CHANNEL STATE FEEDBACK FOR DIGITAL COMMUNICATIONS IN A FADING ENVIRONMENT

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SUMMARY

Signal fading due to multipath effects causes severe performance deterioration in a digital transmission system. The usual way of combatting these effects is by the use of diversity techniques. However, if a feedback link is available, as is usually the case in a duplex communication system, the transmitter can vary parameters in accordance with receiver decisions sent via this link to combat the effects of such fading. These systems are called channel state feedback systems and their use to combat slow, non-selective fading is the general topic of this thesis.

In the literature three different system types are identified depending on the variable transmitter parameter in question. However, such systems operate within the constraints of a practical system and it is shown that the performance can be quite dependent on such constraints. Thus it is concluded that only the simpler system configurations which consider two channel states are practically viable. One such system, an intermittent system, where the transmitter stops sending data when the signal level is too low, is shown to have a potential performance better than fourth order diversity. The effects of system constraints on an intermittent system are then considered in detail where general implications for other channel state feedback systems are also considered.

One such constraint is the limited control information reliability due to feedback information error and overall control delay. In this thesis a simple sub-optimal strategy, which is only marginally inferior to the optimum one, is derived to minimise the effects of feedback and feedforward error for an intermittent system. However, the reliability is limited due to the need to keep control delay below a critical level. The effect of control delay is considered along with its interaction with feedback error and some suggested methods to minimise these effects. An alternative method of control is suggested for the case of reciprocal fading.

Such an intermittent system requires a buffer at transmitter and receiver to interface with a uniform data source and sink. In this thesis, an expression is derived for performance of an intermittent system subject to a finite buffer size. It is shown that for best performance an optimum threshold must be used. In order to minimise the amount of buffer required, a variable threshold scheme is suggested which a simulation study indicates, could halve the buffer requirements. A general analytical formulation of this problem is then derived to allow for a hysteresis control function and variable transmission rate. A hardware model of an intermittent system was constructed and the results of subjective tests on speech indicate the potential of an intermittent system to remove the effects of fading.

Finally, the total effect of all the system constraints on an intermittent system is analysed. It is shown that the feedback strategy does not affect the buffer requirements to a great extent and that overall, constraint interaction is a second order effect. This theoretical model is validated using computer simulation and the implications for two case studies are discussed thus illustrating the general system design problem.

CORRIGENDA

Page	Line	Details
3	16	"selection" should be "selective"
53	13	"low" should be "law"
69	4	"fedback" should be "feedback"
32	2	"$p(a_\tau/a_o)$" should be "$p(a_o/a_\tau)$"
32	18	The integral in equation 3.28 should be written: $\int_o^\infty P_\varepsilon(a_o, r)\, p(a_o/a_\tau) da_o$ where $x = a_\tau^2/2\sigma^2$
19	26	The following sentence should be added: "The term 'Average Error Probability' applies to the sink data and is equal to 'The Average Transmission Error Rate'."

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STATEMENT

This thesis describes work I performed in the Department of Electrical Engineering, University of Adelaide from 1/2/73 to 20/2/76.

I declare that, to the best of my knowledge, the research described herein is original except where otherwise acknowledged and that the thesis has not, either in whole or in part, been submitted for another degree at this or any other University.

Reginald Paul Coutts

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Lastly, I should like to thank Telecom Australia for allowing me to finish this thesis and in particular Mr. Roger Smith of the Research Laboratories for making the last stages of preparation easier, and Mrs. I. Brandt and Judy Wilkinson for typing this thesis.

LIST OF SYMBOLS

W	Message Bandwidth
T_M	Time Delay spread
P_ε	Error Probability in additive noise
γ	Signal to Noise Ratio at Decision Instant
S	Received Signal Power
N_o	Single sided Noise Power Density
R	Transmission Data Rate
γ_o	Mean SNR over fading
x	Normalised random value of SNR
r_1	Transmission Ratio which is the ratio of Transmission Data Rate to Throughput Data Rate
x_{th}	SNR threshold above which transmission occurs
r_f	Feedback Information Data Rate
P_{ij}	Service Information Error Probability where $i,j \in \{0,1\}$
n_p	Power Ratio which is the ratio of main signal power to service information power
n_r	Rate Ratio which is the ratio of main transmission data rate to service information data rate
γ_m	Main signal SNR (not including r_1)
γ_f	Service information SNR (includes r_f)
g	Adaptive Power Gain of Feedback Information Receiver
y_{th}	Feedback SNR threshold below which transmission ceases
$\hat{x}_{th}$	Feedforward SNR threshold below which the receiver does not receive data
$k(\tau)$	Normalised Autocorrelation Coefficient of Lag τ
$\hat{P}_e$	Predicted Error Probability
τ	Control Delay
r	Variable normalised data rate
V	3 dB Fading Bandwidth in rad/s
τ_p	Propagation Delay
τ_f	Dilter Delay
τ_d	Decision Delay

$\bar{p}_\varepsilon$	Mean Error Probability over Fading
f_r	Average frequency of downward zero crossings across a threshold x_{th}
f_{rms}	RMS frequency of spectrum of fading process
λ	Average number of bits added to Tx buffer during a fade
μ	Average number of bits removed during a transmission period
ρ	Traffic intensity which is the ratio of mean arrivals to mean departures
P_o	Probability of empty buffer condition when $x \geq x_{th}$
P_c	Probability of full buffer condition when $x < x_{th}$
P_o'	Probability of non-transmission when $x \geq x_{th}$
P_c'	Probability of transmission when $x < x_{th}$
C	Normalised Buffer Capacity
z	Exponential function of SNR γ_o
$\bar{r}$	Mean data rate of unity
B	Actual Buffer size in bits
R_i	Actual throughput data rate of system
ω	Normalised Buffer occupancy
$G(\omega)$	Buffer Control Function (value of x_{th})
$g_o(\omega)$	Buffer Control Function for below threshold
$g_1(\omega)$	Buffer Control Function for above threshold
P_t	Probability of Transmission
P_f	Probability of Fade
T_D	Message Delay in sec
f_e	Average fade rate which equals f_r for $x_{th} = \ln 2$
v	Vehicle velocity in km/hr
f_o	Carrier frequency in Hz
c	Velocity of light in km/hr
$F^J(\omega)$	Distribution Function of buffer distribution defined at end of transmission periods
$F^I(\omega)$	Distribution Function at the beginning of transmission periods
$F(\omega)$	Distribution Function at an arbitrary time point
D	Distance in km.

LIST OF FIGURES

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LIST OF PRESENTATIONS AND PUBLICATIONS

1. "Effect of Feedback Error on Two Rate Data Transmission over a Rayleigh Fading Channel". Proceedings of 1975 Annual Engineering Conference, Hobart, Feb 1975, pp. 269-276.
2. "Channel State Feedback for Data Transmission in a Fading Environment". IREE International Electronics Convention, Sydney, Aug 1975, pp.327-329.
3. "Buffer Requirements for Intermittent Data Transmission in a Fading Environment". IREE International Electronics Convention, Sydney, Aug 1975, pp. 353-355.
4. with B.R. Davis, C.T. Beare and N.H. Le
   "Computer Simulation of Communication Systems". Research Report No. 2/75, Elec Eng Dept, University of Adelaide.
   Digest presented at IREE International Electronics Convention, Sydney, Aug 1975, pp. 330-332.
5. with B.R. Davis
   "Buffer Requirements for Intermittent Data Transmission over a Rayleigh Fading Channel". Research Report No. 4/75, Elec Eng Dept, University of Adelaide.
   (Accepted for publication in the IEEE Transactions on Communications).
6. with C.T. Beare
   "Simulation of Data Communication Systems".
   Submitted to Conference on Computers in Engineering, Perth, Sept 1976.

1.1.1 Models of Fading

Depending on the situation of interest, one uses different models to describe this fading characteristic. For a full discussion on the characterisation of fading one should consult Kennedy (1969) or Schwartz et al (1966). However, for this discussion the various models will be briefly described introducing the relevant terms to describe this situation.

If a sine wave is transmitted over such a medium, the various received waves combine to form a fluctuating signal which, for as few as six components, has essentially a Rayleigh distributed amplitude and a uniformly distributed phase. Furthermore, it is further assumed that this signal can be resolved into direct and quadrature components which are independent, Gaussian processes. This is called the multi-variate Gaussian model. However, in practice one transmits a band of frequencies; thus one must consider the effect over the band of interest of this fading.

Fading is caused by signals arriving at different times at the receiver. One can identify a range of times of arrival of these signals and define a "time delay spread T_M". When transmitting a narrowband signal of bandwidth W over such a medium, the receiver will be unable to resolve the various arriving components in the time domain unless:

$$W \gg 1/T_M \qquad 1.1$$

However, where the reverse is true, i.e.

$$W \ll 1/T_M \qquad 1.2$$

then the delays are sufficiently small with respect to the message bandwidth such that all frequencies within the band fade together. This sort of fading is called "flat" or "non-selective" fading and the Rayleigh fading model is applicable. The maximum value of W for which this approximation is valid is termed the "coherence bandwidth".

Flat Rayleigh fading is the most commonly used model of fading and is used extensively in this thesis. Another parameter of interest is the rate of fading f_e which depends on the carrier frequency and the relative velocity of the scatterers. The fading is termed "slow" if:

$$W >> f_e \qquad 1.3$$

In a digital situation, this means that the signal level can be considered constant over a bit interval. If this approximation is not valid one has "fast" or "time-selective" fading. This thesis is concerned with slow, flat fading.

If the message bandwidth does not satisfy the approximation given by 2 (i.e. greater than the coherence bandwidth), then one obtains "frequency selective" fading. In this situation, different parts of the message bandwidth fade by different amounts. One way of describing this situation is by a time varying transfer function. However, since this situation is a random process one needs to know the two dimensional co-variance function discussed by Kennedy (1969) where the multivariate Gaussian assumption is also evoked. In the situation where the approximation given by 3 is valid, the time term can be omitted. In most practical situations, the fading phenomenon is not usually time selective and frequency selection at the same time.

Throughout this thesis, the assumption of slow, non-selective Rayleigh fading is used, for this is the main context where channel state feedback as discussed in Chapter 2 has most potential. However, the analysis could have considered a Rician distribution, as done by Lindsey (1964), which considers a direct signal plus the scattered signal. Rayleigh fading, however, can be considered the worst case situation where the direct component is zero.

1.1.2 Applications

It is now worthwhile to consider the common fading channels of which some mention will be made in the thesis in order to describe the relevant fading characteristics and mention some of the main transmission problems.

The most used part of the radio spectrum is the HF area going from about 3 - 30 MHz. Propagation in this environment over long distances is via the ionosphere. The resultant fading phenomenon is quite complicated. In most instances one could consider it to exhibit slow, selective fading over a message bandwidth of 4 kHz. Digital transmission in such an environment is usually at low rates (e.g. about 100 - 1000 bits/s). Many systems using diversity and coding methods to overcome the effect of fading are discussed in a survey of the literature by Brayer (1975). However, this area of application will not be discussed in any more detail here as it does not lend itself to the application of channel state feedback due to excessive control delay (see Chapter 3) due to propagation time.

An area of particular interest is the Tropospheric scatter system[1]. This channel is observed to undergo Rayleigh fading of about 1 - 10 fades/s with a coherence bandwidth of several MHertz. Thus transmission rates of about 1 Mbit are seen to undergo slow, non-selective fading. Data rates higher than this but using an adaptive equaliser like that of Monsen (1971) have also been proposed. Because the fading is slow and has a wide coherence bandwidth, it is the most likely application for channel state feedback. In most present Troposcatter systems, diversity is used to cope with the fading.

In a mobile radio telephony situation at UHF as discussed by Clarke (1968), rapid fading occurs due to the motion of the mobile relative to the scatterers (e.g. buildings). Depending on the vehicle's velocity, the fading can be up to 100 fades/s. However, in comparison with a typical data rate of 56 kbit/s, and with a typical coherence bandwidth of 100 kHz this consitutes a slow, non-selective fading environment. It has been proposed by Clarke (1968) and others that diversity be used to overcome the fading in such an environment. However, as will be discussed later, several channel state feedback systems might find suitable application in this area.

Line of sight microwave systems also undergo fading where the envelope would be Rician distributed with usually a large specular to fading component ratio. However, problems such as "ducting" make this area quite different in its characteristics. Most consideration will be given to the former two applications in this thesis.

1.2 DIGITAL TRANSMISSION

Digital transmission in a fading environment has received extensive treatment in the literature and a recent publication edited by K. Brayer (1975) attempts to draw together the various aspects of the prob-

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1. see Schwartz et al (1966) and Gunther (1966).

lem and its solution. As a survey of the literature it is incomplete, giving an excessive bias to coding solutions in the HF environment and completely omits any discussion of channel state feedback. Presumably, this is because it has not been exploited yet. It is useful, therefore to briefly cover some of the basic aspects of the modulation methods used, the effects of fading and a survey of some of the methods used to solve the problem.

1.2.1 Effects of Fading

One of the most obvious effects of fading is the varying signal level and in most applications this is the dominant problem in a digital communication system in such an environment. Thus linear modulation methods which carry information in the signal level are particularly vulnerable to such signal variations. In spite of this obvious problem, it is interesting to note that some authors[2] still consider this form of modulation with its requirement for a signal dependent decision threshold. However, in most situations, angle modulation is preferable.

The most simple modulation method available is probably non-coherent FSK which will be considered exclusively throughout this thesis because of its simplicity. However, better performance can be obtained for coherent demodulation or by using a phase modulation method where these latter methods do involve extra implementation problems. Fundamentally, the performance of the system is dominated by the effect of the "deep" fade of more than 20 dB below the mean level. In this situation the error probability approaches 1/2. Thus, as will be explained in the next section, most solutions to the problem are directed at such deep fades which affect most angle modulation methods in a similar fashion. It is worth noting that using a phase locked loop in such a situation further aggrevates the problem by the loss of lock during deep fades.

If there is selective fading as is the case when the data rate exceeds the coherence bandwidth, the performance is further impaired by intersymbol interference. This leads to the so-called "irreducible error probability" mentioned by Schwartz et al (1966) where no better performance can be obtained with increasing signal level. A similar phenomenon is observed for linear modulation (fixed threshold) in a flat fading environment.

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2. see Painter and Wilson (1974).

However, if this intersymbol interference can be removed by suitable equalisation such as that suggested by Monsen (1973) or Price & Green (1958), the performance can be improved beyond that of the flat fading channel. This improvement has been attributed by Monsen (1973) to a form of "implicit" diversity. An interesting simple approach suggested by Perry (1969) for improving the performance for selective fading with a dual filter receiver is to use adaptive gains on the filter outputs to take advantage of the variation of signal level due to frequency selectivity. Chadwick (1971) shows that slow fading is a greater limitation to performance than fast or time selective fading. However, if no integrator is used to integrate over these variations as treated by Bello & Nelin (1962) fast fading can also produce an "irreducible error probability" as for frequency selective fading.

From these observations, it can be seen that slow, non-selective fading places a fundamental limitation on performance of non diversity systems unless channel state feedback is used. Frequency selectivity and time selectivity, if exploited at the receiver, can lead to better performance over flat fading. This is the reason why interest in this thesis centres on slow, non-selective fading for the application of channel state feedback.

1.2.2 Solutions to Fading Problems

In the literature, one can identify essentially three main methods for combatting the effects of fading where combinations of these methods are also used. These are coding, diversity and channel state feedback. More will be said of diversity techniques in the next section and channel state feedback is the subject of Chapter 2.

In the HF environment, since one is dealing with low data rates subject to frequency selective fading, a common method to improve the performance is to use coding methods[3]. Usually, these methods do not use any information about the channel state other than by initial design to allow for error bursts. Several methods, however, have been suggested[4] which combine coding methods with channel measurement so as to make better use of the information available at the receiver. However, in applications using much higher data rates (e.g. Troposcatter), coding methods are not suffi-

3. see Pierce et al (1970), Ralphs (1971), Goodman & Farrell (1975).
4. see Chase (1973), Lerner & Nicholson (1973), McCarthy (1975), Brayer (1975).

cient by themselves to remove the effect of deep fades due to the long coding distances that would be required. In such an environment, coding is best used as a second order solution to improve performance after diversity techniques have yielded a sufficiently low initial error probability and broken up the error bursts.

Before going on to discuss diversity, which at present is the main method used in a fading environment, one should mention the systems which make use of "implicit" diversity in a frequency selective fading situation. The first type discussed extensively by Schwartz et al (1966) is the Rake technique which is essentially an adaptive transversal filter equaliser. The second method by Monsen (1971) uses a decision feedback equaliser. Thus essentially the problem of frequency selective fading is a problem in adaptive equalisation.

The most common way of combatting fading is by the use of diversity techniques which are discussed by Schwartz et al (1966). The principle of diversity is that the receiver has available several statistically independent versions of the faded noisy signal. Thus the receiver can combine these signals, where the probability of all these branches being poor simultaneously is small, to improve performance. The types of diversity and combining methods will be discussed in the next section. As stated previously, the main cause of performance degradation is the deep fade. By use of diversity, the probability of such fades is greatly reduced, hence improving the performance.

However, in the case of a duplex communication system, the receiver can inform the transmitter of the channel state (e.g. the SNR) via feedback information multiplexed with the return information. Using this knowledge to vary transmitter parameters, the system can combat the effects of the time varying channel state. Such systems are called channel state feedback systems. The two main system types are power control suggested by Hayes (1968) and rate control by Cavers (1972). The various versions suggested in the literature will be discussed in Chapter 2 where it will be shown that these systems also depend on overcoming the deep fades to improve the performance.

All the methods to overcome fading have the common property that they must overcome the effect (or probability of occurrence) of a deep fade. In the case of the coding methods, this means effective burst error correcting properties over the fade lengths in question. Secondly, frequency selectivity can be thought of as introducing a level of diversity but which is hard to exploit in practice.

1.3 DIVERSITY

Diversity is the most common way of combatting the effects of fading. As comparison will be made during this thesis between diversity methods and channel state feedback, it is necessary to consider the various properties and practical constraints of diversity and the associated combining methods. It must also be pointed out that diversity can be implemented at the transmitter using channel state feedback as will be discussed in Chapter 2 as a third type of channel state feedback system.

1.3.1 Types of Diversity

The various types of diversity are quite well discussed by Schwartz et al (1966) and consist of:

1. Space Diversity
2. Frequency Diversity
3. Time Diversity
4. Angle Diversity
5. Polarisation Diversity.

In order to illustrate some of the facets of diversity, the first three types of diversity are discussed with respect to a Troposcatter system application.

Space diversity consists of antennae spaced a sufficient distance apart so that the fading on each antenna is independent. For space diversity, an extra advantage is an increase in received signal power which in the case of MRPDC combining (see next section) amounts to a 3 dB gain. However, in a Troposcatter system, receiver antennae are 10 m diameter dishes which are quite heavy and expensive. Thus space diversity, which is used quite usually on such systems does require expensive duplication of antennae and signal feeding.

Frequency diversity requires two transmitters operating at frequency separations greater than the coherence bandwidth. For second order diversity one uses twice as much transmission bandwidth and must duplicate the transmitting and receiving equipment. However, this has the advantage of a parallel reliability of transmitters and receivers. Secondly, if switching is done at the transmitter using channel state feedback instead of at the receiver, the system has a standby bearer capability. In practice this is usually done and, combined with space diversity, is used to achieve quadruple diversity.

In the case of digital transmission, one can also use time diversity. The simplest form of such diversity is to repeat each bit, spaced by a time greater than the fading time constant. A more sophisticated approach is the use of an interleaved code of similar length. However, such a system requires storage and extra bandwidth. Variable rate channel state feedback systems (see Chapter 3) are also a time diversity approach but are far more effective at improving performance with more efficient use of bandwidth.

1.3.2 Combining Methods

Given a particular type of diversity, there is a variety of ways in which these signals can be combined. Schwartz et al (1966) covers the most common combining techniques where a more complete discussion may be found in Brennan (1959). Some of the methods are:

1. Maximal Ratio
2. Equal Gain
3. Selection
4. Switch.

It is useful to briefly examine some of the important constraints of such combining methods.

The most commonly quoted diversity performance curves are usually for maximal ratio predetection combining, since this is the optimum type of combining for slow non-selective fading. However, to achieve this sort of combining, one must be able to estimate the channel state, which includes amplitude and phase, on the various diversity branches in order to optimally combine them. However, even when all of this information is not available or not used as in equal gain or switch diversity, the performance is only about 1.5 dB inferior. Thus, in practice, it is often better to use the simpler combining methods and yet still gain the main diversity improvement due to a decreased probability of deep fades.

Work on even simpler combining methods such as Switch Diversity treated by Davis (1970) and Rustako (1973) for a mobile radio situation re-inforce the principle that the main improvement is due to the effect on deep fades. It is also an advantage of such simple methods that no duplication of front end receivers is required.

However, for comparison purposes with channel state feedback systems, it is useful to use the performance of the optimum diversity system. When making these comparisons, it must be borne in mind that the detailed comparison depends on more specific information of the situation involved.

1.4 CONCLUSIONS

Depending on the application, there are different aspects of the fading problem which must be considered. However, the dominant problem in any fading environment are the large amplitude variations and, in particular, the deep signal fades. The most common model of such fading is that of slow, non-selective Rayleigh fading or sometimes Ricean fading to include the effect of a direct signal component.

Digital transmission performance in such an environment is dominated by the large number of errors occurring during a deep fade and so any solution relates to these deep fades. Time selective and frequency selective fading usually introduce further degradation in performance in the form of an irreducible error probability. However, such effects can be exploited as a form of implicit diversity. Of course, this may prove too complex and cannot be done in the rare cases where a channel is both time and frequency selective.

In the literature, many methods are suggested to overcome the effects of fading where the three basic methods are coding, diversity and channel state feedback. These three categories are of course not mutually exclusive. Coding is most useful in low data rate applications such as in the HF environment and as a supplementary solution in most other applications. Diversity is the most useful accepted method of overcoming fading. However, channel state feedback is another possibility which has not been significantly exploited to date. It is the only alternative to diversity to improve performance in a slow, non-selective fading environment.

CHAPTER 2

2.0 INTRODUCTION

In a duplex communication system, feedback information concerning the channel state can be multiplexed with the return information to control transmitter parameters in such a way as to counteract the effect of the time varying channel state. Such systems intended to combat slow, non-selective fading are called channel state feedback systems. Unlike information feedback systems treated by Turin (1969) or decision feedback schemes such as that treated by Metzner et al (1968), channel state feedback only uses the information learnt at the receiver concerning the channel state.

In the literature three types of such systems can be identified and their potential performance assessed. However, such systems must operate within the constraints of a practical system. In the literature, many authors have neglected or understated the effects of important constraints on a channel state feedback system such as feedback delay and/or limited bandwidth. The main system constraints can be identified and their effect on performance assessed so as to predict a more realistic measure of the performance potential of the various systems. This work has been reported by the author Coutts (1975b).

From such considerations of the various system types possible, attention has centred on an intermittent data transmission system and performance of the ideal system is evaluated. In chapters 3 to 5, this system is treated in more detail.

2.1 SYSTEM TYPES

There are three distinct parameters which may be varied at the transmitter to combat the effects of fading. These are transmitter power, channel selection and data rate where a system may incorporate a combination of such parameter variations. The various system types are discussed with reference to how they are treated in the literature and the basic reasons for such performance is discussed.

2.1.1 Variable Power

Hayes (1968) treats the case of a variable power system used with diversity where the power on all diversity branches vary together. Skinners and Cavers (1973) treat the case where the power on the diversity branches vary independently and come to the conclusion that only the best branch should be used with the total power available. Hentinen[1] (1974) shows the variation of optimum control law with the average power constraint. This control law concentrates most power at below the median signal level but which decreases for very low levels and the higher levels.

However, none of the above authors properly considers the effect of a peak power constraint. In the case where no diversity is used, a 30 dB[2] peak to average power ratio was required to give a decrease in average error probability for increasing signal to noise ratio as predicted by Hayes (1968) for a non-diversity situation. In most applications one is interested in the peak power requirement, since this is what limits the capability of the transmitter. Thus in practice a 30 dB peak power requirement would be out of the question. If the peak to average power ratio is restricted to a more reasonable figure of 3 dB as in a situation to save power in a remote repeater, the average error probability reduces as the reciprocal of the mean signal to noise ratio (SNR) for high SNR values in the same way as the case for a fixed power system.

In the situations where variable power is used in conjunction with diversity or variable rate, the restriction on peak power is no longer critical. In fact for "Selective Diversity" treated by Skinners and Cavers (1973), a 10 dB ratio is typically required. Similarly, Hentinen (1974) combines variable power with variable rate and introduces up to a 10 dB peak power limitation with marginal decrease in performance. A quite different system, using decision feedback and channel state feedback treated by Glave (1972), exhibits a similar limited performance with a peak power (maximum repetition number) constraint when no diversity is used.

When used in isolation of diversity or variable rate, a variable power system requires a high peak to average power value to achieve app-

1. Derived first by Hayes (1968).
2. From simulation results.

reciable performance improvement. The reason is that a variable power system cannot cope with deep signal fades other than by using large peak powers and it is these situations that dominate the system performance. Thus variable power should be used with other methods such as diversity so as to reduce the average power requirement where this is a design criterion such as in remote site applications.

2.1.2 Channel Selection

For a number of reasons, it is desirable in some situations to switch between diversity channels at the transmitter rather than at the receiver. One such obvious advantage is with frequency diversity as mentioned in Chapter 1, to provide a multiple user capability. This method is used on microwave links to improve reliability and is a special case of "Selective Diversity" treated by Skinners and Cavers (1973) who also combine this system with variable power. Another advantage is applicable if it is easier to use multiple antennae for space diversity at the transmitter rather than the receiver. Such a system, called "Switch Diversity" treated by Davis (1970) and then by Rustako et al (1973) for use in a mobile radio system, is only slightly inferior to selection diversity provided the control delay is small. This problem will be discussed in section 2.2.2.

Skinners and Cavers (1973) claim a most dramatic performance potential for "Selective Diversity" to the extent that a fourth order diversity system offers better performance than the equivalent non-fading channel. However, the system relies on using the total available power (varying) on the best channel at any given time and an 8 dB peak to average power ratio. Another advantage of this system is the multiple user capability. This system illustrates the potential of variable power to save on average power when used effectively with diversity.

Along with other channel selection methods such as that treated by Bitler et al (1973), the main advantage of channel selection systems is the ability to have the multiple antennae at the transmitter in the case of space diversity and to allow multiple users in the case of frequency diversity.

2.1.3 Variable Data Rate

A slow fading channel can be considered to be a channel of time varying channel capacity. Thus, one effective method of improving performance is to match this channel with a corresponding time varying information rate. In practice, the easiest way to implement this principle is by a variable data rate system of the type suggested by Cavers (1972) which has a variable bandwidth. In order to match the channel, one would really want to use the full bandwidth all the time and use a variable number of levels as well as variable data rate. Such a variable number of levels scheme has been suggested by Hentinen (1974) but could prove difficult in practice to implement.

Such variable rate schemes have been shown to be extremely effective at coping with fading. In fact, the variable bandwidth scheme of Cavers (1972) can completely eliminate the effects of fading and this result has been shown by Srinivasan (1973) and Hentinen (1974) to be true for a broad class of modulation methods. If used with variable power or variable number of levels, such variable rate schemes offer potential performance better than the equivalent non-fading channel. However, in practice, such systems must operate within certain constraints such as finite bandwidth which will be discussed later.

In a Rayleigh fading situation, the main performance improvement is due to the low data rate (near or equal to zero) used during deep fades which are the conditions that dominate the performance. If a bandwidth limitation is imposed by restricting the maximum date rate, then the performance decreases by about 1 dB for a rate constraint of double the average rate. Comparison with an intermittent system (Two rate scheme) treated by the author Coutts (1975a) and Martin (1967) reveal that a fully variable rate system is only 2.5 dB superior in performance. Thus, an intermittent system overcomes the dominant source of error and, in practice, would be far simpler to implement. This system will be discussed in detail in section 2.4.

Hentinen (1974) and Srinivasan & Brewster (1974) have discussed the possibility of using variable power with variable rate to overcome the effects of limited bandwidth. Such schemes do offer a reduction in average power over variable rate schemes but in most situations this is not as important a criterion as peak power.

2.2 SYSTEM CONSTRAINTS

Channel State Feedback systems have a wide variety of implementations and claims about their potential performance must be seen in the light of several system constraints which apply to all such systems. Some of these constraints have already been mentioned (e.g. peak power). Detailed consideration of these constraints with specific detail concerning an intermittent system will be discussed in Chapters 3 and 4. Here the general problem will be qualitatively discussed with reference to the mentioned types of system.

2.2.1 Peak Power and Maximum Bandwidth

One of the most obvious constraints imposed on any communication system is the finite dimensionality of the signal space with respect to peak power and bandwidth.

Peak power, as mentioned previously, imposes a severe limitation on variable power systems where no diversity or variable rate is used. To cope with deep fades of 30 dB, the transmitter power would have to be capable of a similar dynamic range. This usually would lead to extremely inefficient transmitter design only to save on power consumption. However, as is illustrated by Hentinen (1974), such peak powers are not necessary when combined with any method that can cope with the deep fades such as a variable rate system.

The variable rate system suggested by Cavers (1972) is a variable bandwidth system and in most applications such a system must operate within a finite bandwidth. Thus, this constraint results in a reduction in the potential performance of such a system. It is in the light of this constraint and obvious implementation difficulties that a variable rate system does not offer as much improvement over an intermittent system as at first thought.

2.2.2 Feedback Reliability

A very important limitation on all such channel state feedback systems is the reliability of the service information used to control the system. This problem may be resolved into two component but related parts; namely, probability of service information error and control delay.

Cavers (1972) discusses the problem of control delay with respect to a variable rate system and shows that the system performance deteriorates quite rapidly if the delay becomes too large with respect to the fading bandwidth. Skinners & Cavers (1973) also show that control delay, which includes propagation time and filter delay, must not be too large. Because of the sensitivity of system performance to control delay, the optimum control law, as for the variable rate system (see Chapter 3) is very dependent on control delay. In applications such as a Troposcatter system, propagation delay is the dominant source of delay and this implies a maximum distance over which such systems will operate.

If the fading is non-reciprocal, then feedback (and sometimes feedforward) information must be sent over a similar, fading noisy channel with the resultant possibility of making control errors. In order to increase the reliability of this service information, one must increase the control decision delay which constitutes another component of control delay. Thus, due to this interaction (see Chapters 3 and 5), the fading rate to data rate must be sufficiently low to obtain reliable service information. Another important aspect of this problem is that service information must be protected from deep fades as well as the main information and in Chapter 3 this will be shown to be possible by using a weighted decision which is dependent on the feedback SNR. Lastly, all of these aspects are accentuated as the amount of service information increases.

The problem of feedback reliability affects all channel state feedback systems in a similar way. Due to propagation delay there is an upper bound on the maximum distance over which such systems will work and, secondly, due to the necessity for reliable service information, the fading rate must be very slow with respect to the data rate.

