OPTIMAL DESIGN OF
GAS PIPELINE NETWORKS

by

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SUMMARY

In constructing a pipeline network to collect gas from a number of fixed wells and transport it to a factory, prior to entry into a trunk main for onward transmission, several interdependant optimization problems arise. The configuration of the network, i.e. the number of junction nodes and the links of the network, has to be determined. Then the junctions have to be located and finally the diameter of each link of the network has to be selected. The thesis studies this area of problems and investigates optimization models that may be of use in the design of such networks. The models are tested on data from a developing gas field at Moomba, in the desert center of Australia.

Optimal networks are trees. Under fairly general assumptions it is proved that the optimal network is a tree. This justifies the policy in this thesis of considering only tree networks and ignoring all other types.

Optimal diameter selection. Two optimization models are developed for this problem. Firstly a Dynamic Programming formulation, secondly a Linear Programming model. Properties of an optimal diameter assignment are proved. Computational results are given for both models and the models compared.

Junction location problem. For a given configuration, the

problem of locating junctions in a pipeline network is considered. The problem is shown to be convex. The problem is solved and computational results presented. The case of 3 given nodes is analysed in depth and some properties of an optimal solution for this case are derived. Experimental results based on a parameter study are presented.

Configuration Problem. This is a complex, combinatorial problem. It is shown that for a small number of given nodes, the positions of these nodes are far more important than the flows in determining the optimal configuration.

SIGNED STATEMENT

This thesis contains no material which has been accepted for the award of any other degree or diploma in any University. To the best of my knowledge and belief, the thesis contains no material previously published or written by any other person, except where due reference is made in the text of the thesis.

SITA BHASKARAN

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CHAPTER 1

INTRODUCTION

This thesis studies the problems that arise in the construction of a gas pipeline collection system and discusses the possibility of applying mathematical optimization techniques. This study started with an investigation of a developing gas field at Moomba, in the desert centre of Australia where a network of pipes has to be constructed to collect gas from a number of wells and transport it to a gas plant prior to dispatch along a main trunk line.

The cost of the network depends on the lengths and diameters of the links. Different pipes can be joined together at junction nodes. The problem is to determine the number and positions of the junctions, the links of the network and the diameters of the links in an optimal way.

The problems are related to a class of problems of which the oldest is the Steiner problem [25]: the construction of a minimal length tree connecting a set of given nodes. This problem has been well studied by Gilbert and Pollak [10]. The Weber problem, studied by Kuhn [13], Kuhn and Kuenne [14], Kuenne and Soland [12] and Palermo [20] is the problem of locating a source so as to minimize the sum of weighted distances between a number of fixed sinks and the source. The work that has been done on Steiner and Weber problems is reviewed in Appendix II. The pipeline problem is much more complex than both the Steiner and Weber problems.

Papers that deal with pipeline construction are Rothfarb *et al.* [22] and Zadeh [28]. They, however, consider an offshore pipeline system, for which junctions other than at existing well rigs are not possible. The problem of finding the optimal junction locations is therefore, in their case, irrelevant. Both Rothfarb *et al.* and Zadeh use for the optimal assignment of diameters, a Dynamic Programming approach and Zadeh has also used a minimum cost flow approach.

In Chapter 2 we prove the important result that under fairly general assumptions, which we claim not to be unrealistic, the optimal network is a tree. This result justifies our policy in the rest of the thesis of restricting attention to trees. We show how the problems that arise in the design of pipeline systems simplify on considering trees. An exact formulation of the problems that concern us is given in Chapter 2. This chapter also sets up notation and terminology to be used throughout the thesis unless otherwise specified.

Chapter 3 deals with a situation that is somewhat more general than that discussed in other chapters, in that we consider the network over a number of years when the rate of gas production at the wells vary over the years. We solve the diameter problem for this more general case. We first present a Dynamic Programming formulation. If, instead of requiring the entire length of a link to be of one uniform diameter, we can assign over the length of a link two or more pipe sizes connected in series then the cost of the network may be reduced. We have the facility to do this in the Linear Programming formulation we develop. We also prove some properties of an optimal diameter assignment. They are

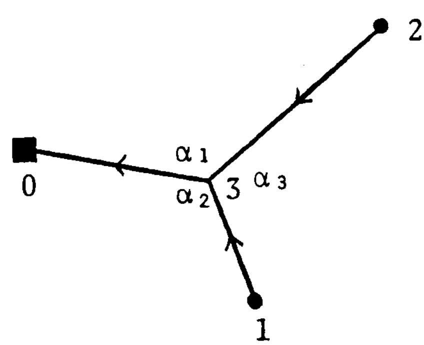
(i) At most two pipe sizes are used for one link.

(ii) Under fairly general conditions the following result holds: If for any link two pipe sizes are used, then they must be of consecutive sizes.

(iii) If the specific gravity of gas is the same for all wells then there exists an optimal diameter assignment in which diameters do not decrease in the direction of flow.

The optimization models have been applied to data from the Moomba gas field and computational results presented.

In Chapter 4 we attack the junction location and configuration problems. The problem of locating junctions arises in both the Steiner and Weber problems. In the pipeline case we show that the junction location problem is a convex programming problem. Thus, there is a unique solution - the global minimum and the problem of dealing with local minima does not arise. We present methods of solution of the junction location problem together with computational results.



We analyse in depth the junction location problem in the case of 3 fixed nodes {0, 1, 2} as shown in the figure, where the position of node 3 is to be determined. We discuss the degenerate cases when node 3 coincides with one of the fixed nodes 0, 1 or 2. We prove 4 properties of the angles at the junction node 3 when it is optimally located. They are

(i) The ratio of the cost per unit length of a link to the Sine of the angle opposite is the same for all 3 links.

(ii) The angle at a node between an incoming and an outgoing link is at least 90°, i.e. $\alpha_1, \alpha_2 \geqslant \pi/2$.

(iii) $\alpha_3 \leqslant \alpha_1$ and $\alpha_3 \leqslant \alpha_2$.

(iv) $\alpha_3 \leqslant 120°$.

We also point out a duality and prove that when node 3 is optimally located, the maximum available pressures at nodes 1 and 2 are completely utilised. We present empirical results based on a parameter study.

The configuration problem is a hard, combinatorial problem. Its complexity is illustrated by comparing it with the Steiner problem. In the Steiner problem the goal is to minimize a sum of distances. In the pipeline problem, we have to minimize a weighted sum of distances. However, the weights depend on diameters which in turn depend on configuration. The pipeline problem is therefore of a much greater complexity than the Steiner problem. But even the Steiner problem cannot effectively be solved for any reasonable number of nodes (say, 7 or more) as the solution methods discussed in the literature are based on enumeration. We have studied the configuration problem in the pipeline case simply by solving a large number of small examples.

We have given a guideline for ruling out nonoptimal configurations. We have also shown that for the cases of 4 or 5 fixed nodes, the positions of the nodes are far more important than the flows in determining the optimal configuration.

Throughout this thesis some basic assumptions have been made. First of all, it is assumed that a complete forecast is available for the production of each of the wells together with the composition of produced gas. The forecast will depend upon the characteristics of each of the wells and company policy. The cost of the network has been calculated in current prices and no account has been taken of discounts, of inflation, for construction in later years. We also assume that there are no physical obstacles to the positioning of pipes or junctions.

The thesis illustrates that certain aspects of optimal pipeline construction lend themselves to mathematical analysis and that the application of mathematical optimization techniques is a practical proposition, and can result in large savings, for a small amount of computer costs.

We denote the end of a proof by the symbol ***.

CHAPTER 2

PIPELINE NETWORKS

We will study gas pipeline networks that transport gas from a set of wells to a factory. The network is composed of pipes. Its shape has to be determined and the diameter of each pipe selected from a given finite set of pipe sizes. The diameters of the various links have to be chosen such that the drop in pressure of gas between any well and the factory does not exceed a certain given limit. Subject to this constraint we want to design the cheapest network.

A tree is a connected network without meshes, where a mesh is a path of links connecting a node with itself. In this chapter we will prove the important result that under fairly general assumption, which we claim not to be unrealistic, the optimal network is a tree. This result justifies our policy in the remaining chapters of this thesis to consider only tree-networks and ignore all other types.

This chapter will also set up notation and terminology used throughout the thesis, unless otherwise specified, and present mathematical formulations of the problems that will be discussed.

§'s 2.1 and 2.2 describe the given data and flow formula. In § 2.3 we prove that optimal networks are trees. Tree networks are described in § 2.4 and the optimization problems that arise stated in § 2.5.

The network terminology we use - node, link, path, mesh and tree is the same as in Potts and Oliver [21].

2.1 <u>Given Data</u>

We are given a gas plant, denoted by i=0 with fixed position (x(0),y(0)), and a number of gas wells i=1,2, ..., m, also with fixed positions (x(i),y(i)). We are also given the following information:

(1) Q(i) - the rate of flow of gas produced at well i,

(2) S(i) - the specific gravity of gas produced at well i,

(3) P_0 - a fixed pressure at which the gas must be delivered at the plant, and

(4) P_1 - the maximum pressure at which gas can be provided at the wells.

The wells have to be connected to the plant by a network of pipes. The diameters of each link of the network is selected from a finite set of diameters

$$D = \{d_1, d_2, \ldots, d_n\}.$$

For each $d \in D$, a <u>cost per unit length</u> C(d) is specified. C(d) is always a monotonically increasing function of d.

It is often advantageous from both an analytic and a computational point of view to consider d as a continuous variable in the range $0 \leq d < \infty$ and fit a continuous cost function C(d), $0 \leq d < \infty$ to the cost data. Interpreting d as a continuous variable is not unrealistic even when only discrete pipe sizes are considered. A link does not have to be of one uniform diameter. If two or more discrete diameter pipes are connected in series along the length of a link then the average diameter of the link is a continuous variable, its value depending on the pipe sizes used and the proportion of the length of the link they cover. Thus, in our work, we will sometimes consider d as a discrete variable with C(d) the given cost data; or, when we need greater flexibility, be more general and consider d as a continuous variable and C(d) a cost function of a continuous variable. In every section it should be obvious from the context whether we are considering d to be discrete or continuous.

One form of the cost function which gives a reasonable approximation to the cost data and which is also convenient to deal with is

$$C(d) = K\, d^{\mu} \qquad (2.1.1)$$

where K, μ are suitably chosen positive constants. This form also has the advantage of being differentiable any number of times.

2.2 Flow Formula

In the design of gas pipeline networks the formula used to describe the flow of gas through pipelines is of vital importance. There exist empirical formulae giving the pressure square drop per unit length of pipe as a function of flow, specific gravity and diameter:

$$\frac{pp}{\ell} = f(q,s,d) \qquad (2.2.1)$$

where

q = rate of gas flow,

s = specific gravity of flowing gas,

ℓ = length of link,

d = internal diameter of pipe,

pp = pressure square drop between ends of pipe.

All variables q, s, d, pp, ℓ are nonnegative and will be assumed to be so throughout this thesis.

We are not going into the experimental details of deriving the formula f. We assume it is given. f is usually a monomial and in our work we will assume it to be. Thus,

$$f(q,s,d) = M\,\frac{q^{\alpha_1}\, s^{\alpha_2}}{d^{\alpha_3}} \quad , \qquad (2.2.2)$$

where $M,\alpha_1,\alpha_2,\alpha_3$ are positive constants. The value of M depends upon the units of measurement used. (2.2.1) can also be written as

$$d = h(q,s,\ell,pp) \quad , \qquad (2.2.3)$$

where h is again a monomial:

$$h(q,s,\ell,pp) = N\, q^{\beta_1}\, s^{\beta_2} \left(\frac{\ell}{pp}\right)^{\beta_3} \qquad (2.2.4)$$

for appropriate positive constants $N,\beta_1,\beta_2,\beta_3$.

Of the existing empirical formulae, we have used, for our numerical computations, the Weymouth formula which is popular for the range of pressures we have to deal with.

Weymouth Formula: $\frac{pp}{\ell} = f(q,s,d) = M \frac{q^2 s}{d^{5\frac{1}{3}}}$ (2.2.5)

The value of M, for the units we use, is given with the computational results in Chapter 3.

To maintain generality, we will, where possible use the general form (2.2.2) of the monomial f instead of the specific formula (2.2.5).

2.3 Optimal networks; tree design

In this section we will assume that the cost function C(d) is a twice differentiable function of d. We also assume that the specific gravity of gas is the same for all wells. This is not a very demanding assumption because the specific gravity of gas only marginally affects the cost of the network and does not, in practice, vary much over the wells.

A pipeline network consists of:

(1) A set N of nodes,

(2) A set A of links

$$A \subset \{ (i,j) \mid i,j \in N , i \neq j \} .$$

Let us define for all $(i,j) \in A$:

d(i,j) - diameter of link (i,j),

q(i,j) - rate of flow of gas on link (i,j),

pp(i,j) - pressure square drop on link (i,j),

ℓ(i,j) - length of link (i,j),

s - specific gravity of gas.

The flows $q(i,j)$ and pressure square drops $pp(i,j)$ have to satisfy the following constraints:

1. <u>Conservation of flow</u>: The flow entering every node must equal the flow leaving it. Thus, for every node $i \in N/\{0\}$ we have a constraint:

$$\sum_{k:(k,i)\in A} q(k,i) = \sum_{j:(i,j)\in A} q(i,j) \quad , \qquad (2.3.1)$$

where $q(i,j) \geqslant 0 \quad \forall\ (i,j)\in A$.

2. <u>Uniqueness of pressure at a point</u>: The pressure square drops around every mesh must be zero.

3. <u>Upper bound on pressure square drop</u>: The pressure square drop between any well and the factory must be within the prescribed limit, $P_1^2 - P_0^2$.

The total cost of the pipeline network is given by

$$COST = \sum_{(i,j)\in A} C(d(i,j))\ \ell(i,j) \quad .$$

Our aim is to find a network which has minimum total cost - an <u>optimal network</u>. Using formulae (2.2.3) and (2.2.4) we can express the cost of a network in terms of flows, pressure square drops, specific gravity and lengths.

$$COST = \sum_{(i,j)\in A} H(q(i,j), s, \ell(i,j), pp(i,j)) \quad , \qquad (2.3.2)$$

where

$$H(q,s,\ell,pp) = \ell\ C(h(q,s,\ell,pp)) = \ell\ C\left(N\, q^{\beta_1}\, s^{\beta_2}\left(\frac{\ell}{pp}\right)^{\beta_3}\right) \quad . \qquad (2.3.3)$$

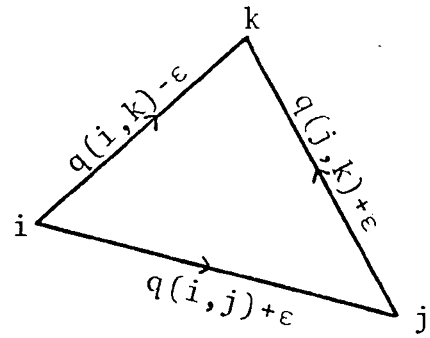
The following theorem is of central importance.

Theorem 2.1. *If H is a strictly concave function of q, then the optimal network is a tree.*

The proof is as follows:

Suppose we have an optimal network with node set N*, link set A*, and, for any link (i,j)∈A*, diameter d*(i,j), flow q*(i,j), pressure square drop pp*(i,j), length ℓ*(i,j). By allowing the flows q(i,j) to vary, but keeping all other variables fixed we can consider the problem of minimizing the cost as a function of the q(i,j)'s under conditions (2.3.1). As H is a strictly concave function of q, COST is a strictly concave function of the q(i,j)'s. Hence we are considering the problem of minimizing a strictly concave function under linear constraints. As is well known the minimum of a strictly concave function is attained at an extreme point of the polygon determined by (2.3.1). The set of extreme points is identical with the set of basic feasible solutions of (2.3.1). Hence, the optimal network corresponds to a basic feasible solution of (2.3.1).

It is easily seen that the subnetwork corresponding to a set of basic variables is a tree. The following proof is taken from Dantzig [5]: There are |N*| - 1 constraints. Hence there are |N*| - 1 basic variables q(i,j). If the subnetwork of |N*| - 1 links corresponding to the basic variables contains no meshes, it is a tree. It remains to be shown that no basic set can give rise to a mesh. Suppose,



for example, q(i,j), q(j,k) and q(i,k) are in the basic set and form a mesh as in the figure alongside. Then the values of the basic variables around the mesh could be altered as indicated, while the values of all other variables would remain the same, yielding a second solution with the same basic variables. This contradicts the uniqueness property of basic solutions.

This concludes the proof of the theorem.

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Theorem 2.1 can be stated in a slightly different form as follows:

Theorem 2.2. *If*

$$d\,\frac{C''(d)}{C'(d)} < \frac{1-\beta_1}{\beta_1} \quad , \tag{2.3.4}$$

where β_1 *is defined in* (2.2.4), *then, the optimal network is a tree.*

This can be shown as follows:

H is a strictly concave function of q iff

$$\frac{\partial^2 H}{\partial q^2} = \frac{\partial^2}{\partial q^2}\,[\ell\, C(h(q,s,\ell,pp))] < 0 \quad ,$$

i.e., as $\ell > 0$:

$$C''(d)\left(\frac{\partial h}{\partial q}\right)^2 + C'(d)\,\frac{\partial^2 h}{\partial q^2} < 0 \quad .$$

Using the expression (2.2.4) for h, we see that this is equivalent to

$$N\ \beta_1\ q^{\beta_1 - 2}\ s^{\beta_2} \left(\frac{\ell}{pp}\right)^{\beta_3} \ (\ d\ C''(d)\ \beta_1 + C'(d)\ (\beta_1 - 1)\ ) < 0 \quad ,$$

which is true if and only if (2.3.4) holds. The theorem is therefore a direct result of *Theorem 2.1.*

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If we use a function of the form

$$C(d) = K\ d^{\mu}$$

for C, then condition (2.3.4) of *Theorem 2.2* reduces to

$$\mu\beta_1 < 1\ . \tag{2.3.5}$$

If we use the Weymouth Formula for the pressure square drop we have $\beta_1 = {}^3/_8$, and the condition is

$$\mu < {}^8/_3\ . \tag{2.3.6}$$

This is not a very severe condition and in practice will not usually be violated. For the data we obtained μ was near 1 and the condition was therefore satisfied easily. There is therefore sufficient theoretical justification for concentrating on tree networks for gas pipeline systems and ignoring all other designs. This is what we will do from now on.

2.4 Tree Networks

The elements of a tree network are:

A: The nodes of the tree, which as before, can be classified as follows:

(1) The set $N_f = \{0,1,\ \ldots,\ m\}$ of fixed nodes, i.e. the plant and the wells, for which the positions $(x(i),y(i))$,

$i \in N_f$ are given.

(2) The set N_s of junction nodes, where different pipe sections are joined together; the number $|N_s|$, and positions $(x(i),y(i))$, $i \in N_s$ of these nodes are not specified *a priori*.

$$\text{Let } N = N_f \cup N_s \text{ and } \bar{N} = N/\{0\} \quad .$$

B: The links of the tree, which, can be specified by defining a predecessor function

$$a: \bar{N} \to N \quad .$$

The set of links is then

$$A_a = \{ (a(i),i) \mid i \in \bar{N} \} \quad . \qquad (2.4.1)$$

The function a must be such that the graph $[N,A_a]$ contains no meshes.

For a tree network we define

d(i) - diameter of link (a(i),i),

q(i) - rate of flow of gas on link (a(i),i),

pp(i) - pressure square drop on link (a(i),i),

$\ell(i)$ - length of link (a(i),i). Thus

$$\ell(i) = \sqrt{[(x(i) - x(a(i)))^2 + (y(i) - y(a(i)))^2]} \quad , \qquad (2.4.2)$$

s(i) - specific gravity of gas on link (a(i),i).

The cost of a tree network is thus,

$$\text{COST} = \sum_{i \in \bar{N}} \ell(i) \; C(d(i)) \quad . \qquad (2.4.3)$$

The set of successor nodes of node i is defined by

$$B(i) = \{j \in N \mid a(j) = i\} \quad , \forall \; i \in N \qquad (2.4.4)$$

and

$$\theta = \{j \in N \mid B(j) = \phi\} \tag{2.4.5}$$

denotes the set of extremal nodes of the tree. Note that $\theta \subset N_f$, i.e. all extremal nodes are wells.

The constraints on the flows and pressure square drops simplify considerably for the case of trees.

1. <u>Conservation of flow</u>: The constraints (2.3.1) reduce in the case of trees to the recursive equations:

$$\left.\begin{array}{lll} q(i) = Q(i) & , \text{ if } i \in \theta \\ q(i) = \sum\limits_{j \in B(i)} q(j) + Q(i) & , \text{ if } i \notin \theta \end{array}\right\} \tag{2.4.6}$$

$$q(i) \geq 0 \, , \forall \; i \; .$$

Thus for trees flows can be calculated directly from the above recursive equations (2.4.6), unlike the case of networks with meshes where the flows have to be calculated by using the iterative techniques of numerical approximation.

2. <u>Uniqueness of pressure</u>: As a tree does not contain meshes, this condition is always satisfied and does not have to be considered.

3. <u>Upper bound on pressure square drops</u>: Because the network is a tree with the plant $i = 0$ at its origin, each of the other nodes can be reached from $i = 0$ by exactly one path. Consider the nodes of the set $\theta = \{i \in N \mid B(i) = \phi\}$ i.e. the extremal nodes of the tree. Let $i \in \theta$ and let

$$0 = i_0, i_1, \ldots, i_k = i$$

be the path linking i with 0, which is the case if

$$a(i_j) = i_{j-1}$$

The upper bound on the pressure square drops can then be expressed as:

$$\sum_{j=0}^{k} pp(i_j) \leqslant P^* \quad , \tag{2.4.7}$$

where $P^* = P_1^2 - P_0^2$.

There is one such constraint for every $i \in \theta$. Also

$$pp(i) > 0 \quad , \quad i \in \overline{N} \quad .$$

For the specific gravities in each of the links we can derive the following recursive equations:

$$\left.\begin{aligned} s(i) &= S(i) \quad , \text{ if } i \in \theta \\ s(i) &= [\sum_{j \in B(i)} (s(j)q(j)) + S(i)Q(i)]/[\sum_{j \in B(i)} q(j) + Q(i)], \text{ if } i \notin \theta \text{ .} \end{aligned}\right\} \tag{2.4.8}$$

2.5 <u>Optimization Problems</u>

To construct an optimal pipeline network the elements of the optimal tree have to be specified. To specify the nodes of the tree the number and positions of the nodes N_s have to be determined; to specify the links of the tree the predecessor function a has to be defined. This task can be divided over two phases in a natural way:

1. <u>The configuration problem</u>: Determine the number of junction nodes $|N_s|$ and the predecessor function a.

