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GENERALISATIONS OF MINKOWSKI'S

THEOREM IN THE PLANE

by

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ABSTRACT

Several Generalisations of Minkowski's Convex Body Theorem, in the Euclidean Plane, are obtained. In the first, the perimeter of an O -symmetric convex set containing no interior lattice points besides O is used as an additional parameter in bounding the area of the set. Secondly, the number of chords of a convex set bisected by O is introduced as a measure of symmetry of the set about O . With this measure of symmetry, a simultaneous generalisation of both Minkowski's Theorem and Ehrhart's Theorem is obtained. Similar generalisations are shown for a number of related symmetry conditions. Finally, the same symmetry measure is used to extend the Minkowski-Van der Corput Theorem in the plane, for sets containing at most four interior lattice points.