2.2.3 Buffer Requirements

Another constraint which is only relevant to variable rate systems is the necessity for a finite buffer at both transmitter and receiver to interface with a uniform data source and sink. With a finite buffer, the possibility of the buffer filling and emptying will degrade the performance where this effect gets worse with decreasing buffer size. Also, in such a buffered system, the digital message is delayed and this delay is directly proportional to buffer size.

Most authors in discussing variable rate systems have neglected to consider the importance of the buffer size as it affects the performance in terms of higher error probability and also message delay. A similar buffer problem exists for the meteor burst systems discussed by Forsyth et al (1957). However, this application is quite different from the Rayleigh fading environment discussed in this thesis. Campbell (1957) discusses the buffer requirements for such a non-synchronous system but does not consider the effect on error probability. The author Coutts (1975c) has applied this analysis to a synchronous intermittent system to consider the required buffer size for a given performance level. This work is the subject of Chapter 4.

The results for an intermittent system suggest that the control law for a variable rate system would be quite dependent on the buffer size used and the performance level required. Thus, the results of Cavers (1972) and Hentinen (1974) must be seen as upper bounds on the performance where buffers much larger than would be necessary in practice were used. However, it is reasonable to say that a variable rate system would require roughly the same size buffer as an intermittent system due again to the dominant problem of buffering against deep fades. Thus, for some applications such as digitised speech (telephony) over a Troposcatter system, the resultant message delay would be unacceptable for any variable rate system to be used.

2.3 INTERMITTENT SYSTEM

In an intermittent system, the receiver measures the signal level of a pilot (or main signal) and makes a two state decision, depending on whether or not this level is above a threshold which, it will be assumed here (see Chapter 3), can be sent reliably over a feedback channel to control the output bit stream of the transmitter. If the signal to noise ratio (SNR) is below threshold, data is stored in a buffer which is here assumed to be infinite (see Chapter 4).

As was discussed in Chapter 1, one will assume here that one is using non-coherent FSK and the system is to operate in a slow, non-selective Rayleigh fading environment where there is no great loss in generality and the analysis is simplified. For non-coherent FSK in an additive Gaussian noise environment, Schwartz et al (1966) give the average error probability as

$$P_{\varepsilon}(\gamma) = \tfrac{1}{2} \exp(-\gamma/2) \qquad 2.1$$

where γ is the SNR of the filter output at the sampling instant which is given by:

$$\gamma = \frac{S}{N_o R} \qquad 2.2$$

S - Received Signal Power

N_o - Single sided Noise Power Density

R - Transmission Data Rate (Determines filter Bandwidth)

In the case of slow, non-selective fading this SNR is a random variable which is exponentially distributed as:

$$p(\gamma) = \frac{1}{\gamma_o} e^{-\gamma/\gamma_o} \qquad o < \gamma < \infty \qquad 2.3$$

where $\gamma_o = < \gamma >$ is the mean value of the SNR averaged over the fading. One can then define the normalised SNR variable x as:

$$x = \gamma/\gamma_o \qquad 2.4$$

For an intermittent system, transmission of actual data only occurs when the SNR is above a normalised threshold x_{th}. However, to maintain the same average data rate as a continuous system, the transmission rate must be higher (increase in bandwidth) by a factor r_1 called the transmission ratio. Thus, for a particular value of normalised SNR x, the actual SNR at detection is given by:

$$\gamma = \frac{\gamma_o x}{r_1} \qquad 2.5$$

The average error probability is thus given by

$$\bar{P}_{\varepsilon} = \int_{x_{th}}^{\infty} r_1 P_{\varepsilon}(\gamma_o x/r_1)\, p(x)\, dx \qquad 2.6$$

and the average normalised data rate is

$$\bar{r} = \int_{x_{th}}^{\infty} r_1\, p(x)\, dx = 1 \qquad 2.7$$

where these integrals assume infinite full buffers and perfect feedback control.

Evaluation of 2.6 gives

$$\bar{P}_\varepsilon = \frac{r_1^2}{\gamma_o+2r_1} \exp\left[-\frac{(2r_1+\gamma_o)\, x_{th}}{2r_1}\right] \qquad 2.8$$

where $x_{th} = \ln r_1$ for an average normalised rate of unity. Thus on substitution for x_{th}, equation 2.8 can be simplified to

$$\bar{P}_\varepsilon = \frac{r_1}{\gamma_o+2r_1} \left(\frac{1}{r_1}\right)^{\gamma_o/2r_1} \qquad 2.9$$

In fact there exists an optimum value of r_1 (and thus x_{th}) to minimise equation 2.9 which must satisfy the necessary condition

$$\frac{\partial \bar{P}_\varepsilon}{\partial r_1} = 0 \qquad 2.10$$

This leads to an intrinsic function for r_1 relating to the mean SNR γ_o given by

$$2r_1 \ln r_1 = \gamma_o (1 - \ln r_1) \qquad 2.11$$

where, since r_1 is greater than unity, one has the inequality

$$1 < r_1 < e \qquad \text{for all } \gamma_o \qquad 2.12$$

The solution of equation 2.11 is shown in Fig. 2.1 for different values of SNR γ_o. Fig. 2.2 is the corresponding optimum value of average error probability versus mean SNR. Thus an intermittent system gives an exponential fall off in error probability for increasing SNR.

However, in a practical system, the dominant consideration regarding the choice of the transmission ratio is the required bandwidth. Thus $r_1 = 2.0$ is the maximum ratio that would be considered reasonable in terms of spectrum utility. In Fig. 2.3 is plotted the performance of the intermittent system for different transmission ratios along with performance of the optimum frequency diversity system.

From these results it can be seen that such an intermittent system compares favourably with fourth order diversity. Secondly, the effect of restricting the transmission ratio to less than $r_1 = 2.0$ is negligible but gets progressively worse as this ratio is decreased even further. During most of this thesis $r_1 = 1.6$ is taken as a reasonable

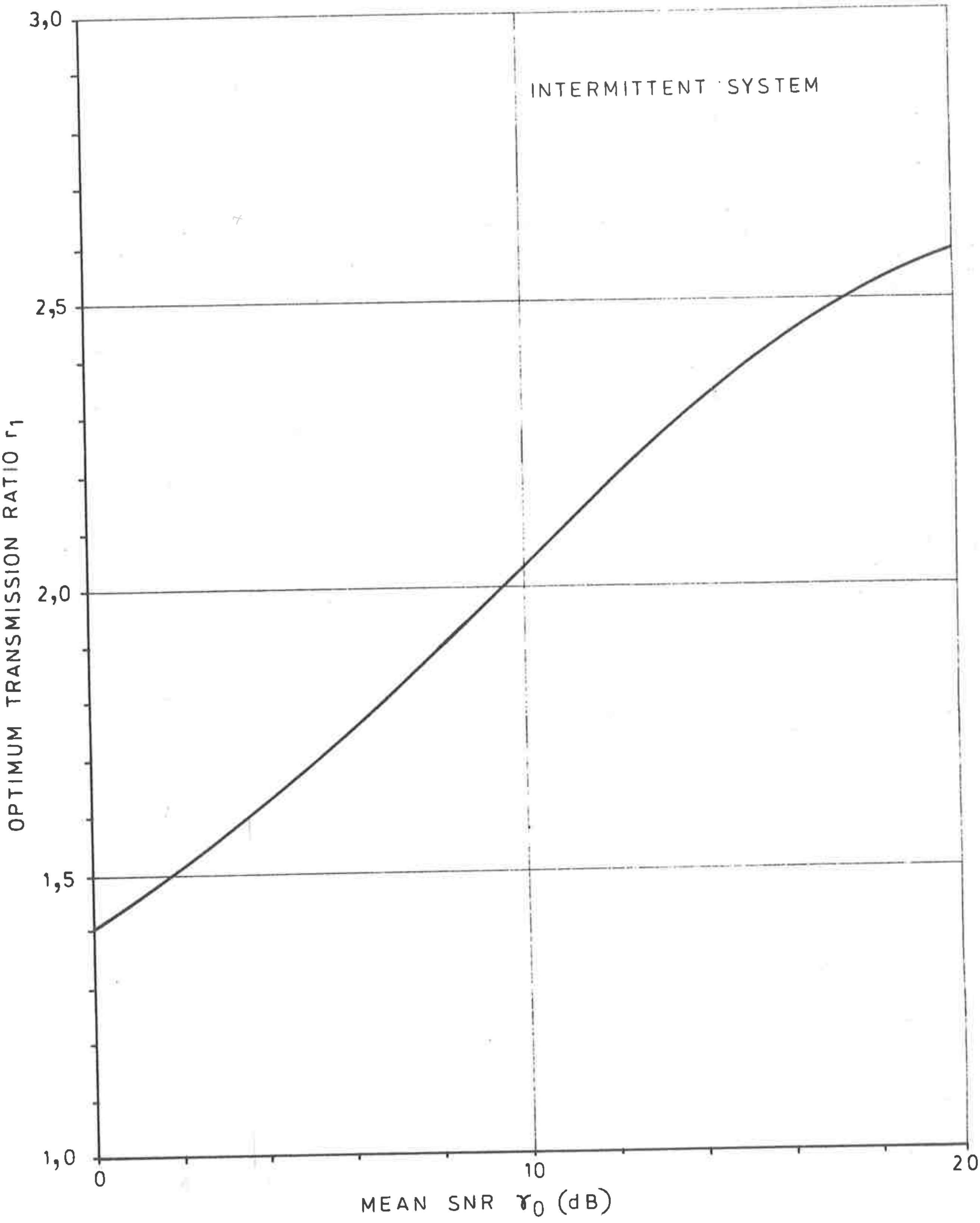

FIG. 2.1 OPTIMUM TRANSMISSION RATIO

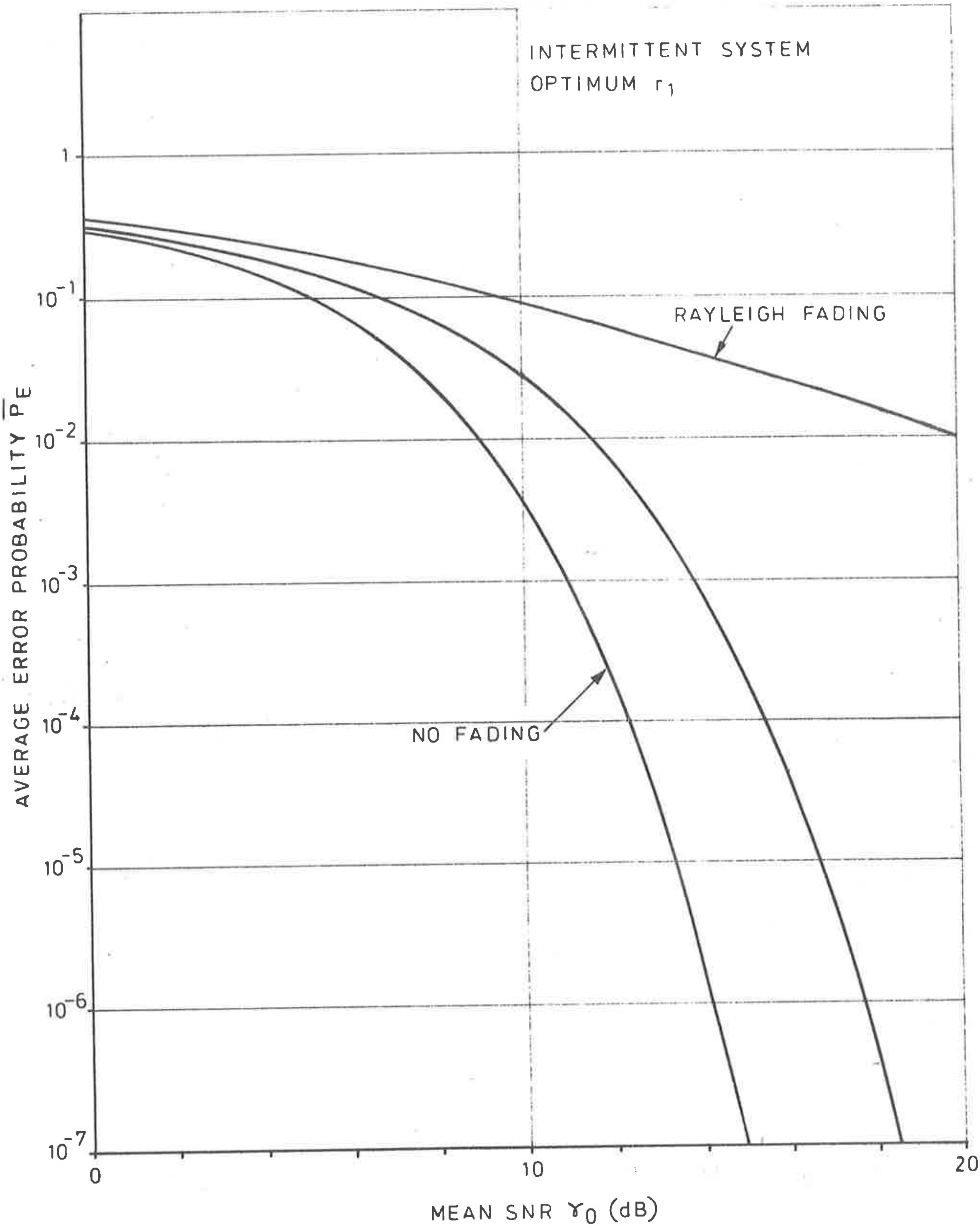

FIG. 2.2 OPTIMUM INTERMITTENT SYSTEM PERFORMANCE

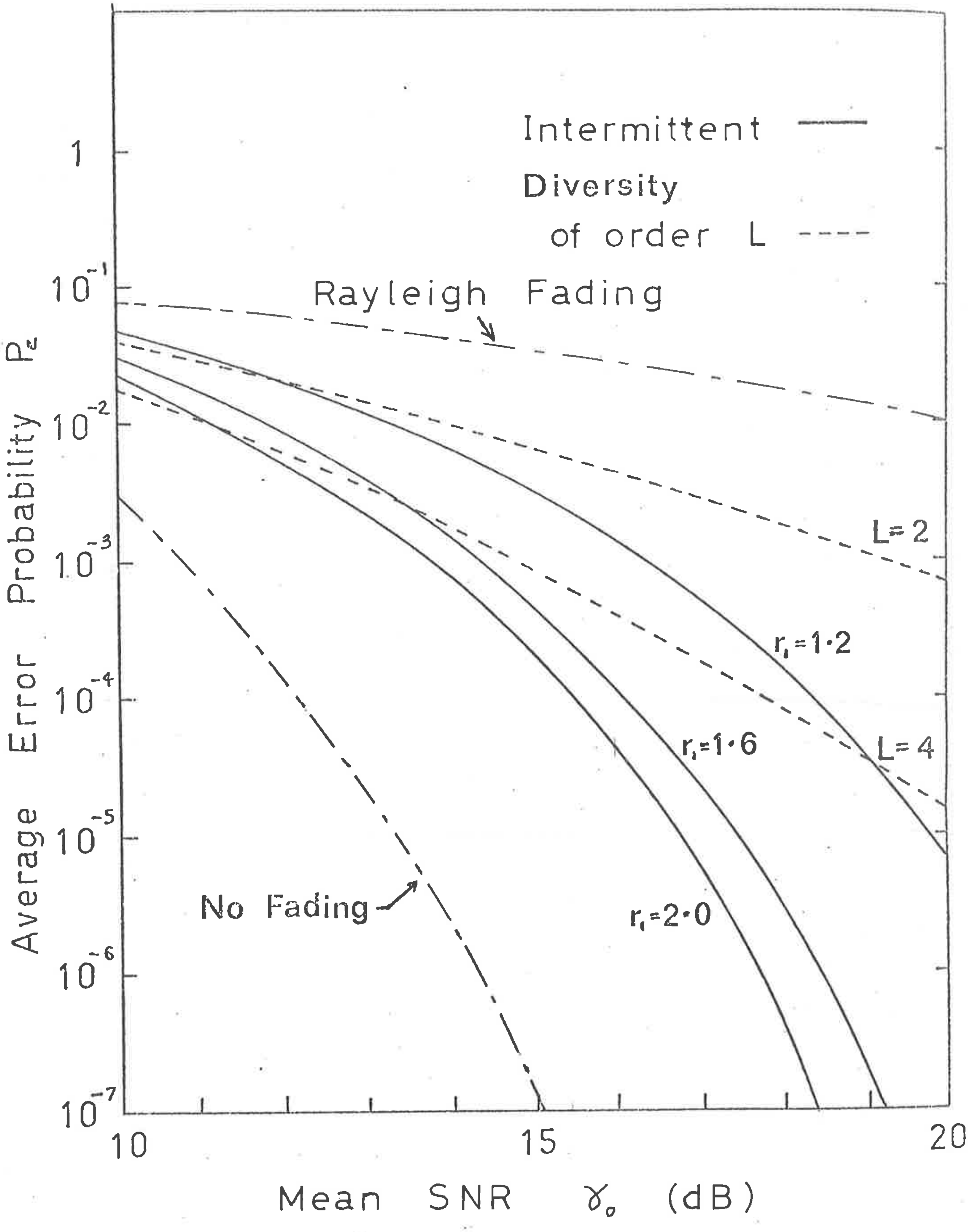

FIGURE 2.3
INTERMITTENT SYSTEM PERFORMANCE FOR DIFFERENT TRANSMISSION RATIOS R_1

compromise value.

The performance of an intermittent system is subject to the constraints of limited control reliability and a finite buffer. These constraints which are applicable to any variable rate system are discussed in detail with respect to an intermittent system in Chapters 3 and 4.

2.4 CONCLUSIONS

Consideration of the performance of the various types of channel state feedback systems subject to practical constraints leads to the general conclusion that the performance of such systems is very dependent on the constraints.

More specifically, a variable power system alone cannot cope with the central problem of deep fades unless a prohibitive peak-to-average power ratio is permitted. However, if used with diversity or variable rate this constraint is much less serious and does offer a reduction in the average power requirement. This assumes of course that reliable feedback information is available.

A variable rate system in the light of a bandwidth constraint offers only about 2.5 dB improvement over an intermittent system but is potentially a more powerful scheme. However, such variable rate schemes are more complex in that they require a buffer resulting in message delay. Also, feedforward information is required when the feedback information has only limited reliability.

In general, only the simple forms of channel state feedback systems such as the intermittent system and the simple channel selection schemes appear to offer any significant practical system potential due to system sensitivity to the practical constraints. It is in the light of this conclusion that interest has centred mainly on an intermittent system which will be considered in much more detail in the following chapters.

CHAPTER 3

3.0 INTRODUCTION

One of the most important constraints on a channel state feedback system of the type discussed in Chapter 2 is the reliability of the service information. The analysis so far has assumed that the transmitter has perfect, instantaneous knowledge of the channel. This chapter investigates this problem[1] with specific reference to an intermittent system and draws some general conclusions applicable to all such feedback systems. Also discussed is an alternative control method which can be used if the fading phenomenon is reciprocal.

3.1 FEEDBACK AND FEEDFORWARD ERROR

For an intermittent system, the receiver must inform the transmitter whether the receiver SNR is above or below threshold and this information must usually be sent over a similar noisy, fading feedback channel. Possible feedback errors result in an increase in the number of errors in the main data stream. Closely coupled with this problem is the necessity for the receiver to know when to receive data. Thus, the receiver is informed of these decisions over the main channel. The ultimate reliability of this service information depends on the allowable control delay which is dependent on the fading rate as discussed in section 3.3.

3.1.1 Feedback Error

In Fig. 3.1 is illustrated the model of an intermittent system. It is assumed that this feedback information is sent at an average rate r_f which is much slower than main data rate by a factor n_r called the rate ratio defined as the ratio of the main transmission rate to the service information rate. The probability of such feedback errors depends on the mean SNR γ_f of the service information which is a function of r_f.

One can define the feedback error probability P_{ij} as the probability that the demodulator at A gives a decision j when the decision i is sent from B where:

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1. This work has been reported by the author Coutts (1975a).

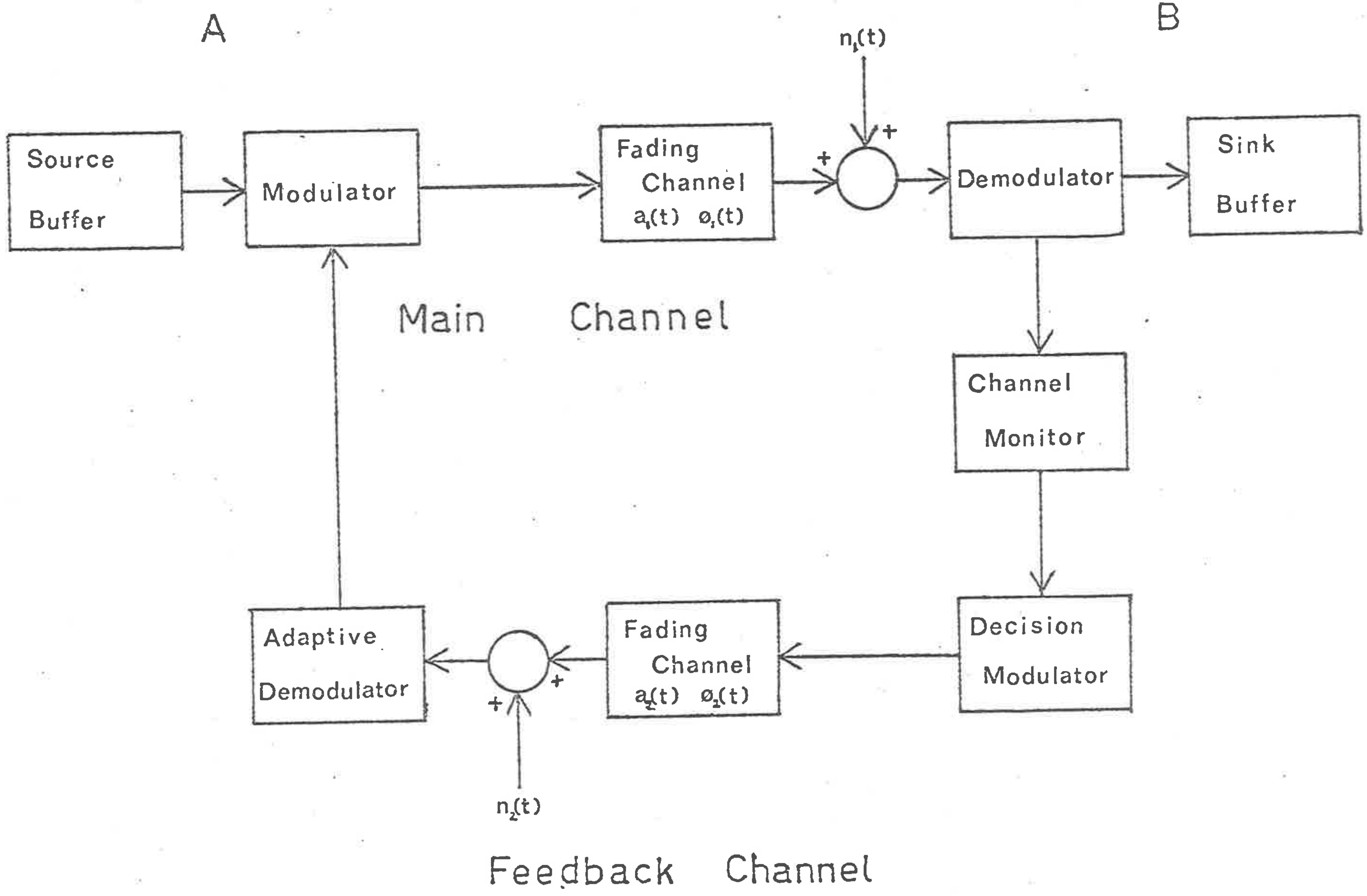

FIGURE 3.1
MODEL OF INTERMITTENT SYSTEM

$$i,\ j = \begin{cases} 0 \text{ if Tx is off} \\ 1 \text{ if Tx is on} \end{cases} \qquad 3.1$$

where $P_{i0} + P_{i1} = 1 \qquad i = 0,\ 1 \qquad 3.2$

The feedback channel is also Rayleigh fading with normalised random channel gain y which is also exponentially distributed. Thus for given values of SNR x and y, which are assumed to be independent, the error probability is given by:

$$P_\varepsilon(x,y) = \{P_{11}(y)\ r_1\ P_\varepsilon(x)\}\ x \geq x_{th}$$

$$+ \{P_{01}(y)\ r_1\ P_\varepsilon(x)\}\ x < x_{th} \qquad 3.3$$

and the rate is given by:

$$r(x,y) = \{P_{11}(y)\ r_1\}\ x \geq x_{th} + \{P_{01}(y)\ r_1\}\ x < x_{th} \qquad 3.4$$

where r_1 is the transmission ratio (see Chapter 2) and it is assumed the receiver at B has perfect knowledge of the transmitter decisions. This assumption will be removed in section 3.1.4.

In a duplex communication system, this feedback information is multiplexed (TDM or FDM) with the return information and thus the transmitter power and the bandwidth must be shared between the main and service information. Thus one defines n_p as the ratio of main signal power to the service signal power. The mean SNR γ_o is now reduced to γ_m and the service channel SNR is γ_f (includes r_f). If this service information is sent by non-coherent FSK, the feedback error probabilities become:

$$P_{01}(y) = P_{10}(y) = \tfrac{1}{2} \exp\left[-\frac{\gamma_f y}{2}\right] \qquad 3.5$$

Substitution of 3.5 into equations 3.3, 3.4 and then averaging over the fading statistics gives the average error probability as:

$$\bar{P}_\varepsilon = \frac{r_1^2}{\gamma_m + 2r_1}\left\{z + \frac{1 - 2z}{2 + \gamma_f}\right\} \qquad 3.6$$

and the average rate is constrained as:

$$\bar{r} = r_1\left\{e^{-x_{th}} + \frac{1 - 2e^{-x_{th}}}{2 + \gamma_f}\right\} = 1 \qquad 3.7$$

where

$$z = \exp\left[-\frac{(\gamma_m + 2r_1)}{2r_1} x_{th}\right] \qquad 3.8$$

If this service information is FDM multiplexed with the return information, the mean SNRs γ_m and γ_f are given by:

$$\gamma_m = \frac{n_p}{n_p+1}\gamma_o \qquad 3.9$$

$$\gamma_f = \frac{1}{n_p+1}\cdot\frac{n_r}{r_1}\gamma_o \qquad 3.10$$

Thus using equations 3.5, 3.8, 3.9 and 3.10, the expression for the average error probability of equation 3.6 was minimised [2] subject to the rate constraint of equation 3.7 for a given value of n_r. The effect of feedback error is shown in Fig. 3.2 for different rate ratios n_r.

The average error probability for high SNR departs quite markedly from the ideal curve meaning n_r is required to be quite large to improve the reliability. This fall off is seen in equation 3.6 as only an inverse dependence on γ_f. As will be explained in section 3.4, n_r is limited above due to the effect of control delay which is directly dependent on n_r. However, an adaptive receiver at A will be shown to greatly improve this position.

3.1.2 Feedback Strategy Optimisation

There are three reasons why the FSK demolulator at A should use weighted decisions:

(i) Unequal a priori decision probabilities;

(ii) Unequal costs associated with either error;

(iii) Time varying SNR makes the optimum weighting a function of this feedback SNR.

Thus the aim is to choose this weighting function so as to optimise the performance of the forward channel.

The demodulator structure shown in Fig. 3.3 has a variable gain k after the '0' envelope detector which is directly controlled by the measured

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2. see Note 3.

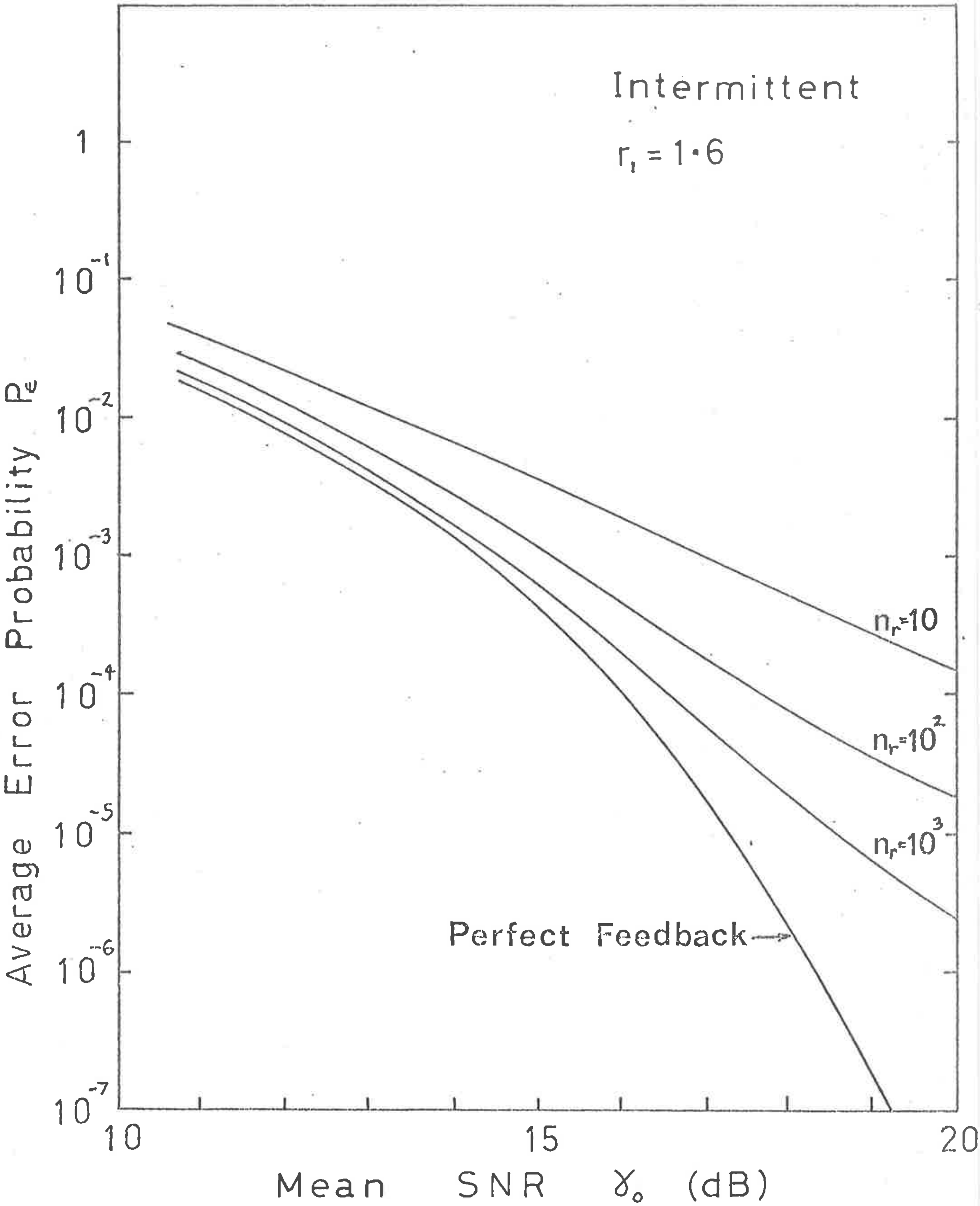

FIGURE 3.2

EFFECT OF FEEDBACK ERROR FOR A NON-ADAPTIVE RECEIVER FOR DIFFERENT RATE RATIOS N_R

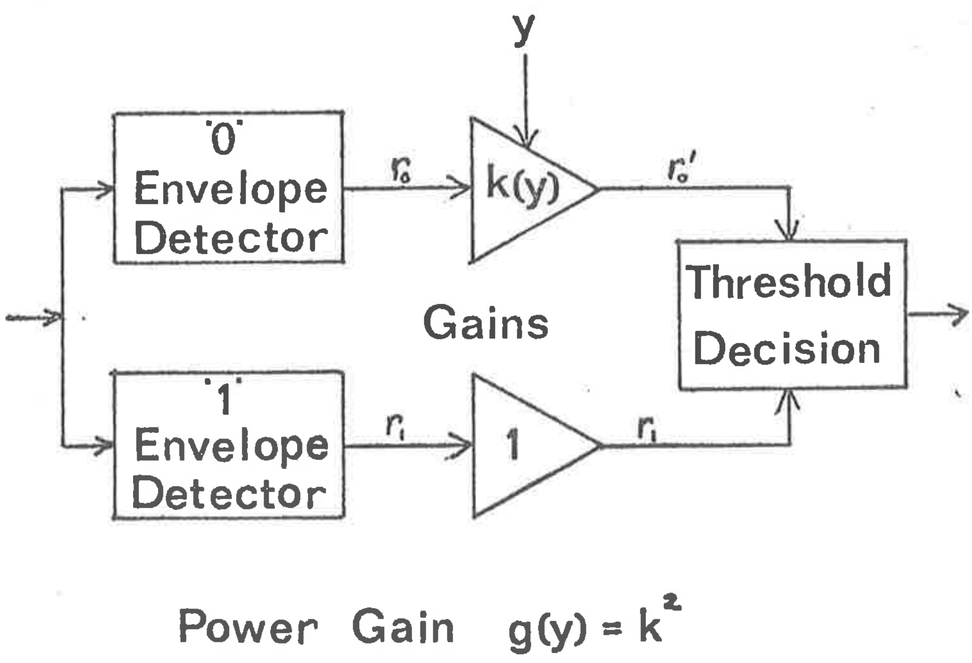

FIGURE 3.3

ADAPTIVE FSK DEMODULATOR

SNR on the feedback channel. For this demodulator it is shown (Appendix A) that the error probabilities P_{ij} are a function of the adaptive power gain $g = k^2$ given by:

$$P_{01}(\gamma) = \frac{1}{1+g} \exp\left[- \frac{g}{1+g} \gamma\right] \qquad 3.11$$

$$P_{10}(\gamma) = \frac{g}{1+g} \exp\left[- \frac{1}{1+g} \gamma\right] \qquad 3.12$$

where γ is the SNR of the feedback.