2. <u>The junction location problem</u>: Determine the position of the junction nodes, $i \in N_s$.

To calculate the cost of the tree and thus completely specify it there remains

3. <u>The diameter problem</u>: Select for each link of A_a a diameter d (or, equivalently a pressure square drop pp).

These problems will be discussed in more detail in the following chapters.

CHAPTER 3

DIAMETER ASSIGNMENT: MATHEMATICAL PROGRAMMING MODELS

In this Chapter only tree designs are considered. Also, except for the last section, the diameter set is considered to be finite, i.e. d is a discrete, not a continuous variable.

For the diameter problem the geometric properties of the tree network are assumed to be given, i.e. the number $|N_s|$ of junction nodes, together with their positions, and the predecessor function a which gives the links of the tree. The only variables that have to be determined are the diameters of the links.

This Chapter deals with a situation that is somewhat more general than that discussed in other Chapters, in that we will consider the network over a number of years and the rate of gas production at the wells will vary over the years. We assume that a complete forecast is available for the production at each of the wells. Thus, the rate of gas flow and specific gravity of gas on all the links are known for all the years. The lengths of all links are also known. We wish to select for each link a diameter $d \in D$. The diameters of the links must be large enough to handle the varying gas flows over the period considered, i.e. diameters must be chosen such that the pressure constraints hold through all the years. And subject to this constraint we wish to choose the cheapest combination of diameters. We will call such a combination of diameters an optimal diameter assignment.

In this Chapter we present two optimization models. In § 3.2 we give a Dynamic Programming formulation. If, instead of requiring the entire length of a link to be of one uniform pipe size we can assign over the length of any link two or more pipe sizes connected in series then the cost of the network may be reduced. We have the facility to do this in the Linear Programming formulation we present in § 3.3. Some properties of an optimal diameter assignment will be proved in § 3.4.

The two models have been applied to data from the Moomba gas field and computational results are given in § 3.5. Finally in § 3.6 it is shown that for series networks, for one time period, the optimal diameter of a link can be expressed as an explicit function of flows, specific gravities and lengths.

3.1 Notation

Most of the notation of Chapter 2 can be used in this Chapter also, but for some of the symbols a time index has to be added.

Let a period of T years be considered. For year t, t=1,2, ..., T we define:

Q(i,t) - rate of flow of gas produced at well i in year t, where $i \in N_f / \{0\}$, S(i,t) - specific gravity of gas produced at well i in year t, where $i \in N_f/\{0\}$,

q(i,t) - rate of flow of gas in link (a(i),i) in year t, where $i \in \overline{N}$,

s(i,t) - specific gravity of gas in link (a(i),i) in year t, where $i \in \overline{N}$,

p(i,t) - pressure at node i in year t, where $i \in N$,

pp(i,t) - pressure square drop on link (a(i),i) in year t, where $i \in \overline{N}$.

Thus, $pp(i,t) = p^2(i,t) - p^2(a(i),t)$.

As before, for each link $(a(i),i) \in A_a$, for every year t=1,2, ..., T the flow rates are given by

$$q(i,t) = Q(i,t) \;, \qquad \text{for extreme nodes } i \in \theta,$$

$$q(i,t) = \sum_{j \in B(i)} q(j,t) + Q(i,t) \;, \text{ if } i \notin \theta$$

and the specific gravities

$$s(i,t) = S(i,t) \;, \qquad \text{if } i \in \theta,$$

$$s(i,t) = \left[\sum_{j \in B(i)} (s(j,t)\; q(j,t)) + S(i,t)\; Q(i,t) \right] \Big/ \left[\sum_{j \in B(i)} q(j,t) + Q(i,t) \right] \;, \text{ if } i \notin \theta \;.$$

The <u>pressure constraints</u> the network has to satisfy are the upper bound on pressure square drop. By (2.2.1), the pressure square drop on link (a(i),i), in year t, is given by

$$pp(i,t) = \ell(i)\; f(q(i,t), s(i,t), d(i)) \;.$$

As lengths, flows and specific gravities are given quantities, the diameter of a link determines the pressure square drop. The diameters have to be chosen such that for every extreme node $i \in \theta$,

$$\sum_{j=0}^{k} pp(i_j,t) \leq P_1^2 - P_0^2 \ , \text{ for } \forall \text{ year } t$$

where $0=i_0, i_1, \ldots, i_k=i$ is the path linking i with 0.

3.2 Dynamic Programming Model

In this model we will express the pressure constraints in terms of the variables p(i,t) instead of the variables pp(i,t), as follows: To each node $i \in N$ is assigned a pressure p(i,t), for all years t, by means of a recursive formula

$$p^2(i,t) - p^2(a(i),t) = \ell(i)\ f(q(i,t),s(i,t),d(i)), \text{ if } i \neq 0$$

$$p^2(0,t) = P_0^2.$$

Because of the tree structure of the pipeline network the pressures are now completely specified in terms of lengths, flows, specific gravities and diameters. As lengths, flows and specific gravities are given quantities, the diameters must be chosen such that

$$P_0 \leq p(i,t) \leq P_1, \text{ for } \forall\ i \in N \text{ and } \forall\ t \in \{1,\ldots,T\}.$$

For this model each link must consist of only one pipe size. We will consider the problem as a multistage decision process. The <u>stages</u> of this model are represented by the links of our network. At every stage the decision to be taken is selecting a diameter for the link at that stage.

The resulting model is not sequential as in usual Dynamic Programming but a branch system. For the stage corresponding to link (a(i),i) we have

Input: $p^2(a(i),t) \quad , \quad \forall\, t$

the pressure squares must be in the range

$$P_0^2 \leqslant p^2(a(i),t) \leqslant P_1^2$$

Decision: $d = d(i) \in D$

Output: $p^2(i,t) = p^2(a(i),t) + g_{i,t}(d) \quad , \quad \forall\, t$

where

$$g_{i,t}(d) = \ell(i)\ f(q(i,t),s(i,t),d)$$

as in formula (2.2.1). The functions q,s,ℓ are known and the decision d must be such that again

$$p^2(i,t) \leqslant P_1^2$$

Cost: $C(d)\ \ell(i)$

Let, for $\forall\, i \in N$

$C_i^*(\{p^2(i,t)\})$ = Minimum cost of the subtree originating at node i, if the pressure squares at i are $p^2(i,t)$ for $t=1, \ldots, T$.

Then the following recursive equations can be derived in the usual Dynamic Programming way:

For nodes $i \notin \theta$

$$C_i^*(\{p^2(i,t)\}) = \sum_{j\in B(i)} \ \operatorname*{Min}_{d\in D} \left[C(d)\ \ell(j) + C_j^*(\{p^2(i,t) + g_{j,t}(d)\})\right] ;$$

for $i \in \theta$

$$C_i^*(\{p^2(i,t)\}) = \begin{cases} 0 & \text{if } p^2(i,t) \leqslant P_1^2 \\ \infty & \text{if } p^2(i,t) > P_1^2 \end{cases} .$$

These equations cannot be solved analytically. To enable numerical methods we have to adopt a grid-system for the allowable pressure square range. By a grid of size M on an interval [a,b] we mean that we consider only M points in the interval, namely, the points

$$a + \frac{(b-a)}{(M-1)} i \quad ; \quad i=0,1, \ldots, M-1 \quad .$$

Thus for node $i \in \bar{N}$ we have to choose a grid in the range

$$P_0^2 \leqslant p^2 \leqslant P_1^2$$

and the output $p^2 + g_{j,t}(d)$ has to be rounded off to the next pressure square in the grid. Then the recursive equations can be solved in the usual Dynamic Programming fashion and the optimal diameters obtained.

As mentioned in the Introduction, Rothfarb *et al.* [22] and Zadeh [28] have also used Dynamic Programming approaches to solve the diameter problem.

3.3 Linear Programming Model

Instead of requiring the entire length of a link to be of the same diameter, if we can assign to a link, sections of different pipe sizes connected in series then the cost of the network may be reduced. The model we now present has the facility to do this.

The decision variables are defined as

$\delta(d,i)$: proportion of length of link $(a(i),i)$ that has diameter $d \in D$.

These variables must satisfy

$$\left.\begin{aligned} \sum_{d\in D} \delta(d,i) &= 1 \quad , \quad \forall\ i \in \overline{N} \\ \delta(d,i) &\geq 0 \quad , \quad \forall\ i,d \end{aligned}\right\} \qquad (3.3.1)$$

For every extreme node $i \in \theta$ let

$$0=i_0, i_1, \ldots, i_k=i$$

be the path linking i with 0. Using (2.2.1) and (2.2.2) we can express the pressure square drop on link (i_{j-1}, i_j) in terms of the variables $\delta(d,i_j)$ as follows:

$$pp(i_j,t) = r(i_j,t) \sum_{d\in D} (\delta(d,i_j)/d^{\alpha_3})$$

where

$$r(i_j,t) = M\ q^{\alpha_1}(i_j,t)\ s^{\alpha_2}(i_j,t)\ \ell(i_j) \quad .$$

As the quantities $r(i_j,t)$ are known, the pressure constraints can be expressed as linear constraints in the δ's:

$$\sum_{j=1}^{k} \sum_{d\in D} \delta(d,i_j)\ (r(i_j,t)/d^{\alpha_3}) \leq P_1^2 - P_0^2 \quad . \qquad (3.3.2)$$

We get one such constraint for each $i \in \theta$ and for each $t \in \{1, \ldots, T\}$.

The total cost of the network can also be expressed linearly in the δ's:

$$\text{COST} = \sum_{i\in\overline{N}} \sum_{d\in D} C(d)\ \ell(i)\ \delta(d,i) \qquad (3.3.3)$$

An optimal diameter assignment is therefore found by solving the Linear Program: Minimize (3.3.3) under the constraints (3.3.2) and (3.3.1). We will call this linear program LP.

If each link is required to have one diameter over its whole length then we must add the constraint:

$$\delta(d,i) = 0 \text{ or } 1$$

and the problem turns into a 0-1 Program. We call this integer program IP.

This linear programming approach has distinct advantages over the Dynamic Programming formulation.

1) It has the facility to assign over the length of one link two pipes of different diameter sizes connected in series,

2) As a grid of pressures is not considered it will give an accurate result,

3) In the Dynamic Programming model the number of possible inputs increases exponentially with T and thus becomes very large. It can therefore only be used for small values of T and the computational results we present are only for one year, i.e. T=1. The Linear Programming model can, however, easily handle the realistic horizon of T=10.

When Dynamic Programming can be used, however, it is faster than Linear Programming.

3.4 Properties of an optimal diameter assignment

In this section we assume that a link can consist of two or more pipe sizes connected in series. Thus, notation is that of the linear programming model.

We define the average diameter $\bar{d}$ of a link (a(j),j) by

$$1/\bar{d}^{\alpha_3} = \sum_{d \in D} (1/d^{\alpha_3})\ \delta(d,j) \quad .$$

Thus, a pipeline with diameter $\bar{d}$ would give the same pressure square drop on link (a(j),j) as the combination of pipes with diameter d on a fraction $\delta(d,j)$ ($\forall\ d \in D$) of the length of the link.

The cost per unit length, $C(\bar{d})$, corresponding to the average diameter $\bar{d}$ of link (a(j),j) is given by

$$C(\bar{d}) = \sum_{d \in D} C(d)\ \delta(d,j) \quad .$$

We will now state and prove some properties of an optimal diameter assignment.

1. *In an optimal diameter assignment at most two pipe sizes are used for one link.*

Consider the link (a(j),j) and the corresponding optimal diameter selection $\{\delta^*(d,j)\}$. We will show that the set

$$\{d \in D \mid \delta^*(d,j) \neq 0\}$$

contains at most 2 elements.

The average diameter $\bar{d}$ of the link is given by

$$1/\bar{d}^{\alpha_3} = \sum_{d \in D} (1/d^{\alpha_3})\ \delta^*(d,j) \quad .$$

Therefore, for link (a(j),j), $\{\delta^*(d,j)\}$ must be a solution of the following linear program in variables $\{\delta(d,j)\}$, $d \in D$.

$$\text{Minimize} \sum_{d \in D} C(d)\, \ell(j)\, \delta(d,j)$$

subject to

$$\left.\begin{aligned} &\sum_{d \in D} (1/d^{\alpha_3})\, \delta(d,j) = 1/\bar{d}^{\alpha_3} \\ &\sum_{d \in D} \delta(d,j) = 1 \end{aligned}\right\} \quad (3.4.1)$$

$$\delta(d,j) \geq 0 \quad , \quad \forall\, d \in D \quad .$$

Because the cost row is not proportional to either of the constraint rows, the minimum cannot occur at a non basic feasible solution of (3.4.1). Hence, the minimum occurs only at a basic feasible solution of (3.4.1). Thus, an optimal solution has at most two non-zero $\delta(d,j)$'s.

2. *If $C(x^{-1/\alpha_3})$ is a discretely strictly convex function of x, where $x \in \{d_1^{-\alpha_3}, d_2^{-\alpha_3}, \ldots, d_n^{-\alpha_3}\}$, then the following condition holds: If for any link, two pipe sizes are used in the optimal assignment, then they must be of consecutive diameter sizes.*

Consider link $(a(j),j)$ and let the corresponding optimal diameter selection have $\delta^*(d_1,j) \neq 0$, $\delta^*(d_2,j) \neq 0$. Let $\delta = \delta^*(d_1,j)$. Then $1 - \delta = \delta^*(d_2,j)$. The average diameter $\bar{d}$ of the link is given by

$$1/\bar{d}^{\alpha_3} = \delta/d_1^{\alpha_3} + (1-\delta)/d_2^{\alpha_3} \quad .$$

Therefore, for link $(a(j),j)$, the diameters d_1, d_2 must be the solution of the following problem,

$$\left.\begin{array}{l} \text{Minimize } \delta\, C(d_1) + (1-\delta)\, C(d_2) \\ \text{where } \delta \text{ is given by} \\ \delta/d_1^{\alpha_3} + (1-\delta)/d_2^{\alpha_3} = 1/\bar{d}^{\alpha_3} \quad . \end{array}\right\} \quad (3.4.2)$$

If C(d) is a discretely strictly convex function of $d^{-\alpha_3}$(or, equivalently, if $C(x^{-1/\alpha_3})$ is a discretely, strictly convex function of x, where $x \in \{d_1^{-\alpha_3}, \ldots, d_n^{-\alpha_3}\}$) it is obvious from (3.4.2) that it is cheapest to choose d_1, d_2 as the consecutive diameters between which $\bar{d}$ lies.

The condition that $C(x^{-1/\alpha_3})$ is a discretely, strictly convex function of x is not a restrictive condition as we now show. This condition is satisfied

1. if a cost function $C(d) = Kd^{\mu}$, where K, μ are positive constants, is considered.

2. if C(d) is a convex increasing function of d.

Finally we note that for the data we use in § 3.5 the condition is also satisfied.

3. *If the specific gravities are the same for all wells then there exists an optimal diameter assignment in which diameters do not decrease in the direction of flow.*

Proof. We will consider a section of the network as in Fig. (3.a). In this proof we define $\ell(i,j)$: distance between nodes i and j. Let, in year t, the flow on link (1,2) be $q_1(t)$ and the flow on link (2,3) be $q_2(t)$. Thus

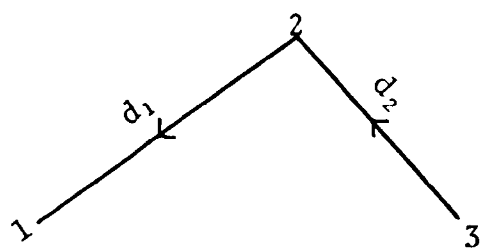


Fig. (3.a)

$$q_1(t) \geq q_2(t) \quad , \quad \forall\ t=1, \ldots, T \ .$$

Suppose in an optimal diameter assignment, the average diameter of link (1,2) is d_1 and that of (2,3) is d_2.

The proof is by construction.

Suppose $d_1 < d_2$.

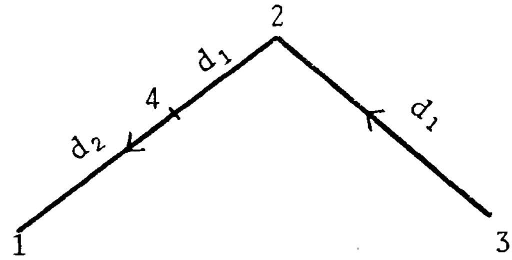


Fig. (3.b)

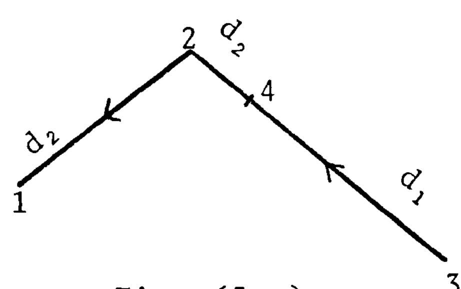


Fig. (3.c)

We select a node 4 either on link (1,2) as in Fig. (3.b), or on link (2,3) as in Fig. (3.c) and reverse the order of the diameters, i.e. assign diameter d_2 to the section of pipe between 4 and 1, and assign diameter d_1 to the section between 3 and 4. We retain the diameters of the optimal diameter assignment on all other links of the network. We call this diameter assignment the <u>reversed diameter assignment</u>. We define

$pp_0(i,j,t)$: pressure square drop between nodes i and j, in year t, in the optimal diameter assignment,

$pp_r(i,j,t)$: pressure square drop between nodes i and j, in year t, in the reversed diameter assignment,

$COST_0(i,j)$: cost of link (i,j), in optimal diameter assignment,

$COST_r(i,j)$: cost of link (i,j) in reversed diameter assignment.

We will construct a node 4 which has the following three properties P1, P2 and P3.

P1 : $pp_r(1,2,t) \leq pp_0(1,2,t) \quad , \quad \forall\ t$

and

P2 : $pp_r(2,3,t) + pp_r(1,2,t) \leq pp_0(2,3,t) + pp_0(1,2,t) \quad , \forall\ t$

i.e. for $\forall$ year t, the pressure square drop between nodes 2 and 1, and between nodes 3 and 1 for the reversed diameter assignment are not greater than the corresponding pressure square drops for the optimal diameter assignment. Also, since the pressure square drops for all other links remain the same as before, the pressure constraints of the network will not be violated under the reversed diameter assignment. Thus, pressure constraints are satisfied.

P3 : $COST_r(1,2) + COST_r(2,3) \leq COST_0(1,2) + COST_0(2,3) \quad ,$

i.e. cost of section considered is not increased.

Thus the reversed diameter assignment will be feasible and not more expensive than the optimal diameter assignment. This construction proves the result if we can show that such a node 4 exists. Now we will show how node 4 can be selected and thus prove that it does exist.

Reversing diameters as described above increases the average diameter of link (1,2). Since pressure square drop is a decreasing function of diameter, we have

$$pp_r(1,2,t) \leq pp_0(1,2,t) \quad , \quad \text{for } \forall \text{ year } t \quad .$$

Thus P1 holds.

To prove P2 and P3 we consider 2 cases.

Case 1

$$\frac{\ell(1,2)}{\ell(2,3)} \geq \frac{q_2^{\alpha_1}(t)\ s_2^{\alpha_2}(t)}{q_1^{\alpha_1}(t)\ s_1^{\alpha_2}(t)} \quad , \tag{3.4.3}$$

for $\forall$ year $t \in \{1, \ldots, T\}$.

We define node 4 by

$$\ell(1,4) = \ell(2,3) \max_{t \in \{1,\ldots,T\}} \frac{q_2^{\alpha_1}(t)\ s_2^{\alpha_2}(t)}{q_1^{\alpha_1}(t)\ s_1^{\alpha_2}(t)} . \tag{3.4.4}$$

Node 4 lies on link (1,2) because from (3.4.3) we see

$$\ell(1,4) \leq \ell(1,2) \quad .$$

From (3.4.4) we have

$$\ell(1,4) \geq \ell(2,3) \frac{q_2^{\alpha_1}(t)\ s_2^{\alpha_2}(t)}{q_1^{\alpha_1}(t)\ s_1^{\alpha_2}(t)} \quad , \quad \forall\ t$$

Multiplying both sides of the above inequality by $(1/d_1^{\alpha_3} - 1/d_2^{\alpha_3})$, it can be rewritten as

$$\ell(1,2) \frac{q_1^{\alpha_1}(t)\ s_1^{\alpha_2}(t)}{d_1^{\alpha_3}} + \ell(2,3) \frac{q_2^{\alpha_1}(t)\ s_2^{\alpha_2}(t)}{d_2^{\alpha_3}}$$

$$\geq \ell(1,4) \frac{q_1^{\alpha_1}(t)\ s_1^{\alpha_2}(t)}{d_2^{\alpha_3}} + \ell(4,2) \frac{q_1^{\alpha_1}(t)\ s_1^{\alpha_2}(t)}{d_1^{\alpha_3}}$$

$$+ \ell(2,3) \frac{q_2^{\alpha_1}(t)\ s_2^{\alpha_2}(t)}{d_1^{\alpha_3}} \quad , \quad \text{for } \forall \text{ year } t \quad ,$$

or, using (2.2.2) and (2.2.1)

$$pp_0(1,2,t) + pp_0(2,3,t) \geq pp_r(1,4,t) + pp_r(4,2,t) + pp_r(2,3,t),$$

for $\forall$ year t .

Thus P2 holds.

P3 remains to be shown. We have, for $\forall$ year t,

$$q_2(t) \leq q_1(t)$$

$$s_2(t) = s_1(t) \quad \text{(by assumption)}.$$

Thus,

$$\frac{q_2^{\alpha_1}(t)\ s_2^{\alpha_2}(t)}{q_1^{\alpha_1}(t)\ s_1^{\alpha_2}(t)} \leq 1 \quad , \quad \text{for } \forall\ t \tag{3.4.5}$$

Using (3.4.4) we have

$$\ell(2,3) - \ell(1,4) \geq 0$$

Also, by monotonicity of cost data

$$C(d_1) - C(d_2) < 0 \quad . \tag{3.4.6}$$

Thus,

$$(C(d_1) - C(d_2))\ (\ell(2,3) - \ell(1,4)) \leq 0$$

or,

$$\ell(1,4)\ C(d_2) + \ell(2,4)\ C(d_1) + \ell(2,3)\ C(d_1) \leq$$
$$\ell(1,2)\ C(d_1) + \ell(2,3)\ C(d_2)$$

or,

$$COST_r\ (1,4) + COST_r\ (4,2) + COST_r\ (2,3) \leq COST_0(1,2) + COST_0(2,3)$$

or,

$$COST_r\ (1,2) + COST_r\ (2,3) \leq COST_0(1,2) + COST_0(2,3) \quad .$$

Thus P3 holds.

