.

The root-mean-square residuals after least squares alignment were of the order of 10mm, 11.27mm with scaling and 11.43mm without scaling. The scaled fit only marginally improves the fit and this indicates that not only is there a size difference but a substantial shape difference between the patient and the experimental reference standard.

7.7.2 Distance and angle analysis of the patient's skull relative to the experimental reference skull standard

The distance and angle analysis of the patient's skull against the experimental reference standard is identical to that for the individual bone distance and angle analysis reported in Sections 7.2 to 7.6. The standards are the same regardless of how the distances and angles are grouped.

7.7.3 Strain analysis of the patient's skull relative to the experimental reference skull standard

A strain analysis of the patient's skull against the average coordinate data of the experimental reference standard showed similar results to those presented in Sections 7.2 to 7.6 and will not be discussed further.

7.8 Comparison Of Analysis Techniques With Qualitative Description

Table 7.18 lists the key skeletal features of Treacher Collins Syndrome described by Gorlin et al. (1976), together with a summary of the corresponding quantitative results found for the patient using each of the analysis techniques. Additionally, this table highlights and enables ready comparison of the information obtainable using each of the analysis techniques.

Table 7.18 **Comparison of analysis techniques with the qualitative description of Treacher Collins Syndrome**

Skeletal Feature	Individual Osseous Landmark		Distance and Angle	Strain
Mandible	**Bone Standard**	**Skull Standard**		
Hypoplastic	Scale factor indicates patient's mandible is smaller than the standard (5.8% least squares, 8.4% repeated median) but not outside the normal population variance. Superimposed wire frame diagrams show, and residuals quantify, · smaller ramus height bilaterally · smaller mandibular body length bilaterally · less developed condylar process	Similar to bone standard comparison but includes effects of position and orientation relative to the rest of the patient's skull. The alignment of the patient relative to the standard accentuates gonial angle and chin differences.	Most distance Z-scores are negative. Fifteen of the distances are significantly smaller than those of the standard · total mandibular length (left and right)* · posterior ramus height (left and right) · gonion right to coronoid tip right · condylion left to coronoid notch left · condylion left to coronoid tip left · lower border of the mandible (left and right)* · dental arch length (left) · dental arch breadth	The external surface of the patient's mandible is 8.8cm^2 smaller than the standard based on the measured triangles in common between the patient and the standard. Individual triangles quantify the extent of hypoplasia in different parts of the mandible (seven triangles showed area decreases, two showed marginal area increases).

* Both gnathion and pogonion were used to define the chin giving another set of measurements

Skeletal Feature	Individual Osseous Landmark		Distance and Angle	Strain
Mandible	Bone Standard	Skull Standard		
Angle of mandible more obtuse	Evident from plots. Landmarks defining the angle differ in position significantly.	Appears more evident than for alignment of only the patient and standard mandibles due to orientation.	Significantly more obtuse (10-14° larger).	Represented by the two triangles sharing the edge gonion left to external oblique point left. The principal directions show dilations in a direction to increase the angle of the mandible.
Ramus may be deficient	The smaller size of the rami are evident from the plots. Landmarks defining the rami differ in position significantly.	The smaller size of the rami are evident from the plots. Landmarks defining the rami differ in position significantly.	Both right and left posterior ramus heights are smaller (12.6mm and 13.0mm respectively).	Left anterior, middle and posterior ramus triangles show increasing underdevelopment from anterior to posterior (that is, 3.1% to 18.8% to 49.0% area decreases).
Coronoid and condylar processes may be flat or aplastic.	Evident from plots. Landmarks differ in position significantly.	Evident from plots. Landmarks differ in position significantly.	A smaller, flatter condylar region is indicated by: · smaller condylion left to coronoid notch left (11.8mm), · smaller condylion left to coronoid tip left (10.2mm), · increased gonion left to condylion left to coronoid notch left angle (10.9°).	The left posterior ramus triangle shows a 49.0% area reduction predominantly due to a 45.0% reduction parallel to a line joining the condylar head left to the coronoid notch left resulting in a flatter condylar process.

Skeletal Feature Mandible	Individual Osseous Landmark Bone Standard	Individual Osseous Landmark Skull Standard	Distance and Angle	Strain
Receding chin	The patient's chin has a similar shape to the standard - evident from scaled plots. The chin appears receding on non-scaled plots due to the smaller mandible size and the increased ramus to body angle.	The patient's chin is very supero-posteriorly displaced relative to the skull standard. For example, the gnathion is displaced by 21mm after repeated median alignment without scaling.	Lower face height measurements are non-significant however lower border of the mandible significantly reduced in length.	Receding chin apparent due to reduction in size of the triangles representing the body of the mandible and the ramus.
Undersurface of the body of the mandible has a concave appearance.	Not measured. Need to define suitable syndrome specific landmarks to measure the undersurface of the mandible.	Not measured. Need to define suitable syndrome specific landmarks to measure the undersurface of the mandible.	Not measured. Need to define suitable syndrome specific landmarks to measure the undersurface of the mandible.	Not measured. Need to define suitable syndrome specific landmarks to measure the undersurface of the mandible.