Make the weighting function g a function of the normalised SNR y, substitute in equations 3.3, 3.4 and average over the fading statistics. The average error probability becomes a functional of g as:

$$\bar{P}_\varepsilon = \frac{r_1^2}{\gamma_m + 2r_1} \left\{ z \int_0^\infty \left(\frac{1-z}{1+g} \exp\left[- \frac{g}{1+g} \gamma_f y\right] - \frac{g\,z}{1+g} \exp\left[- \frac{1}{1+g} \gamma_f y\right] \right) e^{-y}\, dy \right\} \qquad 3.13$$

and average rate is constrained as:

$$\bar{r} = r_1 \left\{ e^{-x_{th}} - \int_0^\infty \left(\frac{g}{1+g} e^{-x_{th}} \exp\left[- \frac{1}{1+g} \gamma_f y\right] - \frac{(1-e^{-x_{th}})}{1+g} \exp\left[- \frac{g}{1+g} \gamma_f y\right] \right) e^{-y}\, dy \right\} = 1 \qquad 3.14$$

This is then a problem of variational calculus to minimise equation 3.13 with respect to g(y) subject to the integral constraint of 3.14. Solutions of the Euler-Lagrange equation leads to the implicit equation (Appendix B)

$$\frac{\gamma_f y + g + 1}{\gamma_f g y + g + 1} \exp\left[- \frac{(g-1)}{g+1} \gamma_f y\right] = k_3 \qquad 3.15$$

where the constant k_3 is chosen to satisfy equation 3.14. Using this equation 3.15, equation 3.13 was minimised with respect to g(y) and also the threshold x_{th}.[3]

In Fig. 3.4. is shown a typical solution of the optimum weighting function g(y). From the form of the solution, the unequal error costs become extremely important only when the feedback SNR drops below a threshold. This means in practice that this optimum strategy sacrifices data rate when the feedback information becomes unreliable.

3.1.3 Sub-Optimal Strategy

It is apparent from the form of the optimum gain function shown in Fig. 3.4 that a simple sub-optimal threshold strategy could be used given by:

$$g(y) = \begin{cases} \infty \text{ if } y < y_{th} \\ 1 \text{ if } y \geq y_{th} \end{cases} \qquad 3.16$$

Substituting this gain function into equations 3.13, 3.14 and integrating one obtains:

$$\bar{P}_\varepsilon = \frac{r_1^2}{\gamma_m + 2r_1} \left\{ z\, e^{-y_{th}} + \frac{(1-2z)}{2+\gamma_f}\, \omega \right\} \qquad 3.17$$

subject to the rate constraint

$$\bar{r} = r_1 \left\{ e^{-(x_{th} + y_{th})} + \frac{(1-2e^{-x_{th}})}{2+\gamma_f} \right\} = 1 \qquad 3.18$$

where

$$\omega = \exp\left[- \frac{(\gamma_f + 2)}{2}\, y_{th} \right] \qquad 3.19$$

Comparison with equation 3.6 shows how this strategy makes $\bar{P}_\varepsilon$ exponentially dependent on γ_f as well as the original inverse dependence.

For a given rate ratio n_r, equation 3.17 was minimised[3] with respect to n_p and y_{th} where x_{th} must satisfy the constraint of equation

3. The equation was minimised numerically using a simple heuristic stepping algorithm.

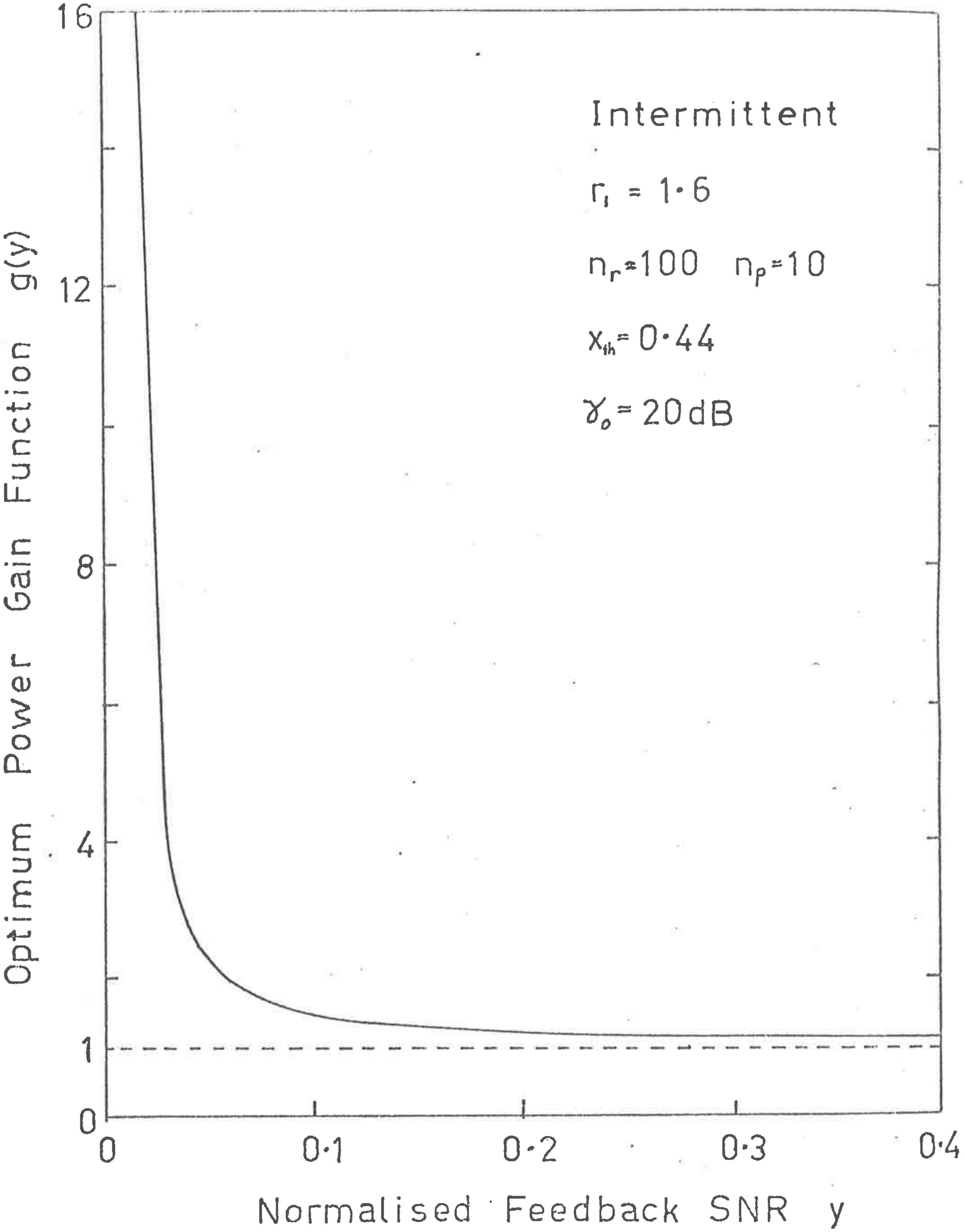

FIGURE 3.4

OPTIMUM GAIN FUNCTION

3.18. These results are shown in Fig. 3.5 for the same three rate ratios. Comparison with the dashed curves (from Fig. 3.2) shows that the threshold strategy greatly improves the performance, especially at low error probabilities. In Table 3.1 is shown a comparison of the results of this sub-optimal strategy and the optimum one corresponding to the optimum gain function of Fig. 3.4. These results show that the sub-optimal strategy is not substantially inferior to the optimum one and is far simpler to implement.

Intermittent System		
$r_1 = 1.6$	$\gamma_o = 20$ dB	$n_r = 100$
System Description	Other Parameters	Average Error Probability $\bar{P}_\varepsilon$
Perfect Feedback	$x_{th} = 0.47$	6.5×10^{-9}
Non-adaptive	$x_{th} = 0.47$ $n_p = 2$	1.9×10^{-5}
Optimum	$x_{th} = 0.44$ $n_p = 10$ g(y) in Fig. 3.4	6.4×10^{-8}
Sup-optimum	$x_{th} = 0.446$ $y_{th} = 0.024$ $n_p = 10$	10.0×10^{-8}

Table 3.1 - Comparison of the Optimal and Sub-optimal Strategy Performance

3.1.4 Feedforward Error

In the previous analysis, it has been assumed that the receiver has perfect knowledge of the transmitter state. However, these decisions must be FDM multiplexed with the A-B information to tell the receiver whether to accept or reject data. If an error occurs in this information, bits are accepted or rejected incorrectly at the receiver. However, it is

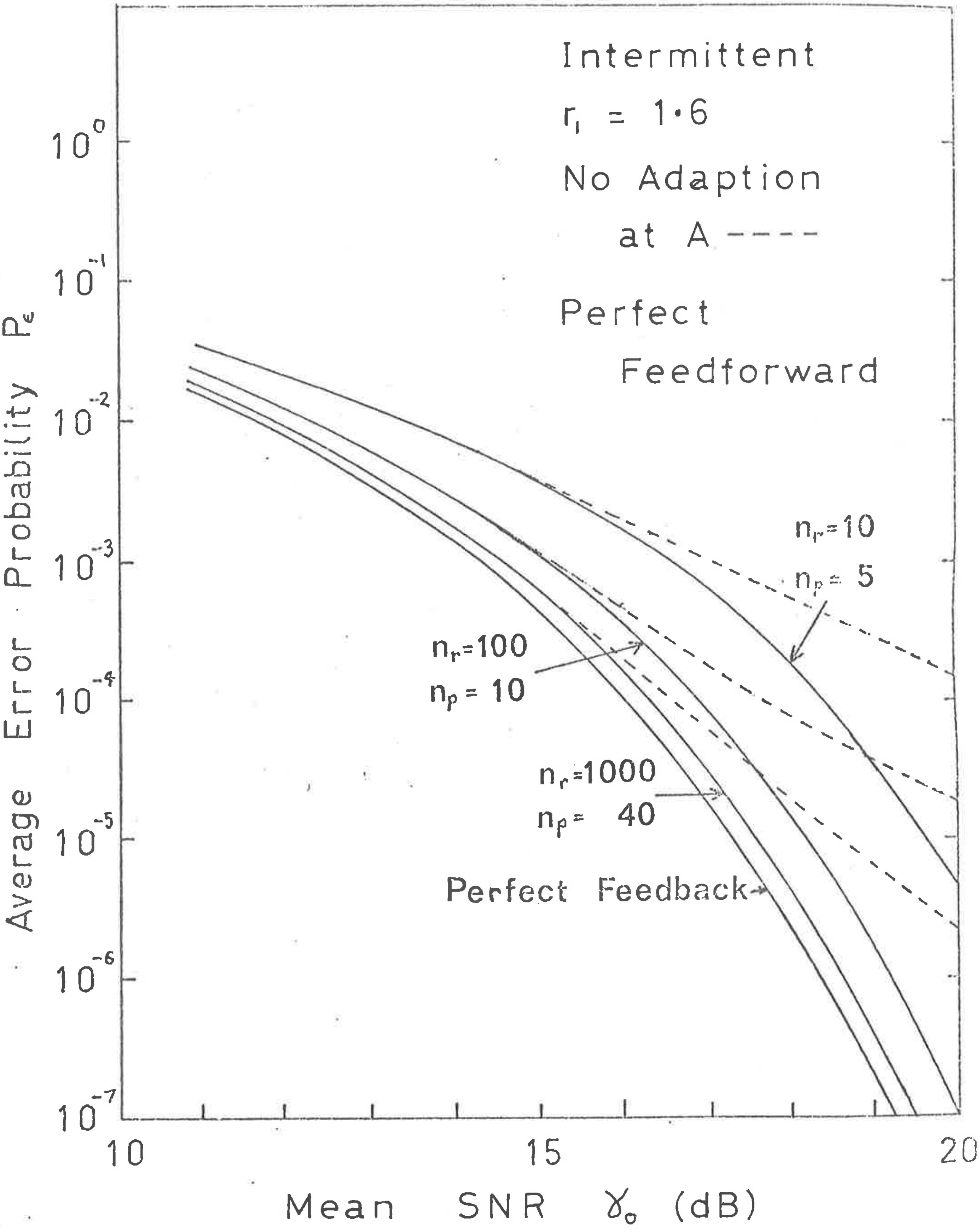

FIGURE 3.5
SUB-OPTIMAL STRATEGY PERFORMANCE

shown here that the use of a similar threshold strategy on the feedforward service information at B makes this problem of secondary importance.

It is assumed this feed-forward information is also sent at rate r_f, the same as the feedback information. However, when the main channel gain x drops below a lower threshold $\hat{x}_{th}$, the receiver ignores this service information and assumes the transmitter is not sending data (no feedback error).

The decisions in the system are:

i : Channel state decision at B

j : The feedback decision received at A

k : The feedforward decision received at B

k': Receiver decision at B

$$\text{where} \quad k' = \begin{cases} k & \text{if } x \leq \hat{x}_{th} \\ 0 \ (i) & x > \hat{x}_{th} \end{cases} \qquad 3.20$$

$$\text{and } i, j, k = \begin{cases} 0 & \text{if decision is not send data} \\ 1 & \text{if decision is to send data} \end{cases}$$

The error probabilities $P_{ij}(y)$ and $P_{jk}(x)$ are defined as before and for typical n_r values, $P_{jk}(x)$ and $P_\varepsilon(x)$ are independent.

Thus assuming x and y are independent as before and simplifying using equation 3.2, the error probability is:

$$P_\varepsilon(x, y) = \{P_{11}(y)\ P_{11}(x)\ r_1\ P_\varepsilon(x)\} \quad x \geq x_{th}$$

$$+ \{P_{01}(y)\ P_{11}(x)\ r_1\ P_\varepsilon(x)\} \quad \hat{x}_{th} \leq x < x_{th}$$

$$+ \{\tfrac{1}{2}\ P_{10}(x)\}\ x \geq \hat{x}_{th} + \{\tfrac{1}{2}\ P_{01}(y)\}\ x < \hat{x}_{th} \qquad 3.21$$

where the first two terms correspond to those in equation 3.3 and the last two terms account for a feedforward error. Using the sub-optimal strategy on the feedback information, integrating over x and y as before and considering only the major terms:

$$\bar{P}_\varepsilon \approx \frac{r_1^2}{\gamma_m + 2r_1} \left\{ z\ e^{-y_{th}} + \frac{(\hat{z}-z)}{2+\gamma_f}\ \omega \right\}$$

$$+ \frac{\hat{\omega}_m}{2(2+\gamma_f)} + \frac{\omega(1-e^{-\hat{x}_{th}})}{2(2+\gamma_f)} - \text{small double error term} \qquad 3.22$$

and $\bar{r}$ is negligibly different from equation 3.18

where $$\hat{z} = \exp\left[-\frac{(\gamma_m + 2r_1)}{2r_1}\hat{x}_{th}\right] \qquad 3.23$$

$$\hat{\omega}_m = \exp\left[-\frac{(\gamma_f + 2)}{2}\hat{x}_{th}\right] \qquad 3.24$$

If the feedforward information is FDM multiplexed with the main information, there is a further division of transmitter power (and bandwidth). If an equal amount of power is devoted to the feedback and feedforward information, then

$$\gamma_m = \frac{n_p}{n_p+2}\gamma_o \qquad 3.25$$

and γ_f remains unchanged.

Given the values of y_{th}, x_{th} and n_p which minimised equation 3.17, equation 3.22 was then minimised[3] with respect to $\hat{x}_{th}$. These results shown in Fig. 3.6 converge to the former curves as n_r increases. As stated previously, n_r has an upper bound since increasing n_r increases the control delay. However, there is another consideration to be considered. As n_r increases the probability of a service bit error decreases but the length of the burst of errors resulting from a service bit error (especially a feedforward error) gets larger. This does not affect the average bit error probability but, in practice, would probably mean worse performance. Thus $n_r = 1000$ would usually be an upper bound on n_r for most cases where there is only 0.3 dB difference in performance with the perfect feedback case. Also shown in Fig. 3.6 is the performance when no strategy is used at transmitter or receiver which is much worse than that due to feedback error alone. In Fig. 3.7 are the parameters x_{th}, y_{th} and $\hat{x}_{th}$ which correspond to the curves of Fig. 3.6.

Thus for the performance of an intermittent system to approach the ideal, such a threshold strategy is essential to minimise the effect of service information errors. The implications this has for other feedback systems are discussed in the next section.

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[3] see Note 3.

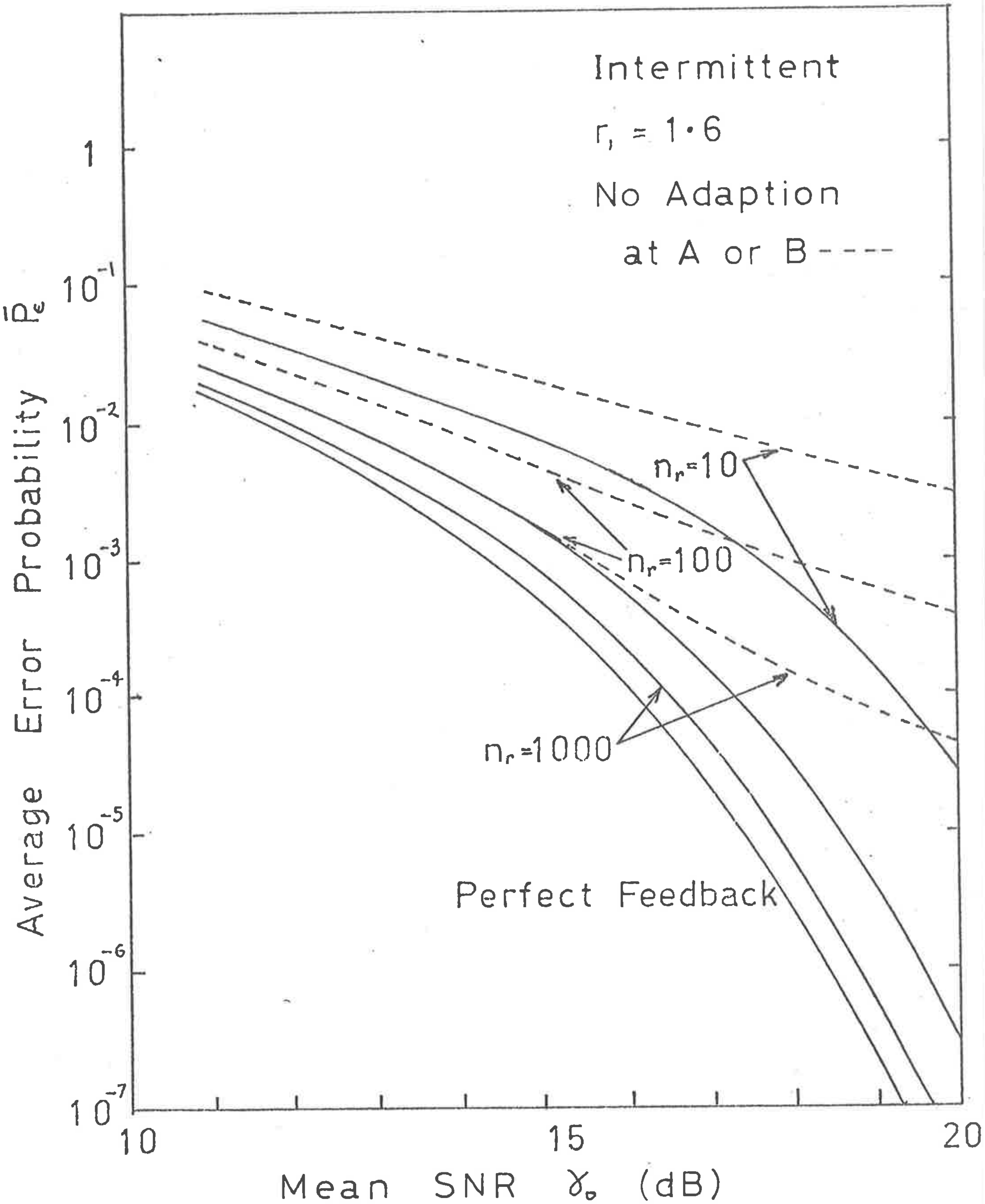

FIGURE 3.6

PERFORMANCE WHEN FEEDFORWARD ERROR INCLUDED

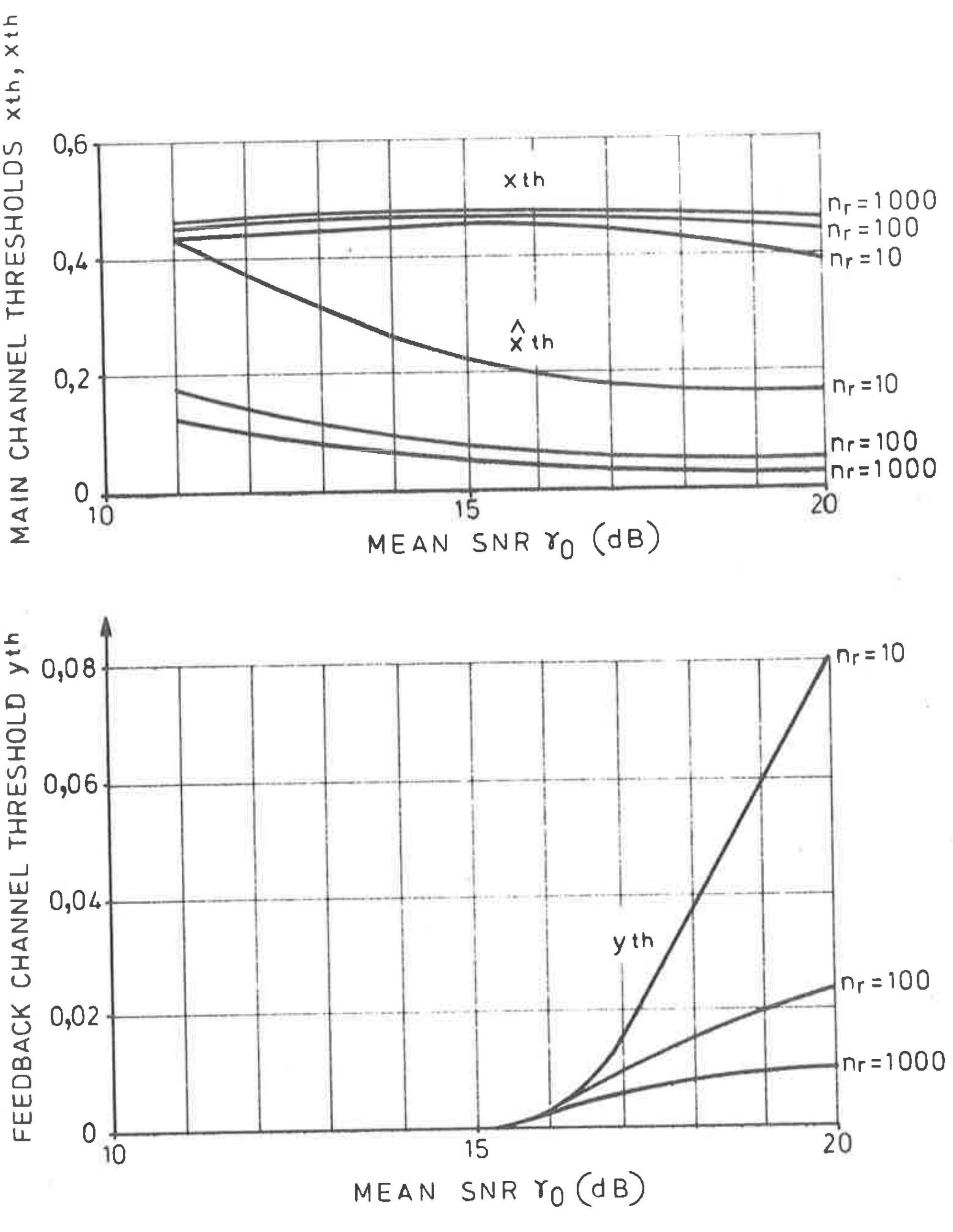

FIG. 3.7 PARAMETER VALUES AT OPTIMUM OPERATING POINTS.

3.1.5 Implications of Results

From the analysis considered for an intermittent system, some useful comments can be made with regards to the general problem of service information reliability for a channel state feedback system of the type discussed in Chapter 2.

One of the most important requirements is the high reliability required of the service information. For the intermittent system discussed, the average SNR of the service information was 14 dB higher than that of the main information. Secondly, the feedback information was regarded as useless for 1 % of the time due to deep fades on the feedback channel. Thus it is a requirement of such systems to have a high service information mean SNR and some strategy to carry the system through fades in the service information SNR.

Cavers (1972) treats a fully variable rate system where he proposes to TDM multiplex the service information with the main return information. Firstly, it would seem difficult to achieve the required reliability by TDM especially in the light of the extra amount of service information envisaged. This comment applies to all fully adaptive systems. Secondly, Cavers neglects the effects of deep fades on the service information when there would be no service information available (rate is near zero) or it would be unreliable.

It would seem that due to these two requirements, the service information should have low information content (i.e. small number of levels), should be FDM multiplexed and that some "play safe" strategy, such as that treated in section 3.1.3, should be used to cope with fades in the service information.

3.2 ALTERNATIVE CONTROL

If the fading phenomenon is reciprocal, the meaning of which is explained in the next paragraph, it is possible to do away with the need for service information via feedback. Thus there would be no source of service information error other than to minimise control delay which will be discussed in the next section.

Reciprocal fading is taken to mean that the instantaneous scattering profile causing the multipath propagation (and thus fading as the scatterers move) is the same for a signal going from A to B via the scatterers as from B to A at the same carrier frequency. One case where this reciprocity is exhibited is in the mobile radio environment where the scatterers are buildings and are thus reciprocal scatterers. This property has been exploited by Bitler et al (1973) in their diversity system. Another possible contender is the Troposcatter medium. However, no experiments in this regard have come to the author's notice to verify this conjecture.

In Fig. 3.8 is a block diagram of a channel state feedback system that does not require a feedback channel. The system transmitter and receiver alternate between sending and receiving at a rate much greater than the fading rate. In this way the transmitter at A uses its channel state estimate at time t (while not transmitting) in order to adapt transmitter parameters for transmission at time $t + t_b$ or t_b seconds later. Similar switching but 180° out of phase is occurring at station B. In other words the transmitter and receiver are isolated by TDM rather than the usual FDM method.

However, such a system configuration is complex in terms of the necessary switching control required which must take account of any significant propagation delay τ and must have a sufficient guard time to allow for variations in propagation time. Another implication of such a system is a doubling of the necessary coherence bandwidth for the same data rate, although the amount of bandwidth used is the same. Other ways of avoiding such complexities require extra power or bandwidth.

Thus in the light of the results of section 3.1, it is felt that for the simpler feedback systems, such as an intermittent system, an actual feedback link is the most practical solution to the problem.

3.3 CONTROL DELAY

Control delay is a significant limitation on the performance of any channel state feedback system. Cavers (1972) treats the delay problem as it relates to a variable rate system. This section examines some implications of Cavers' analysis as regards to several channel state feedback systems and then treats this problem in more detail for an intermittent system.