<u>Case 2</u>

$$\frac{\ell(1,2)}{\ell(2,3)} < \frac{q_2^{\alpha_1}(t)\ s_2^{\alpha_2}(t)}{q_1^{\alpha_1}(t)\ s_1^{\alpha_2}(t)} \quad , \tag{3.4.7}$$

for some year $t \in \{1, \ldots, T\}$.

We define node 4 by

$$\ell(4,3) = \ell(1,2) \underset{t\in\{1,\ldots,T\}}{\text{Min.}} \frac{q_1^{\alpha_1}(t)\ s_1^{\alpha_2}(t)}{q_2^{\alpha_1}(t)\ s_2^{\alpha_2}(t)} \qquad (3.4.8)$$

Node 4 lies on link (2,3) because from (3.4.7) we see

$$\ell(4,3) \leqslant \ell(2,3) \quad .$$

From (3.4.8) we have

$$\ell(4,3) \leqslant \ell(1,2) \frac{q_1^{\alpha_1}(t)\ s_1^{\alpha_2}(t)}{q_2^{\alpha_1}(t)\ s_2^{\alpha_2}(t)} \quad , \quad \forall\ t \quad .$$

Multiplying both sides of the above inequality by $(1/d_1^{\alpha_3} - 1/d_2^{\alpha_3})$ it can be rewritten as

$$\ell(1,2) \frac{q_1^{\alpha_1}(t)\ s_1^{\alpha_2}(t)}{d_1^{\alpha_3}} + \ell(2,3) \frac{q_2^{\alpha_1}(t)\ s_2^{\alpha_2}(t)}{d_2^{\alpha_3}} \geqslant$$

$$\ell(1,2) \frac{q_1^{\alpha_1}(t)\ s_1^{\alpha_2}(t)}{d_2^{\alpha_3}} + \ell(2,4) \frac{q_2^{\alpha_1}(t)\ s_2^{\alpha_2}(t)}{d_2^{\alpha_3}} +$$

$$\ell(4,3) \frac{q_2^{\alpha_1}(t)\ s_2^{\alpha_2}(t)}{d_1^{\alpha_3}} \quad , \quad \text{for } \forall \text{ year } t$$

or, using (2.2.2) and (2.2.1)

$$pp_0(1,2,t) + pp_0(2,3,t) \geqslant pp_r(1,2,t) + pp_r(2,4,t) + pp_r(4,3,t),$$
$$\text{for } \forall \text{ year } t \quad .$$

Thus P2 holds.

To show P3, (3.4.5) and (3.4.6) hold as in Case 1. Thus, by (3.4.8)

$$\ell(4,3) - \ell(1,2) \geqslant 0$$

and

$$(C(d_1) - C(d_2))\ (\ell(4,3) - \ell(1,2)) \leqslant 0 \quad ,$$

or,

$$\ell(1,2)\ C(d_2) + \ell(2,4)\ C(d_2) + \ell(4,3)\ C(d_1) \leq \ell(1,2)\ C(d_1) + \ell(2,3)\ C(d_2)$$

or,

$$COST_r(1,2) + COST_r(2,4) + COST_r(4,3) \leq COST_0(1,2) + COST_0(2,3)$$

or

$$COST_r(1,2) + COST_r(2,3) \leq COST_0(1,2) + COST_0(2,3)$$

Thus P3 holds.

This completes the proof.

***

Note: When the Weymouth formula is used the above result is true even if the specific gravity at the various wells are not the same.

The assumption that the specific gravity of gas at all the wells are the same was necessary only for the inequality (3.4.5) to hold. For the Weymouth formula $\alpha_1 = 2$, $\alpha_2 = 1$, let, in year t, $q_3(t)$ and $s_3(t)$ be the rate and specific gravity of gas flowing into node 2 other than through link (3,2). Thus,

$$q_1(t) = q_2(t) + q_3(t) \quad , \quad \forall\ t$$

and

$$s_1(t) = \frac{q_2(t)\ s_2(t) + q_3(t)\ s_3(t)}{q_2(t) + q_3(t)} \quad , \quad \forall\ t$$

Hence (3.4.5) reduces to

$$q_2(t)\ q_3(t)\ s_3(t) + q_3(t)\ (q_2(t)\ s_2(t) + q_3(t)\ s_3(t)) \geq 0$$

which is obviously true as flows and specific gravities are non-negative quantities.

3.5 MOOMBA GAS FIELD: Computational Results

The models have been tested on data from the Moomba Gas field in the South Cooper Basin in the desert center of Australia. This gas field, consisting of 13 gas wells is being developed and is spread over an area of about 4,000 square miles. A map of the gas wells is given in Figure 3.1. In the next 10 years a network of pipes has to be constructed to transport gas from the wells to the factory prior to entry into a trunk main for onward transmission. The critical gas wells which are now under active consideration are the wells numbered 0 to 8. We restrict our computational results to this section of the gas field.

The data relating to the Moomba field is given in Appendix III. The units used are: flow in cu. ft. per 24 hours, pressure in lbs. per sq. inch absolute, length of links in miles, diameter of pipes in inches. An estimated rate of raw gas production during the years 1975-1989 are given in Table III.1. The full well stream gas compositions for the various wells and the specific gravities of the components is given in Table III.2. The specific gravity of emerging gas is the weighted specific gravity of the components.

The pressure limits P_0 and P_1 are given to be 1115 and 1185 lbs./sq. inch respectively. There are 19 pipe sizes, each size being characterised by its internal diameter and the associated cost per mile. This is given in Table III.3.

The Weymouth formula is

$$\frac{pp}{\ell} = f(q,s,d) = M \frac{q^2 s}{d^{5\frac{1}{3}}}$$

The constant M is given by

$$M = \frac{1}{(433.45)^2} \left(\frac{P_s}{T_s}\right)^2 T$$

where

T is absolute temperature of flowing gas (°F + 460),

T_s is standard absolute temperature,

P_s is standard pressure (lbs. per sq. inch absolute).

These constants are in our case $T_s = 520$, $T = 560$ and $P_s = 14.65$. The computer we use is a C.D.C. Cyber 173.

Results for the Dynamic Programming Model

In this approach, the accuracy of the result depends on how fine the grid of pressures chosen is. But, increasing the fineness of the grid also increases the time taken by the program. Thus, it is a question of balancing accuracy against time.

We have considered 3 grid sizes, grids of 20, 40 and 80 in the 70 lb. range between 1115 and 1185 lbs.

We give below, as examples, two trees on which the model has been tried. The year 1986 was chosen as this is a year when there is fairly good flow in all the wells.

Example 3.5.1. The tree considered is shown in Figure 3.2. Table 3.1 gives the cost of and time needed to solve the problem for this network for the 3 grid sizes. Table 3.2 gives the diameters assigned to the various links in the 3 cases.

Example 3.5.2. Figure 3.3, Tables 3.3 and 3.4 give the corresponding results for this tree.

Results for Linear and Integer Programming Models

To solve the linear program LP and the integer program IP, we used APEX III, a large Control Data Corporation Linear Programming package at the University of Adelaide. APEX III has mixed integer facilities and uses a branch and bound technique for integer programming. We solved both LP and IP, for a wide variety of trees, both for $T = 1$ (the year 1986) so it could be compared with the Dynamic Programming results and also for $T = 10$ (the period 1980-1989).

We would like to mention here that for all optimal diameter assignments we obtained by solving LP, diameters never decreased in the direction of flow.

We again consider the same two trees as in Examples 3.5.1 and 3.5.2 and this time apply LP and IP on them.

Example 3.5.1'. The tree is given in Figure 3.2. Table 3.5 gives the cost of this tree and the time taken by the programs when $T = 1$ and when $T = 10$. The various diameters assigned by the various programs are given in Table 3.6.

Example 3.5.2'. Tables 3.7 and 3.8 give the corresponding results for the tree in Figure 3.3.

Comparison of Methods

1. It can be seen that the results of IP for T = 1 are cheaper than the results of the Dynamic Program even for a grid of 80. Thus, the dynamic programming formulation did not reach the exact optimal diameter selection for reasonable grid sizes.

2. For using APEX III the data has to be in MPS format as required by the system and this involves calculating the various coefficients in the Linear Programming model. Thus a matrix generator program has to be used to solve LP and IP. The execution times given in the tables does not include the matrix generation times.

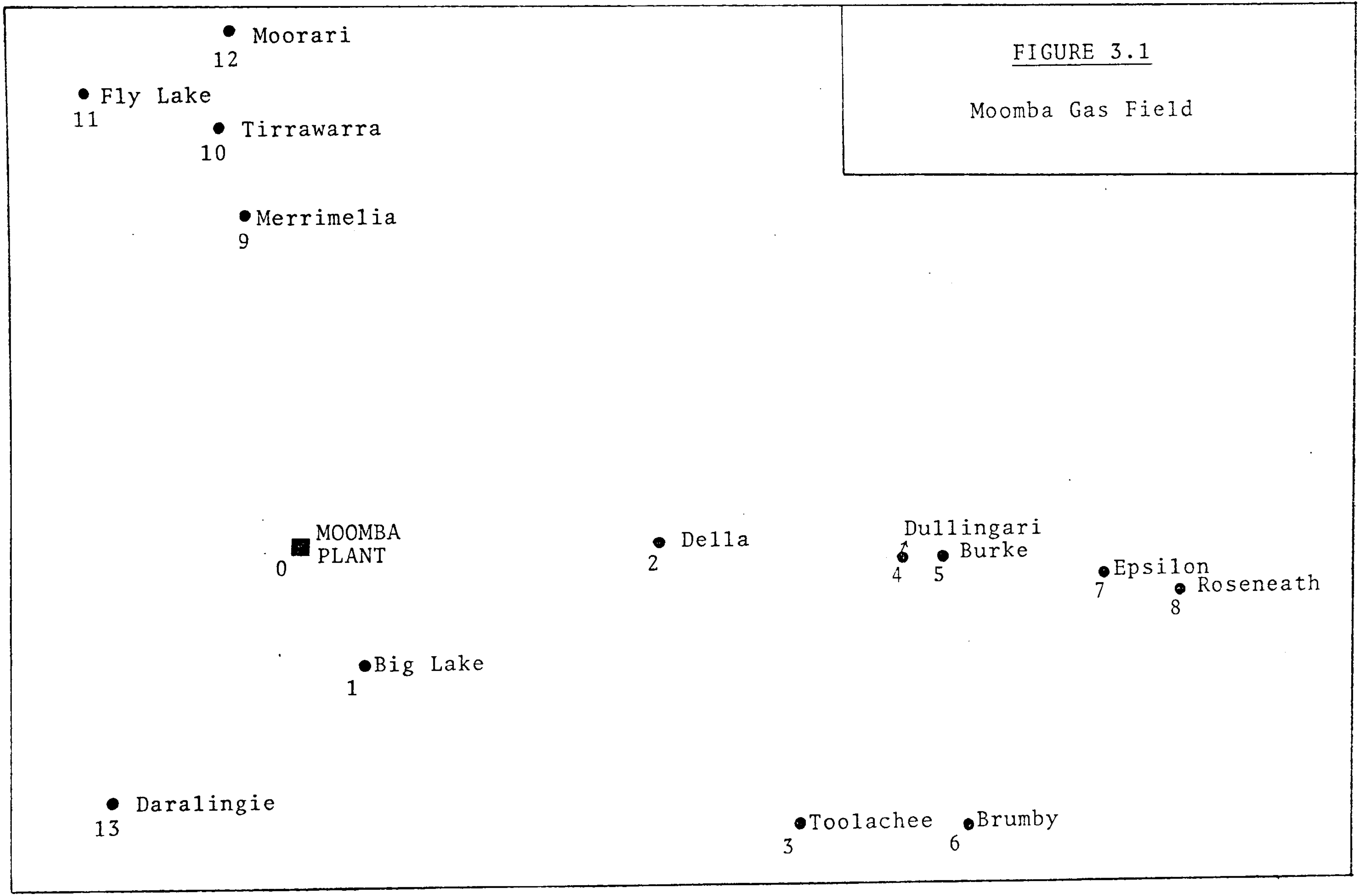


FIGURE 3.1

Moomba Gas Field

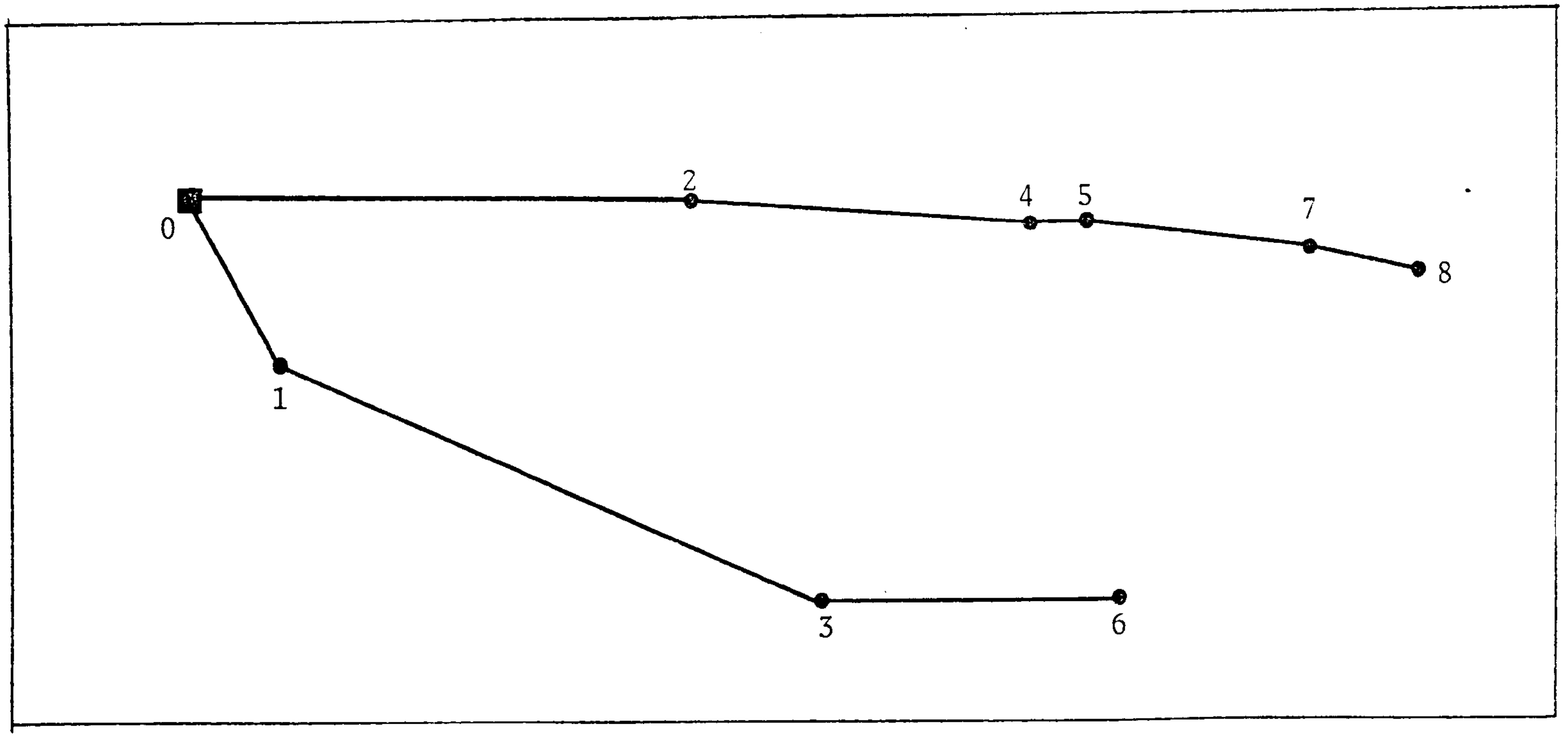


FIGURE 3.2 Tree considered for Diameter Assignment in Example 3.5.1 and in Example 3.5.1'.

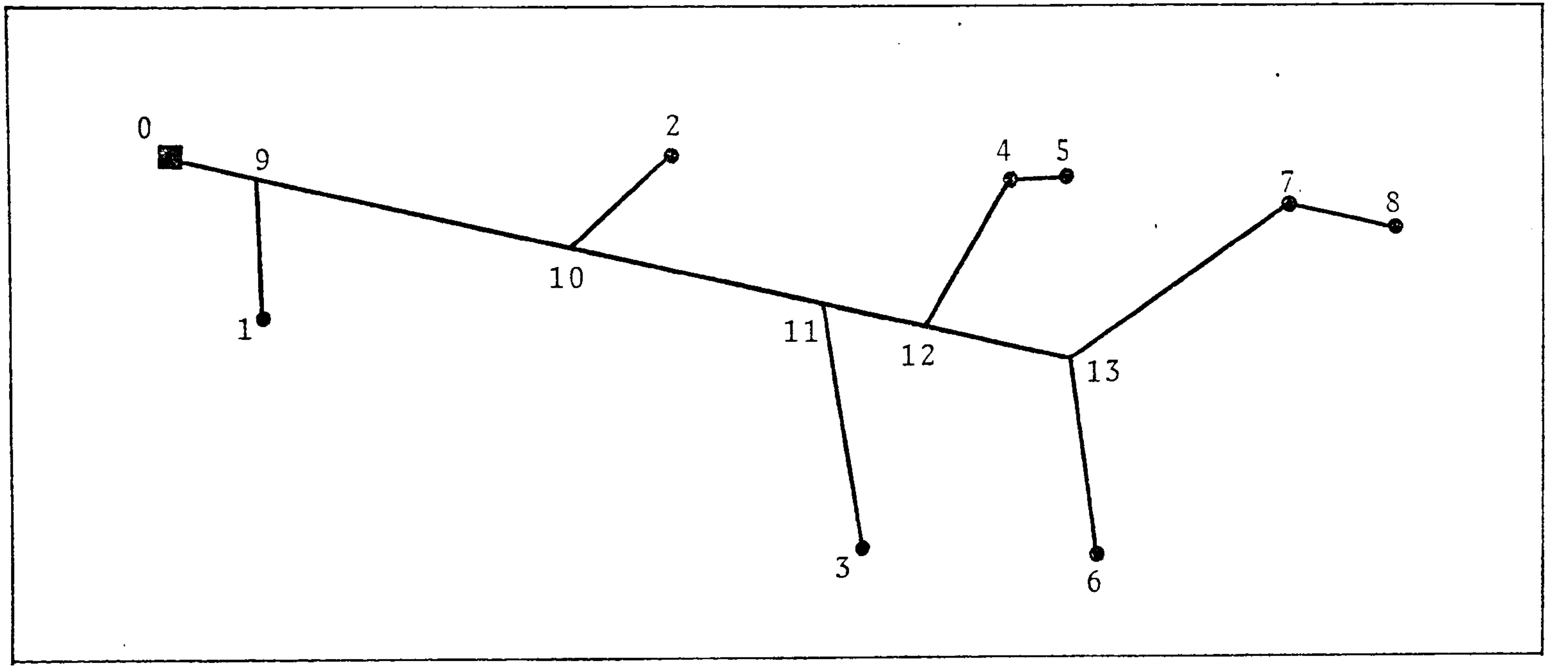


FIGURE 3.3 Tree considered for Diameter Assignment in Example 3.5.2 and in Example 3.5.2'.

TABLE 3.1

D.P. MODEL

GRID SIZE, COST OF NETWORK, AND TIME TAKEN FOR TREE IN FIGURE 3.2

GRID SIZE	COST OF NETWORK ($)	TIME TAKEN (C.P. SECS)
20	36,914,851	1.3
40	36,713,408	2.7
80	36,531,885	8.1

TABLE 3.2

D.P. MODEL

PIPE SIZES ASSIGNED TO LINKS OF TREE IN FIGURE 3.2

LINK	PIPE SIZES ASSIGNED		
	GRID OF 20	GRID OF 40	GRID OF 80
(0,1)	13	13	14
(0,2)	18	18	18
(1,3)	12	12	12
(2,4)	14	13	13
(3,6)	6	6	5
(4,5)	10	12	11
(5,7)	10	10	10
(7,8)	3	4	4

TABLE 3.3

D.P. MODEL

GRID SIZE, COST OF NETWORK AND TIME TAKEN FOR TREE IN FIGURE 3.3

GRID SIZE	COST OF NETWORK ($)	TIME TAKEN (C.P. SECS)
20	35,011,277	1.8
40	33,849,359	4.4
80	33,769,466	14.1

TABLE 3.4

D.P. MODEL

PIPE SIZES ASSIGNED TO LINKS OF TREE IN FIGURE 3.3

LINK	PIPE SIZES ASSIGNED		
	GRID OF 20	GRID OF 40	GRID OF 80
(0,9)	18	19	19
(9,1)	5	5	5
(9,10)	19	19	19
(10,2)	11	11	11
(10,11)	18	19	19
(11,3)	11	10	10
(11,12)	16	16	16
(12,4)	11	10	10
(12,13)	13	11	12
(4,5)	7	7	7
(13,6)	6	6	5
(13,7)	11	10	10
(7,8)	3	4	3

TABLE 3.5

L.P. MODEL

COST OF NETWORK AND TIME TAKEN FOR TREE IN FIGURE 3.2

NUMBER OF YEARS	PROGRAM	COST OF NETWORK ($)	TIME TAKEN (C.P. SECS)
T=1	L.P.	36,118,307	2.1
	I.P.	36,429,252	16.3
T=10	L.P.	37,793,435	4.1
	I.P.	38,041,214	23.1

TABLE 3.6

L.P. MODEL

PIPE SIZES ASSIGNED TO LINKS OF TREE IN FIGURE 3.2

LINK	PIPE SIZES ASSIGNED TO LINK OR PROPORTION OF LINK			
	T = 1		T = 10	
	L.P.	I.P.	L.P.	I.P.
(0,1)	13	14	15(66%),16(34%)	16
(0,2)	17(59%),18(41%)	18	17(18%),18(82%)	18
(1,3)	11(27%),12(73%)	12	13	13
(2,4)	13	13	13	13
(3,6)	6	5	6	6
(4,5)	12	14	11	10
(5,7)	10	9	9(29%),10(71%)	10
(7,8)	4	4	4	4

TABLE 3.7

L.P. MODEL

COST OF NETWORK AND TIME TAKEN FOR TREE IN FIGURE 3.3

NUMBER OF YEARS	PROGRAM	COST OF NETWORK ($)	TIME TAKEN (C.P. SECS)
T=1	L.P.	32,964,110	3.87
	I.P.	33,515,679	17.73
T=10	L.P.	34,559,858	10.4
	I.P.	35,131,750	54.33

TABLE 3.8

L.P. MODEL

PIPE SIZES ASSIGNED TO LINKS OF TREE IN FIGURE 3.3

LINK	PIPE SIZES ASSIGNED TO LINK OR PROPORTION OF LINK			
	T = 1		T = 10	
	L.P.	I.P.	L.P.	I.P.
(0,9)	19	19	19	19
(9,1)	4(7%),5(93%)	5	8(51%),9(49%)	9
(9,10)	19	19	19	19
(10,2)	10(94%),11(6%)	11	10(1%),11(99%)	11
(10,11)	19	18	19	19
(11,3)	9(18%),10(82%)	10	10(34%),11(66%)	11
(11,12)	16	16	15(35%),16(65%)	16
(12,4)	9(35%),10(65%)	10	10(78%),11(22%)	10
(12,13)	11	12	11	12
(4,5)	7	7	7	8
(13,6)	5	5	5	5
(13,7)	9(58%),10(42%)	10	9(57%),10(43%)	9
(7,8)	4	3	4	5

3.6 Series Network

The only situation for which an explicit formula for the optimal diameters can be obtained is when we deal with a series network, and in addition we assume that the diameter variable is continuous in the range $[0,\infty)$ and the cost function C is of the form

$$C(d) = K\,d^{\mu}$$

Also, only one time period is considered, i.e. T=1.