Skeletal Feature	Individual Osseous Landmark		Distance and Angle	Strain
Mandible	Bone Standard	Skull Standard		
Other quantified results	Plots and d/σ-scores indicate that the following are also significant relative to bone standard: · Narrower dental arch, · Coronoid tips more laterally displaced, · Larger bigonial breadth. The root mean square residual is 8.9mm indicative of considerable shape and size differences.	Plots and d/σ-scores indicate that the following are also significant relative to skull standard: · Narrower dental arch, · Coronoid tips more laterally displaced, · Larger bigonial breadth. The condylar heads are positioned more antero-superiorly and medially. The patient's mandible relative to the skull standard is rotated to move the chin posteriorly and superiorly.	Arch breadth was 8.5mm smaller and arch angle was 5.5° smaller. Left dental arch length smaller than right and significantly smaller than standard (7.8mm). Coronoid tip width was significantly increased (9.7mm). Larger bigonial breadth (5mm) but this was not significant. Lower border angle as measured from gonion right to gnathion to gonion left is significantly increased (18.7°). Angular relationship of the mandibular plane to the cranial base and the hard palate are also significantly more obtuse (16-21° larger). The angle of the left anterior border of the ramus to the dental alveolar crest in the molar region is also significantly more obtuse.	Contraction along left dental arch length of 18.3% but no significant change in right dental arch length. Area reduction of the left body of the mandible of 25.3%. Contraction along right mandible body length of 25.5%.

Skeletal Feature	Individual Osseous Landmark		Distance and Angle	Strain
Maxilla	Bone Standard	Skull Standard		
Poor Development	Scale factors indicate that the patient's maxilla is smaller than the standard (8.4% least squares and 4.7% repeated median) but not outside the population variance. Absence of landmarks due to thinner or poorer quality bone.	Similar to bone standard comparison but includes effects of position and orientation relative to the rest of the patient's skull. The orientation of the patient relative to the standard emphasises that the patient's lateral face heights are substantially reduced relative to the standard.	Of the 21 determined distances, 16 were smaller (5 significantly). Lateral maxillary height was significantly reduced (by 16.2mm, right and by 11.0mm, left).	The external surface of the patient's maxilla is $4.5cm^2$ smaller than the standard based on the measured triangles in common between the patient and the standard. The individual triangles quantify the magnitude and direction of hypoplasia in the different parts of the maxilla. Of the 7 triangles relating specifically to the maxilla, six show area decreases and one a marginal area increase. All the minor principal strain directions for the maxilla were approximately vertical indicating the smaller height of the patient's maxilla relative to the standard, and this was more so laterally.
High arched palate, cleft in 30% of cases	Not measured. Need to define suitable landmarks.	Not measured. Need to define suitable landmarks.	Not measured. Need to define suitable landmarks.	Not measured. Need to define suitable landmarks.

Skeletal Feature	Individual Osseous Landmark		Distance and Angle	Strain
Maxilla	Bone Standard	Skull Standard		
Dental Malocclusion	Superimposed wire frame diagrams show, and residuals quantify, - • reduced dental arch breadth • reduced dental arch angle	In this case the orientation of the hard palate is more closely aligned with that of the standard and thus the dental arch appears more steeply inclined. The dental arch depth, breadth and width are all reduced relative to the standard.	Significantly reduced: • dental arch length bilaterally by 6.2mm, right and by 5.2mm, left. • dental arch breadth by 15.9mm. • dental arch angle by 16.1°.	Contraction along the right dental arch length of 15.3%. Both right and left dental alveolar complexes showed area reductions of $1.7cm^2$ and $0.8cm^2$ respectively.
Nasofrontal angle usually obliterated and bridge of nose raised	Visible in wire frame models.	Visible in wire frame models.	Significantly more obtuse (155.6°) than standard (138.4°).	No triangles defined for this region.

Maxilla	Bone Standard	Skull Standard		
Other quantified results	The landmarks that define the anterior face height do not differ significantly in position from the standard, however the landmarks that define the lateral face heights (orbitale and upper molar point) differ significantly in a direction such that the face height is smaller. The nasal aperture (defined by nasale, nasal breadth points, right and left, and anterior nasal spine) appears more prominent at nasale but medio-posteriorly displaced at the nasal breadth point right giving the structures defining the nasal complex a more beaked appearance. The landmark posterior nasal spine is significantly displaced from the standard (possibly due to preferential anterior alignment due to more anterior landmarks being identified). The angle between nasion, medial orbitale right and orbitale right appears more acute for the patient than for the standard, indicating a change in angulation of the right lower orbital rim. The root mean square residual is 7.0mm and this is indicative of the extent of the shape and size differences.	The patient's nasal line is closely aligned with that of the standard, emphasising the reduction in lateral face height. This suggests that the nasal line is less affected, and other features of the maxilla more affected, by the syndrome than the individual bone analysis might have indicated. This is in accord with the lateral clefting/hypoplastic nature of this syndrome. When the patient's skull is aligned with the standard's skull the position of the posterior nasal spine is virtually superimposed on that of the standard.	The height of the anterior maxilla was within normal variance, as were the nasal cavity lengths, and the nasopharynx measurements.	Triangles representing the lateral aspect of the maxilla (right and left) demonstrated contractions of 39.3% and 26.9% respectively, indicating reduced height between the orbits and the upper molar teeth. Of the 5 triangles which specifically describe the nasal cavity, 3 show area increases and 2 decreases. The nasal aperture showed a net area decrease of $0.6cm^2$.