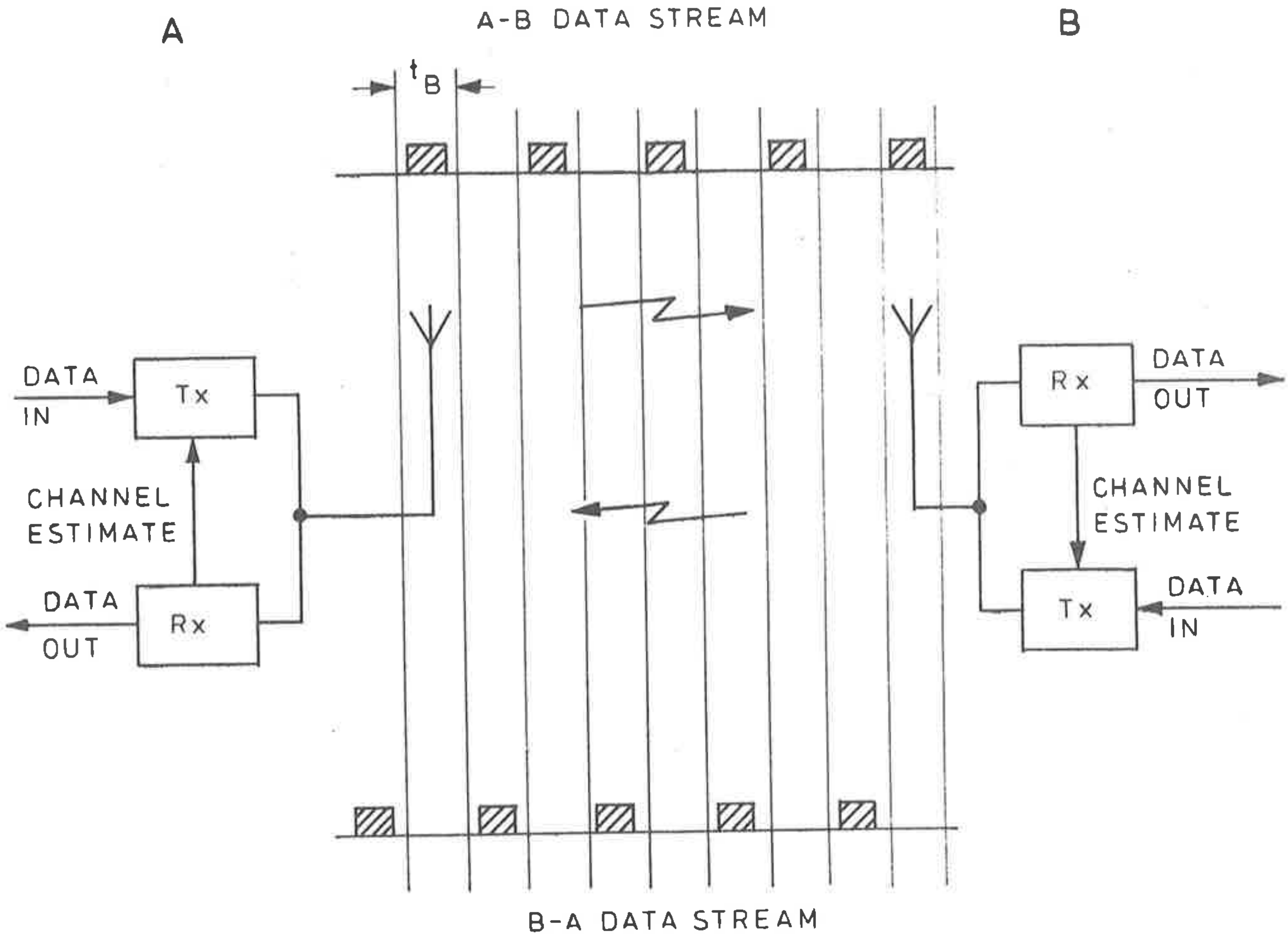

FIG. 3.8 ALTERNATIVE SWITCH CONTROL

3.3.1 Variable Rate System

Cavers (1972) derives the conditional probability density function $p(a_\tau/a_o)$ as:

$$p(a_o/a_\tau) = \frac{a_o}{\sigma^2(1-k^2)} . \exp\left[- \frac{a_o^2 + k^2 a_\tau^2}{2\sigma^2(1-k^2)} \right] I_o\left[\frac{k a_o a_\tau}{\sigma^2(1-k^2)} \right] \qquad 3.26$$

where

a_o : Envelope value of receiver at detection

a_τ : Envelope value τ sec previous to a_o according to which the control decision is made

k : Normalised autocorrelation coefficient of time lag τ of the direct (or quadrature) component of the fading process so that

$$k = \frac{R_\mu(\tau)}{\sigma^2} \qquad 3.27$$

σ^2 : Variance of fading process

$I_o[]$: Zero order modified Bessel Function of the first kind.

As $\tau \to \infty$ then $k \to 0$ and equation 3.26 reduces to the Rayleigh probability density function as expected. From this density function (which only depends on τ via equation 3.27), Cavers derives the expected error probability $\hat{P}_\varepsilon(x,r)$ where x is defined in Chapter 2 such that:

$$\hat{P}_\varepsilon(x,r) = \int_0^\infty P_\varepsilon(x,r) p(a_o/a_\tau) da_o \qquad 3.28$$

which after normalisation comes to:

$$\hat{P}_\varepsilon(x,r) = \frac{r}{2r + \gamma_o(1-k^2)} \exp\left[- \frac{k^2 \gamma_o x}{2r + \gamma_o(1-k^2)} \right] \qquad 3.29$$

where r, the rate function (see r, in Chapter 2) is some function of channel gain x.

The average probability $\overline{P}_\varepsilon$ is then given as:

$$\overline{P}_\varepsilon = \int_0^\infty \hat{P}_\varepsilon(x,r) r(x) p(x) dx \qquad 3.30$$

subject to the rate constraint

$$\bar{r} = \int_0^\infty r(x)p(x)dx = 1 \qquad 3.31$$

Adjoining 3.30 and 3.31 with a Lagrange multiplier the optimum rate function is then a solution of the implicit equation

$$\frac{\dfrac{2k^2r^2\gamma_o x}{2r + \gamma_o(1-k^2)} + \gamma_o(1-k^2)r + r(2r + \gamma_o(1-k^2))}{[2r + \gamma_o(1-k^2)]^2} . \exp\left[- \frac{k^2\gamma_o x}{2r + \gamma_o(1-k^2)} \right]$$

$$= \lambda \qquad 3.32$$

where λ is chosen to satisfy 3.31.

Solution of equation 3.32 for the case $\nu\tau = 0.01$ treated by Cavers is plotted in Fig. 3.9 for several values of mean SNR. This rate function shows a distinct threshold phenomenon similar to that obtained in section 3.1.3. In Fig. 3.10 is plotted the performance curves of a variable rate system treated by Cavers for the optimum and non-optimum rate functions. In fact the optimum can be well approximated by:

$$r(x) = \begin{cases} Cx & \text{if } x \geq x_{th} \\ 0 & \text{if } x < x_{th} \end{cases} \qquad 3.33$$

where x_{th} is a threshold value of SNR and C is a constant to satisfy equation 3.31.

Thus,

$$C = e^{x_{th}} / (1+x_{th}) \qquad 3.34$$

These results indicate how sensitive system performance is to control delay, especially if the control function is not optimised. A similar sensitivity to control delay is observed for a selective diversity system treated by Skinners and Cavers (1973). In fact, a clear observation from these results is the futility of trying to use the channel during deep fades and this is why an intermittent system performs so well in comparison with the more complex variable rate system.

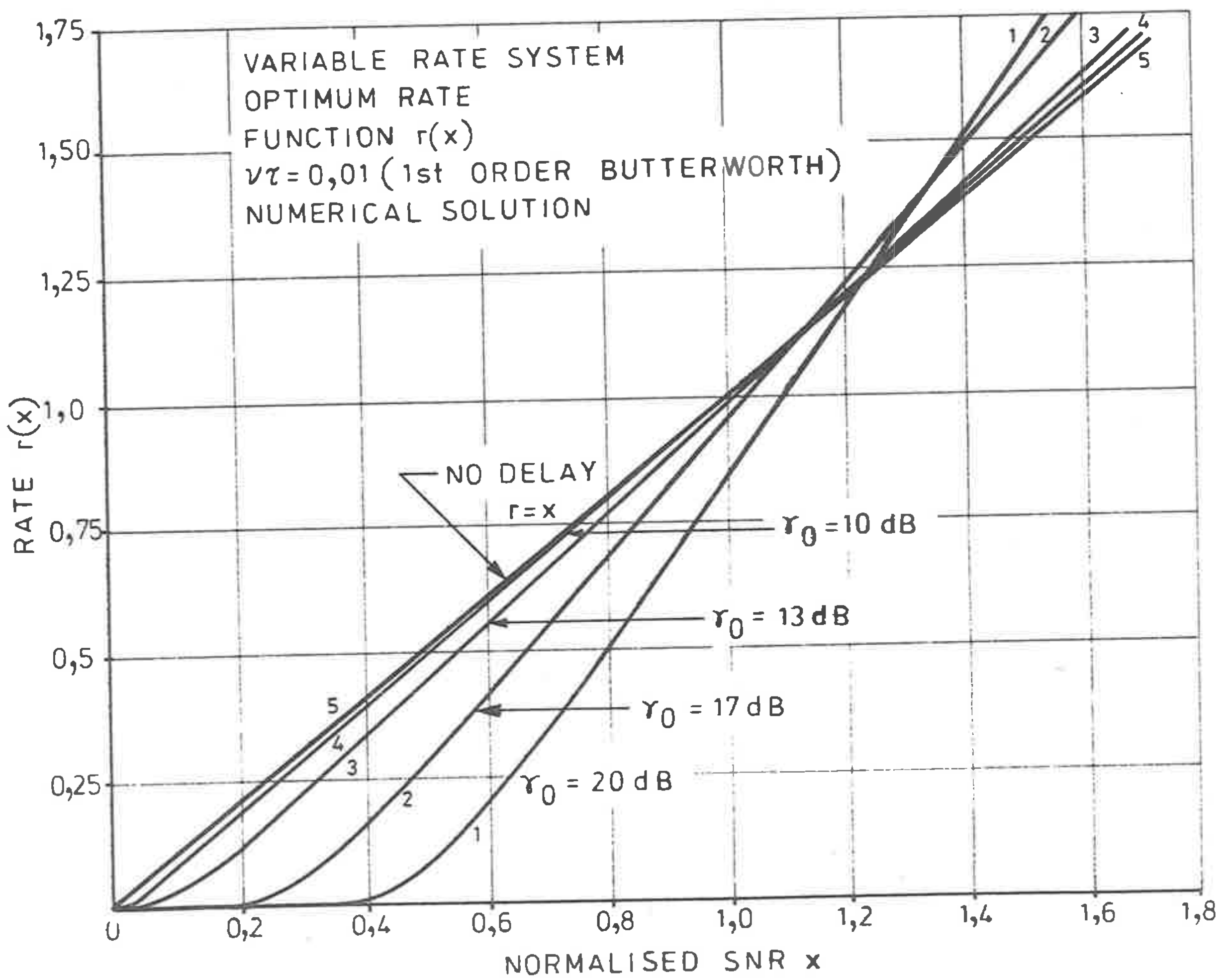

FIG. 3.9 OPTIMUM RATE FUNCTION FOR VARIABLE RATE SYSTEM WITH CONTROL DELAY.

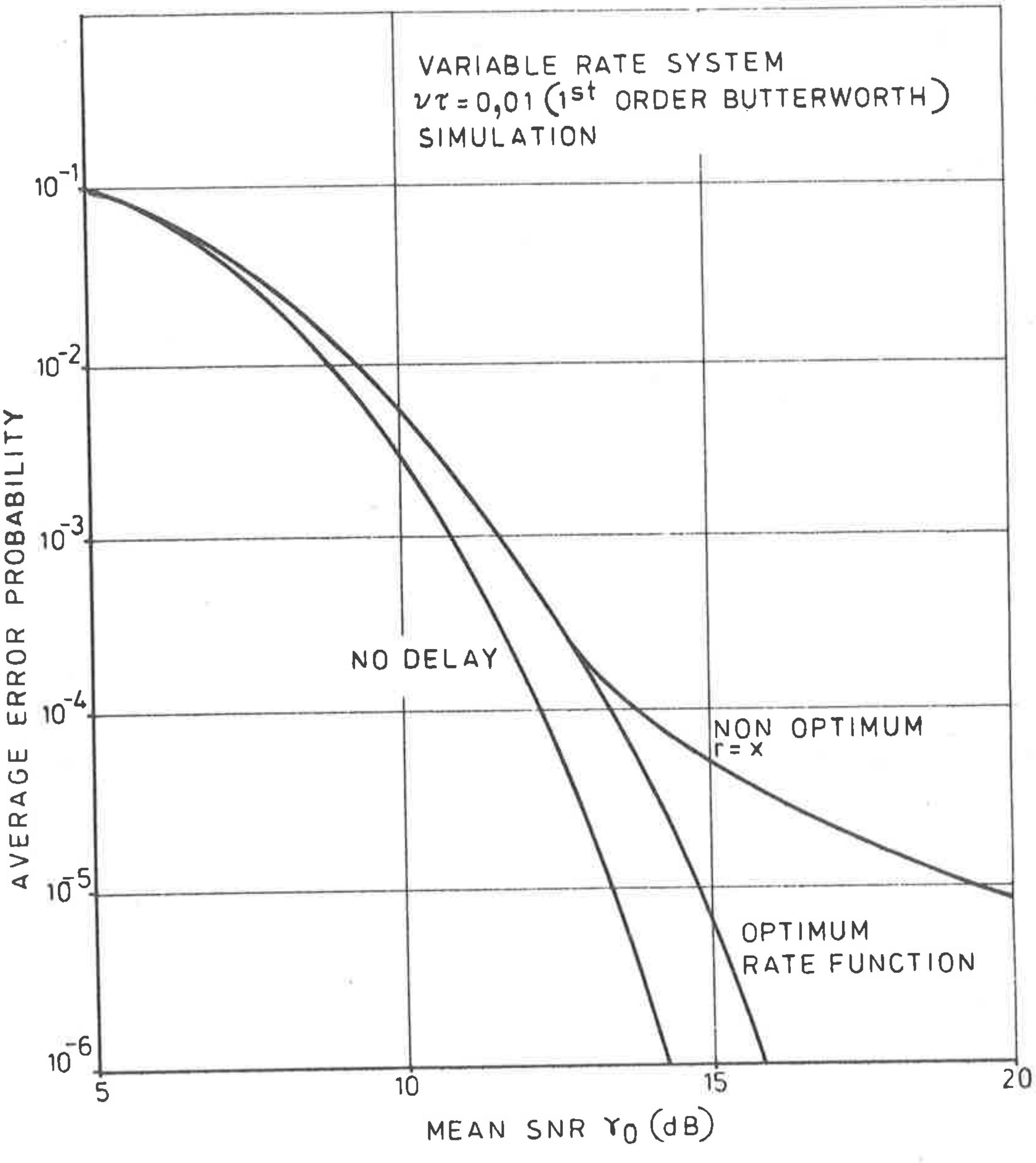

FIG. 3.10 EFFECT OF NON-OPTIMUM RATE FUNCTION FOR FINITE CONTROL DELAY.

3.3.2 Intermittent System

The analysis of Cavers (1972) is easily applied to the two rate case to give an average error probability given by:

$$\overline{P}_{\varepsilon} = \frac{r_1^2}{2r_1+\gamma_o} \cdot \exp\left[- \frac{(2r_1 + \gamma_o)x_{th}}{2r_1 + \gamma_o(1-k^2)} \right] \qquad 3.35$$

where $x_{th} = \ln r_1$ to maintain unity data rate. The effect of delay is shown in Fig. 3.11 for different values of delay bandwidth product. Thus, as for all such feedback systems, the $\nu\tau$ product must be kept below a critical level. The specific sources of contribution to the control delay will be discussed in section 3.3.3.

Cavers assumes for his work a specific correlation function, namely an exponential function such that:

$$R_{\mu}(\tau) = \sigma^2 \exp(-\nu|\tau|) \qquad 3.36$$

One property that this correlation function implies is that the first derivative is unbounded! Such a property (cf Markov Process) is impossible in practice since this implies scatterers having unbounded velocity. However, a fading process must be strictly bandlimited. Papoulis (1965) shows that for such processes, perfect prediction is possible by the use of a sufficient number of derivatives. This of course assumes no additive noise. The choice of the "best" channel state predictor is a problem which will not be discussed in detail here, and Caver's analysis will be used for simplicity.

For the JANET meteor burst system[4] it was observed that a double threshold, with a higher value to "stop" transmission, helped to alleviate the control delay problem. This observation has some interesting ramifications for channel state feedback systems. The reason for a double threshold is that it is more important for the system to cope with the decline into a fade than to catch the rise out of it. This is similar to the cost difference analysed in section 3.1.3. Thus, a double threshold would undoubtedly decrease the effect of control delay in an intermittent system. An extension of this technique (basically hysteresis) to a more general channel state feedback

4. See Forsyth et al (1957)

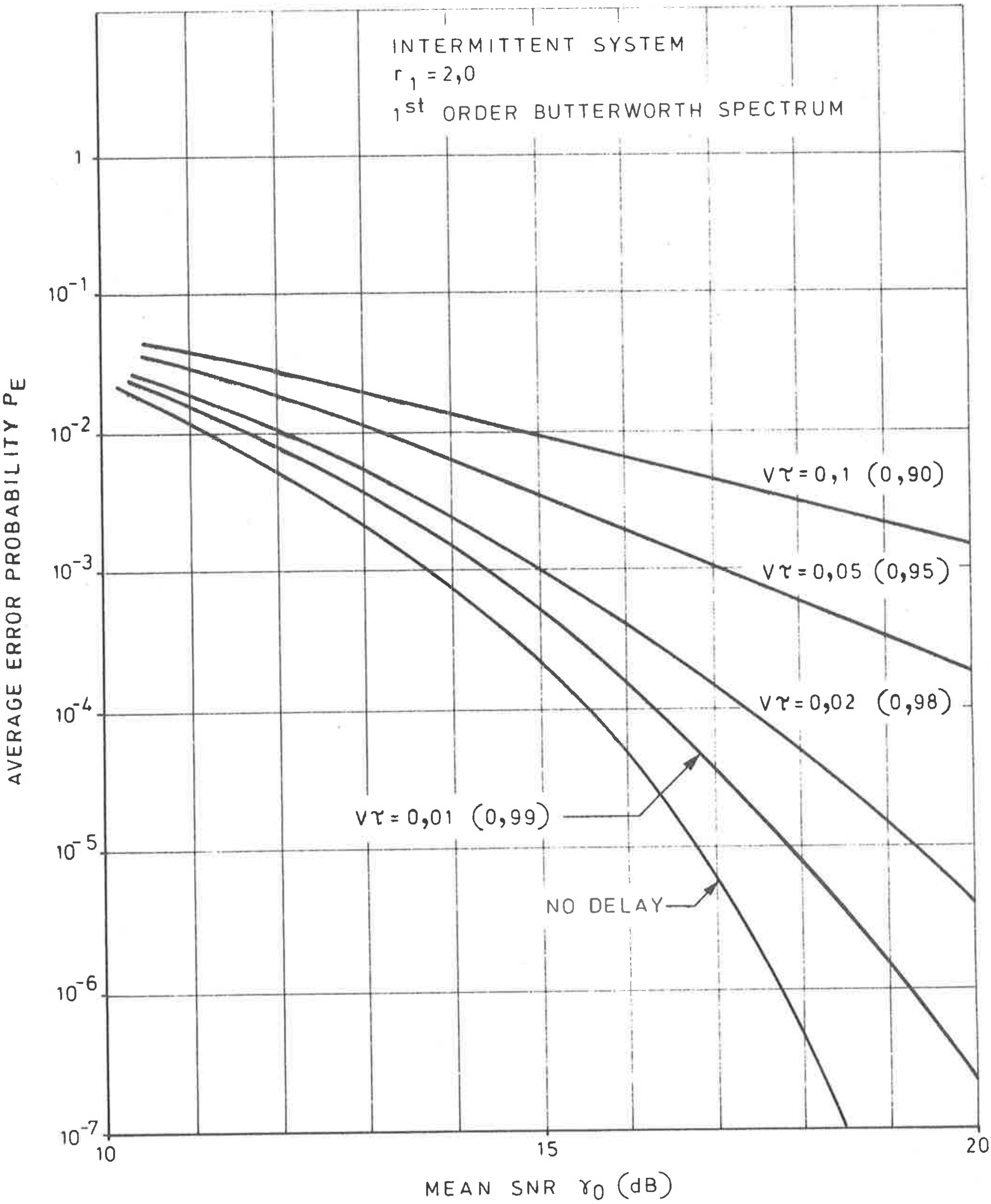

FIG. 3.11 EFFECT OF CONTROL DELAY ON INTERMITTENT SYSTEM

system such as a variable rate system of section 3.3.1 would imply the optimum control law would be a hysteresis curve. The analysis, however, has not been attempted, but these comments shed some light on the problem of defining the "best" predictor mentioned in the previous paragraph.

In section 4.2.2 the question of hysteresis is again raised as it relates to the buffer control problem. However, the above discussion raises some interesting estimation theory problems to be solved.

3.3.3 Interaction with Feedback Error

It was mentioned previously in section 3.1 that the rate ratio n_r is limited due to its relationship to control delay . This section identifies the sources of this control delay and discusses its relative importance on performance.

Basically, one can identify three such sources of delay. These are propagation delay, filter delay (ie the delay in measuring the SNR of the channel at B) and lastly, decision delay. This decision delay τ_d is given by: (see section 3.1.1)

$$\tau_d = \frac{n_r}{r_1 . R_i} \qquad 3.37$$

Thus, as n_r increases, so improving the reliability of the service information, the control delay increases correspondingly. The total control delay is given by:

$$\tau = 2\tau_p + \tau_f + 2\tau_d \qquad 3.38$$

τ_p: Propagation delay

τ_f: Filter delay

τ_d: Decision delay.

Which source of delay is dominant depends on the application. For example, in a mobile environment, the effect of propagation delay would be negligible, but in a Troposcatter system the reverse would be the case. (see section 5.4)

Using the analagous case of Cavers (1972) equation 3.29 for an intermittent system becomes:

$$\hat{P}_\varepsilon(x) = \frac{r_1}{2r_1 + \gamma_m(1-k^2)} .\exp\left[\frac{k^2\gamma_m x}{2r_1 + \gamma_m(1-k^2)}\right] \tag{3.39}$$

Thus, this expression can be substituted for $P_\varepsilon(x)$ in 3.21 and integrated over x and y to give:

$$\overline{P}_\varepsilon \simeq \frac{r_1^2}{\gamma_m + 2r_1}\left\{ z'e^{-y_{th}} + \frac{\hat{z} - z'}{2 + {}_f}\,\omega \right\} + \frac{\hat{\omega}_m}{2(2 + \gamma_f)} + \frac{\omega(1 - e^{-x_{th}})}{2(2 + \gamma_f)} \tag{3.30}$$

where

$$z' = \exp\left[-\frac{(2r_1 + \gamma_m)x_{th}}{2r_1 + \gamma_m(1-k^2)}\right] \tag{3.31}$$

Thus, the optimisation of section 3.1.4 can be extended now to include the minimisation with respect to n_r. However, to do this, one must be more specific about the other parameters of the system such as the data rate and operating distance.

Therefore, specific results illustrating the interaction between control delay and service information will be discussed in Chapter 5 along with the interaction with the finite buffer constraint.

3.4 CONCLUSIONS

The effect of control reliability on channel state feedback systems has been discussed with specific interest centred on an intermittent system. Thus specific conclusions reached with respect to the intermittent system illustrate the general principles of all such systems.

If the feedback information is sent over a similar fading channel (as is usually the case), then it must firstly have a high mean reliability and secondly, the system must have a "play safe" strategy to cope with the situation of a deep fade on the feedback channel. In the light of these comments, it would seem difficult to achieve these aims using TDM methods and minimise control delay at the same time. If however, the fading is reciprocal, then an alternative switched control method could be used.

Control delay places two important constraints on a channel state feedback system. As mentioned in the introduction, the allowable control delay (depends on fading rate) ultimately determines the possible reliability of the service information. Secondly, due to propagation time, control delay places an upper limit upon the distance over which such systems can be used. For example, it is impossible to use such schemes over most HF circuits due to this limitation.

In Chapter 5, the implications of the feedback strategy of section 3.1 on the required buffer will be discussed.

4.0 INTRODUCTION

Variable rate systems, unlike the other channel state feedback systems discussed in Chapter 2, require a buffer at the transmitter and the receiver to interface with a uniform data source and sink.These buffers are finite in size and so possible overflow conditions degrade performance. This chapter discusses this problem with specific reference to an intermittent system described by Coutts (1975c). Also described is the buffer synchronisation requirement so that the system can still operate during overflow conditions and regain this synchronisation if a feedforward error occurs (see section 3.1.4). Finally, the experiences gained from a hardware model of an intermittent system are discussed to illustrate the practical realisability of such a buffer system for intermittent transmission.

4.1 BUFFER ANALYSIS

In Fig. 4.1 is illustrated the model of an intermittent system where it is assumed here that one has perfect feedback and feed-forward decisions. (This assumption is removed in Chapter 5) As mentioned previously, since these buffers are finite, there exists the possibility of buffer overflows. In particular, when the A station buffer fills, the transmitter sends data (at the input rate on the average) even though the SNR is below threshold causing a large increase in the error probability. Similarly, if the buffer empties the date rate reduces to the input rate (on average) resulting in a loss of possible throughput.

For this anaysis, it is assumed that the buffer occupancy can be considered continuous which is commensurate with the assumption of slow fading. Secondly, it is assumed that the times between fades and the durations of fades are exponentially distributed. The actual distributions for different fading power spectra are discussed by Rice (1958). However, it is contended that the exact form of the distribution is unimportant for this application since one is interested in the longer fades (distribution tail) for which an exponential distribution is appropriate. Thus the only dependence of these distributions on the fading power spectrum is through the mean fade length.

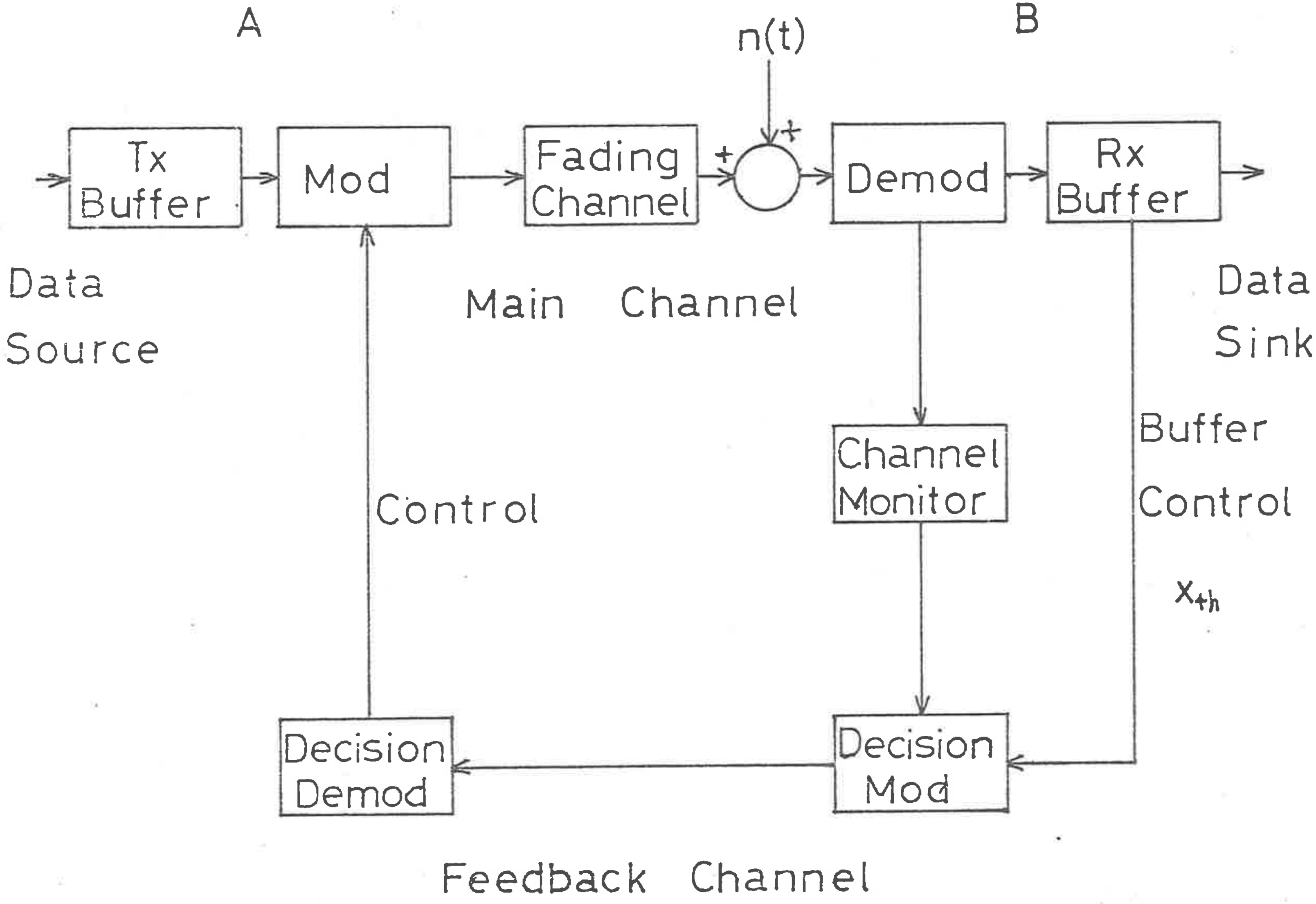

FIGURE 4.1 - MODEL OF INTERMITTENT SYSTEM

For a Rayleigh distributed variable, the average frequency of downward zero crossings across a threshold x+h is given by Rice (1958) as:

$$f_r = f_{rms} 2 \sqrt{\pi x_{th}} \; e^{-x_{th}} \qquad 4.1$$

where f_{rms} is the rms frequency defined as:

$$f_{rms} = \left[\frac{\int_{-\infty}^{\infty} f^2 G(f) df}{\int_{-\infty}^{\infty} G(f) df} \right]^{\frac{1}{2}} \qquad 4.2$$

and G(f) is the equivalent low pass power density spectrum of the fading process. The value of f_{rms} for several types of power spectra G(f) are given in Table 4.1.

One can now define λ and μ, the mean arrival and departure rates in terms of the mean fade time and mean transmission time to give:

$$\lambda = \frac{1 - e^{-x_{th}}}{f_r} \qquad 4.3$$

$$\mu = \frac{e^{-x_{th}}(r_1 - 1)}{f_r} \qquad 4.4$$

λ - average number of bits added to buffer during a fade.

μ - average number of bits removed during a transmission period.

Thus, the traffic intensity ρ which is independent of the fading time constant is given by:

$$\rho = \frac{\lambda}{\mu} = \frac{e^{x_{th}} - 1}{r_1 - 1} \qquad 4.5$$

and goes to unity for $x_{th} = \ln r_1$ as for the infinite buffer case of Chapter 2. However, for a finite buffer capacity $\rho < 1$ is the usual operating condition.