In a series network the pipes are connected end to end as shown in Fig.(3.d), i.e. every node i=0,1, ..., n-1 has only one successor. The problem is to determine diameters d(i), i=1, ..., n which

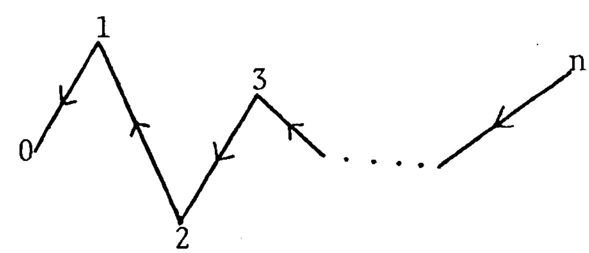


Fig. (3.d)

$$\text{Minimize COST} = \sum_{i=1}^{n} \ell(i)\; C(d(i)) \tag{3.6.1}$$

subject to

$$\sum_{i=1}^{n} pp(i) \leq P^* \quad , \tag{3.6.2}$$

where

$$P^* = P_1{}^2 - P_0{}^2 \quad ,$$

and

$$d(i) = h(q(i),\, s(i),\, \ell(i),\, pp(i))$$

Since COST is a decreasing function of the pressure square drops pp(i), it is trivial to see that, at optimality, (3.6.2) is satisfied as an equality. Thus

$$pp(n) = p^* - \sum_{i=1}^{n-1} pp(i) \quad .$$

Substituting

$$d = h(q,s,\ell,pp) = N\, q^{\beta_1}\, s^{\beta_2} \left(\frac{\ell}{pp}\right)^{\beta_3} \qquad (3.6.3)$$

in the stationary conditions

$$\frac{\partial\ COST}{\partial\ pp(1)} = \ldots = \frac{\partial\ COST}{\partial\ pp(n-1)} = 0 \quad ,$$

we have:

$$\left[\frac{pp(i)}{pp(n)}\right]^{\beta_3+1} = \frac{C'(d(i))\ q^{\beta_1}(i)\ s^{\beta_2}(i)\ \ell^{\beta_3+1}(i)}{C'(d(n))\ q^{\beta_1}(n)\ s^{\beta_2}(n)\ \ell^{\beta_3+1}(n)} \quad .$$

For general cost functions this does not give pp(i) as an explicit function of the flows, specific gravities and lengths. But, for $C(d) = Kd^{\mu}$, we have

$$pp(i) = \frac{q^{\frac{\mu_1}{\mu_3+1}}(i)\ s^{\frac{\mu_2}{\mu_3+1}}(i)\ \ell(i)}{\sum_{j=1}^{n} q^{\frac{\mu_1}{\mu_3+1}}(j)\ s^{\frac{\mu_2}{\mu_3+1}}(j)\ \ell(j)}\, p^* \quad , \quad i=1, \ldots, n \qquad (3.6.4)$$

where

$$\mu_i = \mu\, \beta_i \quad , \quad i=1,2,3 \quad .$$

Using (3.6.3) we find the optimal diameters:

$$d(i) = \frac{N}{(p^*)^{\beta_3}}\, q^{\frac{\beta_1}{\mu_3+1}}(i)\ s^{\frac{\beta_2}{\mu_3+1}}(i)\ \left(\sum_{i=1}^{n} q^{\frac{\mu_1}{\mu_3+1}}(i)\ s^{\frac{\mu_2}{\mu_3+1}}(i)\ \ell(i)\right)^{\beta_3}$$

Since COST is a convex function of the variables pp(i), i=1, ..., n-1, as will be shown in § 4.1, these diameters do indeed give the minimum cost.

Thus, we see that in the case of series networks, for the cost function $C(d) = Kd^{\mu}$, the diameter problem can be completely solved by purely analytic methods if $T = 1$.

For a branching network, however, the equations corresponding to the stationary conditions cannot be solved and no explicit solutions for the diameter problem can therefore be obtained.

CHAPTER 4

JUNCTION LOCATION AND CONFIGURATION

This chapter discusses the geometrical properties of an optimal pipeline tree-network; in our terminology, the Junction Location Problem and the Configuration Problem. Throughout the chapter we assume d to be a continuous variable, $0 \leq d < \infty$ and as in (2.1.1)

$$C(d) = K\, d^{\mu} \tag{4.0.1}$$

where K, μ are positive constants. Only one year will be considered, i.e. in the notation of Chapter 3 we take T=1, and therefore the notation of Chapter 2 is valid.

In § 4.1 we formulate the junction location problem and show it to be a convex programming problem. In § 4.2 we present a method of solution of the junction location problem together with computational results; other methods of solution are given in § 4.3. In §'s 4.4 and 4.5 we analyse in depth the junction location problem in the case $|N_f| = 3$. In § 4.4 we prove 4 properties of the angles at an optimally located junction node. In § 4.5 we prove 2 more properties related to the solution of the problem and present empirical results based on a parameter study.

§ 4.6 states the Configuration Problem and establishes its complexity.

4.1 The Junction Location Problem : Convexity

For the junction location problem we assume that the configuration is fixed, i.e. the number of junction nodes $|N_s|$ is given and so is the predecessor function a. Thus the rate of gas flow and specific gravity of flowing gas on each of the links is known. Still to be determined are the positions $(x(i),y(i))$, $i \in N_s$, of the junction nodes and the diameters $d(i)$ for all links $(a(i),i)$. The positions of nodes determine the lengths of links. The length of link $(a(i),i)$, is as in (2.4.2) given by

$$\ell(i) = \sqrt{[(x(i) - x(a(i)))^2 + (y(i) - y(a(i)))^2]} \ . \qquad (4.1.1)$$

From formula (2.2.3) we know that the diameters can be expressed in terms of the pressure square drops and lengths:

$$d(i) = h(q(i),\ s(i),\ \ell(i),\ pp(i)) \ . \qquad (4.1.2)$$

The diameters are therefore completely determined by the $\ell(i)$'s and the $pp(i)$'s as flows and specific gravities are fixed. For simplicity we define

$$d(i) = g_i\left(\frac{\ell(i)}{pp(i)}\right) = N\ q^{\beta_1}(i)\ s^{\beta_2}(i)\left(\frac{\ell(i)}{pp(i)}\right)^{\beta_3} \qquad (4.1.3)$$

where we have used expression (2.2.4) for h.

The variables of our problem are therefore

$$x(i),\ y(i) \quad , \quad \forall\ i \in N_s$$

$$pp(i) \quad , \quad \forall\ i \in \overline{N} \ .$$

The objective is

$$\text{Minimize COST} = \sum_{i \in \overline{N}} \ell(i)\ C\left(g_i\left(\frac{\ell(i)}{pp(i)}\right)\right) \qquad (4.1.4)$$

where $\ell(i)$ is defined in (4.1.1), and the constraints as in (2.4.7) are: for every $i \in \theta$ (the extreme nodes)

$$\sum_{j=0}^{k} pp(i_j) \leq P^* \tag{4.1.5}$$

where

$$P^* = P_1^2 - P_0^2$$

and

$$0 = i_0, i_1, \ldots, i_k = i$$

is the path linking node i with node 0, and

$$pp(i) \geq 0 \quad , \quad \forall\, i \in \overline{N} .$$

As COST is a monotonically decreasing function of the variables pp(i), at optimality, inequality (4.1.5) is always satisfied as a strict equality. Thus (4.1.5) becomes

$$\sum_{j=0}^{k} pp(i_j) = P^* \quad . \tag{4.1.6}$$

The junction location problem thus includes the diameter assignment problem. However, devising methods to solve the junction location problem does not make Chapter 3 redundant. This is because assumption (4.0.1) which is made in this Chapter was not made in Chapter 3, and in Chapter 3 we did not restrict ourselves to one time period. Thus the methods of Chapter 3 are more general and the diameter problem is solved more accurately.

Theorem 4.1. COST = $\sum_{i \in \overline{N}} \ell(i)\, C\left(g_i\left(\frac{\ell(i)}{pp(i)}\right)\right)$ *is a convex function of the variables* $x(i)$, $y(i)$, *for* $i \in N_s$ *and* $pp(i)$, *for* $i \in \overline{N}$.

We first prove a lemma.

Lemma: $\phi = \ell\, C\left(g\left(\frac{\ell}{pp}\right)\right)$, *where* $\ell = \sqrt{[(z_1-z_2)^2 + (z_3-z_4)^2]}$ *is a convex function of the variables* pp *and* z_i, i=1, ..., 4 *if* $F(t) = t\, C(g(t))$ *is a convex, monotonically increasing, twice differentiable function of t.*

Proof of Lemma. We use the following result: If $f(z_1,\ldots z_n)$ is a twice differentiable function on an open convex set D, then it is convex on D iff the quadratic form

$$QQ(f) = \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial^2 f}{\partial z_i\, \partial z_j} \lambda_i\, \lambda_j \geq 0 \quad , \quad \forall\, \lambda_i\, ,\, \lambda_j$$

for every point in D.

Thus the quadratic form to be considered for ϕ is

$$QQ(\phi) = \sum_{i=1}^{4} \alpha_i^{\,2} \frac{\partial^2\phi}{\partial z_i^{\,2}} + \gamma^2 \frac{\partial^2\phi}{\partial pp^2} + 2 \sum_{i\neq j} \alpha_i\, \alpha_j \frac{\partial^2\phi}{\partial z_i\, \partial z_j} + 2\gamma \sum_{i=1}^{4} \alpha_i \frac{\partial^2\phi}{\partial pp\, \partial z_i} \tag{4.1.7}$$

where γ and α_i, i=1, ..., 4 are multipliers.

Now

$$\phi(z_1, \ldots, z_4, pp) = pp\, F(\ell/pp)$$

hence

$$\frac{\partial^2\phi}{\partial z_i\, \partial z_j} = \frac{\partial^2 \ell}{\partial z_i\, \partial z_j} F'(\ell/pp) + \frac{1}{pp} \frac{\partial \ell}{\partial z_i} \frac{\partial \ell}{\partial z_j} F''(\ell/pp) \quad ,$$

$$\frac{\partial^2\phi}{\partial pp^2} = \frac{\ell^2}{pp^3} F''(\ell/pp) \quad ,$$

and

$$\frac{\partial^2\phi}{\partial pp\, \partial z_i} = - \frac{\partial \ell}{\partial z_i} \frac{\ell}{pp^2} F''(\ell/pp) \quad .$$

Substituting these values in (4.1.7) we find

$$QQ(\phi) = F'(\ell/pp)\ QQ(\ell) + \frac{1}{pp} F''(\ell/pp) \left(\sum_{i=1}^{4} \alpha_i \frac{\partial \ell}{\partial z_i} - \frac{\gamma \ell}{pp} \right)^2 .$$

Since ℓ is a convex function of the z_i 's, we have

$$QQ(\ell) \geqslant 0 \quad ,$$

and the conditions of the lemma imply

$$F'(\ell/pp) \geqslant 0 \text{ and } F''(\ell/pp) \geqslant 0 \quad .$$

Hence,

$$QQ(\phi) \geqslant 0 \quad .$$

This proves the lemma.

***

Proof of Theorem 4.1. We will prove convexity of every term of (4.1.4). Then, (4.1.4), a sum of convex functions will be convex.

There are 3 types of links,

Type 1: A link connecting fixed nodes, i.e. a link between gas wells or a well and the plant.

Type 2a: A link draining gas from a fixed to a junction node.

Type 2b: A link draining gas from a junction to a fixed node.

Type 3: A link connecting junction nodes.

We will prove convexity for a link of Type 3, and proof for other types will follow as a special case.

We consider the link (a(i),i).

Let

$$F_i(t) = t\ C(g_i(t)) \ . \qquad (4.1.8)$$

From (4.1.3) we have

$$g_i(t) = N_i \ t^{\beta_3} \qquad (4.1.9)$$

where

$$N_i = N\ q^{\beta_1}(i)\ s^{\beta_2}(i) \ . \qquad (4.1.10)$$

Substituting (4.1.9) and (4.0.1) in (4.1.8) we have

$$F_i(t) = (K\ N_i^{\ \mu})\ (t^{1+\beta_3\mu}), \text{ where } K,\ N_i > 0 \ .$$

As $\beta_3\ \mu \geq 0$ we have

$$\frac{d^2F_i(t)}{dt^2} = K\ N_i^{\ \mu}\ (1+\beta_3\mu)\ \beta_3\ \mu\ t^{\beta_3\mu-1} \geq 0, \text{for } t \geq 0 \ .$$

Hence $F_i(t)$ is a convex, increasing function of t, for $t \geq 0$.

By the Lemma, the term of (4.1.4) corresponding to the link (a(i),i) is convex.

This proves the Theorem.

***

Hence, the Junction Location Problem is to minimize the convex function (4.1.4) subject to the linear constraints (4.1.6). Thus the problem of dealing with local minima does not arise and there is only one minimum - the global minimum.

4.2 Solution of the Junction Location Problem : Computational Results

Substituting for d and C(d) from (4.1.3) and (4.0.1) in (4.1.4) the objective function of the junction location problem can also be written as

$$\text{COST} = \sum_{i \in \overline{N}} c_i \frac{\ell^{1+\mu_3}(i)}{pp^{\mu_3}(i)} \tag{4.2.1}$$

where

$$c_i = K\, N^{\mu}\, q^{\mu_1}(i)\, s^{\mu_2}(i) \quad , \quad i \in \overline{N} \tag{4.2.2}$$

and

$$\mu_i = \mu\, \beta_i \quad , \quad i=1, \ldots, 3 \quad .$$

The quantities c_i , $i \in \overline{N}$ depend on the flows and specific gravities and hence are known.

In this section we use unconstrained programming to solve the problem and present computational results.

Substituting

$$pp(i_k) = P^* - \sum_{j=0}^{k-1} pp(i_j)$$

for every $i_k \in \theta$ from (4.1.6) in (4.2.1) the constraints reduce to

$$\left.\begin{aligned} P^* - \sum_{j=0}^{k-1} pp(i_j) &\geq 0 \quad , \quad \forall\, i_k \in \theta \\ pp(i) &\geq 0 \quad , \quad \forall\, i \in \overline{N} - \theta \quad . \end{aligned}\right\} \tag{4.2.3}$$

Reducing Problems and Degenerate Configurations

If in the optimal solution of the junction location problem $\ell(i) = 0$ for some i, we call it a reducing problem. A problem that is not reducing is called nonreducing.

As in Steiner's problem [10], an upper bound on the number of junction nodes is given by

$$|N_s| \leq |N_f| - 2 \quad .$$

Configurations with $|N_s| = |N_f| - 2$ are called full configurations, and those with $|N_s| < |N_f| - 2$ are called degenerate configurations. A degenerate configuration can be obtained either from a full configuration or from another degenerate configuration by shifting the position of a junction node to coincide with an adjacent node, thus reducing to zero the length of the link in between, i.e. by identifying 2 nodes and deleting a link. When a junction location problem is reducing, its optimal solution has degenerate configuration.

For a nonreducing problem starting from a point where

$p^* - \sum_{j=0}^{k-1} pp(i_j) > 0$ for $\forall\, i_k \in \theta$ and $pp(i) > 0$, $\forall\, i \in \bar{N}/\theta$ and

using an iterative procedure to improve the solution, the boundaries of (4.2.3), i.e.

$$p^* - \sum_{j=0}^{k-1} pp(i_j) = 0 \text{ and } pp(i) = 0$$

are never reached let alone crossed for any $i_k \in \theta$ or any $i \in \bar{N}/\theta$ because COST is ∞ at the boundary. Hence the inequalities (4.2.3) are redundant.

We deal with reducing problems as follows: Every reducing problem is a nonreducing problem for some degenerate configuration. The procedure we use is that when during the computations, for some i, $\ell(i)$ becomes less than a pre-specified small positive number ε and continues to remain so, we change the configuration to the new degenerate configuration that is approached and then solve the junction location problem for this configuration. Thus we restrict our computation to solving nonreducing problems.

We cannot avoid changing configurations as explained above, because for reducing problems, we cannot guarantee that the boundaries of (4.2.3) are not crossed if $\ell(i) \to 0$ for some i. This is because $\frac{\ell^{1+\mu_3}}{pp^{\mu_3}}$ may not become arbitrarily large as ℓ, pp tend to zero. Hence the constraints (4.2.3) cannot be ignored in the case of reducing problems.

Thus, for nonreducing problems we have an unconstrained problem in the variables

$$x(i), y(i) \quad , \quad i \in N_s$$

and

$$pp(i) \quad , \quad \underline{i \in \bar{N}/\theta} \quad .$$

The objective is differentiable and methods of unconstrained nonlinear programming utilising derivatives can be used. We have used the conjugate direction technique of Fletcher and Reeves (§ 3.6 in [11]) combined with the Fibonacci one dimensional search to solve this problem. We wrote a computer program to implement this technique.

Computational results

For the computer program a starting solution had to be provided and convergence criteria specified. The program was a convenient tool to study the junction location problem. Using it we could solve a real size problem (approximately $|N_f| = 9$, $|N_s| = 6$ or 7) in about 10 secs. Thus it was fast enough to run interactively.

We found that in the neighbourhood of the optimal junction locations, small changes in the positions of the junction nodes does not produce a great change in the cost.

Thus, for practical purposes undue importance need not be given to the exact location of the junction nodes.

We present two examples. The data is from the Moomba gas field, the year considered is 1986 and the Weymouth formula (2.2.5) is used. The values of K and μ we use (obtained by a least squares fit) are K = 4603.4 and μ = 1.28. In both examples we graphically present both the starting solutions and the optimal solution, and display the difference in costs. The proportion of the total pressure square drop P* that occurs over the various links in both the starting solutions and the optimal solution are presented in tabular form.

<u>Example 4.2.1</u>. $N_f = \{0,1, \ldots, 8\}$. This example displays the convexity of the junction location problem by showing two different starting solutions which give the same optimal solution. The number and positions of the junction nodes for the two starting solutions are shown in Figure 4.1(a) and Figure 4.1(b) and for the optimal solution in Figure 4.1(c). The proportion of the total pressure square drop that occurs over the various links in the different solutions are shown in Table 4.1.

It is noted that the assumed configuration in Figure 4.1(a) is not the same as in Figure 4.1(c). The problem was reducing and in the computations node 10 and 10' approached each other, so did nodes 11 and 11', node 12 approached node 5 and node 13 approached node 7. We changed the configuration by identifying the various pairs of nodes made the problem nonreducing and solved it to obtain Figure 4.1(c).

Similarly, with starting Solution 2 nodes 12 and 5, and nodes 13 and 7 were identified. The time taken to solve the junction location problem with Starting Solution 1 was 8.1 C.P. secs. and with Starting Solution 2 was 6.2 C.P. secs.

Example 4.2.2. $N_f = \{0,1, \ldots, 8\}$. We start with the network shown in Figure 4.2(a). On solving the junction location problem (which took 12.3 C.P. secs.), the new positions of the junction nodes are shown in Figure 4.2(b). The proportion of the total pressure square drop that occurs over the various links in the two solutions are shown in Table 4.2. This network was in fact the cheapest one we could construct for this section of the Moomba field. We note that theoretically as will be shown in §'s 4.5 and 4.6 the nodes 13 and 4 and also the nodes 14 and 5, and nodes 15 and 7 cannot coincide in the optimal configuration. However, these links become so small, that for both practical and computational purposes we may allow them to vanish.