Skeletal Feature	Individual Osseous Landmark		Distance and Angle	Strain
Orbits	Bone Standard	Skull Standard		
Lower margin of the orbit noted to be defective	The patient's left and right lower orbital rims are displaced posteriorly relative to the standard.	The orientation of the orbits after alignment shows that the lower orbital rims and the frontal processes of the zygomas (lateral orbital walls) are displaced in an infero-lateral posterior direction bilaterally relative to the standard.	The distances from optic foramen right to lateral orbitale right, opposite orbitale right and orbitale right are all significantly reduced (3.0mm, 6.1mm, 5.5mm) indicative of underdevelopment of the orbital rim.	The right lateral orbital floor triangle showed a large area reduction of 20.8% (8.4% reduction in depth and 13.6% in the width). The right lateral tetrahedron showed a contraction of 17.9% along the direction from right orbitale to the right supero-lateral orbital roof and wall indicating a lack of development in the lateral orbital wall.

Skeletal Feature	Individual Osseous Landmark		Distance and Angle	Strain
Orbits	Bone Standard	Skull Standard		
Roof inclining downward and outward	The lateral aspect of the left and right orbital roofs are longer than the standard.	The lateral aspect of the orbital roofs are longer, and in the direction from superior orbitale to lateral orbitale are more inclined infero-laterally for the patient than for the standard.	The angle at superior orbitale was significantly reduced (11.4°). The lateral orbital rim (superior orbitale to lateral orbitale right) is increased (8mm for the right and 9.3mm for the left). The measurements indicate an infero-lateral-posterior displacement of the patient's lateral orbital wall relative to the standard.	The right supero-lateral orbital roof and wall showed a 30.4% dilation across the width of the triangle and the right lateral tetrahedron showed a large dilation of 30.4% approximately in the direction from right superior orbitale to the right lateral orbital wall and this coupled with the above observation of a smaller right lateral orbital floor accounts for the observation that the lateral orbital roof in the direction from superior orbitale to lateral orbitale is more inclined infero-laterally than for the standard.

Skeletal Feature	Individual Osseous Landmark		Distance and Angle	Strain
Orbits	Bone Standard	Skull Standard		
Orbital cavity, oval shaped	The plots show that both orbits are more elongate than the standard.	The patient's orbits are more elongated in an infero-lateral posterior direction relative to the standard.	The medial orbitale to superior orbitale right is significantly reduced (by 8mm) while superior orbitale to lateral orbitale right and left were increased (by 8mm and 9.3mm).	The directions and magnitudes of the triangles defining the anterior border of the right and left orbital cavities indicate that the patient's orbital rims are skewed infero-laterally. The right lateral and medial tetrahedra showed that there was a height reduction between the supero-lateral orbital roof and wall and orbitale, a slight posterior displacement of the lateral anterior orbital plane and an increased separation of the lateral orbital wall and superior orbitale, indicating that the patient's right orbit was also skewed in a posterior direction leading to a net elongation in an infero-lateral-posterior direction.

Skeletal Feature Orbits	Individual Osseous Landmark Bone Standard	Skull Standard	Distance and Angle	Strain
Other quantified results	The patient's orbits are significantly smaller than the standard (by 5.6% least squares and 7.0% repeated median). The root mean square residual was 7.4mm.	The relationship of the patient's medial orbitale to nasion is similar to that of the standard but the position of the other landmarks defining the patient's orbit become increasingly infero-laterally posteriorly displaced the more laterally the orbit is transversed.	Many of the distance Z-scores are negative. Breadth measurements revealed that the separation of the patient's orbits was reduced by approximately 5mm relative to the standard. The right and left frontal processes of the zygomas (lateral orbital walls) were significantly reduced in width (by 12.1mm on the right and by 7.7mm on the left).	The surface area and volume of the patient's right orbital cone were 91.6mm^2 and 1.5ml smaller than the standard respectively.

Zygomas	Bone Standard	Skull Standard		
Most often it is grossly and symmetrically underdeveloped	The scale factors indicate that the patient's zygomas, as defined by the selected osseous landmarks, are significantly smaller by approximately 10%. The superimposed wire frame diagrams show, and the residuals quantify: · marked narrowing of the frontal process of the zygomatic bone right and left (essentially the lateral orbital walls) as the zygomatic corners are located more anteriorly while opposite orbitales more posteriorly, more so on the right than the left. · orbitale, right and left, are located more posteriorly.	Patient's zygomas are much smaller, differently shaped and displaced in an inferolateral direction.	All of the distances used to define the zygomas of the patient were smaller than those of the standard (Z-scores all negative). The following measurements are indicative of the extent of the underdevelopment of the patient's zygomas relative to the standard: · both the distances zygomatic corner right to lateral orbitale right and zygomatic corner right to opposite orbitale right reflect the reduced width of the right zygoma (smaller by 6.3mm and 12.1mm respectively) and therefore its underdevelopment. Consistent with these measurements is the significantly smaller angle at lateral orbitale (47° smaller). · the width of the left zygoma was also significantly smaller than the standard (by 7.7mm from opposite orbitale left to zygomatic corner left). · the separation of the zygomatic corners was reduced by 20.2mm relative, to the standard.	Only the triangles representing the left and right frontal processes of the zygomas could be determined for the patient. Both of these triangles showed large area reductions (of 67.5%, right, and 43.3%, left, leading to a total decrease of 1.2cm^2 relative to the standard).