Extending the analysis of Campbell (1957) it is shown (Appendix C) that the probabilities of empty and full buffer are given by:

TABLE 4.1 - RMS FREQUENCY

Power Spectrum G(f)	R.M.S. Frequency f_{rms}
1. Ideal Low Pass $G(f) = \begin{cases} G_o; & f<f_c \\ 0; & f>f_c \end{cases}$	$f_c/\sqrt{3}$
2. Linear Fall Off $G(f) = \begin{cases} G_o(1-f/f_c); & f<f_c \\ 0; & f>f_c \end{cases}$	$f_c/\sqrt{6}$
3. Gaussian $G(f) = \frac{2}{\sqrt{\pi}} e^{-f^2}$	$1/\sqrt{2}$
4. 2nd Order Butterworth $G(f) = \frac{V^4}{V^4 + \omega^4}$	$V/2\pi$
5. 1st Order Butterwoth $G(f) = \frac{V^2}{V^2 + \omega^2}$	Undefined[1.]
6. Mobile $G(f) = \begin{cases} G_o(1-(f/f_c)^2)^{-\frac{1}{2}}; & f<f_c \\ 0; & f>f_c \end{cases}$	$f_c/\sqrt{2}$

1. For the overflow probabilities predicted from equations 4.6 and 4.7 to agree with those from simulation, $f_{rms} = 0.6V$.

$$P_o = \frac{1 - \rho}{1 - \rho e^{-\beta C}} \qquad 4.6$$

$$P_c = P_o e^{-\beta C} \qquad 4.7$$

$$\text{where } \beta = \frac{1 - \rho}{\lambda} \qquad 4.8$$

$P_o = P_r$ { empty buffer condition / $x \geq x_{th}$ }

$P_c = P_r$ { full buffer condition / $x < x_{th}$ }

and C is the normalised buffer capacity.[2.,3.]
During these overflow conditions, the transmitter sends bits intermittently or an average, at the input rate. Thus it is useful to define the probabilities of transmission during these conditions as:

$$P_o' = P_r \{ \text{not sending} / x > x_{th} \} = \frac{r_1 - 1}{r_1} . P_o \qquad 4.9$$

$$P_c' = P_r \{ \text{sending} / x < x_{th} \} = \frac{1}{r_1} . P_c \qquad 4.10$$

It is assumed that the times between fades and transmission periods are independent. Further, assuming that the channel SNR is varying much faster than the buffer occupancy, the average error probability can be shown to be: (Similar to analysis of Coutts (1975a))

$$\overline{P}_\varepsilon = \frac{r_1^2}{\gamma_o + 2r_1} \{ (1 - P_o')z + P_c' (1 - z)\} \qquad 4.11$$

$$\text{where } z = \exp\left[- \frac{(\gamma_o + 2r_1)}{2r_1} x_{th} \right] \qquad 4.12$$

---

2. Note that these results are analgous to those of a m/m/1 queue with a finite waiting room.
3. C is Buffer size in bits for a unity data rate. Also note that $\lim_{\rho \to 1} P_o = \lim_{\rho \to 1} P_c = \frac{\lambda}{\lambda + C}$

and thus,

$$\bar{r} = r_1 \{ (1 - P_o') e^{-x_{th}} + P_c' (1 - e^{-x_{th}}) \} = 1 \qquad 4.13$$

These results are plotted in Fig. 4.2 against threshold for a given mean SNR with different normalised buffer capacities C for a given value of RMS fading frequency. In Fig. 4.3 are the overflow probabilities that correspond to the same variation of threshold. As expected, there is an optimum threshold x_{th} to compromise between reduced data rate and increased error probability during full buffer conditions. As C increases, this optimum threshold moves closer to the condition $\rho = 1$ corresponding to the value of threshold for an infinite full buffer. As x_{th} is decreased below the optimum for a given value of C, the error probability approaches the infinite capacity asymptote for $\rho < 1$ given by:

$$\bar{P}_\varepsilon = \frac{r_1}{\gamma_o + 2r_1} \exp\left[- \frac{\gamma_o x_{th}}{2r_1} \right] \qquad 4.14$$

Thirdly, the normalised buffer capacity C required for the performance to approach the theoretical lower bound ($C = \infty$) grows exponentially. The actual buffer size B which is required depends on the data rate and fading rate giving:

$$B = \frac{R_i}{2\pi . f_{rms}} C \qquad 4.15$$

B - Actual Buffer size in bits.

R_i - Data Rate of system in bits/s.

In order to validate these theoretical results, the system was simulated on a digital computer using a partial simulation technique[4.] for the case of a 2nd order Butterworth fading spectrum with a 3dB cut-off of 1 rad/sec. The results for $C = 10,20$ are shown in Fig. 4.2 and 4.3 which closely agree with those predicted from the approximate theoretical model.

As the SNR γ_o varies, the optimum threshold changes. In Fig. 4.4 the error probability is plotted against mean SNR for several normalised buffer capacities with the optimum threshold. This optimum threshold was

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4. See Davis et Al (1975a) and Beare & Coutts (1976).

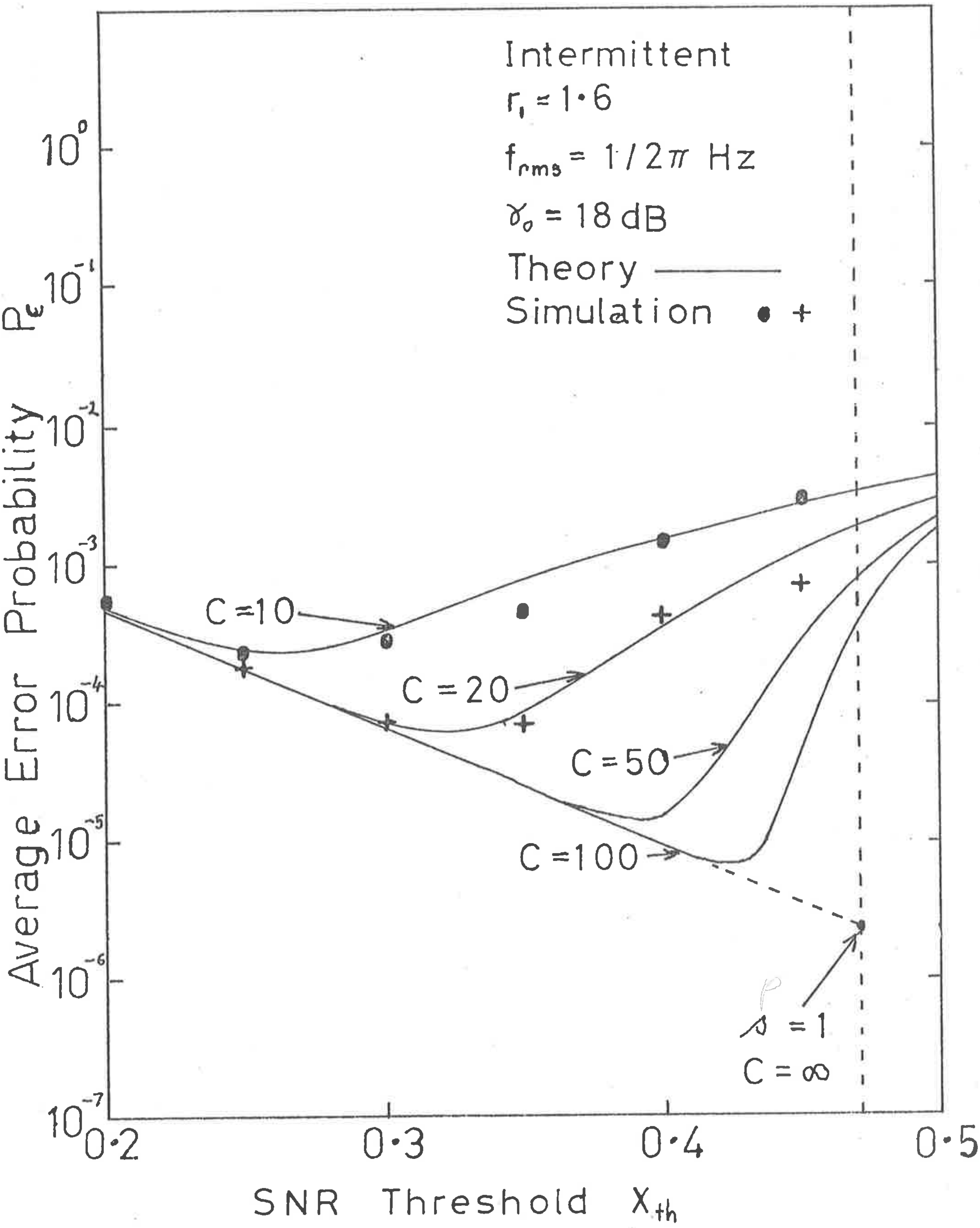

FIGURE 4.2

EFFECT OF THRESHOLD ON PERFORMANCE FOR DIFFERENT NORMALISED BUFFER CAPACITIES C

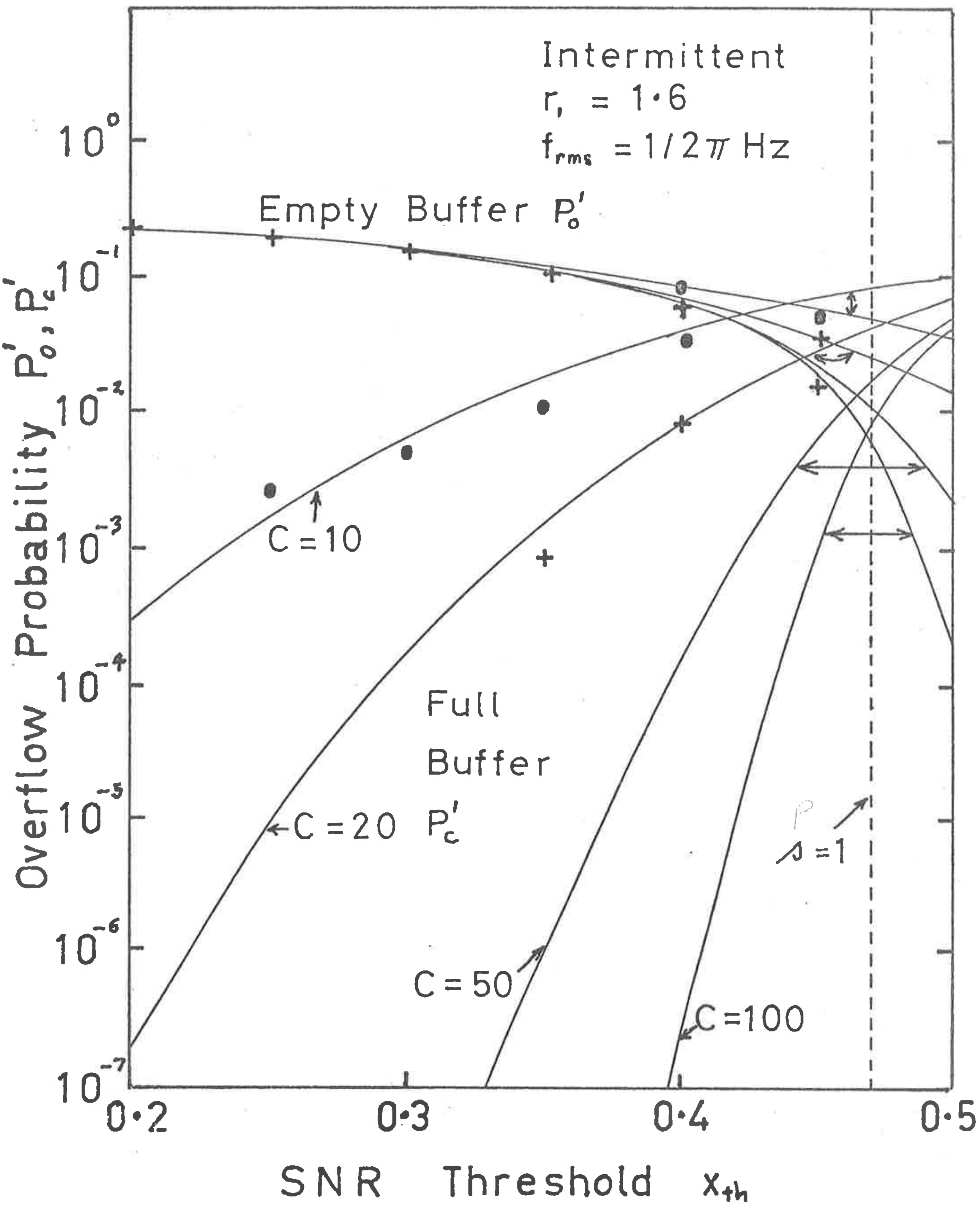

FIGURE 4.3

VARIATION OF OVERFLOW PROBABILITIES WITH THRESHOLD found from equation 4.11 and is shown in Fig. 4.5. From these curves, it can be seen that the ideal performance can be approached with increasing buffer size and that the error probability falls exponentially for a finite but large buffer. Shown in Fig. 4.6 is a similar set of results for a higher transmission ratio r_1 indicating no great advantage in using a higher transmission ratio.

These results are optimum for a time independent threshold. However, this threshold may be made variable so as to make more effective use of a given buffer capacity. Two methods of such control are examined in section 4.2.

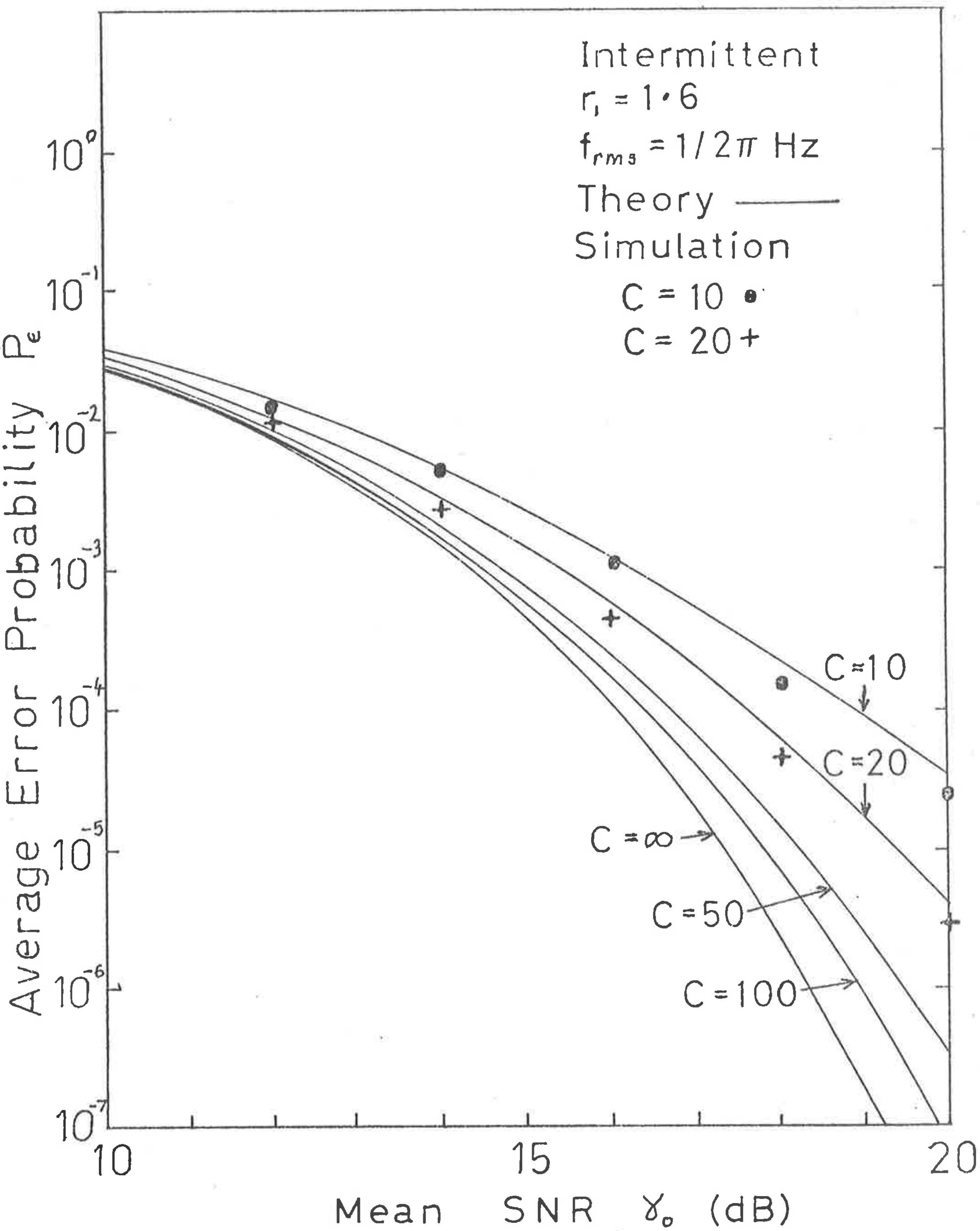

FIGURE 4.4
EFFECT OF A FINITE BUFFER USING THE OPTIMUM THRESHOLD

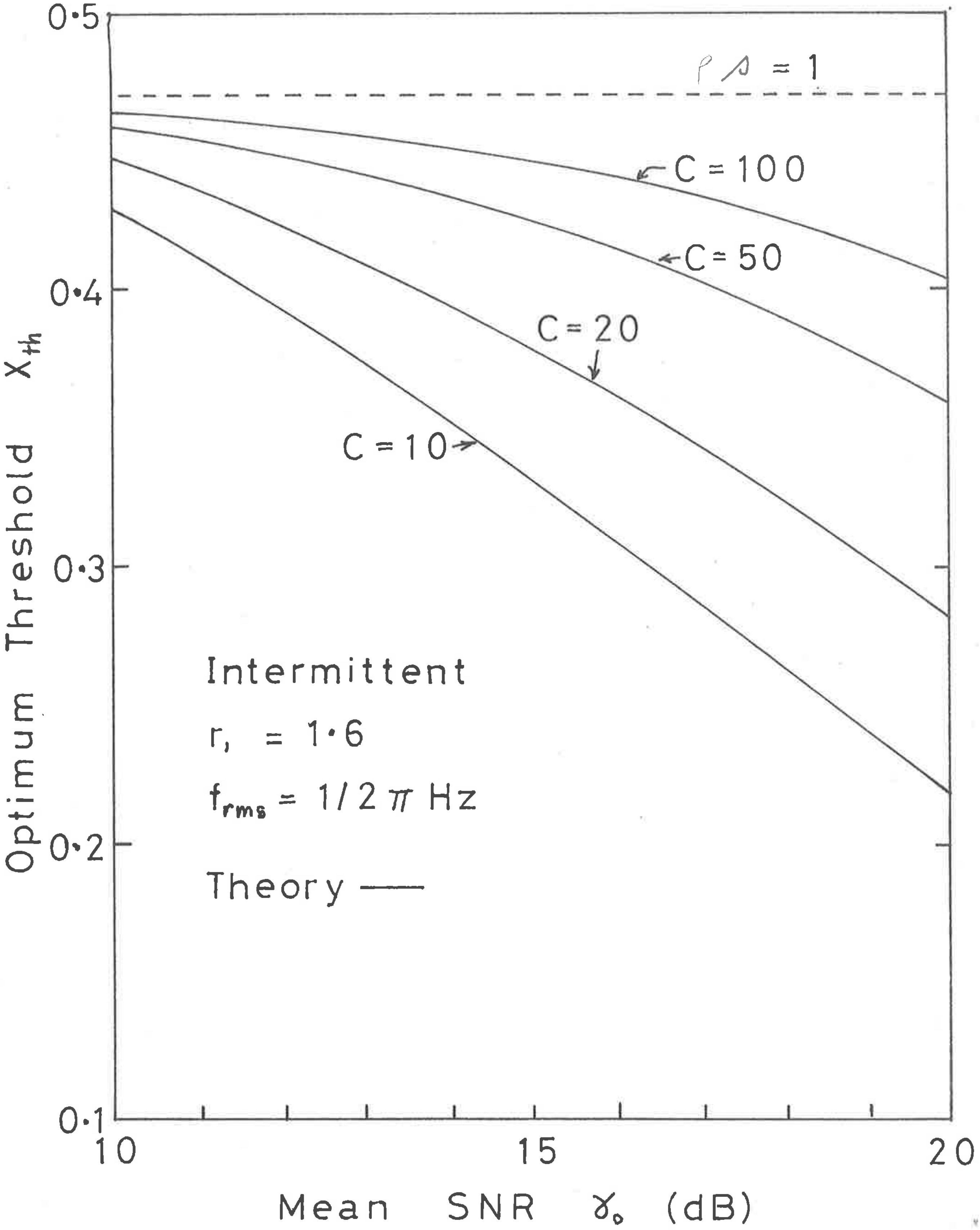

FIGURE 4.5

OPTIMUM THRESHOLD AS A FUNCTION OF MEAN SNR

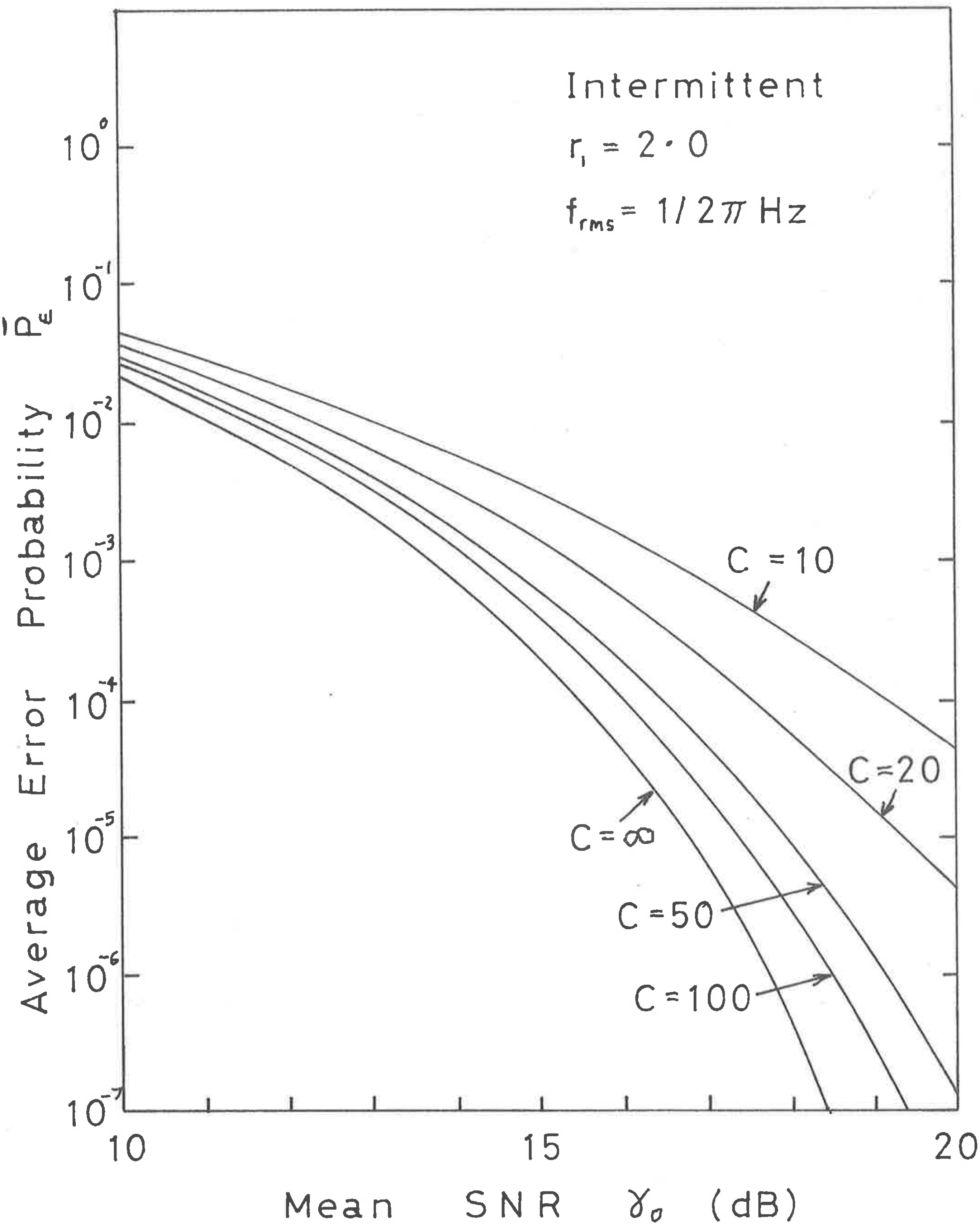

FIGURE 4.6

EFFECT OF A FINITE BUFFER USING THE OPTIMUM THRESHOLD

4.2 ADAPTIVE THRESHOLD

The results of section 4.1 indicate that the effect of a full buffer condition is a sharp increase in the error probability. For this reason, the optimum threshold is much lower than that for the infinite buffer case and decreases with decreasing buffer size. Thus, it would seem desirable to make the threshold variable to make more efficient use of a given buffer. Two forms of control will be discussed; namely filter and buffer occupancy control. The results of a simulation study will be presented and then some thoughts on an analytical treatment on the latter control method will be discussed.

4.2.1 Simulation Study

Filter control consists of an RC filter following a square low device to estimate a short term measure of the mean SNR of the fading process. This form of filter would be necessary in practice anyway for a time varying situation. However, simulation results indicate that for a stationary fading process no significant performance improvement can be gained.

As stated in Section 4.1, there is already an intrinsic buffer control in operation. However, to improve performance, the threshold can be made a direct function $G(\omega)$ of the buffer occupancy ω. This function would result in a lowering threshold as the transmitter buffer filled, (Control actually at receiver B) so as to decrease the probability of a full buffer condition. In this way the buffer control smoothes the effects of a full buffer.

The choice of this control function has been investigated by a random search technique [5.] using a short simulation run. The results of several such runs are then averaged to give the control function shown in Fig. 4.7 for a particular value of SNR γ_o and normalised buffer capacity C. From this result, the function (dotted curve) which is of form:

$$G(\omega) = k\sqrt{C - \omega} \qquad 4.16$$

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5. Reported by the author Coutts (1975c).

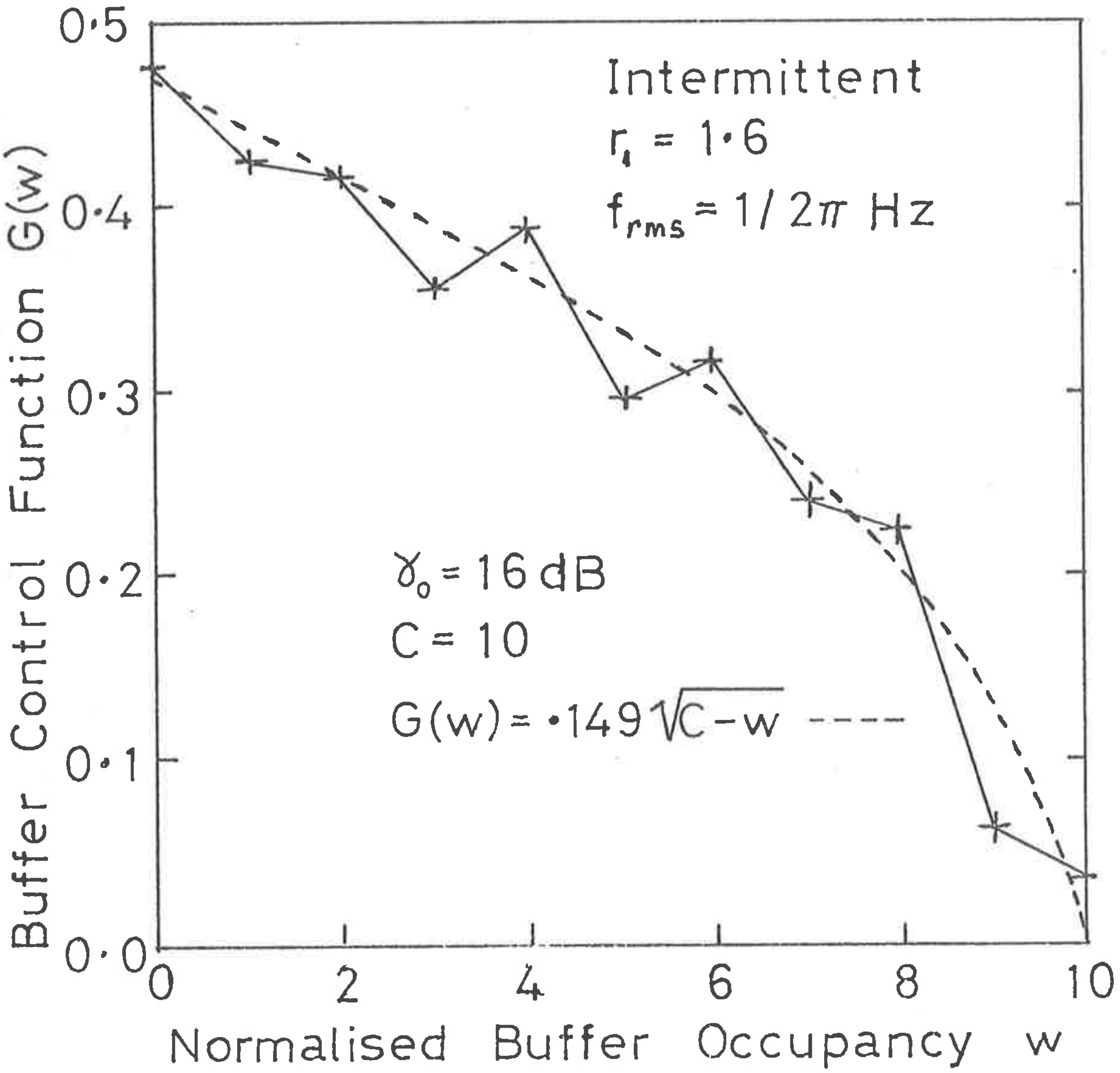

FIGURE 4.7

BUFFER CONTROL FUNCTION G(ω) FROM A SIMULATION STUDY was used and the simulation result along with that for a fixed threshold is given in Table 4.2.

These results indicate that buffer control achieves the same improvement in performance as doubling the buffer capacity. This means a reduction in cost and halving the message delay which is given by:

$$T_D = \frac{C}{2\pi f_{rms}} \qquad 4.17$$

However, the full potential of an adaptive threshold is still unknown as will be discussed in section 4.2.2.

4.2.2 Analytical Problem

This section discusses the theoretical formulation of the buffer occupancy control problem treated in section 4.2.1 in a quite general way so as to allow the use of two separate control functions depending on whether the buffer is filling or emptying. This solution is a hysteresis control characteristic. The specific results for the constant threshold case are also shown to be a special case.

It is assumed for this analysis, as before, that the start and end of fade follow a Poisson process, where, however, the actual fade length distribution is no longer exponential due to the varying threshold. In particular, define:

$$P_\lambda(s,t) = P_r \{\text{increase of contents from s to more than t bits during a fade}\}.$$

$$= e^{-\int_s^t 1/\lambda(\omega)\, d\omega} \qquad 4.18$$

and

$$P_\mu(s,t) = P_r \{\text{ decrease of contents from s to less than t bits during a transmission period}\}.$$

$$= e^{-\int_t^s 1/\mu(\omega)\, d\omega} \qquad 4.19$$

where $\lambda(\omega)$, $\mu(\omega)$ are as defined in section 4.1 but are now functions[6].