Starting Solution 1
COST = $54.90 million

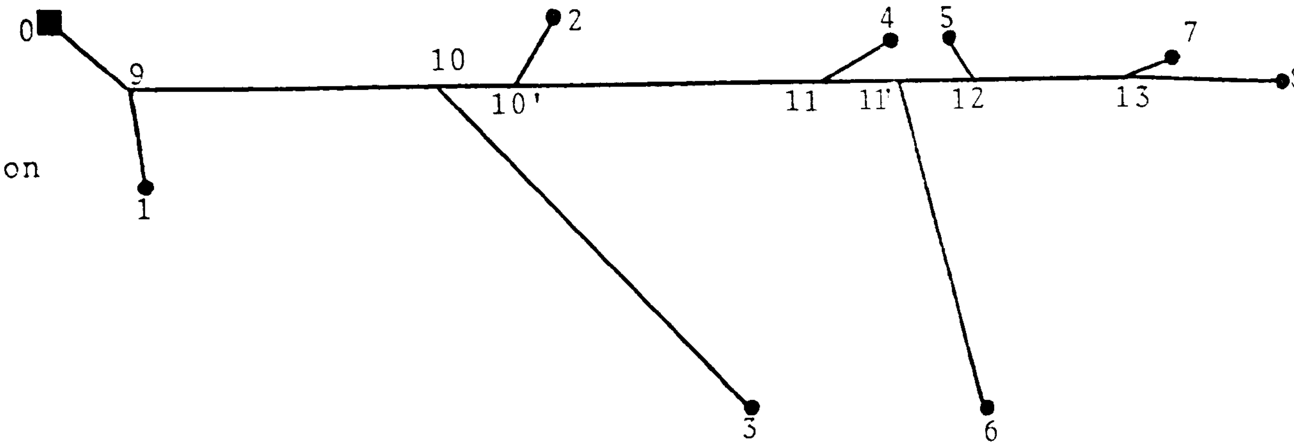


FIGURE 4.1(a)

Starting Solution 2
COST = $44.19 million

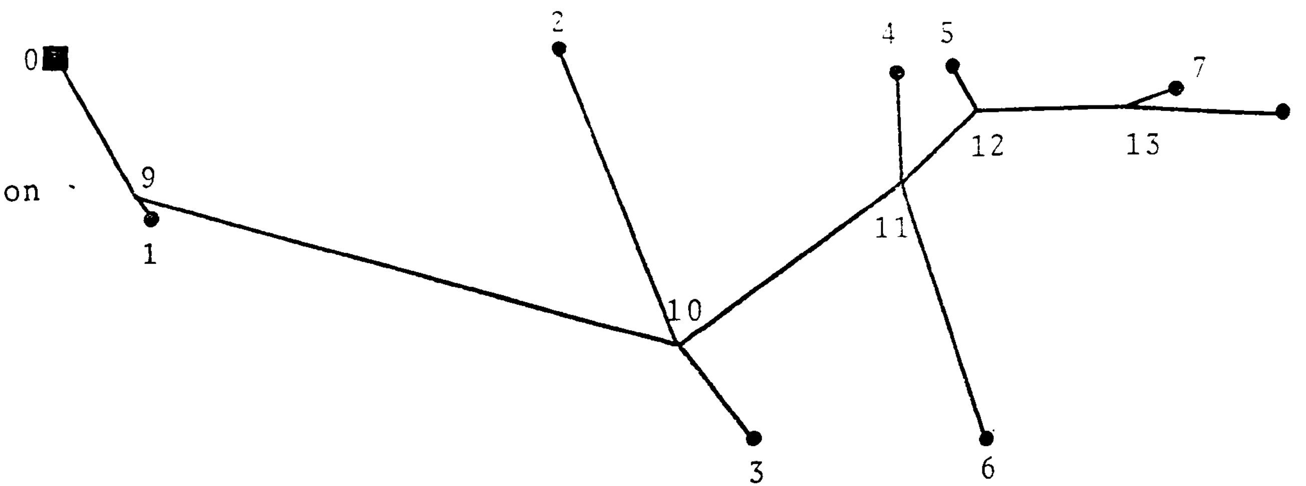
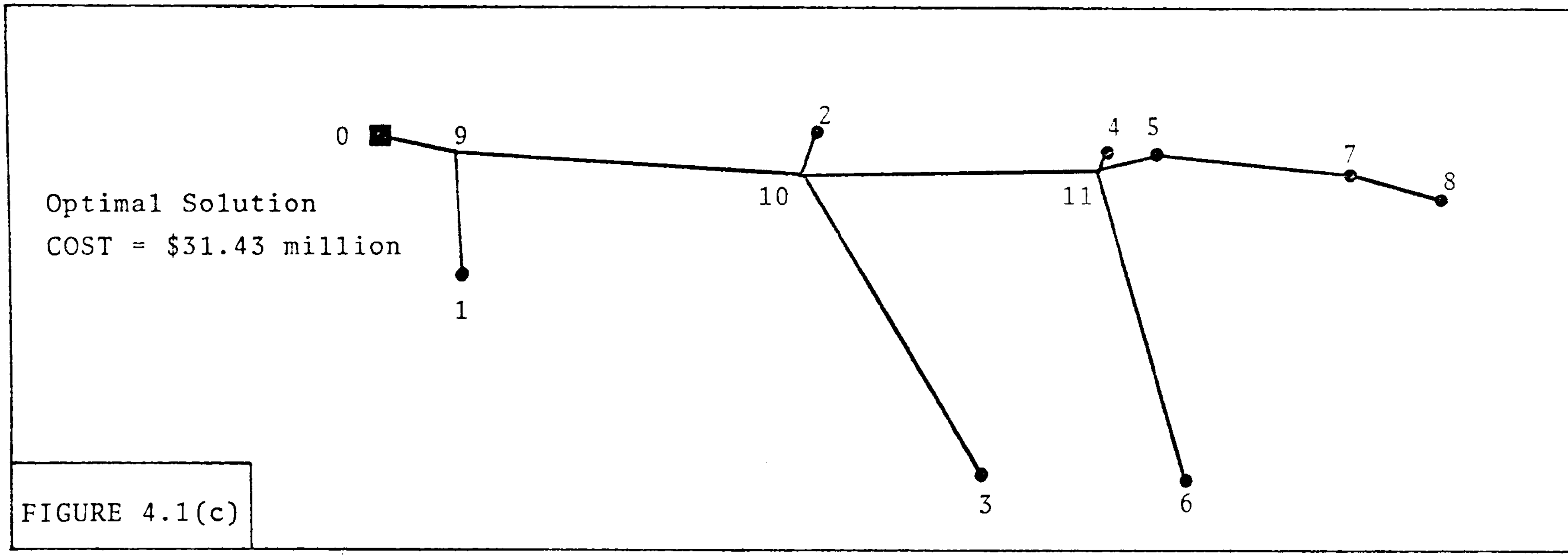


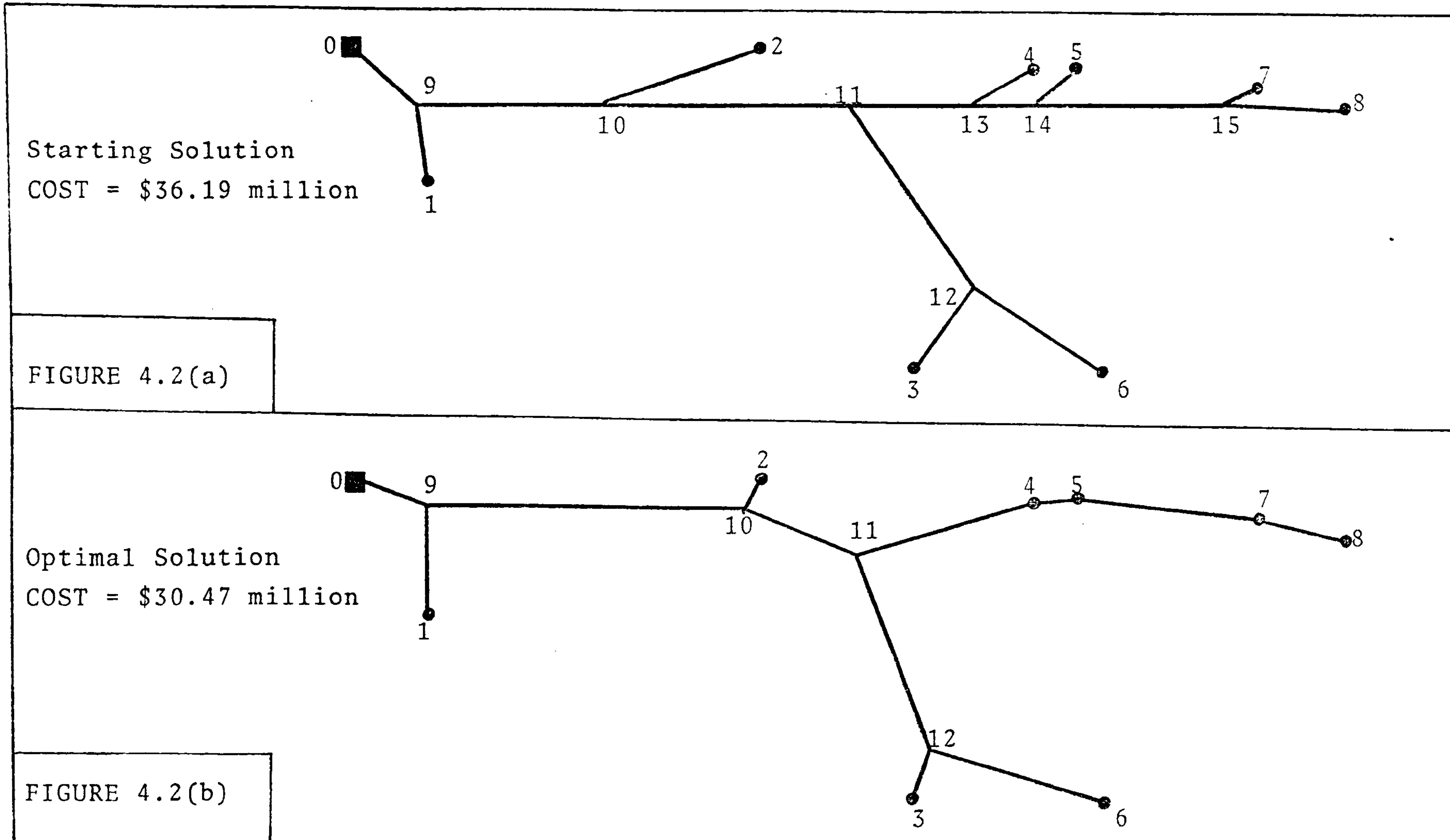
FIGURE 4.1(b)



FIGUFES 4.1 Junction Location Problem in Example 4.2.1

TABLE 4.1

Starting Solution 1		Starting Solution 2		Optimal Solution	
Link (a(i),i)	pp(i)/P*	Link (a(i),i)	pp(i)/P*	Link (a(i),i)	pp(i)/P*
(0,9)	.1	(0,9)	.1	(0,9)	.07818
(9,1)	.9	(9,1)	.9	(9,1)	.92182
(9,10)	.1	(9,10)	.1	(9,10)	.35151
(10,3)	.8	(10,2)	.8	(10,2)	.57031
(10,10')	.1	(10,3)	.8	(10,3)	.57031
(10',2)	.7	(10,11)	.1	(10,11)	.27946
(10',11)	.1	(11,4)	.7	(11,4)	.29085
(11,4)	.6	(11,6)	.7	(11,5)	.06454
(11,11')	.1	(11,12)	.1	(11,6)	.29085
(11',6)	.5	(12,5)	.6	(5,7)	.18609
(11',12)	.1	(12,13)	.1	(7,8)	.04022
(12,5)	.4	(13,7)	.5		
(12,13)	.1	(13,8)	.5		
(13,7)	.3				
(13,8)	.3				



FIGURES 4.2 Junction Location Problem in Example 4.2.2

TABLE 4.2

Starting Solution		Optimal Solution	
Link (a(i),i)	pp(i)/P*	Link (a(i),i)	pp(i)/P*
(0,9)	.1	(0,9)	.07708
(9,1)	.9	(9,1)	.92292
(9,10)	.1	(9,10)	.34836
(10,2)	.8	(10,2)	.57456
(10,11)	.1	(10,11)	.11375
(11,12)	.1	(11,12)	.26383
(11,13)	.1	(11,4)	.23093
(12,3)	.6	(12,3)	.19698
(12,6)	.6	(12,6)	.19698
(13,4)	.6	(4,5)	.04574
(13,14)	.1	(5,7)	.15110
(14,5)	.5	(7,8)	.03304
(14,15)	.1		
(15,7)	.4		
(15,8)	.4		

4.3 Other methods of solution

The method of the previous section is not the only method that can be used to solve the junction location problem. We present two others.

Geometric Programming

Condensation techniques of Geometric Programming can be used. We used two different condensation procedures:

Condensation technique 1. Using (4.2.1) the junction location problem can be expressed as a signomial program in variables x_0; $x(i)$, $y(i)$, $i \in N_s$; $\ell(i)$, $i \in \bar{N}$; $pp(i)$, $i \in \bar{N}$ in the following manner,

$$\text{Minimize } x_0$$

subject to

$$x_0 \geq \sum_{i \in \bar{N}} \frac{c_i \ \ell^{1+\mu_3}(i)}{pp^{\mu_3}(i)} \tag{4.3.1}$$

$$\ell^2(i) \geq (x(i) - x(a(i)))^2 + (y(i) - y(a(i)))^2$$

and constraints (4.1.5).

We now apply the condensation technique of Avriel, Dembo and Passy [1], consisting of monomial condensations and cutting planes in which the above signomial is solved as a sequence of posynomials and each posynomial as a sequence of linear programs.

Condensation technique 2. To convert the problem into a signomial program by Technique 1, extra variables x_0; $\ell(i)$, $i \in \bar{N}$ were added and a new constraint added for every $i \in \bar{N}$. The resulting increase in the size of the program makes its numerical solution difficult. Avriel and Gurovich [2] devised a way of getting over this problem and using their 'direct' method of solution without increasing the number of variables we solved the problem as a sequence of piecewise geometric programs with exponents (GPE's), and each GPE as a sequence of linear programs.

We wrote computer programs to implement both condensation techniques. However, when it came to computational experience unconstrained programming was far superior to both Geometric Programming techniques where both C.P. time and computer storage space were considered. The two condensation techniques were very comparable and Condensation technique 2 was not a clear winner over Condensation technique 1 despite its theoretical attractiveness. The inefficiency of the Geometric Programming techniques was due to the large number of cuts that had to be added. Thus Geometric Programming is an unsuitable technique for this problem.

Solving a system of nonlinear equations

Stationary conditions for (4.1.4) are

$$\left.\begin{aligned} \frac{\partial\ COST}{\partial x(i)} = \frac{\partial\ COST}{\partial y(i)} &= 0 \quad , \quad i \in N_s \\ \frac{\partial\ COST}{\partial pp(i)} &= 0 \quad , \quad i \in \overline{N} \end{aligned}\right\} \qquad (4.3.2)$$

Thus, the junction location problem is equivalent to solving the system of equations given by (4.3.2) and (4.1.6). We, however, did not computationally use this approach.

4.4 The case $|N_f| = 3$. Part 1 - Angles

In this section and the next, we study in depth the simplest, nontrivial case of the junction location problem - the case $|N_f| = 3$. Solutions to Steiner's and Weber's problems for this case are given in Appendix II.

In these sections we assume that the specific gravity of gas is constant, i.e.

$$S(i) = \text{constant} \quad , \quad i \in N_f / \{0\} \quad .$$

Also, for the sake of clarity we sometimes use the slightly different notation

$\ell(j,i)$: length of link (j,i),

$pp(j,i)$: pressure square drop on link (j,i), and

$p(i)$: pressure at node i.

Let $N_f = \{0,1,2\}$. There are 4 possible configurations and these are shown in Fig. (4.a).

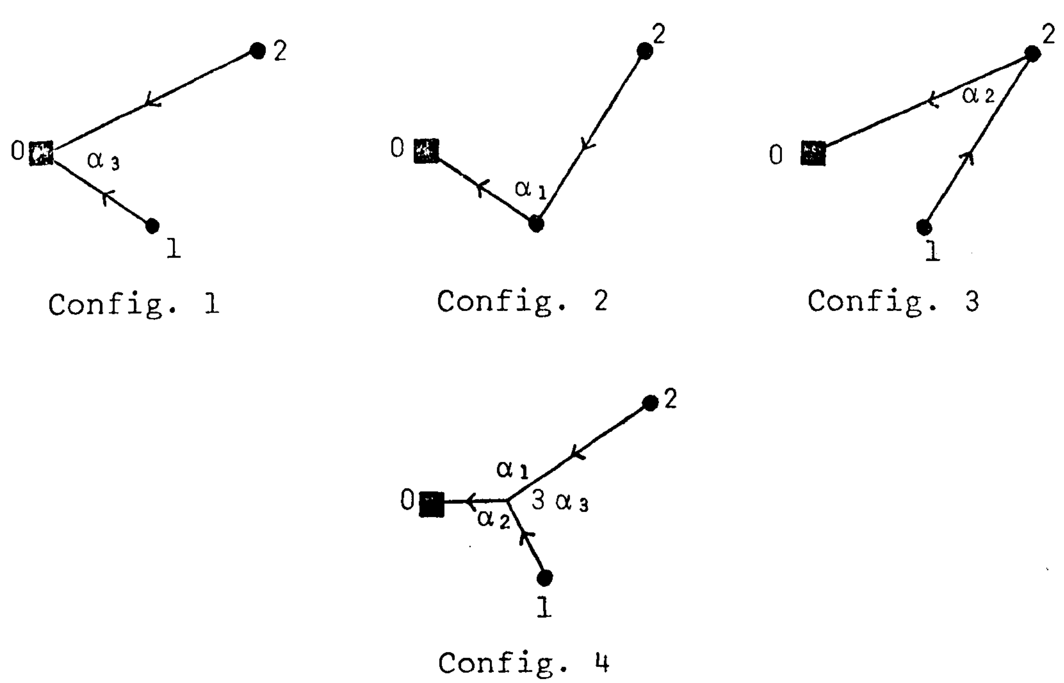


Fig. (4.a)

Config. 1, Config. 2 and Config. 3 are degenerate cases of the only full configuration Config. 4, and occur when node 3 coincides with node 0, 1 and 2 respectively.

A network with $|N_f| = 3$ is often part of a bigger network and in this case pressures at nodes 1 and 2 need not be equal. We will study this more general case and assume that the maximum available pressures at nodes 1 and 2 are P_1 and P_2 respectively, where P_1, P_2 need not be equal.

Considering an optimal network it can be shown that the angles between incoming and outgoing links at a node cannot take on arbitrary values. In this section we prove four properties that must be satisfied by these angles if the network is optimal. Throughout we will use the convention: α_i is the angle opposite the link originating at node i, i.e. α_1 is the angle opposite link (3,1), that is angle between (3,2) and (0,3) etc. In Config. 1 we will denote the angle between links (0,1) and (0,2) by α_3 and use similar definitions for the other degenerate configurations.

Property 1. *If the full configuration is optimal and node 3 is in the optimal position then*

$$\frac{C(d(1))}{\sin \alpha_1} = \frac{C(d(2))}{\sin \alpha_2} = \frac{C(d(3))}{\sin \alpha_3} \quad . \tag{4.4.1}$$

For optimality we have

$$\left.\begin{aligned} pp(3) + pp(1) &= P^* \\ pp(3) + pp(2) &= P^{**} \end{aligned}\right\} \tag{4.4.2}$$

where

$$P^* = P_1^2 - P_0^2 \quad , \quad P^{**} = P_2^2 - P_0^2 \quad . \tag{4.4.3}$$

Hence the only independant variables for the junction location problem are x(3), y(3) and pp(3).

Stationary conditions

$$\frac{\partial\ \text{COST}}{\partial x(3)} = \frac{\partial\ \text{COST}}{\partial y(3)} = \frac{\partial\ \text{COST}}{\partial pp(3)} = 0$$

give

$$\frac{c_1(x(3) - x(1))}{\ell^{1-\mu_3}(1)\ pp^{\mu_3}(1)} + \frac{c_2(x(3) - x(2))}{\ell^{1-\mu_3}(2)\ pp^{\mu_3}(2)} + \frac{c_3(x(3) - x(0))}{\ell^{1-\mu_3}(3)\ pp^{\mu_3}(3)} = 0 \qquad (4.4.4)$$

$$\frac{c_1(y(3) - y(1))}{\ell^{1-\mu_3}(1)\ pp^{\mu_3}(1)} + \frac{c_2(y(3) - y(2))}{\ell^{1-\mu_3}(2)\ pp^{\mu_3}(2)} + \frac{c_3(y(3) - y(0))}{\ell^{1-\mu_3}(3)\ pp^{\mu_3}(3)} = 0 \qquad (4.4.5)$$

$$\frac{c_1\ \ell^{1+\mu_3}(1)}{pp^{1+\mu_3}(1)} + \frac{c_2\ \ell^{1+\mu_3}(2)}{pp^{1+\mu_3}(2)} = \frac{c_3\ \ell^{1+\mu_3}(3)}{pp^{1+\mu_3}(3)} \qquad (4.4.6)$$

where we have made use of formula (4.2.1) for COST. Multiplying (4.4.5) by (x(3) - x(1)) and subtracting it from (4.4.4) multiplied by (y(3) - y(1)) and using the formulae

$$\text{Sin}\ \alpha_3 = \frac{(y(2) - y(3))\ (x(1) - x(3)) - (y(1) - y(3))\ (x(2) - x(3))}{\ell(1)\ \ell(2)} \quad \text{etc.}$$

we have

$$\frac{c_2\ \ell^{\mu_3}(2)}{pp^{\mu_3}(2)\ \text{Sin}\ \alpha_2} = \frac{c_3\ \ell^{\mu_3}(3)}{pp^{\mu_3}(3)\ \text{Sin}\ \alpha_3} \qquad (4.4.7)$$

Similarly we find

$$\frac{c_1\ \ell^{\mu_3}(1)}{pp^{\mu_3}(1)\ \text{Sin}\ \alpha_1} = \frac{c_2\ \ell^{\mu_3}(2)}{pp^{\mu_3}(2)\ \text{Sin}\ \alpha_2} \qquad (4.4.8)$$

(4.4.7) and (4.4.8) give the result.

***

Property 3. In an optimal network, the angle at a node between an incoming and outgoing link is at least 90°.

In other words α_1, $\alpha_2 \geq \pi/2$ *for those configurations where these angles are defined.*

Proof. We will only prove it for Config. 4. The proof for other configurations is similar. The proof is by contradiction. Let for the optimal location of node 3, $\alpha_2 < \pi/2$. Then the circle with node 1 as centre and radius the length of the link (3,1) will cut the line joining nodes 0 and 3. We call this point of intersection node 4. There are 2 cases possible.

Case 1. Node 4 lies on the same side of node 0 as node 3 as shown in Fig. (4.b). Let p(4) be the pressure at node 4. We will prove that the network $N_f = \{0,1,2\}$, $N_s = \{4\}$ shown in Fig. (4.c) with the same pressure p(4) as before at node 4 is cheaper than the network in Fig. (4.b).

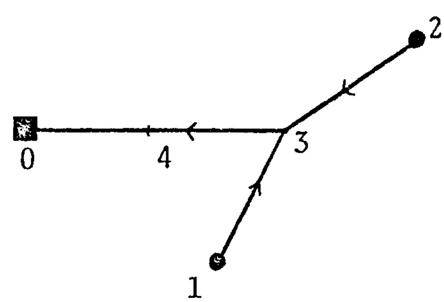


Fig. (4.b)

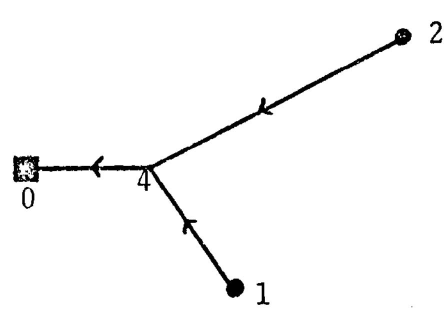


Fig. (4.c)

Obviously,

Cost of link (0,4) in Fig. (4.c) = Cost of segment (0,4) in Fig. (4.b).

Also, since

$$p(4) < p(3)$$

and

$$\ell(4,1) = \ell(3,1)$$

we have

$$\frac{Q^{\mu_1}(1)\ \ell^{1+\mu_3}(4,1)}{pp^{\mu_3}(4,1)} < \frac{Q^{\mu_1}(1)\ \ell^{1+\mu_3}(3,1)}{pp^{\mu_3}(3,1)} .$$

Thus, by (4.2.1) and (4.2.2)

Cost of link (4,1) in Fig. (4.c) < Cost of link (3,1) in Fig. (4.b).