Skeletal Feature	Individual Osseous Landmark		Distance and Angle	Strain
Zygomas	Bone Standard	Skull Standard		
Non fusion of the zygomatic arches	Inspection of the patient's radiology revealed both zygomatic arches to be absent. Those landmarks associated with the zygomatic arch were accordingly absent.	Inspection of the patient's radiology revealed both zygomatic arches to be absent. Those landmarks associated with the zygomatic arch were accordingly absent.	Inspection of the patient's radiology revealed both zygomatic arches to be absent. Those landmarks associated with the zygomatic arch were accordingly absent.	Inspection of the patient's radiology revealed both zygomatic arches to be absent. Those landmarks associated with the zygomatic arch were accordingly absent.
Maybe totally absent	Zygomas not totally absent for this patient.	Zygomas not totally absent for this patient.	Zygomas not totally absent for this patient.	Zygomas not totally absent for this patient.
Other quantified results	The root mean square of the residuals for the zygomas is 9.2mm, indicative of the substantial shape and size differences between the patient and the standard.			

Skeletal Feature	Individual Osseous Landmark		Distance and Angle	Strain
Calvaria	Bone Standard	Skull Standard		
Essentially Normal	Scale factor indicates that the patient's calvaria is slightly larger than the standard (least squares 3.8%, repeated median 3.3%) but not outside the population variance. This is in contrast to the scale factor results observed for the facial components which showed the patient's face to be smaller than the standard. Many landmarks of the patient's calvaria were positioned at significantly different locations to the standard although the length dimension was not significantly different. The landmarks defining the foramen magnum were not significantly different in position.	Similar to the bone standard but includes the effects of position and orientation relative to the rest of the patient's skull. Even though the patient's calvaria was found to be larger than the standard, the smaller nature of the patient's facial regions relative to the standard dominates, and alignment with scaling results in the patient's skull being enlarged to match the standard accounting for the larger size difference between the patient's and the standard's calvaria.	The traditionally defined cranial base measurements were all non-significant. Also the distance measurements defining the perimeter of the foramen magnum were non-significant as were its length and breadth measurements. The length of the cranium was found not to differ significantly from the standard. The following distances for the patient were found to differ significantly from the standard: · n-g 18.0mm larger, · g-zfr 15.3mm larger, · op-o 16.6mm larger, · op-mtl 18.7mm larger, · eamr-v 13.3mm larger, · eamr-eaml 6.4mm larger, · mtr-mtl 8.4mm larger, · eamr-zfr 24.6mm smaller, · eaml-zfl 22.3mm smaller. These measurements indicate that the patient's calvaria is larger than the standard and also an hypoplastic condition in the region of the temporal fossae.	The external surface of the patient's calvaria was found to be $356cm^2$ while that of the standard was found to be $321cm^2$, and this shows that the patient's calvaria has a larger surface area by $35cm^2$.

Skeletal Feature	Individual Osseous Landmark		Distance and Angle	Strain
Calvaria	Bone Standard	Skull Standard		
Supra-orbital ridges may be poorly developed	From the plots the major observed differences involve the patient's forehead giving the impression of it being more vertical in the mid-line and underdeveloped laterally.	The infero-posterior displacement of the zygomatic frontal points left and right, coupled with the infero-lateral inclination of the supra-orbital roofs from superior orbitale are indicative of lack of supra-orbital ridge development bilaterally.	Glabella to the zygomatic frontal point right was significantly increased (15.3mm). Bilaterally the distance between the landmarks external auditory meatus and the zygomatic frontal were significantly smaller (by 24.6mm on the right and 22.3mm on the left). These measurements are indicative of an hypoplastic condition in the temporal fossae.	The right and left frontal triangles showed large area increases (57.4% and 29.5% for the right and left respectively) between the patient and the standard due to displacement of the landmarks zygomatic frontal right and left infero-laterally, which reflects an hypoplastic condition in the region of the anterior temporal fossae.
May be increased digital markings on radiographs in the presence of normal sutural relationships	Not metrically assessed	Not metrically assessed	Not metrically assessed	Not metrically assessed
Other quantified results	The root mean square of the residuals for the calvaria was 12.3mm and this is indicative of the large variation in landmark positions of the patient relative to the standard.			The most striking feature of the strain analysis is the very symmetric orientation of the principal strain directions about the mid-sagittal plane.

7.9 Summary

In this chapter, several techniques have been employed to describe and quantify the differences between an individual with Treacher Collins Syndrome and the set of experimental reference standards that were developed in Chapter 6.

Both least squares and repeated median alignment procedures were used to compare individual landmark locations between a patient and the standards. The least squares approach orients, scales and translates the patient relative to the standard, so as to minimize the sum of squares of differences between homologous landmarks. The repeated median approach essentially positions the patient to a median alignment, by calculating the median translation, scale factor and orientation. The median orientation and median scale factor was based on the relative orientation of all homologous line segments. By alignment on the "middle" orientation, the repeated median approach has the potential of exact alignment on those landmarks that do not differ in shape between the homologues provided they number more than 50% of the landmarks.

For this patient, the results derived from the least squares and repeated median alignments relative to the standards were very similar. This can be attributed to the considerable difference between the patient and the standards, so that the repeated median technique cannot "lock onto" enough similar landmarks to produce distinctively different solutions to the alignments. This shows that when there *are* many differences, the repeated median alignment produces a viable alignment for comparison of the shapes. The similarity between the alignment techniques in this case is a reflection that the median of a normally distributed variable is a close approximation of the mean.

In spite of the similarity between the alignment techniques, the use of the repeated median approach was continued for evaluation because of its potential for exact alignment on landmarks that do not differ between the homologues. It was anticipated that this approach would be most suited to the comparison of the same individual pre and post operatively. Unfortunately, this was not able to be assessed as the patient was unavailable (inhabitant of Hong Kong) for follow-up radiography.