---

6. It can further be noted that λ and μ can also include r_1 as a variable as well to allow for a variable rate.

INTERMITTENT SYSTEM		
r_1 = 1.6	γ_o = 16dB	C = 10
System Description	Other Parameters	Average Error Probability $\overline{P}_\varepsilon$
Infinite Buffer	x+h = 0.47	1.0×10^{-4}
Fixed Opt Threshold		
Theory	x+h ≃ 0.306	1.27×10^{-3} $1.17 \pm 0.09 \times 10^{-3}$*
Simulation	x+h ≃ 0.306	
Buffer Control	As in Fig. 4.7 $G(\omega) = .149\sqrt{10-\omega}$	$6.02 \pm 2 \times 10^{-4}$*

* From 10 simulation runs.

TABLE 4.2 - Performance Using Buffer Control.

of buffer occupancy ω through the two unknown control functions

$$x_{th} = g_o(\omega) \quad \text{during a fade}$$

and

$$x_{th} = g_1(\omega) \quad \text{during transmission.}$$

In Appendix C is derived an integral equation for the distribution function of the buffer occupancy at the end of transmission points given as:

$$F^J(z) = H_1(z) - \int_{x=o}^{C} K_1(x,z)F^J(x)dx \qquad 4.20$$

where

$$H_1(z) = P_\mu(C,z) \qquad 4.21$$

and

$$K_1(x,z) = \int_{\max(x,z)}^{C} {}^1/_{\lambda(x)} \cdot {}^1/_{\lambda(y)} P_\lambda(x,y)P_\mu(y,z)dy - {}^1/_{\lambda(x)} \{ U(x-z)P_\mu(x,z) + U(z-x)P_\lambda(x,z) - P_\lambda(x,c)P_\mu(C,z)\} \qquad 4.22$$

It must be noted that:

$$F^J(0) = P_o \qquad 4.23$$

and

$$F^J(C) = 1 \qquad 4.24$$

Similarly, the distribution function of the buffer occupancy at the beginning of transmission points $F^I(z)$ could be derived. However, contributions to the error probability only occur during transmission periods and during full buffer conditions. Thus, since the end of a transmission period is the same as any other point during the actual transmission period for a Poisson process, $F^J(z)$ also describes the buffer distribution at an arbitrary point during a transmission period.

It is derived in Appendix C that the distribution function at the beginning of a transmission period (end of fade) $F^I(z)$, which by similar argument is the distribution function during a fade, is derivable from $F^J(z)$ as:

$$F^I(z) = \int_{x=o}^{C} \int_{y=0}^{\min(x,z)} 1/\mu(y)(1-P_\lambda(y,z))P_\mu(x,y)dy\ dF^J(x) + (1-P_\lambda(0,z))P_o \qquad 4.25$$

However, one is interested only in P_c, the probability of full buffer where

$$F^I(0) = 0 \qquad 4.26$$

$$F^I(C) = 1 - P_c \qquad 4.27$$

Thus, equation 4.25 need only be evaluated for z = C which is given by:

$$F^I(C) = \int_{x=0}^{C} \int_{y=o}^{x} 1/\mu(y)(1 - P_\lambda(y,C))P_\mu(x,y)dy\ dF^J(x) + (1-P_\lambda(0,C))P_o \qquad 4.28$$

where it must be noted that

$$\left.\frac{dF^J(x)}{dx}\right|_{x=0} = P_o\delta(0) \qquad 4.29$$

One can now define the modified overflow probabilities P_o' , P_c' as in section 4.1. If now one assumes that x is varying much faster than the buffer occupancy then the average error probability is given by: (Appendix C)

$$\overline{P}_\varepsilon \simeq \frac{r_1^2}{\gamma_m + 2r_1} \{ (1 - P_o') P_t \int_0^C z(g_1(\omega))dF^J(\omega) + P_c' P_f(1 - z(g_o(\omega))) \} \qquad 4.30$$

where P_t and P_f are found from the unity data rate condition.

It will also be assumed that the amount of hysteresis is small so that the probability of the SNR x being between the hysteresis limits is small (a few percent). The actual hysteresis characteristic is expected to be in the same direction as that mentioned to reduce the effect of control delay (Chapter 3) where a higher threshold is needed during transmission than during a fade. However, this situation leads to multiple switching instability until the signal crosses the hysteresis distance. In practice, this is no real problem but does involve an oversimplification in the analysis.

To solve this problem analytically, appears difficult except for the simple case of constant thresholds treated by Campbell (1957). Numerical solution of the integral equation 4.20, which is a Fredholm equation of the second kind, is still under investigation. By choosing break points in g_o and g_1 as for the simulation study, it would seem possible to then minimise equation 4.30 for these values by a non-linear optimisation method as in Chapter 5.

A solution to this adaptive threshold problem is still to be found and is an area of future research. It must also be pointed out that such a control problem arises in data compression systems[7.] so that perhaps the problem could be usefully generalised. Another possible generalisation is how the transmitter and receiver at the same station serving opposite going data streams could use a common buffer. This would mean a further reduction in buffer size and thus resultant message delay.

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7. See Dosik (1974) and Schwartz (1968).

4.3 SYSTEM IMPLICATIONS

Such an intermittent system has application in any slow fading environment. However, depending on the data rate and fading rate, the size of the required buffer can be very different. To illustrate the results of section 4.1, in terms of buffer size, two different areas of application will be discussed. These two examples, namely a Tropscatter system and a mobile radio system, will be discussed in more detail in Chapter 5.

Consider a Troposcatter system operating on a 1 GHz carrier frequency at an average data rate of 1 Mbit/s in a slow fading environment of average fade rate f_e of 1 fade/s. The buffer size for a given performance requirement depends on the fade rate since:

$$f_{rms} = f_e / \sqrt{\pi \ln 2} \qquad 4.31$$

Substitution in 4.15 gives:

$$B = 0.235\ CR_i / f_e \qquad 4.32$$

In Table 4.2 are given the buffer size and resultant message delay for two cases of mean SNR and capacity. For such a system a large store is required but with modern storage techniques in mind, this system could have large advantages over the present systems using large and expensive diversity antennae providing the delay was allowable as in a non-voice application.

Another interesting application is for a digital mobile radio system [8.] operating on a carrier frequency of 1 GHz at an average data rate of 56 kbit/s. In such an environment, the buffer size depends on the vehicle velocity v in km/hr since f_{rms} is given by: (Table 4.1)

$$f_{rms} = \frac{f_o}{c\sqrt{2}}\ v \qquad 4.33$$

f_o - Carrier frequency (Hz).

c - velocity of light (km/hr).

Substitution in 4.15 gives:

$$B = 0.243\ CR_i / v \qquad 4.34$$

In Table 4.3 the buffer size and resultant message delay are given for two typical parameter conditions. For such a system, a message delay of about 0.1 seconds would be perfectly acceptable for voice applications and the digital circuitry (see section 4.5) could be put onto one chip. There is a problem of course, if the vehicle moves very slowly or stops in a signal null; however, this example illustrates a novel

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8. See Clarke (1968).

INTERMITTENT SYSTEM	
Troposcatter System	Mobile Radio System
R_i = 1Mbit/s $\overline{P}_\varepsilon \approx 1.0 \times 10^{-6}$ 1. γ_o = 19dB C = 50 f_e = 1 B ≃ 12 Mbit T_D = 12 s. 2. γ_o = 20dB C = 20 f_e = 1 B = 4.7Mbit T_D = 4.7 s.	R_i = 56 kbit/s $\overline{P}_\varepsilon \approx 1.0 \times 10^{-4}$ 1. γ_o = 17dB C = 20 v = 30km/hr B = 9.0 kbit T_D = 0.16 s. 2. γ_o = 18dB C = 10 v = 30km/hr B = 4.5kbit T_D = 0.08 s.

TABLE 4.3 - SYSTEM BUFFER REQUIREMENTS

approach to the moving mobile situation.

In any application the compromise between transmitter power and buffer size for a given performance measure must be weighted in some fashion. However, these examples illustrate that such an intermittent system with an adequate buffer (and hence message delay) can achieve performance comparable to an optimum fourth order diversity system.

4.4 SYNCHRONISATION PROBLEM

As in any synchronous digital transmission system, there is a need for synchronisation. However, in a variable rate transmission system, this problem is further complicated by the need for buffer synchronisation. However, one must first briefly discuss bit and block synchronisation. This treatment is confined to an intermittent system and no attempt is made to cover the added problems with multi-rate transmission.

It is assumed for this work that near perfect bit synchronisation is possible and block synchronisation (block size of n_r bits) is also near perfect except for a loss of a block of data due to a feedforward error. (see section 3.1.4) To achieve synchronism the transmission clock is phase locked at a multiple (namely the transmission ratio r_1) of the main data clock . The receiver then locks onto this transmission clock and derives the main data clock in reverse manner. To maintain this lock during non-transmission periods, a one-zero sequence would be transmitted so as to maximise this locking capability.

However, a unique requirement of such a system is the need for buffer synchronisation. This means that the buffer occupancy at the transmitter and receiver (plus bits in transit) must total one complete buffer size. In this way, when the transmitter buffer empties, the receiver buffer fills exactly the number of transit bit times later and vice versa. In this situation the buffers are said to be in "lock". An important requirement of such a "lock" is that it be unique and that the buffers automatically pull into lock at start up or after a feedforward error.

To achieve this lock, the transmitter buffer must transmit when it is full and stop transmission when it is empty. These decisions (slightly modified from section 3.1.4) are then sent to receiver B by feedforward transmission. If $r_1 = 2.0$ or greater, then the receiver requires only one intrinsic control to stop inputing data when the transmitter buffer is full. [9] With this control the buffers pull into lock when the transmitter buffer goes empty, because then the receiver hits full buffer. This locking mechanism is independent of the number of bits in transit. This has been demonstrated with some hardware (see section 4.4) and also with a simulation program.

However, the situation for transmission ratios $r_1 < 2.0$ is more complex to follow in detail. Thus, no specific method is given here to achieve "lock" for this situation for such transmission ratios. It does not seem unreasonable to assume that a similar method (but involving a counter at the receiver) is possible for such situations.

Another problem envisaged is how this feedforward information can be synthesised at the receiver when the main channel SNR goes low (ie $x < \hat{x}_{th}$, see Chapter 3). If this is not done, the receiver will not know to start inputting if the Tx buffer goes full. Similar, to the above situation, the solution would seem to be a counter at the receiver to generate intrinsic control in synchronism with the feedforward information.

The synchronisation problem has still a number of aspects unanswered in detail as already discussed. However, the basic principle involved is very similar to a phase locking problem and is an avenue for further work.

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9. See Cowley (1975).

4.5 HARDWARE MODEL

One of the main requirements of an intermittent system is a buffer at both transmitter and receiver. In section 4.1, the effect of a finite buffer on performance is analysed and thus the size of buffer required for a particular application can thus be predicted. Specific examples will be discussed in Chapter 5. However, how such a buffer system would operate in practice to improve performance will be discussed here with respect to a specific hardware model of the system that was built.

During the course of this work, two buffer systems were designed and built. The first of these consisted of two buffers of 1024 bits using dynamic shift registers. However, such a buffer system would limit the data rate to 1 Kbit/s due to the maximum clock frequency limitation. The second buffer system uses two 4096 bit dynamic RAMs. This system has a data rate lower limit of 32 kbit/s due to refresh requirements and an upper limit of about 320 kbit/s due to cycle time limitations. This particular buffer system reported by Cowley (1975) was then used in the hardware model simulation of an intermittent system suitable for a mobile radio environment. (see section 4.3)

The system model is illustrated in Fig. 4.8 incorporating the two 4 kbit buffers. The analogue message is converted to a digital bit stream by a companded delta modulator [10.] which is then clocked into the transmitter buffer. As can be seen, the higher transmission rate clock is derived from the data clock by a phase locked loop. Then, according to the channel state and whether the buffer is full or empty,[9.] bits are read out of the transmitter buffer into a continuous bit stream to drive an FM modulator. The RF signal at about 1 MHz goes into a Rayleigh fading simulator [11.] which gives an output which has a random (actually pseudo random) Rayleigh distributed amplitude distribution and a random uniformly distributed phase distribution. One output goes to an envelope detector and comparator to derive the channel state. (ie. whether the SNR is above or below a threshold) . The other output

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10. See Davis (1974).
11. See Davis (1975).

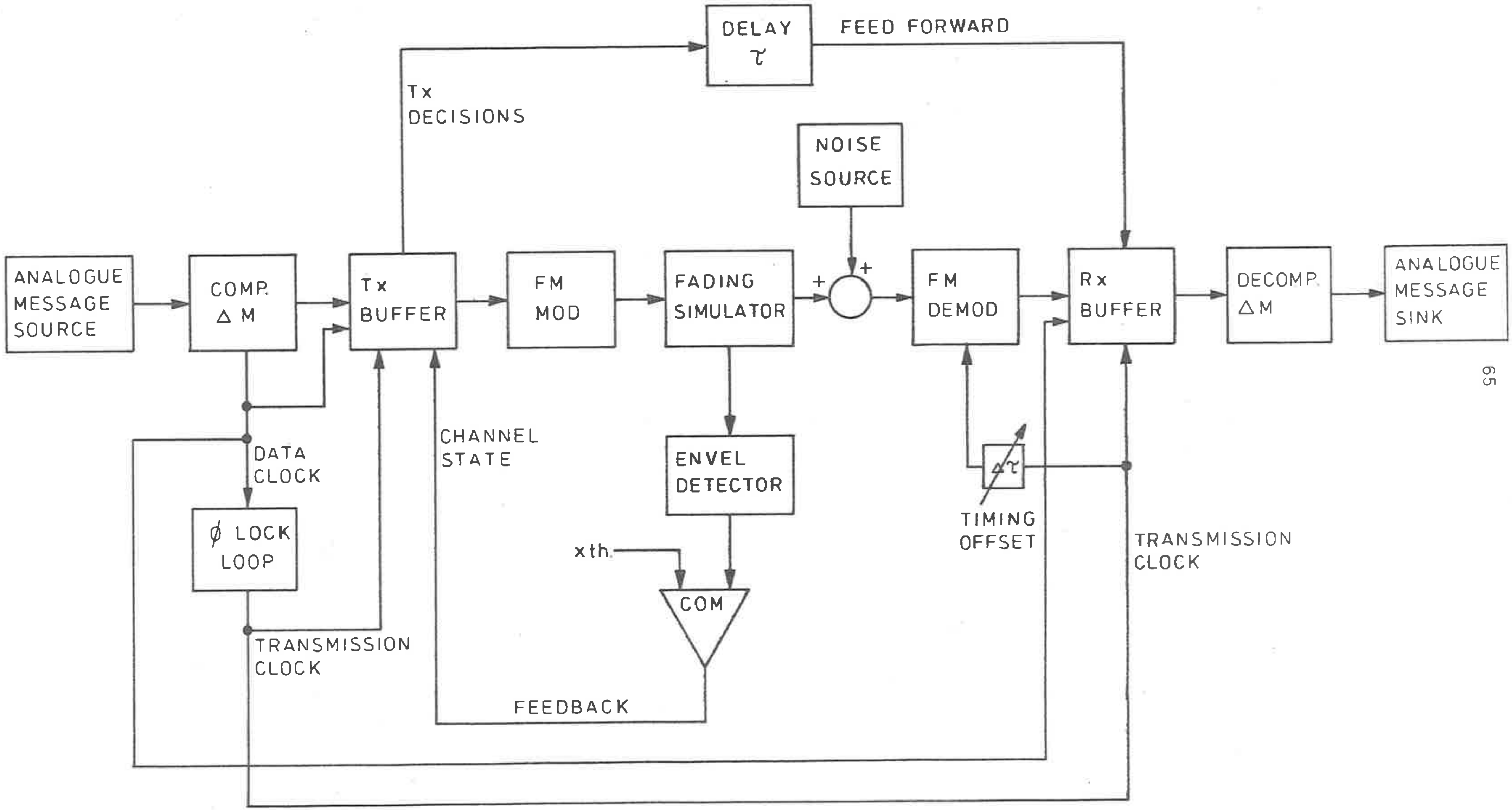

FIG. 4.8 HARDWARE MODEL OF INTERMITTENT SYSTEM

plus noise goes to a non-coherent FSK demodulator to give a continuous bit stream. Then, according to the state of the delayed fed-forward decision of the transmitter and whether the receiver buffer is full, (see section 4.4) bits are inputed to the receiver buffer. Bits are read out of the receiver into the delta demodulator to yield the received analogue message.

Using this system model, the behaviour of the system was observed for different values of threshold, SNR and fading rate. For a given value of SNR and fading rate, the threshold x_{th} is variable manually. When $x_{th} = 0$, the system operates as a normal single rate system, but with a 3 dB inferior SNR (Here $r_1 = 2.0$). As the threshold is increased, the performance dramatically improves until occasional full buffer conditions occur which can be observed visually on a full buffer indicator light at the transmitter and is noticeable audibly as sharp pops in the signal. This threshold for best performance is quite easily found manually using the indicator light. One of the most interesting observations is the effect of different fade rates on speech.

If the vehicle moved very slowly at about 5 km/hr, the fading[12.] results in a swishing sound on the signal so that complete words can be lost in the fade. This of course, is unacceptable. If the vehicle moves at about 30 km/hr, the fading results in plops in the signal so marring syllables but still giving understandable but poor speech. However, if the fading is fast corresponding to a vehicle speed of 100 km/hr, the speech becomes unintelligible. The signal seems to be totally immersed in noise rather than undergoing fading as such. When an intermittent system is used to combat the fading, the performance improvement is most dramatic for the fast fading situation. At 30 km/hr the improvement is still quite marked indicating the buffer size predictions from theory are quite reasonable. However, 4 kbit buffers cannot cope with the very slow fades occuring at 5 km/hr and little improvement with an intermittent system is observed.

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12. This is for a non-intermittent system.

With a continuous system, increasing the mean SNR only gradually improves system performance. However, with an intermittent system the performance improves quite quickly so that for speech applications, no appreciable improvement is noticed (ie. no point in $\overline{P}_{\varepsilon} < 10^{-4}$).

Some of these observations have been recorded on magnetic tape for demonstration purposes. However, the most useful aspect of the hardware model is to illustrate some of the system characteristics such as buffer synchronisation and the overflow conditions.

4.6 CONCLUSIONS

The required buffer size for an intermittent system for a particular performance level has been derived where for optimum performance there is a unique choice for the threshold. However, it was noticed in Chapter 3 that a variable rate system subject to a small amount of control delay did not transmit data below a certain threshold. Thus, it is suggested a similar sized buffer would be required for a variable rate system as for an intermittent system and thus reducing the possible potential of a fully variable rate system.

In order to make better use of a given buffer size, methods to vary the threshold were examined. A simulation study suggests that the buffer size could be halved by making the threshold a suitable function of buffer occupancy. A more general control problem involving a hysteresis control function has been formulated. The solution of this problem, however, is an area for future work.

In order to illustrate the results obtained for the intermittent system, two system examples which are considered in greater detail in Chapter 5, are examined with regard to the required buffer size. These examples show that the buffer size required is not unreasonable in the light of current memory technology. Also discussed is how the two buffers can be synchronised and resynchronised after a feed-forward error.

A hardware model of the mobile system using 4K buffers was tested using a fading simulator. It is found that there are three different fading rate catagories in regard to their effect on speech. An intermittent system is most effective on the very fast fading (vehicle velocity of 100 km/hr) which, subjectively, has the worst effect on speech. However, such a system could not cope with the very slow fading unless a larger buffer were used. It is also found that it is quite easy to manually find the best threshold which, in practice, could be automatically adjusted.

CHAPTER 5

5.0 INTRODUCTION

The work so far has considered the constraints on a channel state fedback system in isolation of each other and thus neglected the effect of interaction. One form of interaction, namely the interaction of feedback error and control delay for an intermittent system has been briefly discussed in Chapter 3. However, this chapter combines the results obtained in Chapter 3 and 4 for an intermittent system so that the total performance can be studied. These results are then applied in detail to two case studies (introduced in Chapter 4) to illustrate more specifically the constraints of design of such a system. Simulation results are also given to validate this analysis.

5.1 COMBINED SYSTEM ANALYSIS

When all of the constraints are included for an intermittent system, as illustrated in a system model in Fig. 5.1, the effect of interactions between the various constraints must be considered. One of the most important interactions is how the feedback strategy, (ie. not sending data when the feedback channel SNR is below a threshold y_{th}) as discussed in Chapter 3, effects the buffer distribution and thus the overflow distributions discussed in Chapter 4. Once this interaction has been included one can derive the expression for the average error probability for the whole system and optimise for the various system parameters.

5.1.1 Effect of Feedback Strategy on Buffer Distribution

In order to combat the effects of feedback error (see Chapter 3) transmission is halted if the feedback channel SNR goes too low. Thus the effective arrival (non-transmission period) distribution is a function of mean fade durations on the main and feedback channels. However, it will be shown that the main effect of the feedback strategy is to increase the proportion of non-transmission time as reflected in the traffic intensity ρ (see Chapter 4) rather than to make the fades longer. Quite to the contrary, the effect of the feedback strategy is to

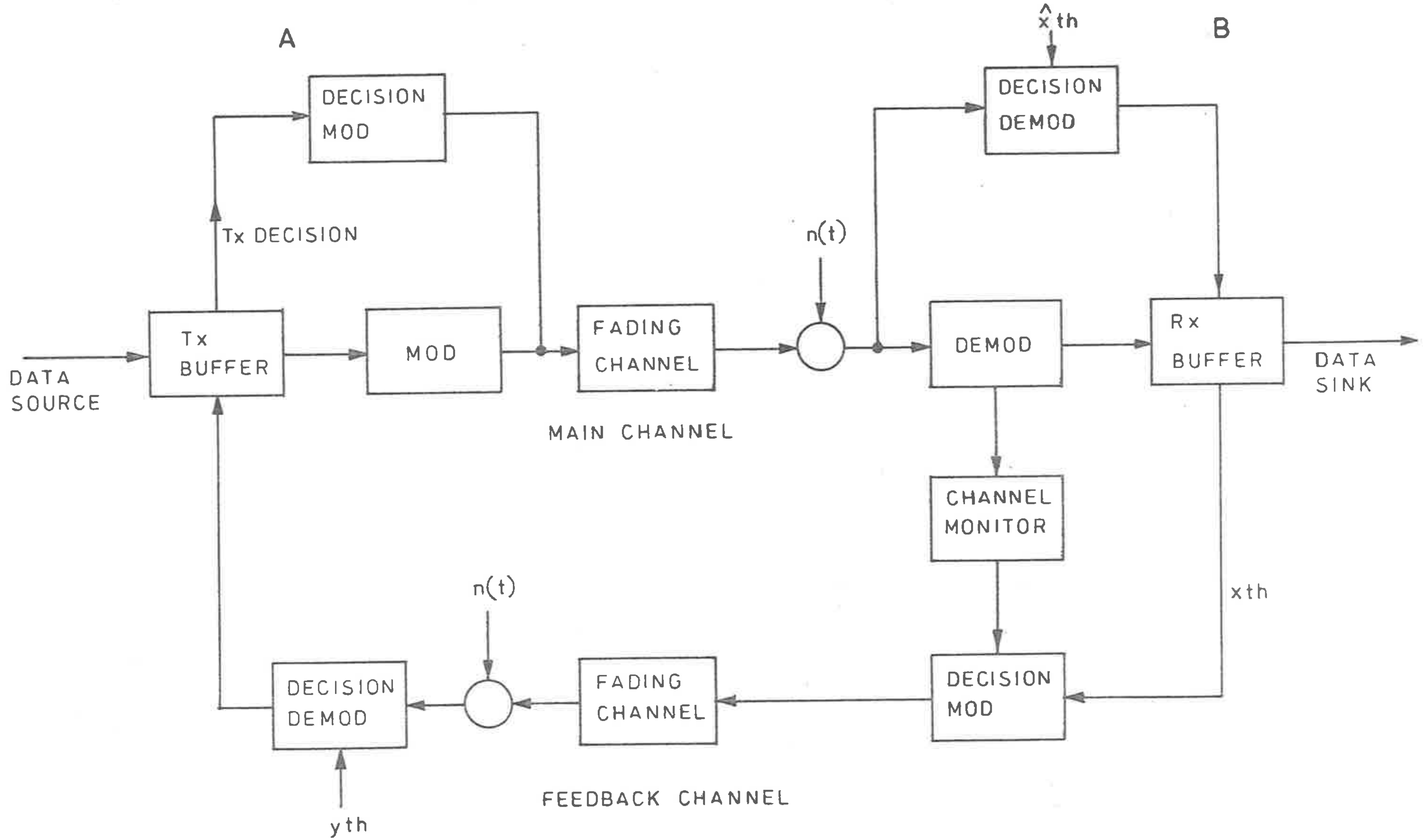

FIG. 5.1 MODEL OF INTERMITTENT SYSTEM

make the mean non-transmission period shorter!

In Appendix D is derived an approximate expression for the non-transmission period distribution function on the assumption of exponential fade durations on main and feedback channels as a function of the two thresholds x_{th} and y_{th}. This is given as:

$$
\begin{aligned}
F(t) = {} & \frac{ad}{(a + b)(b + d)} \left\{ 1 + \frac{b}{a - c + d} \right\} e^{-ct} \\
& - \frac{abcd}{(a + d)(b + d)(a - c + d)(b + c - d)} e^{-(a + d)} \\
& + \frac{bcd}{(a + b)(b + d)(a + b + c)} \left\{ 1 + \frac{a}{b + c - d} \right\} e^{-(a+b+c)t} \\
& + \frac{bc}{(c + d)(b + d)} \left\{ 1 - \frac{d}{a - b - c} \right\} e^{-at} \\
& + \frac{abcd}{(b + c)(b + d)(a - b - c)(a + d - b)} \cdot e^{-(b + c)t} \\
& + \frac{abd}{(c + d)(b + d)(a + c + d)} \left\{ 1 + \frac{c}{a - b + d} \right\} e^{-(a+d+c)t}
\end{aligned}
\qquad 5.1
$$

where a,b,c, and d are reciprocal time constants of the fade distributions on the two channels given by:

$$a = f_{ry}/(1 - e^{-y_{th}}) \qquad 5.2$$

$$b = f_{ry}/e^{-y_{th}} \qquad 5.3$$

applying to the feedback channel and

$$c = f_{rx}/(1 - e^{-x_{th}}) = 1/\lambda \qquad 5.4$$

$$d = f_{rx}/e^{-x_{th}} = (r_1 - 1)/\mu \qquad 5.5$$

applicable to the main channel which are used in the analysis in Chapter 4 where the feedback channel was assumed to be perfect and thus gave the non-transmission distribution as:

$$F(t) = e^{-ct} \qquad 5.6$$

Similarly, f_{rx} and f_{ry} are given by:

$$f_{rx} = 2f_{rms}\sqrt{\pi x_{th}}\ e^{-x_{th}} \qquad 5.7$$

$$f_{ry} = 2f_{rms}\sqrt{\pi y_{th}}\ e^{-y_{th}} \qquad 5.8$$

The distribution given by 5.1 is shown in Fig. 5.2 for several different typical values of threshold y_{th} for a given value of threshold x_{th}. It can be seen that for most cases of interest the distribution is exponential.

For an intermittent system the inequality given by:

$$y_{th} << x_{th} < \ln 2 \qquad 5.9$$

is applicable which leads to the inequality:

$$a > c > d > b \qquad 5.10$$

Thus, considering $t > 1/c$, equation 5.1 can be approximated by the first term:

$$F(t) \approx \frac{ad}{(a + b)(b + d)} \{ 1 + \frac{b}{a - c + d}\} e^{-ct} \qquad 5.11$$

which for low y_{th} simplifies to

$$F(t) \approx \frac{d}{b + d} e^{-ct} \qquad 5.12$$

Thus, if one contends that the buffer behaviour is affected by the distribution tail, then one can assume the worst case of

$$\lambda = {}^{1}/_{c} \qquad 5.13$$

that is, λ is unaffected. However, from the theorem of total probability, the proportion of non-transmission time to transmission time, as reflected in the traffic intensity s, (Chapter 4) is modified to:

$$\rho = \frac{e^{x_{th} + y_{th}} - 1}{r_1 - 1} \qquad 5.14$$

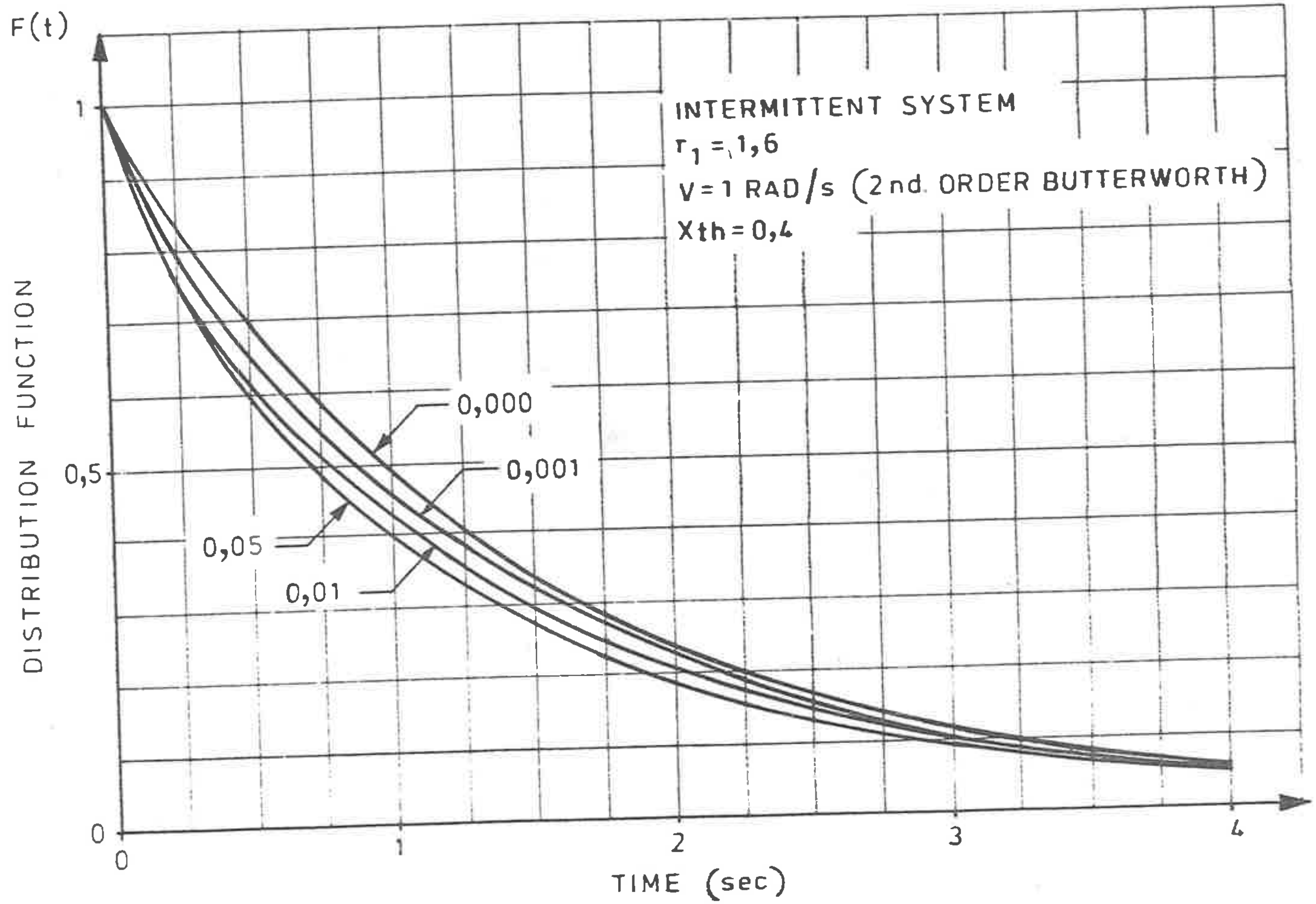

FIG. 5.2 VARIATION OF DISTRIBUTION FUNCTION WITH THRESHOLD y^{th}

Thus, as stated earlier, the effect of this feedback strategy is to increase the traffic intensity and thus increase the required buffer capacity to achieve the same overflow probabilities.