It remains to show

Cost of link (4,2) in Fig. (4.c) ≤ Cost of segment (4,3) in Fig. (4.b) + Cost of link (3,2) in Fig. (4.b).

or,

$$\frac{Q^{\mu_1}(2)\ \ell^{1+\mu_3}(4,2)}{pp^{\mu_3}(4,2)} \leq \frac{(Q(1)+Q(2))^{\mu_1}\ \ell^{1+\mu_3}(4,3)}{pp^{\mu_3}(4,3)} + \frac{Q^{\mu_1}(2)\ \ell^{1+\mu_3}(3,2)}{pp^{\mu_3}(3,2)} \qquad (4.4.9)$$

where

$$pp(4,2) = pp(4,3) + pp(3,2) = P_2^{\,2} - p^2(4) \quad . \qquad (4.4.10)$$

The right hand side of (4.4.9) is

$$\geq \frac{Q^{\mu_1}(2)\ \ell^{1+\mu_3}(4,3)}{pp^{\mu_3}(4,3)} + \frac{Q^{\mu_1}(2)\ \ell^{1+\mu_3}(3,2)}{pp^{\mu_3}(3,2)}$$

$$\geq \min_{pp(4,3)} \left[\frac{Q^{\mu_1}(2)\ \ell^{1+\mu_3}(4,3)}{pp^{\mu_3}(4,3)} + \frac{Q^{\mu_1}(2)\ \ell^{1+\mu_3}(3,2)}{(pp(4,2) - pp(4,3))^{\mu_3}} \right] . \qquad (4.4.11)$$

The above minimum is attained at

$$\frac{pp(4,3)}{pp(4,2)} = \frac{\ell(4,3)}{\ell(4,3) + \ell(3,2)} .$$

Hence

Right hand side of (4.4.11) =

$$\frac{Q^{\mu_1}(2)\ (\ell(4,3) + \ell(3,2))^{1+\mu_3}}{pp^{\mu_3}(4,2)}$$

$\geq$ Left hand side of (4.4.9).

Thus the network in Fig. (4.c) is cheaper than the network in Fig. (4.b). Hence the junction node was not optimally located in Fig. (4.b).

Case 2. When node 4 lies on the other side of node 0 as node 3 as shown in Fig. (4.d). In this case, it can be shown as before that the network shown dotted with $N_f = \{0,1,2\}$ and $N_s = \phi$ is cheaper.

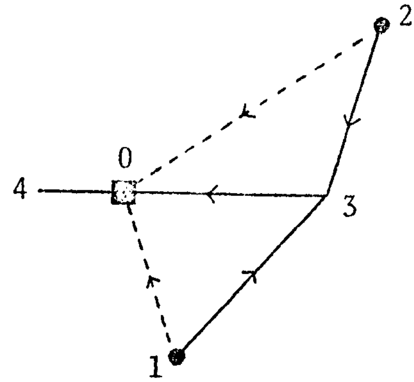


Fig. (4.d)

This proves the property.

***

This property marks out a region within which the optimal location of junction node 3 lies. Draw semicircles on links (0,1) and (0,2) as diameter as shown in Fig. (4.e). Then the junction node 3 has to be in the region shown shaded.

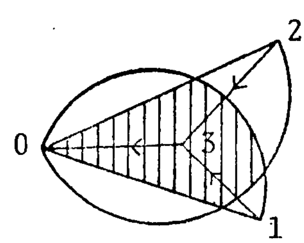


Fig. (4.e)

Property 3. If the full configuration is optimal, and node 3 *is in the optimal position then* $\alpha_3 \leq \alpha_1$ *and* $\alpha_3 \leq \alpha_2$.

This follows from the above *Properties 1* and *2* together with Property 3 of § 3.4. There exists an optimal diameter assignment for which

$$C(d(3)) \geq C(d(1))$$

$$C(d(3)) \geq C(d(2))$$

and hence

$$\text{Sin } \alpha_3 \geq \text{Max } (\text{Sin } \alpha_1, \text{ Sin } \alpha_2) \geq 0 \quad .$$

As α_1, $\alpha_2 \geq \pi/2$, this implies that $\alpha_3 \leq$ Min (α_1, α_2).

***

This property directly leads to the next one, as one of the angles α_1, α_2, α_3 must be less than or equal to 120°.

Property 4. *If the full configuration is optimal and node* 3 *is in the optimal position then* $\alpha_3 \leq 120°$.

4.5 The case $|N_f| = 3$. Part 2 - Experimental Results

To obtain further insight into the problem, we investigated under which conditions each of the four configurations is optimal, simply by solving the junction location problem for a large number of cases, using the algorithm described in § 4.2.

In the junction location problem the form of the objective (4.2.1), the constraints (4.1.6), and the expression for the length (4.1.1) are such that multiplying the flows, pressure square drops or the coordinate system by a scaling factor will not affect the solution. Thus, without loss of generality we will assume

$$(x(0),y(0)) = (0,0)$$

$$(x(1),y(1)) = (1,0) \quad ,$$

and

$$0 < Q(1) < 1$$

$$Q(2) = 1 - Q(1)$$

and

$$P_0 = 0$$

$$0 < P_1 < 1$$

$$P_2 = 1 - P_1 \quad .$$

There are thus 4 independent parameters which determine the solution to the junction location problem, namely, x(2), y(2), Q(1) and P_1. Before presenting experimental results we discuss two more properties, the first of which we call duality.

P1. Duality

Consider $N_f = \{0,1,2\}$ and let

$$Q(1) = q \quad , \quad P_1 = P$$

Consider a node 2' on the line 02 given by

$$\frac{\ell(0,2)}{\ell(0,1)} = \frac{\ell(0,1)}{\ell(0,2')}$$

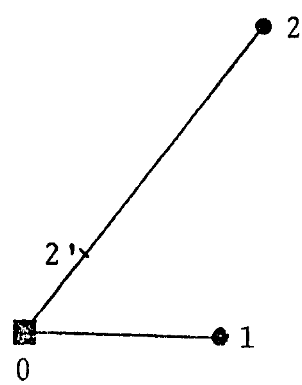


Fig. (4.f)

Because the relative positions of nodes 0,1,2 is the same as that of 0,2',1 except for a scaling factor and a reflection, which does not have any bearing on the configuration that is optimal, the following is true:

The optimal configuration for

$$N_f = \{0,1,2\}$$

with

$$Q(1) = q \quad , \quad P_1 = P$$

is also optimal for

$$N_f = \{0,2',1\}$$

with

$$Q(1) = 1-q \quad , \quad P_1 = 1-P \quad .$$

Node 2 is mapped onto Node 2' by the mapping

$$(x,y) \rightarrow \frac{(x,y)}{x^2 + y^2} \quad . \qquad (4.5.1)$$

The other property is:

P2. *In an optimal configuration the maximum available pressure at every node is completely utilised.*

Proof. In Config. 4 the pressure square drops satisfy the constraints (4.4.2). Because these constraints are equalities, the available pressures P_1 and P_2 at nodes 1 and 2 are completely utilised.

In Config. 1, the pressure square drops on links (0,1) and (0,2) are $P_1^2 - P_0^2$ and $P_2^2 - P_0^2$ respectively. Hence, again all of the available pressure at nodes 1 and 2 is utilised.

It may happen in Config. 2 that the maximum available pressure at node 1 is not utilised. If so, we will prove Config. 2 is not optimal. The case of Config. 3 is similar. Let

$$\bar{Q} = Q(1) + Q(2) .$$

The cost of Config. 2 is given by (4.2.1) subject to the constraints

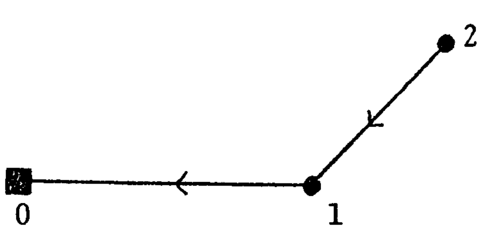


Fig. (4.g)

$$pp(1) \leqslant P^* \tag{4.5.2}$$

$$pp(2) + pp(1) = P^{**} \tag{4.5.3}$$

where P^*, P^{**} are given by (4.4.3).

Stationary conditions

$$\frac{\partial \text{ COST}}{\partial pp(1)} = 0$$

give, as in § 3.6

$$pp(1) = \frac{\bar{Q}^{\frac{\mu_1}{1+\mu_3}} \ell(0,1)\ P^{**}}{\bar{Q}^{\frac{\mu_1}{1+\mu_3}} \ell(0,1) + Q^{\frac{\mu_1}{1+\mu_3}}(2)\ \ell(1,2)} \, . \tag{4.5.4}$$

By (4.5.3), the maximum available pressure at node 2 is utilised. We assume the maximum available pressure at node 1 is not utilised, i.e.

$$pp(1) \lneq P^* \, . \tag{4.5.5}$$

The pipeline network with Config. 2, with optimal diameters is called Network 1.

We will construct a network with Config. 4 which is cheaper than Network 1. This we call Network 2. The geometry of Network 2 is shown in Fig. (4.h): $N_f = \{0,1,2\}$, $N_s = \{3\}$ where node 3 is at a small distance ε from node 1. The diameter of link (0,3) is selected such that the pressure square drop over link (0,3) in Network 2 is equal to the pressure square drop over link (0,1) in Network 1, i.e. equal to pp(1) as given by (4.5.4).

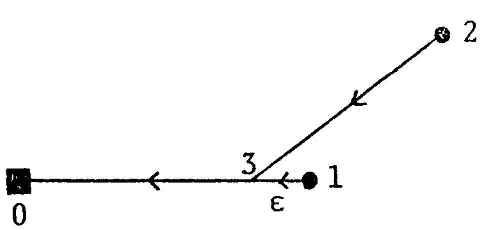


Fig. (4.h)

Cost of Network 2 is, by (4.2.1) and (4.2.2) a constant multiple of

$$\frac{\bar{Q}^{\mu_1} \ell^{1+\mu_3}(0,3)}{pp^{\mu_3}(1)} + \frac{Q^{\mu_1}(1) \ell^{1+\mu_3}(3,1)}{(P^* - pp(1))^{\mu_3}} + \frac{Q^{\mu_1}(2) \ell^{1+\mu_3}(3,2)}{(P^{**} - pp(1))^{\mu_3}}, \quad (4.5.6)$$

and the cost of Network 1 is the same constant multiple of

$$\frac{\bar{Q}^{\mu_1} \ell^{1+\mu_3}(0,1)}{pp^{\mu_3}(1)} + \frac{Q^{\mu_1}(2) \ell^{1+\mu_3}(1,2)}{(P^{**} - pp(1))^{\mu_3}} \quad (4.5.7)$$

where pp(1) is given by (4.5.4).

We have

$$\left.\begin{aligned} \ell(0,3) &= \ell(0,1) - \varepsilon \\ \ell(3,2) &< \ell(1,2) + \varepsilon \quad . \end{aligned}\right\} \quad (4.5.8)$$

Substituting this (4.5.8) in (4.5.6) and using the approximations

$$\left(1 - \frac{\varepsilon}{\ell(0,1)}\right)^{1+\mu_3} \cong 1 - (1+\mu_3) \frac{\varepsilon}{\ell(0,1)}$$

and

$$\left(1 + \frac{\varepsilon}{\ell(1,2)}\right)^{1+\mu_3} \cong 1 + (1+\mu_3) \frac{\varepsilon}{\ell(1,2)} \quad (4.5.9)$$

we have that (4.5.6) is less than

$$\frac{\bar{Q}^{\mu_1} \ell^{1+\mu_3}(0,1)}{pp^{\mu_3}(1)} \left(1 - (1+\mu_3) \frac{\varepsilon}{\ell(0,1)}\right) + \frac{Q^{\mu_1}(1) \varepsilon^{1+\mu_3}}{(P^* - pp(1))^{\mu_3}} +$$

$$\frac{Q^{\mu_1}(2) \ell^{1+\mu_3}(1,2)}{(P^{**} - pp(1))^{\mu_3}} \left(1 + (1+\mu_3) \frac{\varepsilon}{\ell(1,2)}\right) . \quad (4.5.10)$$

ε must be small enough for the two approximations (4.5.9) to be realistic. If in addition it is selected such that

$$\varepsilon^{\mu_3} < (1+\mu_3)\left(\frac{p^* - pp(1)}{pp(1)}\right)^{\mu_3} \frac{\ell^{\mu_3}(0,1)}{Q^{\mu_1}(1)}\, \bar{Q}^{\frac{\mu_1\,\mu_3}{1+\mu_3}} \left[\bar{Q}^{\frac{\mu_1}{1+\mu_3}} - Q^{\frac{\mu_1}{1+\mu_3}}(2)\right] \qquad (4.5.11)$$

then the expression (4.5.10) is smaller than expression (4.5.7) and hence Network 2 is cheaper than Network 1. Such an $\varepsilon > 0$ can always be found as the Right Hand side of (4.5.11) is positive.

***

An immediate consequence of the above property is the following result:

Corollary. Config. 2 *cannot be optimal if* $P_1 \geq P_2$, Config. 3 *cannot be optimal if* $P_2 \geq P_1$ *and neither* Config. 2 *nor* Config. 3 *can be optimal if* $P_1 = P_2$.

Now we will present some examples we have studied. What we have done is to fix Q(1), P_1 and then moved the node (x(2),y(2)) over the half plane y(2) $\geq$ 0 and for each position determined the optimal configuration. The other half plane y(2) $\leq$ 0 is just a mirror image of the one we have considered. The interesting region is where the optimal configurations change. We use the following notation:

A_i is region of the x-y plane such that if $(x(2),y(2)) \in A_i$ then Config. i is optimal, i=1, ..., 4.

Also:

C is the curve separating A_1 and A_4,

D is the curve separating A_2 and A_4,

E is the curve separating A_3 and A_4.

We found that when $P_2 \geqslant P_1$, the curve E vanishes (Fig. (4.i)). This was to be expected from the *Corollary* to property *P2*. Similarly, when $P_1 \geqslant P_2$ the curve D vanishes (Fig. (4.j)). It is sufficient to study the shape of one of the curves D or E as the other can be obtained by the duality property *P1*.

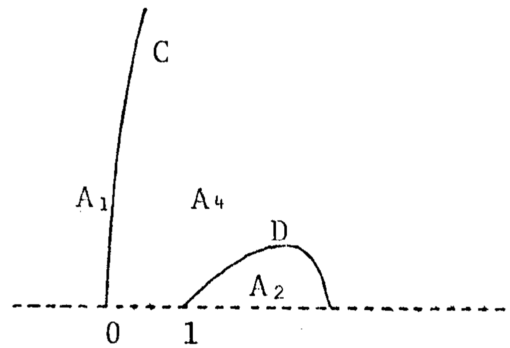


Fig. (4.i)

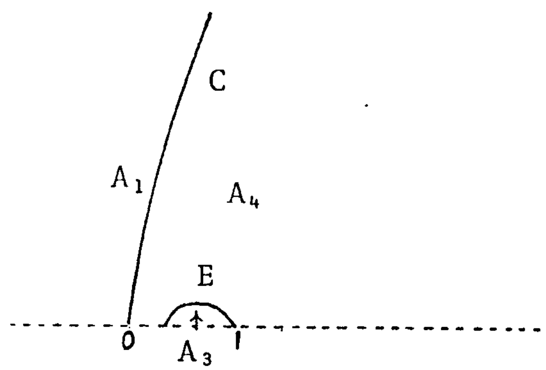


Fig. (4.j)

Example 4.5.1. $P_1 = P_2 = .5$. As Q(1) increases from .1 to .5 to .9 the curve C shifts as shown in Figure (4.1). In Figure (4.2) the region near the origin is greatly magnified. (Note that the two axes have been dilated to different extents to display the shape of the curves), and the curves traced. This figure illustrates the effect of duality, e.g. the curve C(Q(1) = .1) in Figure (4.2) is the image of the curve C(Q(1) = .9) in Figure (4.1) under mapping (4.5.1). Also as was to be expected from the *Corollary* to property *P2*, Config. 2 and Config. 3 are never optimal.

Example 4.5.2. $P_1 = .3$, $P_2 = .7$. As Q(1) increases from .1 to .5 to .9 the curves C and D shift as shown in Figure (4.3). Curve E vanishes.

Example 4.5.3. $P_1 = .7$, $P_2 = .3$. As Q(1) increases from .1 to .5 to .9 the curves C and E shift as shown in Figure (4.4). Curve D vanishes.

For constant flow, we display in the next example, the effect of a change in the pressures P_1, P_2 on the optimal configuration.

Example 4.5.4. Q(1) = Q(2) = .5. As P_1 increases from .1 to .5 the shift in the curves C and D is shown in Figure (4.5). Curve D flattens out from the position $D(P_1 = .1)$ to the x-axis where it vanishes at $P_1 = .5$.

We conclude the section with two observations.

1. As there are 4 configurations, there are 12 possible changes in the optimal configuration for a change in one of the parameters. But, as can be seen from the examples we have given at most 5 of these changes are likely to occur.

2. From the shape of the curves C, unless the node 2 is such that $y(2) >> 1$ or very near the origin we see that α_3 is never much less than $\pi/2$. Thus, in general, we don't expect in an optimal network that the angle at a node between 2 incoming links will be much less than $\pi/2$. A consequence of this remark is that configurations in which 4 links meet at a node are not usually optimal configurations. This is explained in § 4.6.