The value of superimposing the aligned matched least squares and repeated median wire frame diagrams between the patient and standard was such that any differences were immediately visually apparent, allowing for instant qualitative description. Further, knowledge of the landmark coordinates allowed differences in position between the homologous landmarks to be quantified in terms of the population variance of the standard for each landmark. By alignment both on individual bone standards and the skull standard, additional information concerning the orientation and position of neighbouring bones was inferred.

The three dimensional coordinate data was also used to calculate distances and angles between landmarks, including many of those routinely determined cephalometrically (with the inherent distortion) and anthropometrically during the clinical evaluation of the patient. These were compared with corresponding distance and angle standards described in Chapter 6, in order to further quantify the differences between the patient and the experimental reference standards.

The surface of the skull was also described in terms of triangular elements. These elements were sufficiently small that they could be used to describe individual bones of the skull, but not so small that they would be adversely affected by landmark location errors. Strain analyses were performed between the triangular elements of the patient and the homologous elements of the standards. These analyses quantified shape differences between the patient and the standard in terms of dilations and contractions along principal directions. Area differences between the patient and the standard for the five major regions of the skull were determined from the triangular elements representing the bone surfaces. These measures quantify the amount of bone difference, and potentially, they are useful indicators of the amount of bone required for surgical correction.

All of the essential skeletal features of Treacher Collins Syndrome described by Gorlin et al., (1976) were identified and quantified by the above analyses. Moreover, a measure of their significance was determined by comparison with the experimental reference standards.

CHAPTER 8

SUMMARY, GENERAL DISCUSSION AND CONCLUSION

8.1 Introduction

> "With all the sophisticated armamentarium at hand during this age of computerization, how can we approach the problem of abnormal facial form? While description of an object in nearly all fields is initiated by simple words of common parlance, the end point is analytic, i.e., defined in the precise language of mathematics.This concept was voiced most eloquently over 50 years ago by D'Arcy Thompson, who stated that 'if the difficulties and representation could be overcome, it is by means of such coordinates in space that we should at last obtain an adequate and satisfying picture of the process of deformation and direction of growth.' "
>
> Gorlin et al., 1976.

Similarly, the craniofacial surgeon asks a related question -

" How different is this patient from a normal population? "

It is to this issue that the present investigation has been addressed. This thesis has elucidated:

(i) methods for the acquisition of three dimensional craniofacial data,

(ii) the application of mathematical methods to the study of craniofacial shape, shape deformity and shape comparison, and

(iii) how (i) and (ii) can be used to identify and quantify regions of deformity as a basis for understanding the ontogeny of craniofacial dysmorphology.

A further consequence of the present investigation relates to its clinical implications. Clearly, from the methods used for three dimensional data collection and the analytic procedures, the potential exists to bring to the clinical situation, for the first time, a better understanding of the craniofacial morphology of a patient relative to a standard, thereby enabling a more complete qualitative and quantitative description of the craniofacial complex in three dimensions.

8.2 Acquisition Of Three Dimensional Coordinate Data For The Craniofacial Complex

The final three dimensional osseous landmark coordinate data used in this thesis have been derived from the integration of coordinate data obtained from the radiographic techniques of biplanar radiogrammetry and computed tomography.

Two sets of biplanar radiographic equipment have been employed - one at the Adelaide Dental Hospital based on a 90° rotation of the subject (using a single X-ray unit) and the other at the Adelaide Children's Hospital based on two orthogonal X-ray units. Despite the differences between the two radiographic installations, the generation of landmark coordinates in three dimensions is similar, as the data are derived from two orthogonal perspective views of the subject.

There can be little doubt that one of the prime difficulties confronting the continued development of three dimensional biplanar radiogrammetry was originally the inability of the experimenter to satisfactorily locate the same landmark on both films. In order to overcome the problem of landmark location error, Singh and Savara (1966), Tracy and Savara (1966), Savara et al. (1967, 1968, 1979), Sekiguchi and Savara (1972) and Takeguchi, Savara and

Shadel (1980) defined new anatomical landmarks that could be identified on both films. However, other researchers in the field have found that many of these landmarks are not readily identifiable and as a consequence the method of Savara et al. has not gained wide acceptance.

As an alternative to the identification of anatomical landmarks, several workers, Buck and Hodge (1975), Rune, Sarnäs and Selvik (1979), Rune (1980), Adams (1981), and Garrison et al. (1982) have used metallic implants which are easily located on both films. But, the use of metallic implants for routine biplanar radiogrammetry was not thought to be appropriate for providing the quantity of information required for a complete description of the craniofacial complex in this study. (A very large number of implants would be required and their placement would be critical if an homology between the patient and standards was to be established.)

In the present investigation, the difficulties associated with osseous landmark identification for biplanar radiography have been overcome through the development of a technique to facilitate the identification of the same landmark on both films. This technique requires the landmark to be located in one projection and a contour identified on the other projection along which the landmark is known to lie. Advantage is then taken of the redundancy inherent in the four projection equations (Section 2.3) to generate a "projection line" from the two dimensional coordinate position of the landmark on the lateral film, for example, to specify the height of the landmark on the coronal radiograph as a function of horizontal position. The intersection of the contour with the "projection line" uniquely determines the position of the landmark on the coronal film and hence its position in three dimensions.