Thus, the effect of the feedback strategy on the buffer distribution can be included in a complete system analysis which is treated in section 5.1.2.

5.1.2 Performance Optimisation

Combining the results of Chapter 3 with those of section 5.1.1 one can obtain an expression for the average error probability subject to all the constraints discussed. Then the performance can be optimised for the best operating parameters.

Since the overflow probabilities P_o' and P_c' are independent of the SNR values x and y, one can combine equation 3.30 and 4.11 to give:

$$\overline{P}_\varepsilon \simeq \frac{r_1^2}{\gamma_m + 2r_1} \left\{ (1 - P_o')z'e^{-y_{th}} + \left(\frac{\hat{z} - z'}{\gamma_f}\right)\omega + P_c'(1 - z')\right\} - \text{double error term (small)} + \frac{\hat{\omega}_m}{2 + \gamma_f} + \frac{\omega(1 - e^{-x_{th}})}{2 + \gamma_f} \qquad 5.15$$

and thus combining 3.18 and 4.13 one obtains

$$\overline{r} \simeq r_1 \left\{ (1 - P_o')e^{-(x_{th}+y_{th})} + \frac{(1 - 2e^{-x_{th}})}{2 + \gamma_f} + P_c'(1 - e^{-x_{th}})\right\}$$

$$= 1 \qquad 5.16$$

where

$$z' = \exp\left[-\frac{(\gamma_m + 2r_1)x_{th}}{2r_1 + \gamma_m(1 - k^2)}\right] \qquad 5.17$$

$$\hat{z} = \exp\left[-\frac{(\gamma_m + 2r_1)\hat{x}_{th}}{2r_1}\right] \qquad 5.18$$

$$\omega = \exp\left[-\frac{(\gamma_f + 2)y_{th}}{2}\right] \qquad 5.19$$

$$\hat{\omega}_m = \exp\left[-\frac{\gamma_f + 2}{2} \cdot \hat{x}_{th}\right] \qquad 5.20$$

and

$$P_o' = \frac{r_1 - 1}{r_1} \cdot \frac{1 - \rho}{1 - \rho e^{-\beta C}} \qquad 5.21$$

$$P_c' = \frac{1}{r_1 - 1} \cdot P_o' e^{-\beta C} \qquad 5.22$$

where

$$\beta = \frac{1 - \rho}{\lambda} \qquad 5.23$$

and ρ and λ are those derived in section 5.1.1.

These results assume that feedforward and feedback errors (except for when $y < y_{th}$) do not significantly effect the buffer distribution due to their small probability of occurrence. Secondly, the effect of feedforward error, which is represented by the last two terms of equation 5.15, has been doubled due to the loss of data when the buffers resynchronise (see Chapter 4). It has also been noted that the double error term given previously by the author Coutts (1975a) is negligible.

Equation 5.15 can now be optimised with respect to n_p, x_{th}, y_{th} and $\hat{x}_{th}$ given some suitable value of n_r. This is done using the modified Fletcher Powell[1.] algorithm on the digital computer. The results of this optimisation will be considered in sections 5.2 and 5.3.

5.1.3 Simulation Procedure

In order to validate the combined analysis, especially that of section 5.1.1, the system is partially simulated on the digital computer. By partial simulation, it is meant that certain properties of the system are assumed to behave according to a known analytical formula which is a function of system parameters and thus is just calculated during the simulation. Further discussion of this method is covered by Beare and Coutts (1976).

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1. See Lootsma (1971).

For this simulation, the effect of feedback error and feedforward are partially simulated (except for when $y < y_{th}$) according to their pre-integrated analytical probability formulae of Chapter 3 (except the effect of feedforward error is doubled). These are given by:

1. $\{ P_{01}(y) r_1 \hat{P}_\varepsilon(x) \}$ $\quad \hat{x}_{th} \leqslant x < x_{th}; \quad y \geqslant y_{th}$

2. $\{ P_{10}(x) \}$ $x \geqslant x_{th}; \quad y \geqslant y_{th}$

3. $\{ P_{01}(y) \}$ $x < x_{th}; \quad y \geqslant y_{th}$

This program also neglects the effect of buffer synchronisation.

Typically, the feedback (and feedforward) error probabilities are extremely low, so that full simulation is out of the question. This is why only partial simulation is used to estimate these effects. However, this simulation should validate the assumptions of section 5.1.1 where it has been pessimistically approximated that λ is unaffected for the choice of y_{th}. (Actually slightly less) It can also be noticed that the simulation also neglects the double error term when a feedforward error cancels a feedback error for $x < x_{th}$.

This simulation program is used in section 5.3 to validate the results predicted for the case studies.

5.2 CONSTRAINT APPRAISAL

Using the expression for average error probability derived in section 5.1.2, one can now assess the effect of the various constraints individually with the other constraints included but constant. These results can then be compared with the results of Chapters 3 and 4 to assess the validity of the original assumption that the constraint interaction is a second order effect. Then an example is simulated to validate the analysis of section 5.1.1.

In Fig. 5.3 is plotted the variation of performance with increasing rate ratio n_r for several values of normalised buffer capacity C at a particular value of SNR γ_o. The improvement is negligible for

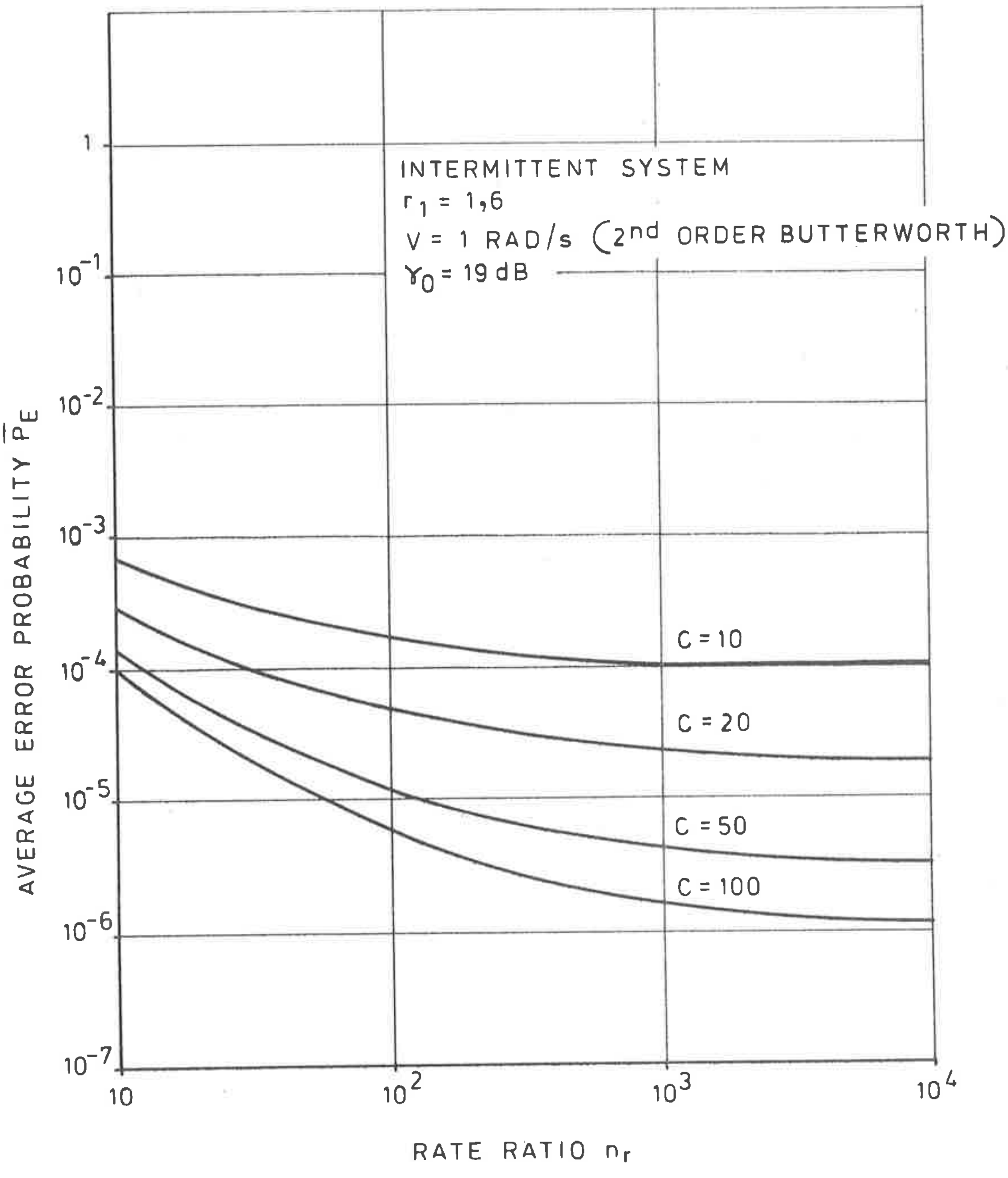

FIG. 5.3 EFFECT OF INCREASING RATE RATIO n_r

$n_r > 1000$ as found in Chapter 3. This corresponds (for $n_p \simeq 50$) to about 11 dB greater SNR on the service information than the main information. However, n_r is limited by the effect of increasing control delay and, as noted in Chapter 3, the possibly undesirable feature of longer (though less probable) error bursts.

However, in order to quantify the effect of control delay on performance, one needs to know the correlation coefficient as a function of the delay-bandwidth product for the particular fading spectrum. In Table 5.1 is the corresponding table to Table 4.1 giving the autocorrelation coefficient for the various spectra G(f). From this table, the 1st order Butterworth spectrum as used by Cavers (see Chapter 3) is the worst case when considering the effect of control delay. Thus, using the second order Butterworth spectrum as an example, Fig. 5.4 shows the effect of control delay limiting the increase of the rate ratio n_r for different ratios of transmission rate to fading rate. From these curves it is again seen the drastic effect of control delay (see Chapter 3) on performance when it exceeds a critical amount. Comparison with the results of Chapter 3 suggests that an exponential correlation function, although more gradual in its effect on performance for increasing delay, is very pessimsitic for small delays such that $V\tau < 0.1$. This order of effect is also noted by Davis (1970) for a feedback diversity system. It can be concluded from these graphs that a ratio of between 100 and 1000 would be used depending on the performance level and allowable control delay. This corresponds to 5 - 11 dB higher mean SNR on the service information than the main information.

In Fig. 5.5 is plotted the effect of increasing buffer size for different values of rate ratio n_r at a given value of mean SNR γ_o. At a low value of rate ratio, the required value of threshold y+h to give adequate service information reliability causes an undue amount of decrease of transmission time resulting in a larger buffer requirement. This is observed for the case of $n_r = 10$. However, for $n_r > 100$ this form of interaction is a second order effect. In practice, especially when buffer control would be used (see Chapter 4), a normalised buffer capacity of about 20 to 50 would be used depending on the performance level required as illustrated in the next section in the case studies.

In order to validate the analysis results, especially those

Power Spectrum $G(f)$	Auto-correlation Coefficient $k(\tau)$
1. Ideal Low Pass $G(f) = \begin{cases} G_o ; & f < f_c \\ 0 ; & f \geq f_c \end{cases}$	$k(\tau) = \text{sinc}(2\pi f_c \tau)$ [2]
2. Linear Fall Off $G(f) = \begin{cases} G_o(1 - \lvert f/f_c \rvert); & f < f_c \\ 0 ; & f \geq f_c \end{cases}$	$k(\tau) = \text{sinc}^2(\pi f_c \tau)$ [2]
3. Gaussian $G(f) = G_o e^{-f^2/2f_c^2}$	$k(\tau) = e^{-(2\pi f_c \tau)^2/2}$
4. 2nd Order Butterworth $G(f) = \dfrac{V^4}{V^4 + \omega^4}$	$k(\tau) \approx 1 - \dfrac{(V\tau)^2}{2}$ for small $V\tau$.
5. 1st Order Butterworth $G(f) = \dfrac{V^2}{V^2 + \omega^2}$	$k(\tau) = e^{-\lvert V\tau \rvert}$
6. Mobile $G(f) = \begin{cases} G_o(1 - (f/f_c)^2)^{-\frac{1}{2}} ; & f < f_c \\ 0 ; & f \geq f_c \end{cases}$	$k(\tau) = J_o(2\pi f_c \tau)$ $\approx 1 - (2\pi f_c \tau)^2/4$ (for small $f_c \tau$)

TABLE 5.1 - AUTO-CORRELATION COEFFICIENTS

2. Here sinc x is defined as $\sin x/x$.

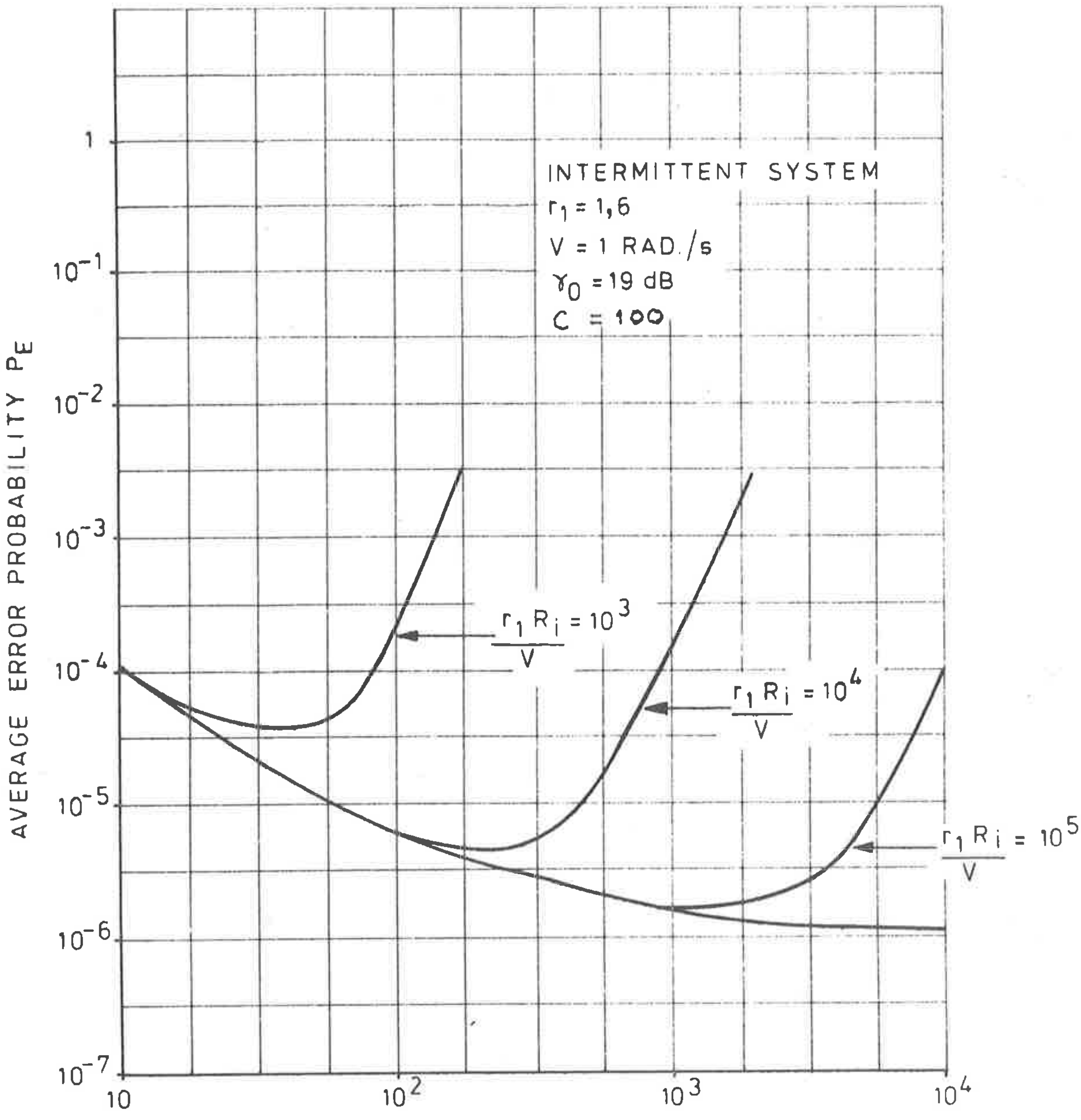

FIG. 5.4 EFFECT OF CONTROL DELAY FOR INCREASING n_r

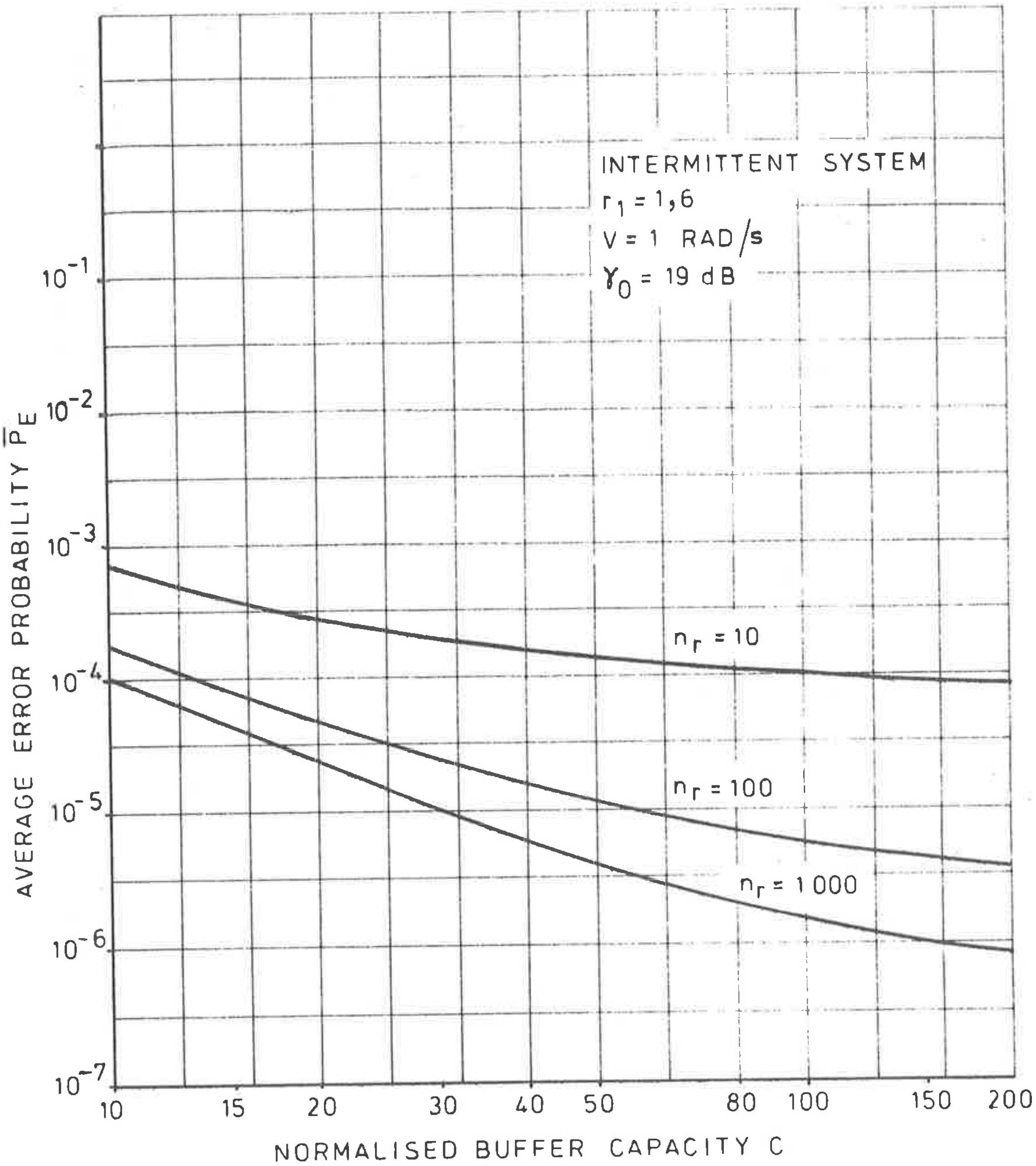

FIG. 5.5 EFFECT OF INCREASING BUFFER SIZE

pertaining to the effect of feedback strategy on required buffer capacity, the system was simulated as discussed earlier using the optimum parameters predicted from the analysis. In Table 5.2 are the results of a simulation program as discussed in section 5.1.3 which uses the optimised parameters predicted. From five independent (but "sufficiently" long) runs one can establish confidence intervals on the result.

INTERMITTENT SYSTEM	
Fixed System Parameters	Optimised Parameters
$r_1 = 1.6$ $C = 20$ $\gamma_o = 18$ dB $r_1 R_i / V = 10^4$ $n_r = 100$	$n_p = 19$ $x_{th} = 0.315$ $y_{th} = 0.033$ $\hat{x}_{th} = 0.088$
Theory: $\overline{P}_\varepsilon = 1.62 \times 10^{-4}$ Simulation: $\overline{P}_\varepsilon = 1.47 \times 10^{-4}$ Sample Standard Deviation $= 0.24 \times 10^{-4}$ 95% Confidence Interval using 5 runs* $= (1.18 \times 10^{-4}, 1.77 \times 10^{-4})$ * Each run is 50,000 points.	

TABLE 5.2 - SIMULATION RESULT

This result indicates quite close agreement with that predicted from theory in spite of the assumptions made. It must be added that this is a partial simulation since full simulation would be totally out of the question due to the small magnitude of the probabilities of some of the events involved (eg. P_r { feedforward error}).

From the results discussed in this section, it is observed that the main threshold x_{th} is set mainly by the buffer size used. The thresholds y_{th}, $\hat{x}_{th}$ are set according to the allowable value of rate ratio n_r consistent with the constraint of maximum control delay. However, these choices are perhaps best illustrated in the case studies of the next section.

5.3 CASE STUDIES

In this section, two case studies are discussed in order to illustrate the results presented so far, and indicate the relevant constraints peculiar to the situation. From the theoretical results, performance of an intermittent system is then compared with alternative systems using diversity pertinent to the application. Discussed in this context is the cost of the memory requirements for such s system in the light of developing memory technology.

5.3.1 Troposcatter System

A Troposcatter[3.] system involves long distance communication via scattered microwave energy off the Troposphere. However, such systems involve high capital cost due to the large antenna dishes required and high transmitter power. Thus, this area seems a likely candidate for the use of an intermittent system for digital applications where the buffer delay would not be of great consequence.

In Table 5.3 is a lsit of relevant parameters applicable to a typical Troposcatter application.

Using these figures in Table 5.3, one can now design an intermittent system using the considerations discussed.

---

3. Schwartz et Al (1966), Gunther (1966).

Carrier Frequency:	1 GHz
Distance:	200 - 500 km
Data Rate:	1 Mbit
Fading Rate:	1 - 10 fades/s.
Spectrum:	2nd Order Butterworth
Transmitter Power:	1 - 30 kW
Performance Level:	$\overline{P}_\varepsilon \sim 10^{-6}$

TABLE 5.3 - TROPOSCATTER SYSTEM PARAMETERS

In Chapter 4, it was suggested that a buffer size of about C >20 would be sufficient. Secondly, in order to have sufficient reliability on the service information a rate ratio n_r of 100 to 1000 is necessary. However, to investigate the possible choices of these parameters, equations can be derived in terms of the normalised variables used previously. The control delay τ is given by:

$$\tau = \frac{2D}{c} + \frac{2n_r}{r_1 R_i} \qquad 5.24$$

where D is propagation distance.
The 3 dB bandwidth of the fading process is related to the fading rate by:

$$V = 2f_e \sqrt{\pi/\ln 2} \quad \text{rad/s} \qquad 5.25$$

$$\text{Thus, } V\tau = \left(\frac{4\sqrt{\pi/\ln 2}}{c}\right)Df_e + \left(\frac{4\sqrt{\pi/\ln 2}}{r_1 R_i}\right)n_r f_e$$

$$= 2.84 \times 10^{-5}\, Df_e + 5.32 \times 10^{-6}\, n_r f_e \qquad 5.26$$

For the maximum fading rate of 10 and for $V\tau < 0.1$ this leads to the requirements

$$D < 350 \text{ km} \tag{5.27}$$

and

$$n_r < 2000 \tag{5.28}$$

Similarly, the normalised buffer capacity is given by:

$$C = \frac{2}{R_i} \sqrt{\pi / \ln 2} \; Bf_e$$

$$= 4.26 \times 10^{-6} \; f_e B \tag{5.29}$$

For the minimum fading rate for C > 20 we have

$$B > 5 \text{ Mbit} \tag{5.30}$$

Thus, one has derived the relevant constraints applicable to a Troposcatter system. The only disturbing observation is the distance limitation of 350 km due to the effect of excessive control delay. Providing this restriction is valid, the typical system parameters would be as shown in Table 5.4 where a distance of 320 km is taken as a reasonable figure.

TABLE 5.4 - PERFORMANCE OF INTERMITTENT TROPOSCATTER SYSTEM

Distance:		320 km
Data Rate:		1 Mbit/s
Transmission ratio r_1:		1.6
Buffer Size:		5 Mbit
Rate ratio n_r:		1000
Mean SNR γ_o:		21dB
Fading Rate f_e:		1 fade/sec.
	Parameters:	n_p = 95
		x_{th} = 0.259
		y_{th} = 0.02
		$\hat{x}_{th}$ = 0.037
	Performance:	$\overline{P}_\varepsilon = 1.05 \times 10^{-6}$ ****

*** Table 5.4 is continued on page 86.

*** Table 5.4 continued.

Fading Rate f_e:		10 fades/s.
	Parameters:	$n_p = 75$
		$x_{th} = 0.423$
		$y_{th} = 0.012$
		$\hat{x}_{th}$ 0.074
	Performance:	$\overline{P}_\varepsilon = 1.93 \times 10^{-6}$

The performance obtained with an intermittent system including all of the discussed constraints as presented in Table 5.4 is about 1 dB inferior to fourth order dual space-frequency diversity [4.] as used on present systems. However, instead of a second large antenna, one requires a 5 Mbit store at transmitter and receiver. It can also be noted that these thresholds must vary with fading rate and mean SNR.

In such a Troposcatter system application, an intermittent system could prove an alternative method to diversity. In the light of the fast reducing cost of memory (eg. 0.02^c/bit for CMOS memory), the economic choice considering equivalent performance potential, favours such systems using buffers as opposed to large secondary antennae of 10 m diameter which are expensive, immobile and require labour intensive construction. An intermittent system, however, requiring buffers would not suffer such disadvantages. On the other hand, 5 seconds delay in the bit stream may not be allowable in some situations thus precluding its use.

5.3.2 Mobile Radio System

Another area of application of an intermittent system, is a mobile radio telephony system operating in the microwave spectrum. In this application, [5.] rapid fading occurs due to the vehicle velocity

4. This figure is for the optimum maximal ratio system.
5. See Clarke (1968).

with respect to the scatterers which are usually buildings, trees and other such objects. The suggested solution to this problem is diversity antennae on the mobile. However, it will be shown that an intermittent system can achieve similar performance by the use of one antenna and a compact memory system.

In Table 5.5 is a list of relevant parameters applicable to this situation.

Carrier Frequency:	1 GHz
Distance:	1 - 20 km
Data Rate:	56 kbit/s.
Mobile Velocity:	30 - 100 km/hr
Spectrum [5.]	
Performance Level:	$\overline{P}_{\varepsilon} \sim 10^{-4}$

TABLE 5.5 - MOBILE SYSTEM PARAMETERS

For such a system, the V product is given by:

$$V\tau = \left(\frac{14400\pi f_o}{c^2} \right) Dv + \left(\frac{4\pi f_o}{c r_1 R_i} \right) n_r v \qquad 5.31$$

$$= 3.88 \times 10^{-5} \, Dv + 1.30 \times 10^{-4} \, n_r v \qquad 5.32$$

For the maximum vehicle velocity of 100 km/hr, and $V\tau < 0.1$, this leads to

$$D < 26 \text{ km} \qquad 5.33$$

and

$$n_r < 8 \qquad 5.34$$

Similarly, the normalised buffer capacity is given by:

$$C = \frac{\pi f_o \sqrt{2}}{c R_i} Bv$$

5. See Clarke (1968).

$$= 7.35 \times 10^{-5} \text{ vB} \quad 5.35$$

Thus for the minimum fading rate of v = 30 km/hr and C > 10 one obtains

$$C > 4.5 \text{ kbit} \quad 5.36$$

These equations thus describe the constraints applicable to the mobile situation. The most drastic constraint is the limitation on rate ratio n_r. Thus two design approaches will be discussed; one uses the alternative control method discussed in Chapter 3. For this system, a buffer size of 8 kbit (8192 bits) would be convenient where one 16 kbit RAM could be used to accomodate both the A-B Tx buffer and the B-A Rx buffer. (See Chapter 3). The possibility of an integrated buffer system will be discussed later.

In Table 5.6, the performance of an intermittent system for a mobile application where feedback information is sent back to the transmitter is summarised. Also given is the performance where the alternative control method is used (Chapter 3) resulting in a 2 dB lower SNR requirement. This performance is about 3.5 dB superior to second order space diversity[4]. as it is proposed to be used to combat fading in such an environment. Thus, instead of using a second antenna on the vehicle, a small buffer (one 16 K RAM) could be integrated with the control circuitry onto one LSI chip in the transceiver.