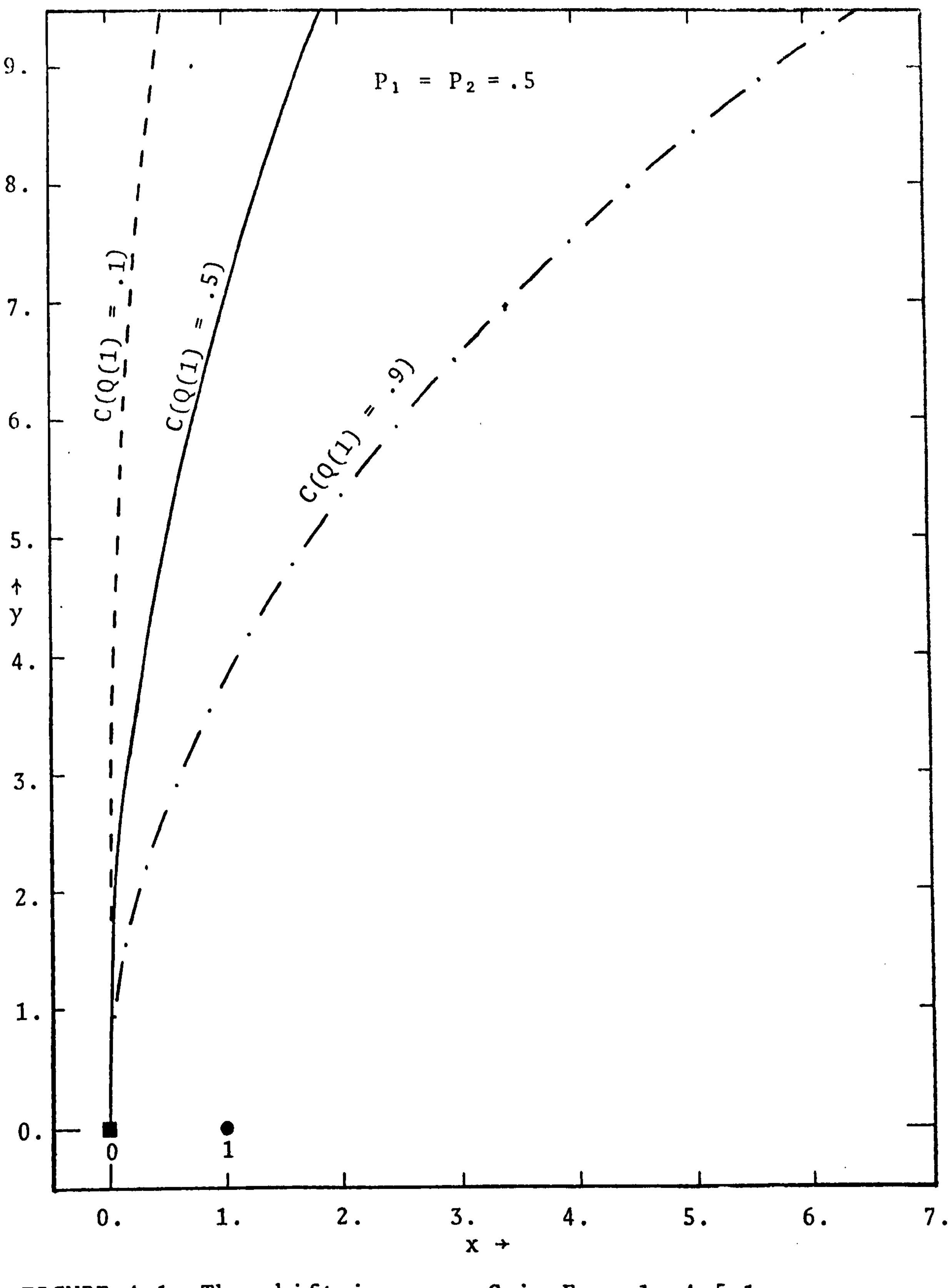


FIGURE 4.1 The shift in curve C in Example 4.5.1

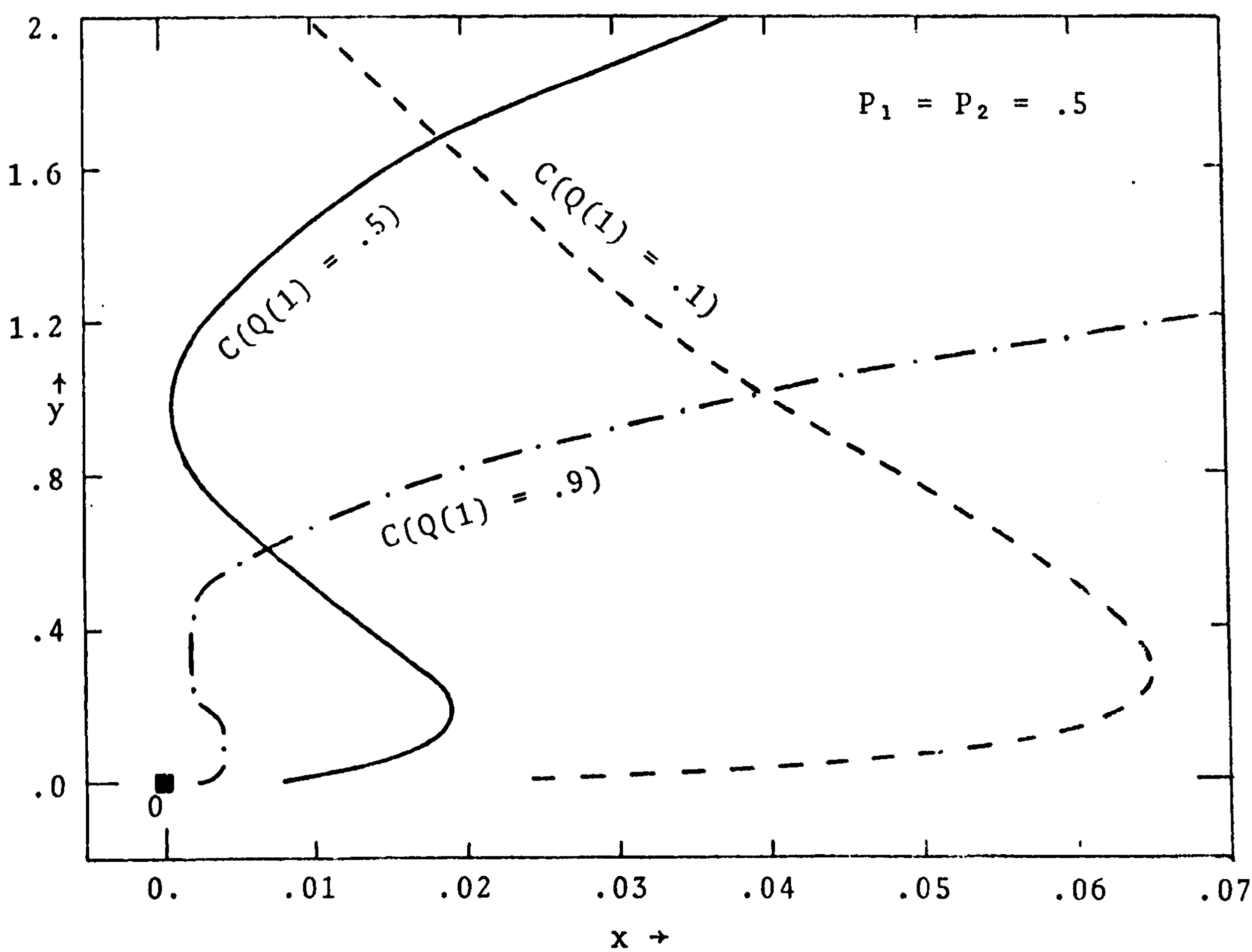


FIGURE 4.2 The effect of duality in Example 4.5.1

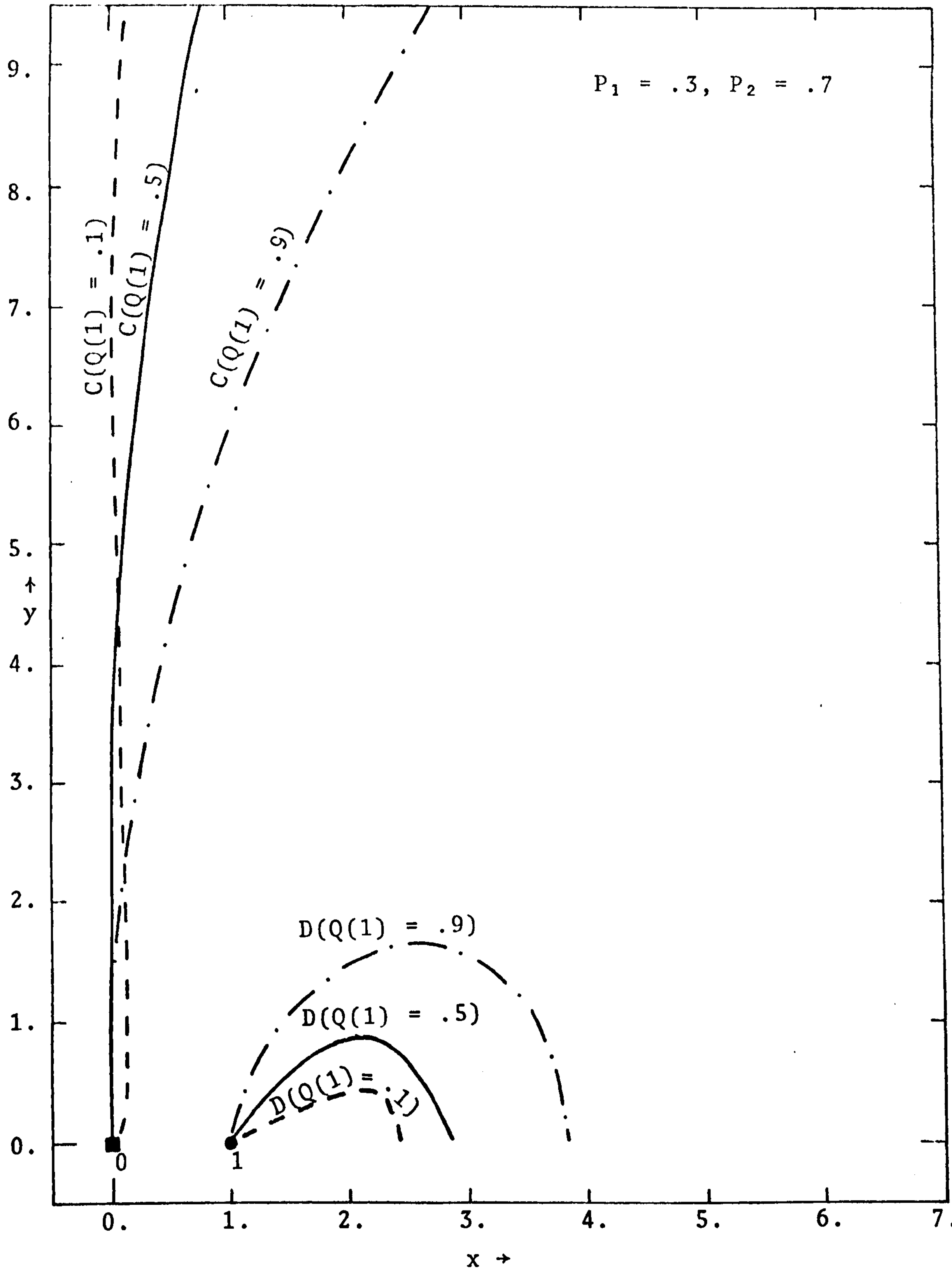


FIGURE 4.3 The shift in curves C and D in Example 4.5.2.

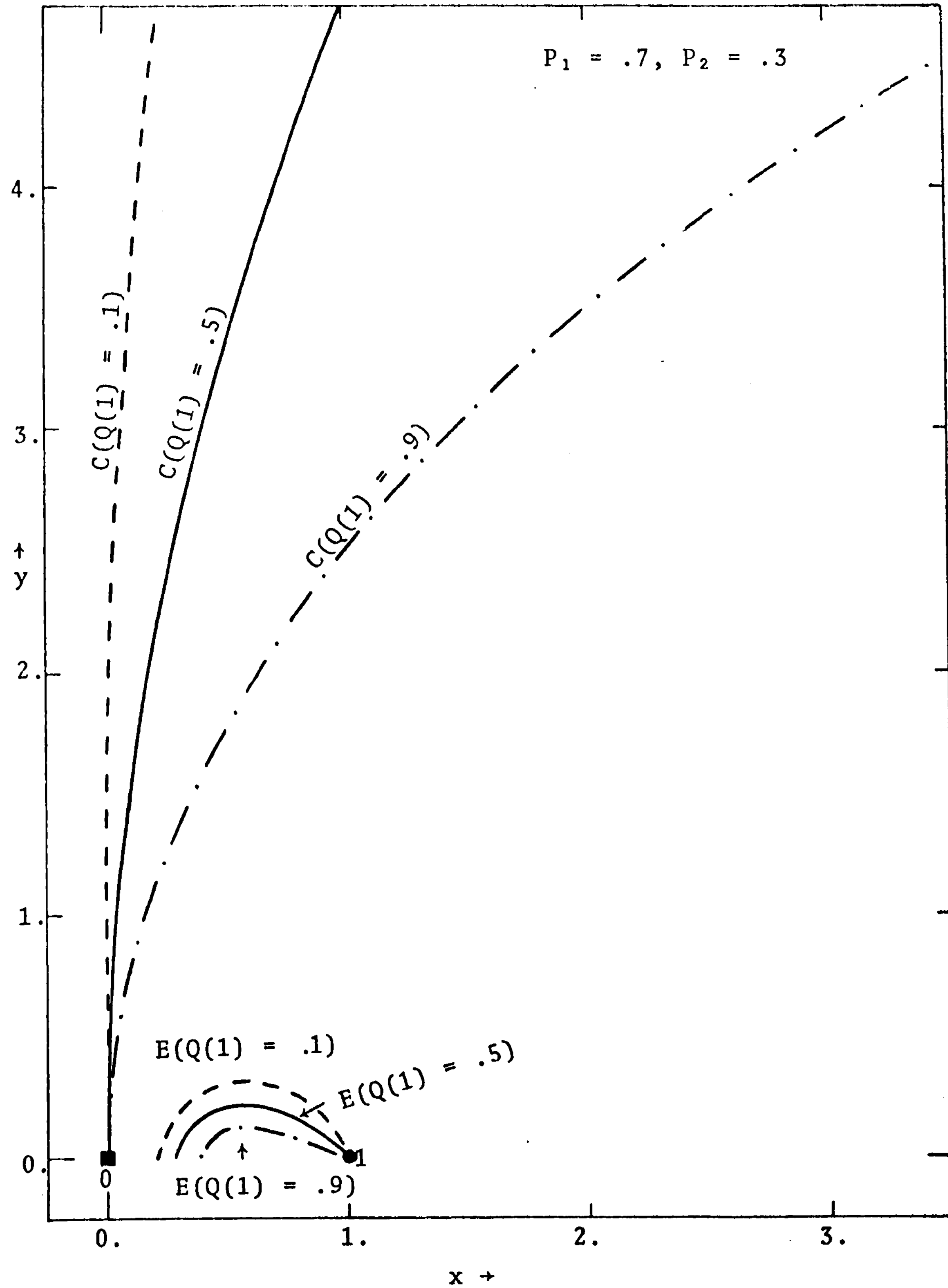


FIGURE 4.4 The shift in curves C and E in Example 4.5.3.

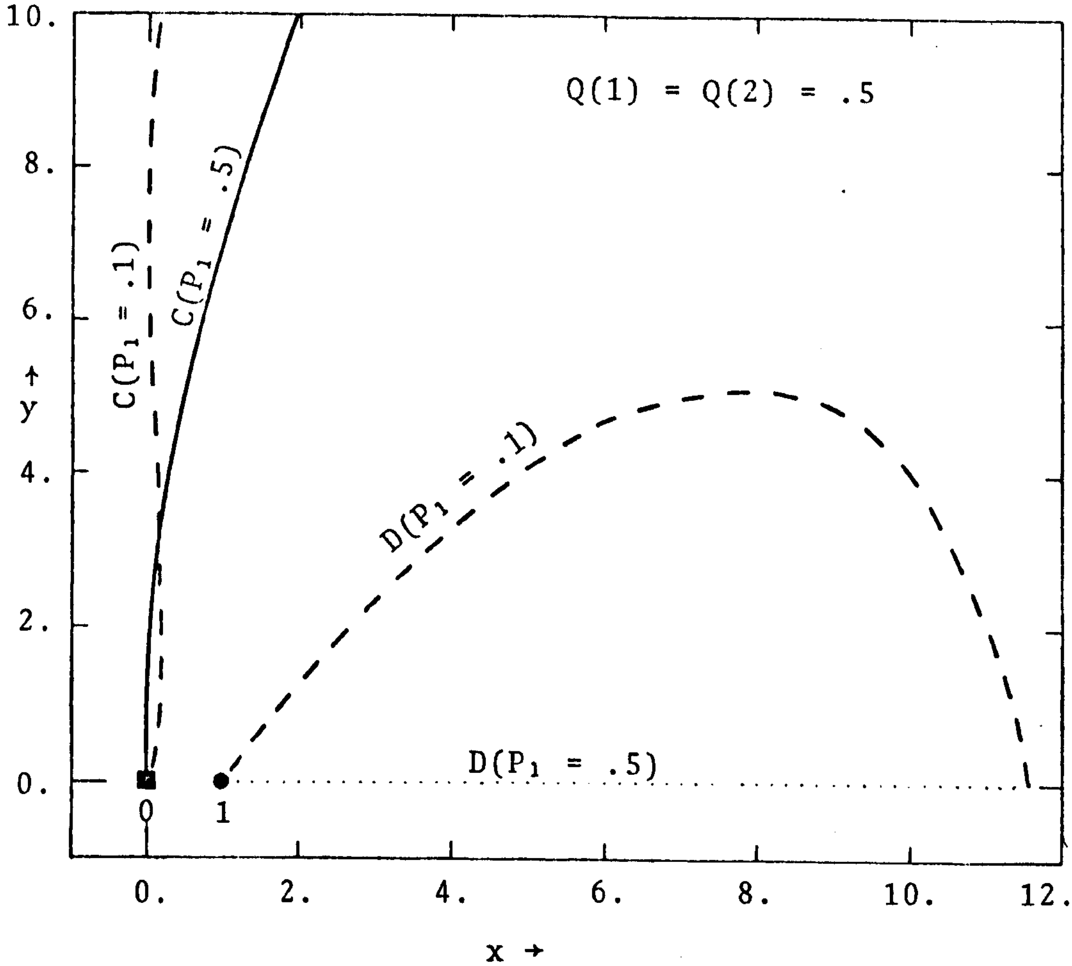


FIGURE 4.5 Figure of Example 4.5.4

4.6 Configuration Problem

This is the only problem remaining from those formulated in Chapter 2. The configuration problem is to determine the number $|N_s|$ of junction nodes and the predecessor function a. When these are known methods like those of §'s 4.2 and 4.3 can be applied to find the positions (x(i), y(i)) of the junction nodes $i \in N_s$ and to select a diameter d(i) for every link (a(i),i) of the network.

The objective is

$$\text{Minimize COST} = \sum_{i \in \overline{N}} \ell(i)\ C(d(i)) .$$

Thus COST is a weighted sum of distances, however, the weights depend on the diameters which in turn depend on the configuration and rate of gas flow at the wells.

Complexity of Problem. This problem is still largely unsolved. The only obvious method to determine which configuration is optimal is to completely enumerate all configurations and then compare the minimum costs corresponding to each of these configurations. The situation is similar to Steiner's problem and it is a hard, combinatorial problem. In attempting to find a 'good' configuration for a case where complete enumeration is not possible we have to use a heuristic approach and combine intuition with partial enumeration. Sometimes it is easy to guess configurations which if not actually best, cost only slightly more than the minimal cost. One should be willing to accept a good network, i.e. the best network obtained for several reasonable configurations, even though it is not proved to be optimal.

As in Steiner's problem, an optimal configuration has to satisfy the condition

$$|N_s| \leq |N_f| - 2 \quad .$$

Computationally, it is sufficient to consider only full configurations (i.e. configurations with $|N_s| = |N_f| - 2$). All other configurations are degenerate cases and do not have to be considered separately. When a degenerate case is optimal the corresponding full configurations reduce to it on locating junctions optimally. The number of full configurations on $|N_f|$ fixed nodes is

$$\frac{(2|N_f| - 4)!}{2^{|N_f|-2}\,(|N_f|-2)!}$$

Some of these numbers can be read from Table II.1. Complete enumeration of all full configurations is prohibitive in most practical cases.

We, however, solved a large number of small examples. We now discuss these simple cases and our empirical findings.

<u>Simple cases</u>.

In the case $|N_f| = 3$ the configuration problem is trivial as there is only one full configuration to be considered and this will give rise to the optimal configuration. Thus the simplest nontrivial case is the following.

<u>$|N_f| = 4$</u>. Consider $N_f = \{0,1,2,3\}$. This is the simplest, nontrivial configuration problem. In this case there are 3 full configurations, shown in Fig. (4.k).

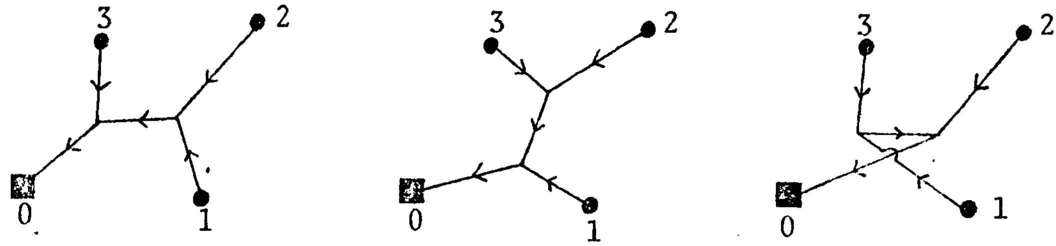


Fig. (4.k)

Because of the many more parameters that determine the optimal configuration it is not possible to numerically analyse this case as thoroughly as was done in § 4.5 for the case $|N_f| = 3$. We only consider the case when the maximum available pressures at the nodes 1, 2, 3 are equal, say, equal to P_1. As before, scaling doesn't affect the optimal configuration, and without loss of generality we will now assume

$$(x(0),y(0)) = (0,0)$$

$$(x(1),y(1)) = (1,0)$$

$$Q(1) + Q(2) + Q(3) = 1 \quad , \quad Q(1),\ Q(2),\ Q(3) > 0$$

$$P_0 = 0 \quad , \quad P_1 = 1 \quad .$$

Thus, the parameters which determine the optimal configuration are Q(1), Q(2), x(2), y(2), x(3) and y(3).

For fixed nodes (x(2), y(2)), (x(3), y(3)) we considered all possible flows at nodes 1, 2 and 3 (i.e. we considered a grid over the open triangle ABC in Fig. (4.ℓ)) and compared the cost of the various configurations.

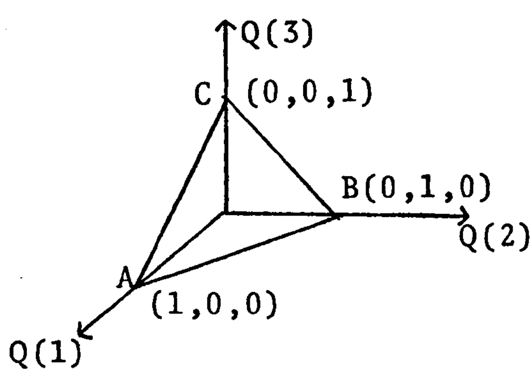


Fig. (4.ℓ)

We did this for several positions of the nodes 2 and 3. The investigation of numerous examples led us to the following conclusions.

1. When the positions of the nodes N_f are not symmetric about one of the lines 01, 02 or 03 then the optimal configuration seems to be independent of flows.

2. When the nodes N_f are positioned symmetrically about one of the lines 01, 02 or 03 then more than one configuration is optimal for all possible flows. This is easily seen because of symmetry as follows.

Consider Fig. (4.m) where the positions of the nodes N_f are symmetric about line 02. Thus if the displayed configuration is optimal for flows $Q(1) = q$, $Q(2) = q'$, $Q(3) = q''$, then the configuration in Fig. (4.n) is optimal for flows $Q(1) = q''$, $Q(2) = q'$, $Q(3) = q$. We found, experimentally, that in this case of symmetrically positioned nodes N_f, two of the three possible full configurations have practically the same optimal cost.

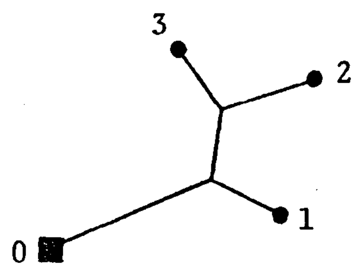


Fig. (4.m)

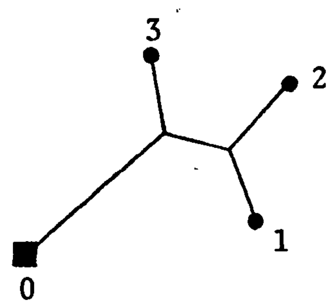


Fig. (4.n)

Thus we make the following remark:

Coordinates of nodes N_f *are far more important than flows in determining the optimal configuration.*

Computations for the case $|N_f| = 5$ also led to the above conclusion.

In addition, we now give an example in which we fix the position of node 3 and all flows:

Example 4.6.1. Let $N_f = \{0,1,2,3\}$ and

$$(x(0),y(0)) = (0,0)$$

$$(x(1),y(1)) = (1,0)$$

$$(x(3),y(3)) = (.25,1.)$$

$$Q(1) = .2 \quad , \quad Q(2) = .3 \quad , \quad Q(3) = .5$$

$$P_0 = 0 \quad , \quad P_1 = 1 \; .$$

Node 2, i.e. (x(2),y(2)) is shifted over the plane and the change in optimal configuration is shown. In Figure (4.6), the boundaries where the optimal configuration changes are traced and the configuration optimal in each region is shown.