This elegant method represents a substantial improvement over that described by Ueda (1983) where the use of direct height transfer (that is, a horizontal "projection line") leads to inaccuracies as the slope of the "projection line" depends on equipment parameters and the position of the landmark relative to the point of intersection of the central beams.

The accuracy and reproducibility of the "projection line" technique using biplanar radiography was assessed by applying it to an acrylic test object (specifically designed for this purpose) with 0.7mm diameter metal markers, accurately calibrated as to position. Consistent location of these markers using the "projection line" technique has been achieved with an accuracy of 0.16mm, well within the diameter of the markers. This level of accuracy indicates that the geometrical parameters of the equipment have been sufficiently well determined.

Having established the precision of the technique with the acrylic test object, a logical extension was to assess the accuracy and reproducibility of the method when applied to the determination of three dimensional coordinate positions of osseous landmarks on dried skulls. Three dimensional coordinates of thirty-four osseous landmarks were determined with a median reproducibility of 0.91mm for the Adelaide Dental Hospital and 0.72mm for the Adelaide Children's Hospital biplanar radiographic equipment. Moreover, there was no significant difference between distances determined from the radiographic coordinate data and distances derived by direct measurement with calipers (except where there was a difference in definition for one landmark).

A further experiment, in relation to the "projection line" technique using biplanar radiographs, was the measurement of a patient presenting with Treacher Collins Syndrome. The objective of this study was the assessment of the influence of soft tissue and pathology on osseous landmark identification. The results reveal that the accuracy and reproducibility of osseous landmark identification in the presence of soft tissue and pathology is consistent with the results derived from the dried skulls.

After development of the biplanar technique and equipment at the Adelaide Dental Hospital and Adelaide Children's Hospital, limited access to the Adelaide Children's Hospital CT scanner became available for non-clinical use. The "life-like" appearance of the three dimensional CT reconstructions showed the potential for three dimensional osseous landmark coordinate determination.

Three dimensional CT reconstructions have created great interest among surgeons and anthropologists interested in human growth and development. Only a few quantitative results, however, have been published (Cutting et al., 1986a; Cutting et al., 1986b; Marsh et al., 1986b; Marsh and Vannier, 1987), and none of these report the accuracy of their measurements or even landmark coordinates. In order to realise the inherent potential of CT derived data, it was necessary to establish the accuracy of CT based techniques for three dimensional landmark identification. A unique opportunity also existed to evaluate the suitability of craniometric and cephalometric definitions of landmarks for use in CT by comparison of the CT derived data with the biplanar coordinate data and the craniometric data determined previously for a number of dried skulls.

Several difficulties were encountered with the on-line determination of the position of landmarks. In particular, access restrictions to the CT scanner made data collection not feasible using this approach. Also, assessment of the initial results using the on-line data collection program (3DMS, as described in Section 3.2) indicated potential problems in transferring many of the cephalometric landmark definitions to the CT environment. To overcome these difficulties, an off-line method was developed to determine landmark coordinate data from multiple sets of stereo images of three dimensional CT reconstructions.

Stereoscopic viewing of the three dimensional CT radiographs gave enhanced definition to the images being analysed and facilitated landmark location. The reproducibility of this method was assessed using the five test skulls and the resulting median landmark relocation error was calculated to be 1.7mm, the width of approximately 2 voxels. Further, there was no significant difference between distances determined from coordinate data derived from three dimensional CT reconstructions and distances measured directly with calipers (except where there were differences in landmark definitions (see Section 3.7)). Similar CT landmark relocation errors were found for the patients, with the median relocation error being 1.2mm.

The difficulty of limited access for the on-line determination of the position of landmarks could be readily circumvented by the availability of a separate computing facility, where CT reconstructions could be analysed away from the clinical environment. Also, a set of algorithmic landmark definitions suitable for CT images is currently being developed and evaluated for the on-line identification of landmarks with Professor Gabor Herman. Further, on-line generation of stereo CT images and viewing facilities have been developed by the Medical Image Processing Group, University of Pennsylvania (Herman, 1988b). In the near future, it should be possible to perform CT data collection and analysis on-line.

While the off-line technique of multiple stereo images has allowed landmarks to be well identified with an accuracy of approximately the width of two voxels and is suitable when additional CT computing facilities are not available, it is forecast the technique will be superseded in the near future by the availability of cheaper, higher powered computers with suitable display facilities.

Of the seventy-six osseous landmarks identified using the CT system and the thirty-four osseous landmarks using the biplanar system, twenty-five were common to both measurement systems. A comparison of these osseous landmarks revealed that ten of the landmarks differed significantly in location between their biplanar and CT determination. These differences could be divided into two groups: those relating to definition differences (four) and those relating to identification difficulty (six) associated with the biplanar method, (in spite of the advantages of the "projection line" technique). The remaining fifteen did not have significant location differences. The landmarks in common were used for alignment of the CT and biplanar data, thereby enabling the independently collected data to be combined (integrated) into a larger and more complete data set.

The value of integration is clear when one is confronted with numerous diagnostic images, with no single image containing the entire complement of information. The ability to integrate the complementary information into one data set allows for a more complete analysis and interpretation of the data. This has obvious advantages in the clinical evaluation of patients.