However, it is felt that considering all of the parameters relevant to a mobile application, diversity is probably the best solution to the fading problem in this situation, since at 1 GHz the antennae required would be quite small, close together and inexpensive. An intermittent system would be a more complex proposition, especially if one considers the alternative control problems and the lower limit on the vehicle velocity. The purpose of examining this area of application is to illustrate the design constraints on an intermittent system and how these vary with the application.

Distance:	5 km
Data Rate:	56 kbit/s.
Transmission Ratio:	1.6
Buffer Size:	8 kbit
<u>System Using Feedback</u> -	
Rate Ratio n_r:	10
Mean SNR γ_o:	20 dB
Vehicle Velocity v:	100 km/hr
Parameters	
	n_p = 5
	x_{th} = 0.336
	y_{th} = 0.088
	$\hat{x}_{th}$ = 0.168
Performance:	$\bar{P}_\varepsilon$ = 7.89×10^{-5}
Vehicle Velocity v:	30 km/hr
Parameters	
	n_p = 5
	x_{th} = 0.255
	y_{th} = 0.093
	$\hat{x}_{th}$ = 0.178
Performance:	$\bar{P}_\varepsilon$ = 1.20×10^{-4}
<u>System Using Alternative Control</u> -	
Mean SNR γ_o:	18 dB
Vehicle Velocity v:	100 km/hr
Parameters:	x_{th} = 0.386
Performance:	$\bar{P}_\varepsilon$ = 1.56×10^{-5}
Vehicle Velocity v:	30 km/hr
Parameters:	x_{th} = 0.310
Performance:	$\bar{P}_\varepsilon$ = 8.45×10^{-5}

TABLE 5.6 - PERFORMANCE OF INTERMITTENT MOBILE SYSTEM

5.4 CONCLUSIONS

This chapter has combined the work of Chapters 3 and 4 regarding an intermittent system to firstly validate the approach of examining the constraints separately and secondly to understand the effect of constraint interaction. It has been found that the main form of interaction is between decreasing information error (increasing n_r) and increasing the resultant control delay which is particularly obvious in the mobile system application. It is also found that for this same situation (low n_r) that the effect on buffer distribution is greatest. In effect, these comments can be generalised to say that for such channel state feedback systems, the fading must be very slow with respect to the data rate.

For the combined analysis and case studies considered in this chapter, no account is taken of the performance improvement possible with buffer adaption and buffer sharing as discussed in Chapter 4. In regard to the design considerations discussed in the case studies, it would probably mean the buffer sizes quoted could be at least halved.

Finally, from consideration of the case studies, application of an intermittent system to a digital troposcatter system is quite feasible and in the light of current memory system development is a viable alternative to diversity techniques. Another application not mentioned previously is to a FDM voice system where data is sent in a voice slot. In this situation, an intermittent system can be interfaced to a current FDM system to improve error performance with a small buffer system (eg. 16 Kbit, one chip) and using the service information already used on most microwave systems. Thus, an intermittent system is not just limited to the areas discussed or to a pure digital system.

CHAPTER 6

6.1 CONCLUSIONS

The central problem in data communications in a fading environment is the effect of deep fades on the error probability. This problem affects most modulation methods in a similar fashion so that in order to do a comparative study of system performance, non-coherent FSK is used and the channel is considered to be a slow, non-selective Rayleigh fading channel.

Channel State Feedback, where the transmitter adapts to the varying signal level by using feedback information from the receiver, has been suggested as a method of combatting the effects of fading. However, consideration of the various system types subject to the operational constraints of any such system shows that system performance is very dependent on these constraints. Thus, it is felt that only the more simple channel state feedback systems such as channel selection and intermittent transmission are practically viable. Variable power is limited to a support role in order to save power consumption for remote repeaters. Thus, an intermittent system emerges as the system with the most practical performance potential as an alternative to diversity.

A fundamental problem of such channel state feedback systems is the possibility of control information error and control delay. From specific investigation of an intermittent system, it has been found that the mean SNR of the control information must be about 10 dB higher than that of the main information and that a "play safe" strategy must be adopted to minimise the effect of deep fades on the control information. In order to track such a time varying channel the control delay must be less than about 10% of the fading time constant. Such control delay is very important in its effect on the system performance and it is conjectured that a hysteresis characteristic is desirable to minimise the effect of such delay. Secondly, in the case of non-reciprocal fading where alternative control delay is not possible, this control delay must be increased to improve the reliability of the service information.

Any variable rate systems require a buffer at the transmitter and receiver to interface with a uniform data source and sink. For an intermittent system, an expression for the average error probability for a specific buffer size, mean SNR and fading spectrum has been derived which shows there is an optimum threshold. Further, for the optimum threshold, the average error probability falls off exponentially even for a finite buffer. In order to make more efficient use of the buffer, this threshold can be made a function of the buffer occupancy. A simulation study indicates that this form of control can lead to a reduction by 50% of the required buffer size. This problem is then formulated analytically in a more general context allowing for a hysteresis control law and variable transmission rate.

In order to better understand the behaviour of such a buffered system, a hardware model of an intermittent system using two 4K RAMs was constructed and then tested using a random fading simulator and random noise generator. Using digitised speech giving a data rate of 56 kbit/s, the system quite dramatically removed the effects of rapid fading. In order to operate such a system, the buffers must be synchronised. For the case $r_1 = 2.0$ this was achieved simply by sending the transmitter decisions as feedforward information. The need for more work in this area is discussed later.

In order to predict the performance of an intermittent system in practice, the combined effects of all the constraints must be considered. It is shown that the feedback strategy affects the number of fades but does not significantly affect the duration of fades. The combined analysis, which is validated using computer simulation, shows that the constraint interaction is a second order effect. These results are then applied to two practical examples. An intermittent system does offer an alternative to a dual space frequency Troposcatter system but only where a message delay of several seconds is of no consequence. It is interesting to see that such an intermittent system is better than dual space diversity in a mobile radio system by using only a single chip memory.

Thus, as a solution to the problem of digital transmission in a fading environment, channel state feedback, in the light of practical constraints, does not offer much improvement over presently used diversity techniques. However, an intermittent system, with the described system methodology could prove a viable alternative to present techniques due to the decreasing cost of memory and the rising number of applications where message delay is no barrier.

6.2 FURTHER WORK

There are several areas in this thesis which, of neccessity, have been partly investigated for want of time and the need to maintain some order of priority in the various areas of work.

The problem of control reliability has a number of avenues for further possible work. Firstly, the formulation of Chapter 3 could be generalised to cover multi-channel state feedback and the required weighting functions derived in order to quantify the necessary conditions for adequate reliability for more complex channel state feedback systems. It is also quite apparent that the use of some sort of non-linear channel predictor could be devised to minimise the effect of control delay. Lastly, some calculation to take into account errors in measurement of the channel state due to additive noise should be done.

However, one of the most interesting problems to be considered is the optimisation of the buffer control law where the problem is generalised to allow a hysteresis control and a variable transmission rate. Whether this problem can be solved analytically or just numerically is unknown at this stage. Another problem concerning the buffer system is the need to develop a method of buffer synchronisation for transmission ratios r_1 of less than two.

The hardware model as constructed did not consider feedback (or feedforward) error or the effect of control delay. Thus, it would be of interest to improve this hardware model and compare both qualitatively and quantitatively the performance of an intermittent system with a hardware model of a diversity system. This work would be a precurser to the design of a practical intermittent system.

APPENDIX A

This appendix derives the average error probability for non-coherent FSK over an additive noise channel where a weighted decision is made.

In Fig. A.1 is the dual filter FSK detector where the '0' filter output is weighted by a gain k. Assume a '0' is sent. An error occurs if $r_1 > r_o'$ where,

$$r_o' = kr_o \qquad 1.$$

For Gaussian distributed additive noise,

$$p(r_o) = \frac{r_o}{N} \exp\left[-\frac{r_o^2 + \mu^2}{2N}\right] I_o\left\{\frac{r_o \mu}{N}\right\} \qquad 2.$$

$$p(r_1) = \frac{r_1}{N} \exp\left[-\frac{r_1^2}{2N}\right] \qquad 3.$$

N: Single sided noise power density.

μ: Signal amplitude.

$I_o(\ )$: Zero order modified Bessel Function.

From 2. one now can derive $p(r_o')$ and substitute in the expression for the error probability P_{01} given as :

$$P_{01} = \int_0^\infty p(r_o')\left\{\int_{r_o'}^\infty p(r_1)dr_1\right\} dr_o' \qquad 4.$$

Substituting $\ell^2 = 1/k^2$ one obtains:

$$P_{01} = \frac{\ell^2}{N} \exp\left[-\frac{\mu^2}{2N}\right] \int_0^\infty \exp\left[-\frac{r_o'(1+\ell^2)}{2N}\right] I_o\left\{\frac{\ell r_o'}{N}\right\} dr_o' \qquad 5.$$

Now using the substitutions

$$x = \sqrt{(1+\ell^2)/N}\; r_o' \qquad 6.$$

$$a = \frac{\ell^2 \mu^2}{N(1+\ell^2)} \qquad 7.$$

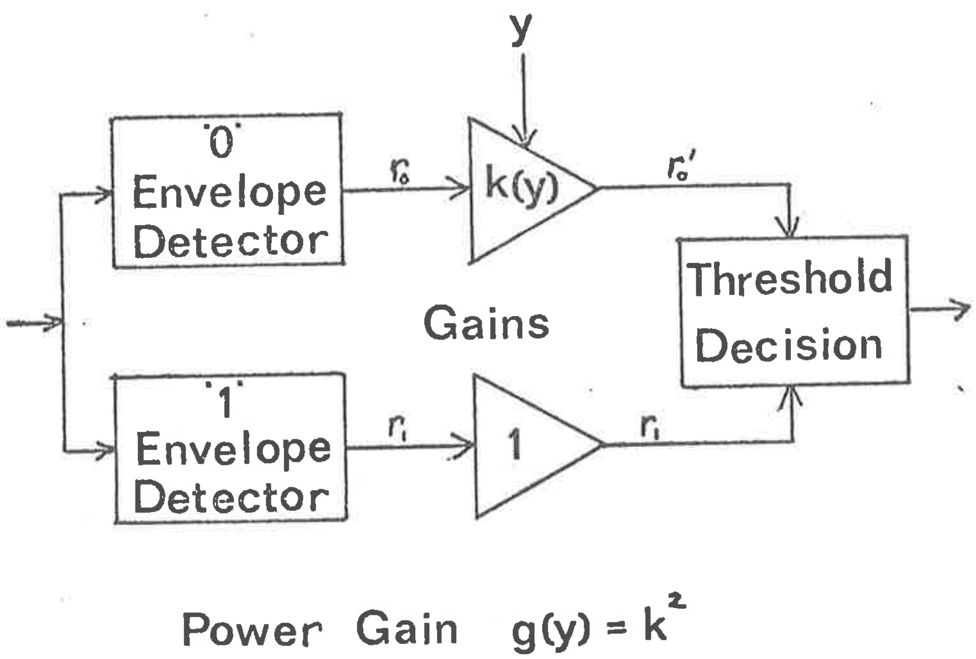

FIGURE A.1

ADAPTIVE FSK DEMODULATOR

$$\gamma = \frac{\mu^2}{2N} \qquad 8.$$

$$g = \frac{1}{\ell^2} = k^2 \qquad 9.$$

one obtains (using the result from Schwartz et Al (1966)).

$$P_{01}(\gamma) = \frac{1}{1+g} \exp\left[-\frac{g}{1+g}\gamma\right] \qquad 10.$$

Similarly,

$$P_{10}(\gamma) = \frac{g}{1+g} \exp\left[-\frac{1}{1+g}\gamma\right] \qquad 11.$$

Note:

(i) $\lim_{g\to 0} P_{01} = 1,$ $\qquad \lim_{g\to 0} P_{10} = 0$

(ii) $\lim_{g\to\infty} P_{01} = 0$ $\qquad \lim_{g\to\infty} P_{10} = 1$

APPENDIX B

This appendix derives the optimality equation for the best gain function to minimise the effect of feedback error discussed in Chapter 3.

The variational problem is written as:

$$\text{Min } J(g) = \int_0^\infty \left\{ \frac{(1-z)}{1+g} \exp\left[-\frac{g}{1+g}\gamma_f y\right] - \frac{gz}{1+g} \exp\left[-\frac{1}{1+g}\gamma_f y\right] \right\} e^{-y}\, dy \qquad 1.$$

subject to the integral constraint

$$\int_0^\infty \left\{ \frac{g}{1+g} e^{-x_{th}} \exp\left[-\frac{1}{1+g}\gamma_f y\right] - \frac{(1-e^{-x_{th}})}{1+g} \exp\left[-\frac{g}{1+g}\gamma_f y\right] \right\}$$

$$\times\ e^{-y}\, dy = \frac{r_1 e^{-x_{th}} - 1}{r_1} \qquad 2.$$

Adjoining equations 1. and 2. with a Lagrange multiplier λ, the Euler-Lagrange equation becomes:

$$\frac{\partial}{\partial g}\left\{ \frac{k_1}{1+g} \exp\left[-\frac{g}{1+g}\gamma_f y\right] + \frac{k_2 g}{1+g} \exp\left[-\frac{1}{1+g}\gamma_f y\right] \right\} = 0 \qquad 3.$$

where

$$k_1 = (1-z) - \lambda(1 - e^{-x_{th}}) \qquad 4.$$

$$k_2 = \lambda e^{-x_{th}} - z \qquad 5.$$

Partially differentiating and interchanging one obtains:

$$\frac{\gamma_f y + g + 1}{\gamma_f g y + g + 1} \exp\left[-\frac{g-1}{g+1}\gamma_f y\right] = k_3(\lambda) \qquad 6.$$

where $k_3(\lambda) = k_1(\lambda) / k_2(\lambda)$ 7.

Thus g(y) is the solution of the implicit equation given by equation 6. where k_3 is such that equation 2. is satisfied. Several things may be noticed about this equation:

(i) The form of the solution of equation 6. is independent of the distribution of y, but k_3 (ie. λ) must satisfy equation 2.

(ii) If $p(y) = \delta(y)$, the solution is optimum for a non-fading feedback channel.

The solution of equation 6. was done numerically on the computer using a simple heuristic stepping algorithm. The evaluation of the integrals used Simpson's rule over the low range of y for which g is evaluated and then a term (where g is a constant) is added to give the overall integral. Typical results are shown in Chapter 3.

APPENDIX C

This appendix derives an integral equation, the solution of which is the distribution function of the buffer occupancy at the end of transmission points. It is assumed that these points constitute a Poisson process but where the mean is variable and dependent on buffer occupancy.

In Chapter 4, one defines the parameters λ and μ where here they are directly related to two Poisson processes which will be defined in terms of the turning points in the buffer occupancy rather than in terms of time crossing points of the signal. (Takes into account the value of r_1) Thus one defines

$$d\omega/_{\lambda(\omega)} = P_r \{ \text{buffer stops filling in next } d\omega \text{ bits} \}$$

Similarly,

$$d\omega/_{\mu(\omega)} = P_r \{ \text{buffer stops emptying in next } d\omega \text{ bits} \}$$

where $\lambda(\omega)$, $\mu(\omega)$ are functions of the buffer occupancy . The nature of the dependency is through two unknown control functions g_o and g_1 which describe the buffer control characteristic (Chapter 4). One can now define

$$P_\lambda(s,t) = P_r \{ \text{increase of contents from } s \text{ to more than } t \text{ bits during a fade} \}$$

Thus considering a small increase in size $d\omega$ bits

$$P_\lambda(s,t+d\omega) \simeq P_\lambda(s,t).P_r \{ \text{increase of } d\omega \text{ bits} \}$$

$$\simeq P_\lambda(s,t).(1 - d\omega/_{\lambda(\omega)})$$

which leads to

$$\frac{dP_\lambda(s,t)}{d\omega} + \frac{P_\lambda(s,t)}{\lambda(\omega)} = 0 \qquad 1.$$

Thus, $P_\lambda(s,t)$ is given by:

$$P_\lambda(s,t) = e^{-\int_s^t 1/\lambda(\omega)\, d\omega} \qquad 2.$$

Similarly, one can define

$$P_\mu(s,t) = P_r\{\text{ decrease of contents from s to less than t bits during a transmission period}\}$$

$$= e^{-\int_t^s 1/\mu(\omega)\, d\omega} \qquad 3.$$

One can derive the distribution function $F^J(z)$ of the buffer contents at the end of a transmission period where

$$F^J(z) = P_r\{\omega < z\} \qquad 4.$$

If y is the buffer contents at the beginning of transmission, then:

$$F^J(z/y) = \begin{cases} 0 & ; z < 0 \\ P_\mu(y,z) & ; 0 < z < y \\ 1 & ; z > y \end{cases} \qquad 5.$$

If x is the buffer contents at the beginning of a fade, and $F^I(z)$ is the corresponding distribution,

$$F^I(y/x) = \begin{cases} 0 & ; y < x \\ 1 - P_\lambda(x,y) & ; x < y < C \\ 1 & ; y > C \end{cases} \qquad 6.$$

Hence,

$$F^J(z/x) = \int_{y=0}^{C} F^J(z/y)\, dF^I(y/x) \qquad 7.$$

where,

$$\frac{dF^I(y/x)}{dy} = P_\lambda(x,y)/\lambda(y) + P_\lambda(x,C)\delta(y-C) \qquad 7.$$

where $x < y < C$

Substitution in 7. leads to:

$$F^J(z/x) = \int_{\max(x,z)}^{C} 1/\lambda(y) P_\lambda(x,y) P_\mu(y,z)\, dy$$

$$+ \; U(z-x)(1 - P_\lambda(x,z)) \; + \; P_\lambda(x,C) P_\mu(C,z) \qquad 9.$$

Thus,

$$F^J(z) = \int_{x=0}^{C} F^J(z/x) \frac{dF^J(x)}{dx} dx \qquad 10.$$

Integrating by parts this leads to the integral equation

$$F^J(z) = H_1(z) - \int_{x=0}^{C} K_1(x,z) F^J(x) dx \qquad 11.$$

where $\quad H_1(z) = P_\mu(C,z) \qquad$ 12.

and

$$K_1(x,z) = \int_{\max(x,z)}^{C} 1/\lambda(x).1/\lambda(y)\; P_\lambda(x,y) P_\mu(y,z) dy$$

$$- \; 1/\lambda(x) \;\{\; U(x-z) P_\mu(x,z) \; + \; U(z-x) P_\lambda(x,z)$$

$$- \; P_\lambda(x,C) P_\mu(C,z) \;\} \qquad 13.$$

For the case treated by Campbell (1957),

$$P_\lambda(s,t) = e^{-(t-s)/\lambda} \qquad 14.$$

$$P_\mu(s,t) = e^{-(s-t)\mu} \qquad 15.$$

Thus, $\quad H_1(z) = e^{-(C-z)/\mu} \qquad$ 16.

and

$$K_1(x,z) = \frac{\mu\lambda}{\lambda + \mu} \left[e^{-(C-x)/\lambda} \quad e^{-(C-z)/\mu} - \begin{cases} e^{-(x-z)/\mu} & ; x > z \\ e^{-(z-x)\;\lambda} & ; x < z \end{cases} \right] \qquad 17.$$

This leads to the solution[1.] of Campbell (1957)

$$F^J(\omega) = \frac{1 - e^{-\beta\omega}}{1 - e^{-\beta C}} \qquad 18.$$

---

1. Note that λ and μ used here are the reciprocals of B and A respectively used by Campbell (1957).

where

$$\rho \quad = \quad \lambda/\mu \qquad 19.$$

and

$$\beta \quad = \quad \frac{1 - \rho}{\lambda} \qquad 20.$$

However, since one has assumed that the probability of ending transmission is independent of previous history (Poisson process), the end of transmission point is no different than any other point during transmission. Thus, $F^J(\omega)$ also describes the distribution function at any point during transmission. The probability of an empty buffer condition during transmission is thus given by:

$$P_o \quad = \quad F^J(0) \quad = \quad \frac{1 - \rho}{1 - \rho e^{-\beta C}} \qquad 21.$$

Now, the distribution $F^I(z)$ can be derived from $F^J(z)$. Similar to equation 7. one has

$$F^I(z/x) \quad = \quad \int_{y=0}^{C} F^I(z/y)\, dF^J(y/x) \qquad 22.$$

where

$$\frac{dF^J(y/x)}{dy} \quad = \quad P_\mu(x,y)/\mu(y) \;+\; P_\mu(0,x)\delta(y) \qquad 23.$$

where $\quad 0 < y < x$

Substituting in 22. gives:

$$F^I(z/x) \quad = \quad \int_{y=0}^{\min(x,z)} 1/\mu(y)(1 - P_\lambda(y,z))P_\mu(x,y)dy \;+\; (1 - P_\lambda(0,z))P_\mu(x,0) \qquad 24.$$

Thus,

$$F^I(z) \quad = \quad \int_{x=0}^{C} F^I(z/x)dF^J(x) \qquad 25.$$

where $F^J(x)$ is known from the solution of the integral equation 11. By similar argument, this distribution function applies to any point during a fade. Thus, for the simple case treated by Campbell (1957),

$$F^I(\omega) \quad = \quad \frac{1 - e^{-\beta\omega}}{1 - \rho e^{-\beta C}} \qquad 26.$$

and thus,

$$P_c = 1 - F^I(C) = \frac{(1-\rho)e^{-\beta C}}{1-\rho e^{-\beta C}} \qquad 27.$$

From these two distributions, one can now calculate the buffer distribution at an arbitrary time point as the weight sum of the two distributions given by:

$$F(\omega) = P_t F^J(\omega) + P_f F^I(\omega) \qquad 28.$$

where $P_t = P_r$ { transmission period }

$P_f = P_r$ { non-transmission period }

However, to maintain the average rate constraint one has:

$$r_1 \{ (1 - P_o')P_t + P_c' P_f\} = 1 \qquad 29.$$

where $P_t + P_f = 1$ 30.

Thus, one can solve for P_t and P_f. For the simple case

$$P_t = e^{-x_{th}} \qquad 31.$$

and

$$P_f = 1 - e^{-x_{th}} \qquad 32.$$

Since the buffer is essentially an integrator, the variation of ω is very much slower than the SNR variation x. Thus it can be approximated that x and ω are independent except to the extent that ω increases if $x < x_{th}$. Thus, the error probability is similar in form to that derived of feedback error (Chapter 3) except one must integrate of the buffer distribution during transmission.

$$\overline{P}_\varepsilon = \frac{r_1^2}{\gamma_m + 2r_1} \{ (1 - P_o')P_t \int_0^C z(g_1(\omega))dF^J(\omega) + P_c' P_f(1 - z(g_o(C)))\} \qquad 33.$$

APPENDIX D

This appendix derives the distribution function of non-transmission times when transmission is halted when $x < x_{th}$ and when $y < y_{th}$. It is assumed the fade durations are exponentially distributed.

A non-transmission period can fall into four cases where the last two are duplicates of the first two but with the role of x and y interchanged. Thus, one can consider two cases as illustrated in Fig. D.1.

Case 1

For this case, the transmission period starts due to x crossing threshold and ends due to x crossing threshold. Firstly, one must define

1/a : Mean fade period of feedback channel.

1/b : Mean transmission period of feedback channel.

1/c : Mean fade period of main channel.

1/d : Mean transmission period of main channel.

One can now identify the sub-cases depending on the number of zero crossings. To derive the distribution function, one must integrate over the various intervals.

For y_o, the ranges of integration are:

$$t_1 + t_2 < t_3 < \infty$$
$$0 < t_1 < \infty$$
$$t < t_2 < \infty$$

Integration leads to the term $\frac{d}{b+d} \cdot \frac{c}{b+c} \quad e^{-(b+c)t}$

For y_1, y_2, ... the ranges of integration are:

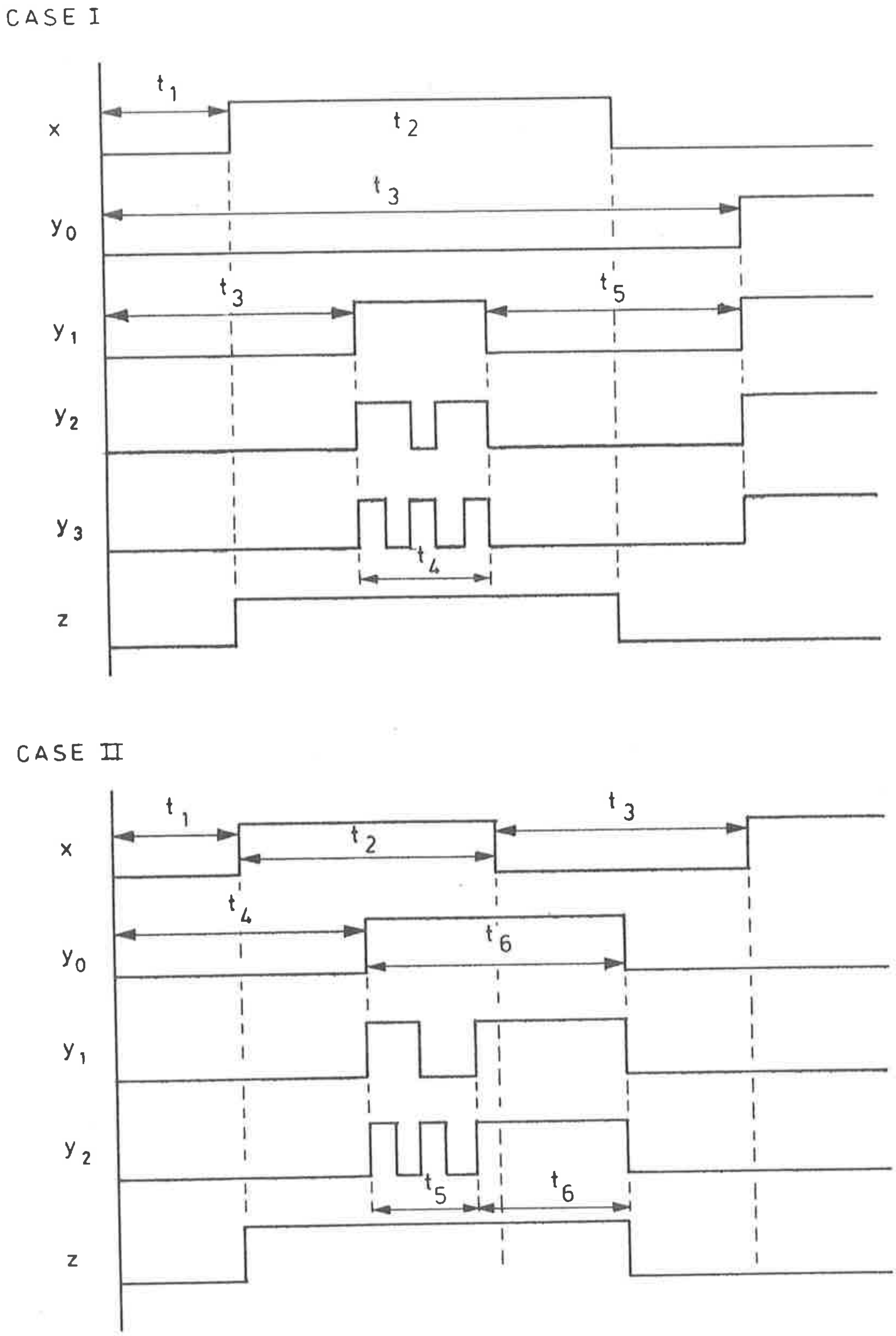

FIG. D.1 NON-TRANSMISSION TIME z : COMPONENT CASES WHERE $z = x + y$

$$t_1 + t_2 - t_3 - t_4 < t_5 < \infty$$

$$t_1 < t_3 < t_1 + t_2 - t_4$$

$$0 < t_1 < \infty$$

$$0 < t_4 < t_2$$

$$t < t_2 < \infty$$

It can be seen that the probability density function of t_4 can be found by convolution. If one now sums the contribution of t_4 over all numbers of zero crossings y_1, y_2, ... , one obtains,

$$\sum_{i=1}^{\infty} p_i(t_4) \xrightarrow{LT} \phi(s) = \frac{a}{a+s}\left[1 + \frac{ab}{(a+s)(b+s)} + (\quad)^2 + \ldots\right]$$

$$= \frac{ab}{(a+b)s} + \frac{a^2}{(a+b)(a+b+s)}$$

$$\therefore \quad p(t_4) = \frac{ab}{a+b} + \frac{a^2}{a+b} e^{-(a+b)t_4}$$

Integration of $t_1 \to t_5$ and then adding the term due to y_o gives:

$$I_i = \frac{ad}{(a+b)(b+d)} e^{-ct} + \frac{bcd}{(a+b)(b+d)(a+b+c)} e^{-(a+b+c)t} \qquad 1.$$

<u>Case II</u>

For this case, the transmission period starts due to x crossing threshold and ends due to y crossing threshold. Again, one can identify the sub-cases. As before, the probability density function of t_5 can be found by convolution. The sum of these contributions is given by:

$$p(t_5) = \sum_{i=1}^{\infty} p_i(t_5) \xrightarrow{LT} \phi(s) = 1 + \frac{ab}{(a+s)(b+s)} + (\quad)^2 + \ldots.$$

$$= 1 + \frac{ab}{a+b}\left\{\frac{1}{s} - \frac{1}{s+a+b}\right\}$$

$$p(t_5) = \delta(t_5) + \frac{ab}{a+b}(1 - e^{-(a+b)t_5})$$

For y_0, ..., y_5 the ranges of integration are:

$$t_4 + t_5 + t_6 - t_1 - t_2 < t_3 < \infty$$

$$t_4 + t_5 - t_1 < t_2 < t_4 + t_5 + t_6 - t_1$$

$$t + t_1 - t_4 - t_5 < t_6 < \infty$$

if $t_4 + t_5 < t + t_1$

$$t_1 < t_4 < t + t_1 - t_5$$

if $t_5 < t$

$$0 < t_1 < \infty$$

$$0 < t_5 < t$$

OR

$$0 < t_6 < \infty$$

if $t + t_1 < t_4 + t_5$

$$t_1 < t_4 < \infty$$

if $t < t_5$

$$0 < t_1 < \infty$$

$$t < t_5$$

OR

$$t + t_1 - t_5 < t_4 < \infty$$

if $t_5 < t$

$$0 < t_1 < \infty$$

$$0 < t_5 < t$$

Integration of $t_1 - t_5$ over the above ranges yields:

$$I_{ii} = \frac{abd}{(a + b)(b + d)(a - c + d)} e^{-ct}$$

$$- \frac{abcd}{(a + d)(b + d)(a - c + d)(b + c - d)} e^{-(a+d)t}$$

$$+ \frac{abcd}{(a + b)(b + d)(a + b + c)(b + c - d)} e^{-(a+b+c)t} \qquad 2.$$

One can now find I_{iii} and I_{iv} by term replacement according to:

$$a \leftrightarrow c \qquad 3.$$

$b \leftrightarrow d$

Thus, $F(t) = I_i + I_{ii} + I_{iii} + I_{iv}$ 4.

Any other cases make neglible contribution.

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