Finally we make a few comments about possible optimal configurations. If 4 links meet at a node it is likely that one of the angles between two incoming links is rather less than $\pi/2$ (Fig. (4.o)). By the previous section the insertion of a new junction

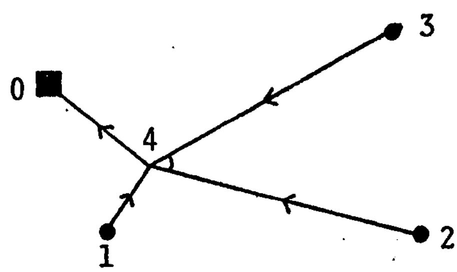


Fig. (4.o)

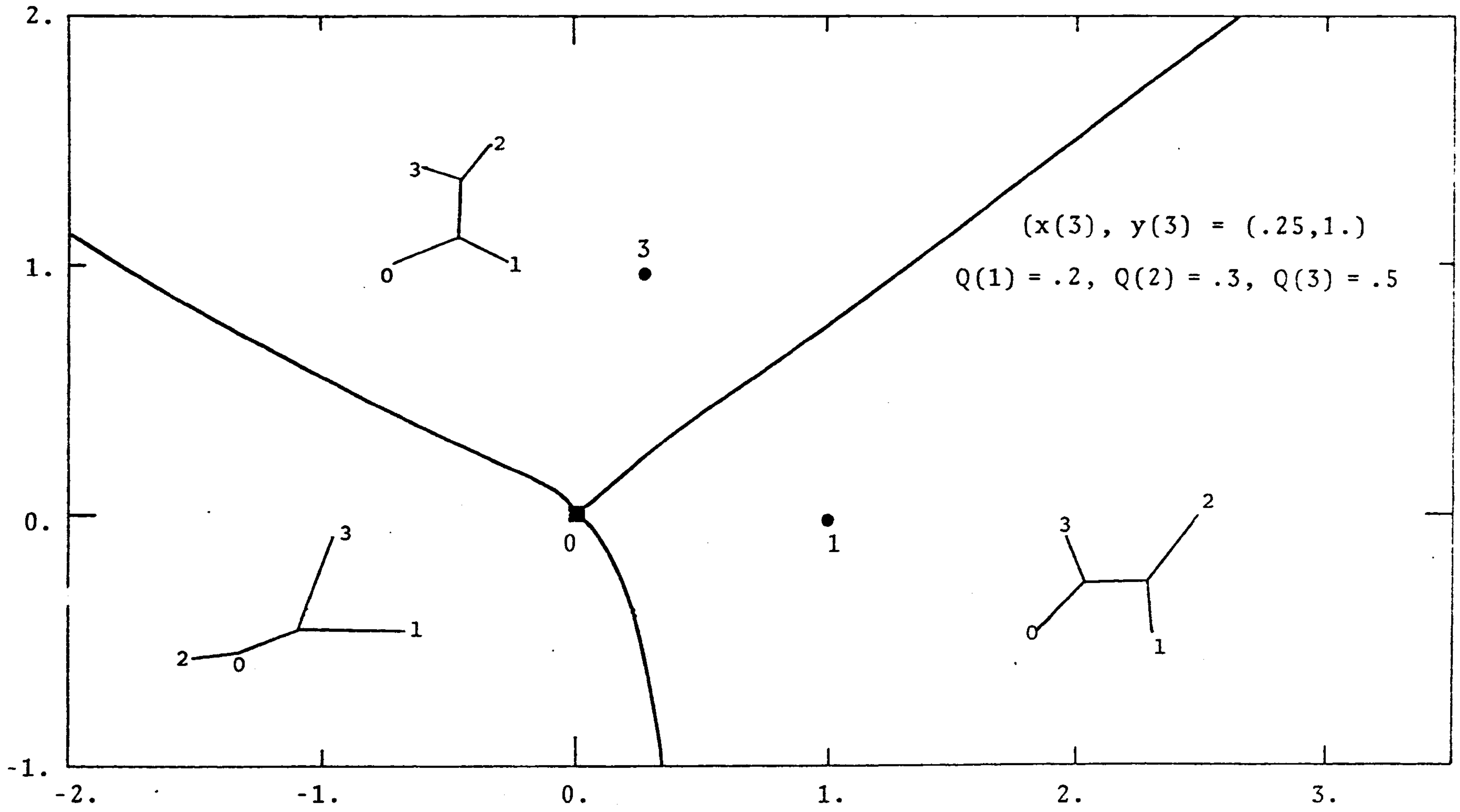


FIGURE 4.6   Figure showing how optimal configurations change in Example 4.6.1

node 5 as shown in Fig. (4.p) would result in a cheaper network. Thus, if on locating junctions optimally to cost a particular full configuration

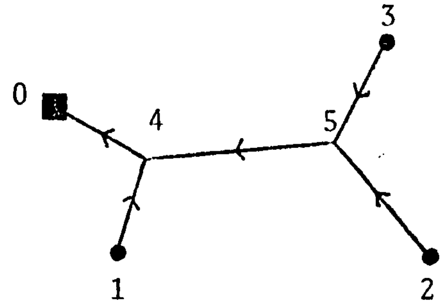


Fig. 4.p)

we find that 2 nonextreme nodes tend to coincide then that full configuration and the degenerate configurations that can arise from it are unlikely to be optimal. When 4 links meet at a node as in Fig. (4.o), splitting the node into 2 junction nodes connected by a link as done in Fig. (4.p) invariably gives a cheaper network. *This is often helpful in ruling out nonoptimal configurations and constructing cheaper ones.* The remark would indicate that the configurations considered in Example 4.2.1 are nonoptimal as indeed is the case.

In the Moomba gas field the maximum available pressures at all the wells are the same. The *Corollary* to property *P2* of § 4.5 implies that a well to well link will never be optimal. This often turns out to be a statement of only theoretical interest. When a junction node gets very close to a well it is practically and computationally convenient to identify the junction with the well - the cost being practically the same. This was done in the Examples in § 4.2.

DISCUSSION

In this thesis we have studied a wide spectrum of problems that arise in the design of pipeline networks. We have tried to mathematically analyse these problems and when this analysis doesn't by itself solve the problem in question we have tried to give practical computational procedures which do so. We have always given examples to illustrate the workings of our procedures.

The two areas where human approximation is used and which hence form a source of inaccuracy are the forms of the flow formula and the cost function. The flow formula we have assumed to be a monomial (2.2.2). For numerical computations we have used the Weymouth formula (2.2.5). We have not gone into the experimental procedures used to derive the flow formula. It is, of course, far more accurate to use the cost data than the cost function (2.1.1) we have used in Chapter 4. But other than the diameter problem we were unable to progress much with the problems without assuming the diameter to be a continuous variable.

In this thesis we have provided solution techniques for the diameter and junction location problems. In the computational results of the junction location problem we remarked that in the neighbourhood of the optimal junction locations small changes in the positions of the junction nodes does not produce much change in cost. This together with the fact that the solution procedure was based on approximating the cost data by a cost function of a continuous variable urges us to remark that undue importance should not be given to the exact location of junction nodes. The computer program for the junction location problem should be used as a tool to compare configurations and to roughly locate the junction nodes. Once this is done, exact optimal diameters for the links can be found by the methods of Chapter 3.

While we have been able to give practical solution techniques to the diameter and junction location problems, the Configuration Problem, as is to be expected, has proved much harder. We have stated in § 4.6 that it is unlikely for 4 links to meet at a node in an optimal network. This has, in practice, proved a useful tool in weeding out nonoptimal networks. When on locating junctions optimally for some configuration we find 4 links meeting at a node, adding a new junction node by splitting as explained in § 4.6 invariably produces a cheaper network. A network can be viewed as being composed of subnetworks with $|N_f| = 3$. This makes the study of §'s 4.4 and 4.5 particularly important. Thus the properties proved for $|N_f| = 3$ would hold in larger networks as well.

Pipelines are not usually built all at the same time, but over a period of time, as the wells come on stream. The approach used in this thesis, to consider the whole pipeline system simultaneously, is therefore a bit unrealistic. Some of the costs are incurred in the future, and one may think of discounting them; it may be cheaper to instal two parallel pipes at different times instead of one broader pipe right in the beginning; in general the whole question of timing the construction of the network has then to be considered. It might also then be economical to instal compressors at strategic points in the network to boost up the pressure of gas. This gives rise to problems that need quite different models and solution methods to those presented here. But it may well be that the complexity then becomes such that any attempts to apply mathematical optimization techniques will produce few practical, meaningful results.

APPENDIX I

PIPELINE NETWORKS

It has been our policy in this thesis to consider only tree networks and ignore all other types. We have shown in Chapter 2 that there is sufficient justification for doing this. In this Appendix we explain the motivation behind restricting attention to trees. We show the problems that arise in the design of pipeline systems simplify on considering trees.

The discussion in this Appendix and the methods cited are applicable not only for gas pipeline networks but also to other pipeline networks, notably water distribution systems. We assume the formulae (2.2.1) and (2.2.2) hold to describe the flow of gas/liquid through pipelines. (In water distribution systems, pp is used to denote pressure drop and not pressure square drop but this does not affect the analysis.) We also assume that the specific gravity of gas/liquid is constant. The notation is the same as in § 2.3.

In the study of pipeline networks 2 cases arise.

Case 1. Study of an existing network.

This study is often conducted in systems which have been in operation for a long time. As the network exists we know the lengths and diameters of connecting pipes. We also know that the gas/liquid enters the network at certain nodes and leaves the network at certain nodes. We want to know how the flows distribute along the various links of the network and what the pressures at intermediate nodes are.

For a network to exist and be in operation, the following two equilibrium conditions have to hold:

1. Node laws or Conservation of Flow: At every node, the total flow entering it is equal to the total flow leaving it. This constraint is given by (2.3.1).

2. Mesh laws or Uniqueness of pressure: The total change in pressure around every mesh is zero, i.e. for every mesh

$$i_0, i_1, i_2, \ldots, i_k, i_{k+1} = i_0$$

we have a constraint

$$\sum_{j=0}^{k} pq(i_j, i_{j+1}) = 0 \tag{I.1}$$

where

$$pq(i_j, i_{j+1}) = \begin{cases} pp(i_j, i_{j+1}) & \text{if } (i_j, i_{j+1}) \in A \\ pp(i_{j+1}, i_j) & \text{if } (i_{j+1}, i_j) \in A \end{cases},$$

A being the set of links.

The variables pressure square drop and rate of gas/liquid flow are by (2.2.1) and (2.2.2) connected by the relation

$$\frac{pp(i,j)}{\ell(i,j)} = M \frac{q^{\alpha_1}(i,j)\ s^{\alpha_2}}{d^{\alpha_3}(i,j)} \quad , \quad \forall\ (i,j) \in A \quad ,$$

or

$$pp(i,j) = M_1\ q^{\alpha_1}(i,j) \quad , \quad \forall\ (i,j) \in A \quad , \tag{I.2}$$

where M_1 depends on the specific gravity of gas/liquid and the dimensions of connecting pipe and is therefore a known quantity.

The problem of analysing an existing network, is thus equivalent to solving the system of equations (2.3.1), (I.1) and (I.2). Methods used to solve this problem fall into two categories:

A. Numeric Analytic methods. The analytic methods that apply are methods of controlled trial and error in which systematic corrections are applied either to an initial set of assumed flows or to an initial set of assumed pressures. They are iterative processes and there is no guarantee that these methods will converge. The widely used methods are:

Hardy Cross Methods [4],
Linear approximation [27], and
Newton Raphson methods [24].

B. Experimental methods. (McIlroy Network Analyser). Other than mathematical methods, direct analogy methods are also used. They are based on the analogy between pipeline and electric networks. The points of analogy between pipeline and electric networks are:

(i) Through variable. Current in an electric circuit behaves similar to flow in a pipeline network.

(ii) Across variable. The passage of electric current through a resistor is accompanied by a drop in voltage which is similar to the drop in pressure when gas/liquid flows through a pipe.

(iii) Node laws. Similar to the node laws for pipeline networks, the sum of currents approaching a terminal in an electric circuit equals the sum of currents leaving it.

(iv) Mesh laws. Similar to the mesh laws for pipeline networks the total voltage drop around a mesh in an electric circuit is zero.

(v) Flow formula. The flow formula in pipeline networks is given by (I.2). In an ordinary resistor, the voltage across it (V) is proportional to the current through it (C),

$$V = RC \tag{I.3}$$

where R is its resistance.

Thus in normal resistors the linear relation (I.3) can't simulate the nonlinear relation (I.2). This has been overcome in the McIlroy Network Analyser [15, 16, 17] where special nonlinear resistors have been developed to simulate the relation (I.2) for the appropriate value of the exponent α_1.

Each pipeline is represented by a nonlinear resistor. A set of these resistors is interconnected according to the plan of the pipeline network. Given conditions of fluid supply to and discharge from the network are then simulated by operating the electric circuit in such a manner that controllable electric quantities are proportional to the known pipeline quantities which they represent. When the circuit is in operation the brightness of the incandescent filaments of the resistors directs the attention of the person conducting the analysis to the pipelines which have the largest head losses. Pressures and flow rates are read directly from special scales mounted on electric instruments.

The advantage of direct analogy is immediate in that in an electric circuit one can read off scales what the various flows and pressures are. The disadvantage is the high setting up cost of the instrument and to a certain extent the inflexibility of such a model.

Case 2. Construction of a new network.

Here, in addition to satisfying the equilibrium conditions stated in Case 1, we have also to specify the nodes and links of the network and select a diameter for each link. Thus Case 2 has all the problems of Case 1 plus some others. Hence, it is a much harder problem. That is why far more research has been done on Case 1 than on Case 2.

In this thesis we have shown that for gas pipeline systems, there is sufficient justification to consider only trees when constructing a new network. When trees are considered, the node laws (2.3.1) simplify to the recursive relations (2.4.6). Also, as there are no meshes, the mesh laws do not apply. Thus the problems of Case 1 are trivially solved and we have, in this thesis, been able to concentrate on the other problems that arise in the construction of a new pipeline network system.

APPENDIX II

STEINER AND WEBER PROBLEMS

The design of pipeline networks has some features in common with the simpler, well-studied Steiner and Weber problems. In this Appendix we review the work that has been done on these problems.

Steiner's Problem. Given n nodes $N_f = \{0,1, \ldots, n\}$ with fixed positions $(x(i),y(i))$, $i \in N_f$ to construct the shortest tree whose vertices contain these n nodes. Junction nodes, N_s, may be added. Their number $|N_s|$, location and the predecessor function have to be determined.

A detailed study of Steiner's problem can be found in Gilbert and Pollak [10]. Other references on Steiner's problem include [18], [25]. It has been shown in an optimal network

$$|N_s| \leq |N_f| - 2 \quad .$$

Junction Location Problem. For a given configuration the junction location problem can be solved by geometrical construction.

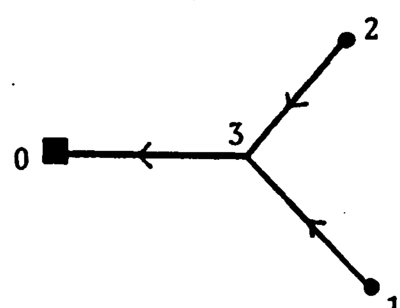


Fig.(II.a)

The case $|N_f| = 3$. This is the case when $N_f = \{0,1,2\}$, $N_s = \{3\}$ with configuration as shown in Fig. (II.a). It is well known that in the solution, node 3 is the unique point in triangle 012 where

$$\angle 031 = \angle 132 = \angle 230 = 120^\circ \qquad (II.1)$$

Geometrical construction of this point is possible. If any of the angles $\angle 102$, $\angle 021$ or $\angle 210$ are $\geqslant 120^\circ$ then it is a degenerate case and node 3 coincides with the vertex at which the angle is $\geqslant 120^\circ$ and thus $N_s = \phi$.

<u>The case $|N_f| > 3$</u>. For a given configuration, the junctions are located optimally by induction on $|N_s|$ in which the construction of the case $|N_f| = 3$ is used several times to locate the junction nodes [10].

<u>Configuration Problem</u>. The number of configurations on $|N_f|$ fixed and $|N_s|$ junction nodes are

$$F(|N_f|, |N_s|) = 2^{-|N_s|}\binom{|N_f|}{|N_s|+2}\frac{(|N_f| + |N_s| - 2)!}{|N_s|!} \qquad (II.2)$$

Table (II.1) gives some of these numbers.

Different configurations may give rise to different locally minimum length trees. The only sure way of solving Steiner's problem is to enumerate all configurations and locate the junctions optimally for every configuration and compare the lengths of all trees thus obtained. As is obvious from Table (II.1), enumeration is not practically a feasible proposition where $|N_f|$ is not small. Gilbert and Pollak [10] have derived a number of necessary conditions which the optimal tree will have to satisfy and these can be used to eliminate many possibilities.

In addition to numeric analytic methods, mechanical link-length minimizers and soap film techniques are also

TABLE II.1

NUMBER OF POSSIBLE TOPOLOGIES FOR STEINER TREES WITH $|N_f|$ GIVEN VERTICES AND $|N_s|$ JUNCTION NODES

$|N_s|$ \ $|N_f|$	3	4	5	6	7
0	3	12	60	360	2,520
1	1	12	120	1,200	12,600
2		3	75	1,350	22,050
3			15	630	17,640
4				105	6,615
5					945
TOTAL	4	27	270	3,645	62,370

used for Steiner's problem (ref. Miehle [19]).

<u>Weber's Problem</u>. Given n nodes $N_f = \{0,1, \ldots, n\}$ with fixed positions $(x(i),y(i))$, $i \in N_f$ and n positive weights $\{w_i\}, i=1, \ldots, n$ to locate a node with position (x,y) to

$$\text{Minimize} \sum_{i=1}^{n} w_i \; d_i$$

where

$$d_i = \sqrt{[(x-x(i))^2 + (y-y(i))^2]} \; .$$

Thus, in this case there is only the junction location problem. Stationary conds. are

$$\left.\begin{aligned} &\sum_{i=1}^{n} \frac{w_i}{d_i} (x-x(i)) = 0 \\ \text{and} \\ &\sum_{i=1}^{n} \frac{w_i}{d_i} (y-y(i)) = 0 \; . \end{aligned}\right\} \quad \text{(II.3)}$$

<u>The case $|N_f| = 3$</u>. This case is again given by Fig. (II.a), and node 3 has to be located. The solution is: Node 3 is the unique point in Δ012 at which

$$\underline{\angle 031} = \text{Sin}^{-1} \frac{\sqrt{k_1}}{2w_1w_3}, \quad \underline{\angle 132} = \text{Sin}^{-1} \frac{\sqrt{k_1}}{2w_1w_2}, \quad \underline{\angle 230} = \text{Sin}^{-1} \frac{\sqrt{k_1}}{2w_2w_3}$$

where

$$k_1 = \sum_{i=1}^{3} w_i^{\,4} - 2 \sum_{\substack{i,j=1 \\ i \neq j}}^{3} w_i^{\,2} \, w_j^{\,2} \; .$$

Again, geometrical construction of the point is possible [9]. If one of the corner angles $\underline{\angle 102}$, $\underline{\angle 021}$ or $\underline{\angle 210}$ are greater than or equal to the corresponding angle at the centre then it is a degenerate case and point 3 coincides with the appropriate vertex.

General case $|N_\ell| > 3$. In this case equations (II.3) don't give explicit values for x and y. Algorithms which locate point (x,y) and which are based on iterative techniques can be found in [12] and [14].

APPENDIX III

DATA FROM MOOMBA GAS FIELD

TABLE III.1

RAW GAS PRODUCTION (μCFD) AT 13 GAS WELLS AT 60°F AND 14.65 PSIA

YEAR	BIG LAKE 1	DELLA 2	TOOLACHEE 3	DULLINGARI 4	BURKE 5	BRUMBY 6	EPSILON 7	ROSENEATH 8	MERRIMELIA 9	TIRRAWARRA 10	FLY LAKE 11	MOORARI 12	DARALINGIE 13
1975	123371	0	0	0	0	0	0	0	0	0	0	0	0
1976	203364	0	0	0	0	0	0	0	0	0	0	0	0
1977	275545	0	0	0	0	0	0	0	0	0	0	0	0
1978	242516	70321	0	0	0	0	0	0	0	0	0	0	0
1979	145832	211091	0	0	0	0	0	0	0	0	0	0	0
1980	207036	208946	137927	0	0	0	0	0	20000	63650	31266	12793	0
1981	238120	199062	208183	0	0	0	0	0	20000	62623	30485	11784	0
1982	203412	195071	225000	0	0	0	0	0	20000	67057	24186	11089	34364
1983	140008	190931	225000	65546	0	35000	0	0	20000	61452	20210	8491	75000
1984	129122	196753	221909	136211	12065	35000	0	0	17995	59141	17084	6139	73847
1985	101622	197156	221644	129079	80409	35000	7411	0	12874	59653	14732	3731	74077
1986	75078	286637	164675	79917	76541	34178	106228	7000	19713	49604	12274	1308	59238
1987	55701	350000	134005	98777	72839	34178	85149	6730	17071	43047	9716	1706	28346
1988	41187	101924	82039	72260	64057	33903	10207	0	13122	32272	17958	2491	14632
1989	31847	137485	50573	51726	42404	23740	73679	6460	10081	26074	15993	4257	0

TABLE III.2

FULL WELL STREAM GAS COMPOSITION (%age)

GAS	SPECIFIC GRAVITY	BIG LAKE	DELLA	TOOLA-CHEE	DULLIN-GARI	BURKE	BRUMBY	EPSILON	ROSE-NEATH	MERRI-MELIA	TIRRA-WARRA	FLY LAKE	MOORARI	DARA-LINGIE
METHANE	0.5539	78.7750	81.2990	71.9020	73.9580	75.9360	68.7760	77.018	74.3500	71.3260	42.6690	51.8820	53.1880	73.2550
ETHANE	1.0382	3.4500	2.8440	9.3860	6.7750	5.5230	8.7520	6.646	6.8700	5.8150	11.5780	10.2570	10.5010	10.2630
PROPANE	1.5225	0.6680	0.7830	3.0230	2.1160	1.3340	2.4670	2.055	2.9600	1.9000	6.4310	4.6430	4.7800	4.3100
ISO.BUTANE	2.0068	0.0730	0.0860	0.3710	0.2820	0.1520	0.3560	0.266	0.4400	0.2150	0.9860	0.7940	0.7880	0.5810
NOR.BUTANE	2.0068	0.1240	0.1580	0.7390	0.4880	0.2600	0.5570	0.510	0.7800	0.4460	1.9180	1.1470	1.2230	1.1670
ISO.PENTANE	2.4911	0.0300	0.0420	0.1880	0.1360	0.0650	0.1730	0.137	0.2000	0.1340	0.5350	0.4160	0.3460	0.3720
NOR.PENTANE	2.4911	0.0390	0.0460	0.2570	0.1390	0.0750	0.2070	0.142	0.2000	0.1660	0.6470	0.4330	0.3560	0.3590
HEXANE	2.9753	0.0430	0.0460	0.2530	0.1380	0.0700	0.2230	0.160	0.1800	0.2040	0.6000	0.5180	0.4040	0.3350
HEPTANE	3.4596	0.0750	0.0830	0.2990	0.1910	0.1050	0.2970	0.180	0.4250	0.2840	0.6660	0.5490	0.3780	0.4810
OCTANE	4.4282	0.1280	0.1340	0.6020	0.4750	0.1780	0.6330	0.472	0.1150	0.7350	1.5250	1.4530	0.7920	0.9000
CO_2	1.5195	16.2990	12.9430	11.8790	14.7400	15.7680	16.8920	11.132	10.9000	18.3790	31.3180	26.6180	25.8930	6.8030
N_2	0.9672	0.2960	1.5360	1.1010	.5620	.5340	0.6670	1.282	2.5800	0.3960	1.1270	1.2900	1.3510	1.1740

TABLE III.3

PIPELINE DIMENSIONS AND COSTS

PIPE SIZE	INTERNAL DIAMETER (inches)	PIPE COST $/mile
1	4.000	28,200
2	6.125	40,500
3	8.001	50,080
4	10.136	59,200
5	12.062	73,680
6	13.250	100,800
7	15.250	135,680
8	17.250	176,220
9	19.188	222,000
10	21.124	250,800
11	23.062	281,760
12	25.062	314,600
13	27.000	339,000
14	28.876	364,000
15	30.876	388,000
16	32.876	411,000
17	34.750	431,000
18	36.750	451,000
19	38.750	470,000

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