Cutting et al. (1986a, 1986b, 1987) is the only group which has reported the integration of CT and biplanar three dimensional coordinate data. Their work uses this expanded data set to demonstrate simulated surgical interaction with a three dimensional diagram of the skull. Unfortunately, it is not clear from their papers how they determined their cephalometric or CT three dimensional coordinate data. Another difficulty is that their method of integration of the two data sets is omitted. Further, the names of many of the osseous landmarks used are not given. Obvious questions arise from this lack of experimental detail, for example, which landmarks were used? Was a least squares alignment procedure employed and if so, what were the residuals?

How close was the fit? What error analysis was performed (landmark accuracy and reproducibility)? Consequently, the value of their work cannot be readily assessed as the landmarks used may not faithfully represent the implied osseous features.

On the other hand, the research presented in this thesis documents the landmarks and their definitions together with details of the data collection methods and mode of integration, as well as tabulating the accuracy and reproducibility of the three dimensional coordinate data.

The methods developed for three dimensional data acquisition enabled between sixty and eighty-five osseous landmarks with known location errors to be identified for each of the five test skulls and the patient. The known accuracy and reliability of this integrated three dimensional coordinate data facilitated the description of the craniofacial complex and provided for the first time, the key to proceed with the analysis and quantification of its shape in three dimensions.

8.3 Methods For Shape And Size Comparison

Three dimensional coordinate data of key landmarks can be used to describe the essential qualities of a subject, as well as being used to generate descriptions relating to the shape and size of that subject. The definition of landmarks provides an immediate basis for an homology to be established between subjects for shape and size comparison. While alignment techniques can be envisaged to align similar shapes without an homology, shape and size comparison are more readily interpreted using landmarks that describe key features. For the three dimensional osseous landmark data described in this thesis, shape and size has been described in terms of:

(i) distance and angles calculated between landmarks,

(ii) constellations of three dimensional landmark coordinates for individual bony elements, and

(iii) triangular elements representing bone surfaces and tetrahedral elements representing bone cavities.

Distance and angle measurements have long been important in the study of human growth and development, and they have been used in this thesis to provide the link with craniometric and cephalometric measurements. Such measurements are interpreted on the basis that distance comparisons indicate size differences, while angular comparisons indicate shape differences. It should be noted that it is not necessary to align the subjects for comparison of distance and angle measurements.

In contrast, comparison of constellations of three dimensional landmark coordinates for individual regions of the craniofacial complex require some prior alignment of the two structures. Following alignment, any dissimilarities in coordinate positions of the landmarks can be used to quantify the magnitude of shape and size differences. By plotting the aligned, superimposed homologous constellations, with lines between landmarks to represent bone edges, differences between the structures are readily visible.

There are several ways in which an alignment between homologous constellations may be achieved. For cross-sectional comparisons of two dimensional cephalometric studies, it is customary to align on "stable" landmarks such as sella to nasion. A similar criterion could be used in three dimensions; however, in this thesis, two general alignment techniques have been developed which treat all landmarks equally. These techniques are based on least squares and an extension of Siegel's (1982a, 1982b) two dimensional repeated median procedure to three dimensions.

The least squares alignment approach orients, scales and translates one subject to the other, so as to minimize the sum of squares of the differences between homologous landmarks. The sum of squares of the residuals can be used as a general indicator of the similarity of the two subjects being compared.

The repeated median approach to three dimensional coordinate alignment, positions one shape relative to the other by calculating the median translation vector, repeated median scale factor, and repeated median orientation between the two shapes. The scale factor and orientation are based on consideration of all homologous line segments. The repeated median is calculated by firstly determining a median scale factor or orientation for each landmark from all line segments associated with that landmark. The repeated median scale factor or orientation is then given by the median of the median values determined for each landmark. By using repeated medians, the technique has the potential for exact alignment on those landmarks that do not differ in shape between the homologues, provided they number more than 50% of the landmarks.

Least squares alignment is most appropriately used when the landmarks of one shape are expected to have the same statistical variation about the homologous landmarks of the other shape (for example, double determination of digitizing error).

Repeated median alignment is most appropriately used when it is expected that only some of the landmarks may differ significantly from the homologous landmarks of the other shape (for example, comparison of data for a patient pre- and post- operatively).

In the present study, least squares alignment (Section 5.3) has been used for:

(i) determination of digitizing error (Section 2.4.2.6),

(ii) comparison of the biplanar radiographic determination of the test object marker location with the calibrated coordinates of the test object (Section 2.5.3),

(iii) determination of the accuracy of CT axial slice data, using the acrylic test object (Section 3.3),

(iv) generation of least squares experimental reference bone standards (Section 6.4.1),

(v) individual osseous landmark analysis of a male skull with the experimental reference bone standards (Section 6.7),

(vi) individual osseous landmark analysis of a patient with Treacher Collins Syndrome, with the experimental reference bone standards (Sections 7.2.1, 7.3.1, 7.4.1, 7.5.1, 7.6.1, 7.7.1, 7.8),

and repeated median alignment (Section 5.4) for:

(i) reproducibility of osseous landmark identification for dried skulls using biplanar radiography (Section 2.6),

(ii) comparison of landmark location between the Adelaide Dental Hospital and the Adelaide Children's Hospital (Section 2.6.2),

(iii) reproducibility of osseous landmark identification for patients using biplanar radiography (Section 2.8),

(iv) reproducibility of osseous landmark identification from multiple stereo pairs of three dimensional CT reconstructions for dried skulls (Section 3.6),

(v) reproducibility of osseous landmark identification from multiple stereo pairs of three dimensional CT reconstructions for patients (Section 3.8),

(vi) comparison of biplanar and CT osseous landmark location (Section 4.2),

(vii) integration of biplanar and CT osseous landmarks (Section 4.3),

(viii) generation of repeated median experimental reference bone standards (Section 6.4.2),

(ix) individual osseous landmark analysis of a male skull with the experimental reference bone standards (Section 6.7), and

(x) individual osseous landmark analysis of a patient with Treacher Collins Syndrome, with the experimental reference bone standards (Sections 7.2.1, 7.3.1, 7.4.1, 7.5.1, 7.6.1, 7.7.1, 7.8).

The perhaps controversial "finite element" method (referred to as strain analysis in this work) has also been utilised and for the first time applied to three dimensional human craniofacial data, and its performance compared with the more familiar distance-angle and landmark configuration analyses for craniofacial description. The phrase "perhaps controversial" relates to the forcefulness with which the proponents (Bookstein, 1982, 1983a, 1983b, 1984a, 1984b, 1985, 1986, 1987; Richtsmeier and Cheverud, 1986; Moss et al., 1985, 1987; Richtsmeier, 1987) advocate the so called finite element method and their derision of techniques based on conventional cephalometrics. Cheverud et al. (1983) have applied the techniques in three dimensions, but have used large volume elements (each encompassing many bones) exclusively, while in this thesis, the advantage of using small triangular elements to describe bone surfaces and tetrahedral elements to characterise bone cavities has been demonstrated.

The strain analysis between homologous elements quantified shape differences in terms of dilations and contractions along principal directions. Additionally, the area and volume differences between homologous elements have been used to provide a measure of differences in the amount of bone present.

8.4 Creation Of Suitable Three Dimensional Reference Standards

While a patient, (or indeed any individual in question), may be compared with another individual using the above techniques, it is preferable to compare the patient with a population of known mean and variability, so that some assessment can be made of the significance of the differences.

As at this time, there are no published reference standards for three dimensional coordinates and it has been necessary to create experimental reference standards. For this purpose, the three dimensional coordinate data collected from the four female skulls has been used to create experimental reference bone standards (mandible, maxilla, orbits, zygomas, cranium, and skull) for each of the analysis techniques (distance and angles, landmark coordinate positions based on least squares and repeated median approaches, and strain analysis). The significance of a difference from the standard is determined by expressing the difference in terms of the population standard deviation.

To demonstrate the use of the standards and determine any possible deficiencies therein, another skull of identical ethnic background, similar age but of a different sex was compared with each of the standards using each analysis technique.

The success of the analyses to identify known male features relative to the experimental reference standards showed that the latter were acceptable for the purpose for which they were established and indicated that the analysis techniques were appropriate methods for shape quantification.

8.5 Application To The Quantitative Description Of A Patient With Treacher Collins Syndrome

Having successfully applied the technique to dried (human) skulls, the next step involved selecting a patient with a congenital syndrome for assessment of the analysis techniques for the quantification of craniofacial deformity. The patient chosen for analysis was afflicted with Treacher Collins Syndrome. This clefting condition (Tessier Clefts 6, 7, and 8) has been well described in the literature (Gorlin et al., 1976) in a qualitative sense and characteristically affects the zygoma, maxilla and mandible.

Comparisons of distances and angles between landmarks, landmark coordinate positions and strains of the patient relative to the standards have been performed. Of significance is that all the essential skeletal features of Treacher Collins Syndrome as described by Gorlin et al. (1976) have been identified and quantified by the analysis techniques, as summarised in Table 7.18. Further, a measure of the significance of the deviations has been determined by comparisons with the experimental reference standards.

Moreover, the analyses have been applied to the same data, and as a consequence, each analysis technique reveals the same structural differences. The different approaches, however, throw the data into different perspectives and while each approach reflects the same differences, the relative ease of interpretation varies depending upon the feature.

8.6 Conclusion

While two dimensional distances and angles derived from cephalometric data for a patient are routinely compared with population standards, the collection of three dimensional data is rare and no three dimensional standards exist. In this investigation, methods for three dimensional data acquisition have been developed for routine use, experimental three dimensional reference bone standards created, and several analysis techniques extended for use in three dimensions.

The merit of the techniques presented in this thesis have been demonstrated by their application to the quantification of the established qualitative features of Treacher Collins Syndrome.

It is from the firm foundation of the quantification of osseous landmark location in three dimensions, with established accuracy, that analysis and statistical techniques for describing craniofacial form and dysmorphology can be further developed. As more three dimensional data are collected for both skeletally normal patients and patients with congenital syndromes, both "normal" and syndrome specific standards will evolve, enabling:

(i) quantitative surgical planning through presentation of patient differences from population standards,

(ii) evaluation and quantification of surgical treatment by comparison of pre- and post- operative data. Follow-up radiography would provide data for assessment of the stability of procedures, including their timing,

(iii) the study of population characteristics of craniofacial conditions, and

(iv) the study of craniofacial symmetry.

Furthermore, the techniques developed and presented in this thesis have relevant application to the study of craniofacial trauma and other regions of the skeleton.

In the more immediate future the techniques will be assessed, to determine their clinical usefulness in the planning stages of patients requiring craniofacial surgery.

The present investigation has opened the way for further studies into the quantification and analysis of biological form in three dimensions. Methods have been developed which are suitable for the routine collection of three dimensional osseous landmark coordinate data, together with several shape analysis techniques. These facilitate the improved description and quantification of the craniofacial complex and therefore lead to better communication of craniofacial form.

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